Heterogeneous Treatment Effects

$$Y = \alpha + \beta_1 \text{ primary2004} + \beta_2 \text{ Neighbors} + \beta_3 \text{ (primary2004} \times \text{Neighbors)} + \epsilon$$

- Average treatment effect for voters in previous election ($\text{primary2004}_i = 1$)
  - $\hat{\alpha} + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3$ $\qquad$ $\hat{\alpha} + \hat{\beta}_1$
  - $\widehat{Y}(\textbf{Neighbors} = 1, \textbf{primary2004} = 1) - \widehat{Y}(\textbf{Neighbors} = 0, \textbf{primary2004} = 1)$
  - $= \hat{\beta}_2 + \hat{\beta}_3$
- Average treatment effect for non-voters in previous election ($\text{primary2004}_i = 0$)
  - $\hat{\alpha} + \hat{\beta}_2$ $\qquad$ $\hat{\alpha}$
  - $\widehat{Y}(\textbf{Neighbors} = 1, \textbf{primary2004} = 0) - \widehat{Y}(\textbf{Neighbors} = 0, \textbf{primary2004} = 0)$
  - $= \hat{\beta}_2$
- Difference in the estimated average treatment effect between voters/non-voters
  - $(\hat{\beta}_2 + \hat{\beta}_3) - \hat{\beta}_2 = \hat{\beta}_3$

```
## lm(formula = primary2008 ~ primary2004 + messages + primary2004:messages,
##     data = social.neighbor)
##
## Coefficients:
##                    (Intercept)
##                        0.23711   α̂
##                    primary2004
##                        0.14870   β̂1
##               messagesNeighbors
##                        0.06930   β̂2
## primary2004:messagesNeighbors
##                        0.02723   β̂3
```

Heterogeneous Treatment Effects

- Interaction model with linear age effect

$$Y = \alpha + \beta_1 \,\textsf{age} + \beta_2 \,\textsf{Neighbors} + \beta_3 \,(\textsf{age} \times \textsf{Neighbors}) + \epsilon$$

  - Average treatment effect difference when age increases by 1 year)
    - Neighbor treatment effect for age x population

$$(\hat{\alpha} + \hat{\beta}_1 x + \hat{\beta}_2 + \hat{\beta}_3 x) - (\hat{\alpha} + \hat{\beta}_1 x) = \hat{\beta}_2 + \hat{\beta}_3 x$$

    - Neighbor treatment effect for age x+1 population

$$\{\hat{\alpha} + \hat{\beta}_1(x+1) + \hat{\beta}_2 + \hat{\beta}_3(x+1)\} - \{\hat{\alpha} + \hat{\beta}_1(x+1)\} = \hat{\beta}_2 + \hat{\beta}_3(x+1)$$

    - Average treatment effect difference when age increases by 1 year

$$\hat{\beta}_3 = \{\hat{\beta}_2 + \hat{\beta}_3(x+1)\} - (\hat{\beta}_2 + \hat{\beta}_3 x)$$

```
## lm(formula = primary2008 ~ age * messages, data = social.neighbor)
##
## Coefficients:
##          (Intercept)                  age
##            0.0894768            0.0039982
##    messagesNeighbors  age:messagesNeighbors
##            0.0485728            0.0006283
```

Heterogeneous Treatment Effects

- Interaction model with linear age and quadratic age effects

$$Y = \alpha + \beta_1 \,\mathsf{age} + \beta_2 \,\mathsf{age}^2 + \beta_3 \,\mathsf{Neighbors} + \beta_4 \,(\mathsf{age} \times \mathsf{Neighbors}) + \beta_5 \,(\mathsf{age}^2 \times \mathsf{Neighbors}) + \epsilon.$$

  - Average treatment effect difference when age increases by 1 year)
    - Neighbor treatment effect for age x population

$$(\hat{\alpha} + \hat{\beta}_1 x + \hat{\beta}_2 x^2 + \hat{\beta}_3 + \hat{\beta}_4 x + \hat{\beta}_5 x^2) - (\hat{\alpha} + \hat{\beta}_1 x + \hat{\beta}_2 x^2) = \hat{\beta}_3 + \hat{\beta}_4 x + \hat{\beta}_5 x^2$$

    - Neighbor treatment effect for age x+1 population

$$\hat{\beta}_3 + \hat{\beta}_4 (x+1) + \hat{\beta}_5 (x+1)^2$$

    - Average treatment effect difference when age increases by 1 year

$$\{\hat{\beta}_3 + \hat{\beta}_4 (x+1) + \hat{\beta}_5 (x+1)^2\} - (\hat{\beta}_3 + \hat{\beta}_4 x + \hat{\beta}_5 x^2) = \hat{\beta}_4 + \hat{\beta}_5 (2x+1)$$

```
##                 (Intercept)                              age
##                  -9.700e-02                        1.172e-02
##                   I(age^2)                 messagesNeighbors
##                  -7.389e-05                       -5.275e-02
##      age:messagesNeighbors  I(age^2):messagesNeighbors
##                   4.804e-03                       -3.961e-05
```

Heterogeneous Treatment Effects

- Interaction model with linear age and quadratic age effects

$$Y = \alpha + \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{Neighbors} + \beta_4 (\text{age} \times \text{Neighbors})$$
$$+ \beta_5 (\text{age}^2 \times \text{Neighbors}) + \epsilon.$$

  - Average treatment effect difference when age increases by 1 year)
    - Neighbor treatment effect for age x population

$$(\hat{\alpha} + \hat{\beta}_1 x + \hat{\beta}_2 x^2 + \hat{\beta}_3 + \hat{\beta}_4 x + \hat{\beta}_5 x^2) - (\hat{\alpha} + \hat{\beta}_1 x + \hat{\beta}_2 x^2) = \hat{\beta}_3 + \hat{\beta}_4 x + \hat{\beta}_5 x^2$$

    - Neighbor treatment effect for age x+c population

$$\hat{\beta}_3 + \hat{\beta}_4 (x+c) + \hat{\beta}_5 (x+c)^2$$

    - Average treatment effect difference when age increases by c years

$$\{\hat{\beta}_3 + \hat{\beta}_4 (x+c) + \hat{\beta}_5 (x+c)^2\} - (\hat{\beta}_3 + \hat{\beta}_4 x + \hat{\beta}_5 x^2)$$

$$= \hat{\beta}_4 c + \hat{\beta}_5 (2cx + c^2)$$

Heterogeneous Treatment Effects

- Interaction model with linear age and quadratic age effects

$$Y = \alpha + \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{Neighbors} + \beta_4 (\text{age} \times \text{Neighbors}) + \beta_5 (\text{age}^2 \times \text{Neighbors}) + \epsilon.$$

$\hat{\alpha} + \hat{\beta}_1 x + \hat{\beta}_2 x^2 + \hat{\beta}_3 + \hat{\beta}_4 x + \hat{\beta}_5 x^2$