09-XC

BST Application

Segment Tree

把一条线分割成不相等的两段，再把这两段按照同样的比例再分成两个部分。假设第一次分出来的两段中，一段代表可见世界，另一段代表理智世界。然后再看第二次分成的两段，他们分别代表清楚与不清楚的程度，你便会发现，可见世界那一段的第一部分是它的影像。

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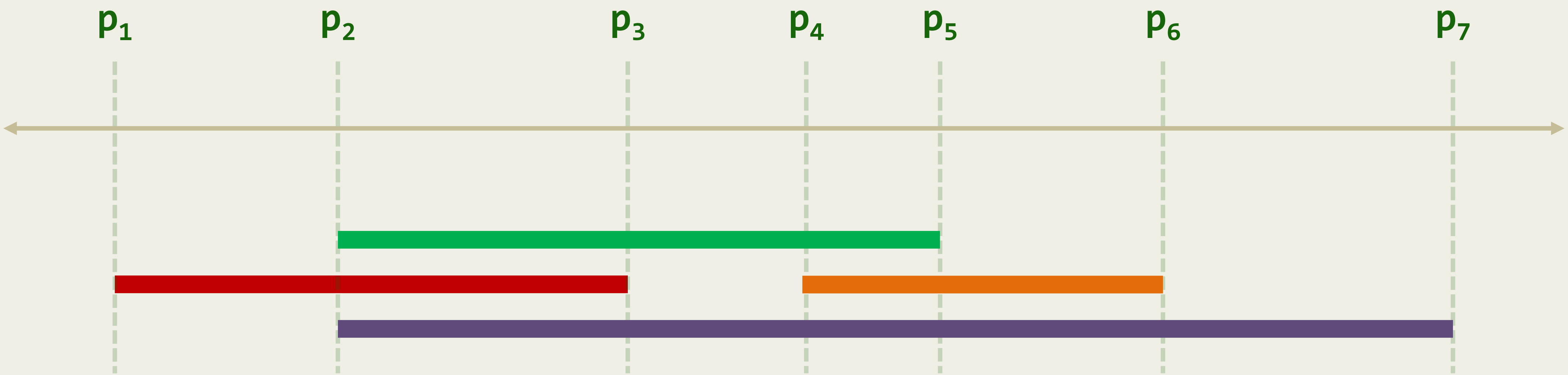
Elementary Intervals

- Let $I = \{ [x_i, x_i'] \mid i = 1, 2, 3, \ldots, n \}$ be n intervals on the x-axis
- Sort all the endpoints into $\{ p_1,\ p_2,\ p_3,\ \ldots,\ p_m \},\ m \leq 2n$



- m+1 elementary intervals are hence defined as:

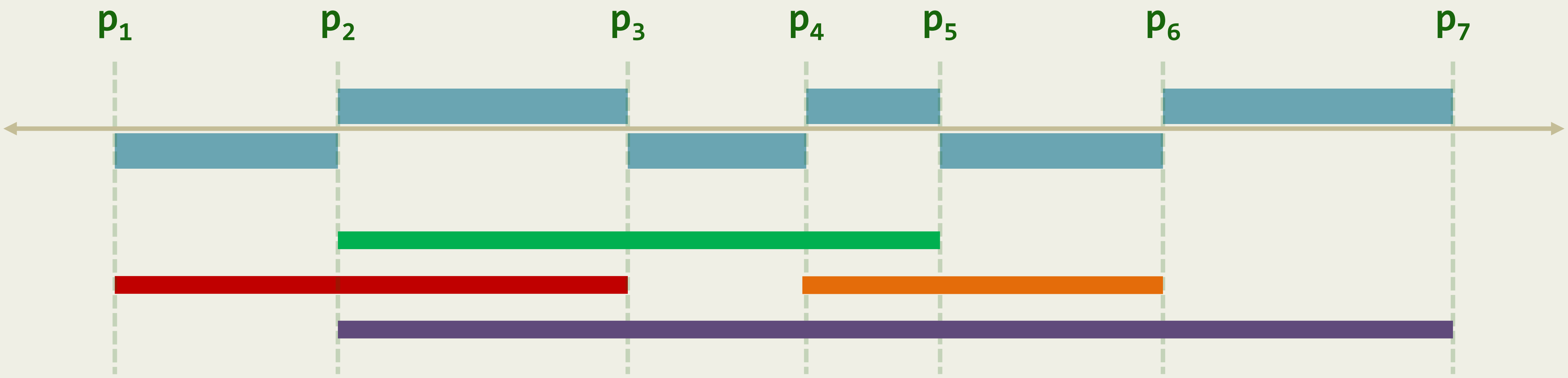
$$(-\infty, p_1],\ (p_1, p_2],\ (p_2, p_3],\ \ldots,\ (p_{m-1}, p_m],\ (p_m, +\infty]$$

Discretization

👁 Within each EI, all stabbing queries share a same output

∴ If we sort all EI's into a vector and

store the corresponding output with each EI, then ...

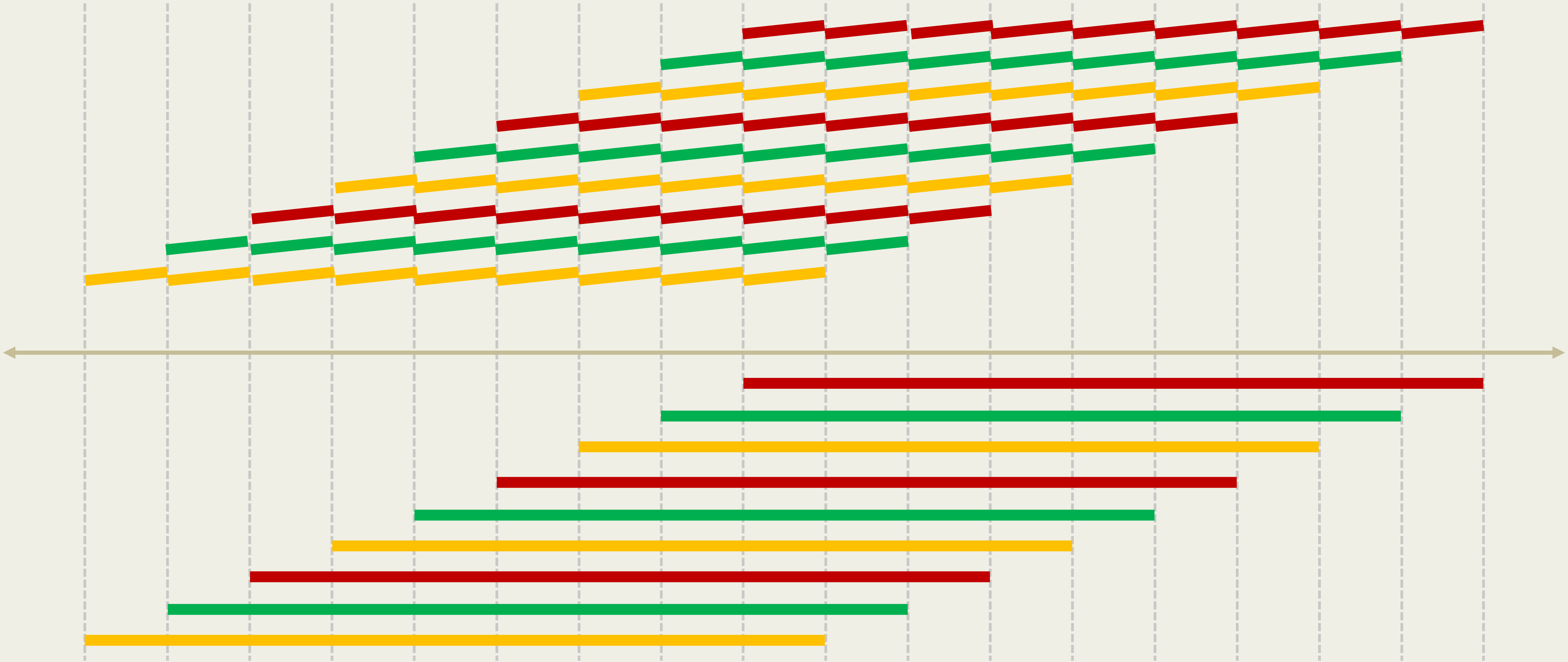


∴ Once a query position is determined, //by an $\mathcal{O}(\log n)$ time binary search

the output can then be returned directly //$\mathcal{O}(r)$

Worst Case

❖ Every interval spans $\Omega(n)$ EI's and a total space of $\Omega(n^2)$ is required

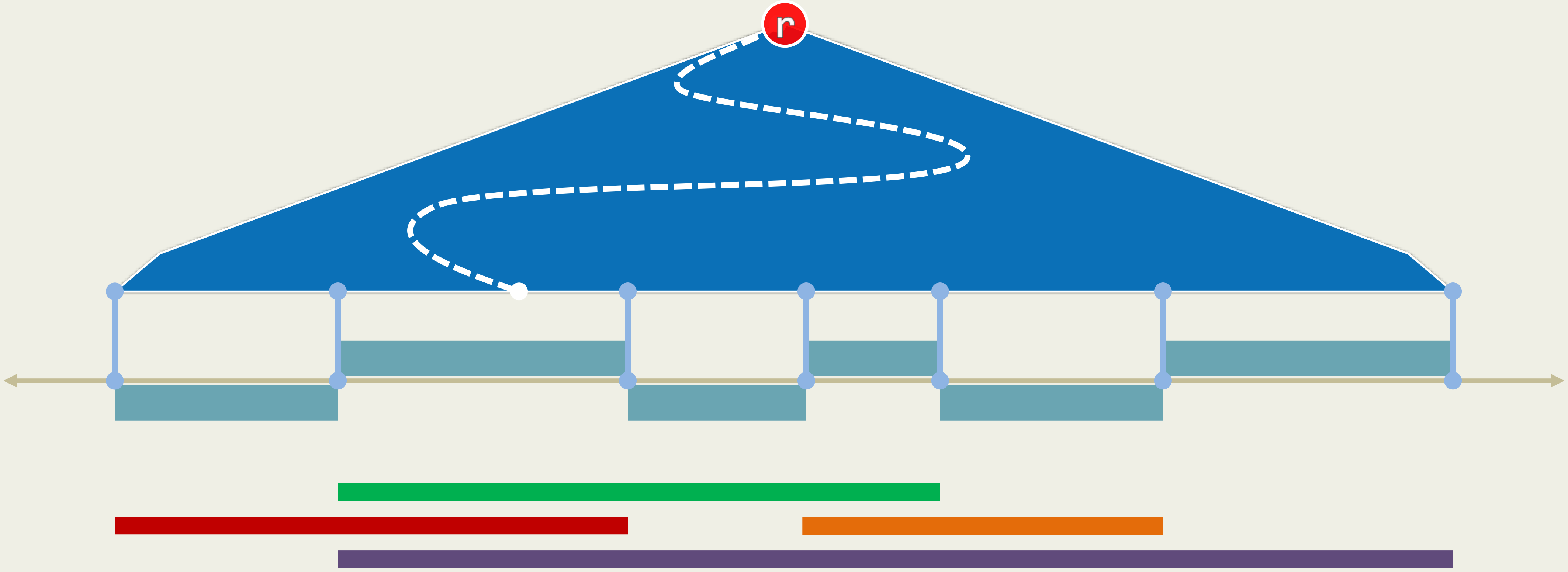


Sorted Vector --> BBST

❖ For each leaf v, denote the corresponding elementary interval as EI(v),

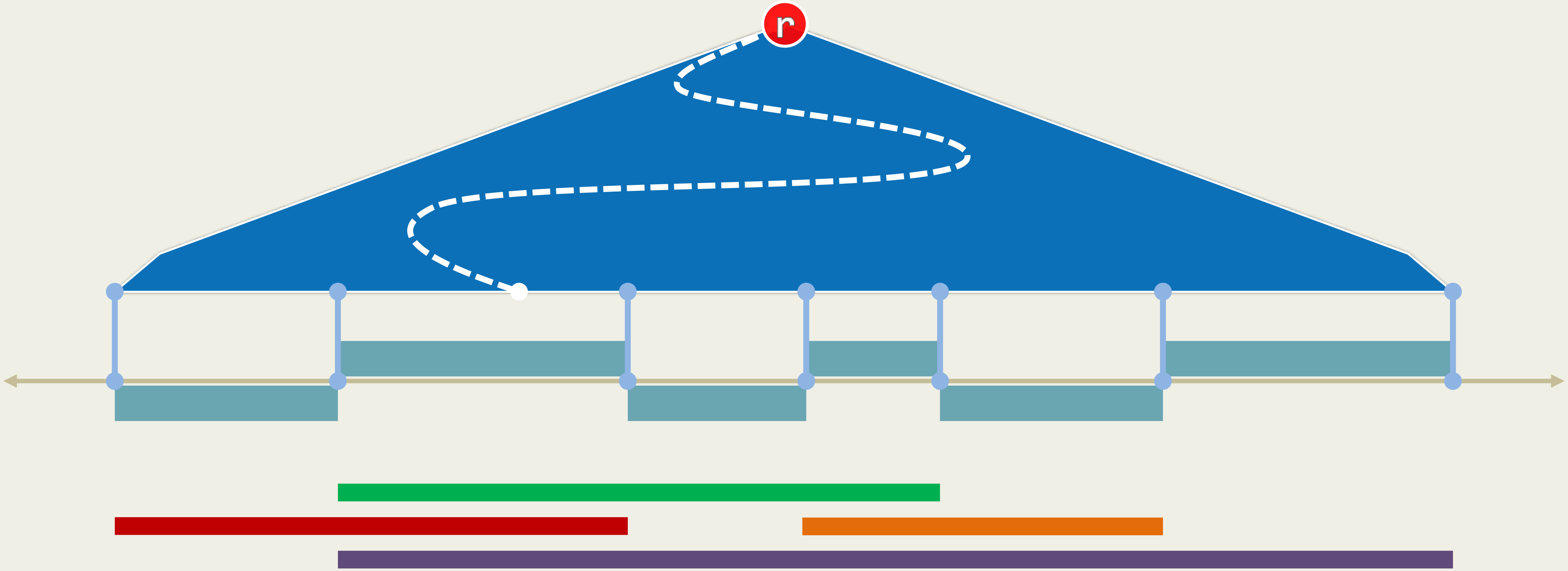
denote the subset of intervals containing EI(v) as Int(v) and

store Int(v) at v

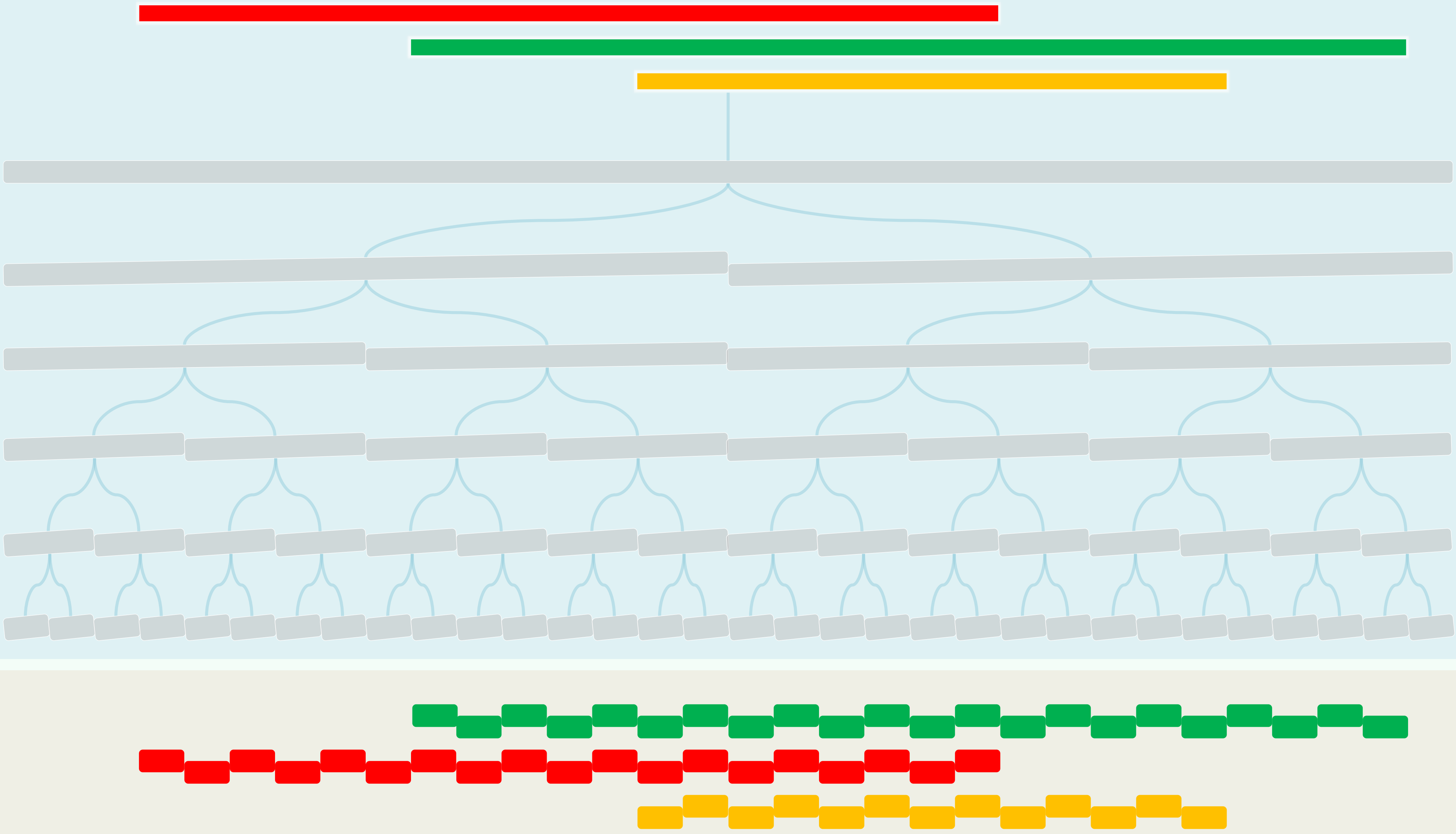


1D Stabbing Query with BBST

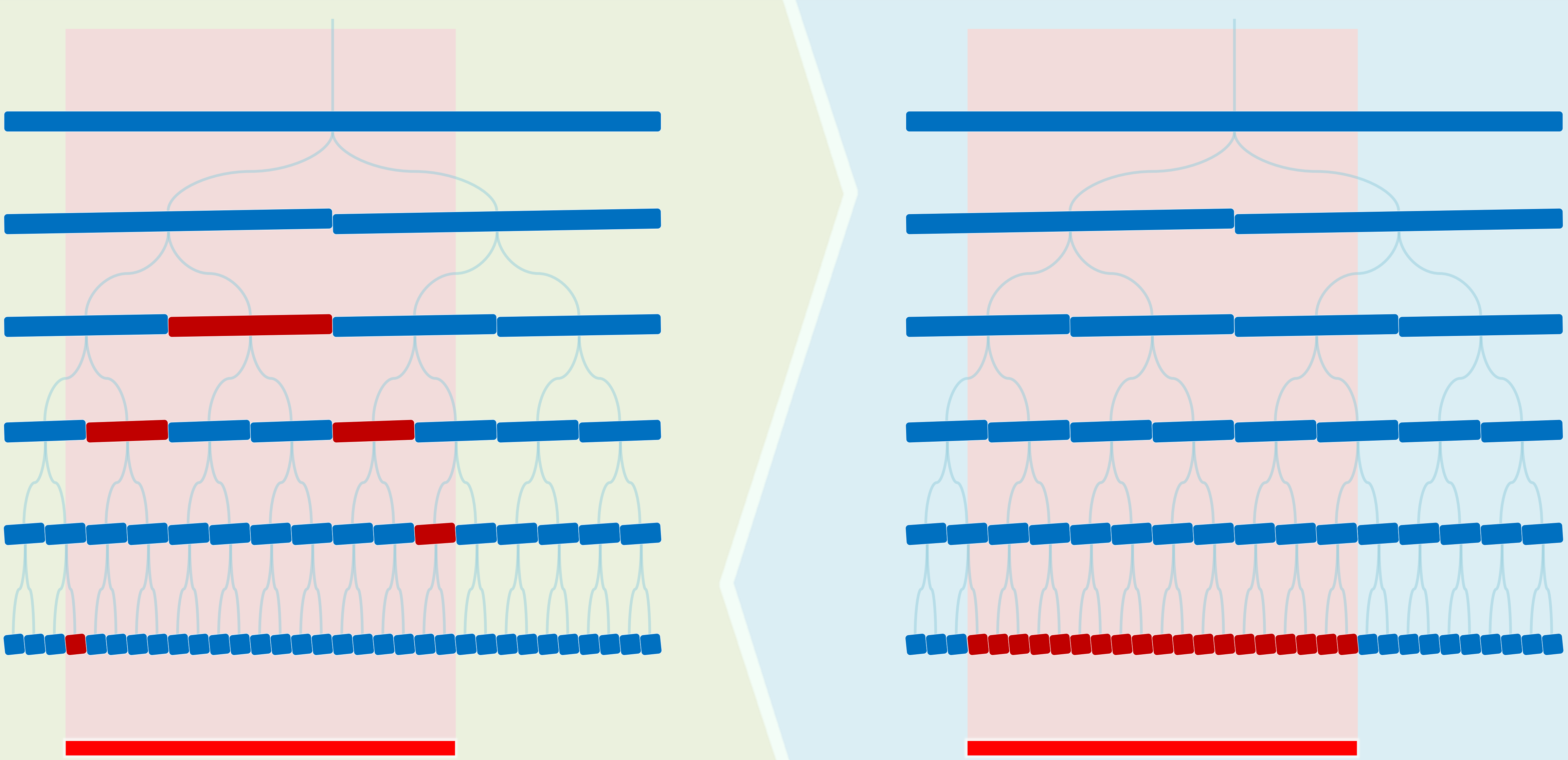
- To find all intervals containing q_x, we can
  - find the EI(v) containing q_x //$\mathcal{O}$(logn) time for a BBST
  - and then report Int(v) //$\mathcal{O}$(1 + r) time



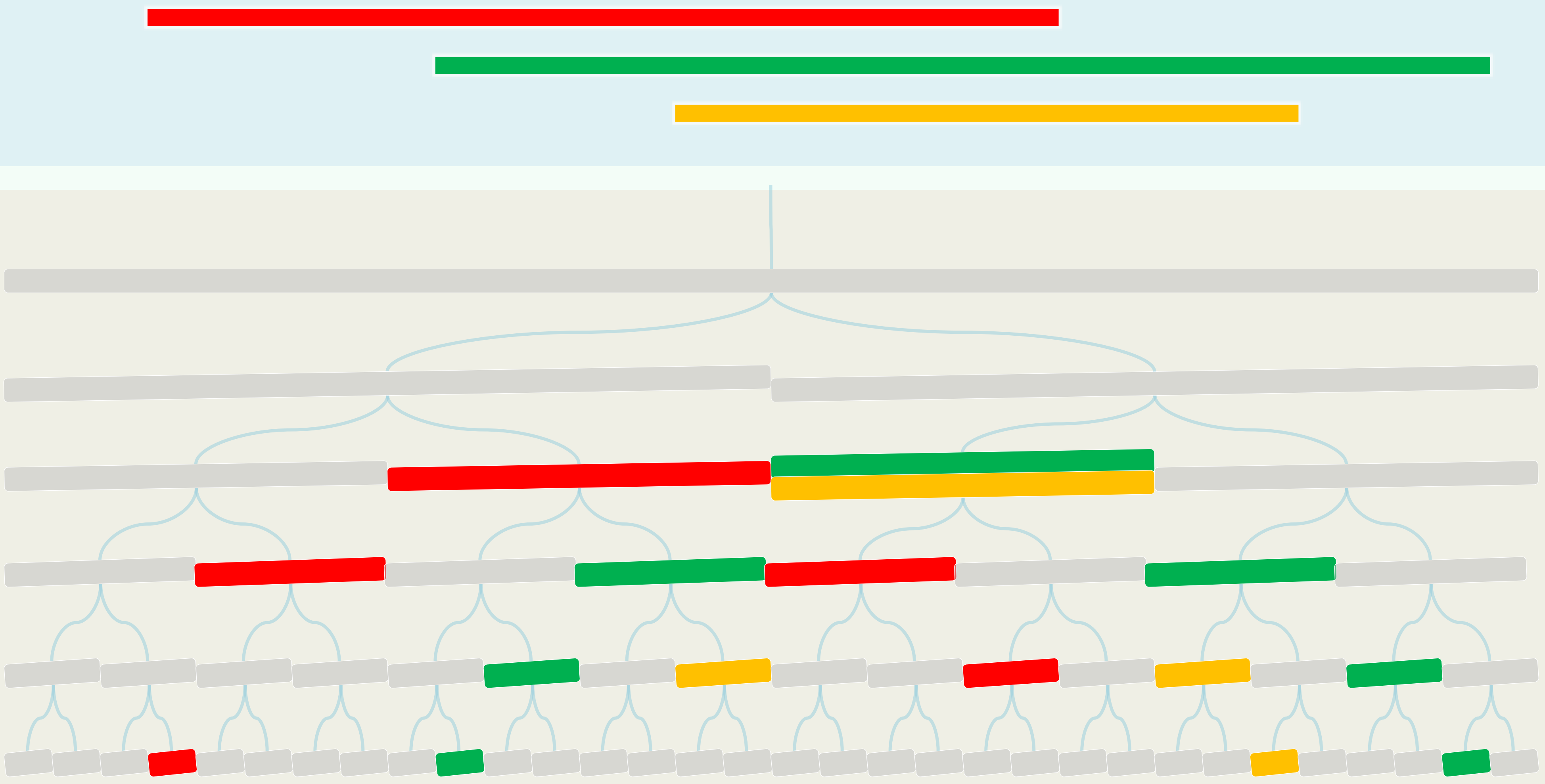
$\Omega(n^2)$ Total Space In The Worst Cases



Merge At Common Ancestors

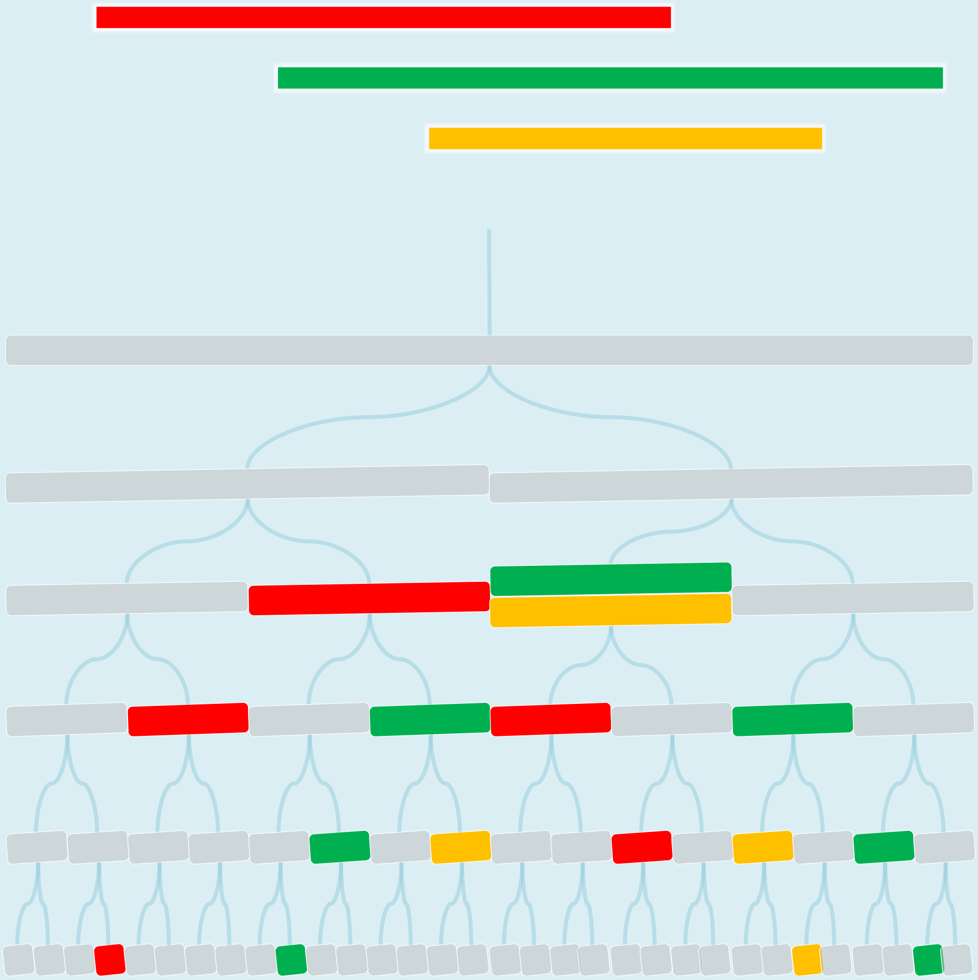


Canonical Subsets with $\mathcal{O}(n\log n)$ Space



BuildSegmentTree(I)

```
❖ //Construct a segment tree on
  //a set I of n intervals
  Sort all endpoints in I before
     determining all EI's //O(nlogn)
  Create T a BBST on all the EI's //O(n)
  Determine Int(v) for each node v
     //O(n) if done in a bottom-up manner
  For each s of I
     call InsertSegmentTree( T.root , s )
```



InsertSegmentTree(v , s)

❖ //Insert an interval s into

//a segment (sub)tree rooted at v

if (Int(v) $\subseteq$ s)

store s at v and return;

if (Int(lc(v)) $\cap$ s $\neq \varnothing$) //recurse

InsertSegmentTree(lc(v), s);

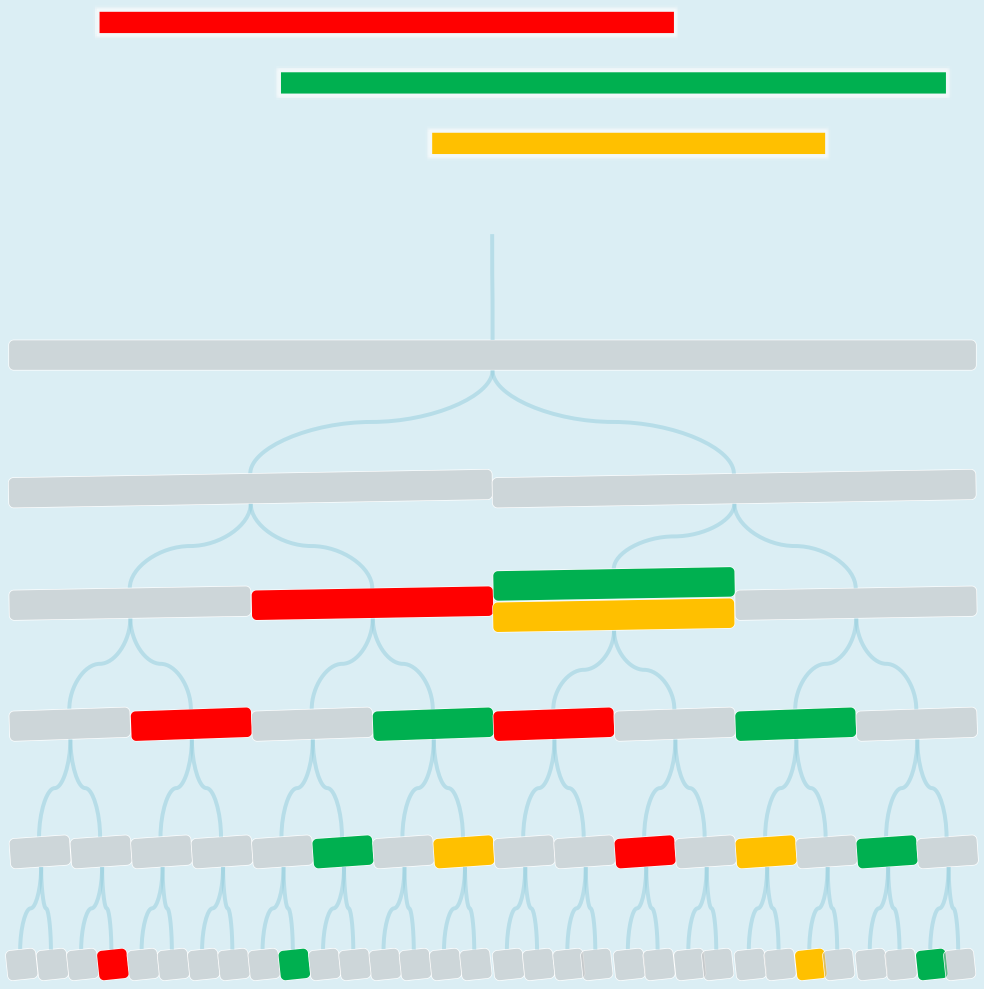
if (Int(rc(v)) $\cap$ s $\neq \varnothing$) //recurse

InsertSegmentTree(rc(v), s);

At each level, $\leq$ 4 nodes are visited

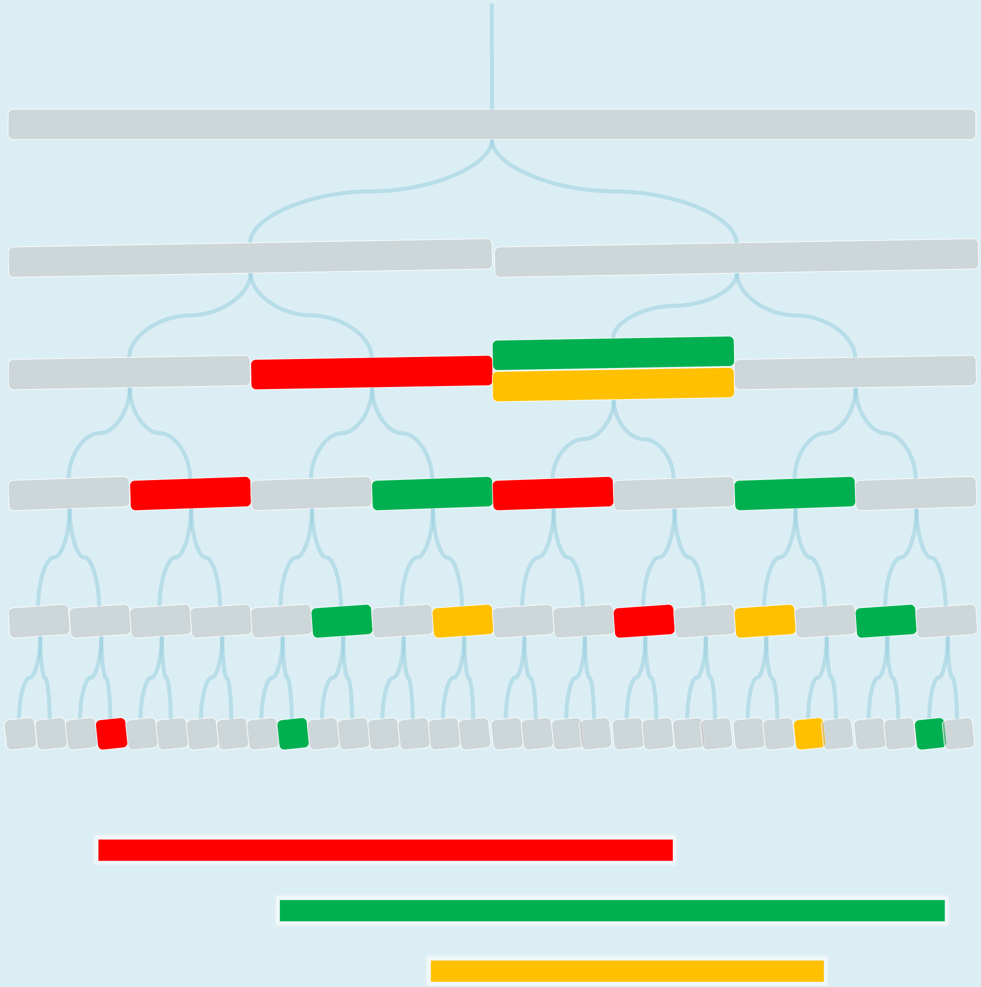
(2 stores + 2 recursions)

$\therefore$ $\mathcal{O}(\log n)$ time



QuerySegmentTree(v , q_x)

```
❖ //Find all intervals
  //in the (sub)tree rooted at v
  //that contain qx
  report all the intervals in Int(v)
  if ( v is a leaf )
     return
  if ( qx ∈ Int( lc(v) ) )
     QuerySegmentTree( lc(v), qx )
  else
     QuerySegmentTree( rc(v), qx )
```



$\mathcal{O}(r + \log n)$

- Only one node is visited per level,

  altogether $\mathcal{O}(\log n)$ nodes

- At each node v

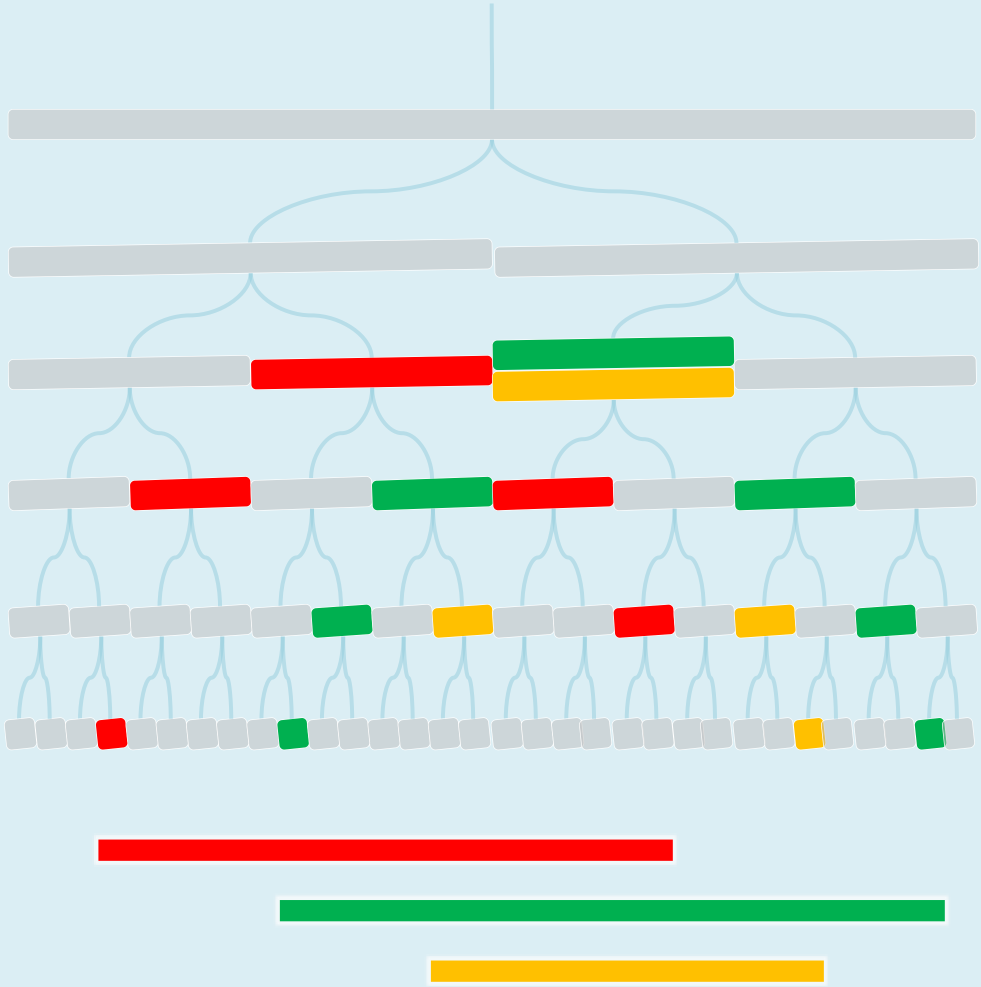
  - the CS Int(v) is reported

  - in time

  $$1 + |\mathrm{Int}(v)| = \mathcal{O}(1 + r_v)$$

∴ Reporting all the intervals

costs $\mathcal{O}(r)$ time



Conclusion

❖ For a set of n intervals,

- a segment tree of size $\mathcal{O}(n\log n)$

- can be built in $\mathcal{O}(n\log n)$ time

- which reports all intervals

  containing a query point

  in $\mathcal{O}(r + \log n)$ time