P2P Incentive Model On Evolutionary Game Theory

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Abstract: Conventional research on P2P incentive mechanism is based on cooperative or non-cooperative game theory. This paper assumes the node in the P2P system is limited rationality, using evolutionary game theory to study P2P incentive mechanism, we first investigate individual population P2P incentive model and model the node replicator dynamics. Mathematics analysis and numerical result shows that the individual population P2P system is stable when the node plays game as the incentive model we designed. Then we investigate the condition that the node in the P2P system with migration ability. We assume the node is divided into the different subpopulation according to the node dynamic property, we model the P2P system replicator dynamics, prove the incentive model is stable even when existing the strong dynamic node in the system. Mathematics analysis result shows that the P2P system is stable when the node plays the game as the incentive model we designed whatever the P2P system with the nature of dynamic and self-organized property.

Keywords: P2P; Incentive Model; Evolutionary game theory;

Replicator dynamics;

I. INTRODUCTION

P2P system is self-organized, distributed network based on network layer, because of the characteristic of self-organized, existing the node in the P2P system is selfish and maximize his own network utility, thus generate the free-riding problem: most users of the system contributes nothing to the system, the system run only on a small quantity of user self-giving contribution. Investigate show that nearly 70% of Gnutella users do not share any files in the P2P system. About 50% of all file searching responses come from the top 1% of information sharing node[1].

To inspirit node to share his own resource to guarantee P2P network performance stable, solving the problem of the P2P system node selfish behavior, there have a series of interrelated research about the topic. The author in [2] proposed a P2P resource share non-cooperative incentive model, the node contributes resource by computing his utility function to maximize his payoff, it also proves the NE of the model. In [3], the author applied economic theory to investigate P2P system and propose a P2P system cooperation framework on cooperation theory. In [4], the author investigates P2P network message relay incentive mechanism, put forward a incentive framework to avoiding "free riding" and make message relay more efficiency. In [5], the author investigates P2P network node collaboration, prove the collaboration model NE. There are other paper in the P2P incentive mechanism, which can be see in [6],[7],[8],[9],[10],[11],[12],[13].

To the best of our knowledge almost all the work about P2P incentive mechanism was done on non-cooperative or cooperative game theory, which assumes the node in P2P system is completely rational. The node is completely rational meaning that the node has the ability to analysis and judge the condition of others player in the game, any player limited rationality and ability can result in not achieving in NE, so the hypothesis that the node in the game is completely rational may not be true in the real environment. In the real P2P system, the player in the game only has limited rationality at most case. Evolutionary game theory is used to model the player having limited rationality in the game, thus there is give it a chance for to analysis P2P system incentive mechanism, so we can use evolutionary game theory to investigate P2P incentive model. In this paper, we assume the node has limited rationality, use evolutionary game theory to investigate P2P network incentive mechanism, and the evolutionary game model on individual population and population with migration was proposed, basis on them we give the replicator dynamics of the models, prove the stability of the model, thus establish a novel P2P system incentive model on evolutionary game theory.

The rest of the paper is organized as follows. Section II introduce the evolutionary game theory and proposes individual population game model, put forward the model replicator dynamics, mathematics and numerical analysis prove the model is stable. Section III proposes node with migration P2P system incentive model, put forward the model replicator dynamics equation, and mathematics analysis the model is stable even the system with strong dynamic node. Section IV conclude the paper and give the future direction.

II. INDIVIDUAL POPULATION INCENTIVE MODEL ON EVOLUTIONARY GAME THEORY

A. Evolutionary game theory model

Evolutionary game theory[14] is first proposed by Maynard Smith, it integrates the game theory analysis with dynamic evolution process to investigate complicated system, thus offer a tool for interpret individual's organic relation in the complicated system. Denote $G=\{I,S,\pi\}$ as a basic evolutionary game model, where I denote the set of player, S denote the player strategy set, π denote the player utility function set. The key concept of the evolutionary game model is evolutionary stable strategy(ESS). If $y \neq x \in S$, exists $\bar{\varepsilon}_y \in (0,1)$, stratify the equation :

$$u[x,\varepsilon y+(1-\varepsilon)x] > u[y,\varepsilon y+(1-\varepsilon)x] \qquad (1)$$

Here $\varepsilon \in (0,\bar{\varepsilon}_y)$, then x is the ESS of the model.

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B. Individual Population P2P Incentive Model

We assume P2P system has N node at period t, they are paired randomly to play at each fixed slot, the node strategy space is {0,1},strategy 0 denote the node contribute his own resource to others simultaneity download resource from others, strategy 1 denote the node only download resource from others but contribute no resource to others. We assume each node contribute D_s unit resource when he playing with strategy 0, the income of contribute one unit is c, assume each node download D_d whatever he play with any strategy, the cost of download one unit is g. According the game rule, we can write node playing with strategy 0 payoff matrix: $A=\begin{pmatrix} -gD_d+cD_s & cD_s \\ -gD_d & 0 \end{pmatrix}$, then the node play with strategy 1 playoff matrix: $B=A^T$. Let $\gamma_0(t)$ denote the node number playing with strategy 0 at period t, let $\gamma_1(t)$ denote the node number playing with strategy 1 at period t, then we can get

$$\gamma(t)=\gamma_0(t)+\gamma_1(t)\,. \tag{2}$$

Let $x(t)=\dfrac{\gamma_0(t)}{\gamma(t)}$ denote the node fraction playing with strategy 0, then $1-x(t)$ denote the node percent playing with strategy 1.

C. Individual Population P2P Incentive Model Replicator Dynamics Equation

We assume each stage game start at period $kt(k\in N)$, end at period $(k+1)t$, individuals node receive average payoffs with respect to all possible opponents , We assume that during the small time interval ε , only an ε fraction of the population plays games. The node number playing with strategy I at time $t+\varepsilon$ can write as follow :

$$\gamma_i(t+\varepsilon)=(1-\varepsilon)\gamma_i(t)+\varepsilon\gamma_i(t)U_i(t);\ i=0,1 \tag{3}$$

$U_0(t)=-gD_dx(t)+cD_s, U_1(t)=-gD_dx(t)$ is node playing with strategy 0 and 1 average payoff. The P2P system node number at time $t+\varepsilon$ can write as follow:

$$\gamma(t+\varepsilon)=(1-\varepsilon)\gamma(t)+\varepsilon\gamma(t)\overline{U}(t) \tag{4}$$

Here $\overline{U}(t)=(cD_s-gD_d)x(t)$. It is the average payoff value at period t. Divided (3) by (4), we obtain an equation for the frequency of the strategy 0 :

$$x(t+\varepsilon)-x(t)=\varepsilon\frac{x(t)[U_0(t)-\overline{U}(t)]}{1-\varepsilon+\varepsilon\overline{U}(t)} \tag{5}$$

Now we divide both sides of (5) by ε , perform then $\lim\varepsilon\to 0$, and get the replicator equation :

$$\frac{dx(t)}{dt}=x(t)[U_0(t)-\overline{U}(t)]=cD_sx(t)(1-x(t)) \tag{6}$$

D. Individual Population P2P Incentive Model Stable Analysis

We assume most node in the P2P system contribute his own resource and download the resource from others, that is say $x(t)\gg 1/2$, but even only $x(t)>0$. According to the node replicator dynamics equation (6) we can get $\frac{dx(t)}{dt}>0$, that is to say $x(t)$ is monotonically increasing function, when the game time is infinite, $x(t)$ is very close to 1, it means that P2P system's node will play the game as the incentive designed.

Theorem 1: P2P system is stable when the node plays the incentive model as we designed.

We assume $c=2$, $D_s=0.5$, using Matlab we can plot the fraction of node playing with strategy 0. From figure 1 we can see when play time increasing, the fraction of node playing with strategy 0 is very close 1.

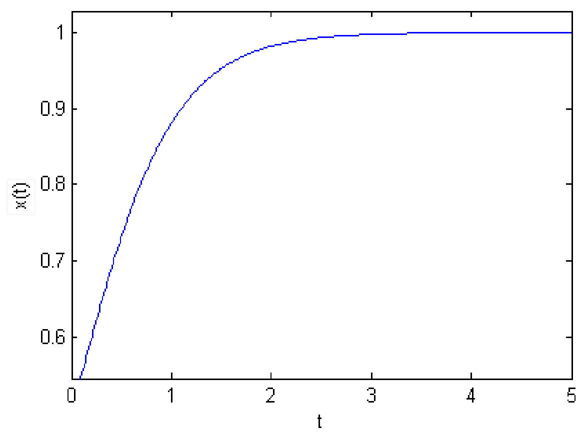

Figure 1 The fraction of node number play with strategy 0

III. P2P INCENTIVE MODEL WITH NODE MIGRATION ON EVOLUTIONARY GAME THEORY

A. Incentive model with node migration

In the previous section we investigate the individual population P2P incentive model, in this section we investigate the P2P system with migration ability. According to the P2P system dynamic, we can model the P2P system as a biology evolution system with migration. Consider P2P system has N node at period t, P2P system node at t+1 period is replicated and migrated by the node at period t. Accordingly to node replicate rate we can divide P2P system node into two subpopulation, $\gamma_1(t)$ denote the first subpopulation (the node is relative stable in the system) node number, $\gamma_2(t)$ denote the second subpopulation(the node is high dynamic in the system) node number, node in different subpopulation can migrate to other class according to the payoff in the stage game. Assumed the node strategy space is {0,1}, strategy 0 denote the node contribute his own resource to others simultaneity download resource from others, strategy 1 denote the node only download resource from others but contribute no resource to others. We assume each node contribute D_s unit resource when he playing with strategy 0, the income of contribute one unit is c, assume each node download D_d whatever he play with any strategy, the cost of download one unit is g. We assume that during any time step ε , a fraction ε of the first subpopulation and a fraction $k\varepsilon$ of the second subpopulation play the game and receive payoffs which are interpreted as the number of their offspring. Moreover, we allow a fraction of individuals to migrate to a subpopulation in which their strategies have higher expected payoffs. The node which play with strategy 0 payoff matrix: $A=\begin{pmatrix} -gD_d+cD_s & -mcD_s \\ -gD_d & 0 \end{pmatrix}$, the node which play with strategy 1 payoff matrix: $B=A^T$, here we assume that $m\le 1$, $cD_s-gD_d>0$.

B. Incentive model with node migration Replicator Dynamics Equation

We let $\gamma_s^i(t)$ denote the number of node which use the strategy s in the subpopulation I. Let $x=\frac{\gamma_0^1(t)}{\gamma_1(t)}$ denote the fraction of node in the first subpopulation of the P2P system playing with strategy 0, let $y=\frac{\gamma_0^2(t)}{\gamma_2(t)}$ denote the fraction of the node in the second subpopulation of the P2P system playing with strategy 0, the first subpopulation and the second subpopulation node number at time t can write: $\gamma_1(t)=\gamma_0^1(t)+\gamma_1^1(t), \gamma_2(t)=\gamma_0^2(t)+\gamma_1^2(t)$.

Let $\alpha=\frac{\gamma_1(t)}{\gamma(t)}$ denote the first subpopulation node fraction of the P2P system, let U_s^i denote the subpopulation I playing with strategy s excepted payoff, then we can get the value of the U_s^i as follow:

$$U_0^1(t)=(x(1+m)-m)cD_s-gD_d x \tag{7}$$

$$U_1^1(t)=-gD_d x \tag{8}$$

$$U_0^2(t)=(y(1+m)-m)cD_s-gD_d y \tag{9}$$

$$U_1^2(t)=-gD_d y \tag{10}$$

We assume the node number change at period $t+\varepsilon$ is due to the node replicator and the node migration. The node number of class I playing with strategy s equal the node number of replicator plus the node migration from others, thus we can deinfed the evolution equation as follow:

$$\gamma_s^i(t+\varepsilon)=R_s^i(t)+\phi_s(t) \tag{11}$$

Here R_s^i denote an increase of the number of the node playing the strategy s in the subpopulation i due to the replication, the rate of the replication of individuals playing the strategy s in the first subpopulation is given by εU_s^1 , and in the second subpopulation by $k\varepsilon U_s^2$ .The parameter k measures the difference of reproduction speeds in both subpopulation. According to the payoff matrix the function can write as follow:

$$R_0^1(t)=(1-\varepsilon)\gamma_0^1(t)+\varepsilon((x(1+m)-m)cD_s-gD_d x)\gamma_0^1(t) \tag{12}$$

$$R_1^1(t)=(1-\varepsilon)\gamma_1^1(t)-\varepsilon gD_d x\gamma_1^1(t) \tag{13}$$

$$R_0^2(t)=(1-\varepsilon)\gamma_0^2(t)+k\varepsilon((y(1+m)-m)cD_s-gD_d y)\gamma_0^2(t) \tag{14}$$

$$R_1^2(t)=(1-\varepsilon)\gamma_1^2(t)-k\varepsilon gD_d y\gamma_1^2(t) \tag{15}$$

Functions $\phi_s(t)$ denote changes of the numbers of the individuals playing strategy s in the relevant subpopulation due to migration. $\phi_s(t)$ will be referred to as the migration of node (who play the strategy s) between two subpopulations. The function can write as follows :

$$\phi_s(t)=\varepsilon\gamma(U_s^1(t)-U_s^2(t))[\gamma_s^2(t)\theta(U_s^1(t)-U_s^2(t))+\gamma_s^1(t)\theta(U_s^2(t)-U_s^1(t))] \tag{16}$$

Here $\theta(x)=\begin{cases}1, x\ge 0\\ 0, x<0\end{cases}$, γ is the migration rate.

We assume $U_0^1(t)>U_0^2(t)$, according to the (11), we can get the P2P system node number function as follow:

$$\gamma_1(t+\varepsilon)=(1-\varepsilon)\gamma_1(t)+\varepsilon\gamma_1(t)[\frac{\gamma_0^1(t)(x(1+m)-m)cD_s}{\gamma_1(t)} -gD_d x+r(x-y)\frac{\gamma_0^2(t)((1+m)cD_s-gD_d)-gD_d\gamma_1^1(t)}{\gamma_1(t)}] \tag{17}$$

$$\gamma_2(t+\varepsilon)=(1-k\varepsilon)\gamma_2(t)+\varepsilon\gamma_2(t)[k\frac{((1+m)cD_s-gD_d)}{\gamma_2(t)} -kgD_d y+r(y-x)\frac{\gamma_0^2(t)((1+m)cD_s-gD_d)-gD_d\gamma_1^1(t)}{\gamma_2(t)}] \tag{18}$$

$$\gamma(t+\delta)=(1-\varepsilon)\gamma_1(t)+(1-k\delta)\gamma_2(t)+\delta\gamma(t)[\alpha((1+m)x-m)cD_s)\frac{\gamma_0^1}{\gamma_1} -gD_d x)+(1-\alpha)k(((1+m)y-m)cD_s)\frac{\gamma_0^2}{\gamma_2}-gD_d y)] \tag{19}$$

We divide (12) by (17), (14) by (18), (17) by (19), then performing $\lim\varepsilon\to 0$ we obtain the following differential equations of node number:

$$\frac{dx}{dt}=x[(1-x)(x(1+m)-m)cD_s+r(x-y) [(\frac{y(1-\alpha)(1-x)}{x\alpha})((1+m)cD_s-gD_d)+gD_d(1-x)]] \tag{20}$$

$$\frac{dy}{dt}=y[kcD_s(1-y)(x(1+m)-m)+r(y-x) [(1-y)((1+m)cD_s-gD_d)+gD_d\frac{(1-x)\alpha}{1-\alpha}]] \tag{21}$$

$$\frac{d\alpha}{dt}=\alpha(1-\alpha)[((x(1+m)-m)cD_s-gD_d)x-$$

$$((y(1+m)-m)cD_s-gD_d)y]+\alpha\gamma[\frac{y(1-\alpha)}{\alpha}(x(1+m)-m)cD_s++gD_d(1-x)(x-y)]+\alpha(1-\alpha)(k-1)(1-((y(1+m)-m)cD_s-gD_d)y) \quad (22)$$

C. Incentive model with node migration stalbe analysis

We assume P2P system is stable at the play start and has more weak dynamic node than strong dynamic node, that is say: $x(0)>y(0)$, according to the differential equations of node fraction get in the previous section we can get equation as follow :

$$\frac{dx}{dt}>x(1-x)[(x(1+m)-m)cD_s+\gamma gD_d(x-y)] \quad (23)$$

We substitute U_s^i value into (23) can get as follow:

$$\frac{dx}{dt}>x(1-x)[(x(1+m)-m)cD_s+\gamma gD_d(x-y)]$$

Because $x(1+m)-m>0$, $x-y>0$, then we can get $\frac{dx}{dt}>0$.

By analogy we can get the second class equation as follow:

$$\frac{dy}{dt}<y(1-y)[k(y(1+m)-m)cD_s-\gamma((1+m)cD_s-gD_d)(x-y)]$$

, for the subpopulation is more dynamic, the replicator rate parameter k<<1, so we can get $\frac{dy}{dt}<0$, integrate two inequality we can show that $x(t)-y(t)$ is increasing function, we can get $x(t)\rightarrow_{t\to\infty}1, y(t)\rightarrow_{t\to\infty}0$ when time is infinite.

By analogy we can get, if $k<(1+gD_d-cD_s)$, then $\alpha(t)\rightarrow_{t\to\infty}1$, the analysis result show, though the P2P system is dynamic, if most of node in the system share the resource and download the resource from other at the same time, the P2P will be stale when the play time is infinite. **Theorem 2:** P2P system is stable when the node plays the incentive model as we designed even if the system existing dynamic and self-organize node.

IV. CONCLUSION

In this paper we use evolutionary game theory investigate P2P network node incentive model, we assume the node in the P2P system is limited rationality, use evolutionary game theory to investigate P2P incentive, we first investigate individual population P2P incentive model and model the node replicator dynamics, prove the model is stable when the P2P system plays game time is infinite. Mathematics analysis and numerical result shows that the individual population P2P system is stable when the node play game as the incentive model we designed. Then we investigate the condition that the node in the P2P system with migration ability. We assume the node is divided into the different subpopulation according to the node dynamic, we model the P2P system replicator dynamics, prove the incentive model is stable even when exist the strong dynamic node in the system. Mathematics analysis result shows that the P2P system is stable when the node plays the game like the incentive model we designed whatever the P2P system with the nature of dynamic and self-organized. Apply evolutionary game theory to investigate complicated P2P system, we only do some elementary work, we will investigate the complicated nature N population P2P system incentive model in the future.

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