排序

希尔排序：Shell序列 + 输入敏感性

14-C2

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Shell's Sequence

- Shell 1959：$\mathcal{H}_{shell} = \{1, 2, 4, 8, 16, 32, 64, \ldots, 2^k, \ldots\}$
- 实际上，采用 $\mathcal{H}_{\text{shell}}$，在最坏情况下需要运行 $\Omega(n^2)$ 时间...
- 考查由子序列 A = unsort[0, 2^{N-1}) 和 B = unsort[2^{N-1}, 2^N) 交错而成的序列

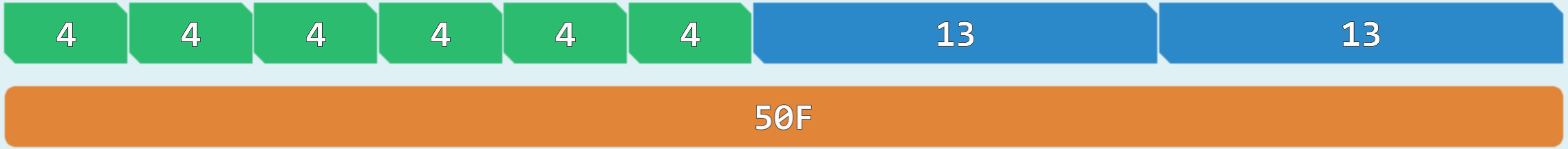
- 在做2-sorting时，A、B各成一列；故此后必然各自有序

- 然而其中的逆序对依然很多，最后的1-sorting仍需$1 + 2 + 3 + \cdots + 2^{N-1} = \Omega(n^2/4)$ 时间
- 根源在于，$\mathcal{H}_{\text{shell}}$中各项并不互素，甚至相邻项也非互素

Postage Problem

- The postage for a letter is 50F, and a postcard 35F

  But there are only stamps of 4F and 13F available

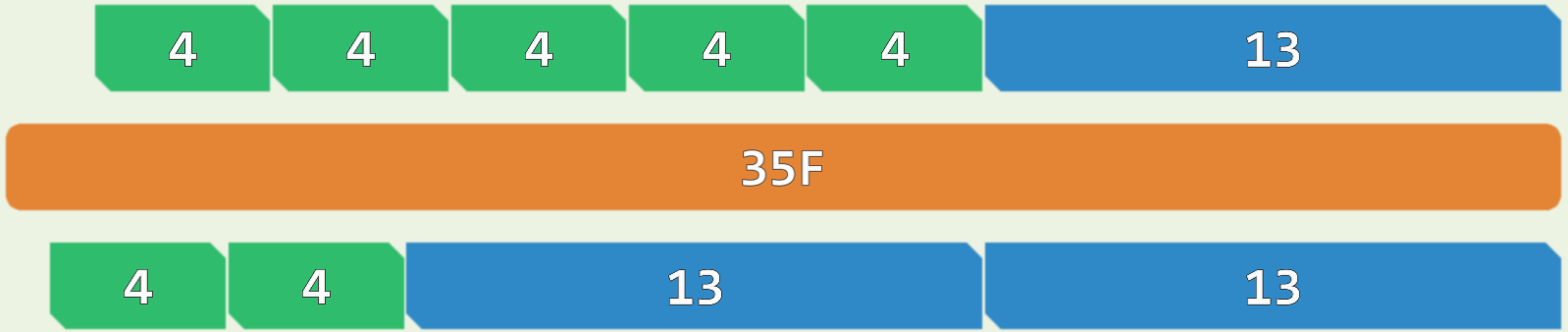


- Possible to stamp

  the letter and

  the postcard

  EXACTLY?



- Given a postage P, determine whether $P \in \{ n \cdot 4 + m \cdot 13 \mid n, m \in \mathcal{N} \}$

Linear Combination

- Let $g, h \in \mathcal{N}$
- For any $n, m \in \mathcal{N}$, $n \cdot g + m \cdot h$ is called a linear combination of g and h
- Denote $\mathbf{C}(g,h) = \{\, ng + mh \mid n, m \in \mathcal{N} \,\}$

  $\mathbf{N}(g,h) = \mathcal{N} \backslash \mathbf{C}(g,h)$ //numbers that are NOT combinations of g and h

  $\mathbf{x}(g,h) = \max\{\, \mathbf{N}(g,h) \,\}$ //always exists?
- Theorem: when g and h are relatively prime, we have

$$\mathbf{x}(g,h) = (g-1)\cdot(h-1) - 1 = gh - g - h$$

e.g. $\mathbf{x}(3,7) = 11$, $\mathbf{x}(4,9) = 23$, $\mathbf{x}(\boxed{4}, \boxed{13}) = \boxed{35}$, $\mathbf{x}(5,14) = 51$

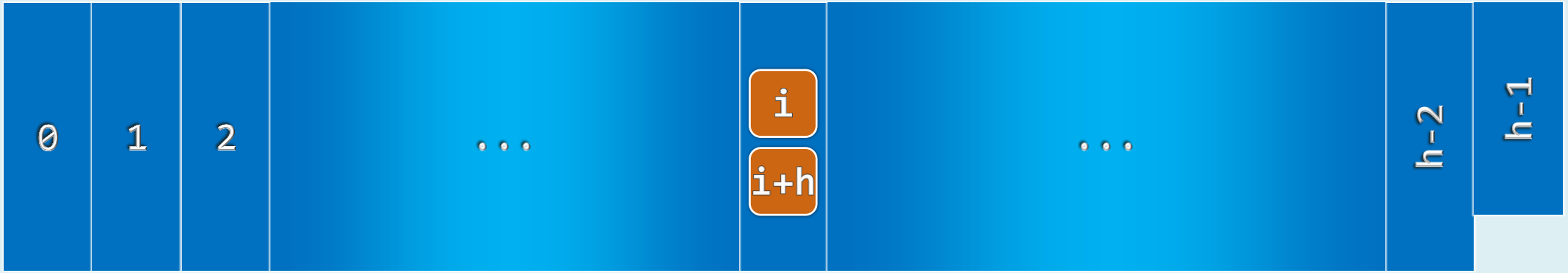
h-sorting & h-ordered

❖ A sequence S[0,n) is called h-ordered if $S[i] \leq S[i+h],\ \forall\ 0 \leq i < n-h$

❖ A 1-ordered sequence is sorted

❖ h-sorting: an h-ordered sequence is obtained by

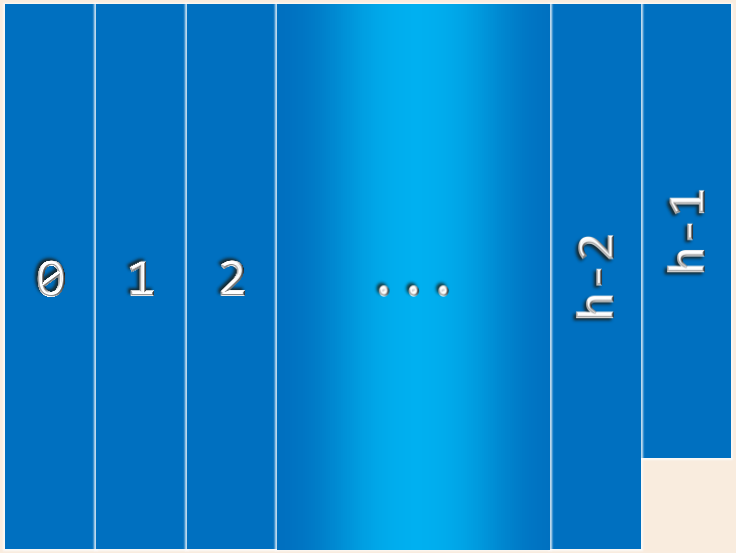
- arranging S into a 2D matrix with h columns and
- sorting each column respectively

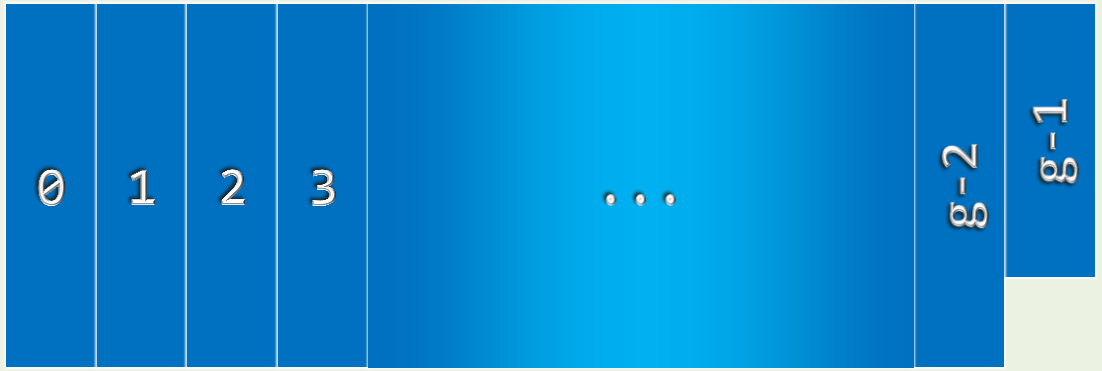


Theorem K

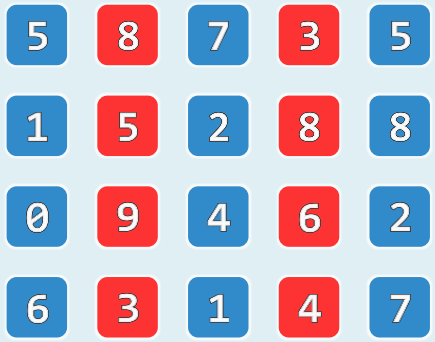
❖ [Knuth, ACP Vol.3 p.90] //习题解析[12-12, 12-13]

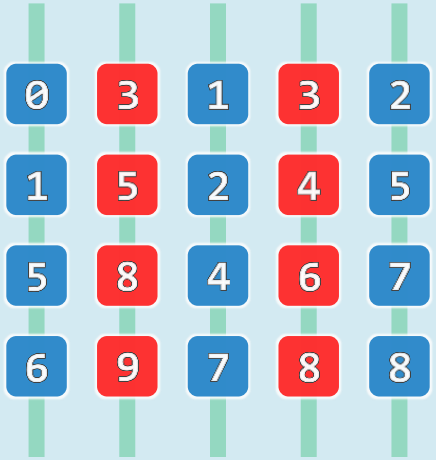
A g-ordered sequence REMAINS g-ordered after being h-sorted.

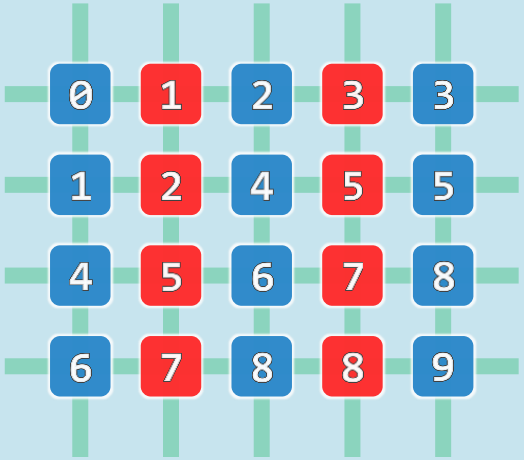


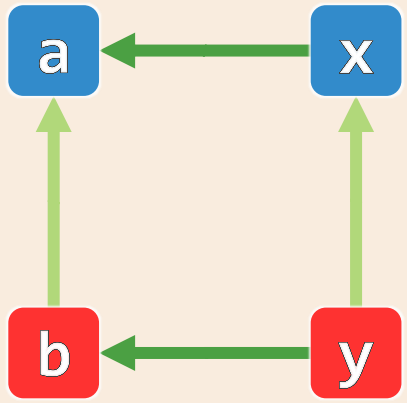


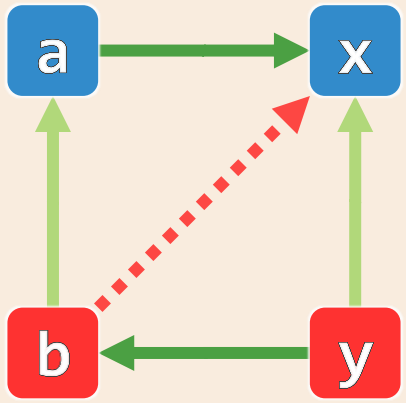
Order Preservation

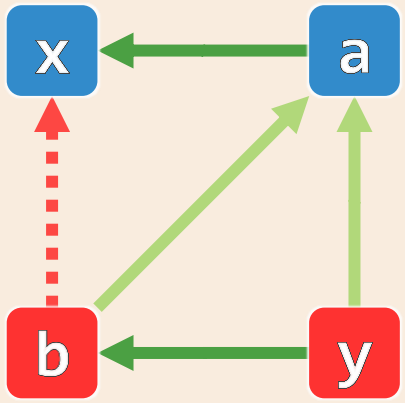


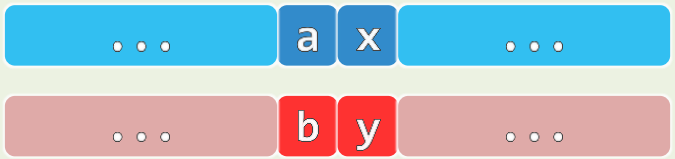




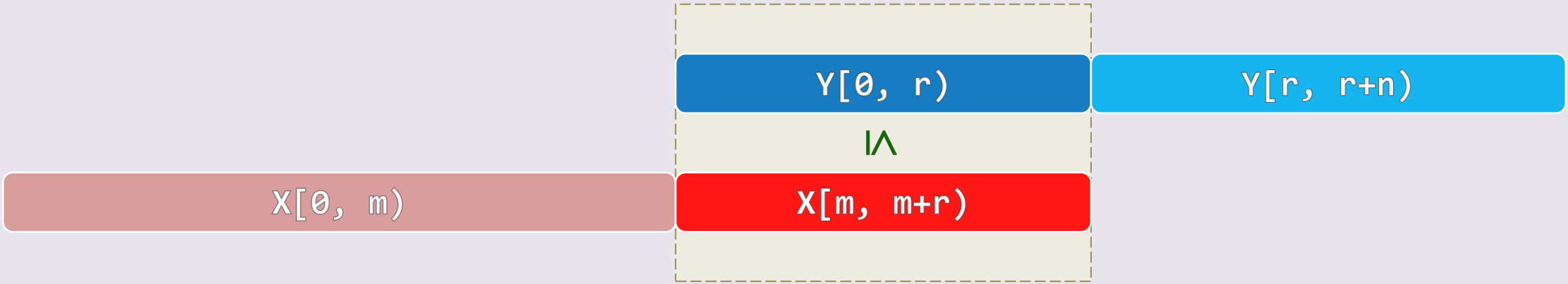


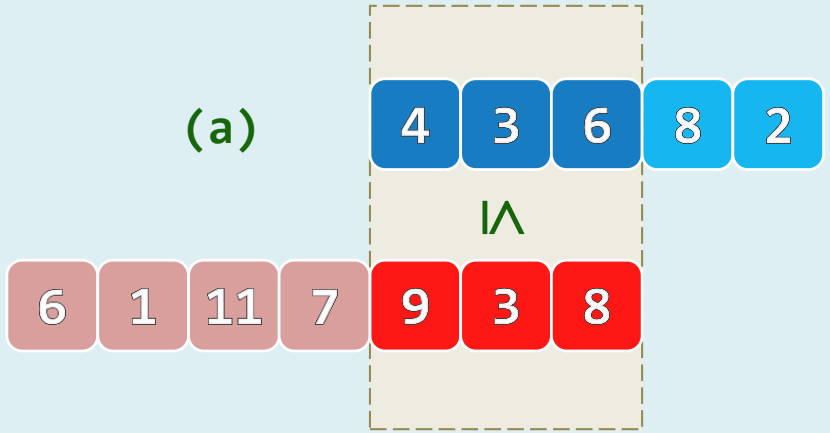


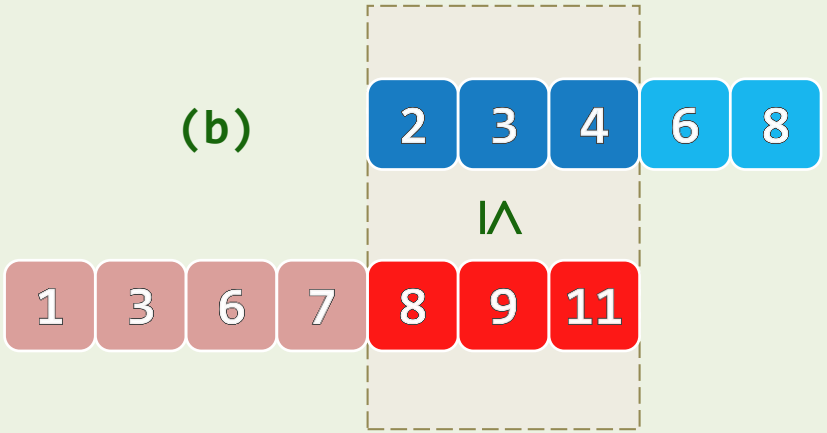




Lemma L





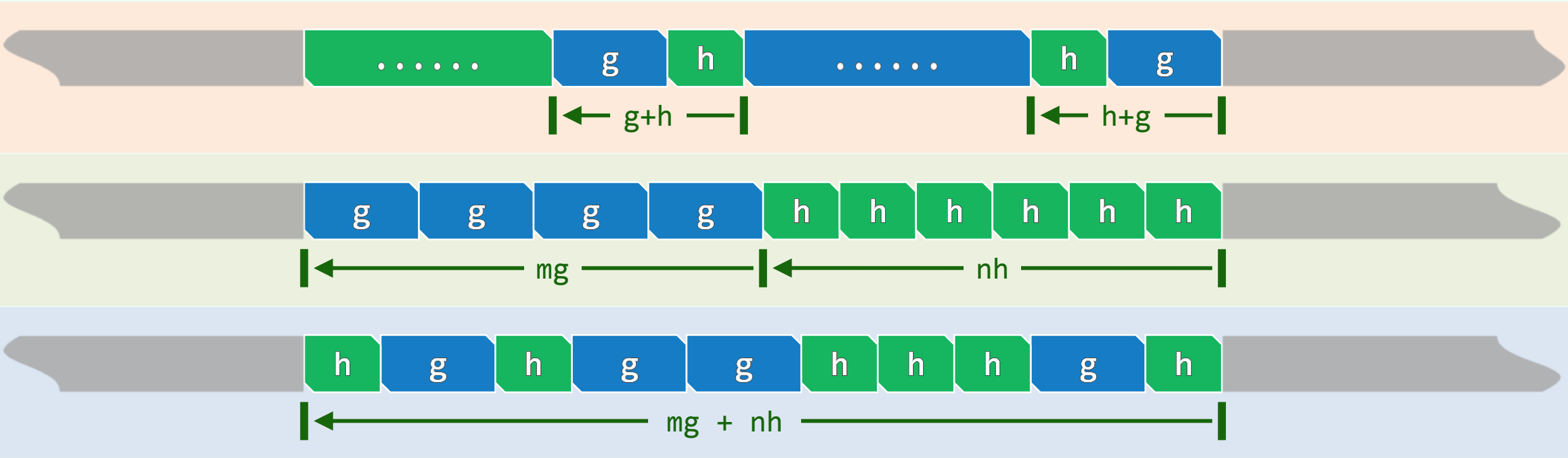


Linear Combination

❖ A sequence that is both g-ordered and h-ordered

is called (g,h)-ordered, which must be both

(g+h)-ordered and (mg+nh)-ordered for any m, n ∈ N



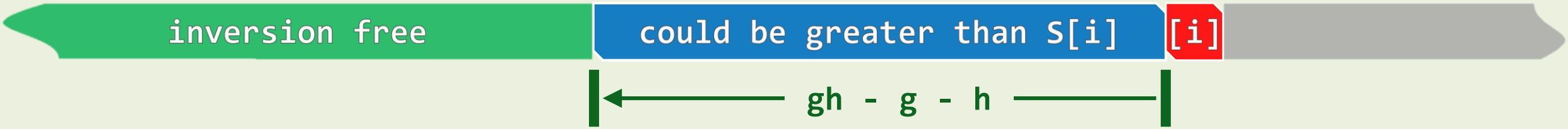
Inversion

- Let S[0,n) be a (g,h)-ordered sequence, where g and h are **relatively prime**
- Then for all elements S[j] and S[i], we have

$$i - j > \mathbf{x}(g,h) \qquad \text{only if} \qquad S[j] \leq S[i]$$

- This implies that to the **LEFT** of each element,

  only the previous $\mathbf{x}(g,h)$ elements could be **GREATER**



- There would be no more than $n \cdot \mathbf{x}(g,h)$ **INVERSIONs** altogether