1. A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1+\frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

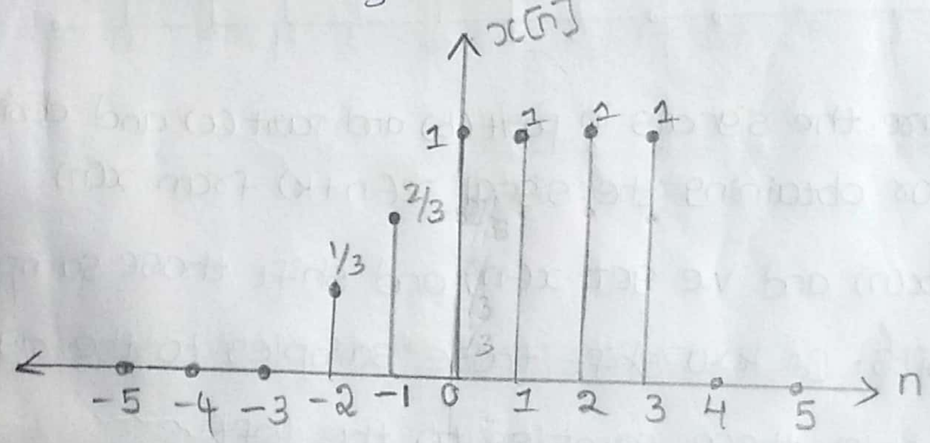
(a) Determine its values and sketch the signal $x(n)$.

$-3 \le n \le -1$ $1+\frac{n}{3}$ $0 \le n \le 3 = 1$

$n = -1$ $1-\frac{1}{3} = \frac{2}{3}$ 0, elsewhere

$n = -2$ $1-\frac{2}{3} = \frac{1}{3}$

$n = -3$ $1-\frac{3}{3} = 0$



(b) Sketch the signals that result if we

(i) First fold $x(n)$ and then delay the resulting signal by four samples.

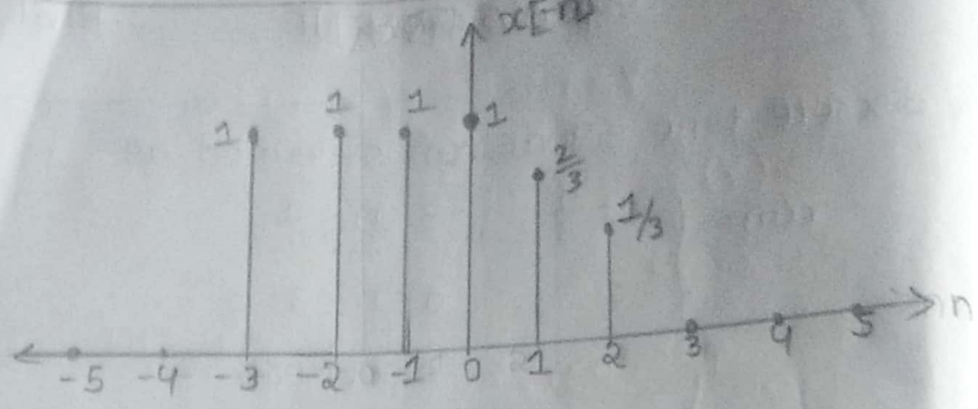
By above picture of signal we have

$$x[n] = \{\ldots 0, \tfrac{1}{3}, \tfrac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots\}$$

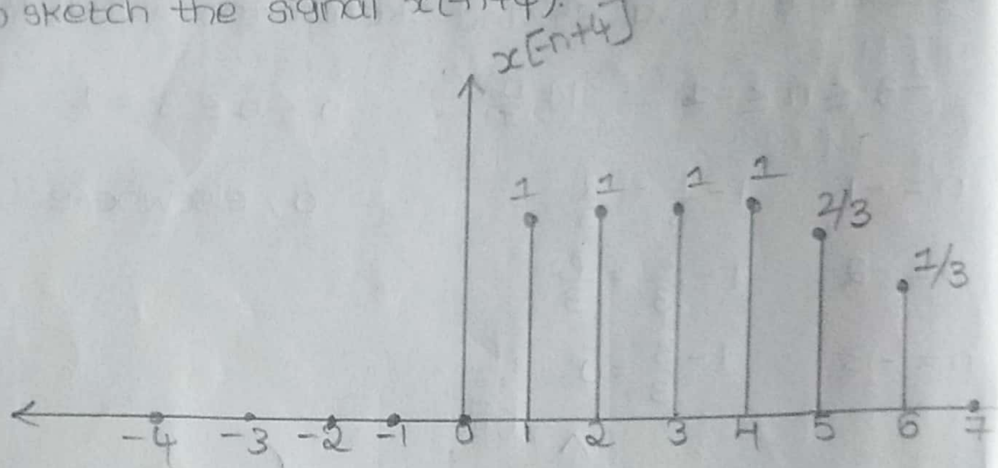
folding $x[-n] = \{\ldots 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0\}$

and then delay the resulting signal by four samples

$$x[-n+4] = \{\ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \tfrac{2}{3}, \tfrac{1}{3}, 0\}$$



(c) sketch the signal $x(-n+4)$.



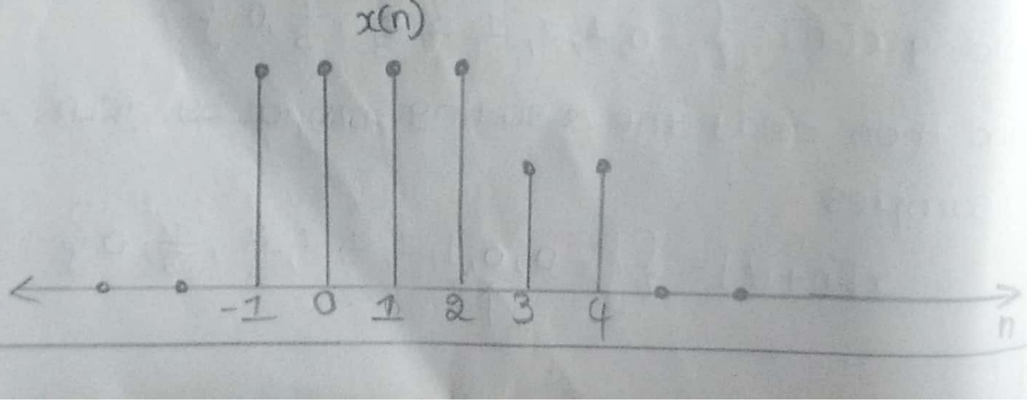
(d) compare the signals in part (b) and part (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

we fold $x(n)$ and we get $x(-n)$ and shift those samples by k units. If $k>0$ shift those samples to the right. If $k<0$ shift those samples to the left.

(e) can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$

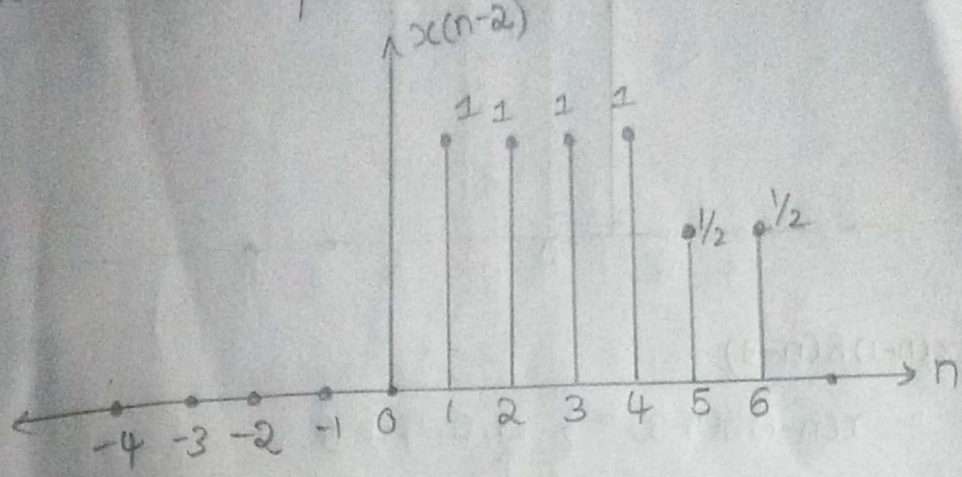
$$x[n] = \frac{1}{3}\delta(n+2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4).$$

(2) A discrete-time signal $x(n)$ is shown in Fig. Sketch and label carefully each of the following signals.



(a) $x(n-2)$

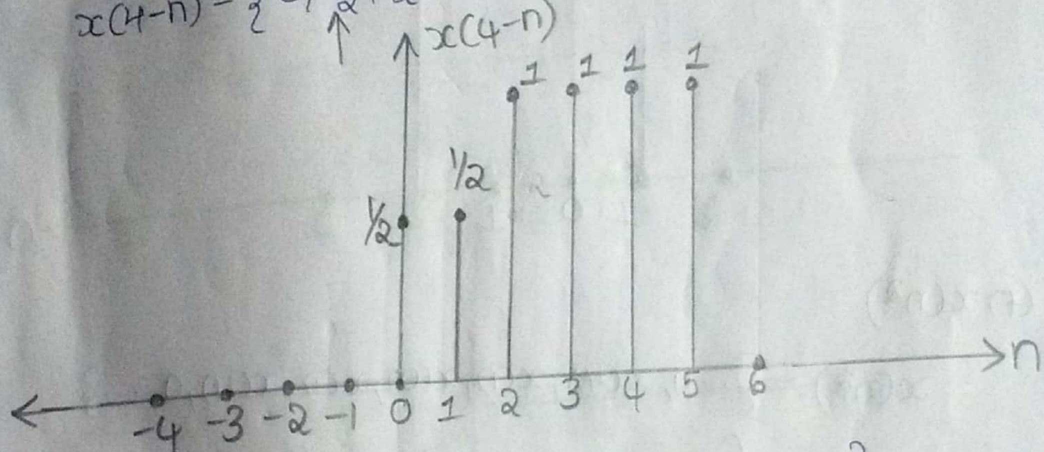
$$x(n-2) = \{\ldots, 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, \ldots\}$$



(b) $x(4-n)$

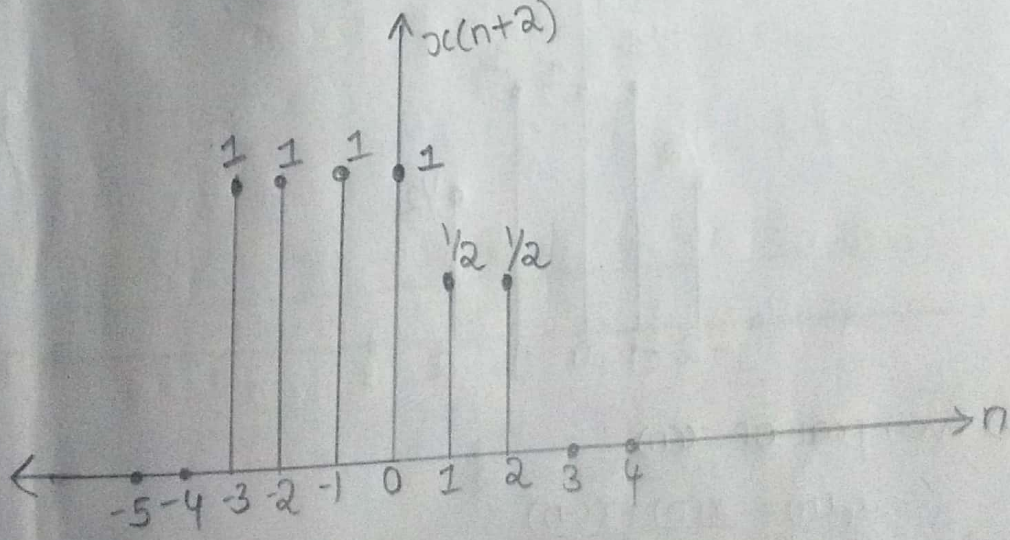
$$x(-n) = \{\tfrac{1}{2}, \tfrac{1}{2}, 1, 1, \underset{\uparrow}{1}, 0, 0, 0\}$$

$$x(4-n) = \{0, \underset{\uparrow}{\tfrac{1}{2}}, \tfrac{1}{2}, 1, 1, 1, 1, 0\}$$



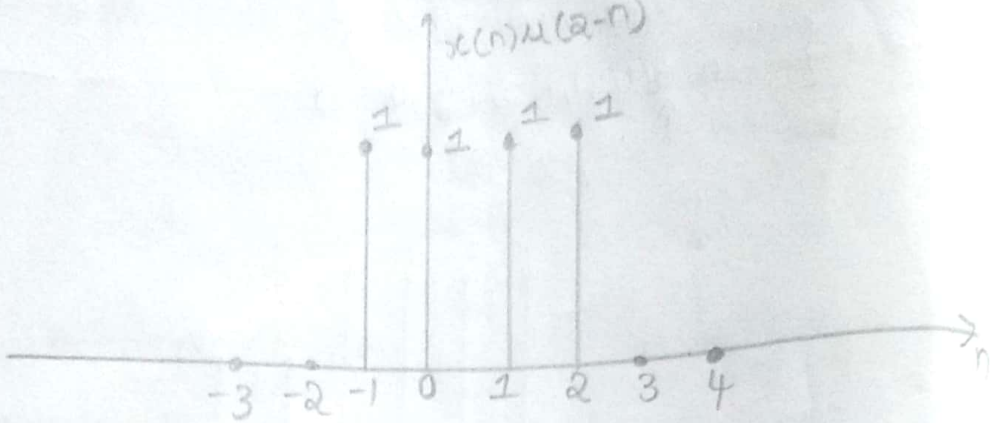
(c) $x(n+2)$

$$x(n+2) = \{\ldots 0, 1, 1, 1, \underset{\uparrow}{1}, \tfrac{1}{2}, \tfrac{1}{2}, 0, \ldots\}$$



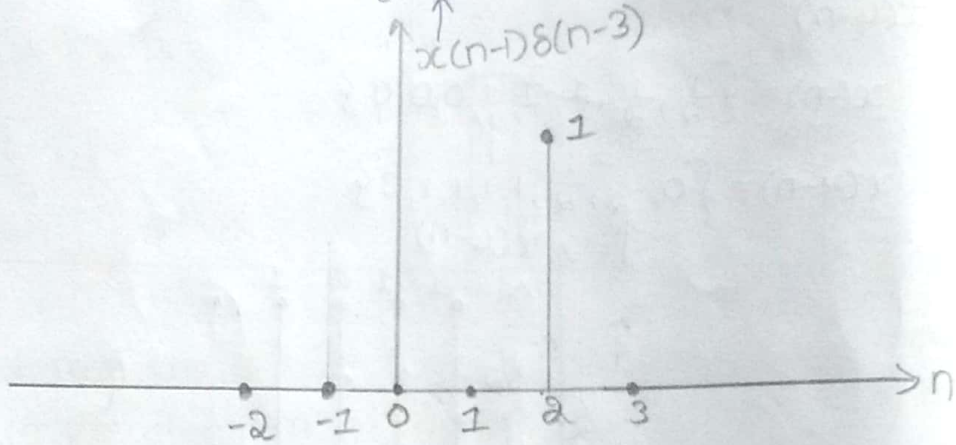
(d) $x(n)\, u(2-n)$

$$x(n)\, u(2-n) = \{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, 0, 0\}$$



(e) $x(n-1)\delta(n-3)$

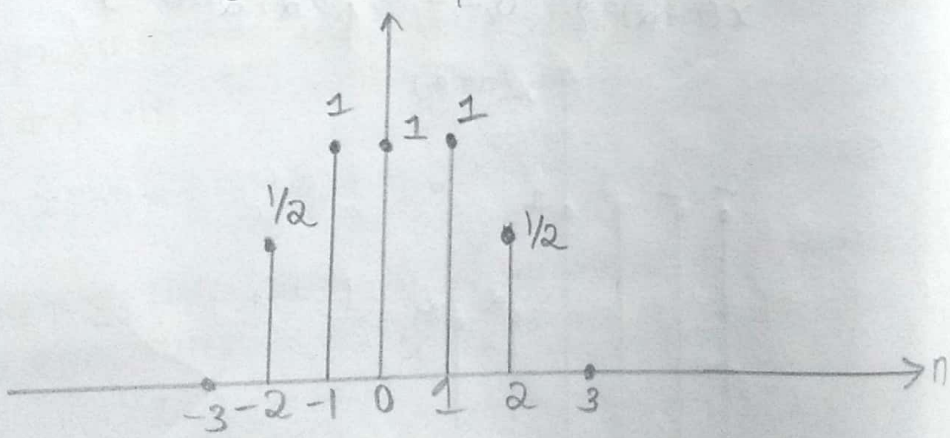
$$x(n-1)\delta(n-3) = \{ \cdot \underset{\uparrow}{0}, 0, 1, 0 \cdot \}$$



(f) $x(n^2)$

$$x(n^2) = \{ \cdots 0, x(4), x(1), x(0), x(1), x(4), 0, \cdots \}$$

$$= \{ \cdots 0, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, 0 \cdots \}$$



(g) even part of $x(n)$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x(-n) = \{ \cdots 0, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0 \cdots \}$$

$x_e(n) = \{\ldots 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \ldots\}$

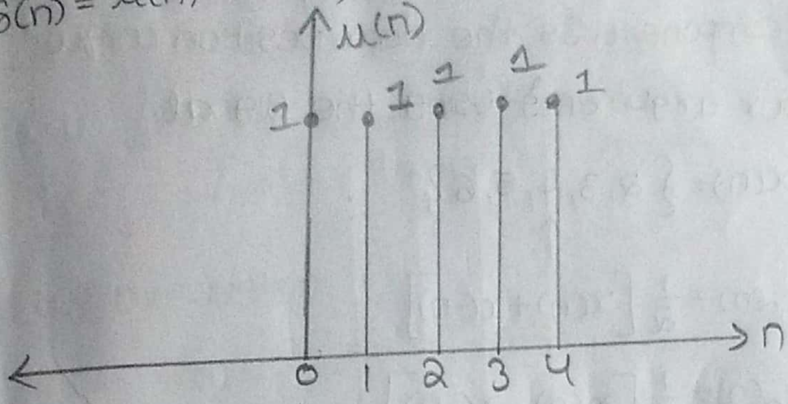
(h) odd part of $x(n)$

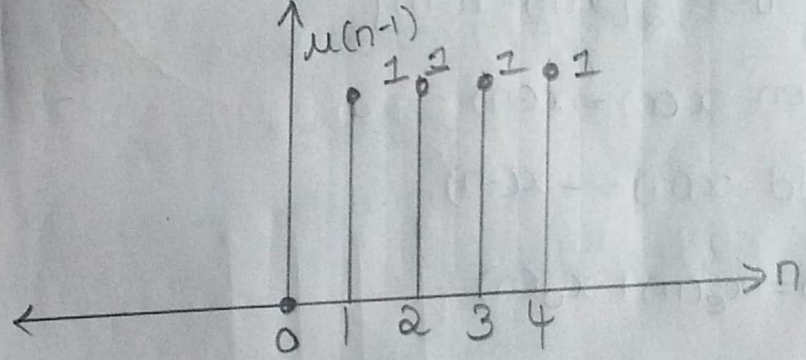
$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

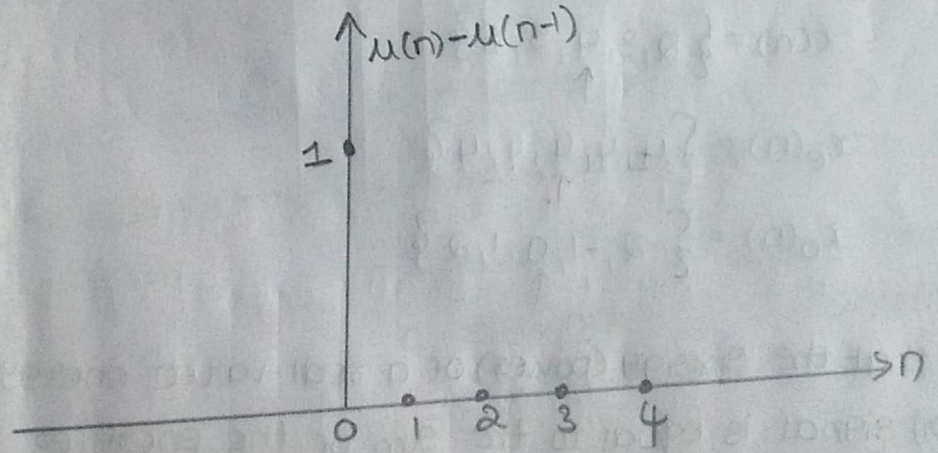
$$x_o(n) = \{\ldots 0, \frac{-1}{4}, \frac{-1}{4}, \frac{-1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \ldots\}$$

3 show that

(a) $\delta(n) = u(n) - u(n-1)$







$$u(n) - u(n-1) = \delta(n) = \begin{cases} 0 & n<0 \\ 1 & n=0 \\ 0 & n>0 \end{cases}$$

(b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$$\sum_{k=-\infty}^{n} \delta(k) = u(n) = \begin{cases} 0 & n<0 \\ 1 & n\geq 0 \end{cases}$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n<0 \\ 1, & n\geq 0 \end{cases}$$

$$\therefore u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$$

4 show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal.

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

For even $x(n) = x(-n)$

For odd $x(n) = -x(-n)$

$$x(n) = x_e(n) + x_o(n)$$

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

$$x_e(n) = \{4, 4, \underset{\uparrow}{4}, 4, 4\}$$

$$x_o(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$$

5 show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (power) of its even and odd components.

$$\sum_{n=-\infty}^{\infty} x_e(n) x_o(n) = 0$$

$$\sum_{n=-\infty}^{\infty} x_e(n) x_o(n) = \sum_{m=-\infty}^{\infty} x_e(-m) x_o(-m)$$

$$= -\sum_{m=-\infty}^{\infty} x_e(m)x_o(m)$$

$$= -\sum_{n=-\infty}^{\infty} x_e(n)x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e(n)x_o(n)$$

$$= 0$$

$$\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n)+x_o(n)]^2$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n)x_o(n)$$

6 consider the system

$$y(n) = T[x(n)] = x(n^2)$$

(a) Determine if the system is time invariant

$$x(n) \rightarrow y(n) = x(n^2)$$

$$x(n-k) \rightarrow y_1(n) = x[(n-k)^2]$$

$$= x(n^2+k^2-2nk)$$

$$\neq y(n-k)$$

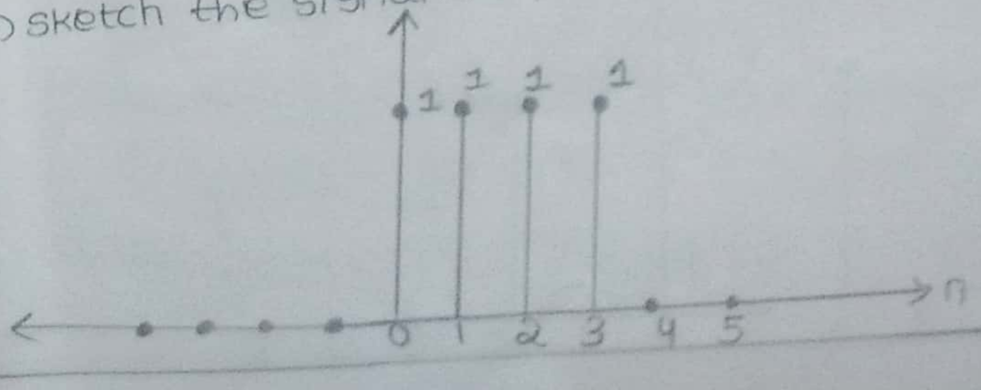
No, the system is time variant.

(b) To clarify the result in part (a) assume that the signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

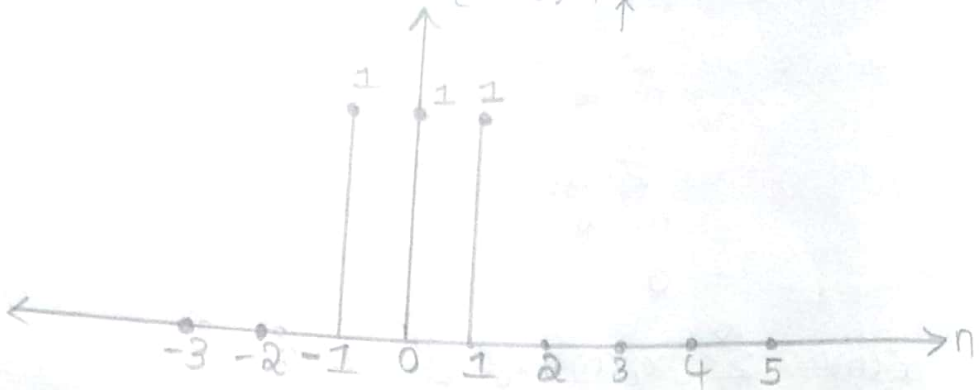
is applied into the system.

(i) sketch the signal $x(n)$



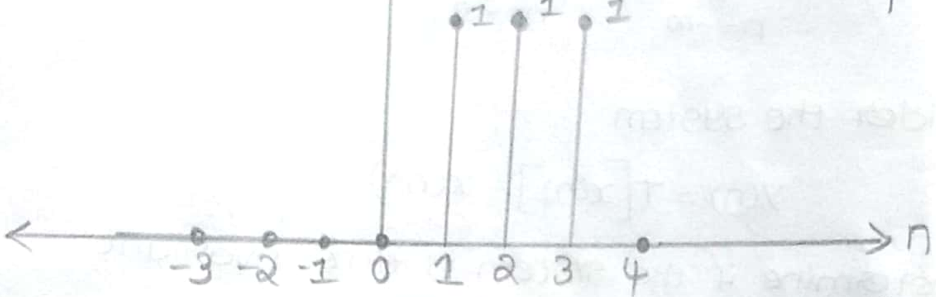
(2) Determine and sketch the signal $y(n) = T[x(n)]$

$$y(n) = x(n^2) = \{0, 1, \underset{\uparrow}{1}, 1, 0, \ldots\}$$



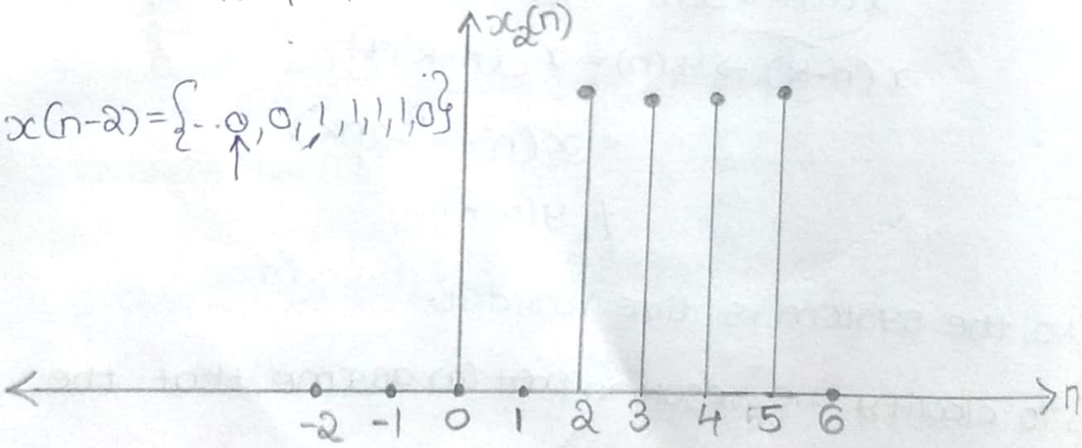
(3) Sketch the signal $y_2'(n) = y(n-2)$

$$y(n-2) = \{\ldots, 0, \underset{\uparrow}{0}, 1, 1, 1, 0\}$$



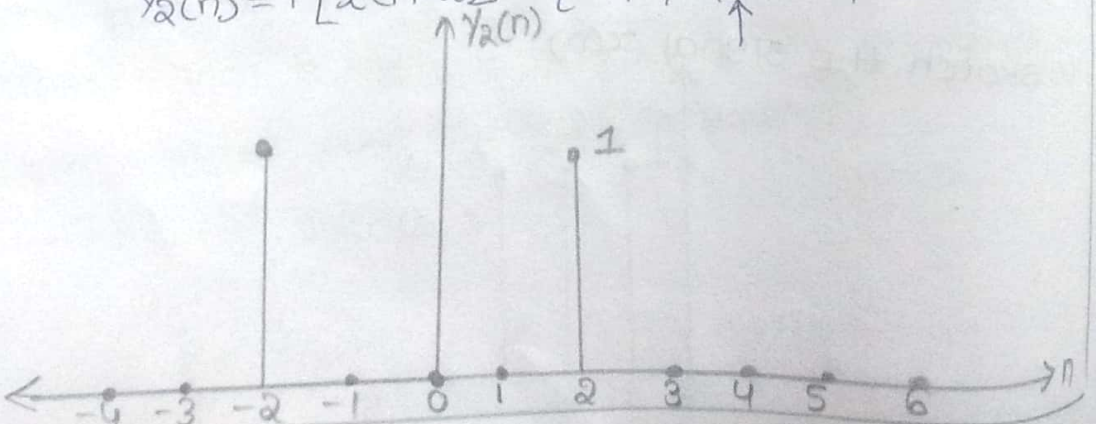
(4) Determine & sketch the signal $x_2(n) = x(n-2)$

$$x(n-2) = \{\ldots \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0\}$$



(5) Determine & sketch the signal $y_2(n) = T[x_2(n)]$

$$y_2(n) = T[x(n-2)] = \{0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0\}$$



(b) compare the signals $y_2(n)$ and $y(n-2)$. What is your conclusion?

$$y_2(n) = \{\cdots 0,1,0,\underset{\uparrow}{0},0,1,0\}$$

$$y(n-2) = \{\cdots,0,\underset{\uparrow}{0},1,1,1,0\}$$

$y_2(n) \neq y(n-2) \Rightarrow$ system is time variant.

(c) Repeat part (b) for the system

$$y(n) = x(n) - x(n-1)$$

can you use this result to make any statement about the time invariance of this system? why?

$$x(n) = \{\underset{\uparrow}{1},1,1,1\}$$

$$y(n) = \{\underset{\uparrow}{1},0,0,0,0,-1\}$$

$$y(n-2) = \{\underset{\uparrow}{0},0,1,0,0,0,0,-1\}$$

$$x(n-2) = \{\underset{\uparrow}{0},0,1,1,1,1\}$$

$$y_2(n) = \{\underset{\uparrow}{0},0,1,0,0,0,0,-1\}$$

$$y_2(n) = y(n-2)$$

system is time invariant; This example alone does not constitute a proof.

(d) Repeat parts (b) & (c) for the system.

$$y(n) = T[x(n)] = n\,x(n)$$

$$y(n) = n\,x(n)$$

$$x(n) = \{\cdots,0,\underset{\uparrow}{1},1,1,1,0\cdots\}$$

$$y(n) = \{\cdots,\underset{\uparrow}{0},1,2,3,\cdots\}$$

$$y(n-2) = \{\cdots\underset{\uparrow}{0},0,0,1,2,3\}$$

$x(n-2) = \{\ldots, 0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

$y_2(n) = T[x(n-2)] = \{\ldots, 0, 0, 2, 3, 4, 5, \ldots\}$

$y_2(n) \neq y(n-2) \rightarrow$ system is time variant.

7\. A discrete-time system can be

(1) static or dynamic

(2) Linear or non Linear

(3) Time invariant or time varying

(4) causal or non causal

(5) Stable or unstable.

Examine the following systems with respect to the properties above.

(a) $y(n) = \cos[x(n)]$

static, nonlinear, time invariant, causal, stable.

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Dynamic, linear, time invariant, non causal and unstable.

(c) $y(n) = x(n)\cos(\omega_0 n)$

static, linear, time variant, causal stable

(d) $y(n) = x(-n+2)$

Dynamic, linear, time invariant, noncausal, stable

(e) $y(n) = trn[x(n)]$ denotes the integer part of $x[n]$, obtainted by truncation.

static, non linear, time invariant, causal, stable

(f) $y(n) = Round[x(n)]$, where $Round[x(n)]$ denotes the intega part of $x(n)$ obtained by rounding.

static, non linear, time invariant, causal, stable

(g) $y(n) = |x(n)|$

static, non linear, time invariant, causal, stable

(h) $y(n) = x(n)u(n)$

static, linear, time invariant, causal, stable

(i) $y(n) = x(n) + n x(n+1)$

Dynamic, linear, time variant, non causal, unstable.

(j) $y(n) = x(2n)$

Dynamic, linear, time variant, non causal, unstable

(k) $y(n) = \begin{cases} x(n) & \text{if } x(n) \geq 0 \\ 0 & \text{if } x(n) < 0 \end{cases}$

static, non linear, time invariant, causal, stable

(l) $y(n) = x(-n)$

Dynamic, linear, time invariant, non causal, stable.

(m) $y(n) = \text{sign}[x(n)]$

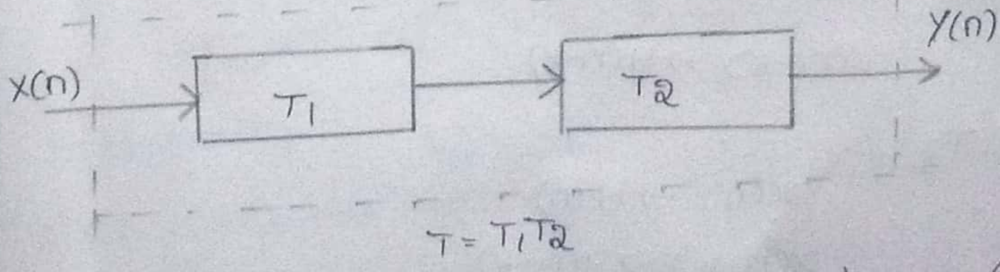
static, nonlinear, time invariant, causal, stable.

(n) The ideal sampling system with input $x_a(t)$ and

output $x(n) = x_a(nT), -\infty < n < \infty$.

static, linear, time invariant, causal, stable.

8. Two discrete-time systems T_1 and T_2 are connected in cascade to form a new system T as shown in Fig. prove or disprove the following statements.



$T = T_1 T_2$

(a) If T_1 and T_2 are linear. then T is linear (ie the

Cascade connection of two linear systems is linear

$$V_1(n) = T_1[x_1(n)]$$

$$V_2(n) = T_2[x_2(n)]$$

$$a_1x_1(n) + a_2x_2(n)$$

$$a_1V_1(n) + a_2V_2(n)$$

By linear property of T_1

$$y_1(n) = T_2[V_1(n)]$$

$$y_2(n) = T_2[V_2(n)]$$

$$\beta_1V_1(n) + \beta_2V_2(n) \rightarrow y(n) = \beta_1y_1(n) + \beta_2y_2(n)$$

By linear property of T_2

$$V_1(n) = T_1[x_1(n)]$$

$$V_2(n) = T_2[x_2(n)]$$

$$A_1T[x_1(n)] + A_2T[x_2(n)]$$

$T = T_1T_2$. Hence T is linear.

(b) If T_1 and T_2 are time invariant then T is time invariant.

for T_1: $x(n) \rightarrow v(n)$

$x(n-k) \rightarrow v(n-k)$

for T_2: $v(n) \rightarrow y(n)$

$v(n-k) \rightarrow y(n-k)$

For T_1T_2

$x(n) \rightarrow y(n)$

$x(n-k) \rightarrow y(n-k)$

$\therefore T = T_1T_2$ is time invariant

(c) If T_1 and T_2 are causal. then T is causal.

True T_1 is causal $\Rightarrow v(n)$ depends only on $x(k)$ for $k \le n$.

T_2 is causal $\Rightarrow y(n)$ depends only on $v(k)$ for $k \le n$.

Therefore $y(n)$ depends only on $x(k)$ for $k \le n$. Hence T is causal.

(d) If T_1 & T_2 are linear and time invariant., the same holds for T.

True by combining a & b.

(e) If T_1 & T_2 are linear and time invariant, then interchanging their order does not change the system T.

True. $h_1(n) * h_2(n) = h_2(n) * h_1(n)$

(f) As in part (e) except that T_1 T_2 are now time varying.

false

$$T_1 : y(n) = n x(n)$$

$$T_2 : y(n) = n x(n+1)$$

$$T_2[T_1[\delta(n)]] = T_2(0) = 0$$

$$T_1[T_2[\delta(n)]] = T_1[\delta(n+1)]$$

$$= -\delta(n+1)$$

$$\neq 0$$

(g) If T_1 and T_2 are nonlinear. then T is nonlinear

False

$$T_1 : y(n) = x(n) + b$$

$$T_2 : y(n) = x(n) - b \text{ where } b \neq 0$$

$$T[x(n)] = T_2\{T_1[x(n)]\} = T_2[x(n) + b] = x(n).$$

T is linear

(h) If T_1 & T_2 are stable. then T is stable.

T_1 is stable $\Rightarrow v(n)$ is bounded if $x(n)$ is bounded

T_2 is stable $\Rightarrow$ $y(n)$ is bounded if $v(n)$ is bounded $y(n)$ is bounded if $x(n)$ is bounded $\Rightarrow$ $T = T_1T_2$ is stable.

(i) show by an example that the inverse of parts (c) and (h) do not hold in general.

Inverse of (c) T_1 and T_2 are noncausal $\Rightarrow$ T is noncausal

$$T_1 : y(n) = x(n+1) \;\&$$

$$T_2 : y(n) = x(n-2)$$

$$T : y(n) = x(n-1)$$

which is causal. Hence, the inverse of (c) is false.

Inverse of (h) T_1 and T_2 is unstable, implies T is unstable.

$T_1 : y(n) = e^{x(n)}$, stable and $T_2 : y(n) = \ln[x(n)]$ which is unstable

But $T : y(n) = x(n)$, which is stable. Hence, the inverse of (h) is false.

Q. Let T be an LTI, relaxed and BIBO stable system with input $x(n)$ and output $y(n)$. show that:

(a) If $x(n)$ is periodic with period N [i.e.. $x(n) = x(n+N)$ for all $n \geq 0$] the output $y(n)$ tends to a periodic signal with the same period.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\,x(n-k), \quad x(n) = 0,\ n<0$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)\,x(n+N-k) = \sum_{k=-\infty}^{n+N} h(k)\,x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\,x(n-k) + \sum_{k=n+1}^{n+N} h(k)\,x(n-k)$$

$$= y(m) + \sum_{k=n+1}^{n+N} h(k)x(n-k)$$

for a BIBO system $\lim_{n\to\infty} |h(n)| = 0$ therefore

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)x(n-k) = 0 \text{ and}$$

$$\lim_{n\to\infty} y(n+N) = y(N)$$

(b) If $x(n)$ is bounded and tends to a constant, the output will also tend to a constant.

$x(n) = x_0(n) + a\,u(n)$, where a is a constant and $x_0(n)$ is a bounded signal with $\lim_{n\to\infty} x_0(n) = 0$

$$y(n) = a\sum_{k=0}^{\infty} h(k)u(n-k) + \sum_{k=0}^{\infty} h(k)x_0(n-k)$$

$$= a\sum_{k=0}^{n} h(k) + y_0(n)$$

clearly $\sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$ from (c) below and thus

Hence $\lim_{n\to\infty} |y_0(n)| = 0$

$$\lim_{n\to\infty} y(n) = a\sum_{k=0}^{n} h(k) = \text{constant}$$

(c) If $x(n)$ is an energy signal. The output $y(n)$ will also be an energy signal.

$$y(n) = \sum_k h(k)x(n-k)$$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty}\left[\sum_k h(k)x(n-k)\right]^2$$

$$= \sum_k\sum_l h(k)h(l)\sum_n x(n-k)x(n-l)$$

But $\sum_n x(n-k)x(n-l) \le \sum_n x^2(n) = E_x$

$$\sum_n y^2(n) \le E_x \sum_k |h(k)| \sum_l |h(l)|$$

For BIBO stable system $\sum_{k} |h(k)| < M$

$E_y \leq M^2 E_x$ so that

$E_y < 0$ if $E_x < 0$

10 The following input-output pairs have been observed during the operation of a time invariant system.

$x_1(n) = [\underset{\uparrow}{1}, 0, 2] \xleftrightarrow{T} y_1(n) = [0, \underset{\uparrow}{1}, 2]$

$x_2(n) = [\underset{\uparrow}{0}, 0, 3] \xleftrightarrow{T} y_2(n) = [0, \underset{\uparrow}{1}, 0, 2]$

$x_3(n) = [0, 0, 0, \underset{\uparrow}{1}] \xleftrightarrow{T} y_3(n) = [1, \underset{\uparrow}{2}, 1]$

can you draw any conclusions regarding the linearity of the system. What is the impulse response of the system.

The system is non linear

$x_3(n) \leftrightarrow y_3(n)$ & $x_2(n) \leftrightarrow y_2(n)$

If the system were linear, $y_2(n)$ would be of the form

$y_2(n) = \{3, 6, 3\}$.

Because the system is time-invariant. However this is not the case.

11. The following input-output pairs have been observed during the operation of a linear system.

$x_1(n) = [-1, \underset{\uparrow}{2}, 1] \xleftrightarrow{T} y_1(n) = [1, \underset{\uparrow}{2}, -1, 0, 1]$

$x_2(n) = [1, \underset{\uparrow}{-1}, -1] \xleftrightarrow{T} y_2(n) = [-1, \underset{\uparrow}{1}, 0, 2]$

$x_3(n) = [0, \underset{\uparrow}{1}, 1] \xleftrightarrow{T} y_3(n) = [\underset{\uparrow}{1}, 2, 1]$

can you draw any conclusions about the time invariance of this systems.

$x_1(n) + x_2(n) = \delta(n)$

The system is linear, the impulse response of the system is

$$y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$$

If system were time invariant, the response of $x_3(n)$

$$\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$$

12 The only available information about a system consists of N-input output pairs, of signals $y_i(n) = T[x_i(n)]$, $i = 1, 2, \ldots N$

(a) what is the class of input signals for which we can determine the output using the information above. if the system is known to be linear?

Any weighted linear combination of the signals

$x_i(n) = 1, 2, \ldots N$

(b) The same as above. if the system is known to be time invariant.

Any $x_i(n-k)$ where k is any integer and $i = 1, 2, \ldots N$

13 show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty \text{ for some constant } M_h$$

A system is BIBO stable if and only if a bounded input produces a bounded output.

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$|y(n)| \leq \sum_k |h(k)|\,|x(n-k)|$$

$$\leq M_x \sum_k |h(k)|$$

where $|x(n-k)| \leq M_x$. Therefore $|y(n)| < \infty$ for all n.

if and only if

$$\sum_k |h(k)| < \infty$$

14 show that:

(a) A relaxed linear system is causal if and only if for any input $x(n)$ such that

$$x(n)=0 \text{ for } n<n_0 \Rightarrow y(n)=0 \text{ for } n<n_0$$

A system is causal $\Leftrightarrow$ the output becomes nonzero after the input the becomes non zero.

$$x(n)=0 \text{ for } n<n_0 \Rightarrow y(n)=0 \text{ for } n<n_0$$

(b) A relaxed LTI system is causal if and only if

$$h(n)=0 \quad \text{for } n<0$$

$$y(n)=\sum_{-\infty}^{n} h(k)x(n-k) \text{ where } x(n)=0 \text{ for } n<0$$

If $h(k)=0$ for $k<0$, then

$$y(n)=\sum_{0}^{n} h(k)x(n-k) \text{ and hence, } y(n)=0 \text{ for } n<0$$

if $y(n)=0$ for $n<0$ then

$$\sum_{-\infty}^{n} h(k)x(n-k) \Rightarrow h(k)=0,\ k=0.$$

15 (a) show that for any real or complex constant, a and any finite integer M and N, we have.

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M-a^{N+1}}{1-a}, & \text{if } a\neq 1 \\ N-M+1, & \text{if } a=1 \end{cases}$$

For $a=1$ $\sum_{n=M}^{N} a^n = N-M+1$

for $a\neq 1$ $\sum_{n=M}^{N} a^n = a^M + a^{M+1} + \cdots + a^N$

$$(1-a)\sum_{n=M}^{N} a^n = a^M + a^{M+1} - a^{M+1} + \cdots + a^N - a^N - a^{N+1}$$

$$= a^M - a^{N+1}$$

(b) show that if $|a|<1$ then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

for $M=0$, $|a|<1$ and $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a|<1$$

16 (a) If $y(n) = x(n) * h(n)$ show that $\Sigma_y = \Sigma_x \Sigma_h$ where

$$\Sigma_x = \sum_{n=-\infty}^{\infty} x(n)$$

$$y(n) = \sum_k h(k)x(n-k)$$

$$\sum_n y(n) = \sum_n \sum_k h(k)x(n-k) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$$

$$= \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$$

(b) compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the test in (a).

(1) $x(n) = [1,2,4]$, $h(n) = [1,1,1,1,1]$

$y(n) = h(n) * x(n) = \{1,3,7,7,7,6,4\}$

$\sum_n y(n) = 35$ $\quad \sum_k h(k) = 5$ $\quad \sum_k x(k) = 7$

(2) $x(n) = [1,2,-1]$ $\quad h(n) = x(n)$

$y(n) = \{1,4,2,-4,1\}$

$\sum_n y(n) = 4$ $\quad \sum_k h(k) = 2$ $\quad \sum_k x(k) = 2$

(3) $x(n) = [0,1,-2,3,-4]$ $\quad h(n) = [\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}]$

$y(n) = \{0, \frac{1}{2}, \frac{-1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, -2\}$

$\sum_n y(n) = -5$ $\quad \sum_n h(n) = 2.5$ $\quad \sum_n x(n) = -2$

(4) $x(n) = [1,2,3,4,5]$ $\quad h(n) = \{1]$

$y(n) = \{1,2,3,4,5\}$

$\sum_n y(n) = 15$ $\quad \sum_n h(n) = 1$ $\quad \sum_n x(n) = 15$

(5) $x(n) = [1,-2,3]$ $\quad h(n) = [0,0,1,1,1,1]$

$y(n) = \{0,0,1,-1,2,2,1,3\}$

$\sum_n y(n) = 8$ $\quad \sum_n h(n) = 4$ $\quad \sum_n x(n) = 2$

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(6) $x(n) = \{0, \underset{\uparrow}{0}, 1, 1, 1, 1\}$ $h(n) = \{1, -\underset{\uparrow}{2}, 3\}$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$ $\sum_n h(n) = 2$ $\sum_n x(n) = 4$

(7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$ $h(n) = \{1, \underset{\uparrow}{0}, -1, -1\}$

$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$

$\sum_n y(n) = -2$ $\sum_n h(n) = 1$ $\sum_n x(n) = 2$

(8) $x(n) = \{1, 1, 2\}$, $h(n) = u(n)$

$y(n) = u(n) + u(n-1) + 2u(n-2)$

$\sum_n y(n) = \infty$ $\sum_n h(n) = \infty$ $\sum_n x(n) = 4$

(9) $x(n) = \{1, 1, 0, 1, 1\}$ $h(n) = \{1, -2, -3, 4\}$

$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$

$\sum_n y(n) = 0$ $\sum_n h(n) = 0$ $\sum_n x(n) = 4$

(10) $x(n) = (\frac{1}{2})^n u(n)$, $h(n) = (\frac{1}{4})^n u(n)$

$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

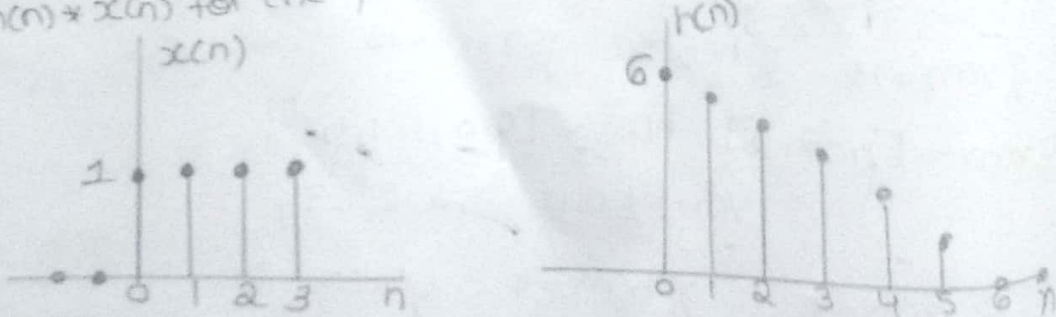
$\sum_n y(n) = 36$ $\sum_n h(n) = 6$ $\sum_n x(n) = 6$

(11) $x(n) = (\frac{1}{2})^n u(n)$, $h(n) = (\frac{1}{4})^n u(n)$

$y(n) = \left[2(\frac{1}{2})^n - (\frac{1}{4})^n\right] u(n)$

$\sum_n y(n) = \frac{8}{3}$ $\sum_n h(n) = \frac{4}{3}$ $\sum_n x(n) = 2$

17. compute and plot the convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signals shown in Fig P2.17



$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$

$h(n) = \{\underset{\uparrow}{6}, 5, 4, 3\ 2, 1\}$

$$y(n) = \sum_{k=0}^{n} x(k)h(n-k)$$

$y(0) = x(0)\,h(0) = 6$

$y(1) = x(0)\,h(1) + x(1)\,h(0) = 11$

$y(2) = x(0)\,h(2) + x(1)\,h(1) + x(2)\,h(0) = 15$

$y(3) = x(0)\,h(3) + x(1)\,h(2) + x(2)\,h(1) + x(3)\,h(0) = 18$

$y(4) = x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0) = 14$

$y(5) = x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + x(4)h(1) + x(5)h(0) =$

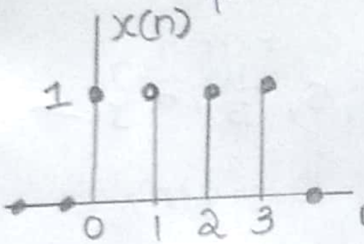
$y(6) = x(1)h(5) + x(2)h(4) + x(3)h(2) = 6$

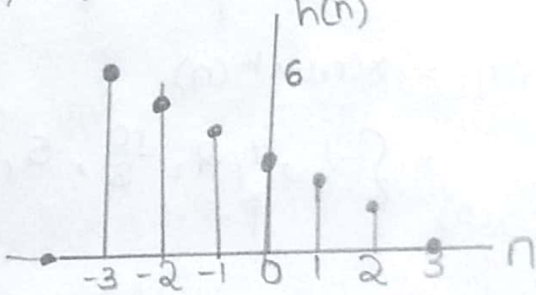
$y(7) = x(2)\,h(5) + x(3)\,h(4) = 3$

$y(8) = x(3)\,h(5) = 1$

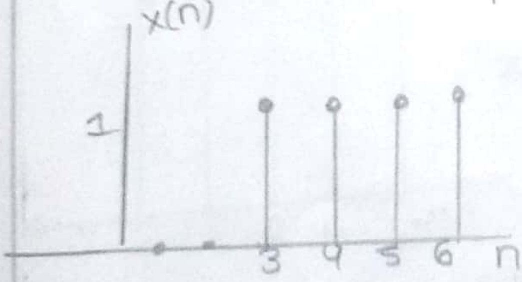
$y(n) = 0, \quad n \geq 9$

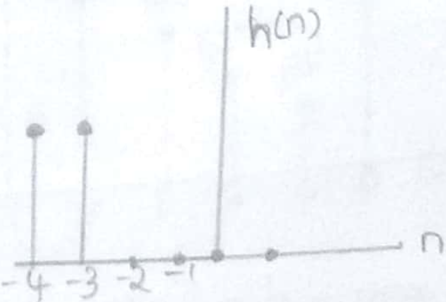
$y(n) = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$



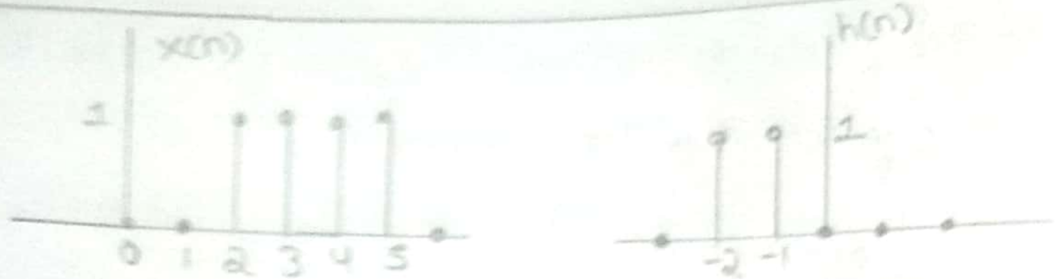


$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$





$y(n) = \{1, \underset{\uparrow}{2}, 2, 2, 1\}$



$y(n) = \{1, 2, 2, 2, 1\}$ (↑ under the first 1)

18. Determine and sketch the convolution y(n) of the signals

$$x(n) = \begin{cases} \frac{1}{3}n & 0 \le n \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

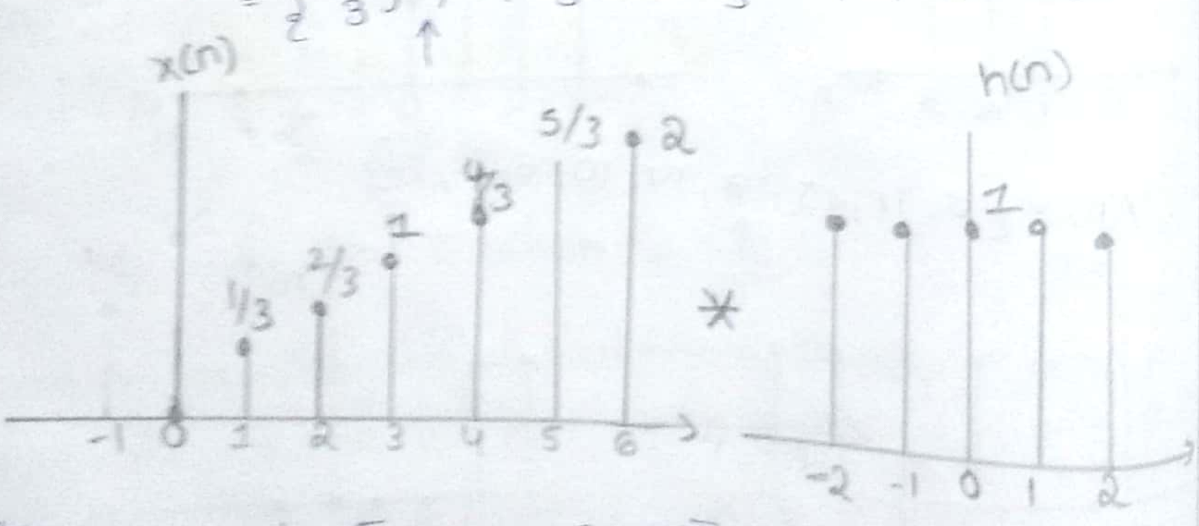
(a) Graphically (b) Analytically.

(a) $x(n) = \{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$y(n) = x(n) * h(n)$

$= \{\frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$



(b) $x(n) = \frac{1}{3}n\left[u(n) - u(n-7)\right]$

$h(n) = u(n+2) - u(n-3)$

$y(n) = x(n) * h(n)$

$= \frac{1}{3} n[u(n) - u(n-7)] * [u(n+2) - u(n-3)]$

$= \frac{1}{3} n[u(n) * u(n+2) - u(n) * u(n-3) - u(n-7) * u(n+2) +$

$u(n-7) * u(n-3)]$

$y(n) = \frac{1}{3} \delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3) +$

$\frac{20}{3}\delta(n-4) + 6\delta(n-5) + 5\delta(n-6) + 5\delta(n-6) + \frac{11}{3}$

$\delta(n-7) + \delta(n-8)$

19 compute the convolution $y(n)$ of the signals.

$$x(n) = \begin{cases} \alpha^n & -3 \le n \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{elsewhere.} \end{cases}$$

$$y(n) = \sum_{k=0}^{4} h(k)x(n-k)$$

$$x(n) = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \ldots \alpha^5\}$$

$$h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$$

$$y(n) = \sum_{k=0}^{4} x(n-k), \quad -3 \le n \le 9$$

$= 0$ otherwise.

$y(-3) = \alpha^{-3}$

$y(-2) = x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}$

$y(-1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1}$

$y(0) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1$

$y(1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha$

$y(2) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^2$

$y(3) = \alpha^{-1} + 1 + \alpha + \alpha^2 + \alpha^3$

$Y(4) = \alpha^4+\alpha^3+\alpha^2+\alpha+1$

$Y(5) = \alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5$

$Y(6) = \alpha^2+\alpha^3+\alpha^4+\alpha^5$

$Y(7) = \alpha^3+\alpha^4+\alpha^5$

$Y(8) = \alpha^4+\alpha^5$

$Y(9) = \alpha^5$

20) consider the following three operations.

(a) Multiply the integer numbers : 131 and 122

$131 \times 122 = 15982$

(b) compute the convolution of signals $\{1,3,1\} * \{1,2,2\}$

$\{1,3,1\} * \{1,2,2\} = \{1,5,9,8,2\}$

(c) Multiply the polynomials $1+3z+z^2$ & $1+2z+2z^2$

$(1+3z+z^2)\cdot(1+2z+2z^2) = 1+5z+9z^2+8z^3+2z^4$

(d) Repeat part (a) for the numbers 1.31 and 12.2.

$1.31 \times 12.2 = 15.982$

(e) comment on your results

Different ways to perform convolution.

21) compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

(a) $x(n) = a^n u(n)$. $h(n) = b^n u(n)$ when $a \neq b$ and when $a = b$

$$y(n) = \sum_{k=0}^{n} a^k u(k) b^{n-k} u(n-k) = b^n \sum_{k=0}^{n} (ab^{-1})^k$$

$$y(n) = \begin{cases} \dfrac{b^{n+1} - a^{n+1}}{b-a} u(n) & a \neq b \\ b^n(n+1)\, u(n) & a = b \end{cases}$$

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(b) $x(n) = \begin{cases} 1 & n = -2, 0, 1 \\ 2 & n = -1 \\ 0 & \text{elsewhere} \end{cases}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$x(n) = \{1, 2, \underset{\uparrow}{1}, 1\}$

$h(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, -1, 0, \underset{\uparrow}{0}, 3, 3, 2, 1\}$

(c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$

$h(n) = [u(n+2) - u(n-3)] \cdot (3 - |n|)$

$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -2, 1\}$

(d) $x(n) = u(n) - u(n-5)$

$h(n) = u(n-2) - u(n-8) + u(n-11) - u(n-17)$

$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h'(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = h'(n) + h'(n-9)$

$y'(n) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

22 Let $x(n)$ be the input signal to a discrete time filter with impulse response $h_i(n)$ and let $y(n)$ be the corresponding output.

(a) compute and sketch $x(n)$ and $y_i(n)$ in the following cases. using the same scale in all figures.

$x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1(n) = \{1, 1\}$

$h_2(n) = \{1, 2, 1\}$

$h_3(n) = \{\frac{1}{2}, \frac{1}{2}\}$

$h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_5(n) = \{\frac{1}{4}, \frac{-1}{2}, \frac{1}{4}\}$

Sketch $x(n), y_1(n), y_2(n)$ on one graph and $x(n), y_3(n)$ $y_4(n), y_5(n)$ on another graph.

$y_1(n) = x(n) * h_1(n)$

$y_1(n) = x(n) + x(n-1)$

$= \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}$

$y_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 15, 21, 25, \cancel{28}\ 24, 9\}$

$y_3(n) = \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\}$

$y_4(n) = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3, 3.25, 3.75, 5.25, 6.25, 7, 6, 2.25\}$

$y_5(n) = \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$

(b) What is the difference between $y_1(n)$ and $y_2(n)$ and between $y_3(n)$ and $y_4(n)$

$y_3(n) = \frac{1}{2} y_1(n)$

$h_3(n) = \frac{1}{2} h_1(n)$

$y_4(n) = \frac{1}{4} y_2(n)$

$h_4(n) = \frac{1}{4} h_2(n).$

(c) comment on the smoothness of $y_2(n)$ and $y_4(n)$ which factors affect the smoothness.

$y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$ but $y_4(n)$ will appear even smoother because of the smaller scale factor

(d) compare $y_4(n)$ with $y_5(n)$ what is the difference? can u explain.

system 4 results in a smoother output; The negative value of $h_5(0)$ i

(e) Let $h_6(n) = \{\frac{1}{2}, \frac{-1}{2}\}$ compute $y_6(n)$. sketch $x(n)$ $y_2(n)$ and $y_6(n)$ on the same figure and comment on the results?

$$y_6(n) = \left\{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, \frac{-1}{2}, \frac{3}{2}, \frac{-9}{2}\right\}$$

$y_2(n)$ is smoother than $y_6(n)$

23 The discrete-time system.

$$y(n) = ny(n-1) + x(n) \qquad n \geq 0$$

is at rest [i.e.. $y(-1) = 0$]. check if the system is linear time invariant and BIBO stable.

$$y_1(n) = ny_1(n-1) + x_1(n)$$

$$y_2(n) = ny_2(n-1) + x_2(n)$$

$$x(n) = ax_1(n) + bx_2(n)$$

$$y(n) = ny(n-1) + x(n)$$

$$y(n) = ay_1(n) + by_2(n)$$

Hence the system is linear. If the input is $x(n-1)$ we have.

$$y(n-1) = (n-1)y(n-2) + x(n-1)$$

$$y(n-1) = ny(n-2) + x(n-1)$$

The system is time variant. If $x(n) = u(n)$ then $|x(n)| \le 1$. For bounded input:

$$y(0) = 1 \quad y(1) = 1+1 = 2, \quad y(2) = 2 \times 2 + 1 = 5$$

which is unbounded. Hence the system is unstable

24 consider the signal $\delta(n) = \alpha^n u(n)$, $0 < \alpha < 1$

(a) show that any sequence $x(n)$ can be decomposed as

$$x(n) = \sum_{n=-\infty}^{\infty} c_k \delta(n-k) \text{ and express } c_k \text{ in}$$

terms of $x(n)$.

$$\delta(n) = \delta(n) - \alpha\delta(n-1) \text{ and}$$

$$\delta(n-k) = \delta(n-k) - \alpha\delta(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\left(\delta(n-k) - \alpha\delta(n-k-1)\right]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) - \alpha \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) - \alpha \sum_{k=-\infty}^{\infty} x(k-1)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left[x(k) - \alpha x(k-1)\right] \delta(n-k)$$

$$c_k = x(k) - \alpha x(k-1)$$

(b) use the properties of linearity and time invariance to express the output $y(n) = T[x(n)]$ in terms of the input $x(n)$ and the signal $g(n) = T[\delta(n)]$ where $T[\cdot]$ is an LTI system.

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{k=-\infty}^{\infty} c_k \delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k T\left[\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k\, g(n-k)$$

(c) Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$.

$$h(n) = T\left[\delta(n)\right]$$

$$= T\left[\delta(n) - a\delta(n-1)\right]$$

$$= g(n) - a g(n-1).$$

25. Determine the zero-input response of the system described by the second-order difference equation.

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

$x(n) = 0$, we have

$$y(n-1) + \frac{4}{3} y(n-1) = 0$$

$$y(-1) = \frac{-4}{3} y(-2)$$

$$y(0) = \left(-\frac{4}{3}\right)^2 y(-2)$$

$$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$$

$$\vdots$$

$$y(k) = \left(-\frac{4}{3}\right)^{k+2} y(-2) \leftarrow \text{zero i/p response}$$

26. Determine the particular solution of the difference equation.

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$$

When the forcing function is $x(n) = 2^n u(n)$.

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$$

$$y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = 0$$

$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0.\ \lambda = \frac{1}{2}, \frac{1}{3}$

$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$

P. solution $\quad x(n) = 2^n u(n)$ is

$y_p(n) = K(2^n), u(n)$

$K(2^n)u(n) - K\left(\frac{5}{6}\right)(2^{n-1})u(n-1) + K\left(\frac{1}{6}\right)(2^{n-2})u(n-2) = 2^n u(n)$

For $n=2$

$4K - \frac{5K}{3} + \frac{K}{6} = 4 \Rightarrow K = \frac{8}{5}$

$y(n) = y_p(n) + y_h(n) = \frac{8}{5}(2^n)u(n) + c_1\left(\frac{1}{2}\right)^n u(n) + c_2\left(\frac{1}{3}\right)^n u(n)$

To find c_1 and c_2 $\quad y(-2) = y(-1) = 0$

$y(0) = 1 \ \& \ y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$

$\frac{8}{5} + c_1 + c_2 = 1 \Rightarrow c_1 + c_2 = -\frac{3}{5}$

$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6} \Rightarrow 3c_1 + 2c_2 = -\frac{11}{5}$

$c_1 = -1 \quad c_2 = \frac{2}{5}$

$y(n) = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n).$

27 Determine the response $y(n), n \geq 0$ of the system described by the second-order difference equation.

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to the input $x(n) = 4^n u(n)$

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda = 4, -1 \qquad y_h(n) = c_1(n)4^n + c_2(-1)^n$

$x(n) = 4^n u(n)$

$$y_p(n) = kn4^n u(n)$$

$$kn4^n u(n) - 3k(n-1)4^{n-1}u(n-1) - 4k(n-2)4^{n-2}u(n-2)$$
$$= 4^n u(n) + 2(4)^{n-1} u(n-1)$$

For $n=2$

$$k(32-12) = 4^2+8 = 24 \rightarrow k = \frac{6}{5}$$

$$y(n) = y_p(n) + y_h(n)$$

$$= \left[\frac{6}{5}n4^n + c_1 4^n + c_2(-1)^n\right]u(n)$$

$y(-1) = y(-2) = 0$ then

$$y(0) = 1$$

$$y(1) = 3y(0) + 4 + 2 = 9$$

$$c_1 + c_2 = 1$$

$$\frac{24}{5} + 4c_1 - c_2 = 9$$

$$4c_1 - c_2 = \frac{21}{5}$$

$$c_1 = \frac{26}{25} \quad c_2 = -\frac{1}{25}$$

$$y(n) = \left[\frac{6}{5}n4^n + \frac{26}{25}4^n - \frac{1}{25}(-1)^n\right]u(n)$$

28 Determine the impulse response of the following causal system.

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

$$\lambda = 4, -1$$
$$y_h(n) = c_1 4^n + c_2(-1)^n$$

$$x(n) = \delta(N)$$
$$y(0) = 1$$
$$y(1) - 3y(0) = 2 \text{ or } y(1) = 5$$

$$c_1 = \frac{6}{5} \ \& \ c_2 = -\frac{1}{5} \qquad h(n) = \left[\frac{6}{5}4^n - \frac{1}{5}(-1)^n\right]u(n)$$

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29 Let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $M_1 \le n \le M_2$ be two finite duration signals.

(a) Determine the range $L_1 \le n \le L_2$ of their convolution in terms of N_1 N_2 M_1 and M_2.

$L_1 = N_1 + M_1 \quad L_2 = N_2 + M_2$

(b) Determine the limits of the cases of partial overlap from the left, full overlap, and partial overlap from the right. For convenience, assume that $h(n)$ has shorter duration than $x(n)$.

partial overlap left low $N_1 + M_1$ high $N_1 + M_2 - 1$

Full overlap low $N_1 + M_2$ high $N_2 + M_1$

partial overlap right

low $N_2 + M_1 + 1$ high $N_2 + M_2$.

(c) Illustrate the validity of your results by computing the convolution of the signals.

$$x(n) = \begin{cases} 1 & -2 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2 & -1 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$$

$$h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$$

$N_1 = -2 \quad N_2 = 4 \quad M_1 = -1 \quad M_2 = 2$

partial overlap from left $n = -3$ $n = -1$ $L_1 = -3$

Full overlap $n = 0$ $n = 3$

partial overlap from right $n = 4$ $n = 6$ $L_2 = 6$

20 Determine the impulse response and the unit step response of the systems described by the difference equation.

(a) $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

$$y(n) - 0.6\,y(n-1) + 0.08\,y(n-2) = x(n)$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$\lambda = 0.2, 0.4$ $\quad y_h(n) = c_1 \frac{1}{5}^n + c_2 \frac{2}{5}^n$

$x(n) = \delta(n)$ $\quad y(0) = 1$

$$y(1) - 0.6\,y(0) = 0 \Rightarrow y(1) = 0.6$$

$$c_1 + c_2 = 1$$

$$\frac{1}{5}c_1 + \frac{2}{5} = 0.6 \Rightarrow c_1 = -1 \quad c_2 = 3$$

$$h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$$

step response $\quad s(n) = \sum_{k=0}^{n} h(n-k) \quad n \geq 0$

$$= \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left[\left(\frac{2}{5}^{n+1} - 1\right) - \frac{1}{0.16}\left[\left(\frac{1}{5}^{n+1} - 1\right)\right]\right]\right\} u(n)$$

(b) $y(n) = 0.7\,y(n-1) - 0.1\,y(n-2) + 2x(n) - x(n-2)$

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ $\quad y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$

$x(n) = \delta(n)$ $\quad y(0) = 2$

$$y(1) - 0.7\,y(0) = 0 \Rightarrow y(1) = 1.4$$

$c_1 + c_2 = 2$ and

$$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$c_1 + \frac{2}{5} c_2 = \frac{14}{5}$

$c_1 = \frac{10}{3} \quad c_2 = -\frac{4}{3}$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n \right] u(n)$$

step response

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n (2^{n+1} - 1)\, u(n) - \frac{1}{3}\left(\frac{1}{5}(5^{n+1} - 1)\, u(n)\right)$$

31 consider a system with impulse response.

$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the input $x(n)$ for $0 \le n \le 8$ that will generate the output sequence

$$y(n) = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0 \cdots\}$$

$$h(n) = \left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right\}$$

$$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0\}$$

$x(0)h(0) = y(0) \Rightarrow x(0) = 1$

$\frac{1}{2}x(0) + x(1) = y(1) \Rightarrow x(1) = \frac{3}{2}$

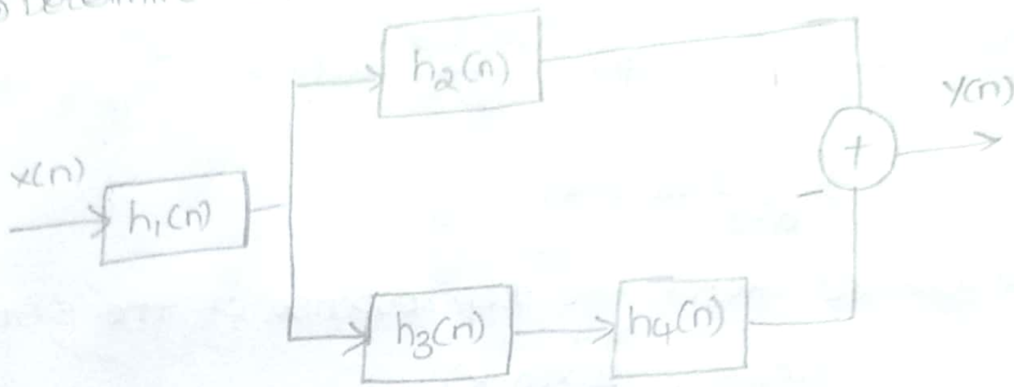
$$x(n) = \left\{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}\right\}.$$

32 consider the interconnection of LTI systems as shown in Fig. P2.32

(a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ and $h_4(n)$.

$$h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$$

(b) Determine $h(n)$ when



$$h_3(n) * h_4(n) = (n-1)u(n-2)$$

$$h_2(n) - h_3(n) * h_4(n) = 2u(n) - \delta(n)$$

$$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$$

$$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * [2u(n) - \delta(n)].$$

$$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$$

(c) Determine the response of the system in part (b) if

$$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3).$$

$$x(n) = \{1, 0, \underset{\uparrow}{0}, 3, 0, -4\}$$

$$y(n) = \left\{\frac{1}{2}, \frac{5}{4}, 2, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \dots\right\}.$$

33A)

$$s(n) = u(n) * h(n)$$

$$s(n) = \sum_{k=0}^{\infty} u(k)h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k}$$

$$= \frac{a^{n+1}-1}{a-1}, \quad n \geq 0$$

$x(n) = u(n+5) - u(n-10)$

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

$$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10) - \frac{a^{n+4}-1}{a-1} u(n$$

$$+ \frac{a^{n-11}-1}{a-1} u(n-12)$$

34. compute and sketch the step response of the system

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

$$h(n) = [u(n) - u(n-M)]/M$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k) h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}$$

35. Determine the range of values of the parameter a for which the linear time-invariant system with impulse response.

$$h(n) = \begin{cases} a^n & n \geq 0, n \text{ even} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0, n \text{ even}}^{\infty} |a|^n$$ stable if $|a| < 1$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$

36. Determine the response of the system with impulse response $h(n) = a^n u(n)$ to the input signal.

$$x(n) = u(n) - u(n-10)$$

$h(n) = a^n u(n)$

$$y_1(n) = \sum_{k=0}^{\infty} u(k) h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1-a^{n+1}}{1-a} u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a}\left[(1-a^{n+1}) u(n) - (1-a^{n-9}) u(n-10)\right]$$

37 Determine the response of the (relaxed) system characterized by the impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to the input signal.

$$x(n) = \begin{cases} 1 & 0 \le n < 10 \\ 0 & \text{otherwise} \end{cases}$$

$h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$y(n) = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u(n) - 2\left[1 - \left(\frac{1}{2}\right)^{n-9}\right] u(n-10)$$

38 Determine the response of the (relaxed) system characterized by the impulse response.

$h(n) = \left(\frac{1}{2}\right)^n u(n)$ to the input signals.

(a) $x(n) = 2^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right]\left(\frac{4}{3}\right)$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$$

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(b) $x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)$$

$$= \sum_{k=}^{\infty} \left(\frac{1}{2}\right)^k = 2, \ n<0$$

$$y(n) = \sum_{k=n}^{\infty} h(k)$$

$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1-\left(\frac{1}{2}\right)^n}{\frac{1}{2}}\right)$$

$$= 2\left(\frac{1}{2}\right)^n, \ n \geq 0$$

39 Three systems with impulse responses $h_1(n) = \delta(n) - \delta(n-1)$, $h_2(n) = h(n)$ $h_3(n) = u(n)$ are connected in cascade

(a) what is the impulse response. $h_c(n)$ of the overall system.

$$h_c(n) = h_1(n) * h_2(n) * h_3(n)$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n)$$

$$= h(n)$$

(b) Does the order of the interconnection affect the overall system?

NO.

40 (a) prove and explain graphically the difference between the relations

$x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$ and $x(n) * \delta(n-n_0) = x(n-n_0)$

(a) $x(n)\delta(n-n_0) = x(n_0)$ Thus, only the value of $x(n)$ at $n=n_0$ is of interest.

$x(n) * \delta(n-n_0) = x(n-n_0)$ Thus, we obtain the shifted version of the sequence $x(n)$

(b) show that a discrete-time system, which is described by a convolution summation is LTI and relaxed.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$= h(n) * x(n)$$

Linearity: $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$

$x(n) = a x_1(n) + \beta x_2(n) \rightarrow y(n) = h(n) * x(n)$

$y(n) = h(n) * [a x_1(n) + \beta x_2(n)]$

$= a h(n) * x_1(n) + \beta h(n) * x_2(n)$

$= a y_1(n) + \beta y_2(n)$

Time Invariance

$x(n) \rightarrow y(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$$= \sum_{k} h(k)\, x(n-n_0-k)$$

$= y(n-n_0)$

(c) what is the impulse response of the system described by a convolution summation. $y(n) = x(n-n_0)$?

$h(n) = \delta(n-n_0)$

4. Two signals $x(n)$ and $v(n)$ are related through the following difference equation.

Design the block diagram realization of:

(a) The system that generates $x(n)$ when excited by $v(n)$.

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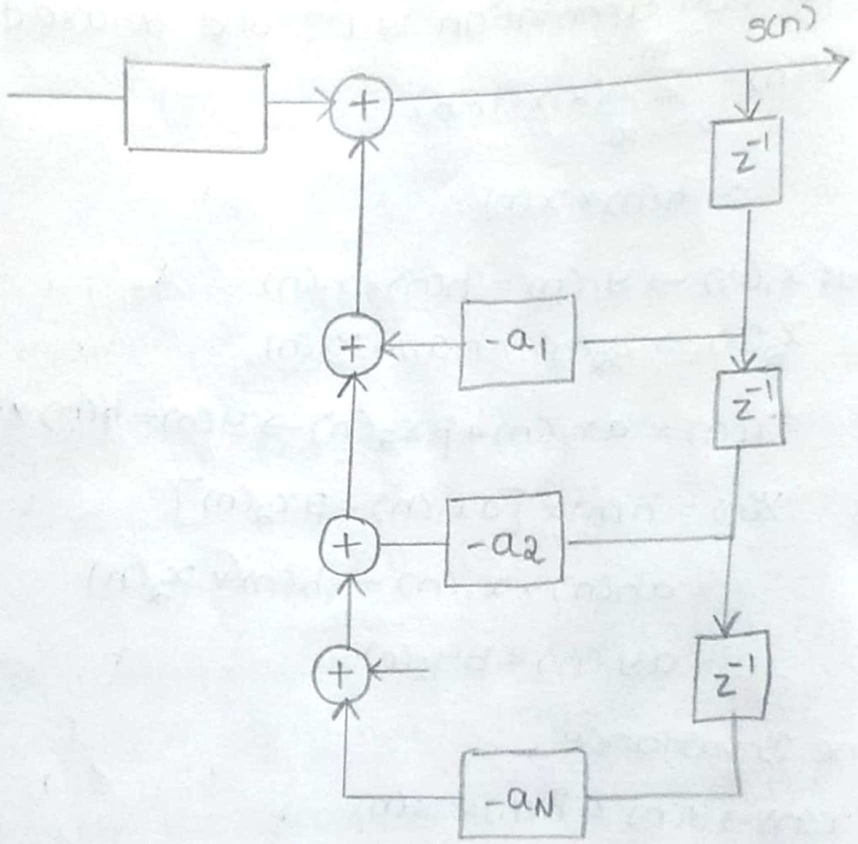
(a) $s(n) = -a_1 s(n-1) - a_2 s(n-2) - \cdots - a_N s(n-N) + b_0 v(n)$.

44

(b) The system that generates $v(n)$ when excited by $s(n)$.

$$v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \cdots + a_N s(n-N)\right]$$



42 compute the zero-state response of the system described by the difference equation

$$y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2) \text{ to the input}$$

$$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$$

$$y(n) = -\frac{1}{2}y(n-1) + x(n) + 2x(n-2)$$

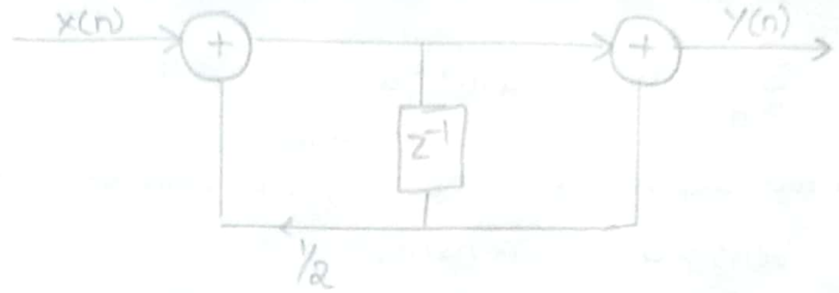
$$y(-2) = -\frac{1}{2}y(-3) + x(-2) + 2x(-4) = 1$$

$$y(-1) = -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = \frac{3}{2}$$

$$y(0) = -\frac{1}{2}y(-1) + 2x(-2) + x(0) = \frac{17}{4}$$

$$y(1) = -\frac{1}{2}y(0) + x(1) + 2x(-1) = \frac{47}{8}$$

44 consider the discrete time system shown.



(a) compute first 10 samples of its impulse response

$$x(n) = \{\underset{\uparrow}{1}, 0, 0\}$$

$$y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$$

$$y(0) = x(0) = 1,$$

$$y(1) = \frac{1}{2}y(0) + x(1) + x(0) = \frac{3}{2}$$

$$y(2) = \frac{1}{2}y(1) + x(2) + x(1) = \frac{3}{4}$$

$$y(n) = \left\{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32} \cdots\right\}$$

(b) Find the input-output relation

$$y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$$

(c) Apply the input $x(n) = \{\underset{\uparrow}{1}, 1, 1 \cdots\}$ and compute the first 10 samples of the output.

$$y(n) = \left\{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16} \cdots\right\}$$

(d) compute the first 10 samples of the output for the input given in part (c) by using convolution

$$y(n) = u(n) * h(n)$$

$$= \sum_{k}^{n} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k)$$

$$y(0) = h(0) = 1$$

$$y(1) = h(0) + h(1) = \frac{5}{2}$$

$$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}.$$

(e) Is the system causal? Is it stable?

$h(n) = 0$ for $n<0$ the system is causal.

$$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}\left(1+\frac{1}{2}+\frac{1}{4}+\cdots\right) = 4 \Rightarrow \text{system is stable}$$

45 consider the system described by the difference equa

$$y(n) = ay(n-1) + bx(n)$$

(a) Determine b in terms of a so that $\sum_{n=-\infty}^{\infty} h(n) = 1$

$$y(n) = ay(n-1) + bx(n)$$

$$\Rightarrow h(n) = ba^n u(n)$$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$$

$$\Rightarrow b = 1-a$$

(b) compute the zero state step response $s(n)$ of the system and choose b so that $s(\infty) = 1$

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= b\left[\frac{1-a^{n+1}}{1-a}\right]u(n)$$

$$s(\infty) = \frac{b}{1-a} = 1$$

$$\Rightarrow b = 1-a$$

(c) compare the values of b obtained in parts (a) and (b). what did you notice?

$b = 1-a$ in both the cases.

46 A discrete-time system is realized by the structure shown

(a) Determine the impulse response.

$$y(n) = 0.8\,y(n-1) + 2x(n) + 3x(n-1)$$

$$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$$

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8$$

$$y_h(n) = c(0.8)^n$$

$$y(n) - 0.8y(n-1) = x(n)$$

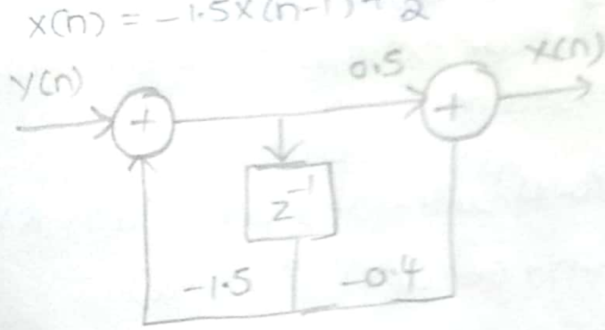
$x(n) = \delta(n) \quad y(0) = 1 \quad c = 1$

$$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$$

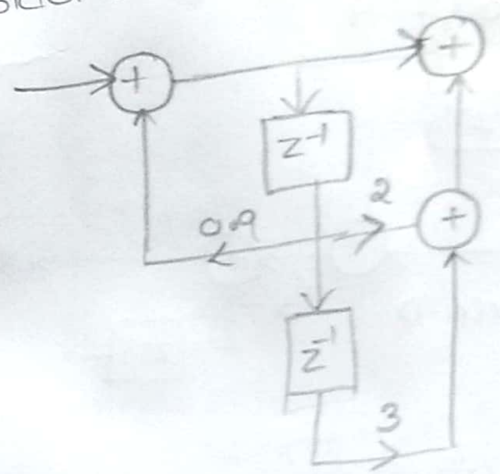
$$= 2\delta(n) + 4.6\,(0.8)^{n-1} u(n-1)$$

(b) Determine the realization for its inverse system, that is, the system which produces x(n) as an output when y(n) is used as an input

$$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1)$$



47 consider the discrete time system shown.



(a) compute the first six values of the impulse response of the system.

$$y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$$

$x(n) = \delta(n)$

$y(0) = 1$

$y(1) = 2.9$

$y(2) = 5.61$

$y(3) = 5.049$

$y(4) = 4.544$

$y(5) = 4.090$

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(b) compute the first six values of the zero-state step response of the system.

$S(0) = Y(0) = 1$

$S(1) = Y(0) + Y(1) = 3.91$

$S(2) = Y(0) + Y(1) + Y(2) = 9.51$

$S(3) = Y(0) + Y(1) + Y(2) + Y(3) = 14.56.$

$S(4) = \sum_0^4 Y(n) = 19.10$

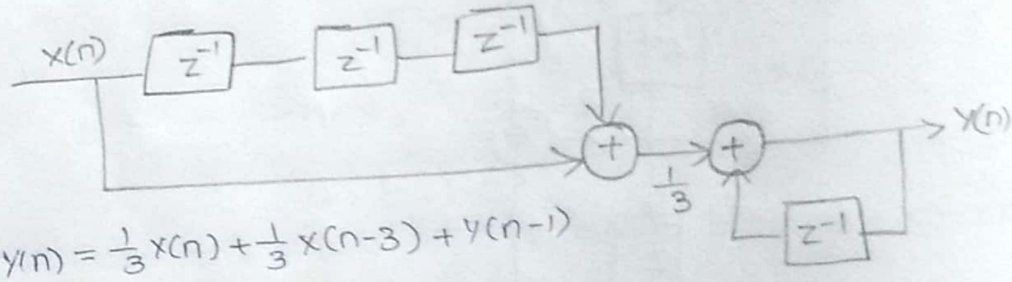
$S(5) = \sum_0^5 Y(n) = 23.19$

(c) Determine the analytical expression for the impulse response of the system.

$h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$= \delta(n) + 2.9\delta(n-1) + 5.61(0.9)^{n-2} u(n-2).$

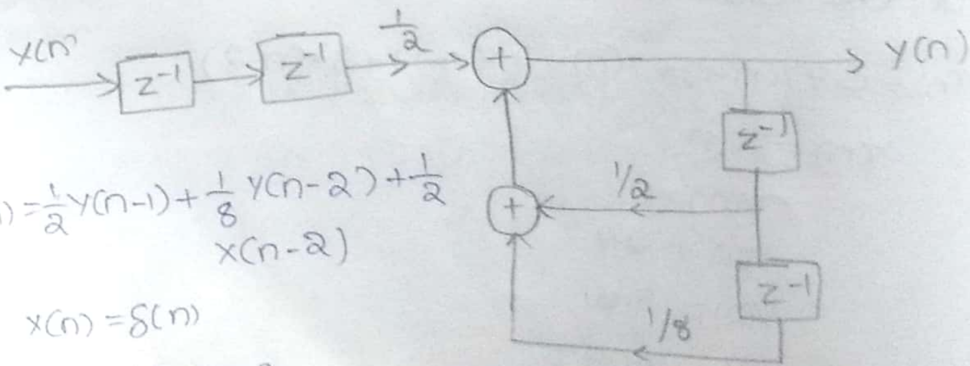
48 Determine and sketch the impulse response of the following systems for $n = 0, 1, \ldots 9.$



$y(n) = \frac{1}{3}x(n) + \frac{1}{3}x(n-3) + y(n-1)$

for $x(n) = \delta(n)$

$h(n) = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3} \ldots\}$

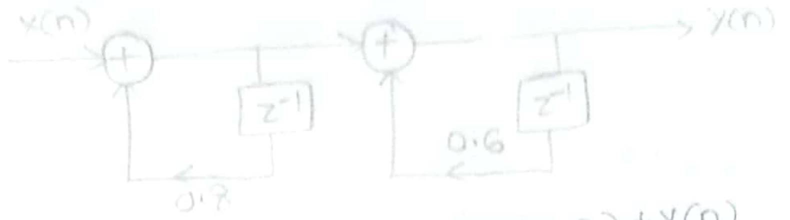


$Y(n) = \frac{1}{2}Y(n-1) + \frac{1}{8}Y(n-2) + \frac{1}{2}x(n-2)$

$x(n) = \delta(n)$

$Y(-1) = Y(-2) = 0$

$h(n) = \{0, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \frac{15}{256}, \frac{41}{1024} \ldots\}$



$y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

$x(n) = \delta(n)$

$y(-1) = y(-2) = 0$

$h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086 \ldots\}$

d. classify the systems above as FIR and IIR.

All three are IIR.

e. Find an explicit expression for the impulse response of the system in part (c).

$y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

$\lambda^2 - 1.4\lambda + 0.48 = 0$

$\lambda = 0.8, 0.6$

$y_h(n) = c_1(0.8)^n + c_2(0.6)^n \quad x(n) = \delta(n)$

$c_1 + c_2 = 1$

$0.8c_1 + 0.6c_2 = 1.4$

$c_1 = 4$

$c_2 = -3$

$h(n) = [4(0.8)^n - 3(0.6)^n]\, u(n)$

49 consider the systems shown in Fig.

(a) Determine and sketch their impulse responses $h_1(n)$ $h_2(n)$ & $h_3(n)$

$h_1(n) = c_0\delta(n) + c_1\delta(n-1) + c_2\delta(n-2)$

$h_2(n) = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2)$

$h_3(n) = a_0\delta(n) + (a_1 + a_0a_2)\delta(n-1) + a_1a_2\delta(n-2)$

(b) Is it possible to choose the coefficients of these systems in such a way that

$h_1(n) = h_2(n) = h_3(n)$

$h_3(n) = h_2(n) = h_1(n)$

$a_0 = c_0$

$a_1 + a_2 c_0 = c_1$

$a_2 a_1 = c_2$

$\frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$c_0 a_2^2 - c_1 a_2 + c_2 = 0$

$c_0 \neq 0$ the quadratic has a real solution if and only if

$c_1^2 - 4 c_0 c_2 \geq 0$

50 consider the system shown

(a) Determine its impulse response $h(n)$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(n) - \frac{1}{2} y(n-1) = \delta(n)$

$h(n) = (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1} u(n-1)$

(b) show that $h(n)$ is equal to the convolution of the following signals.

$h_1(n) = \delta(n) + \delta(n-1)$

$h_2(n) = (\frac{1}{2})^n u(n)$

$h_1(n) * [\delta(n) + \delta(n-1)] = (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1} u(n-1)$

51 compute the sketch the convolution $y_1(n)$ and correlation $r_1(n)$ sequences for the following pair of signals and comment on the results.

(a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$ $h_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

convolution $y_1(n) = \{1, \underset{\uparrow}{3}, 7, 7, 7, 6, 4\}$

correlation $r_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

(b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$ $h_2(n) = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}\}$

convolution $y_2(n) = \{\frac{1}{2}, 0, \frac{3}{2}, -2, \frac{1}{2}, -6, \frac{-5}{2}, -2\}$

correlation $r_1(n) = \{\frac{1}{2}, 0, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\}$

$y_2(n) = r_2(n) \quad \because h_2(-n) = h_2(n)$

conv $y_3(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

corr $r_1(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

(c) $x_3(n) = \{1, 2, 3, 4\}$ $h_4(n) = \{1, 2, 3, 4\}$

$y_4(n) = \{1, 4, 10, 20, 25, 24, 16\}$

$r_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$

52 The zero state response of a causal LTI system to the input $x(n) = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y(n) = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$ Determine its impulse response.

$h(n) = 2$

$h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$

$h_0 = 1 \quad h_1 = 1$

53 prove by direct substitution the equivalence of equation (which describe the direct to the relation which describes the direct form I structure

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$w(n) = -\sum_{k=1}^{N} a_k w(n-k) + x(n)$$

$$y(n) = \sum_{k=0}^{M} b_k w(n-k)$$

$$x(n) = w(n) + \sum_{k=1}^{N} a_k w(n-k)$$

LHS = RHS

54 Determine the response $y(n), n \geq 0$ of the system described

by the second-order difference equation

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ i/p is

$x(n) = (-1)^n u(n)$

and the initial conditions are $y(-1) = y(-2) = 0$

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$

$y_h(n) = c_1 2^n + c_2 n 2^n$

$y_p(n) = K(-1)^n u(n)$

$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) -$

$(-1)^{n-1} u(n-1)$

$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$

$y(0) = 1 \quad y(1) = 2$

$c_1 + \frac{2}{9} = 1$

$c_1 = \frac{7}{9}$

$2c_1 + 2c_2 - \frac{2}{9} = 2$

$c_2 = \frac{1}{3}$

55 Determine the impulse response $h(n)$ for the system described by the second order difference equation

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$h(n) = [c_1 2^n + c_2 n 2^n] u(n)$

$y(0) = 1 \quad y(1) = 3$

$c_1 = 1$

$2c_1 + 2c_2 = 3$

$c_2 = \frac{1}{2}$

$h(n) = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$

56 Show that any discrete-time signal $x(n)$ can be expressed

as $\quad x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$

where $u(n-k)$ is a unit step delayed by k units in time, that is

$$u(n-k) = \begin{cases} 1 & n \geq k \\ 0 & \text{otherwise} \end{cases}$$

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * [u(n) - u(n-1)]$$

$$= [x(n) - x(n-1)] * u(n)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$$

7 show that the output of an LTI system can be expressed in terms of its unit step response $s(n)$ as follows.

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$

$$= \sum [x(k) - x(k-1)] s(n-k)$$

$h(n)$ impulse response

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = s(k) - s(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$

compute the correlation sequences $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequences.

$$x(n) = \begin{cases} 1 & n_0 - N \leq n \leq n_0 + N \\ 0 & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1 & -N \leq n \leq N \\ 0 & \text{otherwise} \end{cases}$$

$$x(n) = \begin{cases} 1 & n_0 - N \leq n \leq n_0 + N \\ 0 & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1 & -N \leq n \leq N \\ 0 & \text{otherwise} \end{cases}$$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

$$n_0 - N \le n \le n_0 + N$$

$$n_0 - N \le n-l \le n_0 + N$$

$$-2N \le l \le 2N$$

$$r_{xx}(l) = \begin{cases} 2N+1-|l| & -2N \le l \le 2N \\ 0 & \text{otherwise} \end{cases}$$

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0| & n_0-2N \le l \le n_0+2N \\ 0 & \text{otherwise.} \end{cases}$$

59 Determine the autocorrelation sequences of the following signals.

(a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

$$r_{xx}(-3) = x(0)x(3) = 1$$

$$r_{xx}(-2) = x(0)x(2) + x(1)x(3) = 3$$

$$r_{xx}(-1) = x(0)x(1) + x(1)x(2) + x(2)x(3) = 5$$

$$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$$

$$r_{xx}(-l) = r_{xx}(l)$$

$$r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$$

(b) $y(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$

$$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)y(n-l)$$

$$r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$$

$$y(n) = x(-n+3)$$

60 What is the normalized autocorrelation sequence of the signal $x(n)$ given by.

$$x(n) = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

$$= \begin{cases} 2N+1-|l| & -2N \le l \le 2N \\ 0 & \text{otherwise} \end{cases}$$

$$r_{xx}(0) = 2N+1$$

$$\rho_{xx}(l) = \frac{1}{2N+1}(2N+1-|l|) \quad -2N \le l \le 2N$$

$$= 0 \quad \text{otherwise.}$$

6 An Audio signal $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coefficients r_1 and r_2. The signal $x(t)$ recorded by a microphone close to the loudspeaker, after sampling is

$$x(n) = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)$$

where k_1 and k_2 are the delays of the two echoes

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

$$= \sum [s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)] * [s(n-l) + r_1 s(n-l-k_1) + r_2 s(n-l-k_2)]$$

$$= (1+r_1^2+r_2^2) r_{ss}(l) + r_1 [r_{ss}(l+k_1) + r_{ss}(l-k_1)] + r_2 [r_{ss}(l+k_2) + r_{ss}(l-k_2)] + r_1 r_2 [r_{ss}(l+k_1-k_2) + r_{ss}(l+k_2-k_1)]$$

(b) can we obtain r_1 r_2 k_1 k_2 by observing $r_{xx}(l)$?

$r_{xx}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$

$k_1 < k_2$

(c) what happens if $r_2 = 0$

the peak occur at $l=0$ and $l = \pm k_1$. easy to obtain r_1 and k_1.

52. Time delay estimation in radar - Let $x_a(t)$ be transmitted signal and $y_a(t)$ be the received signal in a radar system. where

$$y_a(t) = a x_a(t - t_d) + v_a(t).$$

$$x(n) = x_a(nT)$$

$$y(n) = y_a(nT) = a x_a(nT - DT) + v_a(nT)$$

$$\triangleq a x(n-D) + v(n)$$