Normal Distribution

26 June, 2019

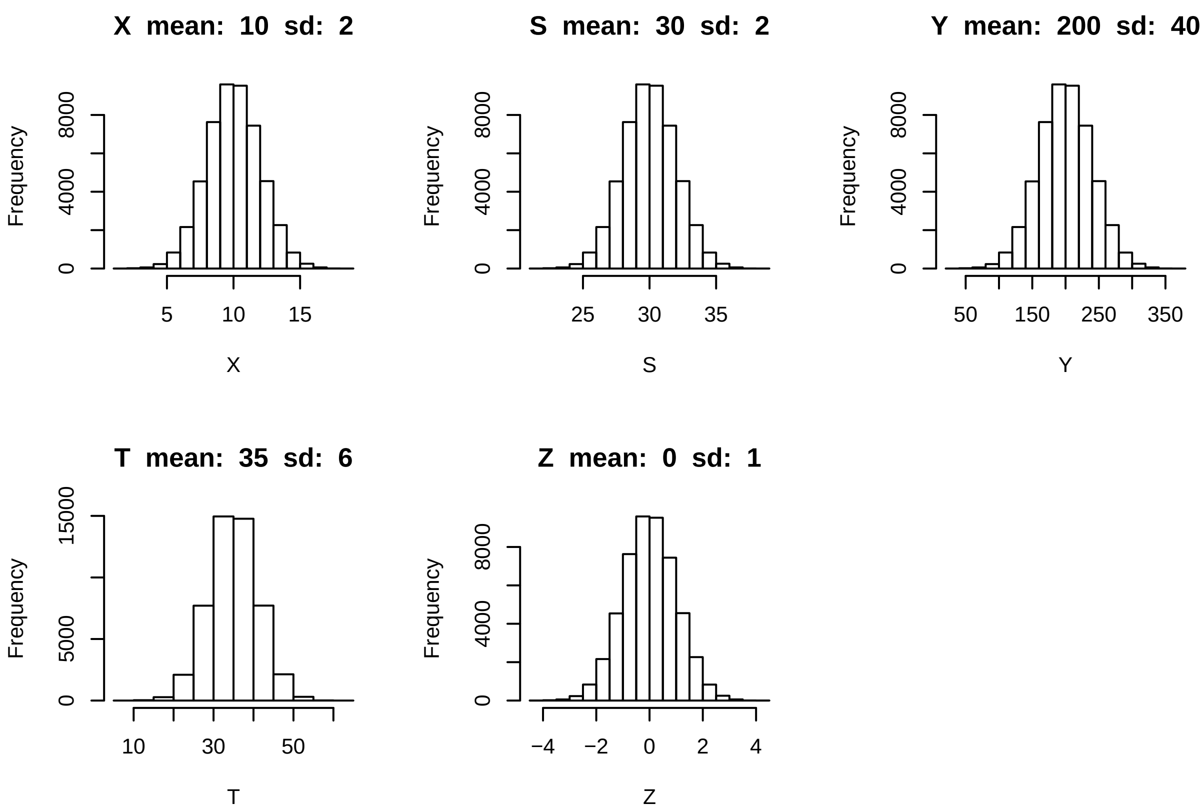
Transformations

$X \sim \mathcal{N}(\mu, \sigma^2)$. Let us draw 50,000 observations each, and draw their histograms.

```
mu <- 10; sigma <- 2
n <- 50000
X <- rnorm(n, mean = mu, sd = sigma)
c <- 20
S <- X + c
Y <- c*X
a <- 3; b <- 5
T <- a*X + b
Z <- (X-mu)/sigma
```

Histograms

```
par(mfrow=c(2,3))
hist(X, main=paste("X  mean: " , round(mean(X))," sd: ", round(sd(X))))
hist(S, main=paste("S  mean: " , round(mean(S))," sd: ", round(sd(S))))
hist(Y, main=paste("Y  mean: " , round(mean(Y))," sd: ", round(sd(Y))))
hist(T, main=paste("T  mean: " , round(mean(T))," sd: ", round(sd(T))))
hist(Z, main=paste("Z  mean: " , round(mean(Z))," sd: ", round(sd(Z))))
```



Probability of being in a range

$P(\mu - k\sigma \leq X \leq \mu + k\sigma) = P(X \leq \mu + k\sigma) - P(\mu - k\sigma \leq X)$
$= P(\mu + c - k\sigma \leq S \leq \mu + c + k\sigma) = P(S \leq \mu + c + k\sigma) - P(\mu + c - k\sigma \leq S)$
$= P(c\mu - kc\sigma \leq Y \leq c\mu + kc\sigma) = P(Y \leq c\mu + kc\sigma) - P(c\mu - kc\sigma \leq Y)$
$= P(a\mu + b - ka\sigma \leq T \leq a\mu + b + ka\sigma) = P(T \leq a\mu + b + ka\sigma) - P(a\mu + b - ka\sigma \leq T)$

```r
k=2
pnorm(mu+k*sigma,mean = mu, sd = sigma)-pnorm(mu-k*sigma,mean = mu, sd = sigma) #X
```

```
## [1] 0.9544997
```

```r
pnorm(mu+c+k*sigma,mean = mu+c, sd = sigma)-pnorm(mu+c-k*sigma,mean = mu+c, sd = sigma) #S
```

```
## [1] 0.9544997
```

```r
pnorm(c*mu+k*c*sigma,mean=c*mu,sd=c*sigma)-pnorm(c*mu-k*c*sigma,mean=c*mu,sd=c*sigma) #Y
```

```
## [1] 0.9544997
```

```r
pnorm(a*mu+b+k*a*sigma,mean=a*mu+b,sd=a*sigma)-pnorm(a*mu+b-k*a*sigma,mean=a*mu+b,sd=a*sigma) #T
```

```
## [1] 0.9544997
```

```r
pnorm(k)-pnorm(-k) #Z
```

```
## [1] 0.9544997
```