

Numerical Exercise

	x	y
Observation	1st sibling age	2nd sibling age
1	20 $\frac{20-19.5}{1} = 0.5$	18 $\frac{18-17}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$
2	18 $\frac{18-19.5}{1} = -1.5$	16 $\frac{16-17}{2/\sqrt{3}} = -\frac{\sqrt{3}}{2}$
3	20 $\frac{20-19.5}{1} = 0.5$	16 $\frac{16-17}{2/\sqrt{3}} = -\frac{\sqrt{3}}{2}$
4	20 $\frac{20-19.5}{1} = 0.5$	18 $\frac{18-17}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$

$$\text{z-score of } x_i = \frac{x_i - \bar{x}}{S_x}$$

$$\text{correlation}(x, y) =$$

$$\frac{1}{n-1} \sum_{i=1}^n (\text{z-score of } x_i \times \text{z-score of } y_i)$$

↳ the use of $n-1$ will be discussed next week.

- z-score(1st sibling age)

$$\bar{x} = (20+18+20+20)/4 = 19.5 \quad S_x = \sqrt{\frac{1}{4-1} \{ (20-19.5)^2 + (18-19.5)^2 + (20-19.5)^2 + (20-19.5)^2 \}} = 1$$

- z-score(2nd sibling age)

$$\bar{y} = (18+16+16+18)/4 = 17 \quad S_y = \sqrt{\frac{1}{4-1} \{ (18-17)^2 + (16-17)^2 + (16-17)^2 + (18-17)^2 \}} = \frac{2}{\sqrt{3}}$$

- Correlation(1st sibling age, 2nd sibling age)

$$\frac{1}{3} \left\{ \frac{1}{2} \times \frac{\sqrt{3}}{2} + \left(-\frac{3}{2}\right) \times \left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{2} \times \left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right) \right\} = \frac{1}{\sqrt{3}}$$