Numerical Exercise

x y

Observation	1st sibling age	2nd sibling age
1	20 $\frac{20-19.5}{1}=0.5$	18 $\frac{18-17}{2\sqrt{3}}=\frac{\sqrt{3}}{2}$
2	18 $\frac{18-19.5}{1}=-1.5$	16 $\frac{16-17}{3\sqrt{3}}=-\frac{\sqrt{3}}{2}$
3	20 $\frac{20-19.5}{1}=0.5$	16 $\frac{16-17}{\sqrt{3}}=-\frac{\sqrt{3}}{2}$
4	20 $\frac{20-19.5}{1}=0.5$	18 $\frac{18-17}{2/\sqrt{3}}=\frac{\sqrt{3}}{2}$

$$z\text{-score of } \mathbf{x}_i = \frac{x_i - \bar{x}}{S_x}$$

$$\text{correlation}(x, y) =$$

$$\frac{1}{n}\sum_{i=1}^{n}(z\text{-score of } x_i \times z\text{-score of } y_i)$$

▶ z-score(1st sibiling age)

$\bar{x} = (20+18+20+20)/4 = 19.5$ $S_x = \sqrt{\frac{1}{4-1}\left\{(20-19.5)^2+(18-19.5)^2+(20-19.5)^2+(20-19.5)^2\right\}}$
$= 1$

▶ z-score(2nd sibiling age)

$\bar{y} = (18+16+16+18)/4 = 17$ $S_y = \sqrt{\frac{1}{4-1}\left\{(18-17)^2+(16-17)^2+(16-17)^2+(16-17)^2\right\}}$
$= \frac{2}{\sqrt{3}}$

▶ Correlation(1st sibiling age, 2nd sibiling age)

$$\frac{1}{4}\left\{\frac{1}{2}\times\frac{\sqrt{3}}{2}+\left(-\frac{3}{2}\right)\times\left(-\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\times\left(-\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\times\left(\frac{\sqrt{3}}{2}\right)\right\} = \frac{\sqrt{3}}{4}$$