Bernoulli random variable: $f(x) \begin{cases} p & \text{if } x=1 \\ 1-p & \text{if } x=0 \end{cases}$

$$E(X) = \sum_x x f(x) = 0 \times f(0) + 1 \times f(1)$$
$$= 0 \times (1-p) + 1 \times p = p$$

$$E(X^2) = \sum_x x^2 f(x) = 0^2 \times f(0) + 1^2 \times f(1)$$
$$= 0 \times (1-p) + 1 \times p = p$$

$$V(X) = E(X^2) - (E(X))^2$$
$$= p - p^2 = p(1-p)$$

Binomial random variable: $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$$E(X) = E\left(\boxed{\sum_{i=1}^{n} Y_i}\right) = \sum_{i=1}^{n} E(Y_i) = np$$

where Y_i is a Bernoulli random variable

WHY? Because n coin flips (Binomial trials) equals $\boxed{\text{the sum of } n \text{ independent Bernoulli trials}}$

We know that the expectation of Bernoulli RV $= p$

Also from $E(X+Y) = E(X) + E(Y)$

$$\Rightarrow E(Y_1 + Y_2 + \cdots + Y_n) = E(Y_1) + E(Y_2) + \cdots + E(Y_n)$$

$$V(X) = E(X^2) - (E(X))^2$$
$$= E\left(\left(\sum_{i=1}^{n} Y_i\right)^2\right) - (np)^2$$

independent

$$= E\left(\sum_{i=1}^{n} Y_i^2 + \sum_{\substack{j \neq k \\ j=1}}^{n} \sum_{k=1}^{n} Y_j Y_k\right) - (np)^2$$
$$= \sum_{i=1}^{n} E(Y_i^2) + \sum_{j \neq k}^{n} \sum_{k=1}^{n} E(Y_j) E(Y_k) - (np)^2$$
$$= np + (n-1)n\,p\,p - (np)^2$$
$$= np + \cancel{n^2p^2} - np^2 - \cancel{n^2p^2} = np(1-p)$$

All I used are equalities for $E(X)$, $V(X)$: P. 294 & 296

Let $X \sim N(\mu, \sigma^2)$

Any transformation of X in the form $aX+b$ will follow a normal distribution

$\Rightarrow$ We all need to calculate the expectation of the new variable $\Rightarrow$ mean & its variance $\Rightarrow$ Variance

$T = aX + b$

$E(T) = E(aX+b)$

$= aE(X) + b = a\mu + b$

$V(T) = V(aX+b)$

$= E((aX+b)^2) - (E(aX+b))^2$

$= E(a^2X^2 + 2abX + b^2) - (aE(X)+b)^2$

$= a^2E(X^2) + \cancel{2abE(X)} + \cancel{b^2} - a^2(E(X))^2 \cancel{- 2abE(X)} \cancel{- b^2}$

$= a^2(E(X^2) - E(X)^2) = a^2V(X) = a^2\sigma^2$

$\Rightarrow T \sim N(a\mu + b, (a\sigma)^2)$

$S = X + C \Rightarrow a = 1 \quad b = C$

$S \sim N(1 \times \mu + C, (1 \times \sigma)^2) = N(\mu + C, \sigma^2)$

$Y = CX \Rightarrow a = C \quad b = 0$

$Y \sim N(C\mu + 0, (C\sigma)^2) = N(C\mu, (C\sigma)^2)$

$Z = (X - \mu)/\sigma = \frac{1}{\sigma}X - \frac{\mu}{\sigma} \Rightarrow a = \frac{1}{\sigma} \quad b = -\frac{\mu}{\sigma}$

$Z \sim N(\frac{1}{\sigma}\mu - \frac{\mu}{\sigma}, (\frac{1}{\sigma}\sigma)^2) = N(0, 1)$