数据挖掘算法

支持向量机(SVM)

0 课程安排及相关知识

- 推导SVM待优化的基本形式（☆）
  - 向量基本知识：向量内积、内积几何意义，向量范数计算方法等。
- 推导SVM待优化的进阶形式（☆☆☆☆）
  - 高数基本知识：函数求导，矩阵乘法。
  - 拉格朗日乘数法和KKT条件：基本原理，表现形式，求解方法。
    - 梯度知识：什么是梯度，梯度方向是什么方向?
- 使用SMO算法进行求解（☆☆☆☆）

基本原理

- 深度学习版本的SVM（☆☆☆）
  - 机器学习知识：损失函数，分类任务、回归任务，神经网络知识。
- SVM的程序调用（☆☆☆）
  - https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html
  - 参数讲解，其他应用中的相关知识。

拓展知识

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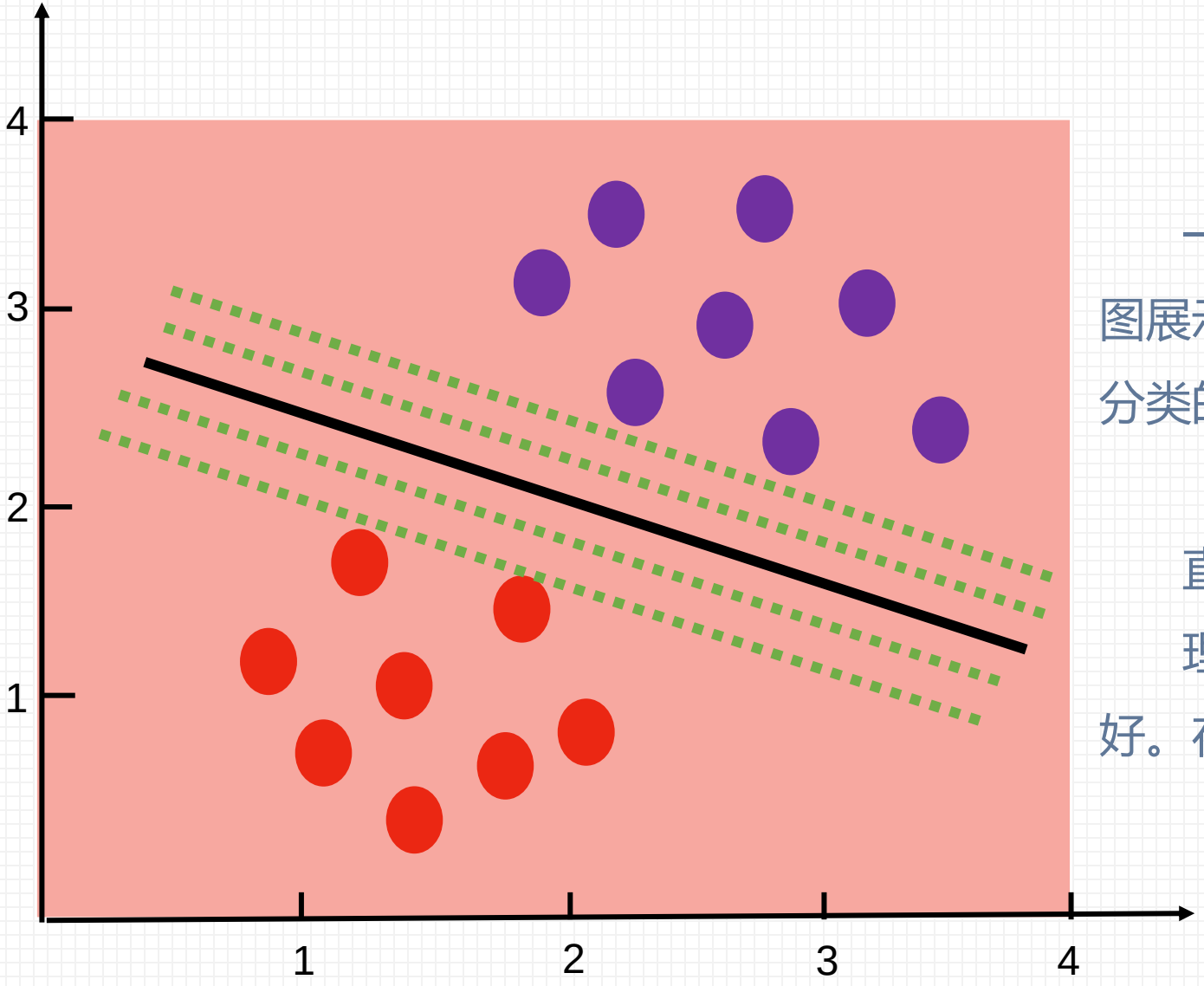
第一章节

推导SVM待优化的基本形式

（☆☆）

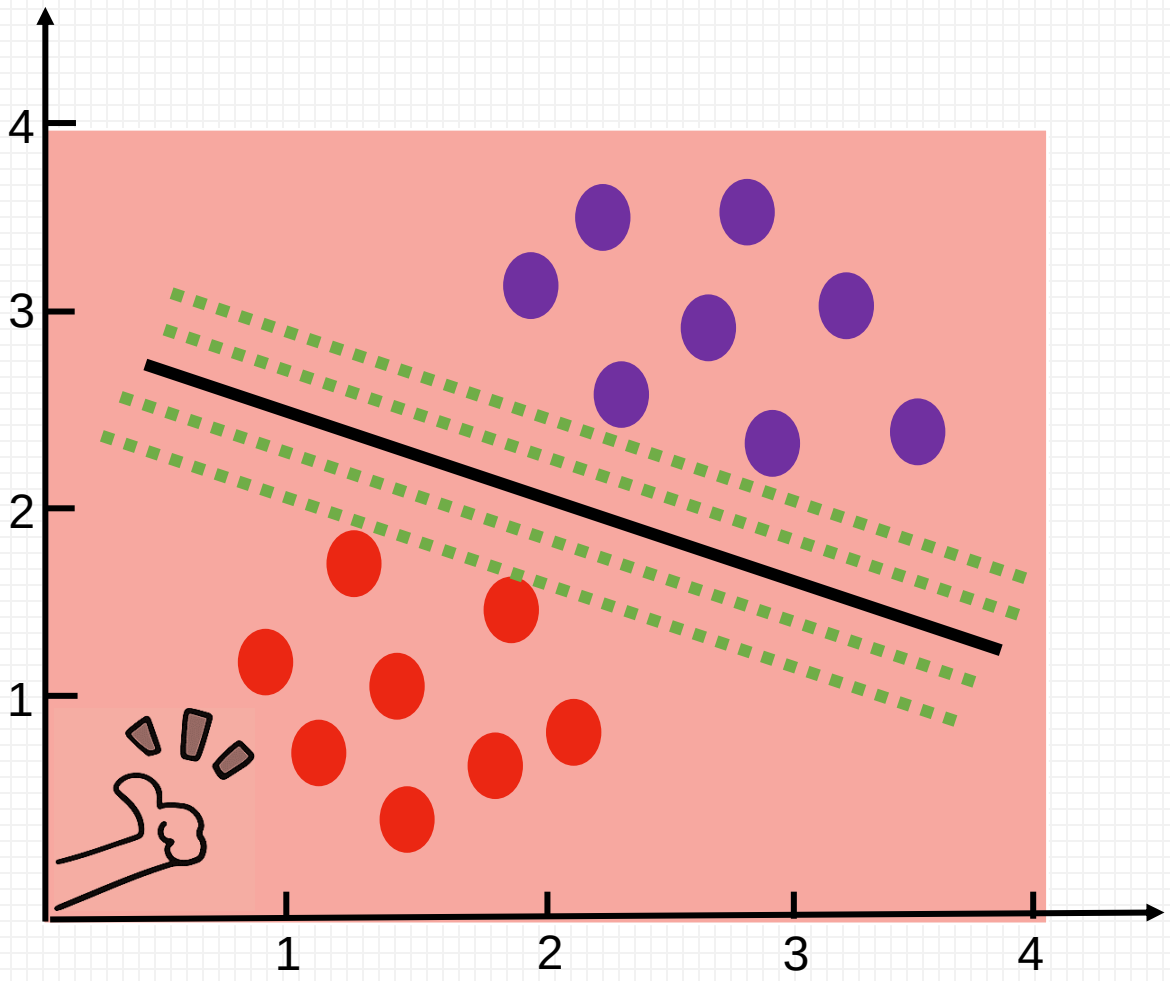


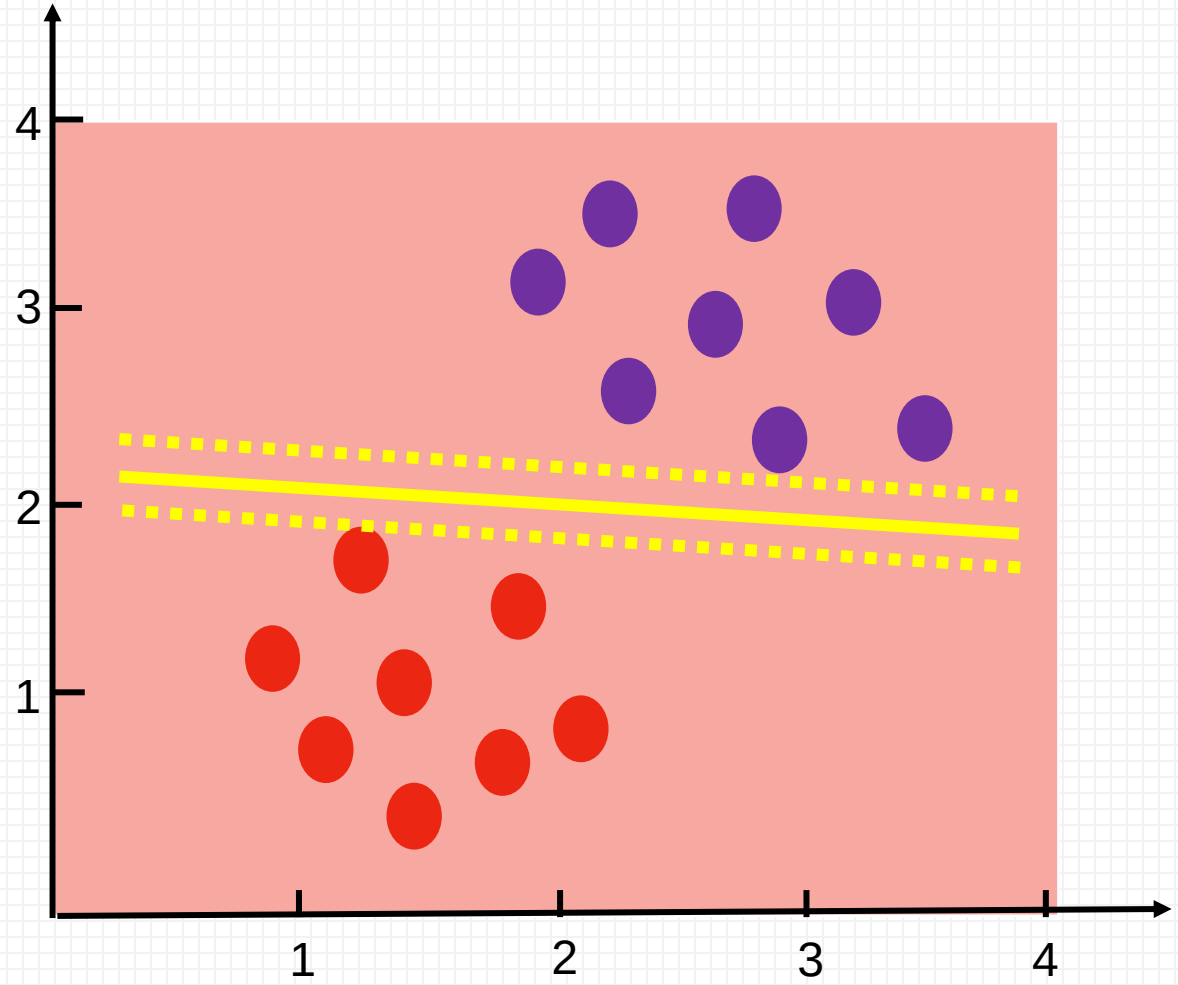
一组**线性可分**的数据，考虑一个问题，如右图展示的几条直线，都是能够将数据进行正确二分类的，问：哪个划分是最好的?

直觉告诉我：**黑色的是最好的。**

理由：黑色的看起来“容错性、健壮性”最好。在机器学习中也称“泛化性”更好 。

分割平面距离边界向量越远越好



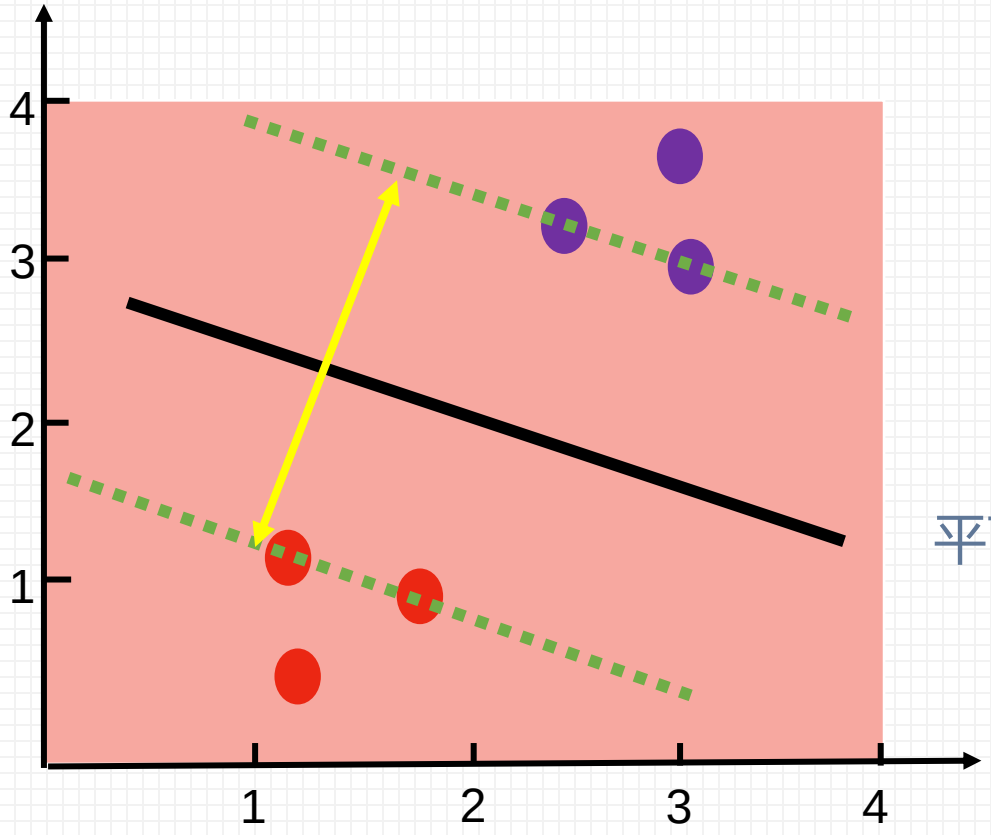


分割平面对应的几何间隔越大越好

N维空间的超平面

$$\boldsymbol{w^T x + b = 0}$$

其中w, x, b均为N维列向量。



绿色虚线：$w^T x + b = \pm k$

分类时：$w^T x + b \geq k$, 归为+1类，

$w^T x + b \leq -k$, 归为−1类。

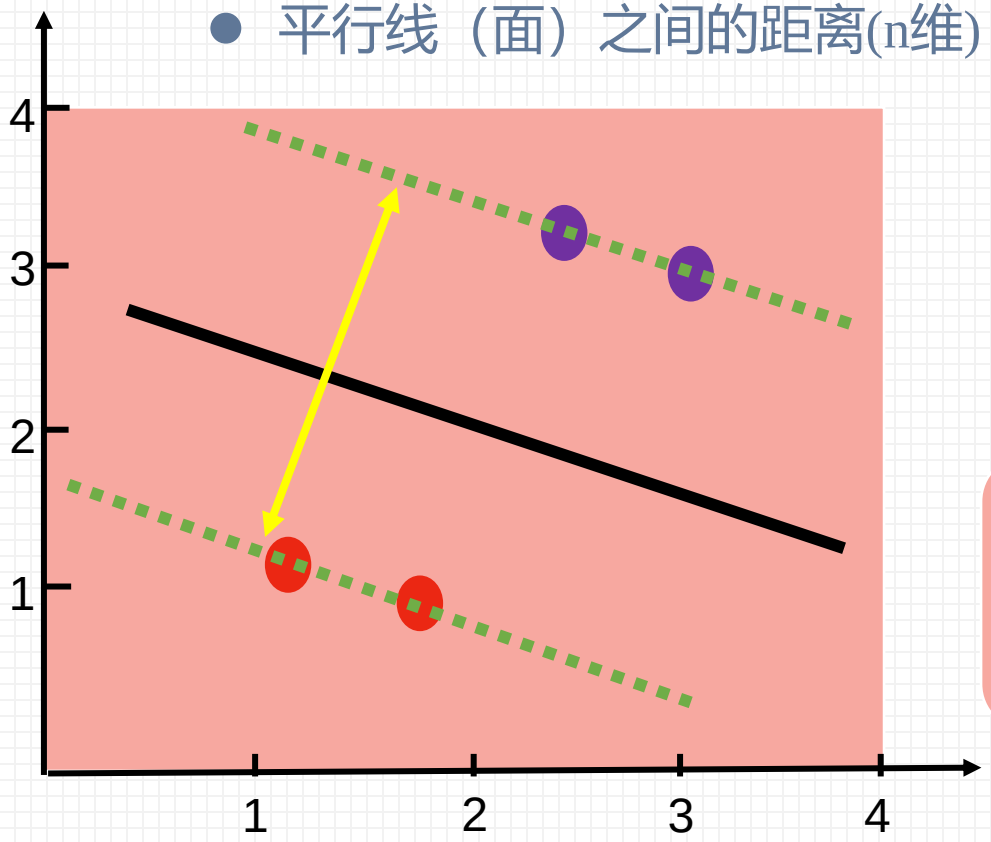
实际中：由于对超平面的权重系数做等规模放缩不影响超平面的性质。因此

$$W^T x + b = \pm k \Rightarrow \boldsymbol{w^T x + b = \pm 1}$$

找一组超平面的 w, b 使得两组绿色虚线间隔最大。

间隔的计算

- 2维距离：$ax + by + c = 0$，$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$
- 平行线（面）之间的距离(n维)：$w^T x + b = \pm 1$，$d = \frac{2}{\sqrt{w^T w}}$

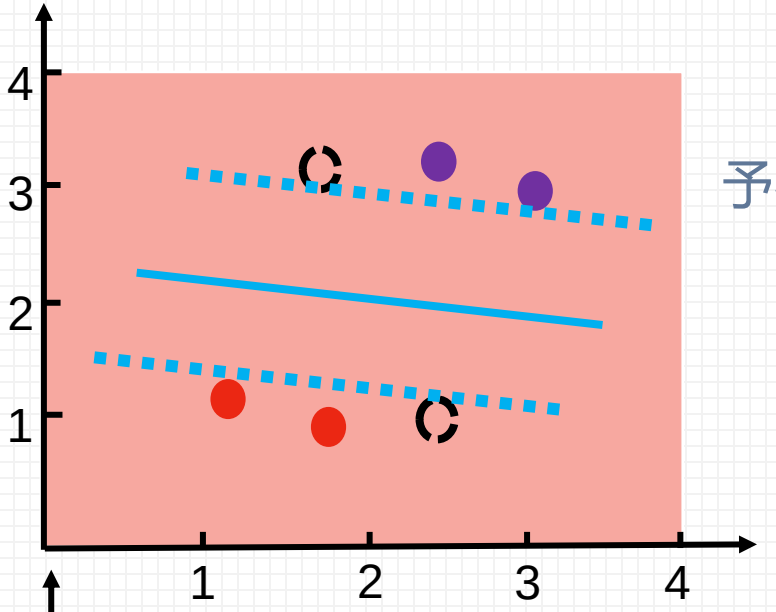


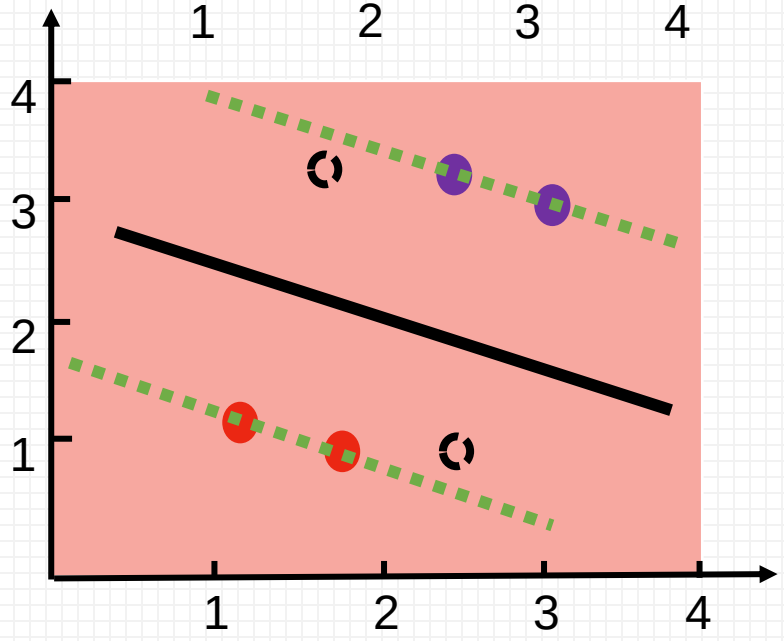
找一组超平面的 $\boldsymbol{w}$ $\boldsymbol{b}$ 使得两组绿色虚线间隔最大。

$$max\ \frac{2}{\sqrt{w^T w}} \rightarrow max\ \frac{2}{w^T w} \rightarrow min\ \frac{1}{2} w^T w$$

SVM基本形式1，n个样本(x_i, y_i)，$x_i \in R^N, y_i \in \{+1, -1\}$：

$$\boldsymbol{min\ \frac{1}{2} w^T w,\ s.t.\ y_i(w^T x_i + b) \geq 1}$$





也许一味地坚持硬性条件：$y_i(w^T x_i + b) \geq 1$，并不是最好的，也许给予模型一点点宽容，反而能带来模型的泛化能力的提高。所以约束条变为：

$$y_i(w^T x_i + b) \geq 1 - \xi_i \ ， \ \xi_i \geq 0$$

但是，ξ_i 整体又不能太大。体现为：

$$min \ \frac{1}{2} w^T w + C \sum_i \xi_i$$

C成为惩罚系数，平衡模型的“软硬程度”。

SVM基本形式2，n个样本(x_i, y_i)，$x_i \in R^N, y_i \in \{+1, -1\}$：

$$\boldsymbol{min \ \frac{1}{2} w^T w + C \sum_i \xi_i}$$

$$\boldsymbol{s.t. \ y_i(w^T x_i + b) \geq 1 - \xi_i \ ， \ \xi_i \geq 0}$$

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第二章节

推导SVM待优化的进阶形式

（☆☆☆☆）

硬间隔SVM目标： $min\ \frac{1}{2}w^Tw,\ \ s.t.\ y_i(w^Tx_i+b)\geq 1$

构造拉格朗日函数：

$$L(w,b,\alpha)=\frac{1}{2}w^Tw+\sum_i\alpha_i[1-y_i(w^Tx_i+b)]\ \ s.t.\ \ \alpha_i\geq 0$$

$$\because \max_{\alpha} L(w,b,\alpha)=\frac{1}{2}w^Tw$$

$$\therefore \min_{w,b}\frac{1}{2}w^Tw=\min_{w,b}\max_{\alpha}L(w,b,\alpha)$$

**对偶： $\boldsymbol{\min_{w,b}\max_{\alpha}L(w,b,\alpha)=\max_{\alpha}\min_{w,b}L(w,b,\alpha)}$

我们不加证明的指出上式在SVM问题中是成立的，并且相应的解满足KKT条件。

[1] 李航. 统计学习方法[M]. 清华大学出版社, 2012. 附录C。

[2] https://zhuanlan.zhihu.com/p/219284970

目标函数：

$$\max_{\alpha} \min_{w,b} L(w,b,\alpha),\ \ s.t.\ \ \alpha_i \geq 0$$

$$L(w,b,\alpha) = \frac{1}{2}w^T w + \sum_i \alpha_i [1 - y_i(w^T x_i + b)]$$

先求L关于w, b的极小值：

$$\frac{\partial L}{\partial w} = 0 \Rightarrow w - \sum_i \alpha_i y_i x_i = 0 \Rightarrow \boldsymbol{w = \sum_i \alpha_i y_i x_i}$$

$$\frac{\partial L}{\partial b} = 0 \Rightarrow -\sum_i \alpha_i y_i = 0 \Rightarrow \boldsymbol{\sum_i \alpha_i y_i = 0}$$

将L拆开，并将相关结果带入：

$$L(w,b,\alpha) = \frac{1}{2}w^T w + \sum_i \alpha_i - \sum_i \alpha_i y_i w^T x_i - b\sum_i \alpha_i y_i$$

$$L(w,b,\alpha)=\frac{1}{2}w^Tw+\sum_i\alpha_i-\sum_i\alpha_iy_iw^Tx_i-b\sum_i\alpha_i\,y_i$$

$$=\frac{1}{2}w^Tw+\sum_i\alpha_i-\sum_i\alpha_iy_iw^Tx_i-0$$

$$=\frac{1}{2}(\sum_i\alpha_iy_ix_i)^T(\boldsymbol{\sum_i\alpha_i\,y_ix_i})+\sum_i\alpha_i-\sum_i\alpha_iy_i(\boldsymbol{\sum_i\alpha_iy_ix_i})^Tx_i$$

$$=\frac{1}{2}(\sum_i\alpha_iy_ix_i)^T\boldsymbol{(\sum_j\alpha_j\,y_jx_j)}+\sum_i\alpha_i-\sum_i\alpha_iy_i\boldsymbol{(\sum_j\alpha_jy_jx_j)}^Tx_i$$

为防止混淆，将w写为$\sum_j\alpha_j\,y_jx_j$

$$=\frac{1}{2}\sum_i\sum_j\alpha_i\alpha_jy_iy_j(x_i^Tx_j)+\sum_i\alpha_i-\sum_i\sum_j\alpha_i\alpha_jy_iy_j(x_i^Tx_j)$$

$$=\boldsymbol{\sum_i\alpha_i-\frac{1}{2}\sum_i\sum_j\alpha_i\alpha_j\,y_i\,y_j(x_i^Tx_j)\,,s.t.\;\;\alpha_i\geq0\,,\sum_i\alpha_iy_i=0}$$

$$w=\sum_i\alpha_iy_ix_i$$

$$\sum_i\alpha_iy_i=0$$

SVM进阶形式 1，n个样本$(x_i\,,y_i)$，$x_i\in R^N$,$y_i\in\{+1,-1\}$:

$$max\;\sum_i\alpha_i-\frac{1}{2}\sum_i\sum_j\alpha_i\alpha_jy_iy_j(x_i^Tx_j)$$

$$s.t.\;\;\alpha_i\geq0\,,\sum_i\alpha_i\,y_i=0$$

软间隔SVM目标：$min\ \frac{1}{2}w^Tw + C\sum_i \xi_i,\ s.t.\ y_i(w^Tx_i + b) \geq 1 - \xi_i\ ,\ \xi_i \geq 0$

构造拉格朗日函数：

$$L(w,b,\alpha,\mu,\xi) = \frac{1}{2}w^Tw + C\sum_i \xi_i + \sum_i \alpha_i[1 - \xi_i - y_i(w^Tx_i + b)] + \sum_i \mu_i(-\xi_i)$$

$$s.t.\ \alpha_i \geq 0\ ,\ \mu_i \geq 0\ ,\ \xi_i \geq 0$$

$$\because \max_{\alpha,\mu} L(w,b,\alpha,\mu,\xi) = \frac{1}{2}w^Tw + C\sum_i \xi_i$$

$$\therefore \min_{w,b,\xi} \frac{1}{2}w^Tw + C\sum_i \xi_i = \min_{w,b,\xi} \max_{\alpha,\mu} L(w,b,\alpha,\mu,\xi)$$

对偶：$\min_{w,b,\xi} \max_{\alpha,\mu} L(w,b,\alpha,\mu,\xi) = \max_{\alpha,\mu} \min_{w,b,\xi} L(w,b,\alpha,\mu,\xi)$

我们不加证明的指出上式在SVM问题中是成立的，并且相应的解满足KKT条件。

目标函数：

$$\max_{\alpha,\mu}\min_{w,b,\xi} L(w,b,\alpha,\mu,\xi),\ s.t.\ \alpha_i \geq 0,\ \mu_i \geq 0,\ \xi_i \geq 0$$

$$L(w,b,\alpha,\mu,\xi) = \frac{1}{2}w^T w + C\sum_i \xi_i + \sum_i \alpha_i[1-\xi_i - y_i(w^T x_i + b)] + \sum_i \mu_i(-\xi_i)$$

先求L关于w, b, ξ的极小值：

$$\frac{\partial L}{\partial w} = 0 \Rightarrow w - \sum_i \alpha_i y_i x_i = 0 \Rightarrow \boldsymbol{w = \sum_i \alpha_i y_i x_i}$$

$$\frac{\partial L}{\partial b} = 0 \Rightarrow -\sum_i \alpha_i y_i = 0 \Rightarrow \boldsymbol{\sum_i \alpha_i y_i = 0}$$

$$\frac{\partial L}{\partial \boldsymbol{\xi_i}} = 0 \Rightarrow \boldsymbol{C - \alpha_i - \mu_i = 0}$$

将L拆开，并将相关结果带入：

$$L(w,b,\alpha)=\frac{1}{2}w^Tw+C\sum_i\xi_i+\sum_i\alpha_i[1-\xi_i-y_i(w^Tx_i+b)]+\sum_i\mu_i(-\xi_i)$$

$\sum_i(C-\alpha_i-\mu_i)\xi_i=0$

$$=\frac{1}{2}w^Tw+\boldsymbol{C\sum_i\xi_i}+\sum_i\alpha_i-\boldsymbol{\sum_i\alpha_i\xi_i}-\sum_i\alpha_iy_iw^Tx_i-b\sum_i\alpha_iy_i-\boldsymbol{\sum_i\mu_i\xi_i}$$

$$=\frac{1}{2}(\sum_i\alpha_iy_ix_i)^T(\boldsymbol{\sum_i\alpha_iy_ix_i})+\sum_i\alpha_i-\sum_i\alpha_iy_i(\boldsymbol{\sum_i\alpha_iy_ix_i})^Tx_i$$

$$=\frac{1}{2}(\sum_i\alpha_iy_ix_i)^T(\boldsymbol{\sum_j\alpha_jy_jx_j})+\sum_i\alpha_i-\sum_i\alpha_iy_i(\boldsymbol{\sum_j\alpha_jy_jx_j})^Tx_i$$

为防止混淆，将w写为$\sum_j\alpha_j\,y_jx_j$

$$=\frac{1}{2}\sum_i\sum_j\alpha_i\alpha_jy_iy_j(x_i^Tx_j)+\sum_i\alpha_i-\sum_i\sum_j\alpha_i\alpha_jy_iy_j(x_i^Tx_j)$$

$\ldots\ldots, s.t.\ \alpha_i\geq 0, \sum_i\alpha_iy_i=0$

$$=\boldsymbol{\sum_i\alpha_i-\frac{1}{2}\sum_i\sum_j\alpha_i\alpha_jy_iy_j(x_i^Tx_j)},\ \boldsymbol{s.t.\ \alpha_i\geq 0,\ \mu_i\geq 0,\ \xi_i\geq 0,\ \sum_i\alpha_iy_i=0,\ C-\alpha_i-\mu_i=0}$$

SVM进阶形式 2，n个样本(x_i, y_i)，$x_i\in R^N, y_i\in\{+1,-1\}$：

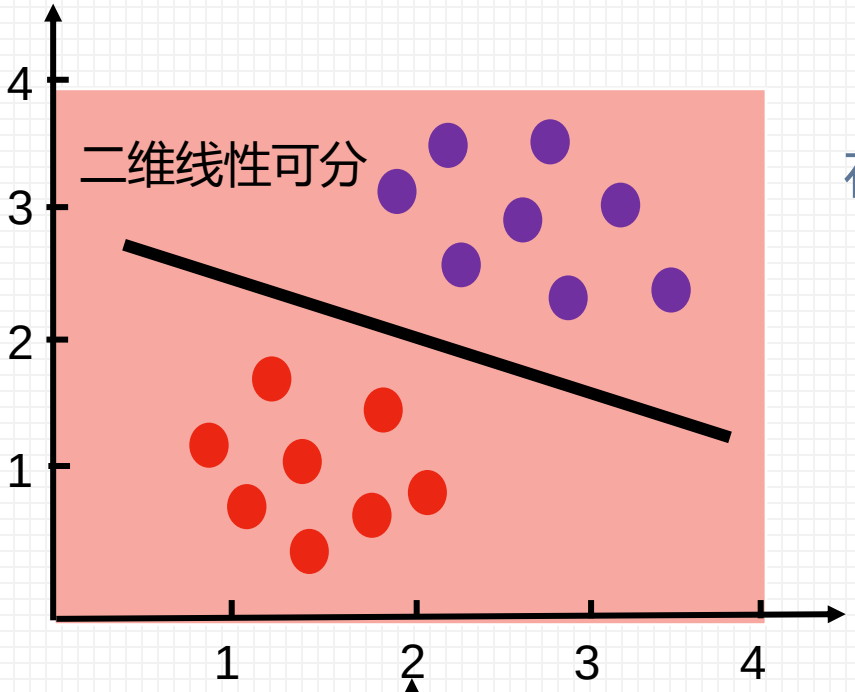
$$\boldsymbol{max}\ \sum_i\alpha_i-\frac{1}{2}\sum_i\sum_j\alpha_i\alpha_jy_iy_j(x_i^Tx_j)$$

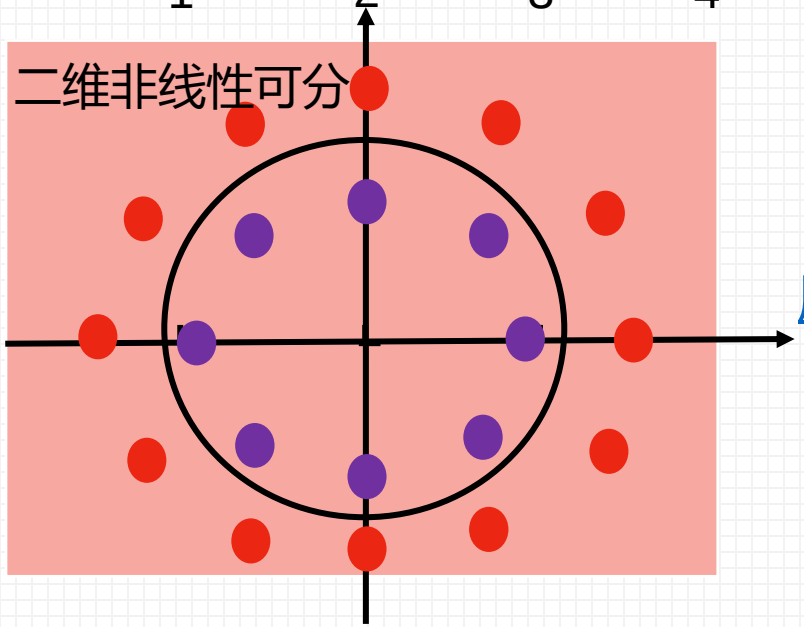
$$s.t.\ \alpha_i\geq 0,\ \mu_i\geq 0,\ \xi_i\geq 0,\ C-\alpha_i-\mu_i=0,\ \sum_i\alpha_iy_i=0$$

$$s.t.\ C\geq\alpha_i\geq 0,\ \mu_i\geq 0,\ \xi_i\geq 0,\ \sum_i\alpha_iy_i=0$$

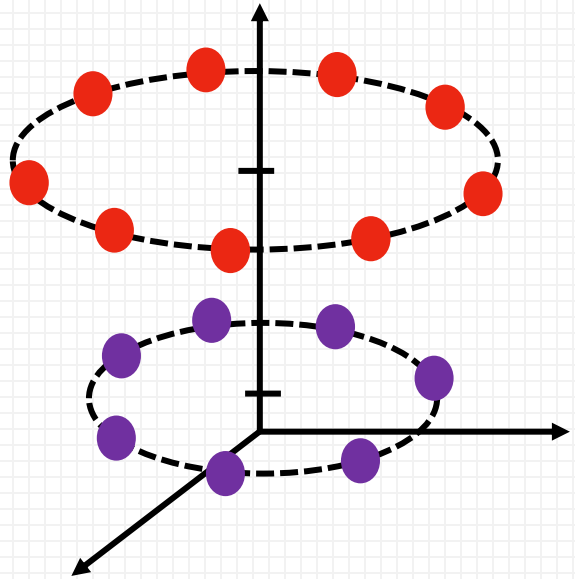
第 2.5 章 节

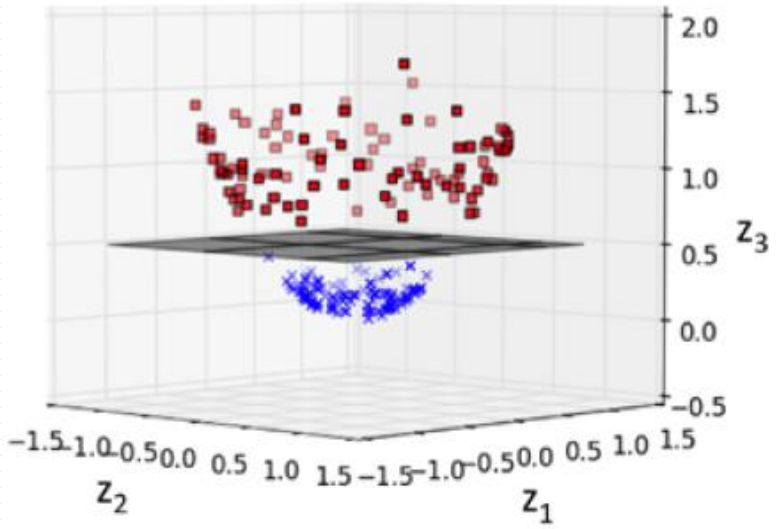
非线性可分与核技巧





在 2 维平面，红色点坐标(a,b)在半径为 1 的圆内部，紫色坐标在圆外部，不是线性可分的。





可以构造 z 纬度：$z=ab$，得到升纬后的新坐标(a,b,ab)。如图所示，可以使用$Z=1$平面进行线性划分。

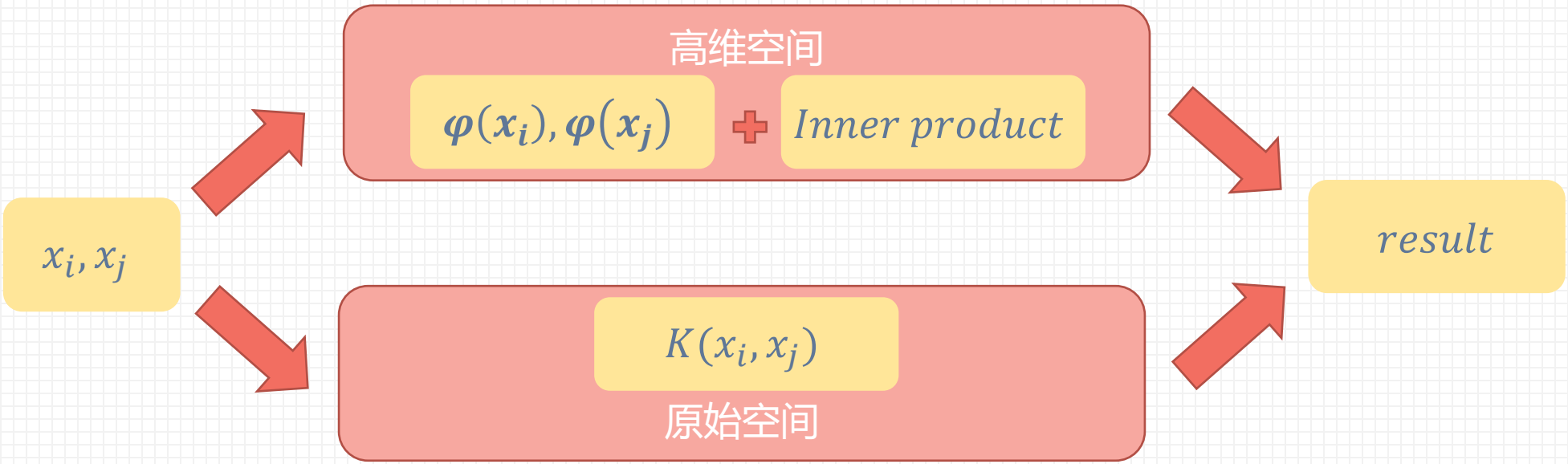
因此，对于线性不可分的情况，我们可以先对样本x升纬度，然后进行后续推导。记升维后的样本为 $\tilde{x}=\varphi(x)$。

$$max \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j (x_i^T x_j)$$

$$max \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j \tilde{x}_i^T \tilde{x}_j$$

1. **使用$\varphi(\cdot)$对变量升维$\Rightarrow \tilde{x}_i = \varphi(x_i)$**
2. **计算升维后的$\varphi(x_i)$与$\varphi(x_j)$的内积**

有没有一种可能，我们可以直接通过一个$K(x_i, x_j)$直接得到x_i, x_j升维后的内积呢？



$$K(x,z) = \phi(x)\cdot\phi(z) = \begin{bmatrix} x_1^2 \\ \sqrt{2}x_1x_2 \\ x_2^2 \end{bmatrix}\cdot\begin{bmatrix} z_1^2 \\ \sqrt{2}z_1z_2 \\ z_2^2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= x_1^2z_1^2 + 2x_1x_2z_1z_2 + x_2^2z_2^2$$

$$\phi(x) = \begin{bmatrix} x_1^2 \\ \sqrt{2}x_1x_2 \\ x_2^2 \end{bmatrix}$$

$$= (x_1z_1 + x_2z_2)^2 = \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\cdot\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}\right)^2$$

$$= (x\cdot z)^2$$

$$K(x,z) = (x\cdot z)^2 \qquad x = \begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix} \quad z = \begin{bmatrix} z_1 \\ \vdots \\ z_k \end{bmatrix}$$

$$= (x_1z_1 + x_2z_2 + \cdots + x_kz_k)^2$$

$$= x_1^2z_1^2 + x_2^2z_2^2 + \cdots + x_k^2z_k^2$$

$$+2x_1x_2z_1z_2 + 2x_1x_3z_1z_3 + \cdots$$

$$+2x_2x_3z_2z_3 + 2x_2x_4z_2z_4 + \cdots$$

$$= \phi(x)\cdot\phi(z)$$

$$\phi(x) = \begin{bmatrix} x_1^2 \\ \vdots \\ x_k^2 \\ \sqrt{2}x_1x_2 \\ \sqrt{2}x_1x_3 \\ \vdots \\ \sqrt{2}x_2x_3 \\ \vdots \end{bmatrix}$$

Radial Basis Function Kernel

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix} \quad z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \end{bmatrix}$$

$$K(x,z) = exp\left(-\frac{1}{2}\|x-z\|_2^2\right) = \phi(x)\cdot\phi(z)?$$

$\phi(*)$ has inf dim!!!

$$= exp\left(-\frac{1}{2}\|x\|_2^2 - \frac{1}{2}\|z\|_2^2 + x\cdot z\right)$$

$$= exp\left(-\frac{1}{2}\|x\|_2^2\right)exp\left(-\frac{1}{2}\|z\|_2^2\right)exp(x\cdot z) \quad = C_xC_zexp(x\cdot z)$$

$$= C_xC_z\sum_{i=0}^{\infty}\frac{(x\cdot z)^i}{i!} = C_xC_z + C_xC_z(x\cdot z) + C_xC_z\frac{1}{2}(x\cdot z)^2\cdots$$

$$[C_x]\cdot[C_z] \qquad \begin{bmatrix} C_xx_1 \\ C_xx_2 \\ \vdots \end{bmatrix}\cdot\begin{bmatrix} C_zz_1 \\ C_zz_2 \\ \vdots \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} C_xx_1^2 \\ \vdots \\ \sqrt{2}C_xx_1x_2 \\ \vdots \end{bmatrix}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix} C_zz_1^2 \\ \vdots \\ \sqrt{2}C_zz_1z_2 \\ \vdots \end{bmatrix}$$

$$\left[C_x, C_xx_1, C_xx_2, \ldots C_xx_1^2, \ldots \sqrt{2}C_xx_1x_2 \ldots C_xx_1^{99} \ldots\right]$$
$$\left[C_z, C_zz_1, C_zz_2, \ldots C_zz_1^2, \ldots \sqrt{2}C_zz_1z_2 \ldots C_zz_1^{99} \ldots\right]$$

李宏毅老师官网主页，B站也有人搬运。

包括线性代数，机器学习，深度学习等。

$$max \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j (x_i^T x_j)$$ 升维 $$max \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j (\varphi(x_i)^T \varphi(x_j))$$

核技巧

SVM进阶形式 3，n个样本(x_i, y_i)，$x_i \in R^N, y_i \in \{+1, -1\}$：

$$max \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j K(x_i, x_j)$$

$$s.t.\ C \geq \alpha_i \geq 0,\ \mu_i \geq 0,\ \xi_i \geq 0,\ \sum_i \alpha_i y_i = 0$$

到此，我们学习了SVM的基本形式，该模型属于凸优化问题，可以使用凸优化包对上述问题进行求解。下面我们将学习针对SVM的更加高效的优化方法：SMO(Sequential minimal optimization).

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第三章节

使用SMO算法进行求解

（☆☆☆☆）

$$max \sum_i \alpha_i - \frac{1}{2}\sum_i \sum_j \alpha_i \alpha_j y_i y_j K(x_i, x_j)$$

$$s.t.\ C \geq \alpha_i \geq 0\ ,\ \mu_i \geq 0\ ,\ \xi_i \geq 0,\ \sum_i \alpha_i\, y_i = 0$$

SMO算法的思想：同时调整n个变量太复杂，可以固定一些变量，调整少量的变量，反正能改善模型性能就好。每次调整少的变量，简单还快。

我们每次调整1个变量，固定n-1个变量。

不行，因为$\sum_i \alpha_i y_i = 0$，确定其余n-1个，那么最后1个变量也就固定了。

那就每次调整2个，固定n-2个吧。

鼠鼠你啊，真滴是太棒辣！

不妨设$\alpha_1\ y_1 + \alpha_2\ y_2 = D$，$\alpha_1 = \frac{(D - \alpha_2\, y_2)}{y_1} = y_1(D - \alpha_2 y_2)$，$\alpha_i \in [0, C]$。

$$\boldsymbol{max}\ \sum_i \alpha_i - \frac{1}{2}\sum_i\sum_j \alpha_i\alpha_j y_i y_j K(x_i, x_j)$$

$$\boldsymbol{s.t.}\ C \geq \alpha_i \geq 0\ ,\ \mu_i \geq 0\ ,\ \xi_i \geq 0,\ \sum_i \alpha_i\, y_i = 0$$

忽略条件 ⇒ $$\boldsymbol{min}\ \frac{1}{2}\sum_i\sum_j \alpha_i\alpha_j y_i y_j k_{ij} - \sum_i \alpha_i$$

$$min\ \frac{1}{2}\sum_i\sum_j \alpha_i\alpha_j y_i y_j k_{ij} - \sum_i \alpha_i$$

$$= \frac{1}{2}\sum_i\sum_j \alpha_i\alpha_j y_i y_j k_{ij} - (\alpha_1 + \alpha_2) - \sum_{i=3} \alpha_i$$

$$= \frac{1}{2}\left(\sum_{i=1}^{2}\sum_j \alpha_i\alpha_j y_i y_j k_{ij} + \sum_{i=3}\sum_j \alpha_i\alpha_j y_i y_j k_{ij}\right) - (\alpha_1 + \alpha_2) - \sum_{i=3} \alpha_i$$

$$= \frac{1}{2}\left[\sum_{i=1}^{2}\left(\sum_{j=1}^{2} \alpha_i\alpha_j y_i y_j k_{ij} + \sum_{j=3} \alpha_i\alpha_j y_i y_j k_{ij}\right) + \sum_{i=3}\sum_j \alpha_i\alpha_j y_i y_j k_{ij}\right] - (\alpha_1 + \alpha_2) - \sum_{i=3} \alpha_i$$

$$= \frac{1}{2}\left[\sum_{i=1}^{2}\left(\sum_{j=1}^{2} \alpha_i\alpha_j y_i y_j k_{ij} + \sum_{j=3} \alpha_i\alpha_j y_i y_j k_{ij}\right) + \sum_{i=3}\left(\sum_{j=1}^{2} \alpha_i\alpha_j y_i y_j k_{ij} + \sum_{j=3} \alpha_i\alpha_j y_i y_j k_{ij}\right)\right] - (\alpha_1 + \alpha_2) - \sum_{i=3} \alpha_i$$

$$= \frac{1}{2}k_{11}\alpha_1^2 + \frac{1}{2}k_{22}\alpha_2^2 + \alpha_1\alpha_2 y_1 y_2 k_{12} + \sum_{i=1}^{2}\sum_{j=3} \alpha_i\alpha_j y_i y_j k_{ij} - (\alpha_1 + \alpha_2) + \frac{1}{2}\sum_{i=3}\sum_{j=3} \alpha_i\alpha_j y_i y_j k_{ij} - \sum_{i=3} \alpha_i$$

$$= \boldsymbol{\frac{1}{2}k_{11}\alpha_1^2 + \frac{1}{2}k_{22}\alpha_2^2 + \alpha_1\alpha_2 y_1 y_2 k_{12} + \alpha_1 y_1 \sum_{j=3} \alpha_j y_j k_{1j} + \alpha_2 y_2 \sum_{j=3} \alpha_j y_j k_{2j} - (\alpha_1 + \alpha_2)}$$

$$\boldsymbol{min}\ \frac{1}{2}k_{11}\alpha_1^2+\frac{1}{2}k_{22}\alpha_2^2+\alpha_1\alpha_2y_1y_2k_{12}+\alpha_1y_1\sum_{j=3}\alpha_jy_jk_{1j}+\alpha_2y_2\sum_{j=3}\alpha_jy_jk_{2j}-(\alpha_1+\alpha_2)$$

记：$v_i=\sum_{j=3}\alpha_jy_jk_{ij}$，又前边推得$w=\sum_j\alpha_j\,y_jx_j$，所以

$$f(x_i)=w^Tx_i+b=\left(\sum_j\alpha_j\,y_jx_j\right)^Tx_i+b=\sum_j\alpha_j\,y_jx_j^Tx_i+b=\sum_j\alpha_j\,y_jk_{ij}+b$$

$$v_i=f(x_i)-\sum_{j=1}^{2}\alpha_j\,y_jk_{ij}-b$$

$$\alpha_1=y_1(D-\alpha_2y_2)$$

$$min\ \frac{1}{2}k_{11}\big(y_1(D-\alpha_2y_2)\big)^2+\frac{1}{2}k_{22}\alpha_2^2+y_1(D-\alpha_2y_2)\alpha_2y_1y_2k_{12}+y_1(D-\alpha_2y_2)y_1v_1+\alpha_2y_2v_2-(y_1(D-\alpha_2y_2)+\alpha_2)$$

$$\frac{1}{2}k_{11}(D-\alpha_2y_2)^2+\frac{1}{2}k_{22}\alpha_2^2+(D-\alpha_2y_2)\alpha_2y_2k_{12}+(D-\alpha_2y_2)v_1+\alpha_2y_2v_2-y_1(D-\alpha_2y_2)-\alpha_2$$

令$\frac{\partial obj}{\partial a_2}=0\Rightarrow k_{11}\alpha_2-k_{11}Dy_2+k_{22}\alpha_2+Dy_2k_{12}-2k_{12}\alpha_2-v_1y_2+v_2y_2+y_1y_2-1=0$

$$(\boldsymbol{k_{11}+k_{22}-2k_{12}})\boldsymbol{\alpha_2}=\boldsymbol{y_2(k_{11}D-k_{12}D+v_1-v_2+y_2\ -y_1)}$$

$$(k_{11}+k_{22}-2k_{12})\alpha_2 = y_2(k_{11}D - k_{12}D + v_1 - v_2 + y_2 - y_1)$$

只看等式右端

将条件带入：$v_i = f(x_i) - \sum_{j=1}^{2}\alpha_j\, y_j k_{ij} - b$，$\alpha_1\, y_1 + \alpha_2\, y_2 = D$

$$\Rightarrow y_2\left[\begin{array}{c} k_{11}(\alpha_1\, y_1 + \alpha_2\, y_2) - k_{12}(\alpha_1\, y_1 + \alpha_2\, y_2) \\ +(f_1 - \alpha_1\, y_1 k_{11} - \alpha_2\, y_2 k_{12} - b) - (f_2 - \alpha_1\, y_1 k_{21} - \alpha_2\, y_2 k_{22} - b) + y_2 - y_1 \end{array}\right]$$

$$\Rightarrow y_2[(y_2 - y_1) + (f_1 - f_2) + k_{11}\alpha_2 y_2 - 2k_{12}\alpha_2 y_2 + \alpha_2\, y_2 k_{22}]$$

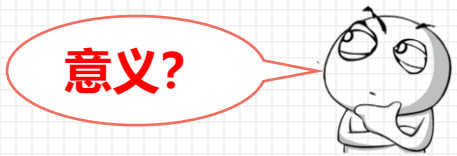
$$\Rightarrow y_2[(f_1 - y_1) - (f_1 - y_2) + \alpha_2 y_2(k_{11} - 2k_{12} + k_{22})]$$

记：$E_i = f_i - y_i$，$\eta = k_{11} - 2k_{12} + k_{22}$

$$\rightarrow \eta\alpha_2 = y_2[(E_1 - E_2) + \alpha_2 y_2 \eta]$$

$$\rightarrow \eta\alpha_2 = \alpha_2\eta + y_2(E_1 - E_2)$$

$$\rightarrow \alpha_2 = \alpha_2 + \frac{y_2(E_1 - E_2)}{\eta}$$



类别迭代方法：比如求解$f(x) = 0$,将其在x_n进行一阶泰勒展开，$f(x) = f(x_n) + f'(x_n)(x - x_n) = 0$

$x = x_n - \frac{f(x_n)}{f'(x_n)}$，将得到的$x$看做新的$x_n$(其实就是$x_{n+1}$)进行迭代: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$\boldsymbol{\alpha_2^{new} = \alpha_2^{old} + \frac{y_2\left(E_1^{old} - E_2^{old}\right)}{\eta}}$$

结合之前的条件: $\alpha_1 y_1 + \alpha_2 y_2 = D$， $\alpha_1 = \frac{(D-\alpha_2 y_2)}{y_1} = y_1(D - \alpha_2 y_2)$

改写为： $\alpha_1^{old} y_1 + \alpha_2^{old} y_2 = D, \alpha_1^{new} = y1(D - \alpha_2^{new} y_2)$

$$\Rightarrow \alpha_1^{new} = y1\left(\alpha_1^{old} y_1 + \alpha_2^{old} y_2 - \alpha_2^{new} y_2\right)$$

$$\Rightarrow \boldsymbol{\alpha_1^{new} = \alpha_1^{old} + y_1 y_2(\alpha_2^{old} - \alpha_2^{new})}$$

上边的迭代是没有考虑边界条件的，回顾我们要优化的问题以及相应的条件：

$$max \sum_i \alpha_i - \frac{1}{2}\sum_i \sum_j \alpha_i \alpha_j y_i y_j K(x_i, x_j)$$

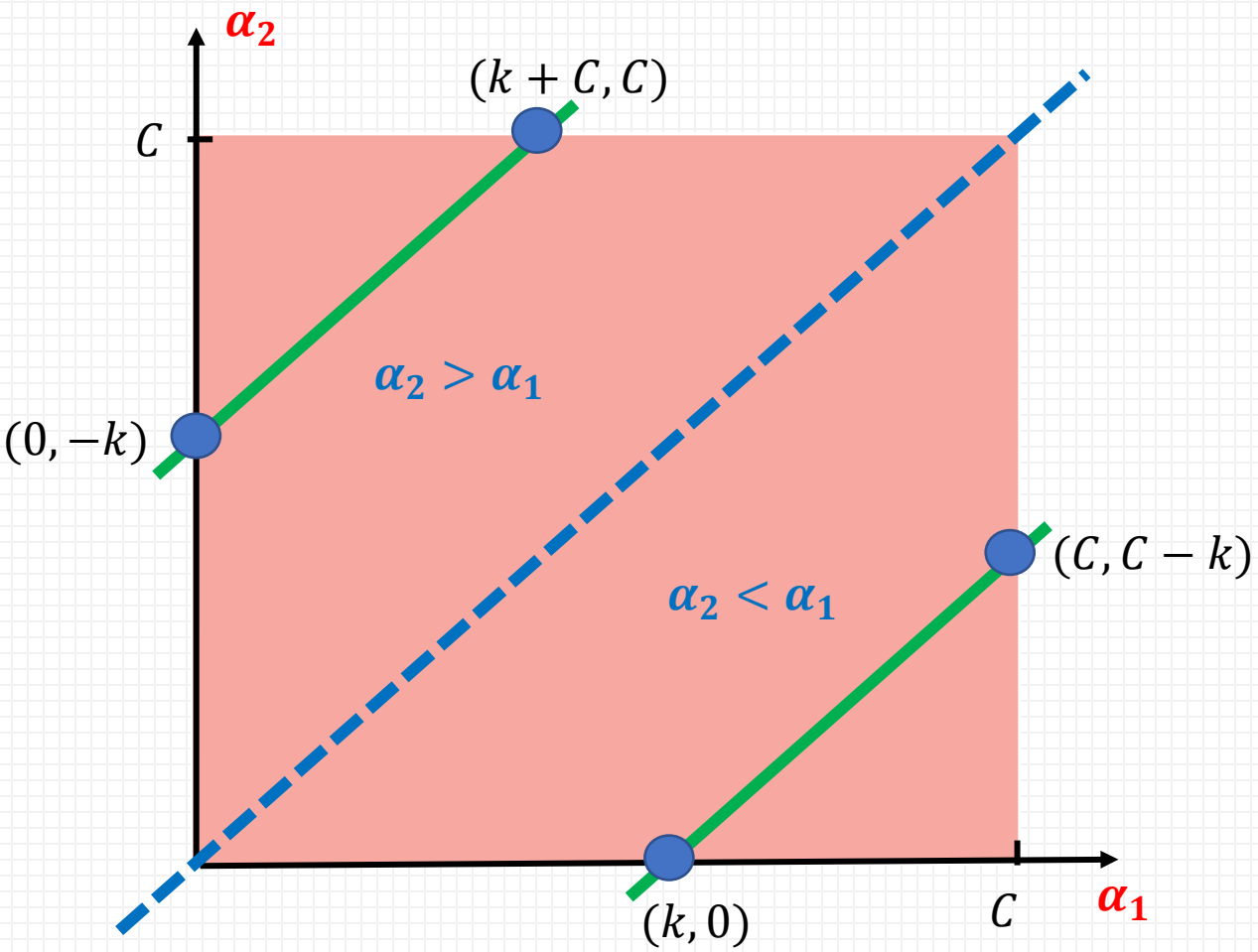
$$s.t.\ \ C \geq \alpha_i \geq 0\ ,\ \mu_i \geq 0\ ,\ \xi_i \geq 0,\ \ \sum_i \alpha_i\, y_i = 0$$

$$\boldsymbol{C \geq \alpha_1 \geq 0, C \geq \alpha_2 \geq 0}$$

$$\boldsymbol{\alpha_1 y_1 + \alpha_2 y_2 = D}$$

当$y_1 \neq y_2$，即二者异号:

$=> \alpha_1 - \alpha_2 = k, k = \pm D$，并考虑$\alpha_1$和$\alpha_2$的大小关系



$$C \geq \alpha_1 \geq 0, C \geq \alpha_2 \geq 0$$

$$\alpha_1 y_1 + \alpha_2 y_2 = D$$

当$\alpha_1 > \alpha_2, \max \alpha_2 = C$，反之$\max \alpha_2 = C - k$，最大值，取交集：$H_2 = \min(C, C - \alpha_1 + \alpha_2)$

当$\alpha_2 > \alpha_1, \min \alpha_2 = -\mathrm{k}$，反之$\min \alpha_2 = 0$，最小值，取交集：$L_2 = \max(\alpha_2 - \alpha_1, 0)$

当$y_1 \neq y_2$，即二者异号:

$$L_2 \leq \alpha_i \leq H_2$$

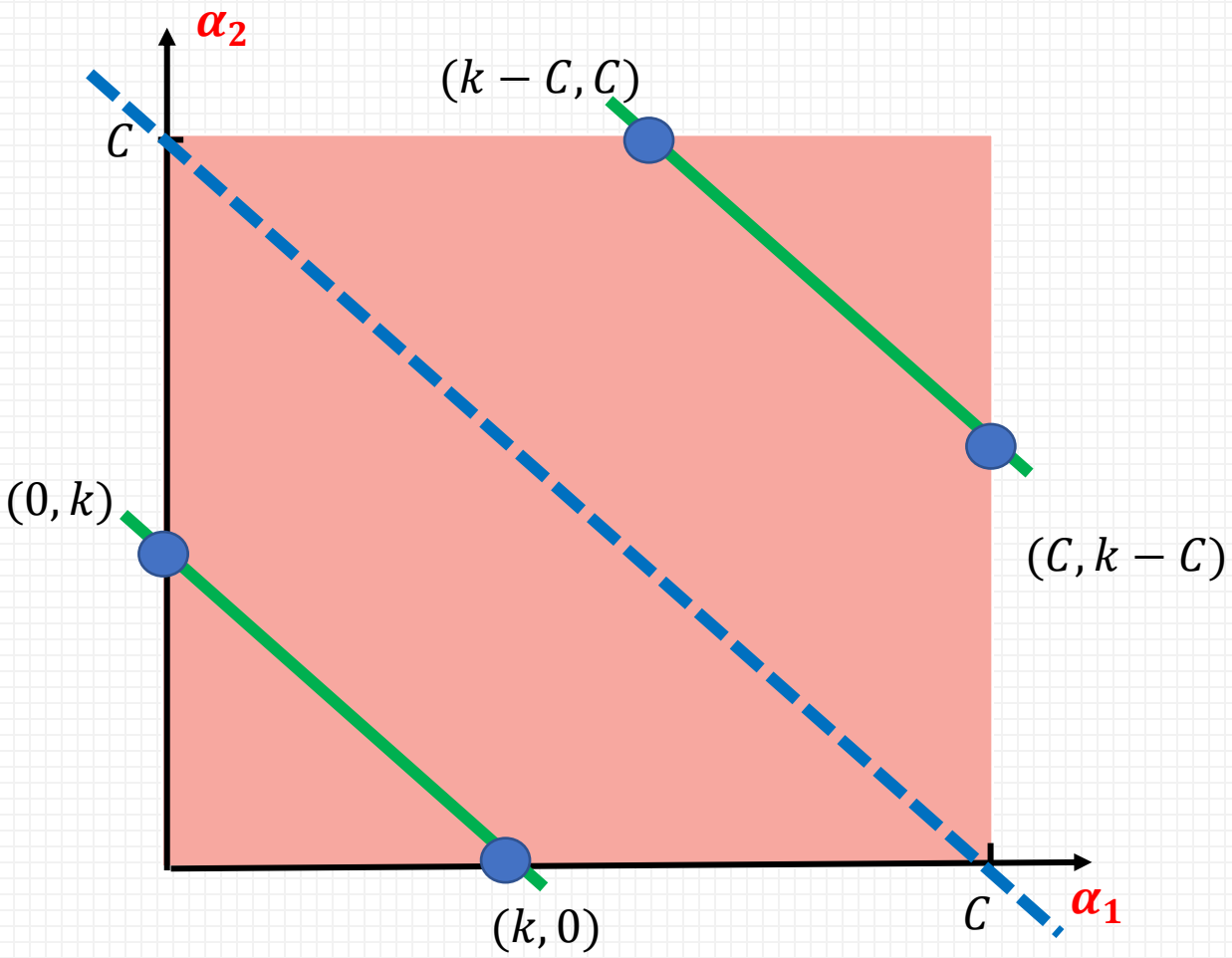
其中:

$$H_2 = \min(C, C - \alpha_1 + \alpha_2)$$

$$L_2 = \max(\alpha_2 - \alpha_1, 0)$$

当$y_1 = y_2$，即二者同号:

$=> \alpha_1 + \alpha_2 = k, k = \pm D$，亦考虑2种情况。



$$C \geq \alpha_1 \geq 0, C \geq \alpha_2 \geq 0$$

$$\alpha_1 y_1 + \alpha_2 \ y_2 = D$$

情况1，$\max \alpha_2 = C$，反之 $\max \alpha_2 = k$，

最大值，取交集：$H_2 = \min(C, \alpha_1 + \alpha_2)$

情况2, $\min \alpha_2 = k - C$，反之 $\min \alpha_2 = 0$，

最小值，取交集：$L_2 = \max(\alpha_1 + \alpha_2 - C, 0)$

当$y_1 = y_2$，即二者同号:

$$L_2 \leq \alpha_i \leq H_2$$

其中:

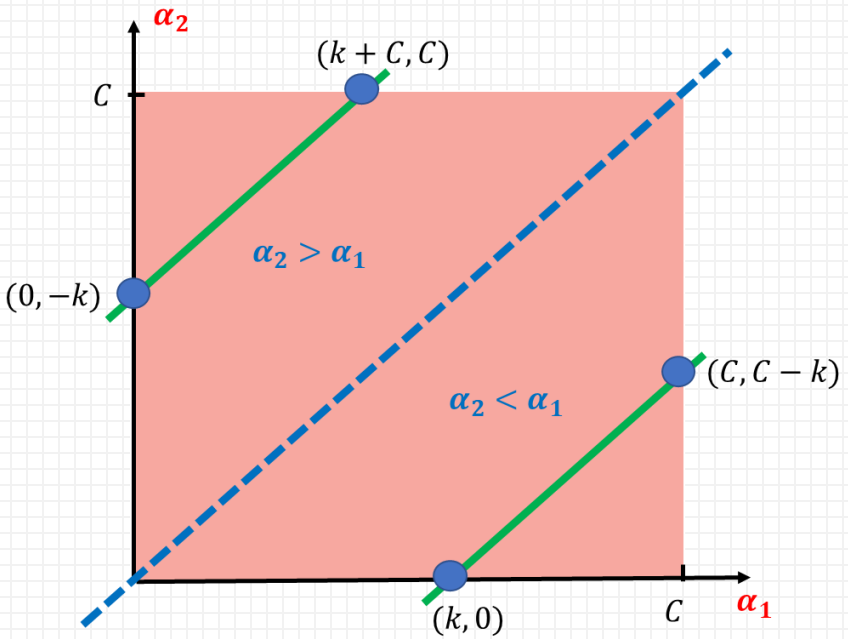
$$H_2 = \min(C, \alpha_1 + \alpha_2)$$

$$L_2 = \max(\alpha_1 + \alpha_2 - C, 0)$$

$$\alpha_2^{new} = \alpha_2^{old} + \frac{y_2(E_1^{old} - E_2^{old})}{\eta}, \quad \alpha_1^{new} = \alpha_1^{old} + y_1 y_2(\alpha_2^{old} - \alpha_2^{new})$$

$$\boldsymbol{\alpha_2^{new} = \begin{cases} H_2, & \alpha_2 \geq H_2 \\ \alpha_2^{old} + \frac{y_2(E_1^{old} - E_2^{old})}{\eta}, & else \\ L_2, & \alpha_2 \leq L_2 \end{cases}}$$

$$\boldsymbol{\alpha_1^{new} = \alpha_1^{old} + y_1 y_2(\alpha_2^{old} - \alpha_2^{new})}$$



为什么α_1不需要裁剪？

答：在$\alpha_1 y_1 + \alpha_2\ y_2 = D$的约束下，$\alpha_1$和$\alpha_2$被约束到了一条直线上。此时要求的就是，目标函数在一条平行于对角线的线段上的最优值。这使得两个变量的最优化问题成为实质上的单变量的最优化问题。不妨考虑为α_2的最优化问题，把握住了α_2，那么α_1自然就定了。

第 3.5 章 节

α_1、α_2的选取与b、E_i的推导

KKT for multiple equality & inequality constraints

Given the constrained optimization problem

$$\min_{\mathbf{x}\in\mathbb{R}^2} f(\mathbf{x})$$

subject to

$$h_i(\mathbf{x}) = 0 \text{ for } i = 1,\dots,l \text{ and } g_j(\mathbf{x}) \le 0 \text{ for } j = 1,\dots,m$$

Define the **Lagrangian** as

$$\mathcal{L}(\mathbf{x},\boldsymbol{\mu},\boldsymbol{\lambda}) = f(\mathbf{x}) + \boldsymbol{\mu}^t\,\mathbf{h}(\mathbf{x}) + \boldsymbol{\lambda}^t\,\mathbf{g}(\mathbf{x})$$

Then $\mathbf{x}^*$ a local minimum $\iff$ there exists a unique $\boldsymbol{\lambda}^*$ s.t.

1. $\nabla_{\mathbf{x}}\mathcal{L}(\mathbf{x}^*,\boldsymbol{\mu}^*,\boldsymbol{\lambda}^*) = \mathbf{0}$
2. $\lambda_j^* \ge 0$ for $j = 1,\dots,m$
3. $\lambda_j^*\, g_j(\mathbf{x}^*) = 0$ for $j = 1,\dots,m$
4. $g_j(\mathbf{x}^*) \le 0$ for $j = 1,\dots,m$
5. $\mathbf{h}(\mathbf{x}^*) = \mathbf{0}$
6. Plus positive definite constraints on $\nabla_{\mathbf{xx}}\mathcal{L}(\mathbf{x}^*,\boldsymbol{\lambda}^*)$.

① $\frac{\partial L}{\partial w} = 0 \Rightarrow w^* - \sum_i \alpha_i^* y_i x_i = 0 \Rightarrow w^* = \sum_i \alpha_i^* y_i x_i$

$\frac{\partial L}{\partial b} = 0 \Rightarrow -\sum_i \alpha_i^* y_i = 0 \Rightarrow \sum_i \alpha_i^* y_i = 0$

$\frac{\partial L}{\partial \xi_i} = 0 \Rightarrow C - \alpha_i^* - \mu_i^* = 0$

KKT条件在SVM中的表现：指出正确的解应该满足何种形式。

② $\alpha_i^* \ge 0,\ \ \mu_i^* \ge 0$

③ $\alpha_i^*\left[1 - \xi_i^* - y_i\left(w^{*T}x_i + b^*\right)\right] = 0,\ \ -\mu_i^*\xi_i^* = 0$

④ $1 - \xi_i^* - y_i\left(w^{*T}x_i + b^*\right) \le 0,\ \ -\xi_i^* \le 0$

根据前边的推导，SVM的解应满足KKT条件，且满足原始约束，如$\alpha_i \ge 0$，$\xi_i \ge 0$ 等。从而得到 $C \ge \alpha_i^* \ge 0$，由此我们有：

① 当 $\boldsymbol{\alpha_i^* = 0}$，$C - \alpha_i^* - \mu_i^* = 0 \Rightarrow \mu_i^* = C \ne 0$，又 $y_i(w^T x_i + b) - 1 + \xi_i \ge 0$，$\mu_i^*\xi_i^* = 0$，得 $\xi_i^* = 0$，从而 $y_i(w^T x_i + b) \ge 1$，即 $\boldsymbol{y_i f(x_i) \ge 1}$。**(对应能够被完美分类的样本)**

在实际中是严格大于（小于）

② 当 $\boldsymbol{0 < \alpha_i^* < C}$，$C - \alpha_i^* - \mu_i^* = 0 \Rightarrow \mu_i^* = C - \alpha_i^* \ne 0$，又$\alpha_i^*\left[1 - \xi_i^* - y_i\left(w^{*T}x_i + b^*\right)\right] = 0$，得 $1 - \xi_i^* - y_i\left(w^{*T}x_i + b^*\right) = 0$，又 $\mu_i^*\xi_i^* = 0$，所以 $\xi_i^* = 0$，从而 $y_i(w^T x_i + b) = 1$，即 $\boldsymbol{y_i f(x_i) = 1}$。**(对应卡在边界的样本，即支持向量)**

③ 当 $\boldsymbol{\alpha_i^* = C}$，$C - \alpha_i^* - \mu_i^* = 0 \Rightarrow \mu_i^* = 0$，又 $1 - \xi_i^* - y_i\left(w^{*T}x_i + b^*\right) = 0 \Rightarrow y_i\left(w^{*T}x_i + b^*\right) = 1 - \xi_i^*$，又 $\mu_i^*\xi_i^* = 0$，得 $\xi_i^* \ge 0$，从而 $y_i(w^T x_i + b) \le 1$，即 $\boldsymbol{y_i f(x_i) \le 1}$。**(对应超平面和边界之间的样本)**

等价KKT条件：① $\boldsymbol{\alpha_i^* = 0 \Leftrightarrow y_i f(x_i) \geq 1}$。② $\boldsymbol{0 < \alpha_i^* < C \Leftrightarrow y_i f(x_i) = 1}$。③ $\boldsymbol{\alpha_i^* = C \Leftrightarrow y_i f(x_i) \leq 1}$。

α_i的更新公式：$\boldsymbol{\alpha_1^{new} = \alpha_1^{old} + y_1 y_2(\alpha_2^{old} - \alpha_2^{new})}$，$\boldsymbol{\alpha_2^{new} = \begin{cases} H_2, & \alpha_2 \geq H_2 \\ \alpha_2^{old} + \frac{y_2(E_1^{old} - E_2^{old})}{\eta}, & else \\ L_2, & \alpha_2 \leq L_2 \end{cases}}$

- α_1、α_2的选取
  - **外循环先选α_1**：目标是尽量选取违反KKT条件的变量，并且优先判断支撑边界上的样本对应的α。具体操作为，先检验$0 < \alpha_i < C$，对应的$y_i f(x_i) = 1$是否成立？选择不成立、结果差的最离谱的 α_i 作为此次的 α_1。如果都满足，则再遍历其余样本。
  - **内循环选择α_2**：思路是，在选定α_1的基础上，选一个α_2能够有足够大的变动：具体为，因为α_2^{new}的变动量是与$|E_1 - E_2|$成正比的，而α_1选定之后，E_1就定了。所以，如果E_1是正的，那么选择最小的E_i的对应的α_i作为α_2，反之选择最大的E_i的对应的α_i作为α_2。
    - 如果上述的选择方式不能使α_2能够有足够大的变动（变动量小于某个阈值），那么启发式的遍历支持向量对应的α_i作为α_2，判断是否能够满足条件。再不行，那就遍历其余样本，最后，实在不行，那就换个α_1。

- 截距 b 的计算：
  - 若 $0 < \alpha_1^{new} < C$，$y_i(w^T x_i + b) = 1 \Rightarrow w^T x_i + b = y_i \Rightarrow y_i = b - w^T x_i$

$$b_1^{new} = y_1 - \sum_{i=3} \alpha_i y_i k_{i1} - \alpha_1^{new} y_1 k_{11} - \alpha_2^{new} y_2 k_{21}$$

$$E_1^{old} = \sum_{i=3} \alpha_i y_i k_{i1} + \alpha_1^{old} y_1 k_{11} + \alpha_2^{nold} y_2 k_{21} + b^{old} - y_1$$

$$\Rightarrow b_1^{new} = \left(-E_1^{old} + \sum_{i=3} \alpha_i y_i k_{i1} + \alpha_1^{old} y_1 k_{11} + \alpha_2^{nold} y_2 k_{21} + b^{old} \right) - \sum_{i=3} \alpha_i y_i k_{i1} - \alpha_1^{new} y_1 k_{11} - \alpha_2^{new} y_2 k_{21}$$

$$\Rightarrow \boldsymbol{b_1^{new} = -E_1^{old} + (\alpha_1^{old} - \alpha_1^{new}) y_1 k_{11} + (\alpha_2^{old} - \alpha_2^{new}) y_2 k_{21} + b^{old}}$$

  - 若$0 < \alpha_2^{new} < C$，同理

$$\Rightarrow \boldsymbol{b_2^{new} = -E_2^{old} + (\alpha_1^{old} - \alpha_1^{new}) y_1 k_{12} + (\alpha_2^{old} - \alpha_2^{new}) y_2 k_{22} + b^{old}}$$

哪个α满足条件，就$b^{new} = b_i^{new}$，否则就取二者均值作为新的b。

- E_i 的计算：

$$\boldsymbol{E_i^{new} = \sum \alpha_j^{new} y_j k_{ji} + b^{new} - y_i}$$

- **算法流程**

- **输入：训练集n个样本(x_i, y_i)，$x_i \in R^N, y_i \in \{+1, -1\}$，精度$\epsilon$，输出：$\alpha$**

① **初始化迭代次数$k=0$，设置$\alpha_i^k = 0$**

② **选择α_1^k、α_2^k，并计算**

$$E_i^k = \alpha_1^k y_1 k_{1i} + \alpha_2^k y_2 k_{2i} + \sum_{j=3} \alpha_j y_j k_{ji} - y_i，\ \eta = k_{11} - 2k_{12} + k_{22}$$

③ **更新计算α_1^{k+1}、α_2^{k+1}**

$$\alpha_2^{k+1} = \alpha_2^k + \frac{y_2(E_1^k - E_2^k)}{\eta}, clip，\ \alpha_1^{k+1} = \alpha_1^k + y_1 y_2 (\alpha_2^k - \alpha_2^{k+1})$$

④ **更新计算b^{k+1}、E_i^{k+1}**

$$b_1^{new} = -E_1^{old} + (\alpha_1^{old} - \alpha_1^{new}) y_1 k_{11} + (\alpha_2^{old} - \alpha_2^{new}) y_2 k_{21} + b^{old}$$

$$b_2^{new} = -E_2^{old} + (\alpha_1^{old} - \alpha_1^{new}) y_1 k_{12} + (\alpha_2^{old} - \alpha_2^{new}) y_2 k_{22} + b^{old}$$

$$E_i^{new} = \sum \alpha_j^{new} y_j k_{ji} + b^{new} - y_i$$

⑤ **是否在ϵ范围内满足KKT条件**

$$\sum_i \alpha_i y_i = 0, y_i f(x_i): \begin{cases} \geq 1, & \{x_i | \alpha_i = 0\} \\ = 1, & \{x_i | 0 < \alpha_i < C\} \\ \leq 1, & \{x_i | \alpha_i = C\} \end{cases}$$

⑥ **$k = k + 1$**

di si zhang jie

第四章节

深度学习版本的SVM

（☆☆☆）

深度学习版本的SVM，指的是使用DL中构造损失函数，利用误差反向传播，梯度下降的思路进行求解。

本节相关内容、图像来自李宏毅老师16年ML课程课件。

Binary Classification

x^1	x^2	x^3	……
$\hat{y}^1$	$\hat{y}^2$	$\hat{y}^3$	

$\hat{y}^n = +1, -1$

- Step 1: Function set (Model)

$$g(x) = \begin{cases} f(x) > 0 & \text{Output = +1} \\ f(x) < 0 & \text{Output = -1} \end{cases}$$

- Step 2: Loss function:

$$L(f) = \sum_n \delta(g(x^n) \neq \hat{y}^n)$$

$$l(f(x^n), \hat{y}^n)$$

The number of times g get incorrect results on training data.

- Step 3: Training by gradient descent is difficult

Gradient descent is possible if $g(*)$ and $\delta(*)$ is differentiable

Step 2: Loss function

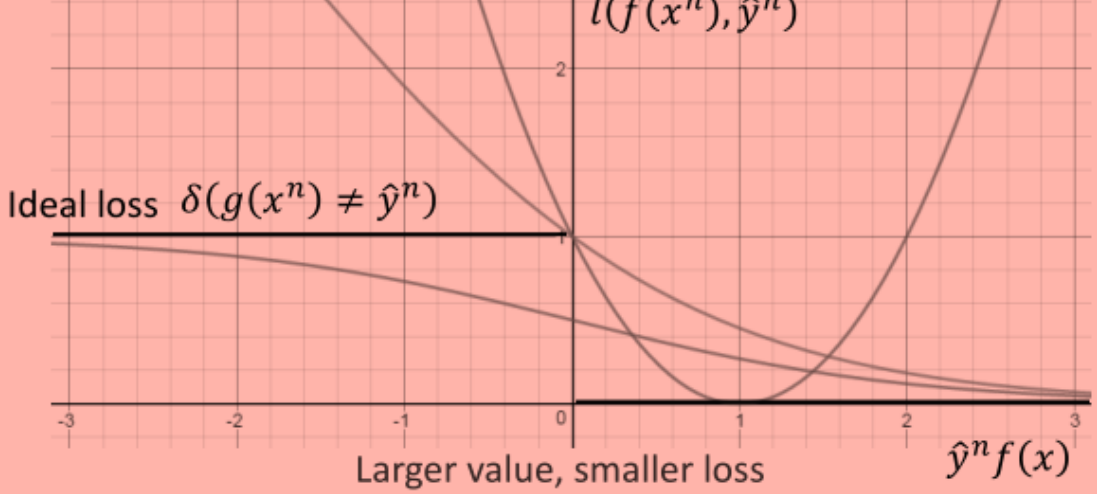
$$g(x) = \begin{cases} f(x) > 0 & \text{Output = +1} \\ f(x) < 0 & \text{Output = -1} \end{cases}$$

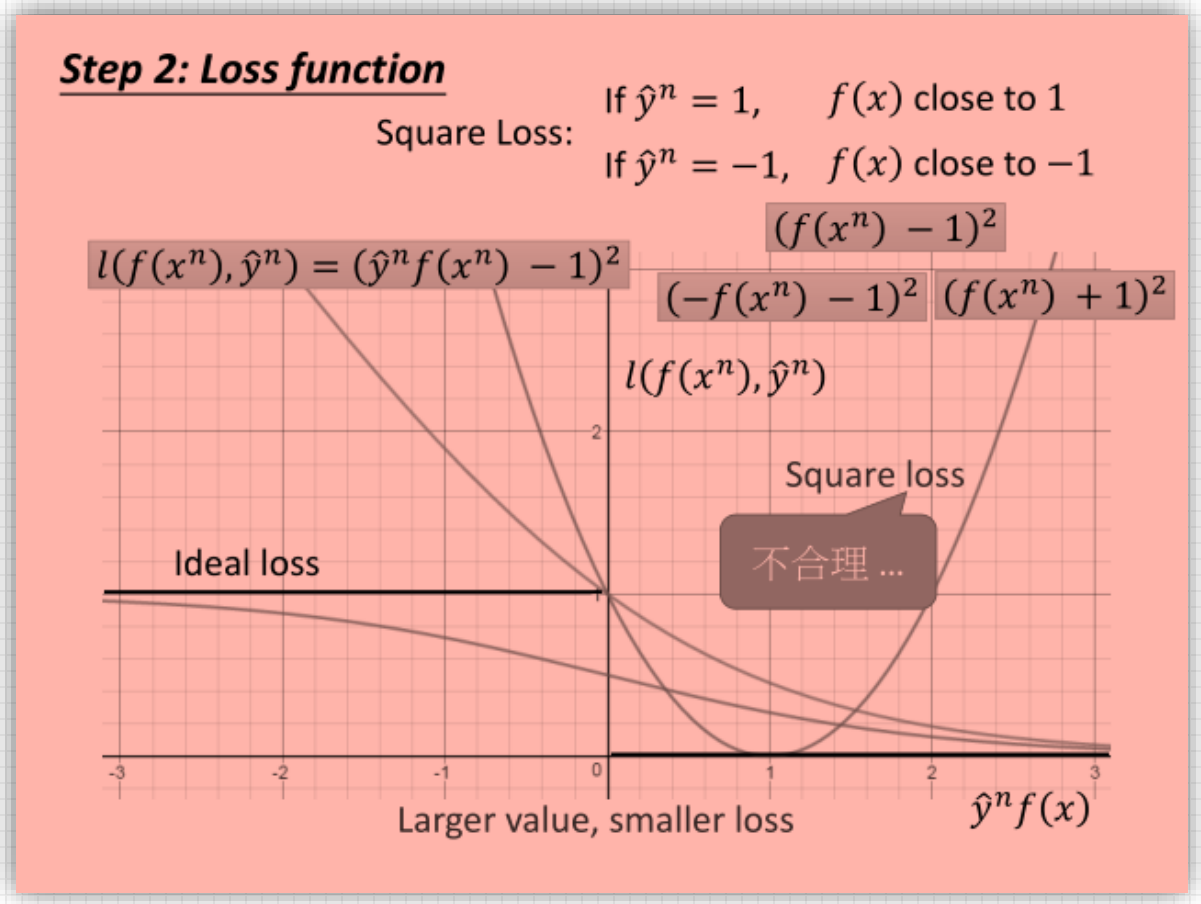
Ideal loss:

$$L(f) = \sum_n \delta(g(x^n) \neq \hat{y}^n)$$

Approximation:

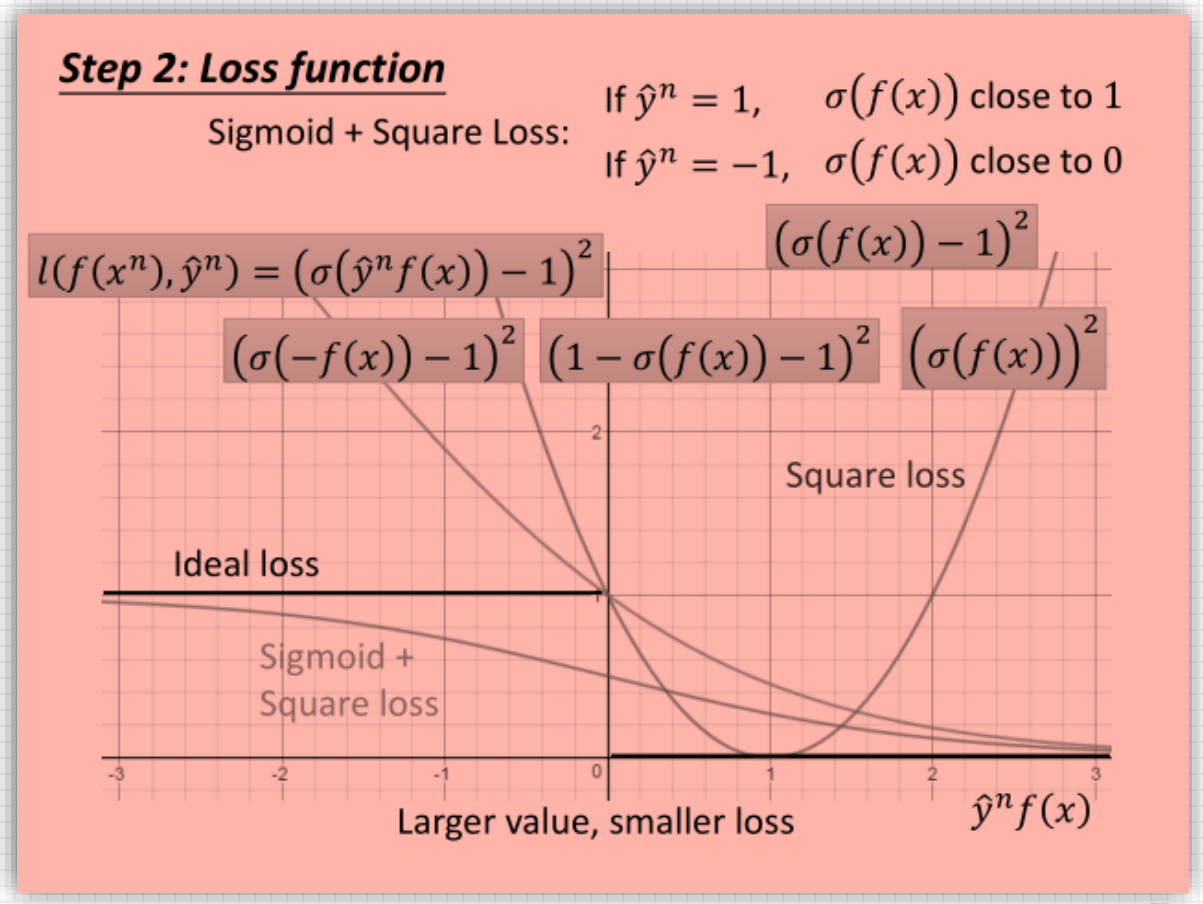
$$L(f) = \sum_n l(f(x^n), \hat{y}^n)$$

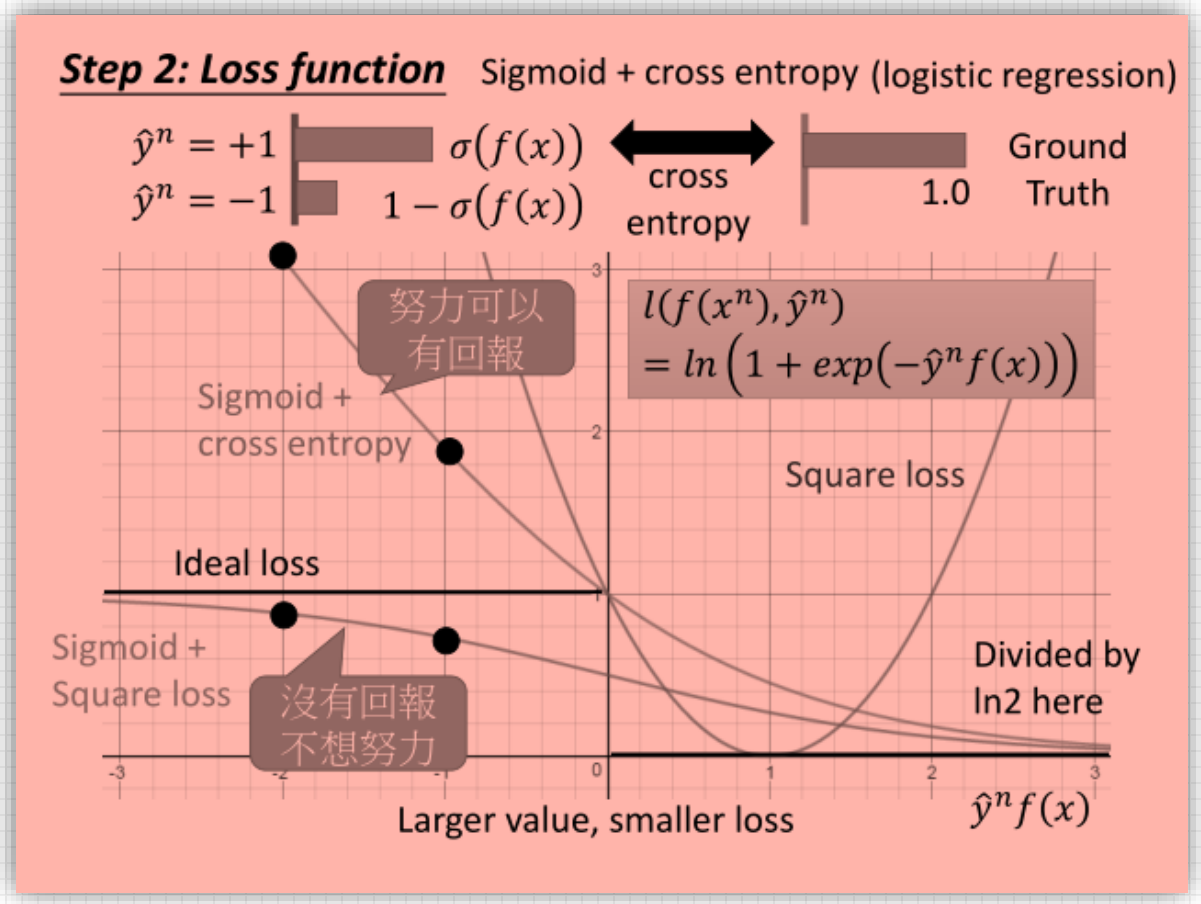




Square Loss

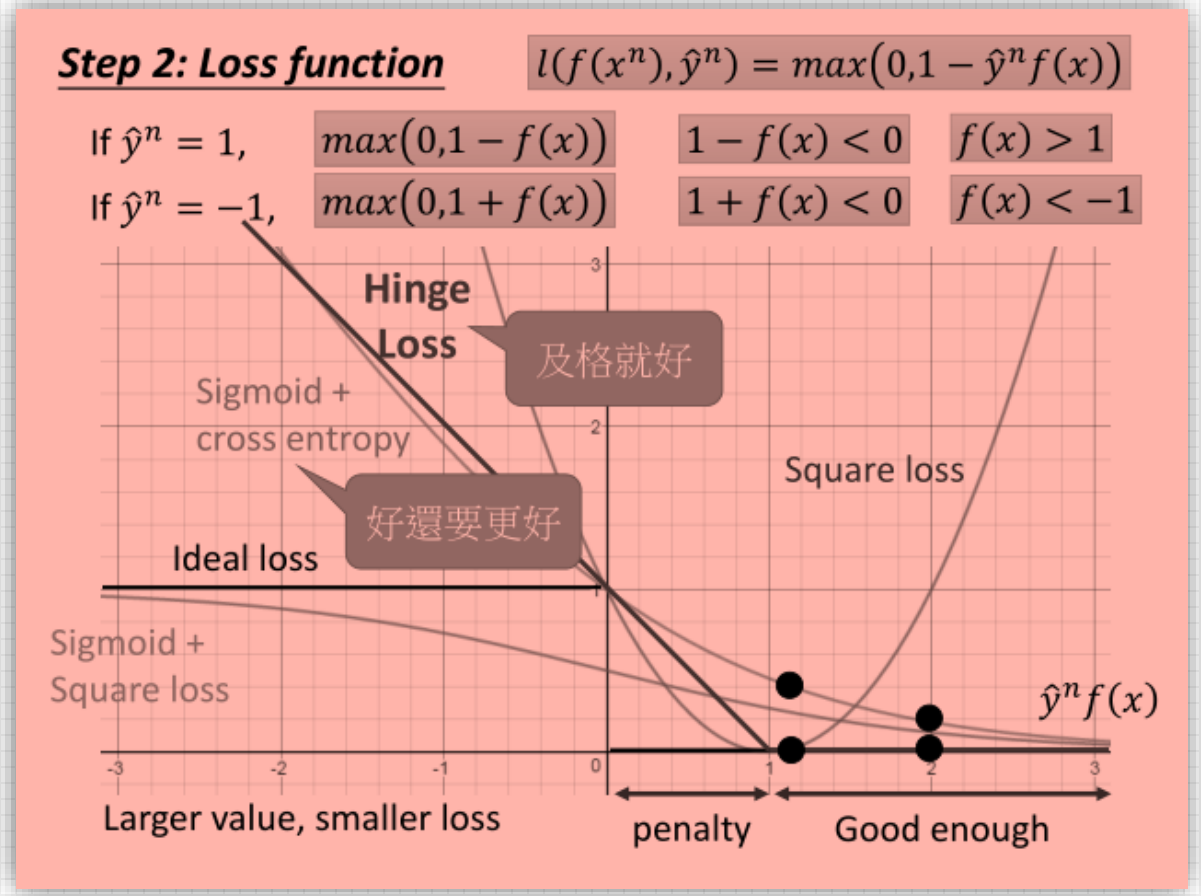
Sigmoid + Square Loss





Sigmoid + Cross Entropy

Hinge Loss



接下来证明，对于n个训练样本，如果对如下函数进行最小化，那么你得到的就是一个SVM。

$$Minimize\ L(f) = \sum l(f(x^n), \hat{y}^n) + \lambda \|w\|_2,\ l(f(x^n), \hat{y}^n) = max\left(0, 1 - \hat{y}^n f(x^n)\right)$$

Proof:

$$Minimize\ L(f) = \sum \boldsymbol{\epsilon^n} + \lambda \|w\|_2,\ \epsilon^n = max\left(0, 1 - \hat{y}^n f(x^n)\right)$$

$\epsilon^n = max\left(0, 1 - \hat{y}^n f(x^n)\right)$ $\not\Leftrightarrow$ $\epsilon^n \geq 0,\ \epsilon^n \geq 1 - \hat{y}^n f(x^n)$

But!!!

$\epsilon^n = max\left(0, 1 - \hat{y}^n f(x^n)\right)$ + **Minimize** $\Leftrightarrow$ $\epsilon^n \geq 0,\ \epsilon^n \geq 1 - \hat{y}^n f(x^n)$ + **Minimize**

$$\boldsymbol{Minimize\ L(f) = \sum \epsilon^n + \lambda \|w\|_2,\ \epsilon^n \geq 0,\ \hat{y}^n f(x^n) \geq 1 - \epsilon^n}$$

PPT13页，软间隔SVM目标：$min\ \frac{1}{2} w^T w + C \sum_i \xi_i,\ s.t.\ y_i(w^T x_i + b) \geq 1 - \xi_i\ ,\ \xi_i \geq 0$

Linear SVM – gradient descent

Ignore regularization for simplicity

$$L(f) = \sum_n l(f(x^n), \hat{y}^n) \qquad l(f(x^n), \hat{y}^n) = max(0, 1 - \hat{y}^n f(x^n))$$

$$\frac{\partial l(f(x^n), \hat{y}^n)}{\partial w_i} = \frac{\partial l(f(x^n), \hat{y}^n)}{\partial f(x^n)} \frac{\partial f(x^n)}{\partial w_i} x_i^n$$

$$f(x^n) = w^T \cdot x^n$$

$$\frac{\partial max(0, 1 - \hat{y}^n f(x^n))}{\partial f(x^n)} = \begin{cases} -\hat{y}^n & \text{If } \hat{y}^n f(x^n) < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial L(f)}{\partial w_i} = \sum_n \underbrace{-\delta(\hat{y}^n f(x^n) < 1)\hat{y}^n x_i}_{c^n(w)} \qquad w_i \leftarrow w_i - \eta \sum_n c^n(w) x_i^n$$

亦可以说明SVM的权重w，是训练集样本的线性组合

Dual Representation

$$w^* = \sum_n \alpha_n^* x^n$$ Linear combination of data points

α_n^* may be sparse ➡ x^n with non-zero α_n^* are support vectors

$$w_1 \leftarrow w_1 - \eta \sum_n c^n(w) x_1^n$$
$$\vdots$$
$$w_i \leftarrow w_i - \eta \sum_n c^n(w) x_i^n$$
$$\vdots$$
$$w_k \leftarrow w_k - \eta \sum_n c^n(w) x_k^n$$

If w initialized as **0**

$$w \leftarrow w - \eta \sum_n c^n(w) x^n$$

$$c^n(w) = \frac{\partial l(f(x^n), \hat{y}^n)}{\partial f(x^n)}$$

Hinge loss: usually zero

c.f. for logistic regression, it is always non-zero

第 4.5 章 节

SVM与逻辑回归、回归任务、NN的关系

$$Minimize\ L(f) = \sum l(f(x^n), \hat{y}^n) + \lambda \|w\|_2$$

使用不同的损失函数对上式进行优化，得到的就是不同的模型。

① 当 $l(f(x^n), \hat{y}^n) = max\left(0, 1 - \hat{y}^n f(x^n)\right)$ 时，得到就是一个SVM。

② 当 $l(f(x^n), \hat{y}^n) = \ln\left(1 + \exp\left(-\hat{y}^n f(x^n)\right)\right)$ 时，得到就是一个 Logistics Regression 。

③ 当 $l(f(x^n), \hat{y}^n) = (f(x^n) - \hat{y}^n)^2$ 时，得到的就是一个带 L2 Regularization 的回归模型，也叫 岭回归。

④ 当 $l(f(x^n), \hat{y}^n) = (f(x^n) - \hat{y}^n)^2$ ，且 $+\lambda\|w\|_1$时，得到是 L1 Regularization 的回归模型，也叫 LASSO回归。

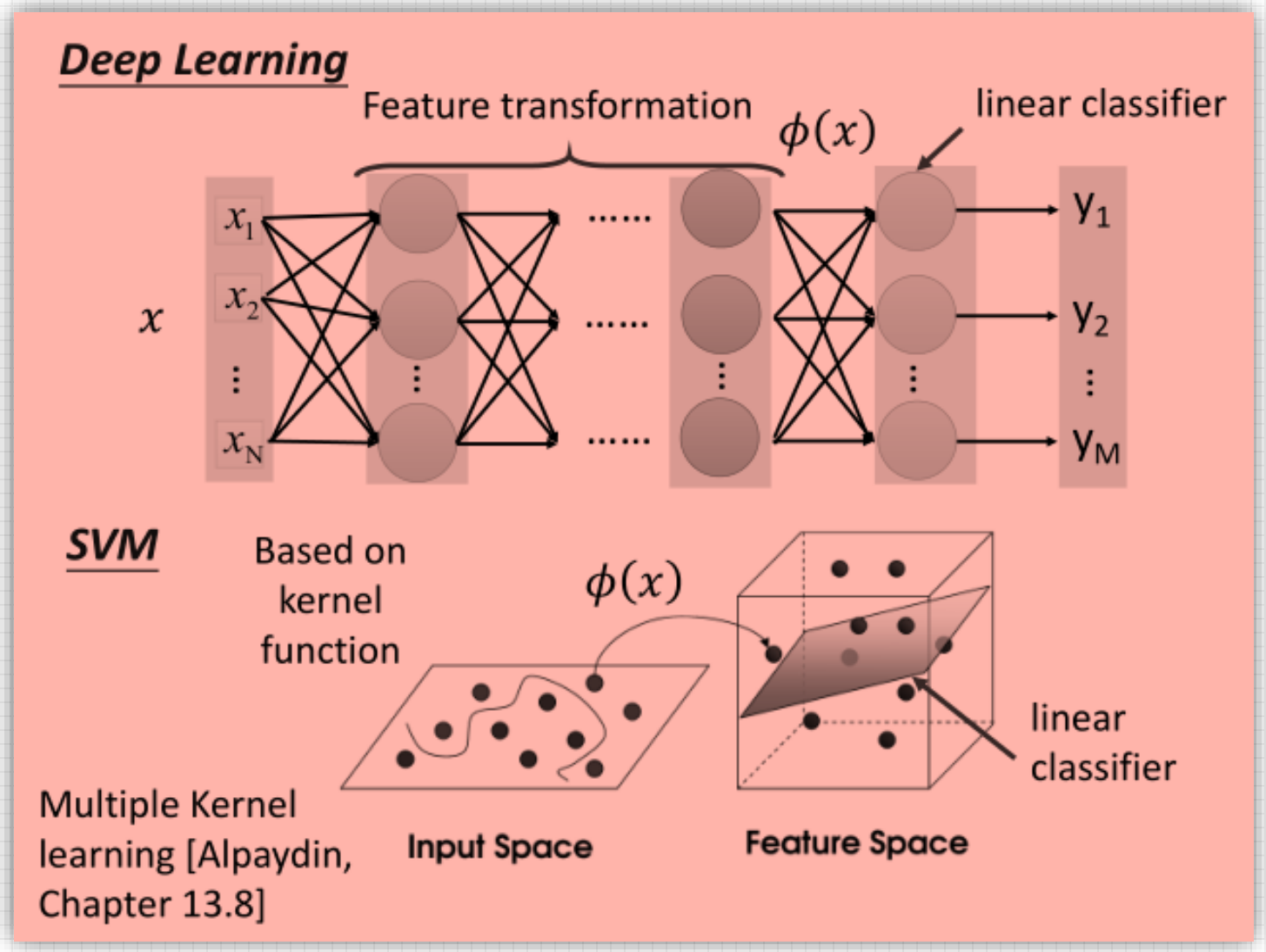
因此，从这个角度，SVM、逻辑回归、带正则项的回归，他们是一样的。区别仅在于选择什么优化函数。

具体L1，L2正则化的原理、理解，读者自行学习。推荐一个B站UP 王木头学科学 的视频：

贝叶斯解释“L1和L2正则化”，本质上是最大后验估

“L1和L2正则化”直观理解(之二)，为什么又叫权重衰

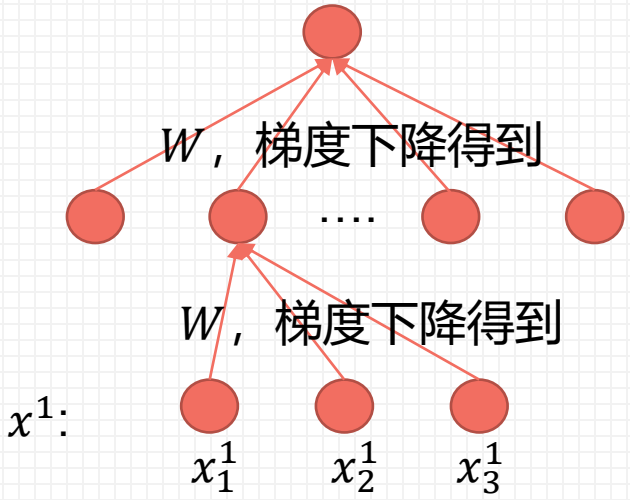
“L1和L2正则化”直观理解(之一)，从拉格朗日乘数法角



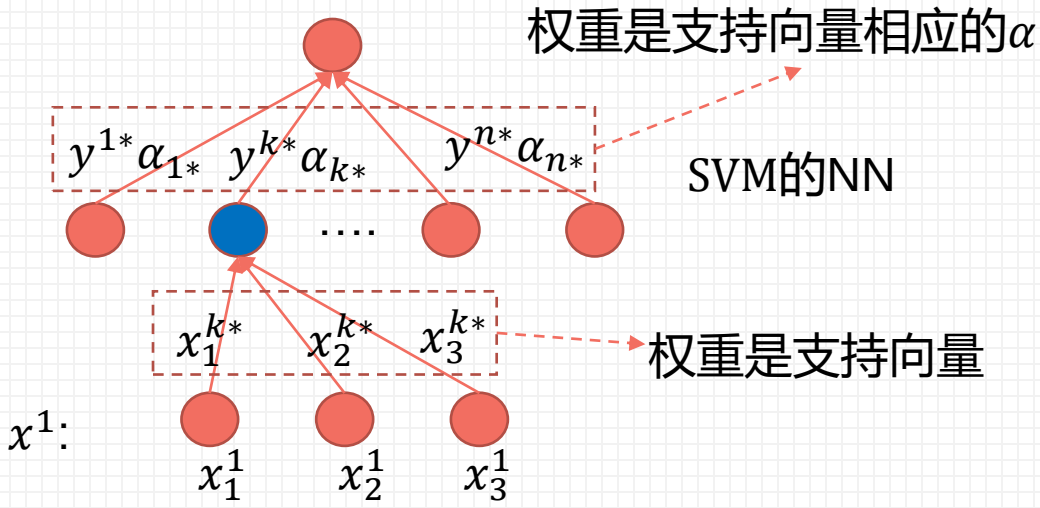
训练的时候：

NN的隐藏层一定程度上要做的事情

和 SVM 中 $\phi(\cdot)$要做的事情类似。



标准3层NN



$$\boldsymbol{f(x^i) = w^T x^i}, \mathrm{w} = \sum \alpha_{n*} y^{n*} x^{n*}, \text{忽略偏置、} \varphi(\cdot)$$

$$f^i = \sum \alpha_{n*} y^{n*} (x^{n*} \cdot x^i)，\ x^{n*}\text{为支持向量}$$

预测的时候：

SVM也是可以写成单个隐含层的神经网络，其中权重是支持向量和其相应的α

di wu zhang jie

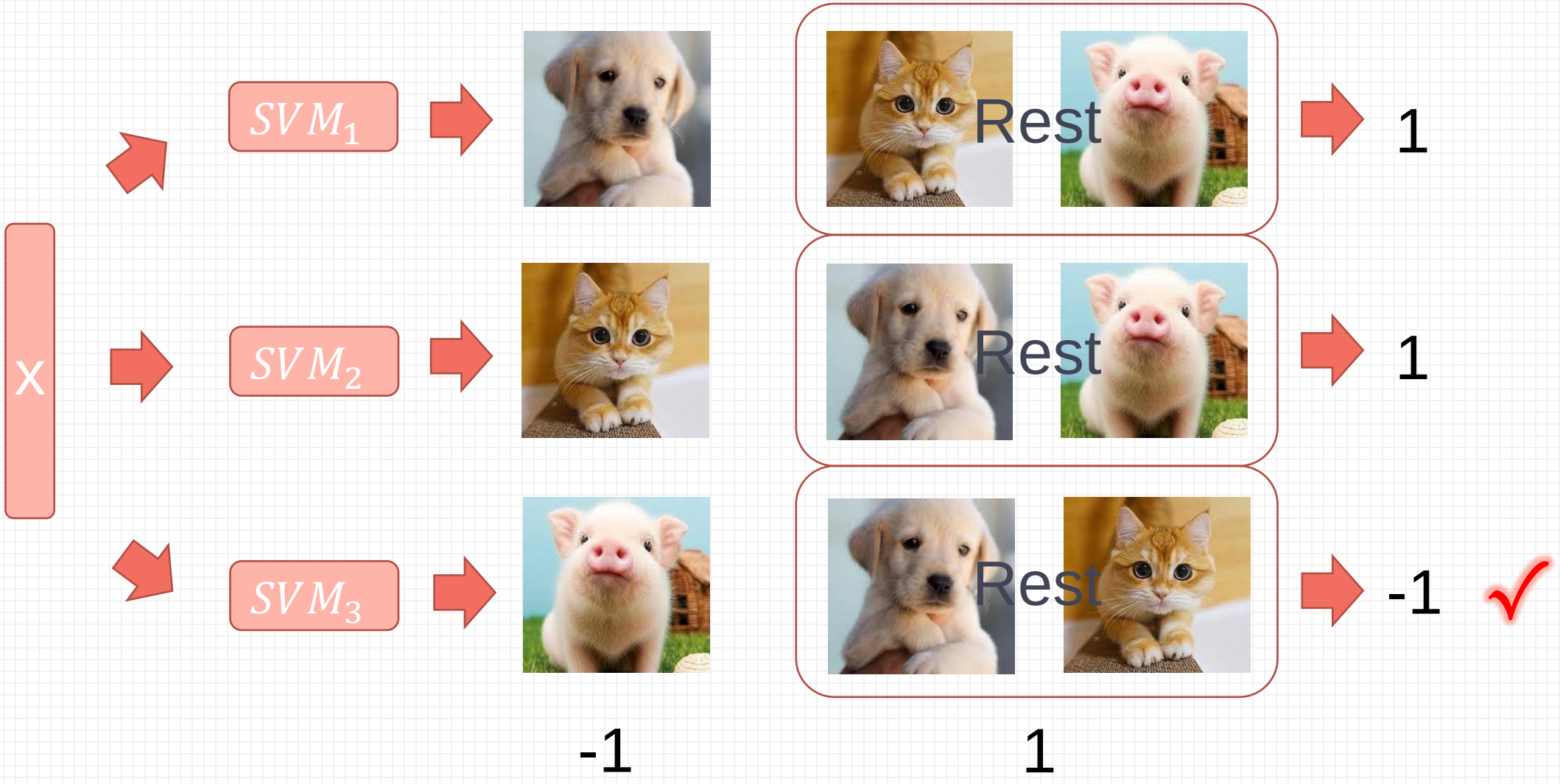
第五章节

SVM的程序调用

（☆☆☆）

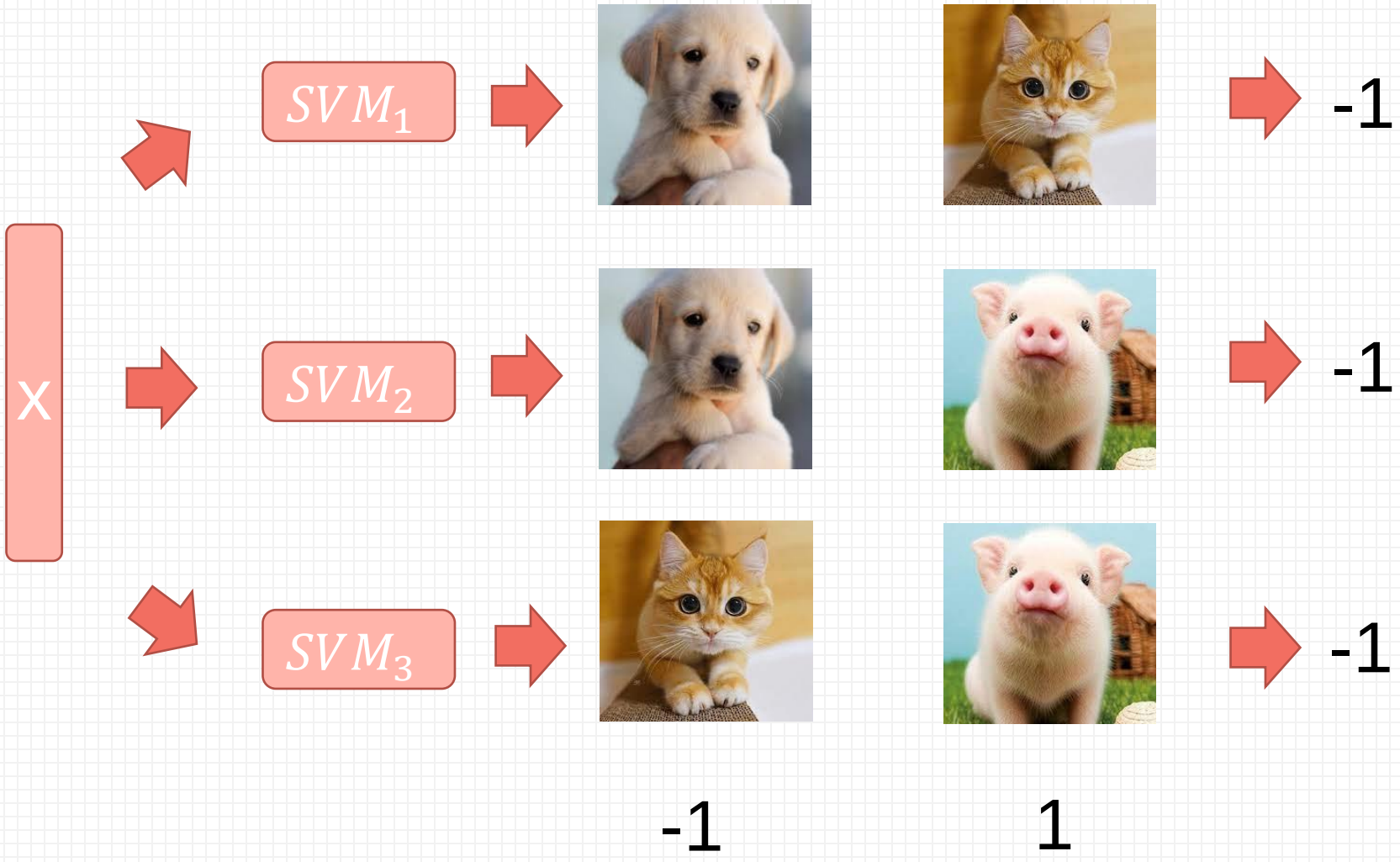
SVM怎么做多分类?

① one-vs-rest（OVR）：思路简单，高效。需要比k次，即训练k个模型，k为类别数目。



SVM怎么做多分类?

② one-vs-one（OVO）：准确，慢。需要比 $k(k-1)/2$次， 即训练个 $k(k-1)/2$ 模型，k为类别数目。



- 顺序要对应：

  比如狗、猫、猪

  最后输出才能对上：

  (-1，-1，-1) 对应

  (狗，猫)、(狗、猪)、

  (猫，猪) 。

- **输出的-1含义不一样，需要额外的信息标识。**
- **最后采用投票法输出最终结果**

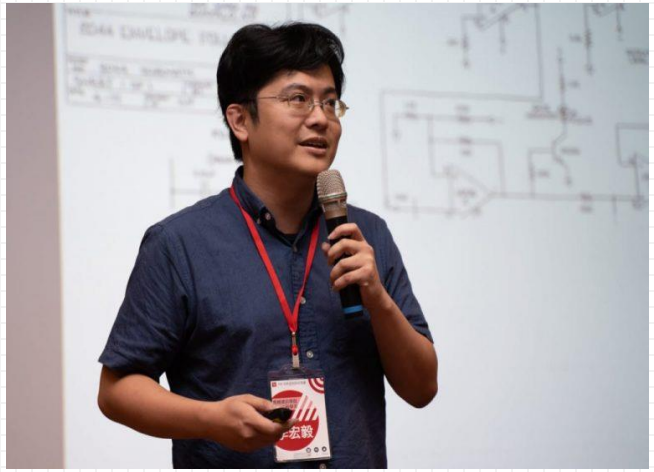
Standard SVM:

https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html

For Large Dataset :

https://scikit-learn.org/stable/modules/generated/sklearn.svm.LinearSVC.html#sklearn.svm.LinearSVC

https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.SGDClassifier.html#sklearn.linear_model.SGDClassifier



本次课程部分图片内容来自李宏毅老师ML课程，
感谢老师的精彩课程讲解及相关开源内容。

所有课程PPT以及code下载链接（或者评论区、视频简介均可下载）：

https://github.com/CHENHUI-X/My-lecture-slides-and-code， 如果对你有帮助，可以 Starred 134 支持我。

UP知识水平有限，且PPT内容公式较多，纯手工敲写，相关内容如有错误及不足，请务必指出，感谢。

THANKS

谢谢观看