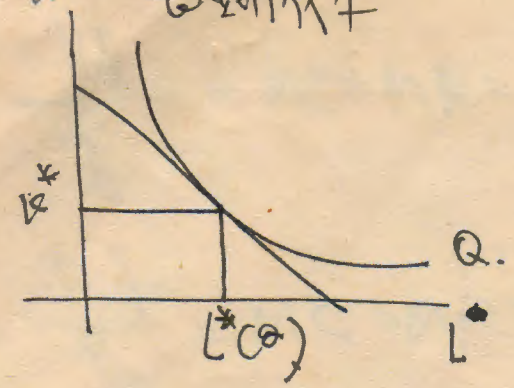
长期成本



$L^*(Q, w_L, w_K)$.

$k^*(Q, w_L, w_K)$.

$TC = w_L L^*(Q, w_L, w_K) + w_K k^*(Q, w_L, w_K)$

$$\begin{cases} \dfrac{w_L}{f_L(L^*, k^*)} = \dfrac{w_K}{f_K(L^*, k^*)} = MC \\ f(L^*, k^*) = Q. \end{cases}$$

Long run

$$MC = w_L \frac{dL^*(Q)}{dQ} + w_K \frac{dk^*(Q)}{dQ}.$$

$$\left(\text{由} \quad f_L \frac{dL^*}{dQ} + f_K \frac{dk^*}{dQ} = 1 \right)$$

$$\frac{dk^*}{dQ} = \frac{1 - f_L \frac{dL^*}{dQ}}{f_K}$$

$$= w_L \frac{dL^*}{dQ} + \frac{w_K}{f_K} - \frac{w_K f_L}{f_K} \cdot \frac{dL^*}{dQ}$$

$$= \left(w_L - \frac{w_K f_L}{f_K}\right) \frac{dL^*}{dQ} + \frac{w_K}{f_K}$$

$$= \frac{w_K}{f_K} = \frac{w_L}{f_L} = \lambda$$

short run MC

短期

$$\begin{cases} C = w_L L^0 \\ f(L^0, \bar{k}) = Q. \end{cases}$$

$f_L dL^0 = dQ$.

$\dfrac{dL}{dQ} = \dfrac{1}{f_L}$

$\dfrac{dC}{dQ} = w_L \dfrac{dL}{dQ} = \dfrac{w_L}{f_L} = MC$

L.R.

$\begin{cases} \min w_L L + w_K k \\ s.t. f(L, k) = Q \end{cases}$

$w_L L + w_K k + \lambda[Q - f(L, k)]$

長期成本(續)

長期平均成本 $LAC = \dfrac{w_L L^*(Q) + w_K K^*(Q)}{f(L^*(Q), K^*(Q))}$

$$\frac{dLAC}{dQ} = \frac{w_L \frac{dL^*}{dQ} + w_K \frac{dK^*}{dQ}}{f} - \frac{w_L L^* + w_K K^*}{f^2}$$

$$= \frac{MC}{f} - \frac{TC}{f^2} \left(= \frac{MC \cdot Q - TC}{Q^2} \right) = \frac{1}{Q}(MC - AC)$$

(當 AC 上昇時 MC > AC；AC 下降時 MC < AC)

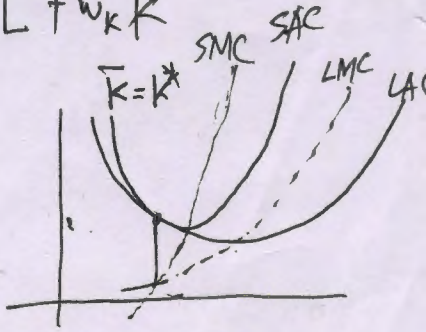
短期平均成本

前述 $C = w_L L^\circ$ 為 K 固定在 $\bar{K}$ 時的變動成本，和平均變動成本

$AVC = \dfrac{w_L L^\circ}{f(L^\circ, \bar{K})}$ 且 $\bar{K}$ 為固定 $F = w_K \bar{K}$ 為固定成本

含固定成本的短期總成本為 $F + w_L L^\circ = w_L L^\circ + w_K \bar{K}$

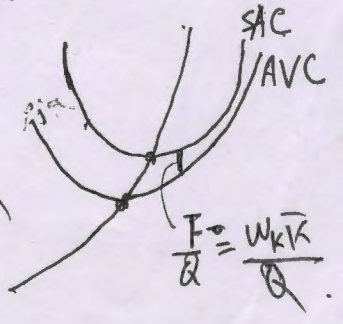
短期平均總成本為



$$\frac{F + w_L L^\circ}{f(L^\circ, \bar{K})} \equiv SAC$$

$$\frac{dSAC}{dQ} = \frac{d}{dQ} \frac{w_L L^\circ + w_K \bar{K}}{f(L^\circ, \bar{K})} = \frac{w \cdot \frac{dL^\circ}{dQ}}{f} - \frac{w_L L^\circ + w_K \bar{K}}{f^2}$$

(固定為常數)



$$= \frac{w_L / f_L}{f} - \frac{w_L L^\circ + w_K \bar{K}}{f^2}$$

當 $\bar{K} = K^*$ 時 $\dfrac{dLAC}{dQ}$