$\boxed{\text{Independently}}$ drawn observations :
$X_1, X_2, \ldots X_n$ from population (probability distribution) with mean $E(X)$ and variance $V(x)$

We are interested in the $\boxed{\text{distribution}}$ of the sample mean $\bar{X}_n$ (i.e. the mean of the observations $X_1, X_2, \ldots X_n$)

Expectation of $\bar{X}_n$:

$$\boxed{E(\bar{X}_n)} = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = E(X)$$

because

$$E(a(X_1+X_2)) = aE(X_1+X_2) = a\,E(X_1) + aE(X_2)$$

And for all i, X_i is drawn from an $\boxed{\text{identical}}$ distribution (population) with mean: $E(X)$

Variance of $\bar{X}_n$

$$\boxed{V(\bar{X}_n)} = V\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} V(X_i) = \frac{1}{n}V(x)$$

because

$$V(a(X_1+X_2)) = a^2V(x_1+x_2) = a^2V(x_1) + a^2V(x_2)$$

And for all i, X_i is drawn from an $\boxed{\text{identical}}$ distribution (population) with Variance $V(X)$

For large enough samples (n is large), the distribution of sample mean $\bar{X}_n$ will converge to normal, and

$$\frac{\bar{X}_n - E(X)}{\sqrt{V(X)/n}} \rightsquigarrow N(0,1)$$

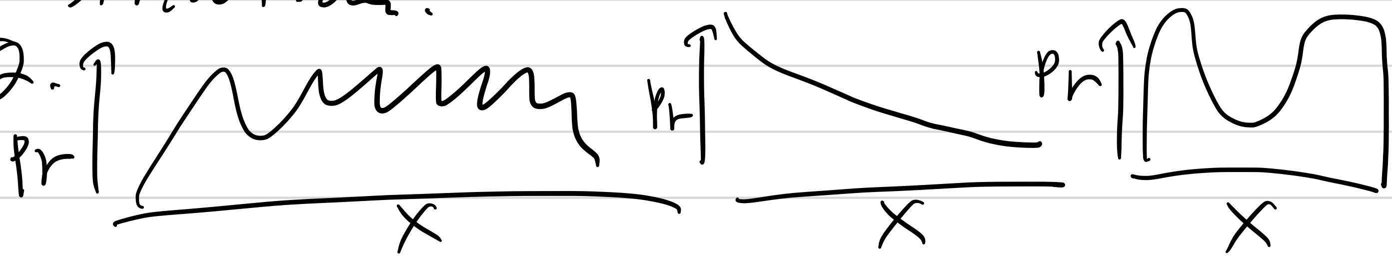
converge in distribution as n grows

This is same as

$$\frac{\bar{X}_n - \boxed{E(\bar{X}_n)}}{\sqrt{\boxed{V(\bar{X}_n)}}}$$

Recall that population can follow ANY distribution!

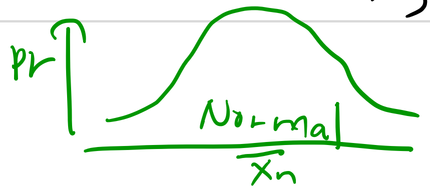
e.g.



HOWEVER once you obtain the samples and draw the distribution of their MEANS, due to Central Limit Theorem:

it will always follow $N(E(X), V(X)/n)$

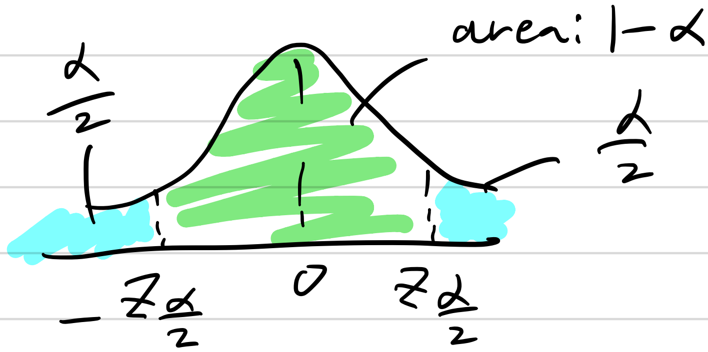
when n is large.



Hypothesis testing

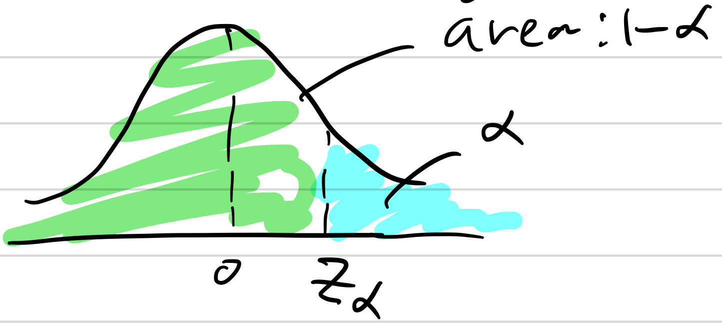
$H_0: \mu = \mu_0 \longrightarrow H_1: \mu \neq \mu_0$

population mean



$H_1: \mu < \mu_0$

$H_1: \mu > \mu_0$

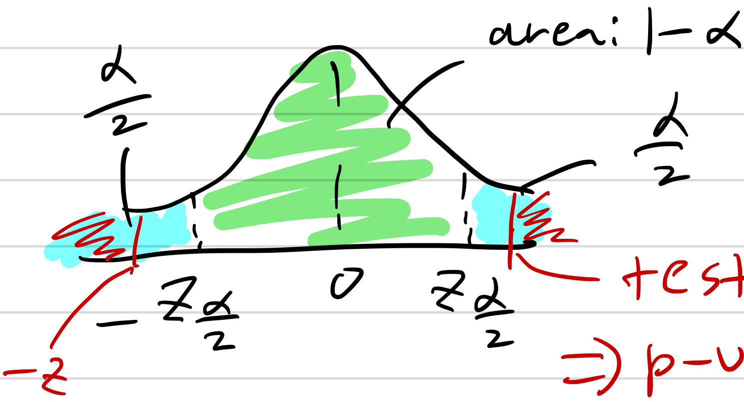


If your test statistic belongs to

- fail to reject the null!
- reject the null

Recall: Hypothesis testing is to check how likely we observe THE sample mean given H_0 is TRUE: $\mu = \mu_0$

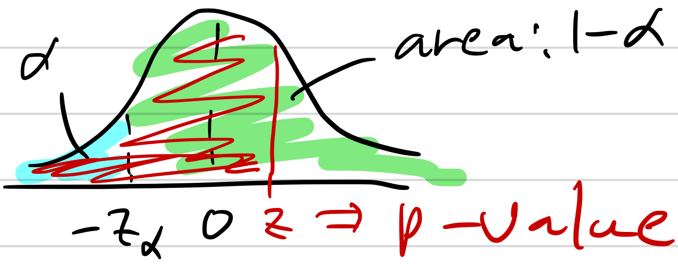
$H_1: \mu \neq \mu_0$



test statistic z

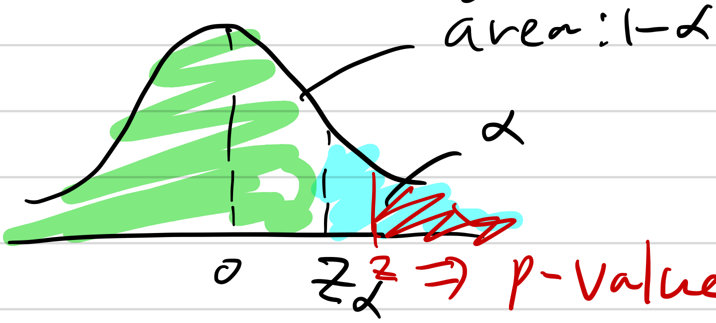
$\Rightarrow$ p-value : $Pr(>|z|) + Pr(<-|z|)$

$H_1: \mu < \mu_0$



$\Rightarrow$ p-value : $Pr(<z)$

$H_1: \mu > \mu_0$



$\Rightarrow$ p-value : $Pr(>z)$

If p-value $\leq \alpha \Rightarrow$ reject the null!
p-value $> \alpha \Rightarrow$ fail to reject the null!

"=" case is negligible since it is almost impossible to get a p-value exactly same as α.

Tea tasting experiment

H_0: θ chance of being correct $= 0.5$

H_1: $\theta > 0.5$

p-value for getting everything correct

if H_0 is true: $\theta = 0.5$

The setting: complete Randomization with M: 4 and T: 4

Let the subject know M: 4 and T: 4

Entire possibilities: ${}_8C_4 = 70$

	# correct choices				
	0	2	4	6	8
# Cases	${}_4C_0\,{}_4C_4$	${}_4C_1\,{}_4C_3$	${}_4C_2\,{}_4C_2$	${}_4C_3\,{}_4C_1$	${}_4C_4\,{}_4C_0$
	1	16	36	16	1
Probability	$\frac{1}{70}$	$\frac{16}{70}$	$\frac{36}{70}$	$\frac{16}{70}$	$\frac{1}{70}$

Sum: 70

Because

If H_0 is true: $\theta = 0.5$, then every case among the 70 cases will have equal probability to occur!

$$p\text{-value}(\#\text{correct choices} = 8) = \frac{1}{70}$$

$$p\text{-value}(\quad '' \quad \geq 6) = \frac{16}{70} + \frac{1}{70} = \frac{17}{70}$$

Because we are conducting right-sided test!

(i.e. $H_1: \theta > \mu_0$)