Online wechat-group

2020 TL COURSE

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Lecture 3: Determination of per-unit-length inductance and capacitance parameters

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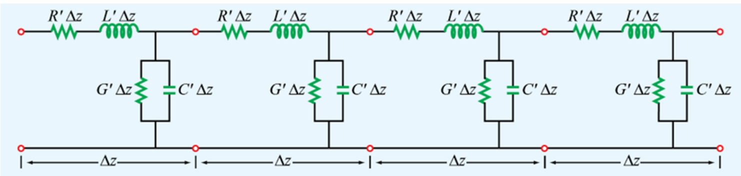
Reviews

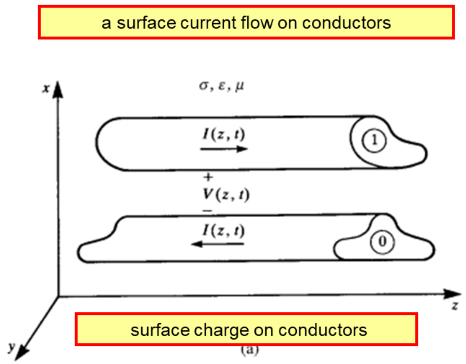


$$-\frac{\partial v(z,t)}{\partial z} = R'\, i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

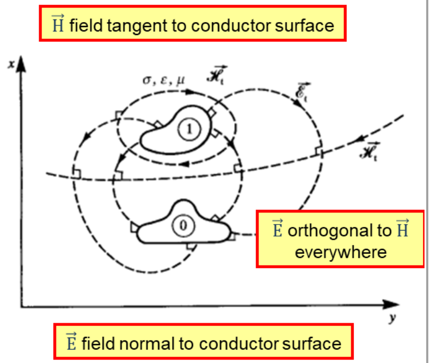
$$-\frac{\partial i(z,t)}{\partial z} = G'\, v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$

Determination of per unit length parameters will be discussed here.

Two-conductor transmission line



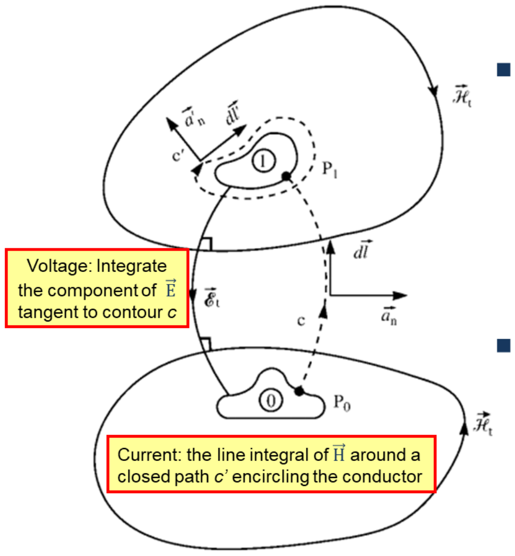
The current and voltage



TEM fields structure

The conductors are considered to be perfect conductors(lossless, $\sigma = \infty$).
The surrounding medium is homogeneous and lossy($\mu = \mu_0, \varepsilon = \varepsilon_r \varepsilon_0, \sigma$).

Voltage and Current for Two-conductor TL



- The voltage between the two conductors

$$V(z,t) = -\int_c \vec{\mathscr{E}}_t \cdot d\vec{l}$$

$$= -\int_{P_0}^{P_1} \vec{\mathscr{E}}_t \cdot d\vec{l}$$

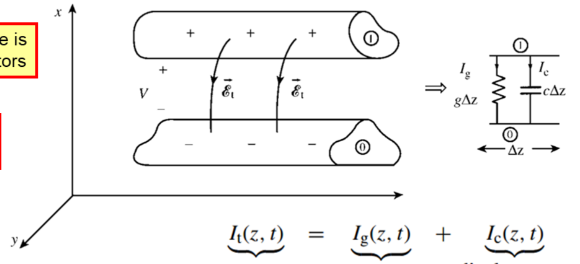
- The current across the line

$$I(z,t) = \oint_{c'} \vec{\mathscr{H}}_t \cdot d\vec{l}'$$

The per-unit-length parameters

Charge is store on two conductors, there is a capacitance between the two conductors

I_g: induced by transverse electric field to flow in lossy medium



EM Field ⟶ **Circuit**

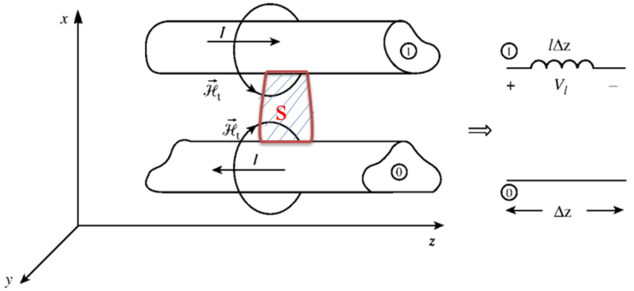
$$\underbrace{I_t(z,t)}_{\text{transverse current}} = \underbrace{I_g(z,t)}_{\text{conduction current}} + \underbrace{I_c(z,t)}_{\text{displacement current}}$$

The conduction current: $\vec{J}_g = \sigma \vec{E}_g$ $\quad g = \lim_{\Delta z \to 0} \frac{G}{\Delta z}$ (S/m) $\quad I_g(z,t) = g\Delta z V(z,t)$

The displacement current: $Q = CV$ $\quad I_c(t) = \frac{dQ}{dt} = \frac{CdV(t)}{dt}$ $\quad c = \lim_{\Delta z \to 0} \frac{C}{\Delta z}$ (F/m)

$$I_c(z,t) = c\Delta z \frac{\partial V(z,t)}{\partial t}$$

The per-unit-length parameters



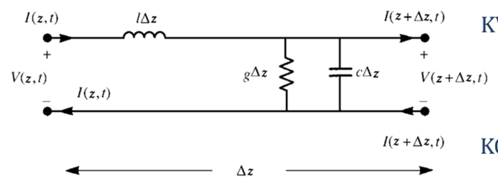
The transverse magnetic flux density $\quad \vec{B}_t = \mu \vec{H}_t$

Magnetic flux $\quad \psi = \int_s \vec{B}_t \cdot d\vec{s} \quad V(t) = \frac{d\Psi}{dt} = \frac{LdI(t)}{dt}$

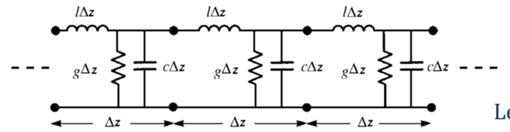
Per-unit-length inductance $\quad l = \lim_{\Delta z \to 0} \frac{L}{\Delta z}$ (H/m)

Longitudinal voltage drop around two conductor $\quad V_l(z,t) = l\Delta z \frac{\partial I(z,t)}{\partial t}$

The Telegrapher's Equation



(a)



(b)

The per-unit-length equivalent circuit for a two-conductor line

KVL at the left and right ends of this circuit

$$V(z+\Delta z,t) - V(z,t) = -l\Delta z \frac{\partial I(z,t)}{\partial t}$$

KCL at the top right node

$$I(z+\Delta z) - I(z,t)$$

$$= -g\Delta z V(z+\Delta z,t) - c\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}$$

Let $\quad \Delta z \to 0$

$$\boxed{\begin{aligned} \frac{\partial V(z,t)}{\partial z} &= -l\frac{\partial I(z,t)}{\partial t} \\ \frac{\partial I(z,t)}{\partial z} &= -gV(z,t) - c\frac{\partial V(z,t)}{\partial t} \end{aligned}}$$

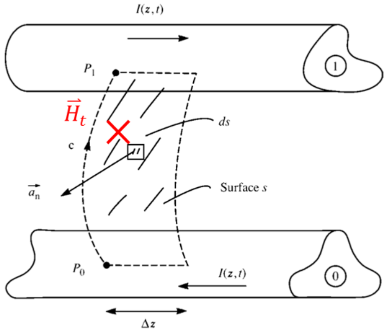
The per-unit-length inductance

The total magnetic flux through the surface

$$\psi = \int_s \mu \vec{\mathscr{H}}_t \cdot \vec{a}_n ds$$

The total inductance

$$l\Delta z = -\frac{\psi = \int_s \mu \vec{\mathscr{H}}_t \cdot \vec{a}_n ds}{I(z,t)}$$



Δz is considered differentially small, the flux doesn't vary with z. $\Delta z \to 0$

or

$$l\Delta z = -\frac{\Delta z \mu \int_c \vec{\mathscr{H}}_t \cdot \vec{a}_n dl}{I(z,t)}$$

$$l = -\frac{\mu \int_c \vec{\mathscr{H}}_t \cdot \vec{a}_n dl}{I(z,t)}$$

Substituting the definition of current

$$I(z,t) = \oint_{c'} \vec{\mathscr{H}}_t \cdot d\vec{l}'$$

$$l = -\mu \frac{\int_c \vec{\mathscr{H}}_t \cdot \vec{a}_n dl}{\oint_{c'} \vec{\mathscr{H}}_t \cdot d\vec{l}'}$$

The total L is the ratio of the magnetic flux through this surface to the current that caused it

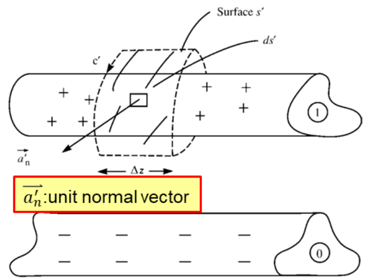
The per-unit-length capacitance

$$\oint_{\partial V} D \cdot dS = q_0$$

Gauss's law: The net electric flux through any hypothetical closed surface is equal $1/\varepsilon_0$ times the net electric charge within that closed surface.

The total capacitance is the ratio of this total charge to the voltage between the two conductor:

$$c\Delta z = \frac{\varepsilon \oint_{s'} \vec{\mathscr{E}}_t \cdot \vec{a}'_n ds'}{V(z,t)}$$



Δz is considered differentially small, the flux doesn't vary with z. $\Delta z \to 0$

$$c\Delta z = \frac{\Delta z \varepsilon \oint_{c'} \vec{\mathscr{E}}_t \cdot \vec{a}'_n dl'}{V(z,t)}$$

or

$$c = \frac{\varepsilon \oint_{c'} \vec{\mathscr{E}}_t \cdot \vec{a}'_n dl'}{V(z,t)}$$

Substituting the definition of voltage

$$V(z,t) = -\int_c \vec{\mathscr{E}}_t \cdot d\vec{l}$$

$$c = \varepsilon \frac{\oint_{c'} \vec{\mathscr{E}}_t \cdot \vec{a}'_n dl'}{-\int_c \vec{\mathscr{E}}_t \cdot d\vec{l}}$$

The total C is the ratio of the total charge to the voltage between the two conductors.

The per-unit-length conductance

The total conductance

$$g\Delta z = \frac{\sigma \oint_{s'} \vec{\mathscr{E}}_t \cdot \vec{a}'_n ds'}{V(z,t)}$$

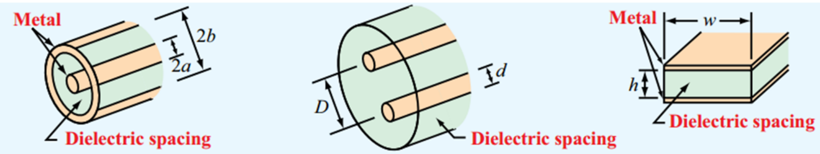
The line Δz is differentially small, the desired transverse current is obtained by simply integrating along the contour c':

$$g\Delta z = \frac{\Delta z \sigma \oint_{c'} \vec{\mathscr{E}}_t \cdot \vec{a}'_n dl'}{V(z,t)}$$

Substituting the definition of voltage

$$g = \sigma \frac{\oint_{c'} \vec{\mathscr{E}}_t \cdot \vec{a}'_n dl'}{-\int_c \vec{\mathscr{E}}_t \cdot d\vec{l}}$$

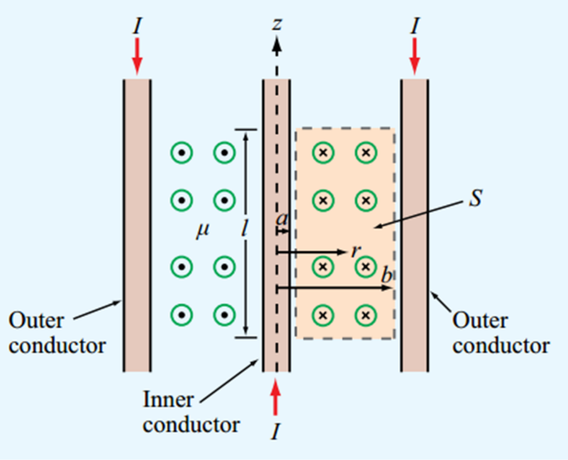
Transmission-line parameters examples



(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\epsilon w}{h}$	F/m

Derivation:Parameters of coaxial line



The per-unit-length inductance

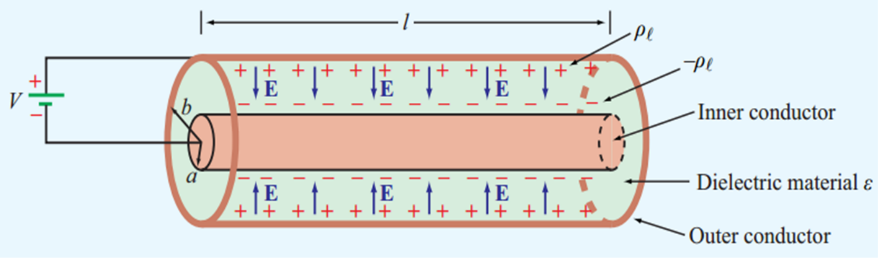
$$\mathbf{B} = \hat{\boldsymbol{\phi}}\,\frac{\mu I}{2\pi r},$$

$$\Phi = l\int_a^b B\,dr$$

$$= l\int_a^b \frac{\mu I}{2\pi r}\,dr = \frac{\mu I l}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$L' = \frac{L}{l} = \frac{\Phi}{lI} = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$$

Derivation:Parameters of coaxial line



The per-unit-length capacitance

$$\mathbf{E} = -\hat{\mathbf{r}}\,\frac{Q}{2\pi\epsilon r l}$$

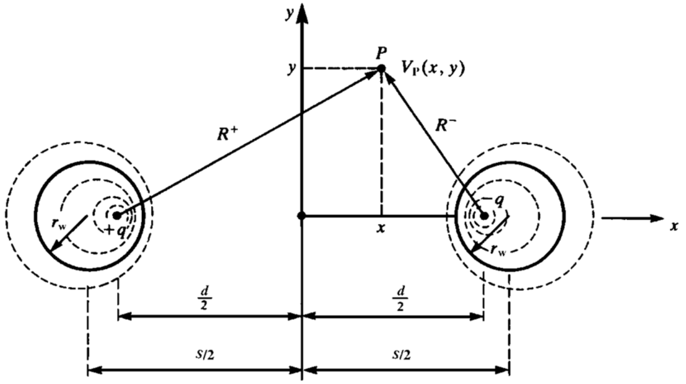
$$V = -\int_a^b \mathbf{E}\cdot d\mathbf{l} = -\int_a^b\left(-\hat{\mathbf{r}}\,\frac{Q}{2\pi\epsilon r l}\right)\cdot(\hat{\mathbf{r}}\,dr)$$

$$= \frac{Q}{2\pi\epsilon l}\ln\left(\frac{b}{a}\right).$$

$$C = \frac{Q}{V} = \frac{2\pi\epsilon l}{\ln(b/a)}$$

$$C' = \frac{C}{l} = \frac{2\pi\epsilon}{\ln(b/a)}$$

Derivation: capacitance of two wires



Point P is equipotential contours, the voltage carried by −q and +q is

$$V_P(x,y) = -\frac{q}{2\pi\varepsilon}\ln\left(\frac{R^+}{d/2}\right) + \frac{q}{2\pi\varepsilon}\ln\left(\frac{R^-}{d/2}\right) = \frac{q}{2\pi\varepsilon}\ln\left(\frac{R^-}{R^+}\right)$$

Points on equipotential contours are such that the ratio

$$\frac{R^-}{R^+} = e^{\left(\frac{2\pi\varepsilon V_P}{q}\right)} = K$$

Derivation: capacitance of two wires

Substituting the equations for R+ and R−

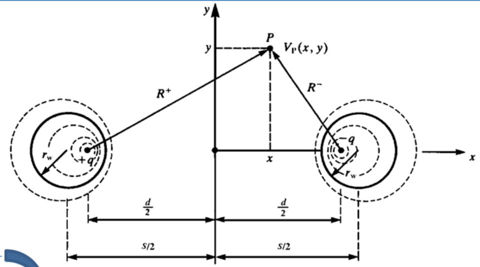
$$R^+ = \sqrt{(x+d/2)^2+y^2} \quad R^- = \sqrt{(x-d/2)^2+y^2}$$

Gives $\dfrac{(x-d/2)^2+y^2}{(x+d/2)^2+y^2} = K^2$

Expanding $x^2 + xd\dfrac{(K^2+1)}{(K^2-1)} + \left(\dfrac{d}{2}\right)^2 + y^2 = 0$

The equation of a circle located at x = -s/2 and having a radius of r_w $\left(x+\dfrac{s}{2}\right)^2 + y^2 = r_w^2$

Solving equations $\dfrac{s}{2} = \dfrac{d}{2}\,\dfrac{K^2+1}{K^2-1} \quad r_w = \dfrac{Kd}{K^2-1}$ $\Leftrightarrow$ $\dfrac{d}{2} = \sqrt{\left(\dfrac{s}{2}\right)^2 - r_w^2} \quad K = \dfrac{s}{2r_w} \pm \sqrt{\left(\dfrac{s}{2r_w}\right)^2 - 1}$



$$V_P = \frac{q}{2\pi\varepsilon}\ln\left[\frac{s}{2r_w} \pm \sqrt{\left(\frac{s}{2r_w}\right)^2 - 1}\right]$$

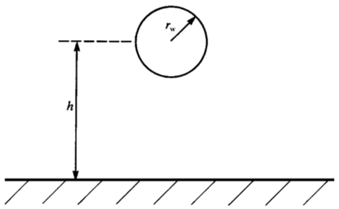
$$V = \frac{q}{\pi\varepsilon}\ln\left[\frac{s}{2r_w} + \sqrt{\left(\frac{s}{2r_w}\right)^2 - 1}\right] = 2V_P$$

the per-unit-length capacitance

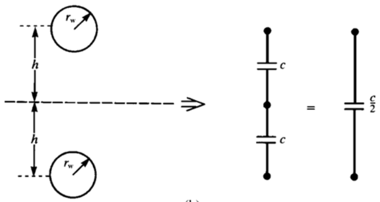
$$c = \frac{q}{V} = \frac{\pi\varepsilon}{\ln\left[\dfrac{s}{2r_w} + \sqrt{\left(\dfrac{s}{2r_w}\right)^2 - 1}\right]} \quad \text{(F/m)}$$

For one certain r_w, there are two symmetrical equipotential contours with difference values

Derivation:One wire above ground



(a)



(b)

$$c_{\text{two-wire}} = \frac{c_{\text{one wire above ground}}}{2}$$

The capacitance of one wire above an infinite, perfectly conducting plane becomes

$$c = \frac{2\pi\varepsilon}{\cosh^{-1}\left(\frac{h}{r_w}\right)} \quad \text{(F/m)}$$

approximately, for $h \gg r_w$:

$$c \cong \frac{2\pi\varepsilon}{\ln\left(\frac{2h}{r_w}\right)} \quad \text{(F/m)}$$

$$\begin{aligned} l &= \mu\varepsilon\ c^{-1} \\ &= \frac{\mu}{2\pi}\cosh^{-1}\left(\frac{h}{r_w}\right) \quad \text{(H/m)} \\ &\cong \frac{\mu}{2\pi}\ln\left(\frac{2h}{r_w}\right) \end{aligned}$$

$$\begin{aligned} g &= \frac{\sigma}{\varepsilon}c \\ &= \frac{2\pi\sigma}{\cosh^{-1}\left(\frac{h}{r_w}\right)} \quad \text{(S/m)} \\ &\cong \frac{2\pi\sigma}{\ln\left(\frac{2h}{r_w}\right)} \end{aligned}$$

Thank you !