Lecture 13: Numerical Time-domain Solutions of Transmission Lines: MOC

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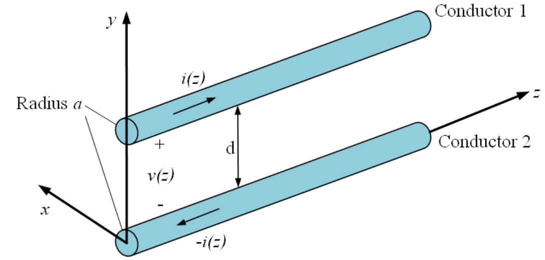
Recall: telegrapher's equations



- Telegrapher's Equations in time domain (for lossless TLs)

$$\frac{\partial V(z,t)}{\partial z} = -L'\frac{\partial I(z,t)}{\partial t}$$
$$\frac{\partial I(z,t)}{\partial z} = -C'\frac{\partial V(z,t)}{\partial t}$$

$$V(z,t) = V^{+}\left(t-\frac{z}{v}\right) + V^{-}\left(t+\frac{z}{v}\right)$$

$$I(z,t) = I^{+}\left(t-\frac{z}{v}\right) + I^{-}\left(t+\frac{z}{v}\right) = \frac{1}{Z_C}V^{+}\left(t-\frac{z}{v}\right) - \frac{1}{Z_C}V^{-}\left(t+\frac{z}{v}\right)$$

Method of Characteristics (MOC)

The method of characteristics seeks to transform the partial differential equations of the transmission line into ordinary differential equations that are easily integrable.

The differential changes in the line voltage and current are:

$$dV(z,t) = \frac{\partial V(z,t)}{\partial z}dz + \frac{\partial V(z,t)}{\partial t}dt$$
$$dI(z,t) = \frac{\partial I(z,t)}{\partial z}dz + \frac{\partial I(z,t)}{\partial t}dt$$

$$\frac{\partial V(z,t)}{\boxed{\partial z}} = -L'\frac{\partial I(z,t)}{\boxed{\partial t}}$$
$$\frac{\partial I(z,t)}{\boxed{\partial z}} = -C'\frac{\partial V(z,t)}{\boxed{\partial t}}$$

Substituting the telegrapher's equations,

$$dV(z,t) = \left(-L'\frac{\partial I(z,t)}{\partial t}\right)dz + \frac{\partial V(z,t)}{\partial t}dt$$

$$dI(z,t) = \left(-C'\frac{\partial V(z,t)}{\partial t}\right)dz + \frac{\partial I(z,t)}{\partial t}dt$$

Along the characteristic curves, we can transform the partial differential equations into ordinary differential equations by introducing the relation of dz/dt.

Characteristic curves

Partial differential equations

$$\frac{\partial V(z,t)}{\boxed{\partial z}} = -L'\frac{\partial I(z,t)}{\boxed{\partial t}}$$

$$\frac{\partial I(z,t)}{\boxed{\partial z}} = -C'\frac{\partial V(z,t)}{\boxed{\partial t}}$$

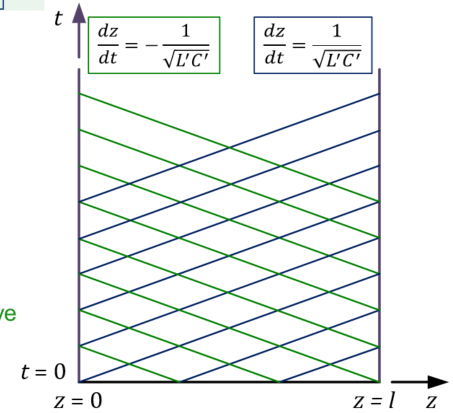
We define the characteristic curves in the $z-t$ plane.

$$\frac{dz}{dt} = \frac{1}{\sqrt{L'C'}}$$

forward characteristic curve

$$\frac{dz}{dt} = -\frac{1}{\sqrt{L'C'}}$$

backward characteristic curve



Along the forward characteristic curves, we can substitute dz with $1/\sqrt{L'C'}dt$.

$$dV(z,t) = \left(-Z_C\frac{\partial I(z,t)}{\partial t} + \frac{\partial V(z,t)}{\partial t}\right)dt$$

$$dI(z,t) = \left(-\frac{1}{Z_C}\frac{\partial V(z,t)}{\partial t} + \frac{\partial I(z,t)}{\partial t}\right)dt$$

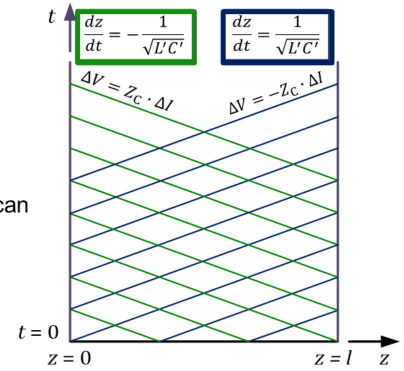
$$\Rightarrow \quad dV(z,t) + Z_C \cdot dI(z,t) = 0$$

Along the backward characteristic curves, we can substitute dz with $-1/\sqrt{L'C'}dt$.

$$dV(z,t) = \left(Z_C\frac{\partial I(z,t)}{\partial t} + \frac{\partial V(z,t)}{\partial t}\right)dt$$

$$dI(z,t) = \left(\frac{1}{Z_C}\frac{\partial V(z,t)}{\partial t} + \frac{\partial I(z,t)}{\partial t}\right)dt$$

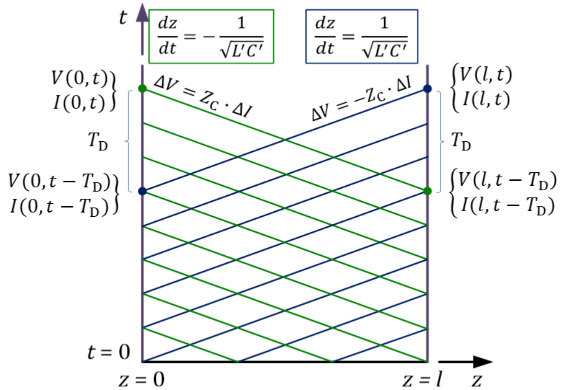
$$\Rightarrow \quad dV(z,t) - Z_C \cdot dI(z,t) = 0$$



The difference between two voltages at two points on a given characteristic curve is related to the difference between two currents on the same characteristic curve.

$$[V(l,t) - V(0,t-T_D)] = -Z_C[I(l,t) - I(0,t-T_D)]$$

$$[V(0,t) - V(l,t-T_D)] = +Z_C[I(0,t) - I(l,t-T_D)]$$



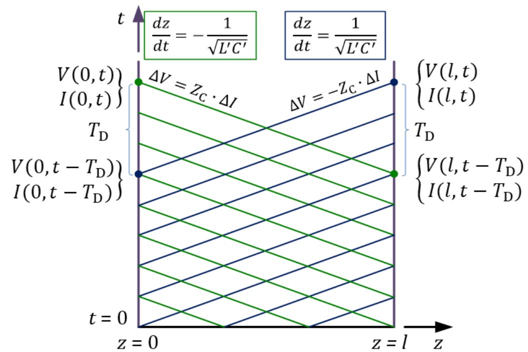
$$T_D = \frac{l}{v}$$

We can rewrite as a two-port model:

$$[V(0,t) - Z_C \cdot I(0,t)] = [V(l,t-T_D) - Z_C \cdot I(l,t-T_D)]$$

$$[V(l,t) + Z_C \cdot I(l,t)] = [V(0,t-T_D) + Z_C \cdot I(0,t-T_D)]$$

Hence, the voltage and current at one end of the line at some present time can be found from the voltage and current at the other end of the line at one time delay earlier.



$$T_D = \frac{l}{v}$$

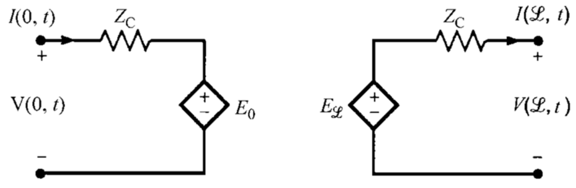
We can get a recursive form:

$$V(0,t) = Z_C \cdot I(0,t) + E_0(l, t - T_D)$$
$$V(l,t) = -Z_C \cdot I(l,t) + E_l(0, t - T_D)$$

where

$$E_0(l, t - T_D) = V(l, t - T_D) - Z_C \cdot I(l, t - T_D)$$
$$E_l(0, t - T_D) = V(0, t - T_D) + Z_C \cdot I(0, t - T_D)$$

The controlled source $E_l(0, t - T_D)$ is produced by the voltage and current at the input to the line at a time equal to a one-way transit delay (T_D) earlier than the present time. Similarly, $E_0(l, t - T_D)$ is produced by the voltage and current at the line output at a time equal to a one-way transit delay earlier than the present time..



$E_0 = V(\mathcal{L}, t\text{-}T_D) - Z_C I(\mathcal{L}, t\text{-}T_D)$ $\quad E_{\mathcal{L}} = V(0, t\text{-}T_D) + Z_C I(0, t\text{-}T_D)$

- F. H. Branin, Jr., Transient analysis of lossless transmission lines, Proceedings of IEEE, 55, 2012-2013, 1967

Thank you!

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