Mathematical Working Principle of the Median Filter

Definition:

The **median filter** is a **non-linear** digital filtering technique used to **remove noise** from an image or signal while **preserving edges**.

Mathematical Steps:

Let the image be represented as a 2D matrix I(x, y), where x and y are the spatial coordinates.

Let the neighborhood (filter window) be of size m×n centered at (x, y).

Step-by-Step:

1. **Select a window** of size m×n centered at pixel (x, y).
2. **Extract the pixel values** within this window:

   W={I(i,j) | i∈[x−a,x+a], j∈[y−b,y+b]}

   where:

   - a = m/2
   - b = n/2
3. **Sort** the pixel values in WW in ascending order:

   Wsorted = {p1,p2,...,pmn}

4. **Compute the median**:

$$\text{median}(W) = \begin{cases} p_{\frac{mn+1}{2}}, & \text{if } mn \text{ is odd} \\ \frac{p_{\frac{mn}{2}} + p_{\frac{mn}{2}+1}}{2}, & \text{if } mn \text{ is even} \end{cases}$$

5. **Replace** the center pixel value I(x,y)I(x, y) with the computed median:

   I′(x,y)=median(W)I'(x, y) = \text{median}(W)

Practical Application of Median Filter

1. Salt-and-Pepper Noise Removal in Images

- Salt-and-pepper noise causes random black (0) and white (255) pixels.

- Median filter removes these outliers by replacing them with the **median of their neighborhood**, which is usually a valid, non-noisy value.

📌 *Example:*

```
Before:
[100, 255, 102]
[ 98,   0, 101]
[ 97, 100, 103]

After Median Filter (3x3):
[100, 100, 102]
[ 98, 100, 101]
[ 97, 100, 103]
```

2. Preserving Edges in Medical Imaging (e.g., MRI, X-ray)

- Unlike averaging filters, which blur edges, the median filter **preserves sharp transitions** while removing noise.
- This makes it useful in medical image pre-processing.

3. Signal Processing (1D median filter)

- In ECG or other biosignals, sudden spikes or dropouts (outliers) are removed using median filtering while keeping the overall waveform shape.

4. Video Surveillance and Traffic Cameras

- Helps in cleaning up noisy frames where lighting or dust causes pixel flickering.

📄 Summary Table

Feature	Median Filter
Type	Non-linear filter
Operation	Replace center value with the median
Preserves edges?	Yes
Good at removing	Salt-and-pepper noise, impulse noise
Common window size	3×3, 5×5, 7×7 (odd-sized, usually square)
Formula for median	See formula above