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Random Variables and Their Distributions

Week 10

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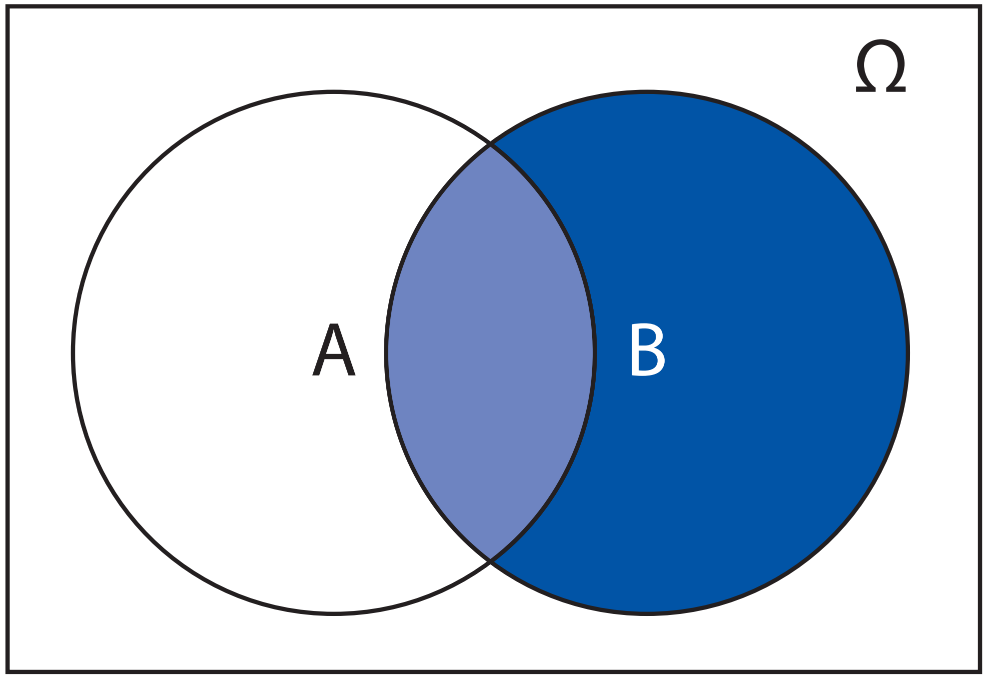
Review of Week 9

- Probability (Chapter 6.1.)
  - Definitions
  - Axioms
  - Permutations
  - Combinations
- Conditional Probability (Chapter 6.2.)
  - Conditional probability
  - Joint probability
  - Independence

Example 1: Write Down Equivalence/Definition and Plug in Values

- Rolling a dice once
  - A: a multiple of 2; B: a multiple of 3
    - $\Omega =$
    - as a set: $A =$ $B =$
    - as a set: $A^c =$ $B^c =$
    - $P(A) =$ $P(B) =$
    - $P(A^c) =$ $P(B^c) =$
    - $P(A \text{ and } B) =$
    - $P(A|B) =$
    - $P(B|A) =$
    - $P(A) = P(A \text{ and } B) + P(A \text{ and } B^c)$



Example 2: Write Down Equivalence/Definition and Plug in Values

- Flipping coin 3 times
  - A: first flip head; B: second flip tail
    - $\Omega =$
    - as a set: $A =$ $B =$
    - as a set: $A^c =$ $B^c =$
    - $P(A) =$ $P(B) =$
    - $P(A^c) =$ $P(B^c) =$
    - $P(A \text{ and } B) =$
    - $P(A|B) =$
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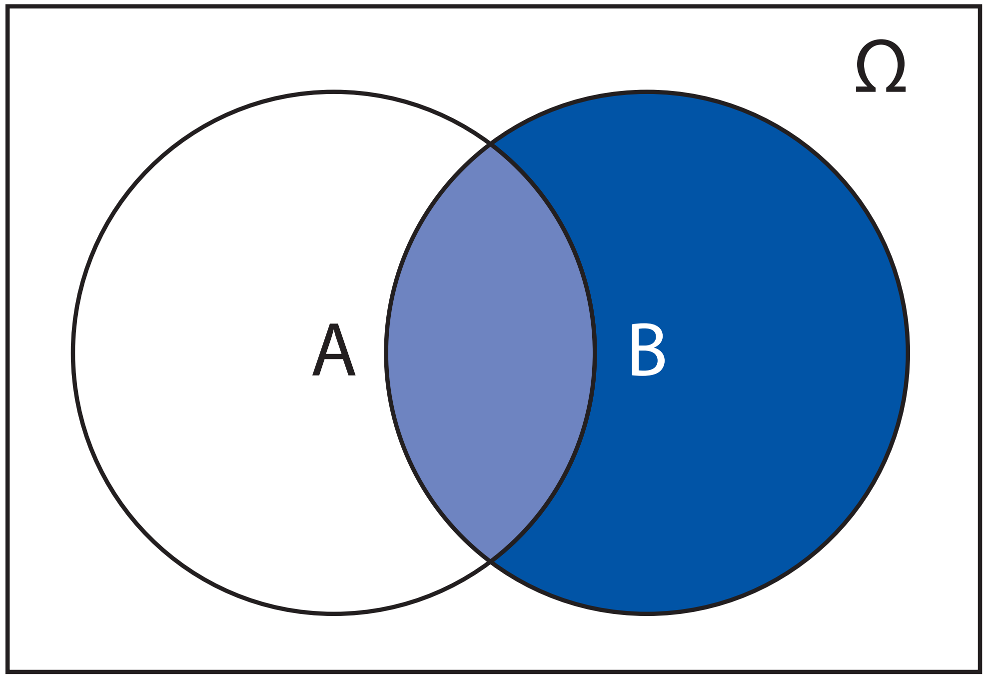

Example 3: Permutations and Combinations

- You are a producer in a large entertainment company
  - You want to form a 5-member unit from IZ*ONE
    - Assume that you are selecting 5 completely at random
    - What is the chance that you select 1 JP member and 4 KR member?
    - What is the chance that you select at least 2 JP members?
  - You are selecting 7 members from IZ*ONE to cover a BTS song
    - Each will be assigned to a different role (e.g. V, Jin, RM ...)
      - How many potential scenarios are there?

Bayes' Rule: Check Textbook 6.2.4

- Bayes' rule (obvious if you know definitions / axioms)

$$\underbrace{\Pr(A \mid B)}_{\text{conditional probability}} = \frac{\overbrace{\Pr(A \text{ and } B)}^{\text{joint probability}}}{\underbrace{\Pr(B)}_{\text{marginal probability}}}$$

$$= \frac{\Pr(B \mid A)\Pr(A)}{\Pr(B \mid A)\Pr(A) + \Pr(B \mid \text{not } A)\Pr(\text{not } A)}$$

- What does Pr(A|B) mean?
  - The chance of A being true given B is observed (i.e. true)
- Why is above the most important result in probability theory?
  - e.g. A is an event happened before B; only B is observed
    - Using Bayes' rule: you can infer about A given B
- Bayesian updating: prior belief Pr(A) -> posterior belief Pr(A|B)

Bayes' Rule: Check Textbook 6.2.4

- Bayes' rule (obvious if you know definitions / axioms)

$$\underbrace{\Pr(A \mid B)}_{conditional\ \ probability} = \frac{\overbrace{\Pr(A \text{ and } B)}^{joint\ \ probability}}{\underbrace{\Pr(B)}_{marginal\ \ probability}}$$

$$= \frac{\Pr(B \mid A)\Pr(A)}{\Pr(B \mid A)\Pr(A) + \Pr(B \mid \text{not } A)\Pr(\text{not } A)}$$

- Posterior probability Pr(A|B): belief of A after observing evidence B
- Prior probability (belief of A happening w/o evidence): P(A)
- Prior probability (belief of A not happening w/o evidence): P(not A)
- Examples (A: prior event; B: data/evidence)

A (prior event)	I voted for A	having a disease	studied hard
B (data)	A won	positive on medical test	high grade in exam

  - Time order is not necessary but good for examples

Example: Monty Hall Problem

- The most famous probability problem
  - http://www.youtube.com/watch?v=mhlc7peGlGg

  - Even a great mathematician failed!

Example: Monty Hall Problem

- You pick door A. Monty opens door C that has a goat.
  - Should you switch to door B?
- Prior beliefs: $P(A) = P(B) = P(C) = 1/3$ (e.g. A: the car is behind door A)
- Data: Monty reveals door C (i.e. MC)
- Posterior belief (inferential goals): $P(A \mid \mathrm{MC})$ and $P(B \mid \mathrm{MC})$
- Question: $P(A \mid \mathrm{MC}) < P(B \mid \mathrm{MC})$
- What do we need? Bayes' rule!
  - Key: Monty's behavior is constrained to open a goat door

- Take home message: Do not believe your intuition, rely on logic.

Contents

Random Variables and Probability Distribution

- Random variable assigns a numeric value to each event of the experiment.
    - Coin flip side: head = 1; tail =0
    - #secs took for commuting: any value greater than 0.
- These values represent mutually exclusive and exhaustive events, together forming the entire sample space Ω.
- A discrete RV takes a finite or at most countably infinite #distinct values.
    - coin flip; race; number of years of education
- A continuous RV assumes an uncountably infinite number of values.
    - height, distance from earth, gross domestic product
- Probability distribution: Probability that a random variable takes a certain value or range of values.
    - P(side): P(side=1) = 0.5; P(side=0) = 0.5
    - P(#secs): P(0<#secs<1000) = 0.3; P(1000<#secs<2000) = 0.4 ....

See you next week.