Solution

DIP Midterm Exam
2020/5/18

1. (20%) Consider $f(x,y)$ and $w(x,y)$ below. (a) Evaluate the linear convolution $g_1(x,y) = f(x,y) * w(x,y)$. (b) Determine $g_2(x,y) = \mathrm{DFT}^{-1}[F(u,v)W(u,v)]$ (3×3 inverse DFT), where $F(u,v)$ and $W(u,v)$ are respectively the 3×3 DFT of $f(x,y)$ and $w(x,y)$, and $g_2(x,y)$ is the inverse 3×3 DFT of $[F(u,v)W(u,v)]$. The shaded grid indicates the origin (0,0) of the image spatial coordinate system.

$f(x,y)$

1	0	0
-1	0	0
0	0	0

$w(x,y)$

1	2	3
4	5	4
3	2	1

Solution:

$g_1(x,y) = f(x,y) * w(x,y)$

1	2	3
4-1	5-2	4-3
3-4	2-5	1-4
-3	-2	-1

=

1	2	3
3	3	1
-1	-3	-3
-3	-2	-1

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$g_2(x,y) = \mathrm{DFT}^{-1}[F(u,v)W(u,v)]$

1-3	2-2	3-1
4-1	5-2	4-3
3-4	2-5	1-4

=

-2	0	2
3	3	1
-1	-3	-3

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2. (15%) A linear, shift-invariant (LSI) system has the impulse response $h(x,y) = \delta(x-a,y-b)$ where a and b are constants, and x and y are discrete quantities. (a) What is the frequency response $H(u,v)$ of this LSI system (assume $M\times N$ 2D DFT)? (b) Given an input, $f_1(x,y) = K$, what is the output of this LSI system? (c) If now the input is $f_2(x,y) = \sin((2\pi u_0 x)/M+(2\pi v_0 y)/N)$, what is the output?

Solution:

(a)

$$H(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}h(x,y)W_M^{ux}W_N^{vy}=\sum_{y=0}^{N-1}\delta(x-a,y-b)W_M^{ux}W_N^{vy}=W_M^{ua}W_N^{vb}=\boxed{e^{-j\frac{2\pi}{M}ua}e^{-j\frac{2\pi}{N}vb}}$$

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(b)

$$f_1(x,y)*h(x,y)=K*\delta(x-a,y-b)=\boxed{K}$$

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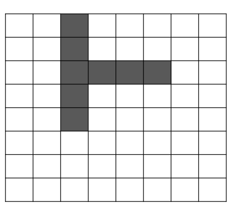
(c)

$$f_2(x,y)*h(x,y)=\sin\left(\frac{2\pi u_0 x}{M}+\frac{2\pi v_0 y}{N}\right)*\delta(x-a,y-b)=\boxed{\sin\left(\frac{2\pi u_0(x-a)}{M}+\frac{2\pi v_0(y-b)}{N}\right)}$$

5

3. (26%)

(a) (10%) Consider $f(x,y)$ below. Let $F(u,v)=DFT\{f(x,y)\}_{8\times 8}$. Plot $g(x,y)=iDFT_{8\times 8}\left\{F(u,v)e^{j\frac{\pi}{4}(u-v)}\right\}$



Solution:

$$(a)\,g(x,y)=iDFT\left\{F(u,v)e^{j\left(\frac{\pi u}{4}-\frac{\pi v}{4}\right)}\right\}=iDFT\left\{F(u,v)e^{j\left(\frac{2\pi u}{8}-\frac{2\pi v}{8}\right)}\right\}=\boxed{f((x+1)_8,(y-1)_8)}$$

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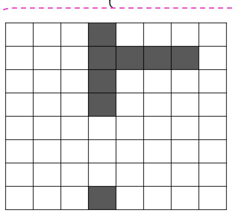

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$$f((x-x_0)_M,(y-y_0)_N)\xleftrightarrow{DFT}F(u,v)e^{-j\frac{2\pi}{M}ux_0}e^{-j\frac{2\pi}{N}vy_0}$$
$$=F(u,v)W_M^{ux_0}W_N^{vy_0}$$

3. (26%)

(b) (16%) Prove the multiplication property of 2D DFT:

$$f(x,y)h(x,y) \xleftrightarrow{DFT} \frac{1}{MN} F(u,v)\otimes H(u,v)$$

where $F(u,v)$ / $H(u,v)$ is the $M\times N$ 2D DFT of $f(x,y)$ / $h(x,y)$. Give two-way proof: $(x,y)\rightarrow(u,v)$, $(x,y)\leftarrow(u,v)$.

Prove: $(x,y)\rightarrow(u,v)$

Let $g(x,y) = f(x,y)h(x,y)$

8%

$$G(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)h(x,y)W_M^{ux}W_N^{vy} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left(\frac{1}{MN}\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} F(\alpha,\beta)W_M^{-\alpha x}W_N^{-\beta y}\right)h(x,y)W_M^{ux}W_N^{vy}$$

$$= \frac{1}{MN}\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} F(\alpha,\beta)\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} h(x,y)W_M^{(u-\alpha)x}W_N^{(v-\beta)y} = \frac{1}{MN}\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} F(\alpha,\beta)H((u-\alpha)_M,(v-\beta)_N)$$

$$= \frac{1}{MN}F(u,v)\otimes H(u,v)$$

$(x,y)\leftarrow(u,v)$

8%

$$G(u,v) = \frac{1}{MN}F(u,v)\otimes H(u,v) = \frac{1}{MN}\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} F(\alpha,\beta)H((u-\alpha)_M,(v-\beta)_N)$$

$$= \frac{1}{MN}\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} F(\alpha,\beta)\left(\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} h(x,y)W_M^{(u-\alpha)x}W_N^{(v-\beta)y}\right) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} h(x,y)\left(\frac{1}{MN}\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} F(\alpha,\beta)W_M^{-\alpha x}W_N^{-\beta y}\right)W_M^{ux}W_N^{vy}$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[h(x,y)f(x,y)]W_M^{ux}W_N^{vy} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} g(x,y)W_M^{ux}W_N^{vy}$$

$$\Rightarrow g(x,y) = f(x,y)h(x,y)$$

4. (30%) Consider a 4 bits/pixel image f_r (size 20×20) with the gray levels $0 \le r_i \le 15$, $i = 0, \ldots, 15$. The number of pixels (n_i) with gray level r_i is: $n_0= n_1= n_2= n_3=0$, $n_4=24$, $n_5= n_6=48$, $n_7= n_8=100$, $n_9=80$, $n_i=0$ for $10\le r_i \le 15$. (a) Determine the intensity transformation function $s = T_1(r)$ for performing histogram equalization (10%), and find out the output histogram (5%).

Solution (a):

r_i	n_i	p_i	$\Sigma(p_i)$	$15\Sigma(p_i)$	s_i
0	0	0.00	0.00	0.00	0
1	0	0.00	0.00	0.00	0
2	0	0.00	0.00	0.00	0
3	0	0.00	0.00	0.00	0
4	24	0.06	0.06	0.90	1
5	48	0.12	0.18	2.70	3
6	48	0.12	0.30	4.50	5
7	100	0.25	0.55	8.25	8
8	100	0.25	0.80	12.00	12
9	80	0.20	1.00	15.00	15
10	0	0.00	1.00	15.00	15
11	0	0.00	1.00	15.00	15
12	0	0.00	1.00	15.00	15
13	0	0.00	1.00	15.00	15
14	0	0.00	1.00	15.00	15
15	0	0.00	1.00	15.00	15

10

r_i	$s_i=T_1(r_i)$
0	0
1	0
2	0
3	0
4	1
5	3
6	5
7	8
8	12
9	15
10	15
11	15
12	15
13	15
14	15
15	15

5

s_i	n_i
0	0
1	24
2	0
3	48
4	0
5	48
6	0
7	0
8	100
9	0
10	0
11	0
12	100
13	0
14	0
15	80

4. (30%) Consider a 4 bits/pixel image f_r (size 20×20) with the gray levels $0 \le r_i \le 15$, $i = 0, \ldots, 15$. The number of pixels (n_i) with gray level r_i is: $n_0 = n_1 = n_2 = n_3 = 0$, $n_4 = 24$, $n_5 = n_6 = 48$, $n_7 = n_8 = 100$ $n_9 = 80$, $n_i = 0$ for $10 \le r_i \le 15$. (b) Determine the intensity transformation function $z = T_2(r)$ to produce the output image with the new histogram (probability density function) {0.2 0 0.2 0 0.1 0 0.1 0 0.1 0 0.1 0 0.1 0 0 0.1} (10%), and find the actual output histogram (5%).

Solution (b):

r_i	n_i	p_{ri}	$\Sigma(p_{ri})$	$\Sigma(p_{zi})$	p_{zi}	z_i
0	0	0.00	0.00	0.20	0.2	0
1	0	0.00	0.00	0.20	0	1
2	0	0.00	0.00	0.40	0.2	2
3	0	0.00	0.00	0.40	0	3
4	24	0.06	0.06	0.50	0.1	4
5	48	0.12	0.18	0.50	0	5
6	48	0.12	0.30	0.60	0.1	6
7	100	0.25	0.55	0.60	0	7
8	100	0.25	0.80	0.70	0.1	8
9	80	0.20	1.00	0.70	0	9
10	0	0.00	1.00	0.80	0.1	10
11	0	0.00	1.00	0.80	0	11
12	0	0.00	1.00	0.90	0.1	12
13	0	0.00	1.00	0.90	0	13
14	0	0.00	1.00	0.90	0	14
15	0	0.00	1.00	1.00	0.1	15

10

r_i	$z_i = T_2(r_i)$
0	0
1	0
2	0
3	0
4	0
5	0
6	2
7	6
8	10
9	15
10	15
11	15
12	15
13	15
14	15
15	15

5

z_i	n_i
0	72
1	0
2	48
3	0
4	0
5	0
6	100
7	0
8	0
9	0
10	100
11	0
12	0
13	0
14	0
15	80

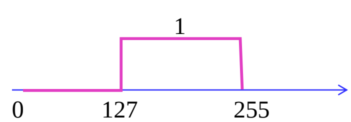
5. (15%) We may design the intensity transformation function $s = T(r)$ to perform the bit-plane slicing. Determine $s = T(r)$ to obtain respectively (a) bit-plane 8 (8%), and (b) bit-plane 6 (7%) image, assume an 8-bit gray-scale image.

Solution:

bit 8	bit 7	bit 6	bit 5	bit 4	bit 3	bit 2	bit 1
128	64	32	16	8	4	2	1

8

$$\text{bit-plane 8: } s = T(r) = \begin{cases} 0, & 0 \le r \le 127 \\ 1, & 128 \le r \le 255 \end{cases}$$



7

$$\text{bit-plane 6: } s = T(r) = \begin{cases} 0, & 0 \le r \le 31,\ 64 \le r \le 95,\ 128 \le r \le 159,\ 192 \le r \le 223 \\ 1, & 32 \le r \le 63,\ 96 \le r \le 127,\ 160 \le r \le 191,\ 224 \le r \le 255 \end{cases}$$