高级搜索树

B-树：插入

10-B4

说再见，在这梦幻国度，最后的一瞥

清醒让我，分裂再分裂

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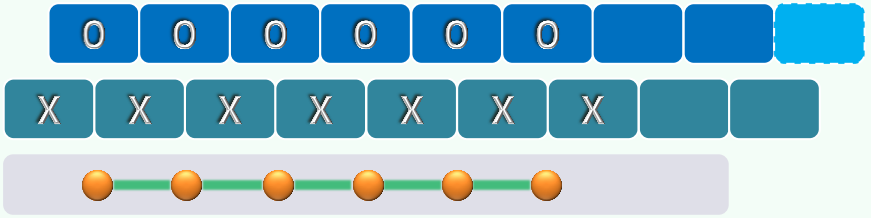
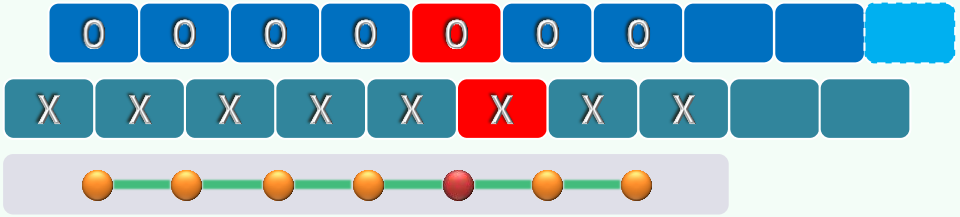
算法

```
❖template <typename T> bool BTree<T>::insert( const T & e ) {
    BTNodePosi<T> v = search( e );
    if ( v ) return false; //确认e不存在
    Rank r = _hot->key.search( e ); //在节点_hot中确定插入位置
    _hot->key.insert( r+1, e ); //将新关键码插至对应的位置
    _hot->child.insert( r+2, NULL ); _size++; //创建一个空子树指针
    solveOverflow( _hot ); //若上溢，则分裂
    return true; //插入成功
}
```





分裂

❖ 设上溢节点中的关键码依次为：

$$\{ k_0, k_1, \ldots, k_{m-1} \}$$

❖ 取中位数 $s = \lfloor m/2 \rfloor$，以关键码k_s为界划分为

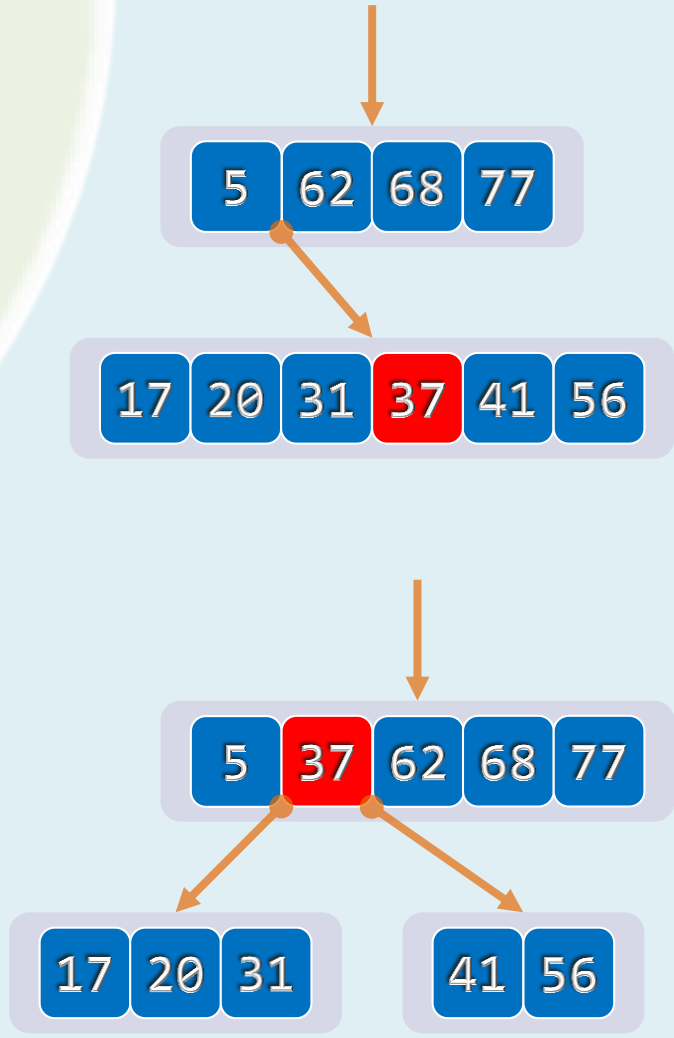
$$\{ k_0, \ldots, k_{s-1} \} \quad \{ k_s \} \quad \{ k_{s+1}, \ldots, k_{m-1} \}$$

❖ 关键码 k_s 上升一层，并分裂（split）

以所得的两个节点作为左、右孩子

❖ 不难验证，如此分裂后

左、右孩子所含关键码数目，依然符合m阶B-树的条件



再分裂

- 若上溢节点的父亲本已饱和，则在接纳被提升的关键码之后，也将上溢

  此时，大可套用前法，继续分裂

- 上溢可能持续发生，并逐层向上传播

  纵然最坏情况，亦不过到根 //若果真抵达树根...

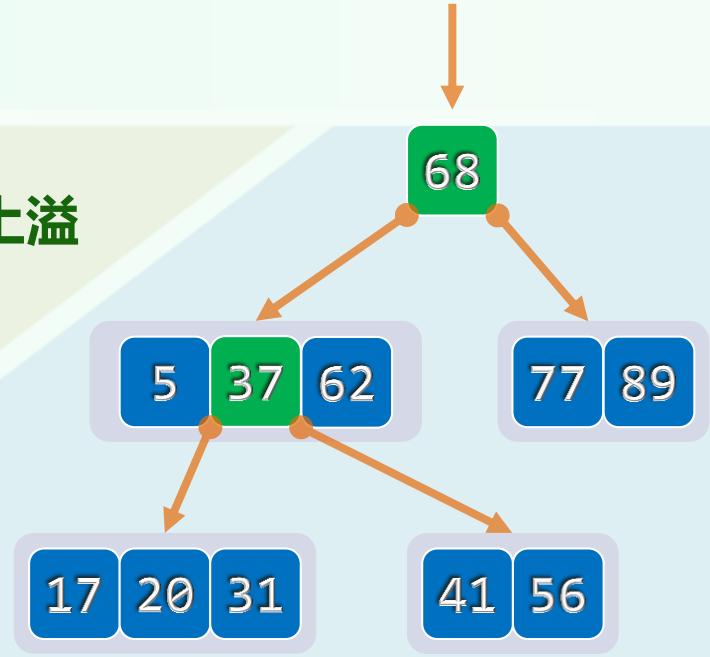
- 可令被提升的关键码自成节点，作为新的树根

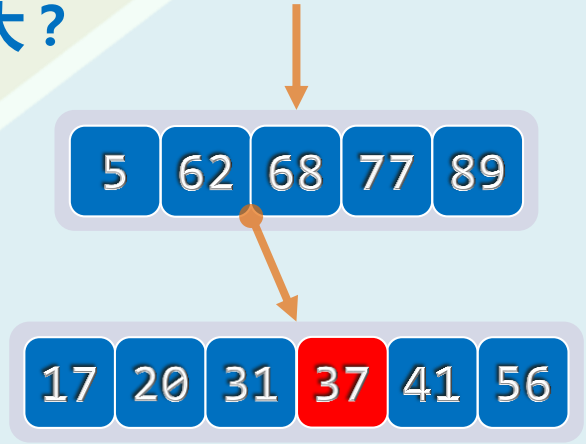
  这是B-树增高的唯一可能 //概率多大？

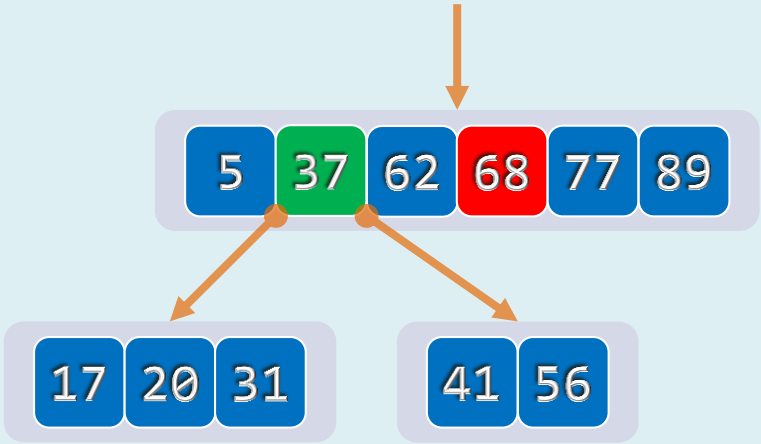
- 注意：新生的树根仅有两个分支

- 总体执行时间正比于

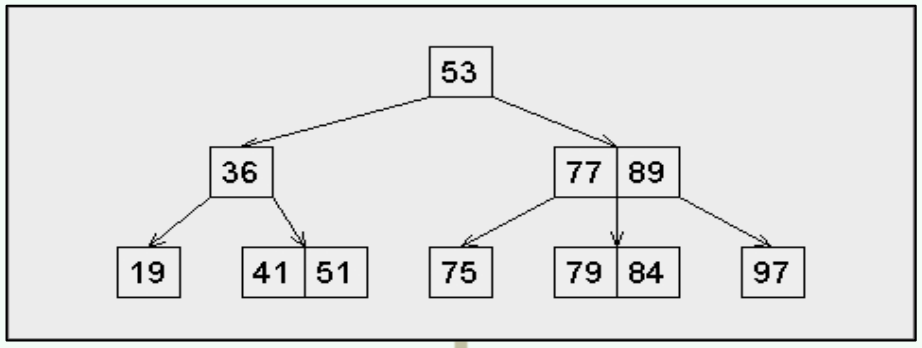
  分裂次数，$O(h)$



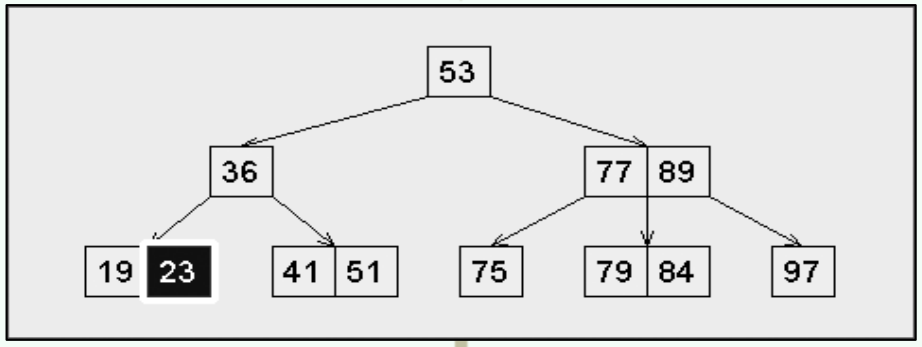




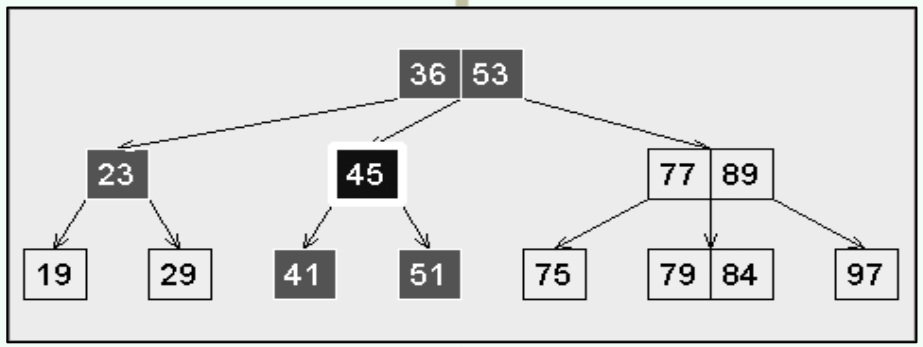
实例：(2,3)-树



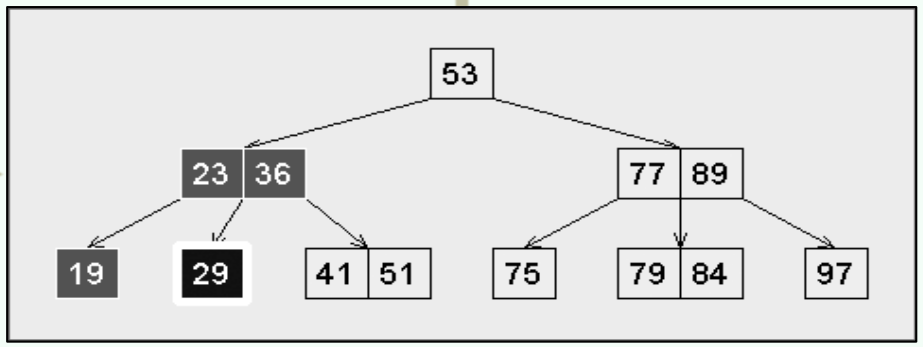
insert(23) //无需分裂



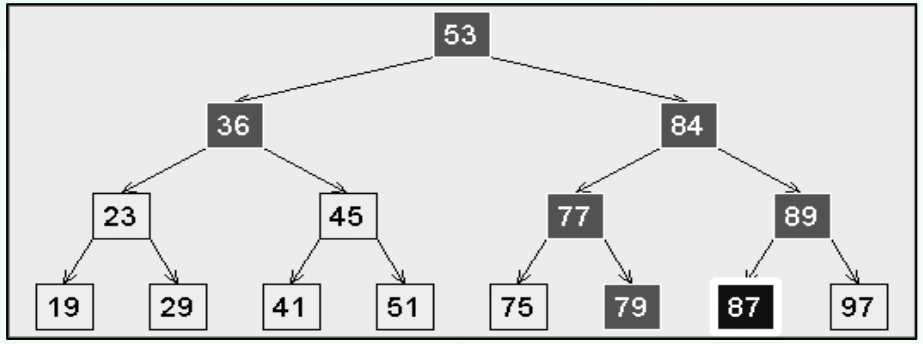
insert(29) //分裂一次



insert(45) //分裂两次



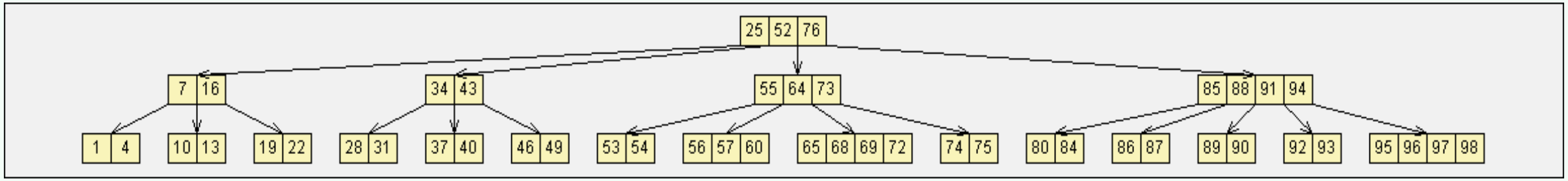
insert(87) //分裂到根



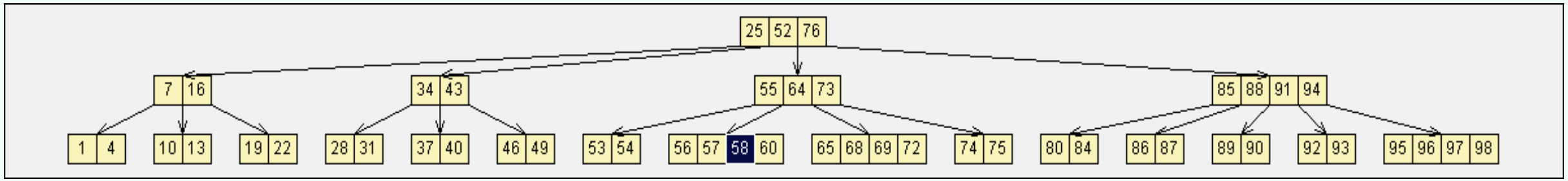
❖ 53 97 36 89 41 75 19 84 77 79 51

实例：(3,5)-树

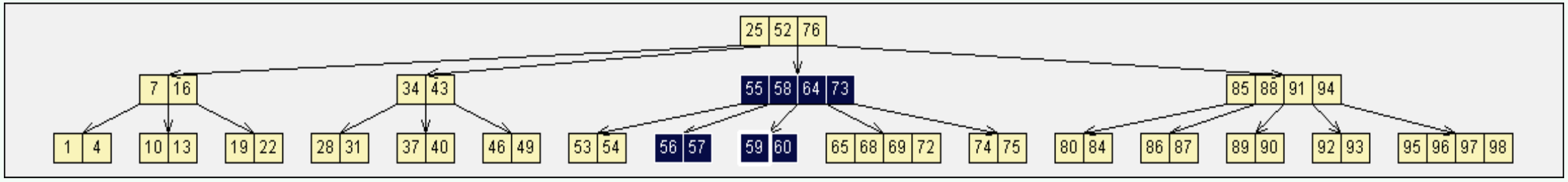
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 56 60 64 68 72 76 80 84 53 54 55 85 86 87 88 89 90 91 92 93 94 95 96 97 98 73 74 75 57 65 69



insert(58)　//无需分裂

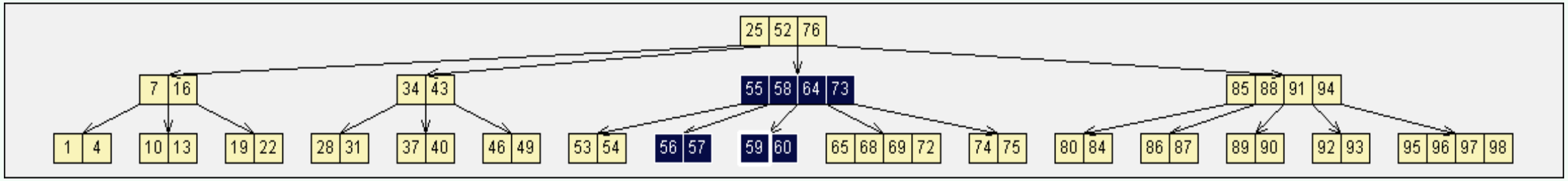


insert(59)　//分裂 1 次

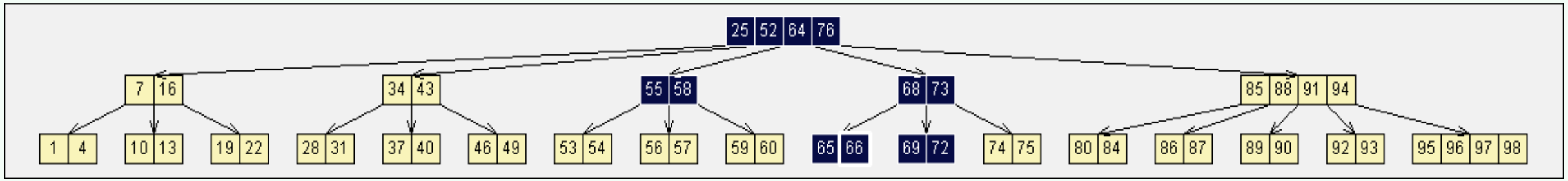


实例：(3,5)-树

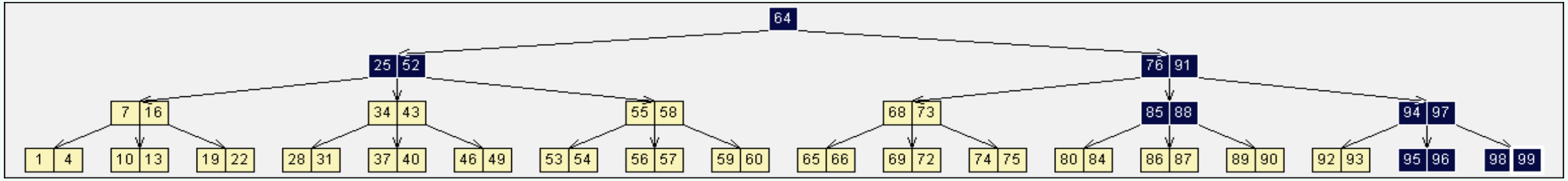
insert(59)　//分裂 1 次



insert(66)　//分裂 2 次



insert(99)　//分裂到根



上溢修复（1/2）

```
❖template <typename T> void BTree<T>::solveOverflow( BTNodePosi<T> v ) {
    if ( _order >= v->child.size() ) return; //递归基：不再上溢
    Rank s = _order / 2; //轴点（此时_order = key.size() = child.size() - 1）
    BTNodePosi<T> u = new BTNode<T>(); //注意：新节点已有一个空孩子
    for ( Rank j = 0; j < _order - s - 1; j++ ) { //分裂出右侧节点u（效率低可改进）
       u->child.insert( j, v->child.remove( s + 1 ) ); //v右侧_order-s-1个孩子
       u->key.insert( j, v->key.remove( s + 1 ) ); //v右侧_order-s-1个关键码
    }
    u->child[ _order - s - 1 ] = v->child.remove( s + 1 ); //移动v最靠右的孩子
    /* TBC */
```

上溢修复（2/2）

```
❖    if ( u->child[ 0 ] ) //若u的孩子们非空，则统一令其以u为父节点
        for ( Rank j = 0; j < _order - s; j++ ) u->child[ j ]->parent = u;
     BTNodePosi<T> p = v->parent; //v当前的父节点p
     if ( ! p ) //若p为空，则创建之（全树长高一层，新根节点恰好两度）
        { _root = p = new BTNode<T>(); p->child[0] = v; v->parent = p; }
     Rank r = 1 + p->key.search( v->key[0] ); //p中指向u的指针的秩
     p->key.insert( r, v->key.remove( s ) ); //轴点关键码上升
     p->child.insert( r + 1, u );  u->parent = p; //新节点u与父节点p互联
     solveOverflow( p ); //上升一层，如有必要则继续分裂——至多（尾）递归O(logn)层
  }
```