

$$1. (a) f(x,y) \otimes w(x,y) = \sum_{m=0}^5 \sum_{n=0}^5 f(m,n) w(x-m, y-n)$$

(假設 $H(x,y)$ 為 convolution 矩陣

$$H(0,0) = f(0,0) \times g(0,0) = 1 \times 1 = 1$$

$$H(0,1) = f(0,0) \times g(0,1) + f(0,1) \times g(0,0) = 1 \times 2 = 2$$

$$H(0,2) = f(0,0) \times g(0,2) = 1 \times 3 = 3$$

$$f(x,y) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$w(x,y) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

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宋其諭

∴ $f(x,y)$ 為 0 元素多可化簡為

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 \\ 1 & 3 & 3 & 0 & 0 \\ 3 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

with (0,0) is origin

$$\begin{array}{ccccc} & 1 & 2 & 3 & 0 & 0 \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & 4-1 & 5-2 & 4-3 & 0 & 0 \\ & 3-4 & 2-5 & 1-4 & 0 & 0 \\ & 3 & -2 & -1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$1-b) F(u,v) = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} f(x,y) w_3^{ux+vy}$$

$$w(u,v) = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} w(x,y) w_3^{ux+vy}$$

$$w(u,v) = w(-1,-1)w_3^{-u-v} + w(-1,0)w_3^{-u} + w(-1,1)w_3^{-u+v}$$

$$+ w(0,-1)w_3^{-v} + w(0,0) + w(0,1)w_3^v$$

$$+ w(1,-1)w_3^{u-v} + w(1,0)w_3^u + w(1,1)w_3^{u+v}$$

$$F(u,v)w(u,v) = w_3^{-u-v} + 2w_3^{-u} + 3w_3^{-u+v}$$

$$+ 3w_3^{-v} + 3 + w_3^v$$

$$+ (-1)w_3^{u-v} + (-1)w_3^u + (-1)w_3^{u+v}$$

$$+ (-1)w_3^{2u+v} + (-2)w_3^{2u} + (-1)w_3^{2u+v}$$

$$= 3w_3^{-v} + 3 + (-3)w_3^v$$

$$-1w_3^{u-v} - 3w_3^u - 3w_3^{u+v}$$

$$-2w_3^{2u+v} - 0 + 2w_3^{2u+v}$$

$$G(u,v) = \sum_{x=0}^2 \sum_{y=0}^2 y_2(x,y)w_3^{ux+vy}$$

3	3	-3
-1	-3	-3
-2	0	2

※ 考試作弊者將受記大過以上處分
 ※ If you cheat in an exam, you will be punished.

Total 11/11

2. a) $H(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \delta(x-a, y-b) e^{-j2\pi \frac{ux}{M}} e^{-j2\pi \frac{vy}{N}}$

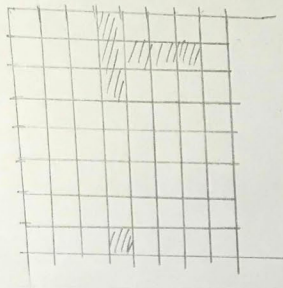
$$= \frac{1}{MN} e^{-j2\pi \frac{ua}{M}} e^{-j2\pi \frac{vb}{N}}$$

$$= \frac{1}{MN} W_M^{ua} W_N^{vb} \text{ 讓原訊號平移 } (a,b)$$

b) $\text{output} = f_1 * h = \text{DFT}^{-1} \{ F_1 \cdot H \} = k$

c) $\text{output} = f_2 * h = \text{DFT}^{-1} \{ F_2 \cdot H \} = \sin \left[\frac{2\pi u_0(x-a)}{M} + \frac{2\pi v_0(y-b)}{N} \right]$

3. (a)



$$F(u, v) e^{j \frac{2\pi}{N} (u-v)} = F(u, v) e^{j \frac{2\pi}{N} (-v+u)} = F(u, v) W_N^{-u+v}$$

$$F(u, v) W_N^{-u+v} \xrightarrow{\text{IDFT}} f((x+1)_8, (y-1)_8)$$

$$b) a[x, y] = \text{IDFT} \{ A(u, v) \} = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} A(u, v) W_{MN}^{-xu-yv}$$

$$\text{and } A(u, v) = \sum_{b=0}^{M-1} \sum_{c=0}^{N-1} F(b) H(u-b) F(c) H(v-c) \Rightarrow (u, v) = k, (b, c) = m$$

$$\therefore a[x, y] = \frac{1}{MN} \sum_{k=0}^{M,N-1} \sum_{m=0}^{M,N-1} F(m) H(k-m) W_{MN}^{-x,y,k}$$

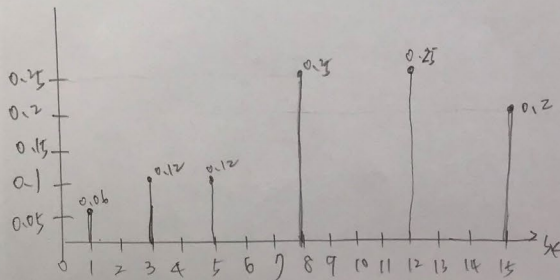
$$\therefore a[x, y] = \frac{1}{MN} \sum_{m=0}^{M,N-1} F(m) W_{MN}^{-x,y,m} \quad MN \frac{1}{MN} \sum_{k=0}^{M,N-1} H(k-m) W_{MN}^{-x,y,(k-m)}$$

$$b) a[x, y] = MN f[x, y] h[x, y]$$

4. (a)

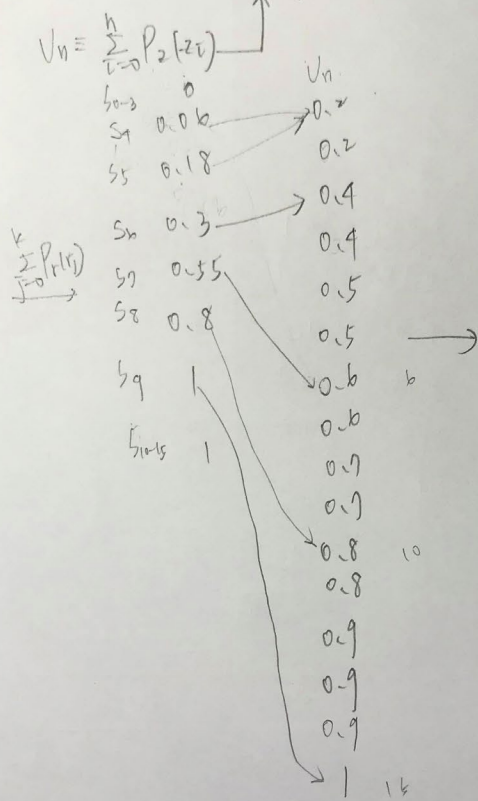
r_k	n_k	$P_r(r_k)$	$\sum P_r(r_k)$	$\sum r_k P_r(r_k)$	$L_k = T_k(n)$
r_0	0	0	0	0	0
r_1	0	0	0	0	0
r_2	0	0	0	0	0
r_3	0	0	0	0	0
r_4	24	0.06	0.06	0.9	1
r_5	48	0.12	0.18	2.7	3
r_6	48	0.12	0.3	4.5	5
r_7	100	0.25	0.55	8.25	8
r_8	100	0.25	0.8	12	12
r_9	80	0.2	1	15	15
r_{10-15}	0	0	1	15	15
					#

L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$P(L)$	0	0.06	0	0.12	0	0.12	0	0	0.25	0	0	0	0.25	0	0	0.2



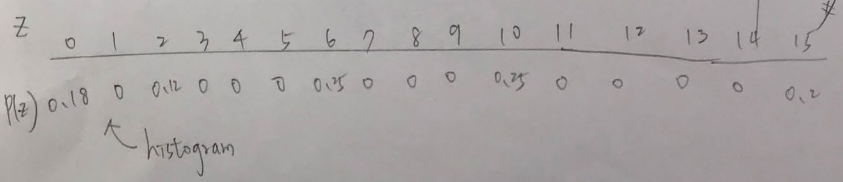
(b) $r_k \rightarrow \sum_{j=0}^k P_r(r_j) \xrightarrow{L_k} \min \{ (V_n - S_k)^2 \} \rightarrow Z_k = G^{-1}(V_n) \rightarrow Z_k$

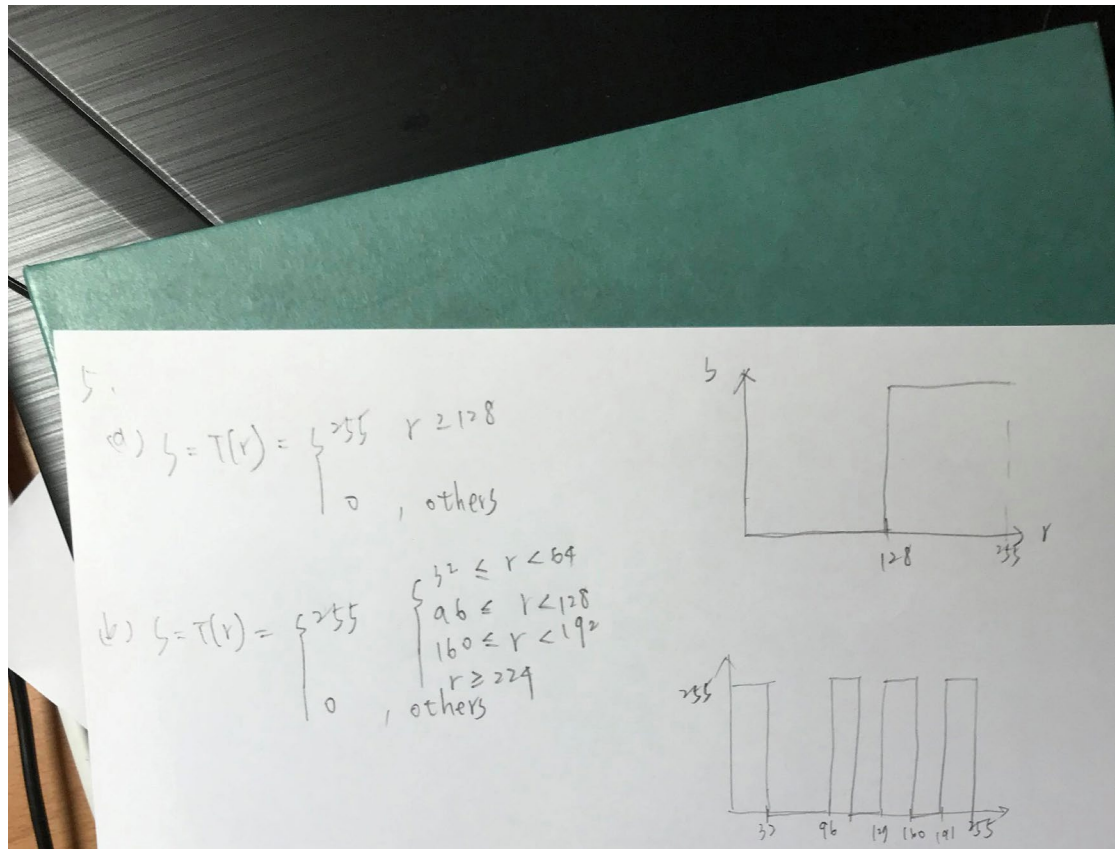
- $r_4 = 0.06$
- $r_5 = 0.12$
- $r_6 = 0.12$
- $r_7 = 0.25$
- $r_8 = 0.25$
- $r_9 = 0.2$
- $r_{10-15} = 0$



actual output

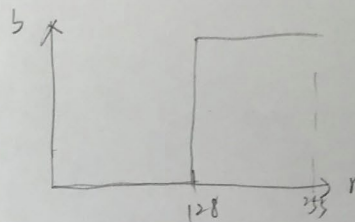
r_k	Z_k
0	0
1	0
2	0
3	0
4	0
5	0
6	2
7	6
8	10
9	15
10	15
11	15
12	15
13	15
14	15
15	15



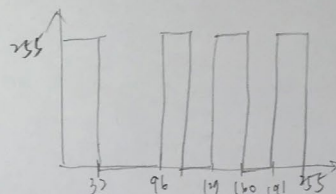


5.

$$a) \zeta = T(r) = \begin{cases} 255 & r \geq 128 \\ 0 & \text{others} \end{cases}$$



$$b) \zeta = T(r) = \begin{cases} 255 & \begin{cases} 32 \leq r < 64 \\ 96 \leq r < 128 \\ 160 \leq r < 192 \end{cases} \\ 0 & \text{others} \end{cases}$$



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$$\begin{aligned} w(u,v) &= w(-1,-1)w_3^{-u-v} + w(-1,0)w_3^{-u} + w(-1,1)w_3^{-u+v} \\ &+ w(0,-1)w_3^{-v} + w(0,0) + w(0,1)w_3^v \\ &+ w(1,-1)w_3^{u-v} + w(1,0)w_3^u + w(1,1)w_3^{u+v} \end{aligned}$$

$$\begin{aligned} F(u,v)w(u,v) &= w_3^{-u-v} + 2w_3^{-u} + 3w_3^{-u+v} = 3w_3^{-v} + 3 + (-3)w_3^v \\ &+ 3w_3^{-v} + 3 + w_3^v \quad -1w_3^{u-v} - 3w_3^u - 3w_3^{u+v} \\ &+ (-1)w_3^{u-v} + (-3)w_3^u + (-3)w_3^{u+v} \quad -2w_3^{2u+v} - 0 + 2w_3^{2u+v} \\ &+ (-3)w_3^{2u+v} + (-2)w_3^{2u} + (-1)w_3^{2u+v} \end{aligned}$$

$$2 \quad 2 \quad u(x,y)w_3^{ux+vy}$$

$$3 \quad 3 \quad -3$$