Topic : Limits & Continuity

Practical No: 1

1) $$\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$$

→

$$= \lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{a+2x-3x}{3a+x-4x} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{a-x}{3a-3a} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \lim_{x\to a} \frac{1}{3} \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$= \frac{1}{3} \frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$= \frac{1}{3} \frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$= \frac{1}{3} \frac{2\sqrt{a}+2\sqrt{a}}{2\sqrt{3a}}$$

$$\frac{2\sqrt{a}}{3} = \frac{2\sqrt{a}}{3\sqrt{3a}} = \frac{2}{3\sqrt{3}}$$

2) $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$$= \lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}}$$

$$= \lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y} \times \sqrt{a+y} + \sqrt{a}}$$

$$= \lim_{y \to 0} \frac{y}{y\sqrt{a+y} \times \sqrt{a+y} + \sqrt{a}}$$

$$= \frac{1}{\sqrt{a} \times \sqrt{a} + \sqrt{a}}$$

$$= \frac{1}{2\sqrt{a}}$$

3) $\lim_{x \to \frac{\pi}{6}} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

put $\cancel{h = x - \frac{\pi}{6}}$ $\qquad x \to \frac{\pi}{6}$

$x = h + \frac{\pi}{6}$

$h \to 0$

$$= \lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

$$= \lim_{h\to 0} \frac{\left(\cosh\cdot\cos\frac{\pi}{6} - \sinh\cdot\sin\frac{\pi}{6}\right) - \sqrt{3}\left(\sinh\cdot\cos\frac{\pi}{6} + \cosh\cdot\sin\frac{\pi}{6}\right)}{\cancel{\pi} - 6h - \cancel{\pi}}$$

$$= \lim_{h\to 0} \frac{\left(\frac{\sqrt{3}}{2}\cosh - \frac{1}{2}\sinh\right) - \sqrt{3}\left(\frac{\sqrt{3}}{2}\sinh + \frac{1}{2}\cosh\right)}{-6h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{\sqrt{3}}{\cancel{2}}\cancel{\cos h} - \sinh\right) - 3\sinh - \sqrt{3}\cancel{\cos h}}{-12h}$$

$$= \lim_{h\to 0} \frac{\cancel{+4}\sinh}{\underset{3}{\cancel{+12}}h} = \frac{1}{3} \qquad \boxed{\frac{\sinh}{h} = 1}$$

4) $$\lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$$

By ratinalization Numerator & Denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)\left(\sqrt{x^2+3} + \sqrt{x^2+1}\right)}{(x^2+3-x^2-1)\left(\sqrt{x^2+5} + \sqrt{x^2-3}\right)}\right]$$

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$$\lim_{x\to\infty} \frac{4\sqrt{x^2+3}+\sqrt{x^2+1}}{4\sqrt{x^2+5}+\sqrt{x^2-3}}$$

$$4\lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

Aftor applijying limit

we get

$= 4$

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Examine the Continuity of the following function at given points

$$f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}} \quad \text{for } 0 < x \le \frac{\pi}{2}$$

$$= \frac{\cos x}{\pi - 2x}, \quad \text{for } \frac{\pi}{2} < x < \pi$$

$\Big\}$ at $x = \frac{\pi}{2}$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos\frac{\pi}{2}}} = 0$$

$f\left(\frac{\pi}{2}\right) = 0$

f. at $x = \frac{\pi}{2}$ define

$$\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$$

put $x - \frac{\pi}{2} = h$

$$x = \frac{\pi}{2} - h$$

$x \to \frac{h}{2} \quad h \to 0$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$$

$$= \cancel{\lim_{x \to \frac{\pi}{2}}} \lim_{h \to 0} \frac{-\sin h}{\pi - \pi - 2h}$$

$$= \lim_{h \to 0} \frac{-\sin h}{-2h} = \frac{1}{2} \quad \text{R.H.S}$$

$$\lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2 \sin x . \cos x}{\sqrt{2 \sin^2 x}}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{2 \sin x . \cos x}{\sqrt{2 \sin^2 x}}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{2 \cancel{\sin x} . \cos x}{\sqrt{2} \cancel{\sin x}}$$

$$= 0$$

$L.H.S \neq R.H.S$

$\therefore f$ is not cts at $x = \frac{\pi}{2}$

ii) $f(x) = \frac{x^2 - 9}{x - 3}$ $\quad 0 < x < 3$

$= x + 3$ $\quad 3 \leq x < 6$

$= \frac{x^2 - 9}{x + 3}$ $\quad 6 \leq x < 9$

at $x = 3$, $x = 6$

i) For $x = 3$

$\cancel{f(x)} = 3 + 3 = 6$

$\therefore f$ is define $f(x) = 3$

ii) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3} (x + 3)$

$= 6$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} f(x) = f(3)$$

$\therefore$ from (i) & (ii)

f is cts at $x = 3$

6) Find value of k so that the function $f(x)$ is cts at indicated point

i) $$f(x) = \left.\begin{matrix} \dfrac{1-\cos 4x}{x^2} & x < 0 \\ = K & x = 0 \end{matrix}\right\} \text{ at } x = 0$$

$\longrightarrow$ f is cts at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$$

$$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)$$

$$2(2)^2 = k$$

$$K = 8$$

iii) $f(x) = \begin{cases} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} & x \neq \dfrac{\pi}{3} \\ & x = \dfrac{\pi}{3} \end{cases}$ at $x = \dfrac{\pi}{3}$

f is cts at $x = \frac{\pi}{3}$

$$\lim_{x \to \frac{\pi}{3}} f(x) = f\left(\frac{\pi}{3}\right)$$

put

$$x - \frac{\pi}{3} = h$$

$$x = \frac{\pi}{3} + h$$

$$x \longrightarrow \frac{\pi}{3}$$

$$h \longrightarrow 0$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan\left(h + \frac{\pi}{3}\right)}{\pi - 3\left(h + \frac{\pi}{3}\right)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan\left(h + \frac{\pi}{3}\right)}{\pi - \cancel{3}\left(\frac{3h + \pi}{\cancel{3}}\right)}$$

$$= \lim_{h \to 0} \frac{\sqrt{3} - \tan\left(h + \frac{\pi}{3}\right)}{-3h}$$

$$\lim_{h \to 0} \frac{\sqrt{3}\left(\dfrac{\tan h + \tan\frac{\pi}{3}}{1 - \tan h \cdot \tan\frac{\pi}{3}}\right)}{-3h} \longrightarrow \text{formula}$$

$$\lim_{h\to 0} \frac{\sqrt{3}\,(1-\tan h.\tan \pi/3) - 1\cdot\tan h \cdot \tan \pi/3}{(-3h)\,(1-\tan h \cdot \tan \pi/3)}$$

$$\lim_{h\to 0} \frac{\sqrt{3}\,(1-\sqrt{3}\tan h) - \sqrt{3}\cdot\tan h}{-3h\,(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3}\lim_{h\to 0}\left(\frac{\tan h}{h}\right)\left(\frac{1}{1-\sqrt{3}\tan h}\right)$$

$$= \frac{4}{3}$$

Discuss the Continuity of following function which of function . have a removable discontinuity ? Redefine the function so as to remove the discontinuity.

$$f(x) = \left.\begin{matrix} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{matrix}\right\} \text{ at } x = 0$$

Redifine function

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

Now line $f(x) = f(0)$

$$\lim_{x\to 0} \frac{2\sin^2 3/2\, x}{x \tan x}$$

$$\lim_{x\to 0} \frac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{\dfrac{x \tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$= 2 \times \frac{9}{4} = \frac{9}{2}$$

ii)

$$f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \dfrac{\pi}{60} & x = 0 \end{cases} \quad \text{at } x = 0$$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin x}{x^2}$$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$3 \lim_{x\to 0} \left(\frac{e^{3x}-1}{3x}\right) \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{(x)}$$

$$3 \log \frac{\pi}{180} = \frac{\pi}{60}$$

$= f(0)$

function cts $x=0$

8) If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is cts at $x=0$ find $f(0)$

→ f is cts at $x=0$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$f(0) = \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2} = \lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{2\sin^2 \cancel{x}\, x/2}{x^2}$$

$$= \log e + 2 \lim_{x\to 0} \left(\frac{\sin^2 x/2}{x^2} \right)$$

$$= 1 + 2 \cdot \frac{1}{4}$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

9) If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \frac{\pi}{2}$ is cts at $x = \frac{\pi}{2}$ find $f(\frac{\pi}{2})$

→ is cts at $x = \frac{\pi}{2}$

$$= \lim_{x\to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} - \sqrt{1+\sin x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{(1+\cancel{\sin x})}{\cancel{(1-\sin x)}(1+\sin x)(\sqrt{2}-\sqrt{1+\sin x})}$$

$$= \frac{1}{2\sqrt{2}+\sqrt{1+1}}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2\times 2\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}}$$

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Practical No : 2

* Derivative

Q.1. show that the following function defined

i) Cot x

$$f(x) = \cot x$$

Df (a) $\lim_{x \to 0} \frac{f(x) - f(a)}{x - a}$

$$= \lim_{x \to 0} \frac{\cot x - \cot a}{x - a}$$

put $x - a = h$ $\quad$ $x \to a$

$x = h + a$ $\quad$ $h \to 0$

$$= \lim_{h \to 0} \frac{\cot(x+h) - \cot(a)}{h + a - a}$$

$$= \lim_{h \to 0} \frac{\frac{\cos(a+h)}{\sin(a+h)} - \frac{\cos a}{\sin a}}{h}$$

$$= \lim_{h \to 0} \frac{\cos(a+h)\sin(a) - \cos a \cdot \sin(a+h)}{\sin(a+h) \cdot \sin a \times h}$$

$$= \lim_{h \to 0} \frac{\sin(a - h - a)}{\sin(a+h) \cdot \sin a \times h}$$

$$= \lim_{h \to 0} \frac{1}{\sin(\cancel{a+h}) \cdot \sin a} - \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{\sin a \cdot \sin a} \times -1$$

$$= \frac{-1}{\sin a^2}$$

$$= -\operatorname{cosec}^2 a$$

ii) Cosec x

$f(x) = \operatorname{cosec} x$

Df (a) $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$\lim_{x \to 0} \dfrac{\operatorname{cosec} x - (\operatorname{cosec} a)}{x - a}$

put $x - a = h$ $\quad h \to 0$

$x \to a \quad x = h + a$

$\lim_{h \to 0} \dfrac{\operatorname{cosec}(h+a) - \operatorname{cosec} a}{h + a - a}$

$\lim_{h \to 0} \dfrac{\frac{1}{\sin(h+a)} - \frac{1}{\sin(a)}}{h}$

$\lim_{h \to 0}$

$\dfrac{\sin a - \sin(h+a)}{\sin(h+a) \cdot \sin(a) \times h}$

$\lim_{h \to 0}$

$\dfrac{2 \cos\left(a + \frac{h}{2}\right) \cdot \sin\left(\frac{-h}{2}\right)}{\sin(h+a) \cdot \sin a \times h}$

$= 2 \lim_{h \to 0} \dfrac{\cos\left(a + \frac{h}{2}\right)}{\sin(h+a) \cdot \sin a} \times -\dfrac{1}{2} \lim_{h \to 0} \dfrac{\sin\left(-\frac{h}{2}\right)}{-\frac{h}{2}}$

$= -\dfrac{\cos a}{\sin a} \times \sin a$

$= -\cot a \cdot \operatorname{cosec} a$

function is differentiable at IR & IR.

iii) $\sec x$

$$Df(a) = \lim_{x \to 0} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to 0} \frac{\sec x - \sec a}{x - a}$$

put $x - a = h$: $x = a + h$ $x \to h$

$h \to 0$

$$\lim_{h \to 0} \frac{\sec(a+h) - \sec a}{h + a - a}$$

$$\lim_{h \to 0} \frac{\frac{1}{\cos(h+a)} - \frac{1}{\cos a}}{h}$$

$$= \lim_{h \to 0} \frac{\cos a - \cos(h+a)}{\cos(h+a) \cdot \cos a \times h}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(a + \frac{h}{2}\right) \cdot \sin\left(-\frac{h}{2}\right)}{\cos(h+a) \cdot \cos a \times h}$$

$$= 2 \lim_{h \to 0} \frac{\sin\left(a + \frac{h}{2}\right)}{\cos(h+a) \cdot \cos a} - \frac{1}{\cancel{2}} \lim_{h \to 0} \frac{\sin\left(-\frac{h}{2}\right)}{-\frac{1}{2}}$$

$$= \frac{\sin a}{\cancel{\cos a}} \times \cos a$$

$$= \tan a \cdot \sec a.$$

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Q.2 If $f(x) = 4x+1 \quad x < 2$

$= x^2+5 \quad x > 2$ at $x=0$ then

find differentiable or not

→ R.H.D $= Df(x^+)$

$$= \lim_{x \to 2^+} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2^+} x+2$$

$$= 4$$

L.H.D $= Df(x^-)$

$$\lim_{x \to 2^-} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}} = 4$$

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R.H.D = L.H.D

hence

function is defferentiable.

5) IF $f(x) = 4x+7 \quad x < 3$

$= x^2+3x+1 \quad x \geq 3$

Then find f is differentiable or not ?

→

R.H.D = Df (3+)

$= \lim_{x \to 3^+} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$

$\lim_{x \to 3^+} \frac{x^2-6x-3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)}$

$= \lim_{x \to 3^+} x+6$

$= 9$

L.H.D = Df (3-)

$\lim_{x \to 3^-} \frac{f(x) - f(a)}{x - a}$

$$= \lim_{x \to 3^-} \frac{4x+3-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$= 4$$

R.H.D $\neq$ L.H.D

function is not differentiable at $x=3$

Q.4 If $f(x) = \begin{cases} 8x-5 & x<2 \\ 3x^2-4x+7 & x>2 \end{cases}$ at $x=2$

find function is differentiable or not

$\longrightarrow$

R.H.D $= Df(2^+)$

$$= \lim_{x \to 2^+} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$\lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$\lim_{x\to 2^+} \frac{(x-2)(3x+2)}{(x-2)}$$

$$= \lim_{x\to 2^+} 3x+2$$

$$= 3\times 2+2$$

$$= 8$$

L.H.D $\quad Df = (2^-)$

$$= \lim_{x\to 2^-} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 8$$

R.H.D = L.HD

hence is differtiable function

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PRACTICAL NO. 3

Topic : Application of Derivative

Q.1. Find the intervals in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$

→ $f'(x) = 3x^2 - 5$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > \frac{5}{3}$

$x > \pm\sqrt{\frac{5}{3}}$

$+ \quad - \quad +$

$\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

$\therefore f$ is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < \frac{5}{3}$

$x < \pm\sqrt{5/3}$

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

$f(x) = x^2 - 4x$

13) $f'(x) = 2x - 4$

$\therefore$ f is incresing iff $f'(x) > 0$

$2x - 4 > 0$

$2x > 4$

$x > \frac{4}{2}$

$x > 2$

$\therefore x \in (2, \infty)$

$\therefore$ f is decresing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x < 2$

$x \in (-\infty, 2)$

14) $f(x) = 2x^3 + x^2 - 20x + 4$

$= 6x^2 + 2x - 20$

$\therefore$ f is incresing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0$ $\quad \therefore x = -2, \; x = \frac{10}{6}$

$- \quad + \quad - \quad +$

$-2 \qquad \frac{10}{6}$

$x \in (-\infty, -2) \cup (\frac{10}{6}, \infty)$

f is decresing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x - 10)(x+2) < 0$

$$\begin{array}{cccc} + & - & + & - \end{array}$$
$$\overline{\quad -2 \qquad\qquad 10/6 \quad}$$

$x \in \left(-2, \frac{10}{6}\right)$

iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$\therefore 3(x^2-9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$$\begin{array}{cccc} - & + & - & + \end{array}$$
$$\overline{\quad -3 \qquad\qquad 3 \quad}$$

$\therefore x \in (-\infty, -3)\ (3, \infty)$

f is decreasing iff $f'(x) < 0$

$3(x^2-9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

$$\begin{array}{cccc} + & - & + & - \end{array}$$
$$\overline{\quad -3 \qquad\qquad 3 \quad}$$

$\therefore x \in (-3, 3)$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24x - 18x + 6x^2$

i.e $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2-4x+x-4>0$

$x(x-4)+1(x-4)>0$

$(x+1)\ (x-4)>0$

$$\begin{array}{ccccc} - & + & - & + \\ \hline & -1 & & 4 \end{array}$$

$x \in (-\infty,-1) \cup (4,\infty)$

f is decreasing iff $f'(x)<0$

$\therefore\ 6(x^2-3x-4)<0$

$x^2-3x-4<0$

$x^2-4x+x-4<0$

$x(x-4)+1(x-4)<0$

$(x+1)\ (x-4)<0$

$$\begin{array}{ccccc} + & - & + & - \\ \hline & -1 & & 4 \end{array}$$

$\therefore x \in (1,4)$

Q.2. Find the intervals is which function is concave upwards are concave downwards

i) $y=3x^2-2x^3$

Let,

$f(x)=y=3x^2-2x^3$

$f'(x)=6x-6x^2$

$f''(x)=6-12x$

$=6(1-2x)$

$f''(x)$ is concave upwards iff.

$f''(x)>0$

$6(1-2x)>0$

If $1-2x>0$

$-2x>-1$

$2x<1$

$x<\frac{1}{2}$

$x\in(-\infty,\frac{1}{2})$

$f''(x)$ is Concave downwards iff,

$f''(x)<0$

$6(1-2x)<0$

$1-2x<0$

$-2x<-1$

$2x>1$

$x>\frac{1}{2}$

$x\in(\frac{1}{2},\infty)$

ii) $y=x^4-6x^3+12x^2+5x+7$

Let,

$f(x)=y=x^4-6x^3+12x^2+5x+7$

$f'(x)=4x^3-18x^2+24x+5$

$f''(x)=12x^2-36x+24$

$=12(x^2-3x+2)$

$f''(x)$ is Concave upwards iff

$f''(x)>0$

$12(x^2-3x+2)>0$

$x^2-3x+2>0$

$x^2-x-2x+2>0$

$x(x-1)-2(x-1)>0$

$(x-2)(x-1)>0$

$+ \quad - \quad - \quad +$

$1 \quad 2$

$x\in(-\infty,1)\cup(2,\infty)$

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$f''(x)$ is Concave downwards iff.

$f''(x) < 0$

$12(x^2 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$x^2 - x - 2x + 2 < 0$

$x(x-1) - 2(x-7) < 0$

$(x-2)(x-1) < 0$

$$\begin{array}{cccc} + & - & + & - \\ \hline & 1 & 2 & \end{array}$$

$\therefore x \in (1,2)$

iii) $y = x^3 - 27x + 5$

Let.

$f(x) = y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is Concave iff upwards iff,

$f''(x) > 0$

$6x > 0$

$x > 0$

$x \in (0, \infty)$

$f''(x)$ is Concave downwards iff.

$f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore x \in (\infty, 0)$

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i)

iv) $y = 69 - 24x - 9x^2 + 2x^3$

Let,

$f(x) = y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$f'(x)$ is concave upwards iff,

$f''(x) > 0$

$-18 + 12x > 0$

$12x > 18$

$x > \frac{18}{12}$

$\therefore x \in (\frac{3}{2}, \infty)$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$-18x + 12x < 0$

$12x < 18$

$x > \frac{18}{12}$

$\therefore x \in (-\infty, \frac{3}{2})$

v)

$y = 2x^3 + x^2 - 20x + 4$

Let,

$f(x) = y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$= 2(6x + 1)$

$\therefore$ $f''(x)$ is Concave upwards iff,

$$f''(x) > 0$$
$$2(6x+1) > 0$$
$$6x+1 > 0$$
$$6x > -1$$
$$x > -\frac{1}{6}$$
$$x \in \left(-\frac{1}{6}, \infty\right)$$

$\therefore$ $f''(x)$ is Concave downward iff,

$$f''(x) < 0$$
$$2(6x+1) < 0$$
$$6x+1 < 0$$
$$6x < -1$$
$$x < -\frac{1}{6}$$
$$x \in \left(-\infty, -\frac{1}{6}\right)$$

20/12/19

Practical No. 4

Topic : Application of Derivative of Newton's Method

Q1. Find maxima and minima

Q.1. Find maxima and minima value of following function.

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now Consider

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

f has minimum voles at $x = 2$

$f(2) = x^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4+4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 2+6$

$= 8>0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum voles at $x = 2$ and $x = -2$

iii) $f(x) = 3 - 5x^3 + 3x^5$

→ $f'(x) = -15x^2 + 15x^4$

Consider

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30+60$

$= 30>0$

f has minimum values at $x=1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum values

at $x = -1$

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has the maximum value 5 at $x = -1$ and has the minimum values 1 at $x=1$

iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ $\quad x = 2$

f has maximum values $x=0$

$f''(x) = 6x - 6$

$= 6(0) - 6$

$= -6 < 0$

f has minimum values $x=2$

$$f(2) = (2)^3 - 3(2)^2 + 1$$
$$= 8 - 3(4) + 1$$
$$= 9 - 12$$
$$= 3$$

f has maximum value 1 of $x = 0$ and

f has minimum value -3 of $x = 2$.

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Consider

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12 \times 2 - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value

$x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

Q.1

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

f has maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

f has maximum value 8 at $x = -1$ and x

f has minimum value -19 at $x = 2$

Q.2.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton Method-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

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$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$
$= 0.0845 - 1.0362 - 55$
$= -55.9467$

$\therefore x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \dfrac{0.0829}{55.9467}$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$= 0.0050 - 0.0879 - 9.416 + 9.5$
$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$= 0.1712 + \dfrac{0.0011}{55.9393}$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$
$= 3x^2 - 4$

$f(2) = 2^2 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$

$$f(3) = 3^3 - 4(3) - 9$$
$$= 27 - 12 - 9$$
$$= 6$$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 3 - \frac{6}{23}$$
$$= 2.7392$$

$$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$$
$$= 20.5528 - 10.9568 - 9$$
$$= 0.596$$

$$f'(x_1) = 3(2.7392)^2 - 4$$
$$= 22.5096 - 4$$
$$= 18.5096$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 2.7392 - \frac{0.596}{18.5096}$$
$$= 2.7071$$

$$F(x_2) = (2.7071)^3 - 4(2.7071) - 9$$
$$= 19.8386 - 10.8284 - 9$$
$$= 0.0102$$

$$f'(x_2) = 3(2.7071)^2 - 4$$
$$= 21.9851 - 4$$
$$= 17.9851$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7071 - \frac{0.0102}{17.9851}$$

$$= 2.7071 - 0.0056$$
$$= 2.7051$$

$$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$$
$$= 19.7158 - 10.806 - 9$$
$$= -0.0901$$

$$f'(x_3) = 3(2.7015)^2 - 4$$
$$= 21.8943 - 4$$
$$= 17.8943$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.7015 + \frac{0.0901}{17.8943}$$

$$= 2.7015 + 0.0050$$
$$= 2.7065$$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17$

$= -2.2$

Let $x_0 = 2$ be initial approximation.

By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{2.2}{5.2}$$

$= 2 - 0.4230$

$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

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$$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$$
$$= 7.4608 - 5.6772 - 10$$
$$= -8.2164$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.577 + \frac{0.6759}{8.2164}$$

$$= 1.577 + 0.0822$$
$$= 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$$
$$= 4.5677 - 4.8553 - 16.592 + 17$$
$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$
$$= 8.2588 - 5.97312 - 10$$
$$= -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.6592 + \frac{0.0204}{7.7143}$$

$$= 1.6592 + 0.0026$$
$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$
$$= 4.5892 - 4.9708 - 16.618 + 17$$
$$= 0.0004$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$
$$= -7.6977$$

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$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.6618 + \frac{0.0004}{7.697}$$

$$= 1.6618$$

$\therefore$ The root of equation is 1.6618

PRACTICAL NO: 5

Integration

i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$I = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

Comparing with $\int \frac{dx}{\sqrt{x^2-a^2}}$: $x^2 = (x+1)^2$, $a^2 = (2)^2$

$I = \log |x+\sqrt{x^2-a^2}| + c$

$= \log |x+1+\sqrt{(x+1)^2-(2)^2}| + c$

ii) $\int (4e^{3x}+1)dx$

$I = \int (4e^{3x}+1)dx$

$= \int 4e^{3x}dx + \int 1 dx$

$= 4\frac{e^{3x}}{3} + x + c$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})dx$

$I = \int (2x^2 - 3\sin x + 5\sqrt{x})dx$

$= 2\int x^2 dx - 3\int \sin x\, dx + 5\int \sqrt{x}\, dx$

$= \frac{2}{3}x^3 dx + 3\cos + 5\times\frac{2}{3}x^{3/2} + c$

$= \frac{2}{3}x^3 + 3\cos + \frac{10}{3}x^{3/2} + c$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right)dx$

$= \int \left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\right)dx$

$= \int \left(x^3 \cdot x^{-1/2} + 3x \cdot x^{-1/2} + 4x^{-1/2}\right)dx$

$= \int x^{5/2}\,dx + 3\int x^{-1/2}\,dx + 4\int x^{-1/2}\,dx$

$= \frac{2}{7}x^{7/2} + 3\cdot\frac{2}{3}x^{3/2} + 4x^{1/2}\cdot 2 + c$

$= \frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + c$

v) $\int t^7 \sin(2t^4)\,dt$

$I = \int t^7 \sin(2t^4)\,dt$

let $t^4 = x$

$4t^3\,dt = dx$

$I = \frac{1}{4}\int 4t^3 \cdot t^4 \sin(2t^4)\,dt$

$= \frac{1}{4}\int x \cdot \sin(2x)\,dx$

$= \frac{1}{4}\left[x\int \sin 2x - \int\left[\int \sin 2x \cdot \frac{d}{dx}(x)\right]\right.$

$$= \frac{1}{4}\left[\frac{-x\cos 2x}{2} + \frac{1}{2}\int \cos 2x \cdot 1\right]$$

$$= \frac{1}{4}\left[\frac{-\cos 2x}{2} + \frac{1}{4}\sin 2x\right] + c$$

$$= -\frac{1}{8} x \cos 2x + \frac{1}{16}\sin 2x + c$$

$$I = -\frac{1}{8} t^4 \cos(2t^4) + \frac{1}{16}\sin(2t^4) + c$$

vii) $\int \sqrt{x}\,(x^2-1)\,dx$

$$I = \int \sqrt{x}\,(x^2-1)\,dx$$

$$= \int (\sqrt{x}\cdot x^2 - \sqrt{x})\,dx$$

$$= \int (x^{5/2} - \sqrt{x})\,dx$$

$$= \frac{2}{7} x^{7/2} - \frac{2}{3} x^{3/2} + c$$

viii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right) dx$

$$I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right) dx$$

let

$$\frac{1}{x^2} = t$$

$$x^{-2} = t$$

$$-\frac{2}{x^3}\,dx = dt$$

$$I = \frac{1}{-2}\int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$$

$$= -\frac{1}{2}\int \sin t \, dt$$

$$= -\frac{1}{2}\left[-\cos t\right] + c$$

$$= \frac{1}{2}\cos t + c$$

Resubstitution $t = \frac{1}{x^2}$

$$I = \frac{1}{2}\cos\left(\frac{1}{x^2}\right) + c$$

viii) $\displaystyle\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

$$I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$$

let, $\sin x = t$

$\cos x \, dx = dt$

$$I = \int \frac{dt}{\sqrt[3]{t^2}}$$

$$= \int \frac{dt}{t^{2/3}}$$

$$= \int t^{-2/3} \, dt$$

$$= 3t^{1/3} + c$$

$$= 3(\sin x)^{1/3} + c$$

$$= 3 \cdot \sqrt[3]{\sin x} + c$$

ix) $\int e^{\cos^2 x} \cdot \sin^2 x \, dx$

$I = \int e^{\cos^2 x} \cdot \sin^2 x \, dx$

let $\cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$-2\sin 2x \, dx = dt$

$I = \int -\sin 2x \, e^{\cos^2 x} dx$

$= -\int e^t dt$

$= -e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

$I = \int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

let,

$x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x) dx = dt$

$3(x^2 - 2x) dx = dt$

$(x^2 - 2x) dx = \frac{dt}{3}$

$I = \int \frac{1}{t} \frac{dt}{3}$

$= \frac{1}{3} \int \frac{dt}{t}$

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03/01/2020

$= \frac{1}{3} \log t + c$ Resubstituting $t = x^3 - 3x^2 + 1$

$I = \frac{1}{3} \log(x^3 - 3x^2 + 1) + c$

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Practical No. 6

Topic :- Application of integration & Numerical Integration

Q.1. Find the length of the following Curve

i) $x = t - \sin t$. $y = 1 - \cos t$ $t \in [0, 2\pi]$

$\rightarrow$ $l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$.

$x = t - \sin t$ $\therefore \frac{dx}{dt} = 1 - \cos t$

$y = \cancel{t}\ 1 - \cos t$ $\therefore \frac{dx}{dt} = 0 - (-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\ dt = \int_0^{2\pi} \sqrt{2(1 - \cos t)} = \int_0^{2\pi} \sqrt{2 \times 2 \sin^2 t/2} = \sqrt{4 \sin^2 t/2}$

$= \int_0^{2\pi} 2 \left|\sin \frac{t}{2}\right| dt \quad \therefore \sin^2 \frac{t}{2} = \frac{1 - \cos t}{2}$

$= \int_0^{2\pi} 2 \sin t/2\ dt$

$= \left(-2 \cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4 \cos \pi) - (-4 \cos 0)$

$2 \left[-\cos\left(\frac{t}{2}\right)\right]_0^{2\pi}$

$-4[-1, -1]$

$= \cancel{4+4}$

$= 8$

$y = \sqrt{4-x^2} \qquad x \in [-2, 2]$

$L = \int_0^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$y = \sqrt{4 \cdot x^2} \quad \therefore \quad \frac{dy}{dx} = \int_{-2}^{2} 1 + \left(\frac{-x}{\sqrt{4 \cdot x^2}}\right)^2 dx$

$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$

$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$

$= 4\left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

$\int_{-2}^{2} 1 + \frac{x^2}{4-x^2}$

$\int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}$

$= 2\pi$

) $y = x^{3/2}$ in $[0, 4]$

$f'(x) = \frac{3}{2} x^{1/2}$

$[f'(x)]^2 = \frac{9}{4} x$

$L = \int_0^b \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^4 \sqrt{1 + \frac{9}{4} x}\, dx$

$\int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$

$= \frac{1}{2} \int_0^4 \sqrt{4+9x}\, dx$

$= \frac{1}{2} \left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$

$$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$$

$$= \frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$= \frac{1}{27}(4)^{3/2} - (40)^{3/2}$$

4) $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

$\frac{dx}{dt} = 3\cos t$ $\frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\, dt$$

$$= \int_0^{2\pi} 3dt \qquad = 3\int_0^{2\pi} dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

Q.2. Using Simpson's Rule solve the following

i) $\int_0^2 e^{x^2}\, dx$ with $n=4$

$a=0 \quad b=2 \quad n=4$

$$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7183	9.4877	54.5982
	y_0	y_1	y_2	y_3	y_4

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{0.5}{3}\left[(1+54.5982)+4(1.2840+9.4877)+2(2.7183 \cancel{+54.5981})\right]$$

$$= \frac{0.5}{3}\left[55.5982+43.0866+5.436\right]$$

$$= 17.3535$$

iii) $\int_0^4 x^2 dx \quad n=4$

$$h = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2\,dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{1}{3}\left[(0+16)+4(1+9)+2(4)\right]$$

$$= \frac{1}{3}\left[16+4(10)+8\right]$$

$$= \frac{1}{3}\left[16+40+8\right]$$

$$= \frac{64}{3}$$

$$= 21.333$$

ii) $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ with $n=6$

$$h = \frac{\frac{\pi}{3}-0}{6} = \frac{\pi}{18}$$

x	0	$\pi/18$	$2\pi/18$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$
y	0	1.4167	0.4585	0.7071	0.8017	0.9752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{\pi/18}{3}\left[(0.4167+0.9306)+4(0.4167+0.7071+0.9752)+2(0.5848+0.8017)\right]$$

$$= \frac{\pi}{54}\left[1.3473+4(1.999)+2(1.3865)\right]$$

$$= \frac{\pi}{54}\left[1.3473+7.996+2.773\right]$$

$$= \frac{\pi}{54}\times 12.1163$$

$$= 0.7049$$

Q.1

5). $x = \frac{1}{6}y^3 + \frac{1}{2y}$ $\quad y = [1,2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4-1}{2y^2}$$

$$= \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\,dy$$

$$= \int_1^2 \frac{y^2+1}{2y^2}\,dy$$

$$= \frac{1}{2}\int_1^2 y^2\,dy + \frac{1}{2}\int_1^2 y^{-2}\,dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12}$$

PRACTICAL NO. 7.

Q1. Topic : Differential Equation.

Q.1. Solve the following differential Equation.

1) $x \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + p(x)y = Q(x)$

Integration formula $(I.F) = e^{\int P dx}$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$p(x) = \frac{1}{x} \quad Q(x) = \frac{e^x}{x}$

$I.F = e^{\int (1/x) dx}$

$= e^{\int 1/x \, dx}$

$= e^{\ln x}$

$= x$

$y(I.F) = \int Q(x) (I.F) \, dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x dx + 1$

$xy = e^x + c$

2) $e^{x}\frac{dy}{dx}+2e^{x}y=1$

$\frac{dy}{dx}+2\frac{e^{x}}{e^{x}}y=\frac{1}{e^{x}}$ ($\div$ by e^{x})

$\frac{dy}{dx}+2y=\frac{1}{e^{x}}$, $\frac{dy}{dx}+2y=e^{-x}$

$p(x)=2$ $\quad Q(x)=e^{-x}$

$\int p(x)\,dx$

$I.f=e^{\int 2dx}$

$=e^{2x}$

$y(I.F)\int Q(x)\,(I.f)\,dx+c$

$y.e^{2x}=\int e^{-x}+2x\,dx+c$

$=\int e^{x}\,dx+c$

$y.e^{2x}=e^{x}+c$

Od

3) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$p(x) = 2/(x) \quad Q(x) = \frac{\cos x}{x^2}$

$T.f = e \int p(x) dx$

$= e \int 2/x \, dx$

$= e^{2/x} x$

$= \ln x^2$

$y(I.f) = \int Q(x) (I.f) dx + c$

$= \int \frac{\cos x}{x^2} \cdot x^2 dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

4) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ (÷ by x on both side

$p(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e \int p(x) dx$

$= e \int 3/x \, dx$

$= e^{3/x} x$

$= e \ln x^3$

$= x^3$

$$y(I.f) = \int Q(x)(I.f)\,dx + c$$
$$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$$
$$= \int \sin x\,dx + c$$
$$x^3 y = -\cos x + c$$

5) $e^{2x}\dfrac{dy}{dx} + 2e^{2x}y = 2x$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$p(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$$I.f = e^{\int p(x)dx}$$
$$= e^{\int 2\,dx}$$
$$= e^{2x}$$

$$y(I.f) = \int Q(x)(I.f)\,dx + c$$
$$= \int 2xe^{-2x}\,e^{2x}\,dx + c$$
$$= \int 2x\,dx + c$$
$$ye^{2x} = x^2 + c$$

6) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$$
$$\frac{\sec^2 x\,dx}{\tan x} = -\frac{\sec^2 y\,dy}{\tan y}$$

$$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$$

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$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = c$

7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Differentiating on both sides

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$

viii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

put $2x+3y = v$

$$\therefore 2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\therefore \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+1}\right)$$

$$\therefore \frac{dv}{dx} = \frac{v-1}{v+1} + 2$$

$$\therefore \frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$\therefore \frac{dv}{dx} = \frac{3v+3}{v+2} = \frac{3(v+1)}{(v+2)}$$

$$\therefore \int\left(\frac{v+2}{v+1}\right) dv = 3dx$$

$$\therefore \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = 3x$$

$$v + \log|x| = 3x + c$$

$$2x+3y + \log|2x+3y+1| = 3x+c$$

$$3y = x - \log|2x+3y+1| + c$$

10/01/2020

Practical No. 8

Topic :- Euler's method

Q.1. $\frac{dy}{dx} = y + e^x - 2$

$f(x,y) = y + e^x - 2$. $y_0 = 2$. $x_0 = 0$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h f(x_n, y_n)$

x	x_n	y_n	$f(x_n, y_n)$	y_{m+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

∴ By Euler's formula

$y(2) = 9.2831$

Q.2. $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$. $y_0 = 0$ $x_0 = 0$ $h = 0.2$

using Euler's iteration formula

$y_{n+1} = y_n + h\ f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6413
3	0.6	0.6413	1.413	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

$\therefore$ By Euler's formula
$y(1) = 1.2942$

0.3 $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $x_0 = 0$ $h = 0.2$

using Euler's iteration formula
$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

$y(1) = ?$

Q.4. $\frac{dy}{dx} = 3x^2+1 \quad y(1) = 2, \quad x_0 = 1 \quad h = 0.25$

for $h = 0.5$

using Euler's iteration formula.

$$y_{n+1} = y_n + h\, f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

$\therefore$ By Euler's formula,

$y(2) = 28.5$

for $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3359	10.1815	8.9048
4	2	8.9048		

$\therefore$ By Euler's formula.

$y(2) = 8.9048$

Q.5 $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y_0 = 1 \quad x_0 = 1 \quad h = 0.2$

using Euler's formula iteration

$$y_{n+1} = y_n + h\, f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

$\therefore$ By Euler's formula

$$y(1.2) = 1.6.$$

17/01/2020

PRACTICAL - 9

Limits & partial order derivatives

Q.1 Evaluate the following limits

1) $$\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy+5}$$

$$\lim_{(x,y)(-4,-1)} \frac{x^3 - 3x - y^2 - 1}{xy+5}$$

Applying limits

$$= \frac{(-4)^3 - 3(-4) + (-1)^2 - 1}{(-4)(-1)+5}$$

$$= \frac{-64+12+1-1}{4+5} = \frac{-52}{9}$$

2) $$\lim_{(x,y)(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

$$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

Applying limit

$$= \frac{(0+1)[(2)^2+(0)^2-4(2)]}{2+(3)(0)}$$

$$= \frac{1(4+0-8)}{2}$$

$$= \frac{4-8}{2} = -\frac{4}{2} = -2$$

$$\lim_{(x,y,z)(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$$

$$\lim_{(x,y,z)(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$$

Applying limit

$$= \frac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)(1)}$$

$$= \frac{1-1}{1-1} = \frac{0}{0}$$

$$= 0$$

Q.2. Find f_x, f_y for each of the following function

i) $f(x,y) = xye^{x^2+y^2}$

$$f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$$

$$= ye^{x^2+y^2}(2x)$$

$$f_x = 2xye^{x^2+y^2}$$

$$f_y = \frac{\partial}{\partial y}(f(x,y))$$

$$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$$

$$= xe^{x^2+y^2}(2y)$$

$$= 2yxe^{x^2+y^2}$$

ii) $f(x,y) = e^x \cos y$

$$f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}(e^x \cos y)$$

$$= (e^x \cos y)$$

$$f_y = \frac{\partial}{\partial y}(e^x \cos y)$$

$$= -e^x \sin y$$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$$f_x = \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$$

$$f_x = 3x^2y^2 - 6xy$$

$$f_y = \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$$

$$= 2x^2y - 3x^2 + 3y$$

Q.3. Using definition find values of f_x . f_y at $(0,0)$ for

$$f(x,y) = \frac{2x}{1+y^2}$$

$$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y - 0}{(1+y^2)^2}$$

$$= \frac{2(1\cancel{+y^2})}{(\cancel{1+y^2})(1+y^2)}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$ $= \frac{2}{1+0}$

$= 2$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2 (0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

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At $(0,0)$ $= \frac{-4(0)(0)}{(40)^2}$

$= 0$

Q4. Find all second order partial derivatives of f. Also verify whether ~~fxy = fyx~~ $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (-x^2 y - 2xy + 2x^2 y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$$

$$= \frac{x^2(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy + 2x^2 y)}{x^6} \quad -(1)$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y + x}{x^2}\right)$$

$$= \frac{2-0}{x^2} = \frac{2}{x^2} \quad — ②$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad — ③$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \frac{\partial}{\partial x}(2y - x) - (2y - x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad — ④$$

from ③ & ④

$$f_{xy} = f_{yx}$$

ii) $f(x,y) = \sin(xy) + e^{x+y}$

$$f_x = y \cos xy + e^{x+y}(1)$$

$$= y \cos xy + e^{x+y}$$

$$f_y = x \cos xy + e^{x+y}(1)$$

$$= x \cos xy + e^{x+y}$$

$\therefore f_{xx} = \dfrac{\partial}{\partial x}\left(y\cos(xy) + e^{x+y}\right)$

$= -y\sin(xy)\cdot(y) + e^{x+y}$

$= -y^2\sin(xy) + e^{x+y} \quad \longleftarrow (1)$

$f_{yy} = \dfrac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x\sin xy\,(x) + e^{x+y}$

$= -x^2\sin xy + e^{x+y} \quad — (2)$

$f_{xy} = \dfrac{\partial}{\partial y}\left(y\cos(xy) + e^{x+y}\right)$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y} \quad — (3)$

$f_{yx} = \dfrac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y} \quad — (4)$

$\therefore$ from (3) & (4)

$f_{xy} \neq f_{yx}$

Q.5 Find the linearization of at given point

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

→ $f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\qquad$ $f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\qquad$ $= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\qquad$ f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \cancel{\frac{2}{\sqrt{2}}}$

$= \cancel{\frac{x+y}{\sqrt{2}}}$

ii) $f(x,y) = 1-x+y\sin x$ at $(\frac{\pi}{2},0)$

$f(\frac{\pi}{2},0) = 1-\frac{\pi}{2}+0$

$= 1-\frac{\pi}{2}$

$f_x = 0-1+y\cos x$ $\qquad$ $f_y = 0-0+\sin x$

$f_x(\frac{\pi}{2},0) = -1+0$ $\qquad$ $f(\frac{\pi}{2},0) = \sin\frac{\pi}{2} = 1$

$= -1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1-\frac{\pi}{2} + (-1)(x-\frac{\pi}{2}) + 1(y-0)$

$= 1-\cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$

$= 1-x+y$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x}+0$ $\qquad$ $f_y = 0+\frac{1}{y}$

f_x (at) $(1,1) = 1$ $\qquad$ $= 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x-1+y-1$

$= x+y-2$

24/07/2020

PRACTICAL 10

Q1. Find the direction derivative of the following function at given points ie in the direction of given vector

i) $f(x,y) = x+2y-3$ $\quad a = (1,-1)$ $\quad u = 3i - j$

→ Here.

$u = 3i - j$ is not a unit vector

$\bar{u} = 3\hat{i} - \hat{j}$

$|u| = \sqrt{10}$

∴ unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3\hat{i} - \hat{j})$

$$= \frac{1}{\sqrt{10}}(3,-1)$$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

Now

$$f(a+hu) = f\left((1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$$

$$= f\left(1+\frac{3h}{\sqrt{10}}, -1-\frac{h}{\sqrt{10}}\right) - 3$$

$$= 1+\frac{3h}{\sqrt{10}} + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 - 2 - 3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$$

$$= -4 + \frac{h}{\sqrt{10}}$$

$\therefore D_u f(a) = \lim_{h \to 0} \dfrac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{-4} + \frac{h}{\sqrt{10}} - (\cancel{-4})}{h}$

$= \lim_{h \to 0} \dfrac{h}{h\sqrt{10}}$

$= \dfrac{1}{\sqrt{10}}$

ii) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$ $u = i + 5j$

Here

$u = i + 5j$ is not a unit vector

$\bar{u} = \bar{i} + 5\bar{j}$

$|\bar{u}| = \sqrt{26}$

$\therefore$ unit vector along u is $\dfrac{\bar{u}}{|\bar{u}|} = \dfrac{1}{\sqrt{26}}(i + 5j)$

$= \dfrac{1}{\sqrt{26}}(1,5)$

$= \left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)$

Now

$f(a+hu) = f\left((3,4) + h\left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)\right)$

$= f\left(3 + \dfrac{h}{\sqrt{26}}, 4 + \dfrac{5h}{\sqrt{26}}\right)$

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$$= \left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right)+1$$

$$= 16 + \frac{25h^2}{26} + \frac{4}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}}$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \lim_{h \to 0} \frac{25h^2}{26} + \frac{36h}{\sqrt{26}}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(\frac{25h^2}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

3) $f(x,y) = 2x + 3y$

→ Here

$u = 3i + 4j$ is not a unit vector.

$u = 3i + 4j$

$|\bar{u}| = \sqrt{25} = 5$

$\therefore$ unit vector along $u = \frac{u}{|\bar{u}|} = \frac{1}{5}(3i + 4j)$

$$= \frac{1}{5}(3,4)$$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

Now

$$f(a+hu) = f\left((1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= 8 + \frac{18h}{5}$$

$$\therefore D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{8 + \frac{18h}{5} - 8}{h}$$

$$= \lim_{h \to 0} \frac{18\cancel{h}}{5\cancel{h}}$$

$$= \frac{18}{5}$$

Q2. Find gradient vector for following function at given point

i) $f(x,y) = x^y + y^x \quad a = (1,1)$

$$f_x = y(x^{y-1}) + y^x \log y$$

$$f_y = x(y^{x-1}) + x^y \log x$$

$\nabla f\ (x,y) = (f_x, f_y)$

$= y^{x^{3-1}} + y^x \log xy\ ,\ xy^{x-1} + x^y \log x)$

$\nabla f\ (x,y)$ at $(1,1)$

$= (1\ x(1)^0 + 1^1 \log(1)\ ,\ 1(1)^{(1-1)} + 1^1 \log\ )$

$= (1,1)$

$f(x,y) = (\tan^{-1} x) \cdot y^2$

$f_x = y^2 \left(\frac{1}{1+x^2}\right) = \frac{y^2}{1+x^2}$

$f_y = 2y \tan^{-1} x$

$\nabla f\ (x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}\ ,\ 2z \tan^{-1} x\right)$

$\nabla f\ (x,y)$ of $(1,-1)$

$= \left(\frac{(-1)^2}{1+(1)^2}\right)\ ,\ 2(-1) \tan^{-1}(1)\ )$

$\nabla f\ (x,y)$ of $(1,-1)$

$= \left(\frac{(-1)^2}{1+(1)^2}\ ,\ 2(-1) \tan^{-1}(1)\right)$

$= \left(\frac{1}{2}\ ,\ \frac{-2\pi}{4}\right)$

$= \left(\frac{1}{2}\ ,\ \frac{-\pi}{2}\right)$

iii) $f(x,y,z)\ xyz - e^{x+y+z}$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0}, 1(0) - e^{1-1+0}, 1(-1) - e^{1-1+0})$

$= (0-1, 0-1, -1, -1)$

$= (-1, -1, -2)$

Q.3. Find the equation of tangent & normal to each of the follow using given function

i) $x^2\cos y + e^{xy} = z$

$f(x,y) = x^2\cos y + e^{xy} - z$

$f_x = 2x\cos y + ye^{xy}$

$f_y = -x^2\sin y + xe^{xy}$

$(x_0, y_0) = (1,0)$

f_x at $(1,0) = 2(1)\cos 0 + 0$

$= 2$

f_y at $(1,0) = -(1)^2\sin(0) + 1(e)^{1(0)}$

$= 1$

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$\therefore \quad 2x - 2 + y = 0$

$2x + y - 2 = 0$

Now,

For equation of Normal;

$bx + ay + d = 0$

$x + 2y + d = 0$

$(1) + 2(0) + d = 0 \quad$ At $(1,0)$

$1 + d = 0$

$d = -1$

$\therefore \quad x + 2y - 1 = 0$ — equation of Normal

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$\therefore$ Equation of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

$2x - 4 - y - 2 = 0$

$2x - y - 6 = 0$

for Equation of Normal

$bx + ay + d = 0$

$-x + 2y + d = 0$

$(-2) + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$d = +6$

$-x + 2y + 6 = 0$

Q.4. Find the equation of tangent normal line & each of the follows.

i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f(x,y,z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x - 0 + 0 + z - 0$ $\quad$ $f_y = -2z + 3 + 0 - 0$

$= 2x + z$ $\quad$ $= -2z + 3$

$f_z = 0 - 2y + 0 + x - 0$ $\quad$ f_x at $(2,1,0) = 2(2) + 0 = 4$

$= -2y + x$ $\quad$ f_y at $(2,1,0) = -2(0) + 3 = 3$

Equation of tangent $\quad$ f_z at $(2,1,0) = -2(1) + 2 = 0$

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$4(x-2) + 3(y-1) + 0(z-0) = 0$

$4x + 3y - 8 = 0$

$4x - 8 + 3y = 0$
$\quad -3$

$4x + 3y - 11 = 0$

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Equation tangent:

$$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$$
$$4(x-2) + 3(y-1) + 0(z-0) = 0$$
$$4x - 8 + 3y - $$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z-0}{0}$$

i) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$f(x,y,z) = 3xyz - x - y + z + 4$

$f_x = 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 - 0$
$= 3xyz$

f_x at $(1,-1,2) = 3(-1)(2) - 1$
$= -7$

f_y at $(1,-1,2) = 3(1)(2) - 1$
$= 5$

f_z at $(1,-1,2) = 3(1)(-1) + 1$
$= -2$

$\therefore$ Equation of tangent

$$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$$
$$-7(x-1) + 5(y-1) + (-2)(z-2) = 0$$
$$-7x + 7 + 5y + 5 - 2z + 4 = 0$$
$$-7x + 5y - 2z + 16 = 0 \quad \text{equation of tangent}$$

Equation of normal

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2} \quad \text{equation of normal}$$

Q.5. Find the local maxima & minima for the following function.

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$\therefore f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6 \quad — (1)$

$\therefore f_y = 2y - 3x + 0 - 4$

$= 2y - 3x - 4 \quad — (2)$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \quad - (3)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \quad - (4).$

Multiplying (3) by (2) and subtracting 4 from 3

$\therefore \quad 4x - 2y = 4$

$\underline{\quad 2y - 3x = 4 \quad}$

$7x = 0$

$x = 0$

Substituting value of x is (3).

$2(0) - y = -2$

$-y = 2$

$y = 2$

Critical points are $(0, 2)$

Now

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$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 - 9$

$= 3 > 0$

Here, $r > 0$ and $rt - s^2 > 0$

f has minimum at $(0,2)$

$\therefore f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 + 8$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy - 0$

$= 8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2y$

$= 3x^2 - 2y$

Now,

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 6y) = 0$

$4x^2 + 6y = 0$ — (1)

multiple in (1) by (2) and substituting (2) from (1).

$12x^2 + 18y = 0$

$\underline{12x^2 - 8y = 0}$

$\quad 2xy = 0$

$y = 10$ (i)

35

Substraction ③ in ②

$3x^2 - 2(0) = 0$

$3x^2 = 0$

$x^2 = 0$

$x = 2$

Critical point are $(0,0)$

Now

$r = f_{xx} = 24x^2 + 6y$

$t = f_{yy} = 1 - 2$

~~$S = f_{xy} = 24x^2 + 6y$~~

& $s = f_{xy} = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$= -48x^2 - 12y - 36x^2$

$= -84x^2 - 12y$

At $(0,0)$

$r = 24(0)^2 + 6(0)$

$= 0$

$s = 6(0) = 0$

$rt - s^2 = -84(0)^2 - 12(0) = 0$

$r = 0$ and $rt - s^2 = 0$

Nothing Can be said

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7½

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \qquad 2x + 2 = 0$

$2(x+1) = 0$

$x = -1$

$f_y = 0$

$-2y + 8 = 0$

$-2(y-4) = 8$

$y = 4$

Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - 0^2$

$= -4 < 0$

$\therefore$ f has minimum at $(-1,4)$

$f(x,y)$ at $(-1,4)$

$= x^2 - y^2 + 2x + 8y - 70$

$= (-1)^2 - (4)^2 + 2 \times -1 + 8 \times 4 - 70$

$= 1 - 16 - 2 + 32 - 70$

$= -88 + 33$

$= -55$

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