01-F1

绪论

动态规划：记忆法

圣人不记事，所以常记得；今人忘事，以其记事

有人建议不妨置备一本签名簿，供来访者留下自己的名字，就像怀特山那样；可是，天哪！我的记性非常好，用不着那个玩意儿。

邓俊辉
deng@tsinghua.edu.cn

fib()：递归

$$fib(n) = fib(n-1) + fib(n-2)$$

0 1 1 2 3 5 8 13 21 34 55 89 ...

❖ `int fib(n) { return (2 > n) ? n : fib(n - 1) + fib(n - 2); }` //为何这么慢？

❖ 复杂度：$T(0) = T(1) = 1; \quad T(n) = T(\boxed{n-1}) + T(\boxed{n-2}) + 1, \ \forall n > 1$

- 令 $S(n) = [T(n) + 1]/2$
- 则 $S(0) = 1 = fib(1), \quad S(1) = 1 = fib(2)$
- 故 $S(n) = S(n-1) + S(n-2) = fib(n+1)$

$$T(n) = 2 \cdot S(n) - 1 = 2 \cdot fib(n+1) - 1 = \mathcal{O}(fib(n+1)) = \mathcal{O}(\phi^n)$$

- 其中 $\phi = (1+\sqrt{5})/2 \approx 1.618$

封底估算

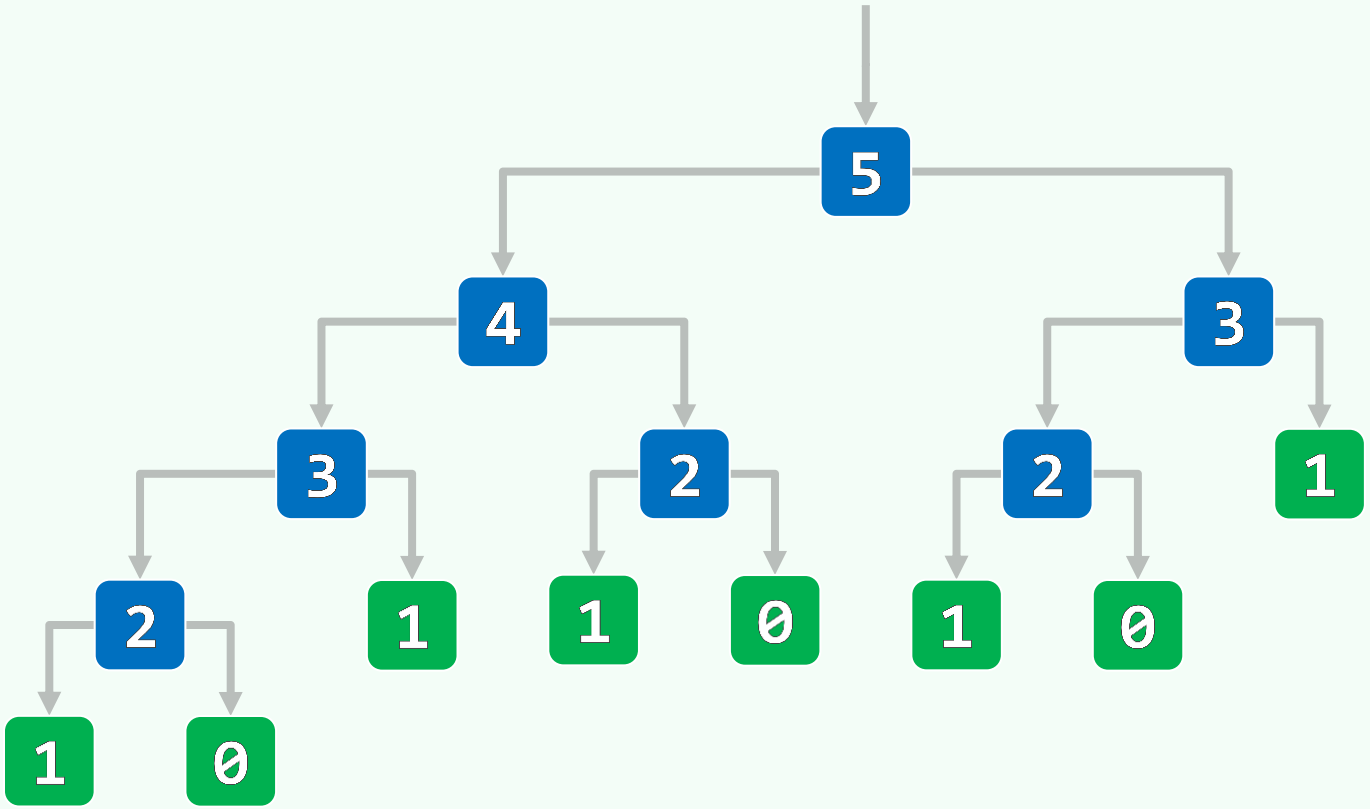
$$\phi^{36} \approx 2^{25} \qquad \phi^{43} \approx 2^{30} \approx 10^9\ flo = 1\ sec$$

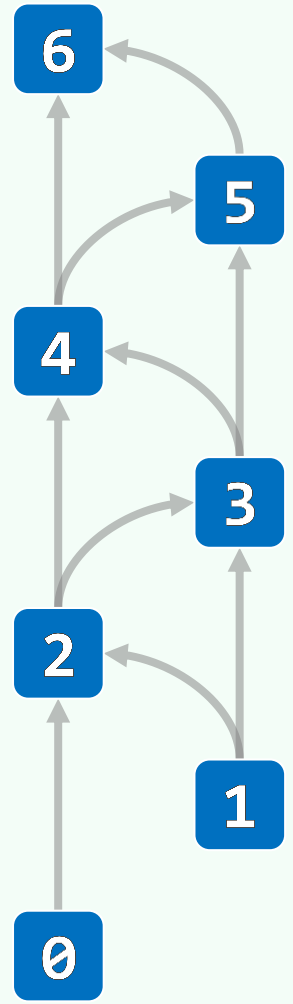
$$\phi^{5} \approx 10 \qquad \phi^{67} \approx 10^{14}\ flo = 10^5\ sec \approx 1\ day$$

$$\phi^{92} \approx 10^{19}\ flo = 10^{10}\ sec \approx 10^5\ day \approx 3\ century$$

递归

❖ 递归版fib()低效的根源在于，各递归实例均被大量地重复调用





❖ 先后出现的递归实例，共计 $\mathcal{O}(\phi^n)$ 个；而去除重复之后，总共不过 $\mathcal{O}(n)$ 种

Memoization

❖

```
def f(n)
    if ( n < 1 ) return trivial( n );

    return f(n-X) + f(n-Y)*f(n-Z);
```

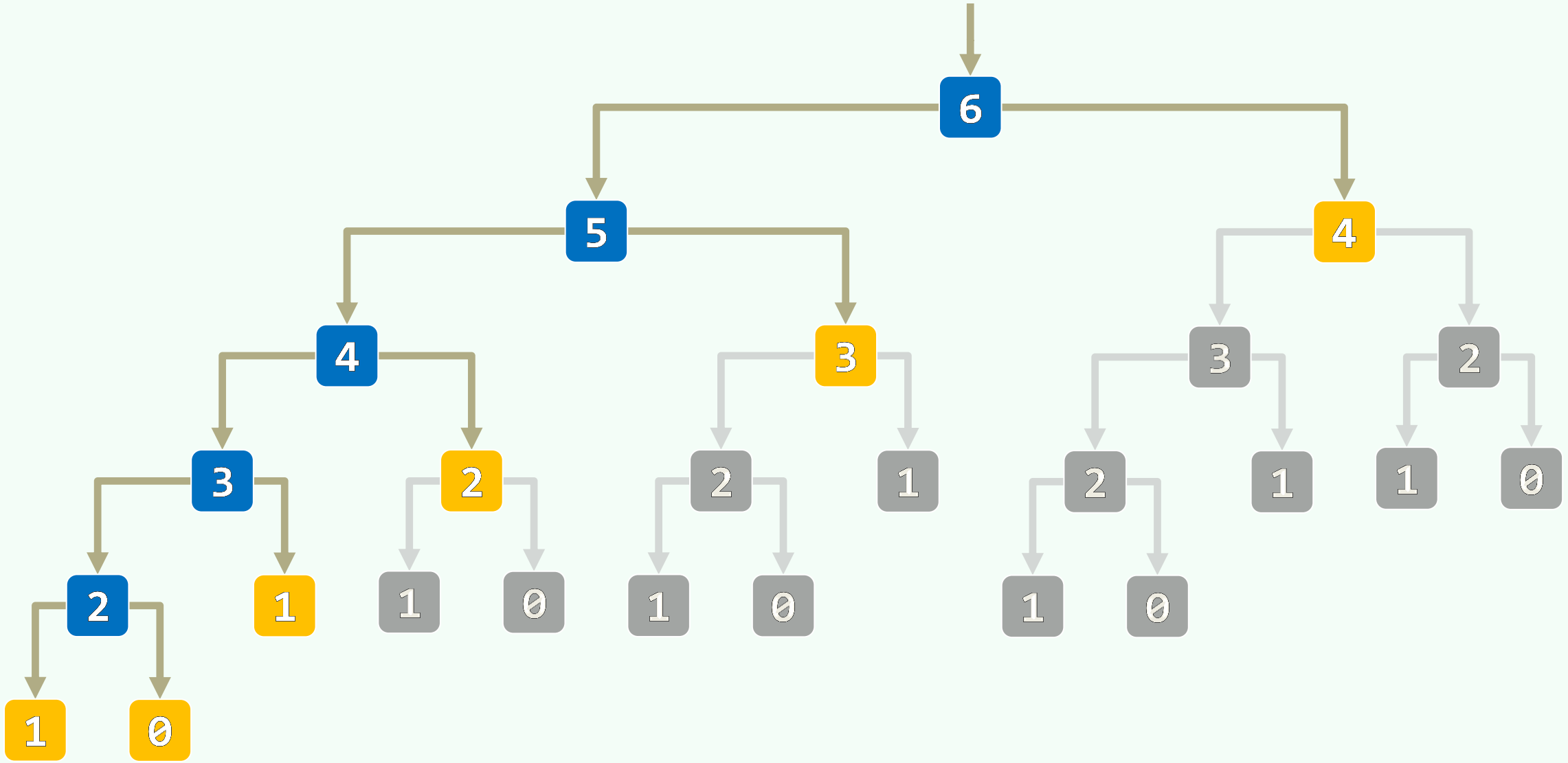
❖ T M[N]; #init. with UNDEFINED

```
def f(n)
    if ( n < 1 ) return trivial( n );

# recur only when necessary &
  always write down the result
  if ( M[n] == UNDEFINED )
     M[n] = f(n-X) + f(n-Y)*f(n-Z);

  return M[n];
```

Memoization: fib()



动态规划

❖ Dynamic programming，颠倒计算方向：

由自顶而下递归，改为自底而上迭代

❖ f = 1; g = 0; //fib(-1), fib(0)

```
while ( 0 < n-- ) {
   g = g + f;
   f = g - f;
}
return g;
```

❖ T(n) = O(n)，而且仅需O(1)空间！