09-XA

BST Application

Range Tree

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BBST<BBST<T>> --> BBST<List<T>>

- Each y-search is just

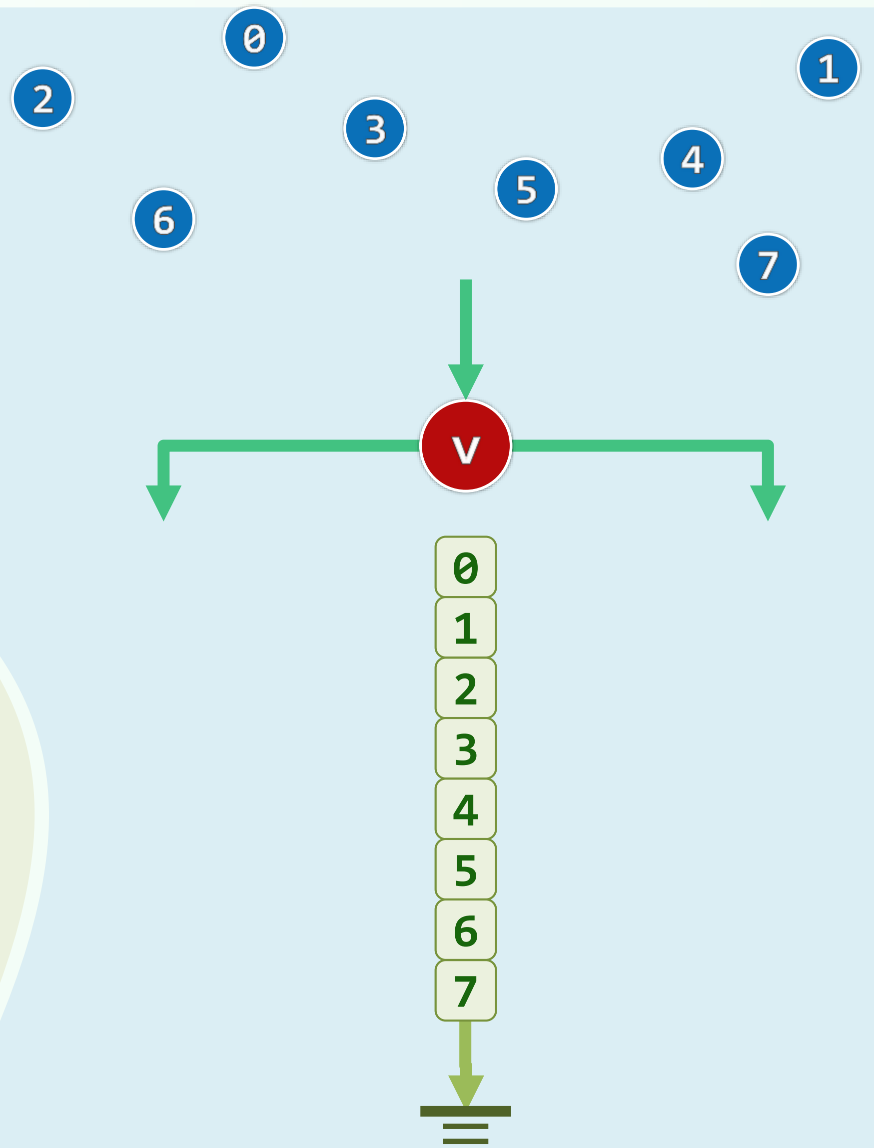
  a 1D query without further recursions

- So it not necessary

  to store each canonical subset

  as a BBST

- Instead, a sorted y-list simply works



Coherence

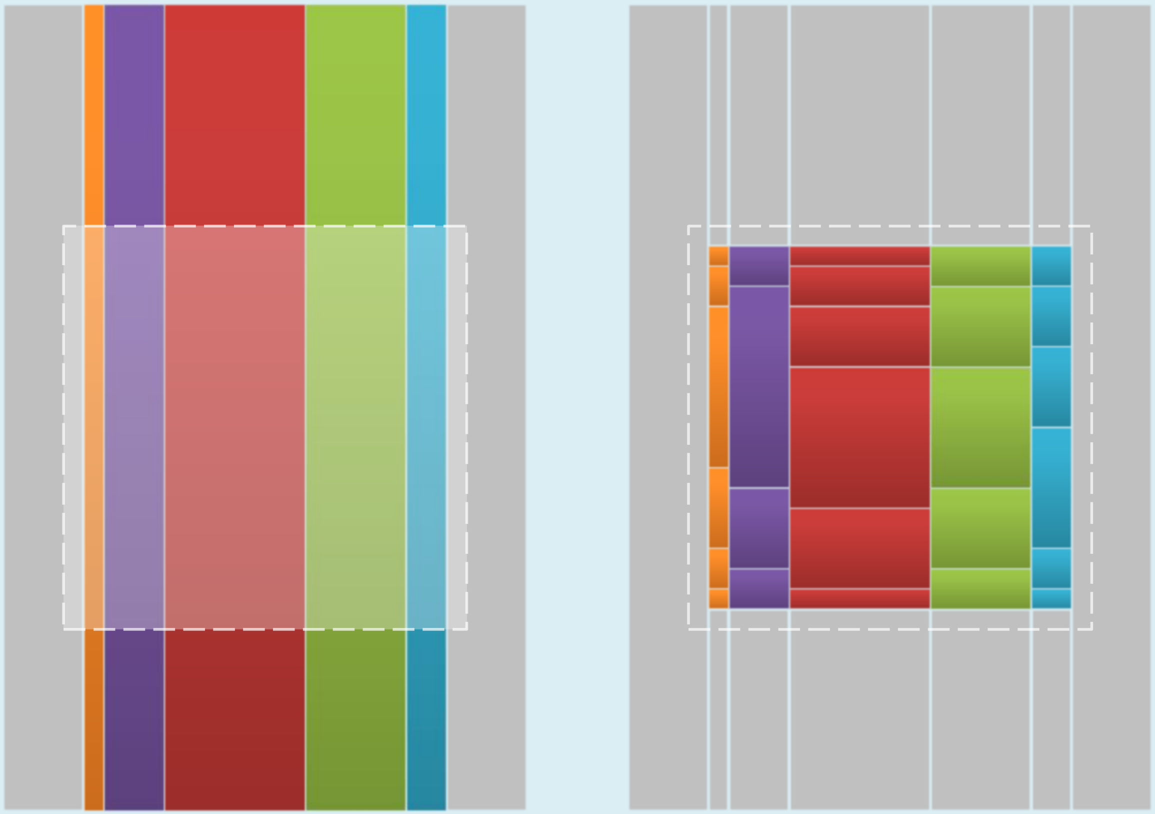
❖ For each query, we

 - need to repeatedly search

 DIFFERENT y-lists,

 - but always with

 the **SAME** key



Links Between Lists

- We can **CONNECT** all the different lists

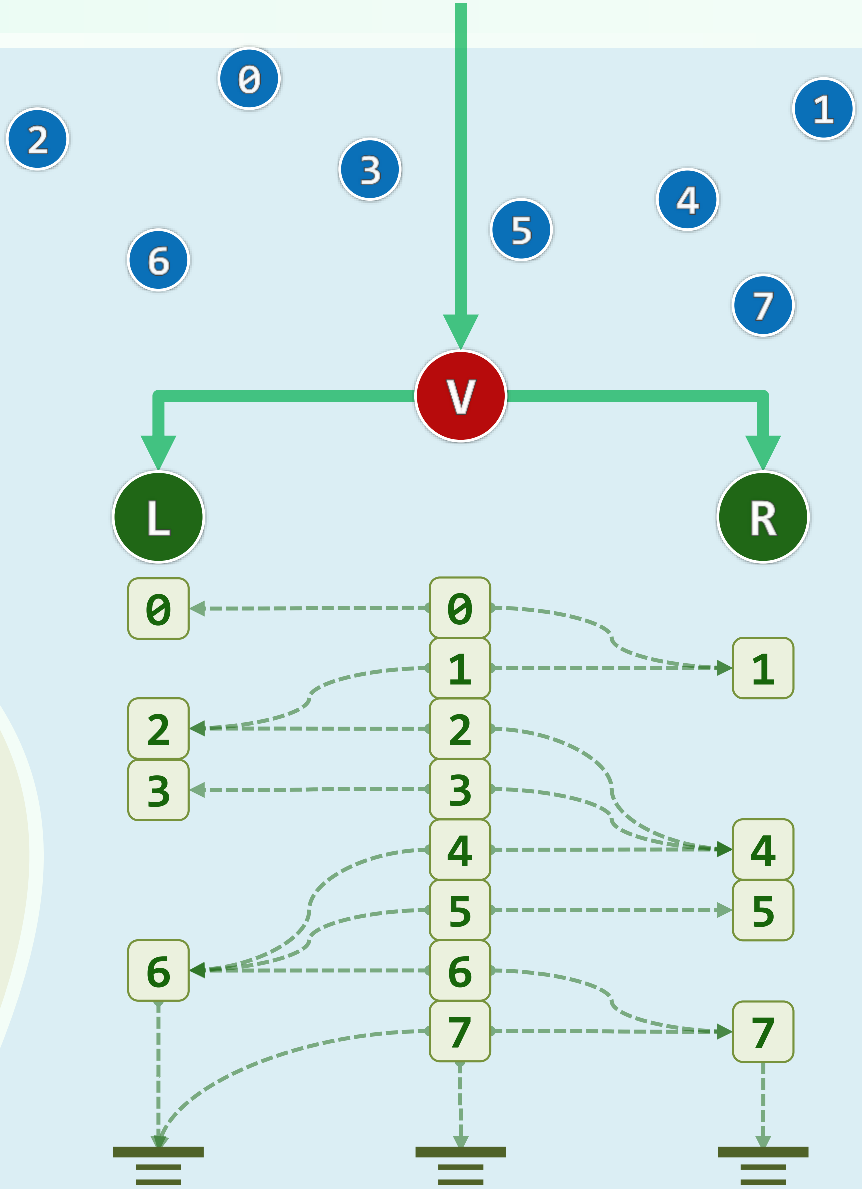
  into a **SINGLE** massive list

- Thus, once a parent y-list is searched,

  we can get, in $\mathcal{O}(1)$ time,

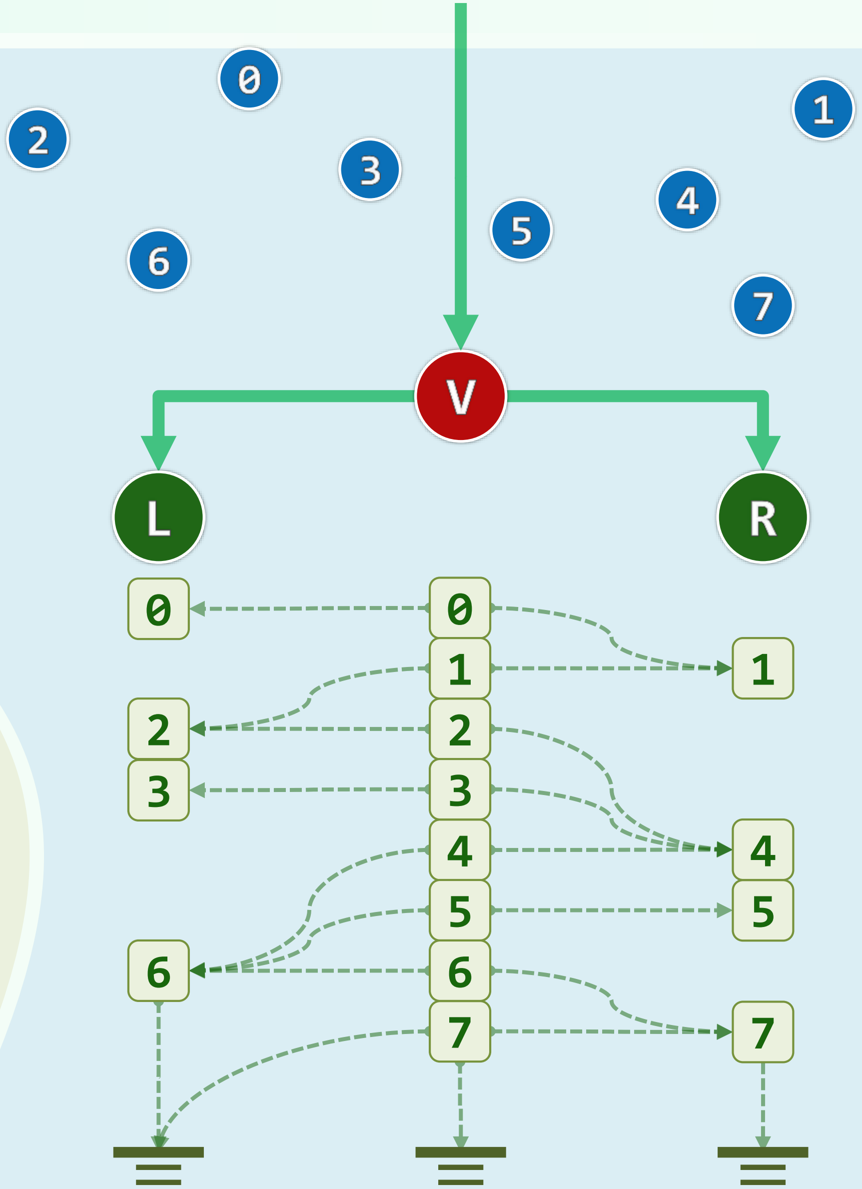
  the entry for child y-list by

  following the link between them



Using Coherence

- To answer a 2D range query, we will
  do an $\mathcal{O}(\log n)$ search
  on the y-list for the root
- Thereafter, while descending the x-tree, we can
  keep track of the position of y-range
  in each successive list in $\mathcal{O}(1)$ time
- This technique is called ......



Fractional Cascading

❖ For each item in A_v,

we store two pointers to

the item of NLT value

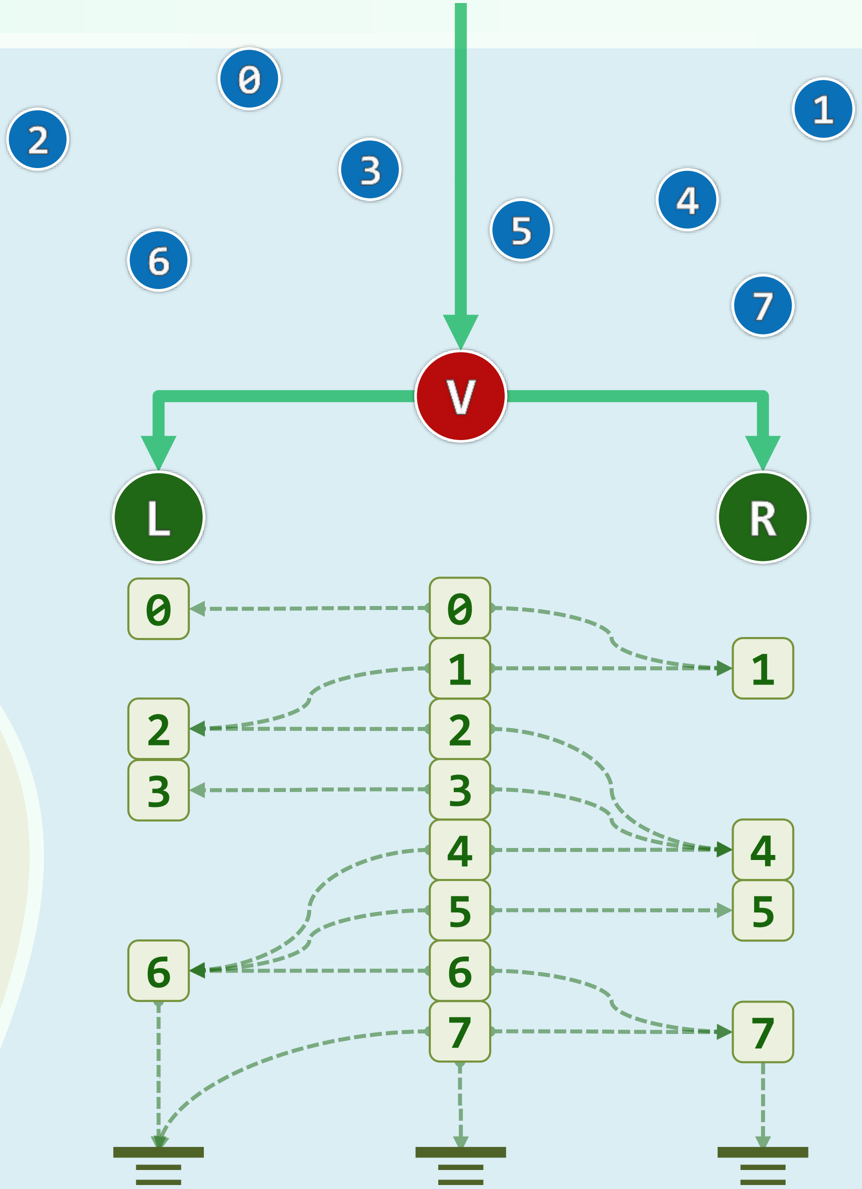
in A_L and A_R resp.

❖ Hence for any y-query with q_y,

once we know its entry in A_v, we can

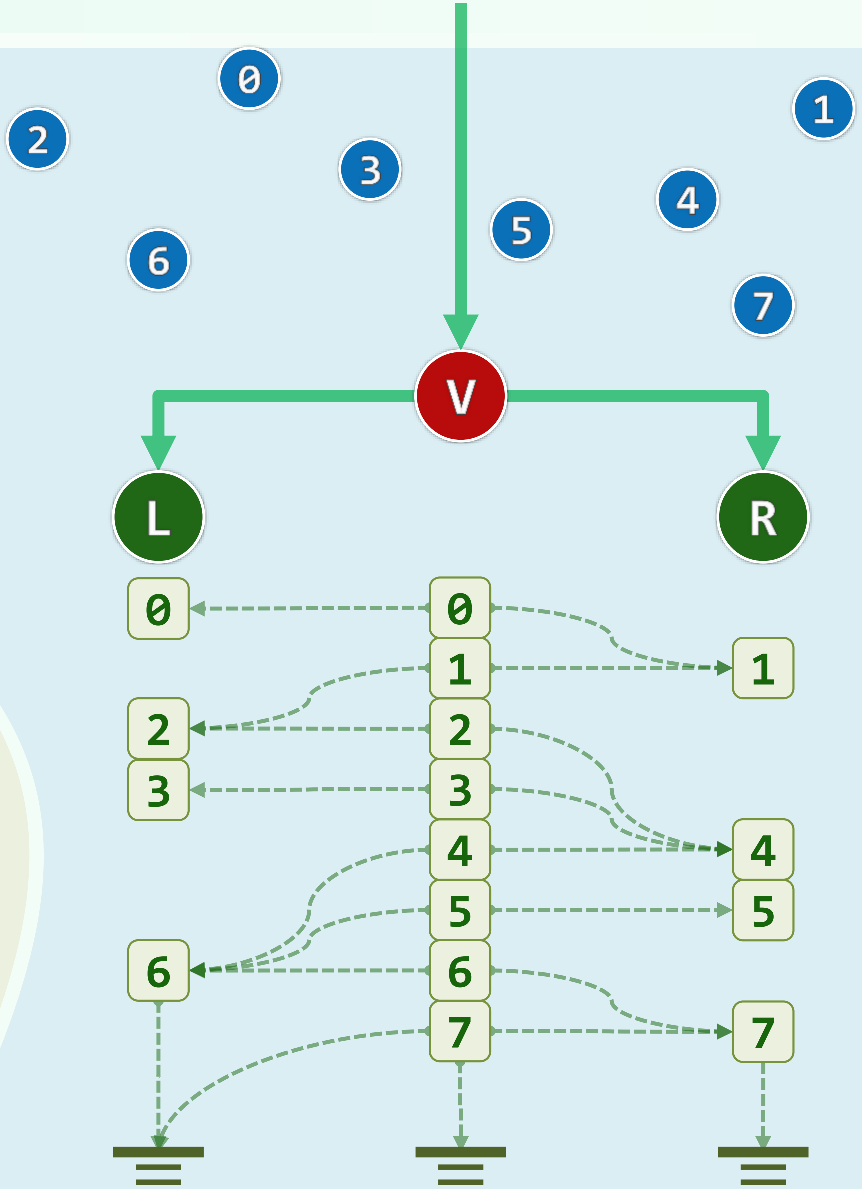
determine its entry in either A_L or A_R

in $\mathcal{O}(1)$ additional time



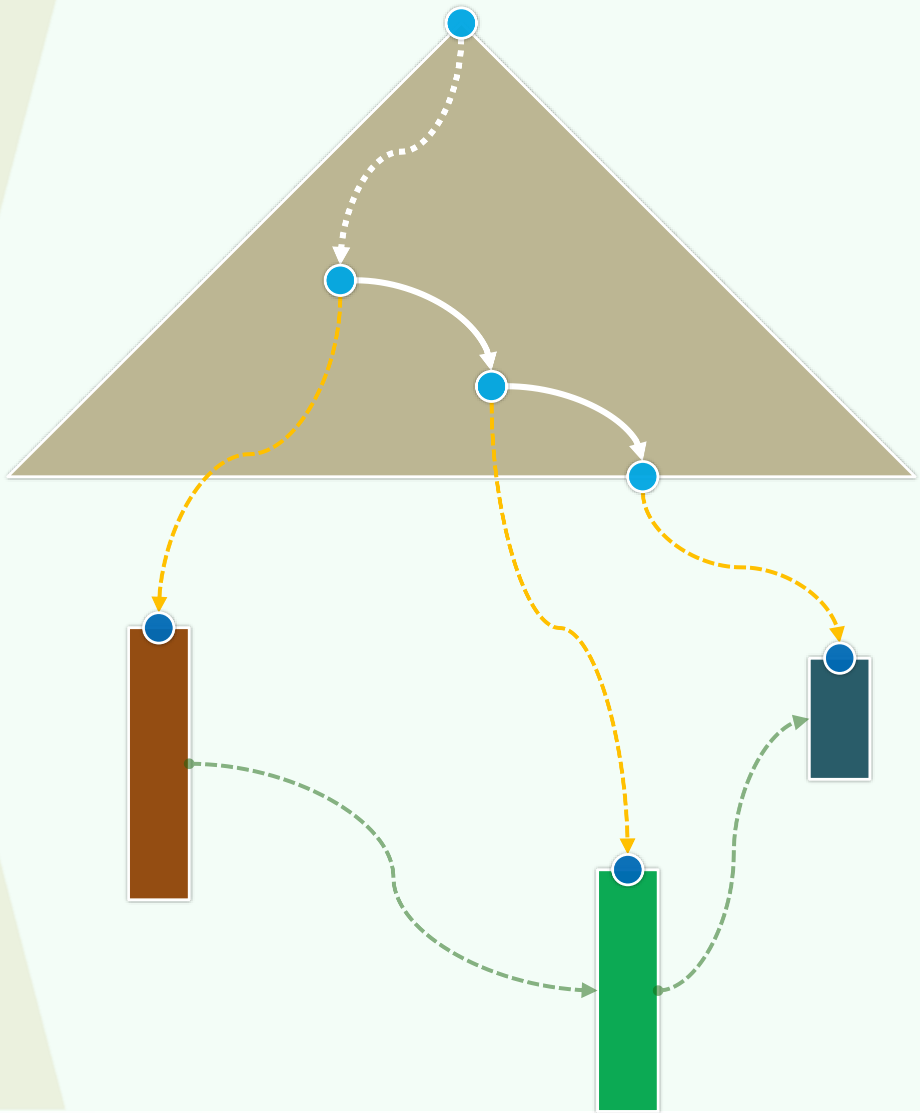
Construction By 2-Way Merging

- Let V be an internal node in the x-tree
  with L/R its left/right child resp.
- Let A_v be the y-list for v and
  A_L/A_R be the y-lists for its children
- Assuming no duplicate y-coordinates, we have
  - A_v is the disjoint union
    of A_L and A_R, and hence
  - A_v can be obtained by
    merging A_L and A_R (in linear time)



Complexity

- An MLST with fractional cascading is called a **range tree**
- A y-search for root is done in $\mathcal{O}(\log n)$ time
- To drop down to each next level, we can get, in $\mathcal{O}(1)$ time, the current y-interval from that of the prior level
- Hence, each 2D orthogonal range query
  - can be answered in $\mathcal{O}(r + \log n)$ time
  - from a data structure of size $\mathcal{O}(n \cdot \log n)$,
  - which can be constructed in $\mathcal{O}(n \cdot \log n)$ time



Beyond 2D

- ❖ Unfortunately, the trick of fractional cascading

  can **ONLY** be applied to

  the **LAST** level the search structure

- ❖ Given a set of n points in $\mathcal{E}^d$,

  an orthogonal range query

  - can be answered in $\mathcal{O}(r + \log^{d-1} n)$ time

  - from a data structure of size $\mathcal{O}(n \cdot \log^{d-1} n)$,

  - which can be constructed in $\mathcal{O}(n \cdot \log^{d-1} n)$ time