=== Part 1: Discrete Least Squares Approximation ===

Given data points:

x = [4.0, 4.2, 4.5, 4.7, 5.1, 5.5, 5.9, 6.3]

y = [102.6, 113.2, 130.1, 142.1, 167.5, 195.1, 224.9, 256.8]

-- (a) Quadratic Fit (y = ax^2 + bx + c) --

Coefficients:

a = 6.691184

b = -1.883746

c = 3.086393

Sum of squared error: 0.005246

-- (b) Exponential Fit (y = b * e^(a * x)) --

Parameters:

b = 21.444544

a = 0.398495

Sum of squared error: 94.983021

-- (c) Power Fit (y = b * x^a) --

Parameters:

b = 6.238952

a = 2.019634

Sum of squared error: 0.011721

=== Part 2: Continuous Least Squares Polynomial Approximation ===

Interval: [-1, 1]

Function: f(x) = 0.5*cos(x) - 0.5*sin(x) + 0.25*x

Degree 2 least squares polynomial:

P2(x) = 0.498279 + -0.201753 * x + -0.232631 * x^2

=== Part 3: Discrete Least Squares Trigonometric Polynomial (S4) ===

Function: f(x) = x^2 * sin(x)

Interval: [0, 1]

Number of discrete points: 16

Trigonometric polynomial degree N = 4

Fourier coefficients:

a_0 = 0.395344

a_1 = 0.072827

b_1 = -0.237249

a_2 = -0.022262

b_2 = -0.123859

a_3 = -0.038390

b_3 = -0.077809

a_4 = -0.043865

b_4 = -0.052223

Integral of S4(x) over [0, 1]: 0.197672

Squared Error E(S4): 0.077402