

Numerical Exercise (Let's Solve Together)

Observation	1st sibling age (X)	2nd sibling age (Y)
1	20	18
2	18	16
3	20	16
4	20	18

$$\bar{X} = 19.5$$

$$\bar{Y} = 17$$

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$$

$$\hat{\beta} = \frac{\sum_{i=1}^n (Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^n (X_i - \bar{X})^2}$$

$$RMSE = \sqrt{\frac{1}{n} SSR} = \sqrt{\frac{1}{n} \sum_{i=1}^n \hat{\epsilon}_i^2}$$

$$TSS = \sum_{i=1}^n (Y_i - \bar{Y})^2 \quad R^2 = 1 - \frac{SSR}{TSS}$$

► Predict Y using X: $\hat{\alpha}, \hat{\beta}$

$$\hat{\beta} = \frac{0.5 + 1.5 - 0.5 + 0.5}{3 \times 0.5^2 + 1.5^2} = \frac{2}{3} \quad \hat{\alpha} = 17 - \frac{2}{3} \times 19.5 = 4$$

► SSR $\frac{2^2 + (-4)^2 + 2^2}{3^2} = \frac{24}{9}$

► RMSE $\sqrt{\frac{1}{4} \times \frac{24}{9}} = \sqrt{\frac{2}{3}}$

► TTS $1^2 + (-1)^2 + (-1)^2 + 1^2 = 4$

► R^2 $1 - \frac{\frac{24}{9}}{4} = \frac{1}{3}$