Title: Understanding Gaussian Filtering in Digital Image Processing

1. Introduction

In digital image processing, noise is a common issue that degrades image quality and affects further analysis such as edge detection, segmentation, and object recognition. One of the most effective techniques to reduce noise while preserving essential features is Gaussian filtering. This report explains the principle, mathematical foundation, kernel computation, and a practical example of Gaussian filtering.

2. Principle of Gaussian Filtering

Gaussian filtering is a type of linear smoothing technique that reduces image noise and detail by averaging pixel values with a Gaussian-weighted kernel. Unlike simple averaging, Gaussian filtering gives more weight to pixels closer to the center, based on the Gaussian (normal) distribution. This helps maintain the structure of the image, especially edges.

3. Mathematical Principle

The 2D Gaussian function used as a kernel is:

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Where:

- x, y are coordinates relative to the kernel center
- σ is the standard deviation
- exp is the exponential function (e ≈ 2.718)

The filtered image is obtained by convolving this kernel with the image:

$$I_{\text{new}}(i, j) = \sum_{x=-k}^{k} \sum_{y=-k}^{k} I(i + x, j + y) \cdot G(x, y)$$

4. Kernel Matrix Computation

To compute a Gaussian kernel matrix:

1. Choose a kernel size (e.g., 3x3, 5x5) and σ

2. Compute G(x, y) for each position using the formula
3. Normalize the matrix so its total sum equals 1

Example (σ = 1, 3x3 kernel):

Kernel:

$$\begin{bmatrix} 0.0585 & 0.0965 & 0.0585 \\ 0.0965 & 0.1592 & 0.0965 \\ 0.0585 & 0.0965 & 0.0585 \end{bmatrix}$$

4. **Practical Example**
   Given a 3x3 image patch:

$$\begin{bmatrix} 52 & 55 & 61 \\ 54 & 59 & 63 \\ 58 & 60 & 65 \end{bmatrix}$$

Multiply each value with the kernel and sum:

New value ≈
(52×0.0585) + (55×0.0965) + (61×0.0585) +
(54×0.0965) + (59×0.1592) + (63×0.0965) +
(58×0.0585) + (60×0.0965) + (65×0.0585)
≈ 58.5

So, the center pixel (originally 59) becomes 58.5.

6. Conclusion
Gaussian filtering is essential for smoothing and denoising images. Its weighted averaging method preserves edges better than simple filters. A clear understanding of how to compute and apply the Gaussian kernel lays a strong foundation for advanced hybrid filtering techniques.