DSP
Assignment

R161561
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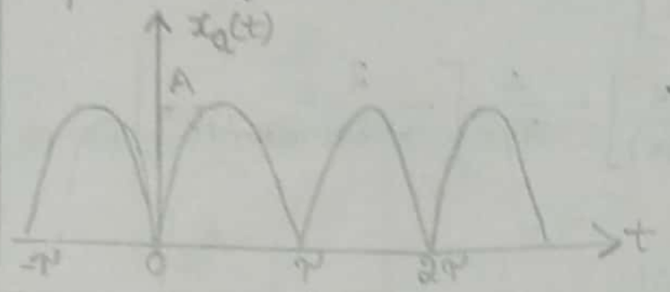
1. consider the full wave rectified sinusoid (a) determine the spectrum $X_a(F)$



∵ It is a periodic signal in continuous time

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi t/T} \qquad T=\tau$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{j2\pi t/\tau}$$

$$c_k = \frac{1}{\tau}\int_0^{\tau} A \sin\left(\frac{\pi t}{\tau}\right) e^{-j2\pi kt/\tau}\, dt$$

$$= \frac{1}{\tau}\int_0^{\tau} A \cdot \left[\frac{e^{\frac{j\pi t}{\tau}} - e^{-\frac{j\pi t}{\tau}}}{2j}\right] e^{-j2\pi kt/\tau}\, dt$$

$$= \frac{A}{2j\tau}\int_0^{\tau}\left[\left(e^{\frac{j\pi t}{\tau}} \cdot e^{-j2\pi kt/\tau}\right) - \left(e^{-\frac{j\pi t}{\tau}} \cdot e^{-j2\pi kt/\tau}\right)\right] dt$$

$$= \frac{A}{2j\tau}\int_0^{\tau}\left(e^{j\pi(1-2k)\frac{t}{\tau}} - e^{-j\pi(1+2k)\frac{t}{\tau}}\right) dt$$

$$= \frac{A}{2j\tau}\int_0^{\tau} e^{j\pi(1-2k)t/\tau}\, dt - \frac{A}{2j\tau}\int_0^{\tau} e^{-j\pi(1+2k)t/\tau}\, dt$$

$$= \frac{A}{2j\tau}\left[\frac{e^{j\pi(1-2k)t/\tau}}{j\pi(1-2k)t/\tau}\right]_0^{\tau} - \frac{A}{2j\tau}\left[\frac{e^{-j\pi(1+2k)t/\tau}}{-j\pi(1+2k)t/\tau}\right]_0^{\tau}$$

$$= \frac{A}{2j\tau}\left[\frac{e^{j\pi(1-2k)} - 1}{j\pi(1-2k)t/\tau}\right] - \frac{A}{2j\tau}\left[\frac{e^{-j\pi(1+2k)} - 1}{+j\pi(1+2k)t/\tau}\right]$$

$$= \frac{A}{2j\tau} \cdot \frac{\tau}{j\pi}\left[\frac{e^{j\pi(1-2k)} - 1}{(1-2k)} + \frac{e^{-j\pi(1+2k)} - 1}{(1+2k)}\right]$$

$$= \frac{-A}{2\pi}\left[\frac{-1-1}{(1-2k)} + \frac{-1-1}{(1+2k)}\right]$$

$$= \frac{-A}{2\pi}\left[\frac{-2}{1-2k} - \frac{2}{1+2k}\right] = \frac{-A}{2\pi}\left[-2\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]\right]$$

$$= \frac{A}{\pi}\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]$$

$$= \frac{A}{\pi}\left[\frac{1+2k+1-2k}{(1-2k)(1+2k)}\right] = \frac{A}{\pi}\left[\frac{2}{1+2k-2k-4k^2}\right]$$

$$= \frac{2A}{\pi(1-4k^2)}$$

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi Ft}\, dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_k e^{j2\pi kF_0 t} \cdot e^{-j2\pi Ft}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} c_k e^{-j2\pi(Ft - kF_0 t)}\, dt \qquad F_0 = \frac{1}{T} = \frac{1}{\tau}$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} c_k e^{-j2\pi(F - \frac{k}{\tau})t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} c_k \int_{-\infty}^{\infty} e^{-j2\pi(F - \frac{k}{\tau})t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} c_k\, \delta\left(F - \frac{k}{\tau}\right)$$

(b) compute the power of the signal

$$P = \frac{1}{T}\int_0^T |x(t)|^2\, dt \qquad T = \tau$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{1}{\tau}\int_0^\tau \left(A \sin\frac{\pi t}{\tau}\right)^2 dt$$

$$= \frac{1}{\tau}\int_0^\tau A^2 \sin^2\frac{\pi t}{\tau}\, dt$$

$$= \frac{1}{\tau}\int_0^\tau A^2\left[\frac{1-\cos 2\frac{\pi t}{\tau}}{2}\right] dt$$

$$= \frac{A^2}{\tau}\int_0^\tau \left[\frac{1}{2} - \frac{\cos 2\left(\frac{\pi}{\tau}\right)t}{2}\right] dt$$

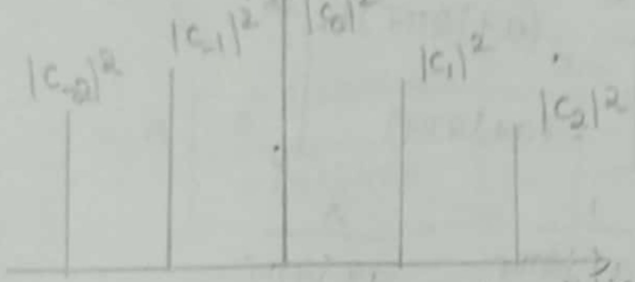
$$= \frac{A^2}{\tau}\int_0^\tau \frac{1}{2}\, dt - \frac{A^2}{\tau}\int_0^\tau \frac{\cos 2\left(\frac{\pi}{\tau}\right)t}{2}\, dt$$

$$= \frac{A^2}{\tau}\left[\frac{\tau}{2}\right] - \frac{A^2}{\tau}\left(\frac{\sin^2\pi}{4}\right)$$

$$= \frac{A^2}{\tau}\cdot\frac{\tau}{2} - 0$$

$$= \frac{A^2}{2}$$

(c) plot the power spectral density.



(d) check the validity of parseval's relation for the given signal.

$$P_x = \sum_{k=-\infty}^{\infty} |C_k|^2$$

$$C_k = \frac{2A}{\pi(1-4k^2)}$$

$$P_x = \sum_{k=-\infty}^{\infty} \left|\frac{2A}{\pi(1-4k^2)}\right|^2$$

$$= \frac{4A^2}{\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{(4k^2-1)^2}$$

$$= \frac{4A^2}{\pi^2}\left[\left.\frac{1}{(4k^2-1)^2}\right|_{k=0} + 2\sum_{k=1}^{\infty}\frac{1}{(4k^2-1)^2}\right]$$

$$= \frac{4A^2}{\pi^2}\left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \frac{2}{35^2} + \cdots\right]$$

$$= \frac{4A^2}{\pi^2}\left[1.041\right]$$

$$= A^2\left[\frac{1.041\times 4}{\pi^2}\right]$$

$$= \frac{A^2[4.164]}{\pi^2} = 0.422A^2$$

2 compute and sketch the magnitude and phase spectra for the following signals ($a>0$)

(a) $x_a(t) = \begin{cases} Ae^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt = \int_{0}^{\infty} A\, e^{-at} \cdot e^{-j2\pi ft}\, dt$$

$$= A \int_{0}^{\infty} e^{-(a+j2\pi f)t}\, dt$$

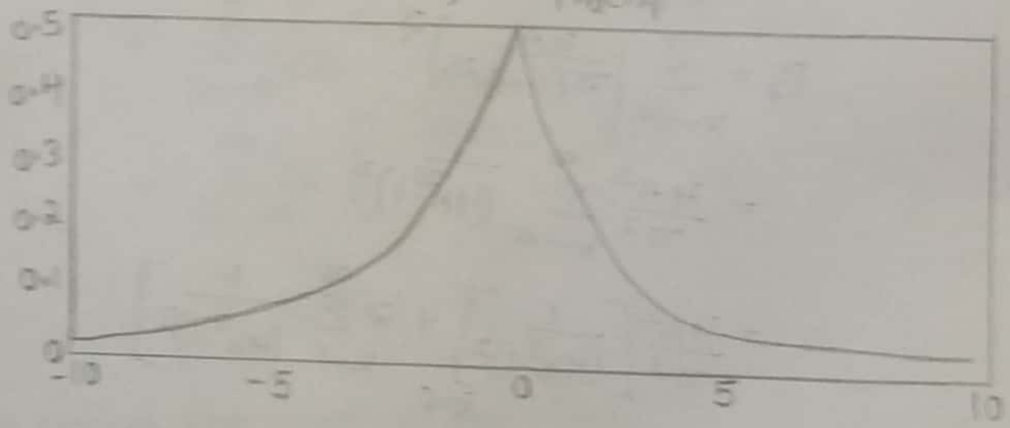
$$= A \left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \right]_0^{\infty}$$

$$= A \cdot \frac{1}{a+j2\pi f} = \frac{A}{a+j2\pi f}$$

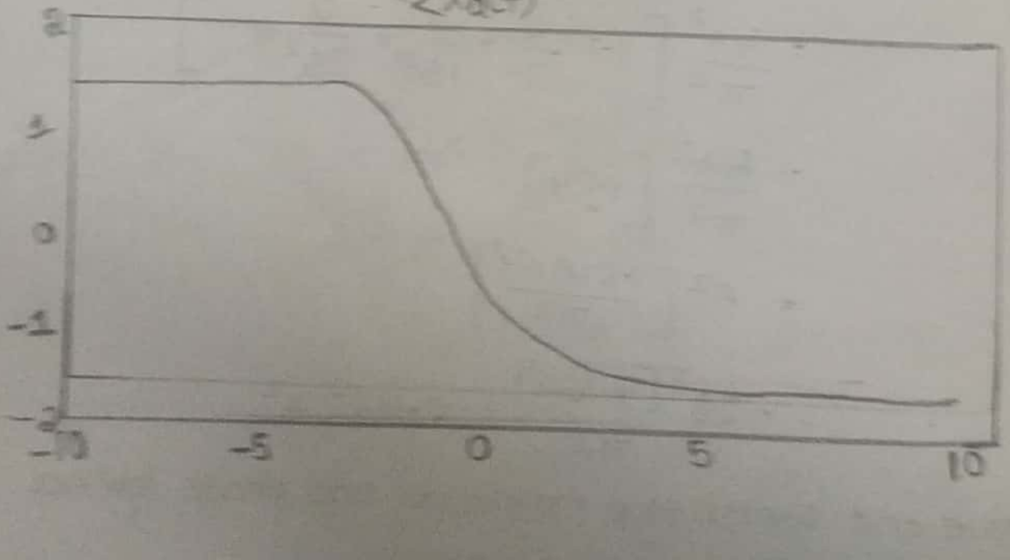
$$|X_a(F)| = \frac{A}{\sqrt{a^2+(2\pi f)^2}} = \frac{A}{\sqrt{a^2+4\pi^2 f^2}}$$

$$\angle X_a(F) = -\tan^{-1}\left(\frac{2\pi f}{a}\right)$$

$|X_a(F)|$



$\angle X_a(f)$



(b) $x_a(t) = Ae^{-a|t|}$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{0} Ae^{at} \cdot e^{-j2\pi ft} dt + \int_{0}^{\infty} Ae^{-at} \cdot e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{0} Ae^{-(j2\pi f - a)t} dt + \int_{0}^{\infty} Ae^{-(a+j2\pi f)t} dt$$

$$= A\left[\frac{e^{-(j2\pi f - a)t}}{-(a+j2\pi f)}\right]_{-\infty}^{0} + A\left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)}\right]_{0}^{\infty}$$

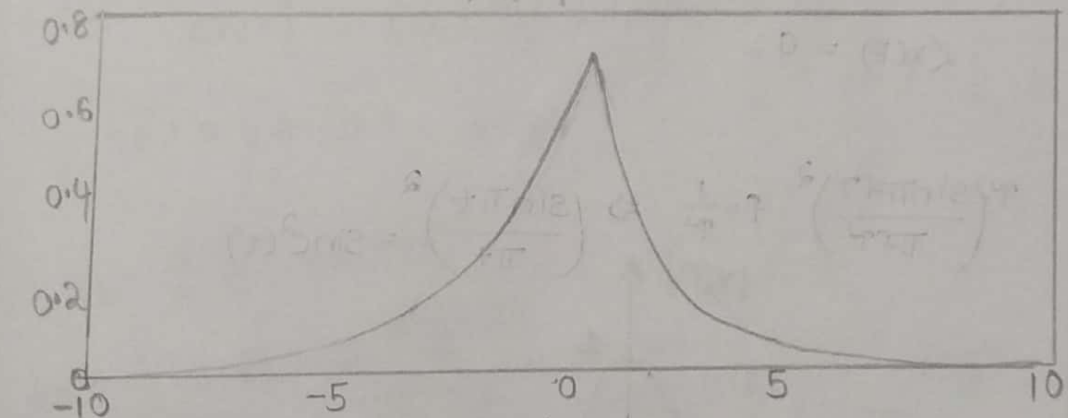
$$= A\left[\frac{1}{a-j2\pi f}\right] + A\left[\frac{1}{a+j2\pi f}\right]$$

$$= \frac{Aa + Aj2\pi f + Aa - Aj2\pi f}{a^2 - (j2\pi f)^2}$$

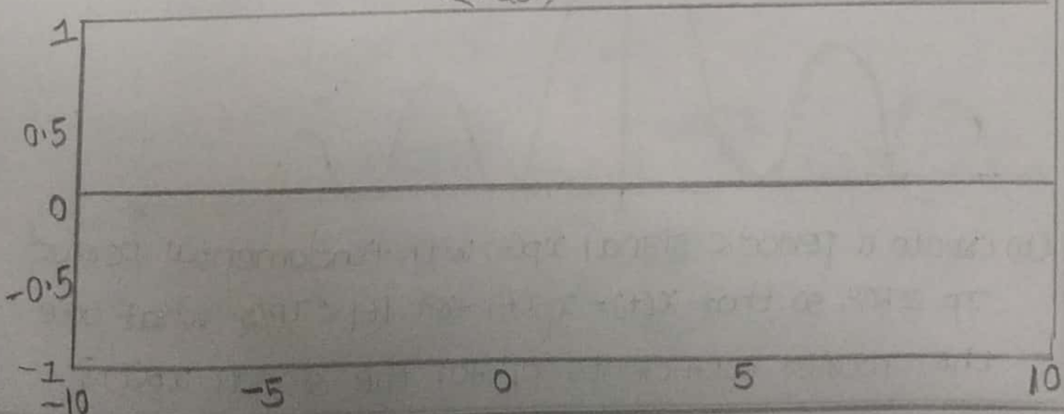
$$= \frac{2aA}{a^2 + (2\pi f)^2}$$

$|X_a(f)| = X_a(f)$ $\quad \angle X_a(f) = \tan^{-1}\left(-\frac{0}{2}\right) = 0$

$|X_a(f)|$



$\angle X_a(f)$



3 consider the signal $x(t) = \begin{cases} 1-\frac{|t|}{\tau}, & |t|\le\tau \\ 0, & \text{elsewhere} \end{cases}$

(a) Determine and sketch its Magnitude and phase spectra $|X_a(f)|$ and $\angle X_a(f)$

$$X(f) = \int_{-\tau}^{0} x(t)e^{-j2\pi ft}dt + \int_{0}^{\tau} x(t)e^{-j2\pi ft}dt$$

First
$$= \int_{-\tau}^{0}\left(1+\frac{t}{\tau}\right)e^{-j2\pi ft}dt + \int_{0}^{\tau}\left(1-\frac{t}{\tau}\right)e^{-j2\pi ft}dt$$

Fourier transform $y(t) = x'(t) = \begin{cases} \frac{1}{\tau} & -T<t\le 0 \\ \frac{-1}{T} & 0<t\le T \end{cases}$

$$Y(f) = \int_{-\tau}^{0}\frac{1}{\tau}e^{-j2\pi ft}dt + \int_{0}^{\tau}\frac{-1}{\tau}e^{-j2\pi ft}dt$$

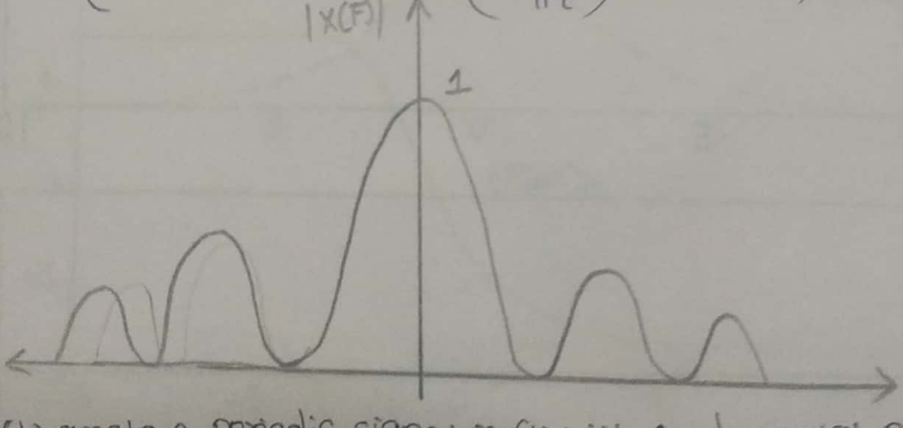
$$= -2\,\frac{\sin^2\pi ft}{j\pi ft}$$

$$X(F) = \frac{1}{j2\pi f}Y(f) = \tau\left(\frac{\sin\pi f\tau}{\pi f\tau}\right)^2$$

$$|X(F)| = \tau\left(\frac{\sin\pi f\tau}{\pi f\tau}\right)^2$$

$$\angle X(F) = 0$$

$$\tau\left(\frac{\sin\pi f\tau}{\pi f\tau}\right)^2 \quad f=\frac{1}{\tau} \Rightarrow \left(\frac{\sin\pi t}{\pi t}\right)^2 = \operatorname{sinc}^2(t)$$



(b) create a periodic signal $x_p(t)$ with fundamental period $T_P \ge 2\tau$, so that $x(t) = x_p(t)$ for $|t| < T_P/2$. What are the fourier coefficients C_k for the signal $x_p(t)$?

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$$C_K = \frac{1}{T_P}\int_{-T_P/2}^{T_P/2} x(t)\, e^{-j2\pi f t/T_P} \cdot dt$$

$$= \frac{1}{T_P}\int_{-\tau}^{0}\left(1+\frac{t}{\tau}\right)e^{-j2\pi k t/T_P}\,dt + \int_{0}^{\tau}\left(1-\frac{t}{\tau}\right)e^{-j2\pi k t/T_P}\,dt$$

$$= \frac{\tau}{T_P}\left[\frac{\sin \pi k\tau/T_P}{\pi k\tau/T_P}\right]^2$$

(c) using the results in part a and b show that $C_K = \left(\frac{1}{T_P}\right)x_a\left(\frac{k}{T_P}\right)$

$$\frac{1}{T_P}x_a\left(\frac{K}{T_P}\right)$$

$$\frac{1}{T_P}\cdot\tau\left(\frac{\sin \pi \frac{K}{T_P}\tau}{\pi \frac{K}{T_P}\tau}\right)^2$$

$$\frac{\tau}{T_P}\left(\frac{\sin \pi k\tau/T_P}{\pi k\tau/T_P}\right)^2 = C_K.$$

$$\therefore C_k = \frac{1}{T_P}x_a\left(\frac{K}{T_P}\right)$$

4 consider the following periodic signal $x(n) = \{\ldots 1,0,1,2,\underset{\uparrow}{3},2,1,0,1\ldots\}$

(a) sketch the signal $x(n)$ and its magnitude and phase spectra.

$$x(n) = \{\ldots\ldots 1,0,1,2,\underset{\uparrow}{3},2,1,0,1\ldots\}$$

$$\underbrace{1\,0\,1\,2\,\underset{\uparrow}{3}\,2}\,1\,0\,1 \qquad N=6$$

$$C_K = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{6}}$$

for $n=0 \rightarrow x(0)\, e^{-\frac{j2\pi k(0)}{6}} = 3\times1 = 3$

$n=1 \rightarrow x(1)\, e^{-\frac{j2\pi k(1)}{6}} = 2e^{-\frac{j2\pi k(1)}{6}} = 2e^{-\frac{jK2\pi}{6}}$

$n=2 \rightarrow x(2)\, e^{-\frac{j2\pi k(2)}{6}} = e^{-\frac{j2\pi k}{3}}$

$n=3 \rightarrow x(3)\, e^{-\frac{j2\pi k(3)}{6}} = x(3)e^{-\frac{j2\pi k}{2}} = x(3)e^{-j\pi k} = 0$

$n=4 \rightarrow x(4)\, e^{-\frac{j2\pi k4}{6}} = 1\cdot e^{-\frac{j2\pi k}{3}}$

$n=5 \rightarrow x(5)e^{-\frac{j2\pi k5}{6}} = 2\cdot e^{-\frac{j10\pi k}{6}}$

$$= \frac{1}{6}\left[3+2e^{-\frac{j\pi 2k}{6}} + e^{-\frac{j2\pi k}{3}} + 0 + e^{-\frac{j4\pi k}{3}} + 2e^{-\frac{j10\pi k}{6}}\right]$$

for $k=0 \Rightarrow \frac{1}{6}[3+2+1+0+1+2]$

$$\Rightarrow \frac{1}{6}[9] = \frac{9}{6} = \frac{3}{2}$$

for $k=1 \Rightarrow \frac{1}{6}\left[3+2e^{-j\pi/3} + e^{-j2\pi/3} + 0 + e^{-j\frac{4\pi}{3}} + 2e^{-j\frac{10\pi}{6}}\right]$

$$\Rightarrow \frac{1}{6}\left[3+2e^{-j\pi/3} + e^{-j2\pi/3} + 0 + e^{-j4\pi/3} + 2e^{-j5\pi/3}\right]$$

$$\Rightarrow \frac{1}{6}\Big[3+2\left(\cos\frac{\pi}{3} - j\sin\frac{\pi}{3}\right) + \cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right) + \cos\left(\frac{4\pi}{3}\right) - j\sin\left(\frac{4\pi}{3}\right) + 2\cos\left(\frac{5\pi}{3}\right) - 2j\sin\left(\frac{5\pi}{3}\right)\Big]$$

$$\Rightarrow \frac{1}{6}\Big[3+2\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + \left(\frac{-1}{2}\right) - j\frac{\sqrt{3}}{2} + \left(\frac{-1}{2}\right) - j\left(-\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{2}\right) - 2j\left(\frac{-\sqrt{3}}{2}\right)\Big]$$

$$\Rightarrow \frac{1}{6}\left[3+1-j\cancel{\sqrt{3}} - \frac{1}{2} - \frac{\cancel{\sqrt{3}j}}{2} - \frac{1}{2} + \frac{\cancel{\sqrt{3}j}}{2} + 1 + \cancel{\sqrt{3}}j\right]$$

$$\Rightarrow \frac{1}{6}[5-1] = \frac{4}{6} = \frac{2}{3}$$

for $k=2$; $c_2 = 0$

$k=3$; $c_3 = 1/6$

$k=4$; $c_4 = 0$

$k=5$; $c_5 = 4/6$

(b) using the results in part (a) verify parseval's relation by computing the power in the time and frequency domains.

$$P_t = \frac{1}{6}\sum_{n=0}^{6} |x(n)|^2$$

$$= \frac{1}{6}\left[1^2+0+1+2^2+3^2+4\right] = \frac{1}{6}[1+1+4+9+4]$$

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$= \frac{19}{6}$

$$P_f = \sum_{n=0}^{5} |c_n|^2 = \left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{4}{6}\right)^2 = \frac{114}{36} = \frac{19}{6}$$

5 consider the signal $x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$

(a) Determine and sketch its PDS.

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

$$= 2 + 2\left[\frac{e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}}}{2}\right] + \left[\frac{e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}}{2}\right] + \frac{1}{2}\left[\frac{e^{\frac{j3\pi n}{4}} + e^{-\frac{j3\pi n}{4}}}{2}\right]$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{-\frac{j\pi n}{2}} + \frac{1}{4}e^{\frac{j3\pi n}{4}} + \frac{1}{4}e^{-\frac{j3\pi n}{4}}$$

$N = 8$

$$c_k = \frac{1}{8}\sum_{n=0}^{7} x(n) e^{-\frac{j\pi k n}{4}}$$

$$x(n) = \left\{\frac{11}{2}, 2+\frac{3}{4}\sqrt{2}, 1, 2-\frac{3\sqrt{2}}{4}, \frac{1}{2}, 2-\frac{3}{4}\sqrt{2}, 1, 2+\frac{3}{4}\sqrt{2}\right\}$$

$c_0 = 2$, $c_1 = c_7 = 1$, $c_2 = c_6 = \frac{1}{2}$, $c_3 = c_5 = \frac{1}{4}$, $c_4 = 0$

(b) Evaluate the power of the signal.

$$\sum_{n=0}^{7} |c_k|^2 \Rightarrow \left[2^2 + 1^2 + 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]$$

$$\Rightarrow \left[4 + 2 + \frac{1}{2} + \frac{1}{8}\right]$$

$$\Rightarrow \frac{32+16+4+1}{8} \Rightarrow \frac{53}{8}$$

6 Determine and sketch the magnitude and phase spectra of the following periodic signals.

(a) $x(n) = 4\sin\frac{\pi(n-2)}{3}$

$$= 4\left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{-\frac{j\pi(n-2)}{3}}}{2j}\right]$$

$$= 4\left[e^{3j\pi(n-2)} - e^{-3j\pi(n-2)}\right]$$

$N = 6$

$$c_k = \frac{1}{6}\sum_{n=0}^{5} x(n) e^{-\frac{j2\pi kn}{6}}$$

$$= \frac{4}{6}\sum_{n=0}^{5}\left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{-\frac{j\pi(n-2)}{3}}}{2j}\right] e^{-\frac{j2\pi kn}{6}}$$

$$= \frac{1}{\sqrt{3}}\left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{-j\pi k/3} + e^{-j2\pi k/3}\right]$$

$$= \frac{1}{\sqrt{3}}(-j2)\left[\sin\frac{2\pi k}{6} + \sin\frac{\pi k}{3}\right] e^{-j2\pi k/3}$$

$c_0 = 0,\ c_1 = -j2e^{-j2\pi/3}$ $\quad c_2 = c_3 = c_4 = 0 \quad c_5 = c_1$

$\angle c_1 = \frac{5\pi}{6} \quad \angle c_5 = -\frac{5\pi}{6} \quad \angle c_0 = \angle c_2 = \angle c_3 = \angle c_4 = 0$

(b) $x(n) = \cos\frac{2\pi}{3}n + \sin\frac{2\pi}{5}n$

$N = 15$

$\cos\frac{2\pi}{3}n$

$$\frac{1}{2}\left[e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right]$$

$$e^{-j\frac{2\pi}{3}n} = e^{-\frac{j2\pi kn}{N}}$$

$$\frac{K}{N} = \frac{1}{3}$$

$$K = \frac{N}{3} = 5$$

$15 - 5 = 10$

$$c_{1k} = \begin{cases} \frac{1}{2} ; & k = 5, 10 \\ 0 ; & \text{otherwise} \end{cases}$$

$\sin\frac{2\pi}{5}n$

$$\frac{1}{2j}\left[e^{j\frac{2\pi}{5}n} - e^{-j\frac{2\pi}{5}n}\right]$$

$$e^{-j\frac{2\pi}{5}n} = e^{-\frac{2j\pi kn}{N}}$$

$$\frac{K}{N} = \frac{1}{5}$$

$5k = N \Rightarrow k = 3$

$15 - 3 = 12$

$$c_{2k} = \begin{cases} \frac{1}{2j} ; & k = 3, \\ \frac{-1}{2j} ; & k = 12 \\ 0 ; & \text{otherwise} \end{cases}$$

$$c_k = c_{1k} + c_{2k}$$

$$C_k = \begin{cases} \frac{1}{2j} & k=3 \\ -\frac{1}{2} & k=5 \\ \frac{1}{2} & k=10 \\ \frac{-1}{2j} & k=12 \\ 0 & \text{otherwise} \end{cases}$$

(c) $x(n) = \cos\frac{2\pi}{3}n \cdot \sin\frac{2\pi}{5}n$

$$\cos a \sin b = \frac{\sin(a+b)}{2} - \frac{\sin(a-b)}{2}$$

$$= \frac{1}{2}\left[\frac{\sin(10\pi n + 6\pi n)}{15} - \frac{\sin(10\pi n - 6\pi n)}{15}\right]$$

$$= \frac{1}{2}\left[\sin\frac{16\pi n}{15} - \sin\frac{4\pi n}{15}\right]$$

$$= \frac{1}{2}\left[\frac{e^{\frac{j16\pi n}{15}} - e^{\frac{-j16\pi n}{15}}}{2j}\right] - \frac{1}{2}\left[\frac{e^{\frac{4\pi n j}{15}} - e^{\frac{-j4\pi n}{15}}}{2j}\right]$$

$$= \frac{1}{4j}\left[e^{\frac{j16\pi n}{15}} - e^{\frac{-j16\pi n}{15}}\right] - \frac{1}{4j}\left[e^{\frac{4\pi n}{15}j} - e^{-j\frac{4\pi n}{15}}\right]$$

$e^{\frac{j16\pi n}{15}} \Rightarrow e^{\frac{j2\pi k}{N}}$ $\qquad$ $e^{\frac{j4\pi n}{15}} \Rightarrow e^{\frac{j2\pi k}{N}}$

$\frac{k}{N} = \frac{8}{15}$ $\qquad$ $\frac{k}{N} = \frac{2}{15}$

$k = 8 \to \frac{1}{4j}$ $\qquad$ $k = 2$

$15-8 = 7 \Rightarrow -\frac{1}{4j}$ $\qquad$ $\frac{-1}{4j} \Rightarrow k = 2$

$15-2 = 13 \to \frac{1}{4j}$

$$C_k = \begin{cases} \frac{1}{4j} & ; 8, 13 \\ -\frac{1}{4j} & ; 7, 2 \\ 0 & \text{otheswise} \end{cases}$$

(d) $x(n) = \{\ldots -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2 \ldots\}$

$x(n) = \{\ldots \underbrace{-2, -1, \underset{\uparrow}{0}, 1, 2}_{N=5}, -2, -1, 0, 1, 2 \ldots\}$

$$c_K = \frac{1}{5}\sum_{n=0}^{4} x(n) e^{-\frac{j2\pi kn}{N}}$$

$$= \frac{1}{5}\sum_{n=0}^{4} x(n) e^{-\frac{j2\pi kn}{5}}$$

$$= \frac{1}{5}\left[0 + e^{-\frac{j2\pi k}{5}} + 2e^{-\frac{j2\pi k(2)}{5}} - 2e^{-\frac{j6\pi k}{5}} - e^{-\frac{j8\pi k}{5}}\right]$$

$$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right)\right]$$

If $k=0$; $c_0 = 0$

$k=1$; $c_1 = \frac{2j}{5}\left[-\sin\left(\frac{2\pi}{5}\right) - 2\sin\left(\frac{4\pi}{5}\right)\right]$

$k=2$; $c_2 = \frac{2j}{5}\left[-\sin\left(\frac{4\pi}{5}\right) - 2\sin\left(\frac{8\pi}{5}\right)\right)$

$c_3 = -c_2$; $c_4 = -c_1$

(e) $x(n) = \{\underbrace{-1, 2, \underset{\uparrow}{1}, 2, -1, 0}, -1, 2, 1, 2 \cdots\}$

$N = 6$

$$c_K = \frac{1}{N}\sum_{n=0}^{5} x(n) e^{-\frac{j2\pi kn}{N}}$$

from 0 to 5 in above eq we get

$$= \frac{1}{6}\left[1 + 2e^{-\frac{j\pi k}{3}} - e^{-\frac{j2\pi k}{3}} - e^{-\frac{j4k\pi}{3}} + 2e^{-\frac{j5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos\frac{\pi k}{3} - 2\cos\frac{2\pi k}{3}\right]$$

$c_0 = \frac{1}{2}$; $c_1 = \frac{2}{3}$; $c_2 = 0$; $c_3 = \frac{-5}{6}$; $c_4 = 0$; $c_5 = \frac{2}{3}$

(f) $x(n) = \{\cdots 0, \underbrace{0, \underset{\uparrow}{1}, 1, 0, 0}, 0, 1, 1, 0, 0 \cdots\}$

$N = 5$

$$c_K = \frac{1}{N}\sum_{n=0}^{4} x(n) e^{-\frac{j2\pi kn}{5}}$$

$$= \frac{1}{5}\left[1 + e^{-\frac{j2\pi k}{5}}\right]$$

$$= \frac{2}{5}\cos\left(\frac{\pi k}{5}\right)e^{-\frac{j\pi k}{5}}$$

$$\therefore c_0 = \frac{2}{5}\ ;\ c_1 = \frac{2}{5}\cos\left(\frac{\pi}{5}\right)e^{-j\pi/5}\ ;\ c_2 = \frac{2}{5}\cos\left(\frac{2\pi}{5}\right)e^{-j2\pi/5}$$

$$c_3 = \frac{2}{5}\cos\left(\frac{3\pi}{5}\right)e^{-\frac{j3\pi}{5}}\ ;\ c_4 = \frac{2}{5}\cos\left(\frac{4\pi}{5}\right)e^{-j4\pi/5}$$

(g) $x(n) = 1 \quad -\infty < n < \infty$

$N = 1$

$c_0 = 1$

$c_k = x(0) = 1$

(h) $x(n) = (-1)^n, \quad -\infty < n < \infty$

$N = 2$

$$c_k = \frac{1}{2}\sum_{n=0}^{1} x(n)e^{-j\pi nk}$$

$$= \frac{1}{2}\left(1 - e^{-j\pi k}\right)$$

$$\therefore c_0 = 0\ ;\ c_1 = 1$$

7 Determine the periodic signals $x(n)$ with fundamental period $N=8$ if their fourier coefficients are given by.

(a) $c_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

$$x(n) = \sum_{n=0}^{7} c_k e^{\frac{j2\pi nk}{N}}$$

Let $c_k = e^{\frac{j2\pi pk}{N}}$

$$\sum_{n=0}^{7} e^{\frac{j2\pi pk}{N}} \cdot e^{\frac{j2\pi nk}{N}}$$

$$\sum_{n=0}^{7} e^{\frac{j2\pi(p+n)k}{N}}$$

it gives 8 ; when $p = -n$

0 ; when $p \neq n$

$$\therefore c_k = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{-\frac{j2\pi k}{8}}\right] - \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{-\frac{j6\pi k}{8}}\right]$$

$$x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\,\delta(n-1) - 4j\,\delta(n+3) + 4j\,\delta(n-3)$$

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$\ddot{j}$ $-3 \le n \le 5$

(b) $c_k = \begin{cases} \sin\frac{k\pi}{3} & ; 0 \le k \le 6 \\ 0 & ; k=7 \end{cases}$

$c_0 = 0$; $c_1 = \frac{\sqrt{3}}{2}$ $c_2 = \frac{\sqrt{3}}{2}$ $c_3 = 0$ $c_4 = \frac{-\sqrt{3}}{2}$ $c_5 = -\frac{\sqrt{3}}{2}$ $c_6 = c_7 = 0$

$$x(n) = \sum_{k=0}^{7} c_k e^{\frac{j2\pi nk}{8}}$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\pi n 2}{4}} - e^{\frac{-j4\pi n}{4}} - e^{\frac{-j5\pi n}{4}}\right]$$

$$= \sqrt{3}\left[\frac{\sin\pi n}{2} + \sin\frac{\pi n}{4}\right] e^{\frac{j\pi(3n-2)}{4}}$$

(c) $c_k = \{\ldots 0, \frac{1}{4}, \frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}, \frac{1}{4}, 0 \ldots\}$

$$x(n) = \sum_{k=-3}^{4} c_k e^{\frac{j2\pi nk}{8}}$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{\frac{-j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{\frac{-j\pi n}{2}} + \frac{1}{4}e^{\frac{j3\pi n}{4}} + \frac{1}{4}e^{\frac{-j3\pi n}{4}}$$

$$= 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{2}$$

8 Two DT signals $s_k(n)$ and $s_l(n)$ are said to be orthogonal over an integral $[N_1\ N_2]$ if $\sum_{n=N_1}^{N_2} s_k(n)\, s_l^*(n) = \begin{cases} A_k & k=l \\ 0 & k \ne l \end{cases}$

If $A_k = 1$ the signals are called orthonormal.

(a) prove the relation $\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \begin{cases} N & k = 0, \pm N, \pm 2N \\ 0 & \text{otherwise} \end{cases}$

$k = 0, \pm N, \pm 2N$

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \text{sum}_{n=0}^{N-1}\ 1 = N$$

If $k \ne 0, \pm N, \pm 2N$ $\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \frac{1 - e^{j2\pi k}}{1 - e^{j2\pi k/N}}$

9. Compute the Fourier transform of the following signals.

(a) $x(n) = u(n) - u(n-6)$

$$x(n) = u(n) - u(n-6)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{5} e^{-j\omega n}$$

$$= \frac{1-e^{-j6\omega}}{1-e^{-j\omega}}$$

(b) $x(n) = 2^n u(-n)$

$$x(n) = 2^n u(-n)$$

$$X(\omega) = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n}$$

$$= \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^n$$

$$= \frac{2}{2-e^{j\omega}}$$

(c) $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$

$$x(n) = \left(\frac{1}{4}\right)^n u(n+4)$$

$$X(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m e^{-j\omega m}) 4^4 e^{j4\omega}$$

$$= \frac{4^4 e^{j4\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

(d) $x(n) = (a^n \sin\omega_0 n) u(n) \quad |a| < 1$

$$x(n) = (a^n \sin\omega_0 n) u(n) \quad |a| < 1$$

$$X(\omega) = \sum_{n=0}^{\infty} a^n \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] e^{-j\omega n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \left[a e^{-j(\omega-\omega_0)}\right]^n - \frac{1}{2j} \sum_{n=0}^{\infty} \left[a e^{-j(\omega+\omega_0)}\right]^n$$

$$= \frac{1}{2j}\left[\frac{1}{1-ae^{-j(\omega-\omega_0)}} - \frac{1}{1-ae^{-j(\omega+\omega_0)}}\right]$$

$$= \frac{a\sin\omega_0 e^{-j\omega}}{1-2a\cos\omega_0 e^{-j\omega}+a^2e^{-j2\omega}}$$

(e) $x(n) = |a|^n \sin\omega_0 n \quad |a|<1$

$$x(n) = |a|^n \sin\omega_0 n \quad |a|<1$$

$$\sum_{n=-\infty}^{\infty}|x(n)| = \sum_{n=-\infty}^{\infty}|a|^n|\sin\omega_0 n|$$

$$\omega_0 = \frac{\pi}{2} \qquad |\sin\omega_0 n| = 1$$

$$\sum_{n=-\infty}^{\infty}|a|^n = \sum_{n=-\infty}^{\infty}|x(n)| \to \infty$$

Fourier transform does not exist

(f) $x(n) = \begin{cases} 2-\left(\frac{1}{2}\right)n & |n|\le 4 \\ 0 & \text{elsewhere}\end{cases}$

$$x(n) = \begin{cases} 2-\left(\frac{1}{2}\right)^n & |n|\le 4 \\ 0 & \text{elsewhere}\end{cases}$$

$$X(\omega) = \sum_{n=-4}^{4} x(n)e^{-j\omega n}$$

$$= \sum_{n=-4}^{4}\left[2-\left(\frac{1}{2}\right)^n\right]e^{-j\omega n}$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}}$$

$$-\frac{1}{2}\left[-4e^{j4\omega}+4e^{-j4\omega}-3e^{j3\omega}+3e^{-j3\omega}-2e^{j2\omega}+2e^{-j2\omega}-e^{j\omega}+e^{-j\omega}\right]$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} + j\left[4\sin 4\omega + 3\sin 3\omega + 2\sin 2\omega + \sin\omega\right]$$

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(g) $x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= -2e^{-j2\omega} - e^{j\omega} + e^{j\omega} + 2e^{-j2\omega}$$

$$= -2j\left[2\sin 2\omega + \sin\omega\right]$$

(h) $x(n) = \begin{cases} A(2M+1-|n|) & |n| \le M \\ 0 & |n| > M \end{cases}$

$$X(\omega) = \sum_{n=-M}^{M} x(n) e^{-j\omega n}$$

$$= A\sum_{n=-M}^{M} (2M+1-|n|) e^{-j\omega n}$$

$$= (2M+1)A + A\sum_{k=1}^{M} (2M+1-k)\left(e^{j\omega k} + e^{j\omega k}\right)$$

$$= (2M+1)A + 2A\sum_{k=1}^{M} (2M+1-k)\cos\omega k$$

10 Determine the signals having the following fourier transforms.

(a) $X(\omega) = \begin{cases} 0 & 0 \le |\omega| \le \omega_0 \\ 1 & \omega_0 < |\omega| \le \pi \end{cases}$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_0} X(\omega) e^{j\omega n} + \int_{\omega_0}^{\pi} X(\omega) e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{-j\omega n}}{jn}\right]_{-\pi}^{-\omega_0} + \left[\frac{e^{j\omega n}}{-jn}\right]_{\omega_0}^{\pi}\right\}$$

$$= \frac{1}{2\pi}\left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{jn} + \frac{e^{j\pi n} - e^{j\omega_0 n}}{jn}\right]$$

$$= \frac{1}{2\pi}\left[2\cdot\frac{e^{-j\omega_0 n} - e^{j\omega_0 n}}{2jn} + 2\cdot\frac{e^{j\pi n} - e^{-j\pi n}}{2jn}\right]$$

2ω

$$= \frac{1}{2\pi}\left[\frac{2}{n} - \sin\omega_0 n + 2\cdot\frac{\sin\pi n}{2jn}\right]^{0} \quad \text{since } \sin\pi n = 0$$

$$= -\frac{\sin\omega_0 n}{n\pi} \;;\; n \neq 0$$

for $n=0$ from eq ①

$$= \frac{1}{2\pi}(\pi-\omega_0) + \frac{1}{2\pi}(\pi-\omega_0)$$

$$= \frac{(\pi-\omega_c)+(\pi-\omega_c)}{2\pi}$$

$$= \frac{2(\pi-\omega_c)}{2\pi} = \pi-\omega_0 \;; \text{when } n=0.$$

(b) $X(\omega) = \cos^2\omega$

$$= \left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)^2$$

$$= \frac{1}{4}\left[(e^{j\omega})^2 + (e^{-j\omega})^2 + 2e^{j\omega}e^{-j\omega}\right]$$

$$= \frac{1}{4}\left[e^{j2\omega} + e^{-j2\omega} + 2\right]$$

$$= \frac{1}{4}e^{j2\omega} + \frac{1}{2} + \frac{1}{4}e^{-j2\omega}$$

$\downarrow$ IFT

$$= \frac{1}{4}\delta(n+2) + \delta(n)\frac{1}{2} + \frac{1}{4}\delta(n-2).$$

(c) $X(\omega) = \begin{cases} 1 \;; & \omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\delta\omega}{2} \\ 0 \;; & \text{elsewhere} \end{cases}$

$$\omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\delta\omega}{2}$$

$$\omega_0 - \frac{\delta\omega}{2} \le -\omega \le \omega_0 + \frac{\delta\omega}{2}$$

$$-\omega_0 + \frac{\delta\omega}{2} \le \omega \le -\omega_0 - \frac{\delta\omega}{2}$$

consider limits $\omega_0 - \frac{\delta\omega}{2} \le \omega \le \omega_0 + \frac{\delta\omega}{2}$

$$= \frac{1}{2\pi}\int_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}} 1\cdot e^{j\omega n}\, d\omega$$

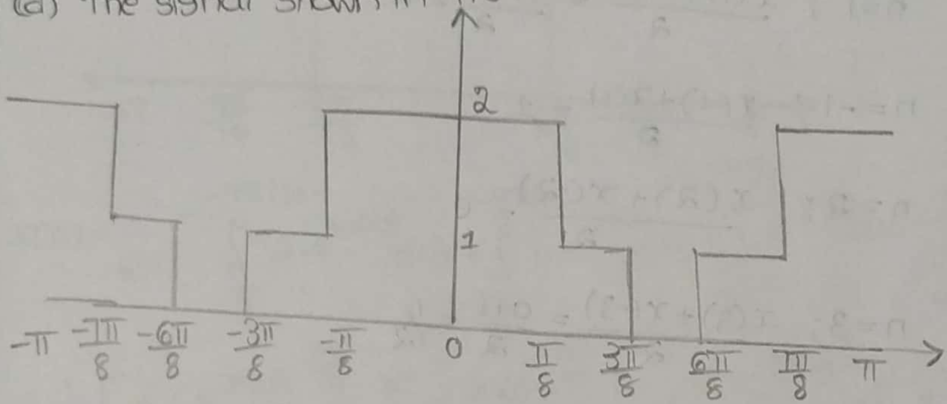
$$\frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}}$$

$$\frac{2}{\pi}\;\frac{e^{j(\omega_0+\frac{\delta\omega}{2})n}-e^{j(\omega_0-\frac{\delta\omega}{2})n}}{2jn}$$

$$\delta\omega\,\frac{2}{\pi}\left[\frac{\sin(\frac{\delta\omega}{2}n)}{\frac{n\delta\omega}{2}}\right]e^{jn\omega_0}.$$

$$\delta\omega\cdot\frac{2}{\pi}\,sa\left(\frac{\delta\omega}{2}n\right)e^{jn\omega_0}$$

(d) The signal shown in fig



$$\frac{1}{2\pi}\int_{-\pi}^{\pi}x(\omega)e^{j\omega n}\,d\omega$$

Let consider limits 0 to π

$$2\cdot\frac{1}{2\pi}\int_0^{\pi/8}2e^{j\omega n}d\omega+\int_{\pi/8}^{3\pi/8}e^{j\omega n}d\omega+\int_{6\pi/8}^{7\pi/8}e^{j\omega n}d\omega+\int_{7\pi/8}^{\pi}2e^{j\omega n}d\omega$$

$$=\frac{1}{\pi}\left[\int_0^{\pi/8}2\cos\omega n\,d\omega+\int_{\pi/8}^{3\pi/8}\cos\omega n\,d\omega+\int_{6\pi/8}^{7\pi/8}\cos\omega n\,d\omega+\int_{7\pi/8}^{\pi}2\cos\omega n\,d\omega\right.$$

$$\because\; j\sin\omega_n \Rightarrow \sin(1n\pi)=0$$

$$=\frac{1}{\pi}\left[(-2\sin\omega n\Big]_0^{\pi/8}+\Big[-\sin\omega n\Big]_{\pi/8}^{3\pi/8}+\Big[-\sin\omega n\Big]_{6\pi/8}^{7\pi/8}+\Big[-2\sin\omega n\Big]_{7\pi/8}^{\pi}\right.$$

$$=\frac{1}{\pi}\left[-2\sin\frac{\pi n}{8}-\sin\frac{3\pi}{8}n+\sin\frac{\pi}{8}n-\sin\frac{7\pi}{8}n+\sin\frac{6\pi n}{8}-2\cancel{\sin}^{0}\right.$$

$$\left.\pi n+2\sin\frac{7\pi}{8}n\right]$$

$= \frac{1}{\pi}\left[\sin\frac{7\pi}{8}n - \sin\frac{\pi n}{8} + \sin\frac{6\pi}{8}n - \sin\frac{3\pi}{8}n \right]$

11 consider the signal $x(n) = \{-1, 0, -1, \underset{\uparrow}{2}, 3\}$ with Fourier transform $X(\omega) = X_R(\omega) + jX_I(\omega)$. Determine and sketch the signal $y(n)$ with Fourier transform $Y(\omega) = X_I(\omega) = X_I(\omega) + X_R(\omega) e^{j2\omega}$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$n = 0$; $x_e(n) = \frac{x(0) + x(-0)}{2} = \frac{2+2}{2} = 2$

$n = 1$; $\frac{x(1) + x(-1)}{2} = \frac{3 + (-1)}{2} = 1$

$n = -1$; $\frac{x(-1) + x(1)}{2} = 1$

$n = 2$; $\frac{x(2) + x(-2)}{2} = 0$

$n = 3$; $\frac{x(3) + x(-3)}{2} = \frac{0+1}{2} = 1/2$

$n = -2$; $\frac{x(-2) + x(2)}{2} = 0$

$n = -3$; $\frac{x(3) + x(3)}{2} = \frac{1}{2}$

$$x_e(n) = \{\tfrac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \tfrac{1}{2}\}$$

$$x_o(n) = \{\tfrac{1}{2}, 0, -2, \underset{\substack{\uparrow \\ n=0}}{0}, 2, 0, \tfrac{1}{2}\}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$X_R(\omega) = \sum_{n=-3}^{3} x_e(n) e^{-jn\omega}$$

$$jX_I(\omega) = \sum_{n=-3}^{3} x_o(n) e^{-jn\omega}$$

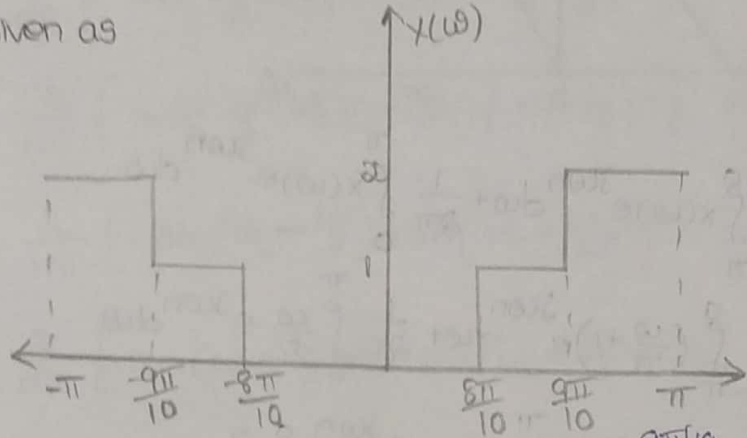
$$Y(\omega) = X_I(\omega) + X_R(\omega) e^{j2\omega}$$

$$= \frac{x_o(n)}{j} + x_e(n+2) \to \text{IFT of } Y(\omega)$$

$$= -j\,x_o(n) + x_e(n+2)$$

$$= \left\{ \frac{1}{2}, 0, 1-\frac{j}{2}, 2, 1+\frac{j}{2}, 0, \frac{1}{2} - j2, 0, \frac{j}{2} \right\}$$

12 Determine the signal $x(n)$ if its fourier transform is as given as



$$x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-9\pi/10} 2 \cdot e^{+jn\omega}\, d\omega + \int_{-9\pi/10}^{-8\pi/10} 1 \cdot e^{jn\omega}\, d\omega + \int_{8\pi/10}^{9\pi/10} e^{jn\omega}\, d\omega + \frac{1}{2}\int_{9\pi/10}^{\pi} 2\, e^{jn\omega}\, d\omega \right]$$

$$= \frac{1}{2\pi}\left[2\left(\frac{e^{jn\omega}}{jn}\right)_{-\pi}^{-\frac{9\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{8\pi/10}^{9\pi/10} + 2\left[\frac{e^{jn\omega}}{jn}\right]_{9\pi/10}^{\pi} \right]$$

$$= \frac{1}{2\pi jn}\left[2\left[e^{-jn\frac{9\pi}{10}} - e^{-jn\pi}\right] + e^{jn-\frac{8\pi}{10}} - e^{-jn\frac{9\pi}{10}} + e^{jn\frac{9\pi}{10}} - e^{jn\frac{8\pi}{10}} + 2e^{jn\pi} - 2e^{jn\frac{9\pi}{10}} \right.$$

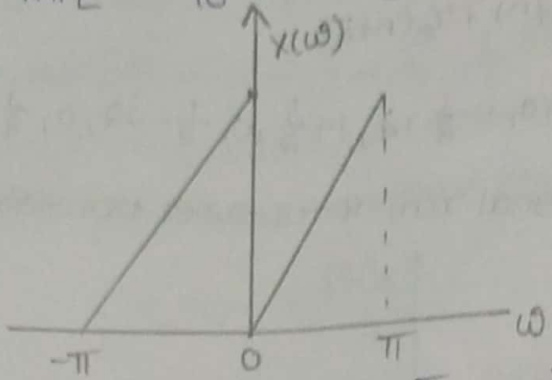
$$= \frac{1}{2\pi jn}\left[2e^{-jn\frac{9\pi}{10}} - 2e^{-jn\pi} + e^{-jn\frac{8\pi}{10}} - e^{-jn\frac{9\pi}{10}} + e^{jn\frac{9\pi}{10}} - e^{-jn\frac{8\pi}{10}} + 2e^{jn\pi} - e^{j\frac{9\pi}{10}} \right]$$

$$= \frac{1}{2\pi jn}\left[e^{-jn\frac{9\pi}{10}} - 2e^{-jn\pi} + 2e^{jn\pi} + e^{-jn\frac{8\pi}{10}} - e^{jn\frac{8\pi}{10}} + e^{jn\frac{9\pi}{10}} + 2e^{jn\frac{9\pi}{10}} \right]$$

$$= \frac{1}{2\pi jn}\left[e^{-jn\frac{9\pi}{10}} - e^{jn\frac{9\pi}{10}} - 2e^{-jn\pi} + 2e^{jn\pi} + e^{-jn\frac{8\pi}{10}} - e^{jn\frac{8\pi}{10}} \right]$$

$$= \frac{+1}{n\pi}\left[-\sin\left(\frac{9\pi n}{10}\right) - \sin\frac{8n\pi}{10} + \sin n\pi \right]$$

$$= \frac{-1}{n\pi}\left[\sin\frac{9\pi n}{10} + \sin\frac{4n\pi}{5}\right]$$

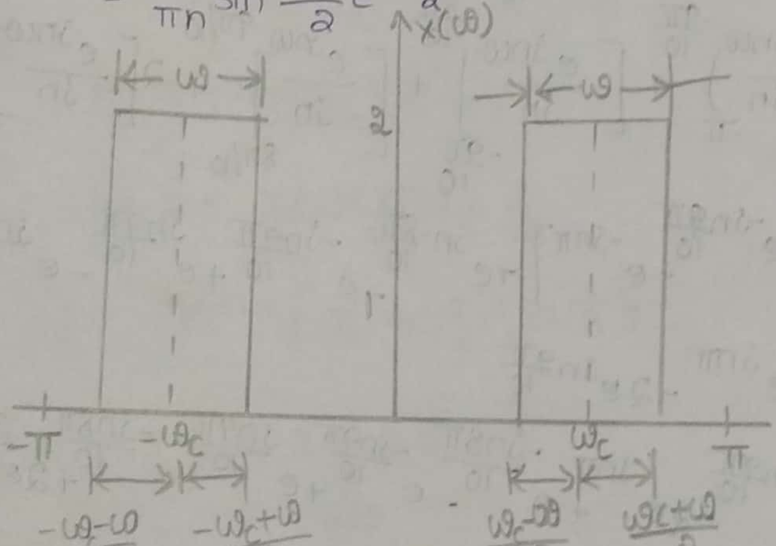


$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} X(\omega)e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{0}^{\pi} X(\omega)e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right)e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{0}^{\pi}\frac{\omega}{\pi}e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\left[\frac{\omega}{jn\pi}e^{j\omega n}\right]_{-\pi}^{\pi} + \left(\frac{e^{j\omega n}}{jn}\right)_{-\pi}^{0}\Bigg]$$

$$= \frac{1}{\pi n}\sin\frac{\pi n}{2}e^{-\frac{jn\pi}{2}}$$



$$x(n) = \frac{1}{2\pi}\left[\int_{\frac{-\omega_c-\omega}{2}}^{\frac{-\omega_c+\omega}{2}} 2\cdot e^{jn\omega}\cdot d\omega + \int_{\frac{\omega_c-\omega}{2}}^{\frac{\omega_c+\omega}{2}} 2e^{jn\omega}\,d\omega\right]$$

$$= \frac{1}{\pi}\left[\left[\frac{e^{jn\omega}}{jn}\right]_{\frac{-\omega_c-\omega}{2}}^{\frac{-\omega_c+\omega}{2}} + \left[\frac{e^{jn\omega}}{jn}\right]_{\frac{\omega_c-\omega}{2}}^{\frac{-\omega_c+\omega}{2}}\right]$$

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$$= \frac{1}{jn\pi}\left[e^{jn(-\omega_c+\frac{\omega}{2})} - e^{jn(-\omega_c-\frac{\omega}{2})} + e^{jn(\omega_c+\frac{\omega}{2})} - e^{jn(\omega_c-\frac{\omega}{2})} \right]$$

$$= \frac{1}{jn\pi}\left[e^{jn(-\omega_c+\frac{\omega}{2})} - e^{jn(-\omega_c-\frac{\omega}{2})} + e^{-jn(-\omega_c-\frac{\omega}{2})} - e^{-jn(\frac{\omega}{2}-\omega_c)} \right]$$

$$= \frac{2}{n\pi}\left[\sin(\tfrac{\omega}{2}-\omega_c)n - \sin(-\omega_c-\tfrac{\omega}{2})n \right]$$

$$= \frac{2}{n\pi}\left[\sin(\tfrac{\omega}{2}-\omega_c)n - \sin(-\omega_c-\tfrac{\omega}{2})n \right]$$

$$= \frac{2}{n\pi}\left[-\sin(\omega_c-\tfrac{\omega}{2})+\sin(\omega_c+\tfrac{\omega}{2})n \right]$$

$$= \frac{2}{n\pi}\left[\sin(\omega_c+\tfrac{\omega}{2})n - \sin(\omega_c-\tfrac{\omega}{2})n \right]$$

13 Given the Fourier transform of the signal

$x(n) = \begin{cases} 1 ; & -m \le n \le m \\ 0 ; & \text{otherwise} \end{cases}$ was shown to be $X(\omega) = 1 + 2\sum_{n=1}^{M} \cos \omega n$ then show that the fourier transform of

$x_1(n) = \begin{cases} 1 ; & 0 \le n \le m \\ 0 ; & \text{otherwise} \end{cases}$ is $X_1(\omega) = \frac{1-e^{-j\omega(m+1)}}{1-e^{-j\omega}}$

$x_2(n) = \begin{cases} 1 ; & -m \le n \le -1 \\ 0 ; & \text{otherwise} \end{cases}$ is $X_2(\omega) = \frac{e^{j\omega} - e^{j\omega(n+1)}}{1-e^{j\omega}}$

$$X_1(\omega) = \sum_{n=0}^{m} 1 \cdot e^{-j\omega n}$$

$$1 + e^{-j\omega} + e^{-j\omega 2} + e^{-j\omega 3} + \dots - \left[e^{-j\omega(m+1)} + e^{-j\omega(m+2)} \right.$$

$$\frac{1}{1-e^{-j\omega}} - \frac{e^{-j\omega(m+1)}}{1-e^{-j\omega}} \qquad \frac{e^{j\omega(m+1)}}{1-e^{-j\omega}}$$

$$X_2(\omega) = \sum_{n=-m}^{-1} e^{-j\omega n} = \sum_{n=1}^{m} e^{j\omega n} = \frac{1-e^{j\omega m}}{1-e^{j\omega}} e^{j\omega}$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$= \frac{1-e^{-j\omega(m+1)}}{1-e^{-j\omega}} + \frac{1-e^{-j\omega m}}{1-e^{j\omega}} e^{j\omega}$$

$$= \frac{1+e^{j\omega}-e^{j\omega}-1-e^{-j\omega(m+1)} \cdot e^{j\omega(m+1)} + e^{j\omega m} + e^{-j\omega m}}{2-e^{-j\omega}-e^{j\omega}}$$

$$= \frac{2\cos\omega m - 2\cos\omega(m+1)}{2-2\cos\omega}$$

$$= \frac{2\sin(\omega m+\frac{\omega}{2})\cos\frac{\omega}{2}}{2\sin^2\frac{\omega}{2}}$$

$$= \frac{\sin(m+\frac{1}{2})\omega}{\sin(\frac{\omega}{2})}$$

$$\therefore 1+2\sum_{n=1}^{M}\cos\omega n = \frac{\sin(m+\frac{1}{2})\omega}{\sin(\frac{\omega}{2})}$$

14 consider the signal $x(n) = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$ with fourier transform $X(\omega)$ compute the following quantities, without explicitly computing $X(\omega)$:

(a) $X(0)$

$$X(\omega)\Big|_{\omega=0} = \sum_{\omega=0} x(n)e^{-j\omega n} \Rightarrow X(0) = -3 \cancel{\varepsilon} = -3$$

(b) $\angle X(\omega) = \pi$ for all 'ω'

(c) $\int_{-\pi}^{\pi} X(\omega)d\omega$ $\quad x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)d\omega$

$$\int_{-\pi}^{\pi} X(\omega)d\omega = 2\pi x(\omega)/\omega=0$$

$$= 2\pi(-3)$$

$$= -6\pi.$$

(d) $x(\pi)$

$$x(\pi) = \sum_{n=-\infty}^{\infty} x(n)e^{-jn\omega}$$

$= \sum_n e^{-jn\pi} \cdot x(n)$

$= \sum_n [\cos(n\pi) - j\sin(n\pi)]x(n)$

$= \sum_n (-1)^n x(n)$

for $n=0$ $\quad (-1)^0 x(0) \Rightarrow 1 \cdot -3 = 3$

$n=1 \quad (-1)^1 x(1) \Rightarrow -1 \cdot 2 = -2$

$n=2 \quad (-1)^2 x(2) \Rightarrow -1 = -1$

$n=-1 \quad (-1)^{-1} x(-1) \Rightarrow -2$

$n=-2 \quad (-1)^{-2} x(-2) \Rightarrow -1$

$\Rightarrow -3-2-1-2-1 \Rightarrow -3-4-2 \Rightarrow -9$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = \sum_n |x(n)|^2$$

$= (-1)^2 + (2)^2 + (-3)^2 + (2)^2 + (-1)^2$

$= 1+4+9+4+1$

$= 19$

$$\int_{-\pi}^{\pi} |X(\omega)|^2 = 19 \times 2\pi = 38\pi$$

15 The center of gravity of a signal $x(n)$ is defined as

$$c = \frac{\sum_{n=-\infty}^{\infty} n x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$ and provides a measure of the "time delay" of the signal.

(a) Express c in terms of $X(\omega)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x(n) e^{0}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x(n)$$

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$$n x(n) \overset{FT}{\longleftrightarrow} j\frac{d}{d\omega}X(\omega)$$

$$-jn\,x(n) \overset{FT}{\longleftrightarrow} \frac{d}{d\omega}X(\omega)$$

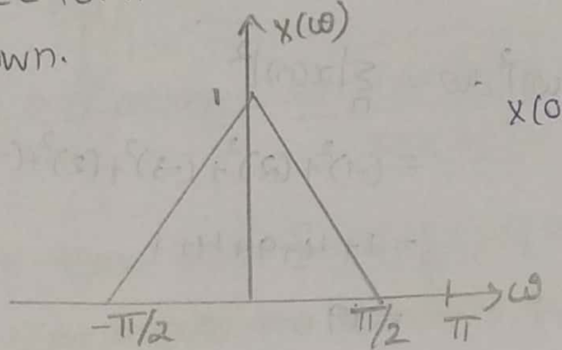
$$\frac{d}{d\omega}X(\omega) = \sum_{n=-\infty}^{\infty} -jnx(n)e^{-jn\omega}\,d\omega$$

$$= -j\sum_{n=-\infty}^{\infty} nx(n)e^{-jn\omega}\,d\omega$$

$$j\frac{d}{d\omega}X(\omega) = \sum_{n=-\infty}^{\infty} nx(n)e^{-jn\omega}\,d\omega$$

$$c = \frac{j\frac{d}{d\omega}X(\omega)}{X(\omega)}\Big|_{\omega=0}$$

(c) compute c for the signal $x(n)$ whose fourier transform is shown.

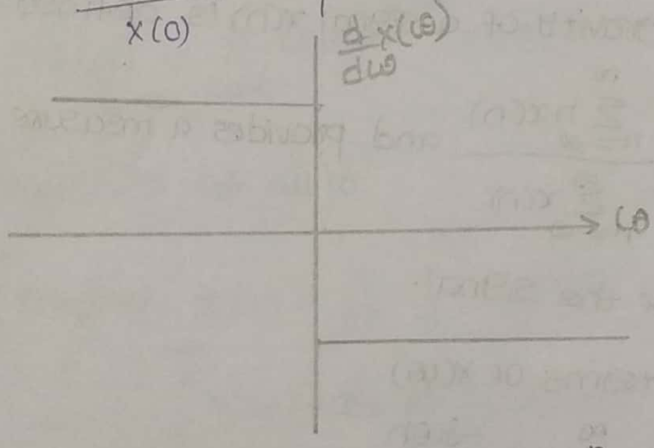


$X(0) = 1$

$A\left(1 - \frac{|t|}{T}\right)$

$1\left(1 - \frac{|t|}{\pi/2}\right)$

$$c = \frac{j\frac{d}{d\omega}X(0)}{X(0)} = \frac{0}{1} = 0$$



16 consider the fourier transform pair $a^n u(n) \overset{FT}{\longleftrightarrow} \frac{1}{1-ae^{-j\omega}}$ and $|a|<1$ use the differentiation in frequency theorem and introduction show that $x(n) = \frac{(n+l-1)!}{n!(l-1)!}a^n u(n) \overset{F}{\longleftrightarrow} X(\omega)$

$$= \frac{1}{(1-ae^{-j\omega})^{l}}$$

Let $l = k+1$

$$x(n) = \frac{(n+k+1-1)!}{n!(k+1-1)!} a^n u(n)$$

$$= \frac{(n+k)!}{n!k!} a^n u(n)$$

$$= \frac{(n+k)(n+k-1)!}{k\,n!\,(k-1)!} a^n u(n)$$

Let $x_k(n) = \frac{(n+k-1)!}{n!(k-1)!} a^n u(n)$ $x_{k+1}(n) = \frac{n+k}{k} x_k(n)$

$$x_{k+1}(\omega) = \sum_{n=-\infty}^{\infty} \frac{n+k}{k} x_k(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{n}{k} x_k(n) + x_k(n)\right] e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\infty}^{\infty} n x_k(n) e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x_k(n) e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\infty}^{\infty} n x_k(n) e^{-j\omega n} + X_k(\omega)$$

$$= \frac{1}{k} \cdot j \frac{d}{d\omega} X_k(\omega) + X_k(\omega)$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^{k+1}} + \frac{1}{(1-ae^{-j\omega})^{k}}$$

17 Let $x(n)$ be a arbitary signal, not necessarily real valued with FT $X(\omega)$ Express the fourier transform of the following signals in terms of $X(\omega)$

(a) $x^*(n)$

$$\sum_{n=-\infty}^{\infty} x^*(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty}\left[x(n)e^{-j\omega n}\right]^*$$

$$X(-\omega)^*$$

(b) $x^*(-n)$

$$\sum_{n=-\infty}^{\infty} x^*(-n)e^{-j\omega n}$$

replace $-n$ with n

$$\sum_{n=-\infty}^{\infty} x^*(n)e^{j\omega n}$$

$$\sum_{n=-\infty}^{\infty}\left(x(n)e^{-j\omega n}\right)^*$$

$$X^*(\omega)$$

(c) $y(n)=x(n)-x(n-1)$

$$\sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x(n-1)e^{-j\omega n}$$

$$X(\omega) - \sum_{n=-\infty}^{\infty} x(n-1)e^{-j\omega n}$$

$l = n-1$

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l)e^{-j\omega(l+1)}$$

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l)e^{-j\omega l}\cdot e^{-j\omega}$$

$$= X(\omega) - e^{-j\omega}\sum_{l=-\infty}^{\infty} x(l)e^{-j\omega l}$$

Replace l by n

$$= X(\omega) - e^{-j\omega}\sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$= X(\omega) - e^{-j\omega}X(\omega)$$

$$= X(\omega)\left[1-e^{-j\omega}\right]$$

(d) $y(n) = \sum_{k=-\infty}^{n} x(k)$

$$= y(n) - y(n-1)$$

$$= x(n)$$

$$X(\omega) = Y(\omega)\left[1-e^{-j\omega}\right]$$

$$Y(\omega) = \frac{X(\omega)}{1-e^{-j\omega}}$$

(e) $y(n) = x(2n)$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} x(2n)e^{-jn\omega}$$

let $l = 2n$

$$= \sum_{l=-\infty}^{\infty} x(l)e^{-j\frac{l}{2}\omega}$$

$$= \sum_{l=-\infty}^{\infty} x(l)e^{-jl\frac{\omega}{2}}$$

$$= X\left(\frac{\omega}{2}\right)$$

(f) $y(n) = \begin{cases} x\left(\frac{n}{2}\right) & \text{'n' even} \\ 0 & \text{'n' odd} \end{cases}$

$$Y(\omega) = \sum_{n} x\left(\frac{n}{2}\right)e^{jn\omega}$$

Let $n = 2l$

$$= \sum_{n} x\left(\frac{2l}{l}\right)e^{jn\omega}$$

$$= \sum_{n} x(2)e^{jn\omega}$$

$$= X(2\omega)$$

18 Determine and sketch the fourier transforms $X_1(\omega), X_2(\omega)$ and $X_3(\omega)$ of the following signals?

(a) $x_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$$X_1(f) = \sum_{n=-\infty}^{\infty} x_1(n) e^{-j\omega n} = \sum_{n=-2}^{2} x_1(n) e^{-j\omega n}$$

$n=-2$; $(1)e^{j2\omega} = e^{j2\omega}$

$n=-1$; $(1)e^{j\omega} = e^{j\omega}$

$n=0$; $(1)e^{0} = 1$

$n=1$; $(1)e^{-j\omega} = e^{-j\omega}$

$n=2$; $(1)e^{-2j\omega} = e^{-2j\omega}$

$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$

$= 2\cos(2\omega) + 2\cos(\omega) + 1$

(b) $x_2(n) = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

$$X_2(f) = \sum_{n=-\infty}^{\infty} x_2(n) e^{-j\omega n} = \sum_{n=-4}^{4} x_2(n) e^{-j\omega n}$$

$n=-4$; $(1)e^{j4\omega} = e^{j4\omega}$

$n=-3$; $(0)e^{j3\omega} = 0$

$n=-2$; $(1)e^{j2\omega} = e^{j2\omega}$

$n=-1$; $(0)e^{j\omega} = e^{j\omega}(0) = 0$

$n=0$; $(1)e^{0} = (1)(1) = 1$

$n=1$; $(0)e^{-j\omega} = 0$

$n=2$; $(1)e^{-j2\omega} = e^{-j2\omega}$

$n=3$; $(0)e^{-j3\omega} = 0$

$n=4$; $(1)e^{-j4\omega} = e^{-j4\omega}$

$= e^{j4\omega} + e^{j2\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$

$= 2\cos(2\omega) + 2\cos(4\omega) + 1$

(c) $x_3(n) = \{1, 0, 0, 1, 0, 0, \underset{\uparrow}{1}, 0, 0, 1, 0, 0, 1\}$

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$$X_3(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-6}^{6} x(n) e^{-j\omega n}$$

$n=-6$; $1 \times e^{j6\omega} = e^{j6\omega}$

$n=-3$; $1 \times e^{j3\omega} = e^{j3\omega}$

$n=0$; $1 \times e^{j0} = 1$

$n=3$; $1 \times e^{-j3\omega} = e^{-j3\omega}$

$n=6$; $1 \times e^{-j6\omega} = e^{-j6\omega}$

$= e^{j6\omega} + e^{-j6\omega} + e^{j3\omega} + e^{-j3\omega} + 1$

$= 2\cos(6\omega) + 2\cos(3\omega) + 1$

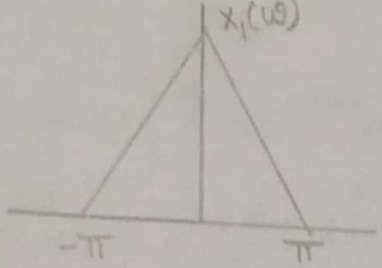
(d) Find the relationship between $X_1(\omega)$, $X_2(\omega)$ and $X_3(\omega)$? What is its physical meaning.

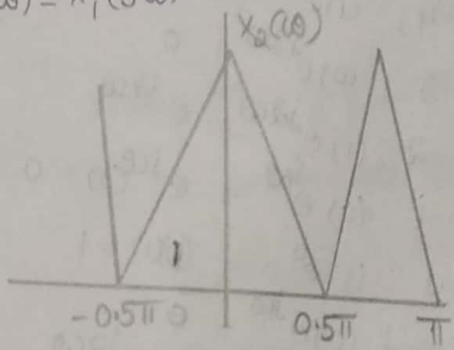
$X_1(\omega) = 2\cos(2\omega) + 2\cos(\omega) + 1$

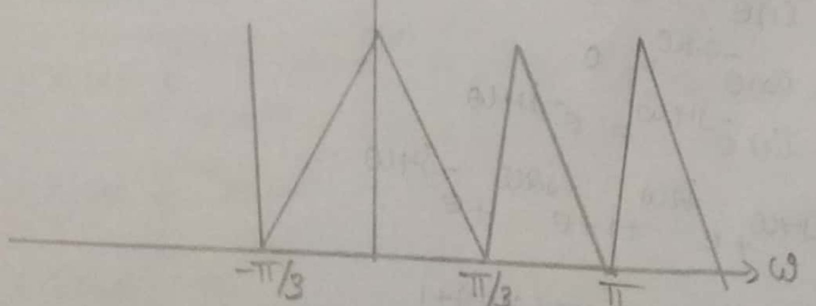
$X_2(\omega) = 2\cos(2\omega) + 2\cos(4\omega) + 1$

$X_3(\omega) = 2\cos(6\omega) + 2\cos(3\omega) + 1$

$X_2(\omega) = X_1(2\omega)$; $X_3(\omega) = X_1(3\omega)$

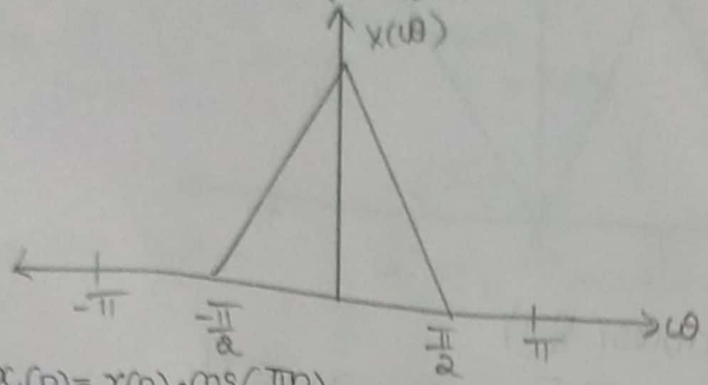






19 Let $x(n)$ be a signal with Fourier transform as shown. Determine and sketch the fourier transforms of the following signals. Note that these signal sequences

are obtained by amplitude modulation of a carrier $\cos\omega_c n$ and $\sin\omega_c n$ by the sequence $x(n)$.



(a) $x_1(n) = x(n) \cdot \cos\left(\frac{\pi n}{4}\right)$

$$x(n)\cos(\omega_0 n) \xleftrightarrow{F.T} \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

$$X_1(\omega) = \frac{1}{2}\left[X\left(\omega+\frac{\pi}{4}\right) + X\left(\omega-\frac{\pi}{4}\right)\right]$$

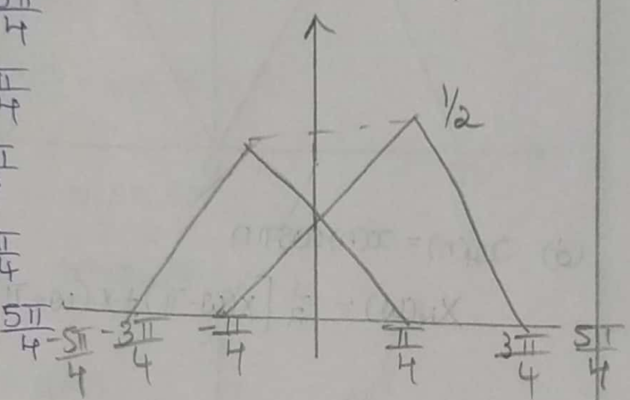
limits: $-\pi + \frac{\pi}{4} = \frac{-3\pi}{4}$

$\frac{-\pi}{2} + \frac{\pi}{4} = \frac{-\pi}{4}$

$\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$0 + \frac{\pi}{4} = \frac{\pi}{4}$

$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$



$-\pi - \frac{\pi}{4} = \frac{-5\pi}{4}$

$-\frac{\pi}{2} - \frac{\pi}{4} = \frac{-3\pi}{4}$

$0 - \frac{\pi}{4} = -\pi/4$

$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

(b) $x_2(n) = x(n)\sin(\pi n/2)$

$$x(n)\sin(n\pi/2) = \frac{1}{2j}\left[X(\omega-\omega_c) + X(\omega+\omega_c)\right]$$

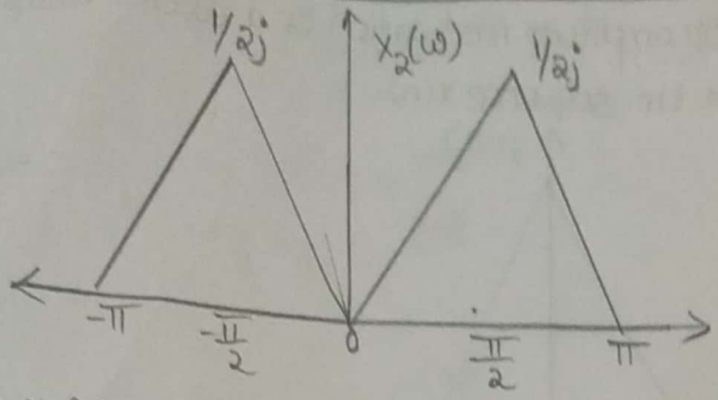
$$X_2(\omega) = \frac{1}{2j}\left[X\left(\omega-\frac{\pi}{2}\right) + X\left(\omega+\frac{\pi}{2}\right)\right]$$

limits:-

$-\pi - \frac{\pi}{2} \Rightarrow \frac{-3\pi}{2}$ $\quad$ $\frac{\pi}{2} - \frac{\pi}{2} = 0$ $\quad$ $-\pi + \frac{\pi}{2} = \frac{-\pi}{2}$ $\quad$ $\frac{\pi}{2} + \frac{\pi}{2} = \pi$

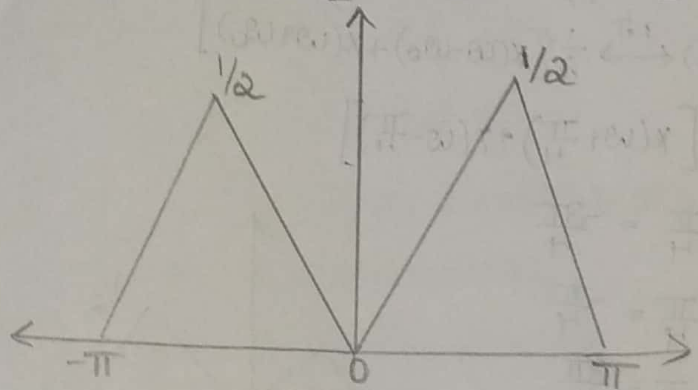
$-\frac{\pi}{2} - \frac{\pi}{2} \Rightarrow -\pi$ $\quad$ $\pi - \frac{\pi}{2} = \frac{\pi}{2}$ $\quad$ $-\frac{\pi}{2} + \frac{\pi}{2} = 0$ $\quad$ $\pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$-\frac{\pi}{2} + 0 \Rightarrow -\frac{\pi}{2}$ $\quad$ $0 + \frac{\pi}{2} = \frac{\pi}{2}$



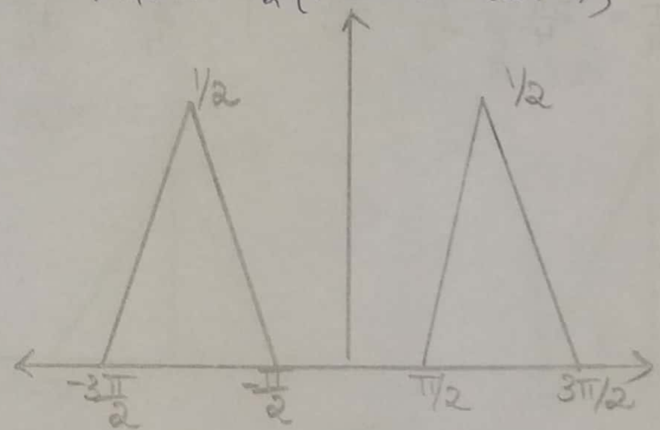
(c) $x_3(n) = x(n)\cos(\frac{n}{2}\pi)$

$$X_3(\omega) = \frac{1}{2}\left[X(\omega-\frac{\pi}{2}) + X(\omega+\frac{\pi}{2})\right]$$



(d) $x_4(n) = x(n)\cos\pi n$

$$X_4(\omega) = \frac{1}{2}\left[X(\omega-\pi) + X(\omega+\pi)\right]$$



20. Consider an aperiodic signal $x(n)$ with FT $X(\omega)$. Show that the fourier series coefficients C_k^y of the periodic signal $y(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$ are given by $C_k^y = \frac{1}{N}X\left[\left(\frac{2\pi}{N}\right)k\right]$

$k = 0, 1, \cdots N-1$

$$C_k^y = \frac{1}{N}\sum_{n=0}^{N-1} y(n)\, e^{-j2\pi kn/N}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{k=-\infty}^{\infty} x(n-lN)\, e^{\frac{-j2\pi kn}{N}}\right.$$

$$\frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-l-lN} x(m)e^{-j2\pi K(m+lN)/N}$$

But $\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-l-lN} x(m)e^{-j\omega(m+N)} = X(\omega)$

$$\therefore C_K^{y} = \frac{1}{N}X\left(\frac{2\pi K}{N}\right)$$

2) prove that $X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin\omega_c n}{\pi n}e^{-j\omega n}$ may be expressed as

$$X_N(\omega) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} \frac{\sin[(2N+1)(\omega-\theta/2)]}{\sin[(\omega-\theta)/2]}\,d\theta$$

Let $x_N(n) = \frac{\sin\omega_c n}{\pi n} \quad -N \le n \le N$

$= x(n)\omega(n)$

$x(n) = \frac{\sin\omega_c n}{\pi n} \quad -\infty \le n \le \infty$

$\omega(n) = 1 \;;\; -N \le n \le N$

$= 0 \;;$ otherwise

$\frac{\sin\omega_c n}{\pi n} \xleftrightarrow{FT} X(\omega)$

$= 1 \;;\; |\omega| \le \omega_c$

$0 \;;$ otherwise

$X_N(\omega) = X(\omega) * W(\omega)$

$= \int_{-\pi}^{\pi} X(\theta)W(\omega-\theta)\,d\theta$

$= \int_{-\omega_c}^{\omega_c} \frac{\sin(2N+1)(\omega-\theta)/2}{\sin(\omega-\theta)/2}\,d\theta$

22 A signal $x(n)$ has the following fourier transform

$$X(\omega) = \frac{1}{1-ae^{-j\omega}}$$

Determine the fourier transform of the following signals.

(a) $x(2n+1)$

$$\sum_{n=-\infty}^{\infty} x(2n+1) e^{-j\omega n} \quad \text{let } 2n+1 = l$$

$$\sum_{l=-\infty}^{\infty} x(l) e^{-j\omega\left[\frac{l-1}{2}\right]}$$

$$\sum_{l=-\infty}^{\infty} x(l) e^{-\frac{j\omega l}{2}} \cdot e^{\frac{j\omega}{2}}$$

$$e^{\frac{j\omega}{2}} \sum_{l=-\infty}^{\infty} x(l) e^{-\frac{j\omega l}{2}}$$

$$e^{\frac{j\omega}{2}} \sum_{l=-\infty}^{\infty} x(l) e^{-\frac{j\omega l}{2}}$$

$$e^{j\omega/2} \sum_{l=-\infty}^{\infty} x(n) e^{-\frac{j\omega}{2} n} \quad l \text{ by } 'n'$$

$$= e^{j\omega/2} X\left(\frac{\omega}{2}\right)$$

$$= e^{j\omega/2} \frac{1}{1 - a e^{-j(\omega/2)}}$$

$$= \frac{e^{j\omega/2}}{1 - a e^{-j(\omega/2)}}$$

(b) $e^{\pi j/2} x(n+2)$

$$e^{j2\omega} X\left(\omega - \frac{\pi j}{2}\right)$$

$$x(n) \longleftrightarrow X(\omega)$$

$$x(n+2) \longleftrightarrow e^{j2\omega} X(\omega)$$

$$e^{\pi j/2} x(n+2) \longleftrightarrow e^{j2\omega} X\left(\omega - \frac{\pi j}{2}\right)$$

$$e^{j2\omega} \cdot X\left(\omega - \frac{\pi j}{2}\right)$$

(c) $x(-2n)$

$$x(n) \longleftrightarrow X(\omega)$$

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$$x(2n) = X(\omega/2)$$

$$x(-2n) = X(-\omega/2)$$

(d) $x(n)\cos(0.3\pi n)$

$$x(n)\cos(0.3\pi n) \leftrightarrow \frac{1}{2}\left[X(\omega+0.3\pi) + X(\omega-0.3\pi)\right]$$

$$x(n)\cos\omega_c n \leftrightarrow \frac{1}{2}\left[X(\omega+\omega_c) + X(\omega-\omega_c)\right]$$

(e) $x(n) * x(n-1)$

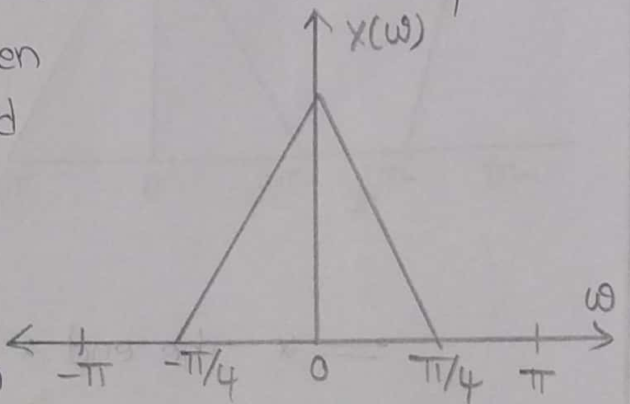
$\cancel{X(-\omega) * X^*(-\omega)}$ $X(\omega) \cdot X^*(\omega)$

$$\frac{1}{1-ae^{-j\omega}} \cdot \frac{1}{1-ae^{j\omega}}$$

$$\frac{1}{1-ae^{j\omega}-ae^{-j\omega}+a^2} \Rightarrow \frac{1}{1+a^2-2\cos\omega}$$

23 From a discrete time signal $x(n)$ with fourier transform $X(\omega)$ shown in figure determine and sketch the fourier transform of the following signals.

Note that $y_1(n) = x(n)s(n)$ where $s(n) = \{\ldots 0,1,0,\underset{\uparrow}{1},0,1,0,1\}$



(a) $y_1(n) = \begin{cases} x(n) & \text{'n' even} \\ 0 & \text{'n' odd} \end{cases}$

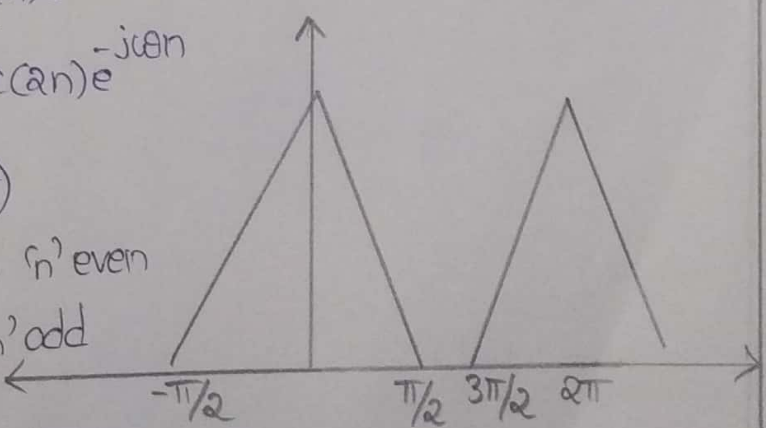
(b) $y_2(n) = x(2n)$

$$y_2(n) = x(2n)$$

$$Y_2(\omega) = \sum_n y_2(n)e^{-j\omega n}$$

$$= \sum_n x(2n)e^{-j\omega n}$$

$$= X\left(\frac{\omega}{2}\right)$$



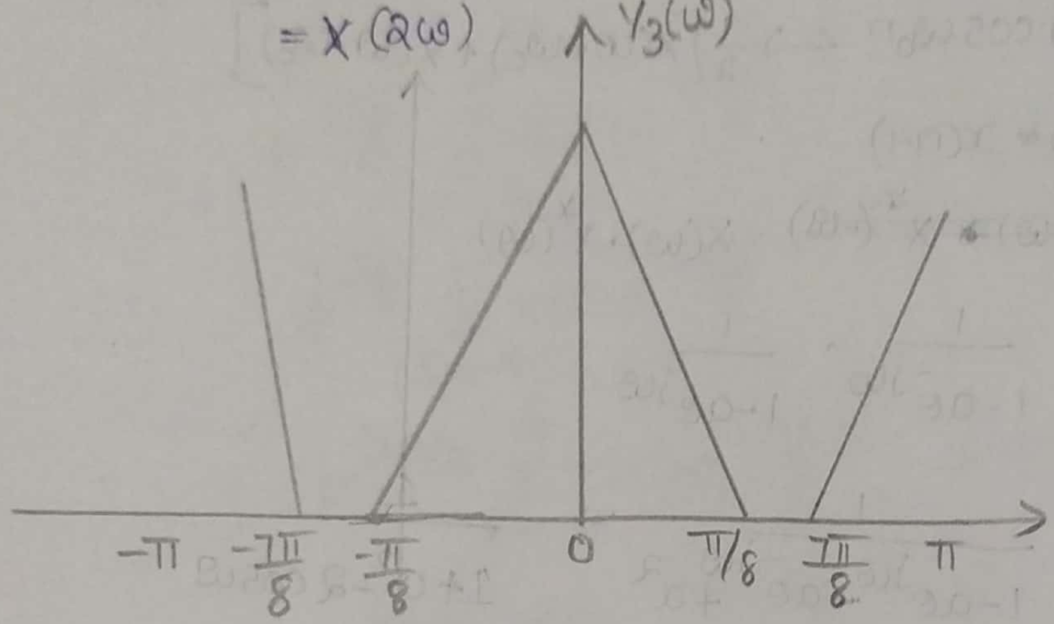
(c) $y_3(n) = \begin{cases} x(n/2) & \text{'n' even} \\ 0 & \text{'n' odd} \end{cases}$

$$Y_3(\omega) = \sum_n y_3(n) e^{-j\omega n}$$

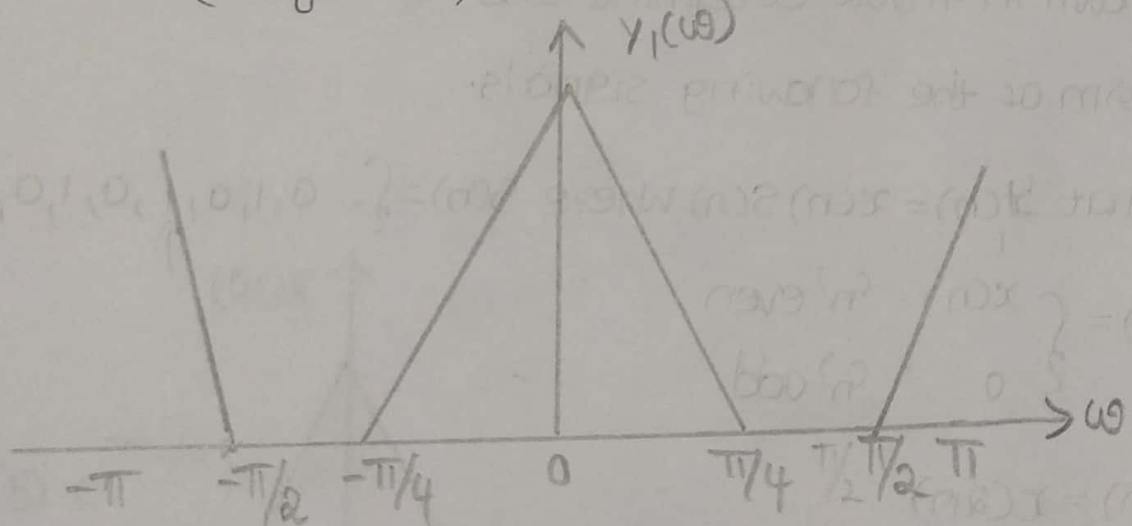
$$= \sum_{n\,\text{even}} x\left(\tfrac{n}{2}\right) e^{-j\omega n}$$

$$= \sum_m x(m) e^{-j2\omega m}$$

$$= X(2\omega)$$



$$y_1(n) = \begin{cases} y_2(n/2) & ; \text{'n' even} \\ 0 & ; \text{'n' odd} \end{cases}$$



== * the end * ==