

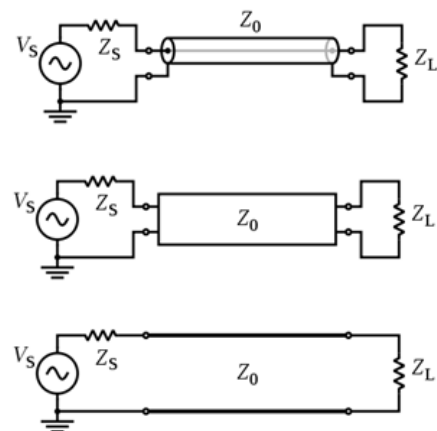
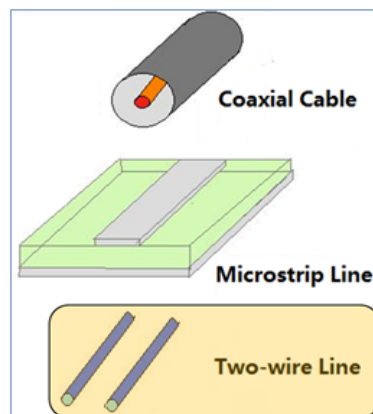
Lecture 2: Telegrapher's Equation for two-conductor system

Yan-zhao XIE

Xi'an Jiaotong University

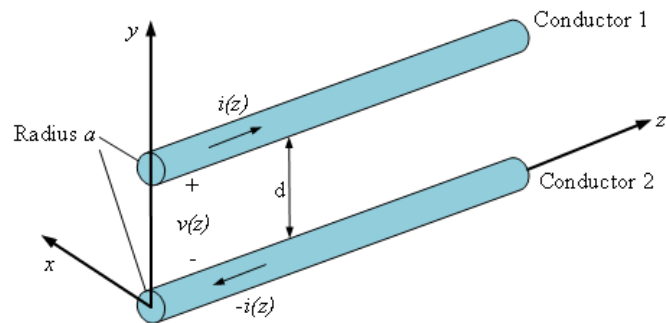
2020.09.15

Configuration of two-conductor TL



A transmission line usually connects a source on one end to a load on the other.

- Line
- Conductor
- Wire
- Cable
- Bundle
- Sheath



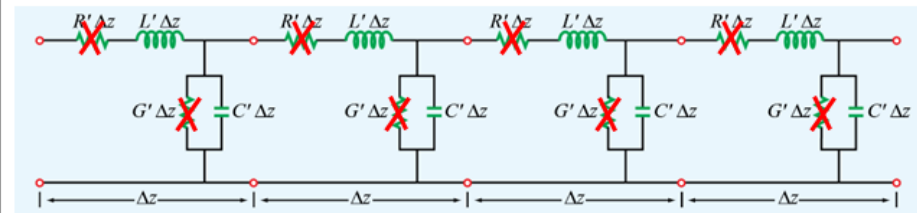
How to build the **relation** between the voltage and current?



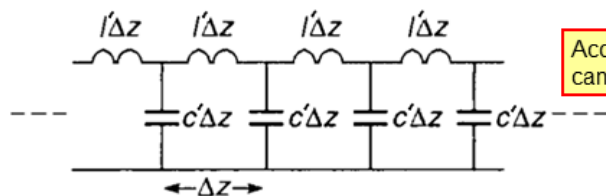
To develop general equations that describe the voltage across and current carried by the transmission line as a function of time t and spatial position z .



Differential sections with each section length of Δz

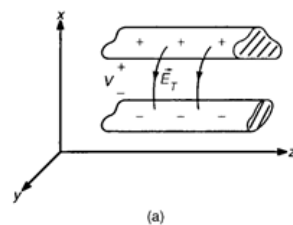


Each section is represented by an equivalent circuit (lossy)

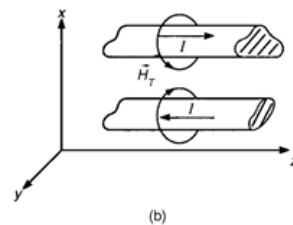


Accordingly, the lossless line can be modeled in this way.

Each section is represented by an equivalent circuit (lossless)

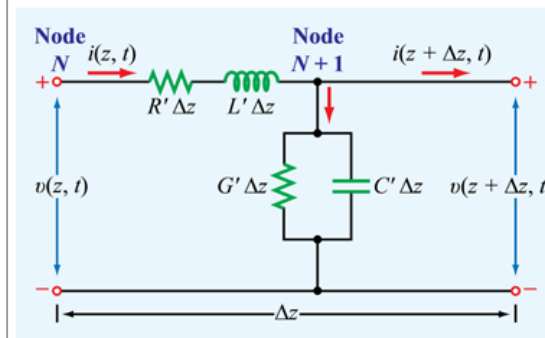


(a) Capacitance per unit length



(b) Inductance per unit length

Determination of per unit length parameters will be discussed further.



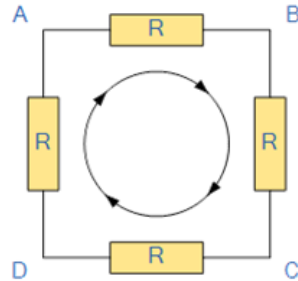
- The quantities $v(z, t)$ and $i(z, t)$ denote the **instantaneous voltage** and **current** at the left end of the differential section (**node N**);
- $v(z + \Delta z, t)$ and $i(z + \Delta z, t)$ denote the **instantaneous quantities** at **node (N+1)**, located at the right end of the section.

Equivalent circuit of a two-conductor transmission line of differential length Δz

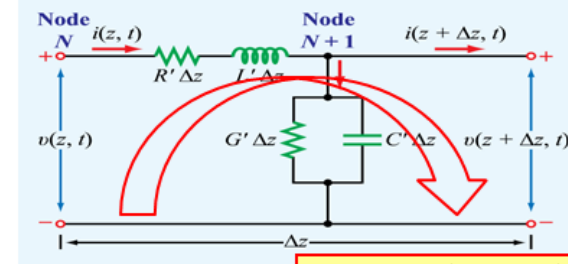


Kirchhoff's voltage law (KVL)

The sum of all the Voltage Drops around the loop is equal to Zero



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$



KVL: The directed sum of the voltages around any closed network is zero.

- The voltage drop across the series resistance $R'\Delta z$ and inductance $L'\Delta z$:

$$v(z, t) - R'\Delta z i(z, t) - L'\Delta z \frac{\partial i(z, t)}{\partial t} - v(z + \Delta z, t) = 0.$$

- Upon dividing all terms by Δz and re-arranging them, we

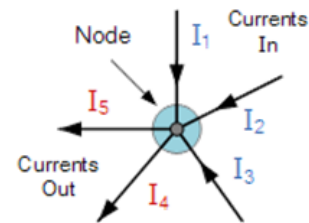
$$-\left[\frac{v(z + \Delta z, t) - v(z, t)}{\Delta z} \right] = R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t}.$$

$$\Delta z \rightarrow 0, \quad -\frac{\partial v(z, t)}{\partial z} = R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t}$$

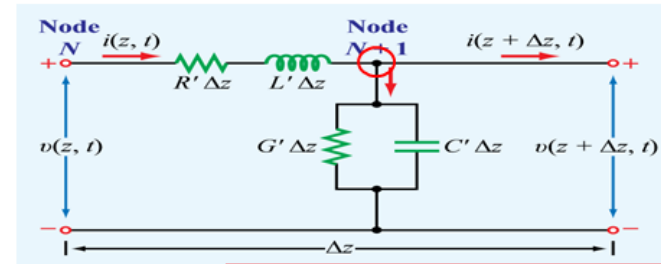


Kirchhoff's current law (KCL)

Currents Entering the Node Equals Currents Leaving the Node



$$I_1 + I_2 + I_3 + (-I_4 - I_5) = 0$$



KCL: The sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

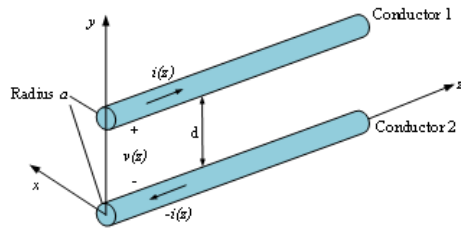
- The current drawn at node (N+1) by the parallel conductance $G'\Delta z$ and capacitance $C'\Delta z$:

$$i(z, t) - G'\Delta z v(z + \Delta z, t) - C'\Delta z \frac{\partial v(z + \Delta z, t)}{\partial t} - i(z + \Delta z, t) = 0.$$

- dividing all terms by Δz and taking the limit $\Delta z \rightarrow 0$,

$$-\frac{\partial i(z, t)}{\partial z} = G' v(z, t) + C' \frac{\partial v(z, t)}{\partial t}$$





■ Telegrapher's Equations
in time domain

$$-\frac{\partial v(z, t)}{\partial z} = R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t}$$

$$-\frac{\partial i(z, t)}{\partial z} = G' v(z, t) + C' \frac{\partial v(z, t)}{\partial t}$$

■ Telegrapher's Equations
in frequency domain

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L') \tilde{I}(z),$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C') \tilde{V}(z).$$



Solicit the comments and questions~



quotes

*A journey of a thousand miles begins
with single step.*



Thank you again !

