14-C3

排序

希尔排序：PS序列

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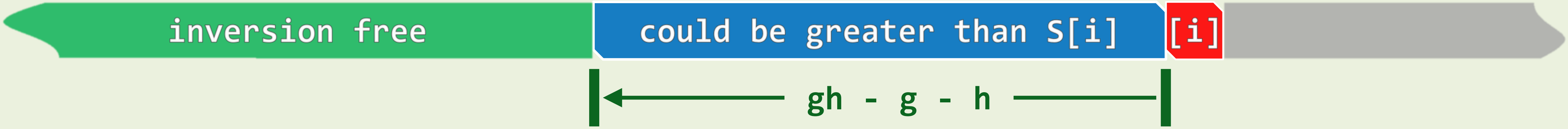
d-Sorting an $\mathcal{O}(d)$-Ordered Sequence in $\mathcal{O}(dn)$ Time

❖ If g and h are relatively prime and are both in $\mathcal{O}(d)$

we can d-sort the sequence in $\mathcal{O}(dn)$ time ...

- re-arrange the sequence as a 2D matrix with d columns
- each element is swapped with $\mathcal{O}((g-1)\cdot(h-1)/d) = \mathcal{O}(d)$ elements



❖ Since this holds for all elements, $\mathcal{O}(dn)$ steps are enough

PS Sequence

- Papernov & Stasevic, 1965 //also called Hibbard's sequence

$$\mathcal{H}_{PS} = \mathcal{H}_{Shell} - 1 = \{\, 2^k - 1 \mid k \in \mathcal{N} \,\} = \{\, 1,\ 3,\ 7,\ 15,\ 31,\ 63,\ 127,\ 255,\ \ldots \,\}$$

- Different items **MAY NOT** be relatively prime, e.g., $h_{2k} = h_k \cdot (h_k + 2)$

  But ADJACENT items **MUST** be, since $h_{k+1} - 2 \cdot h_k \equiv 1$

- Shellsort with $\mathcal{H}_{ps}$ needs

  - $\mathcal{O}(\log n)$ outer iterations and

  - $\mathcal{O}(n^{3/2})$ time to sort a sequence of length n //Why ...

t < k

❖ Let h_t be the h closest to $\sqrt{n}$ and hence $h_t \approx \sqrt{n} = \Theta(n^{1/2})$

1) Consider those iterations for $\{ h_k \mid t < k \} = \{ \overleftarrow{h_{t+1},\ h_{t+2},\ \ldots,\ h_m} \}$

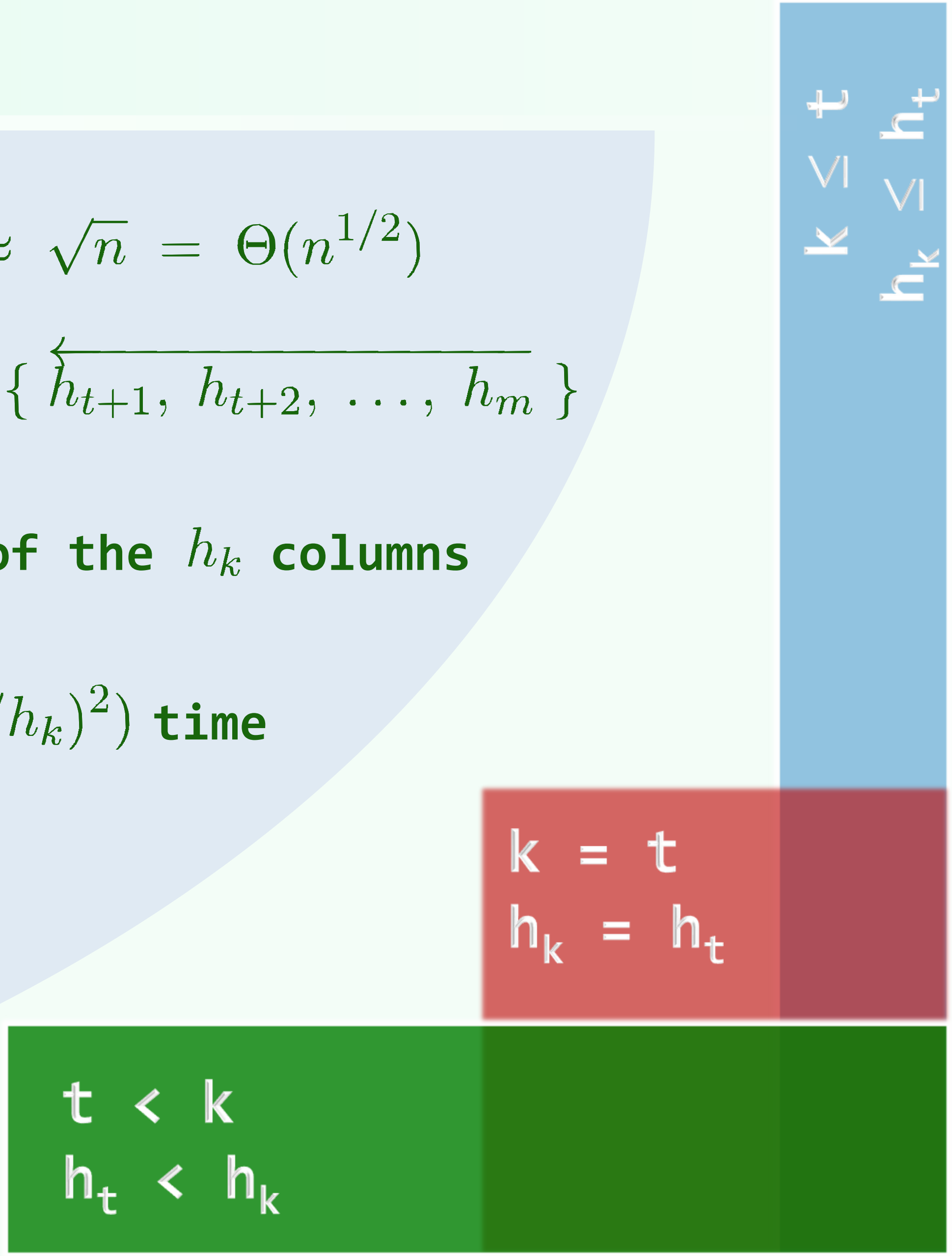
   ∵ there would be $\mathcal{O}(n/h_k)$ elements in each of the h_k columns

   ∴ we can insertionsort each column in $\mathcal{O}((n/h_k)^2)$ time

   ∴ each h_k-sorting costs $\mathcal{O}(n^2/h_k)$ time

   ∴ all these iterations cost time of

   $\mathcal{O}(2 \times n^2/h_t) = \mathcal{O}(n^{3/2})$



$k \le t$

2) Consider those iterations for $\{\, h_k \mid k \le t \,\} = \{\, \overleftarrow{h_1,\ h_2,\ \dots,\ h_t} \,\}$

∵ h_{k+1} and h_{k+2} are relatively prime and are both in $\mathcal{O}(h_k)$

∴ each h_k-sorting costs $\mathcal{O}(n \times h_k)$ time

∴ all these iterations cost $\mathcal{O}(n \times 2 \cdot h_t) = \mathcal{O}(n^{3/2})$ time

❖ This upper bound is TIGHT

❖ What about the average cases?

- $\mathcal{O}(n^{5/4})$ based on simulations
- but not proved yet