Equation:

$$E + S \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_3} E + P$$

8.1 Four equations for the rate of changes of E, S, ES, and P.

$$\frac{d[E]}{dt} = V_E = (k_2 + k_3)[ES] - k_1([E] - [ES])[S]$$

$$\frac{d[S]}{dt} = V_S = k_2[ES] - k_1([E] - [ES])[S]$$

$$\frac{d[ES]}{dt} = V_{ES} = k_1([E] - [ES])[S] - (k_2 + k_3)[ES]$$

$$\frac{d[P]}{dt} = V_P = k_3[ES]$$

Where [*xx*] represents the concentration of components *xx.*

8.2 Code for numerically solving these four equations using the fourth-order RungeKutta method

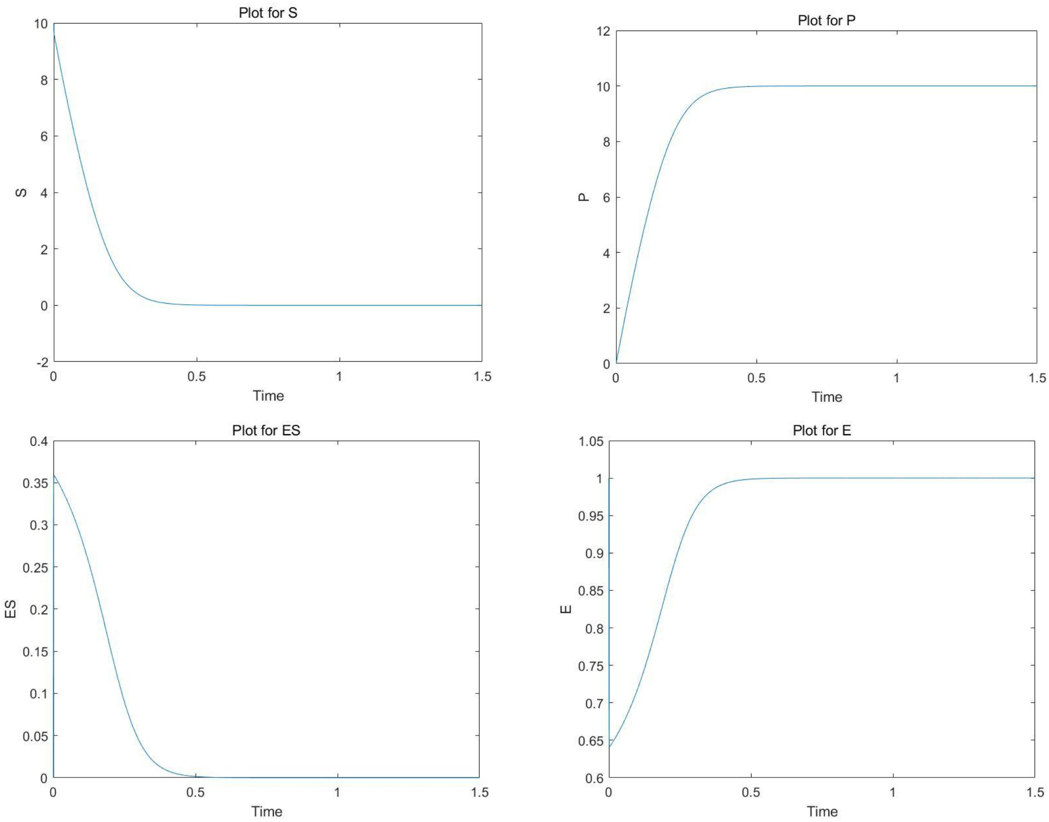
The code was completed in Matlab version 2021a, and a screenshot of the code is shown below, along with running results.

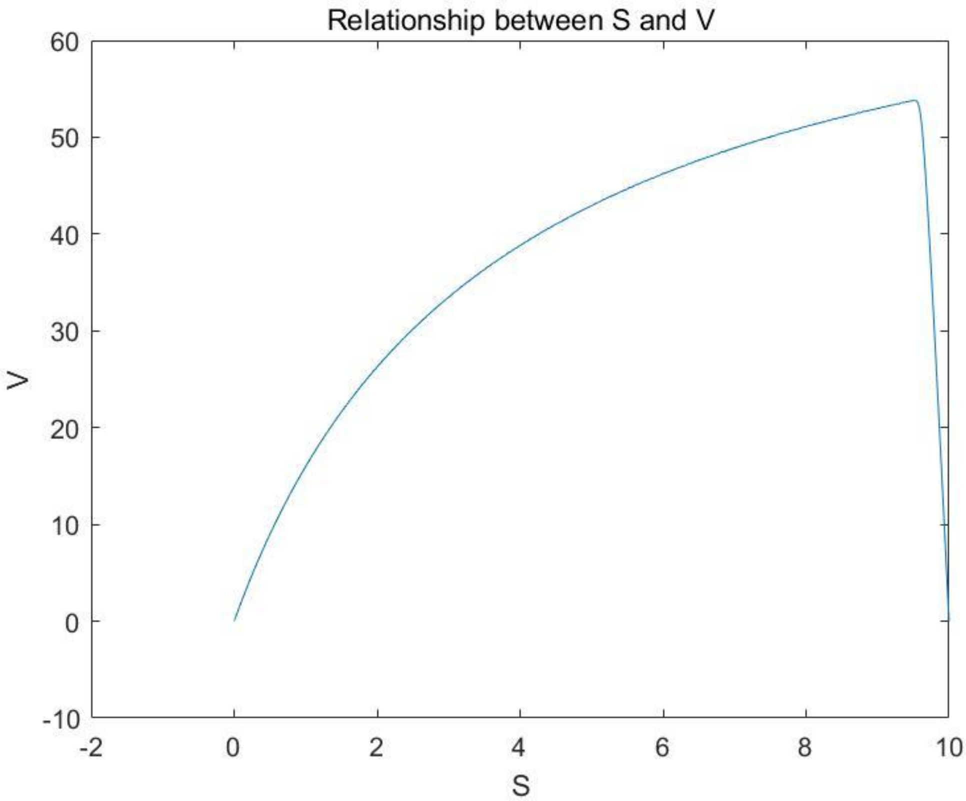
CODE in m:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
% Solve the differential equation
timespan = [0, 1.5];
x0 = [1, 10, 0, 0];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
% Using MATLAB's own function ode45 to solve differential equations
% The usage of ode45 derives from the code posted on the website below:
% https://www.zhihu.com/question/395096211/answer/1227935749
[T, X] = ode45(@f, timespan, x0);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
% Plot Figures of E, S, ES, and P respectively
figure;
```

```
plot(T, X(:, 1))
title('Plot for E');
ylabel('E'); xlabel('Time');

figure;
plot(T, X(:, 2))
title('Plot for S');
ylabel('S'); xlabel('Time');

figure;
plot(T, X(:, 3))
title('Plot for ES');
ylabel('ES'); xlabel('Time');

figure;
plot(T, X(:, 4))
title('Plot for P');
ylabel('P'); xlabel('Time');

V=150*X(:, 3);

figure;
plot(X(:,2),V)
xlabel('S'); ylabel('V')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
% Function defination
function f = f(t, x)

    f = zeros(4, 1);

    % system of four differetial equations
    f(1) =750 * x(3) - 100 * (x(1) - x(3)) * x(2);
    f(2) =600 * x(3) - 100 * (x(1) - x(3)) * x(2);
    f(3) =100 * (x(1) - x(3)) * x(2) - 750 * x(3);
    f(4) =150 * x(3) ;
end
```

Results are shown below：





8.3 Plot the velocity V as a function of the concentration of the substrate S

Premise:

$$\mathrm{V} = \mathrm{V_p}$$

As the reaction reaches dynamic equilibrium, for ES: the rate of production equals to the rate of decomposition. And the numerical relationship is shown as :

$$k_1([E] - [ES])[S] \ = \ (k_2 + k_3)[ES]$$

According to the equation above, we can get:

$$[\mathrm{ES}] = \frac{\mathrm{k_1[E][S]}}{(\mathrm{k_2} + \mathrm{k_3}) + \mathrm{k_1[S]}} = \frac{\mathrm{[E][S]}}{\frac{\mathrm{k_2} + \mathrm{k_3}}{\mathrm{k_1}} + [\mathrm{S}]}$$

Define: $k_m = \frac{\mathrm{k_2+k_3}}{\mathrm{k_1}}$

Then, when [S] is high, all the E can convert into ES, which is $[\mathrm{E}] \ = \ [\mathrm{ES}]$, at this moment, the velocity can reach its maximum V_m, for V_m:

$$V_m = \ k_3[ES] = k_3[E]$$

And we can get:

$$\mathrm{V} \ = \ \mathrm{k_3[ES]} = \frac{\mathrm{k_3[E][S]}}{\mathrm{k_m} + [\mathrm{S}]} = \frac{\mathrm{V_m[S]}}{\mathrm{k_m} + [\mathrm{S}]}$$

Simplify the above equation and we can get the form in:

$$\mathrm{V} = V_m - k_m \times \frac{V}{[S]}$$

Set V as the y-axis, V/[S] as the x-axis, we can get following relation: