

## Lecture 21: Analytic Iterative Approach to Crosstalk Analysis of Multi-conductor Transmission Lines

Yan-zhao XIE

Xi'an Jiaotong University

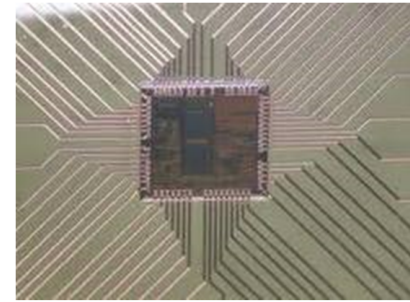
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## INTRODUCTION



Cable bundle

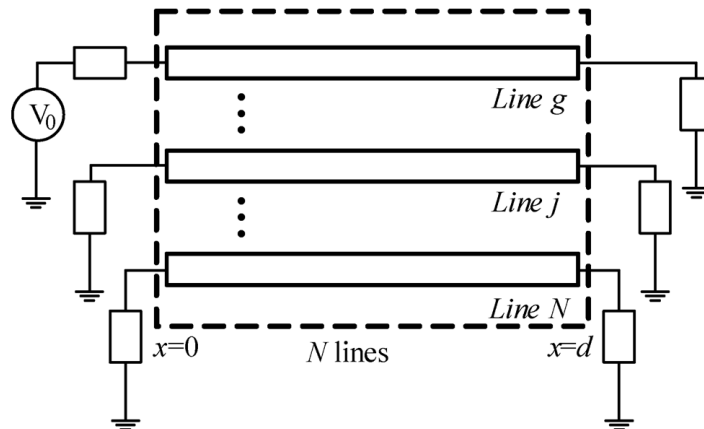


Chip interconnect

With large number of conductors



## INTRODUCTION



Configuration of MTLs to be investigated.



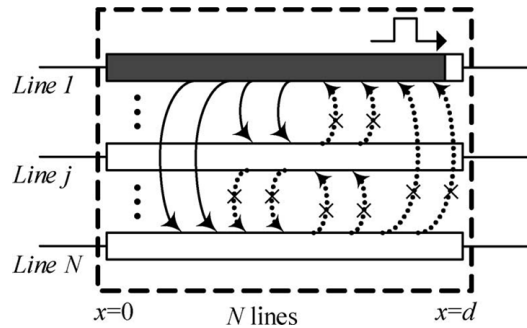
## INTRODUCTION

- Most of the existing methods use some form of decoupling algorithm such as a similarity transformation to convert the set of coupled partial differential equations describing the lines into a set decoupled single modal equations. Subsequently, the line voltages and currents are obtained as a linear combination of the modal variables which have the simple form of exponential functions.
- However, the use of the decoupling matrix and transformation to mode quantities and viceversa will lead to the simulation with low efficiency, especially for a large number of coupled conductors.



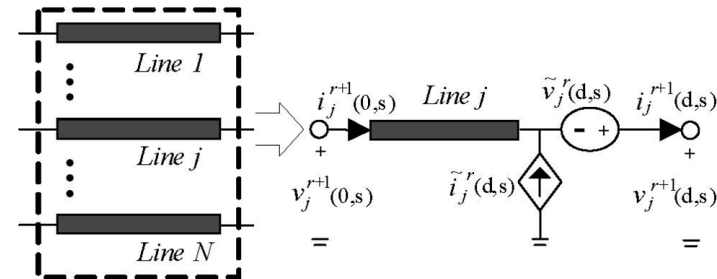
## INTRODUCTION

- C. R. Paul provided an analytical method to get the symbolic solution of transmission line equations under the weak coupling assumption (**WCA**) for a three-conductor lossless line in a nonhomogeneous medium.



## INTRODUCTION

- N. M. Nakhla *et al* proposed a numerical method based on the Waveform Relaxation and Transverse Partitioning (**WRTP**) technique. The single-ended WR-TP method avoids the need for the decoupling matrix by reducing the coupled simulation problem into a series of simulation steps, leading of the capability to handling a large number of coupled lines.



## INTRODUCTION

- A novel approach using the distributed analytical representation and an iterative technique (**DARIT**) was proposed, that makes use of the strong points of the **WCA** method and the **WR-TP** method.
- It starts with the initial state of coupled transmission lines under WCA and employs a Jacobi iteration algorithm which is the same as in the WR-TP to update the state of the transmission lines based on the state of the lines at the previous iteration.



## OUTLINE OF THE PROPOSED APPROACH

$$\begin{aligned} \frac{d\mathbf{V}(x,s)}{dx} + \mathbf{Z}'(s)\mathbf{I}(x,s) &= 0, \\ \frac{d\mathbf{I}(x,s)}{dx} + \mathbf{Y}'(s)\mathbf{V}(x,s) &= 0, \end{aligned} \quad \Rightarrow \quad \begin{aligned} \frac{dv_j(x,s)}{dx} + z'_{jj}(s)i_j(x,s) &= -\sum_{k=1, k \neq j}^N z'_{jk}(s)i_k(x,s), \\ \frac{di_j(x,s)}{dx} + y'_{jj}(s)v_j(x,s) &= -\sum_{k=1, k \neq j}^N y'_{jk}(s)v_k(x,s). \end{aligned}$$

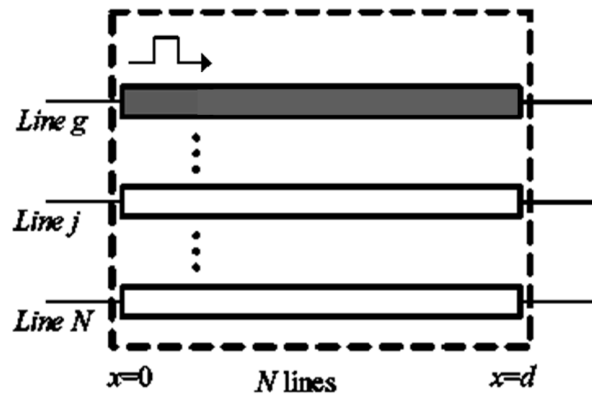
$$\begin{aligned} \frac{dv_j^{(r+1)}(x,s)}{dx} + z'_{jj}(s)i_j^{(r+1)}(x,s) &= e_j^{(r)}(x,s), \\ \frac{di_j^{(r+1)}(x,s)}{dx} + y'_{jj}(s)v_j^{(r+1)}(x,s) &= q_j^{(r)}(x,s), \end{aligned}$$

KEY  
POINT

the proposed algorithm is, that we can adopt these two terms as new *distributed source terms* for the  $(r+1)$ th iteration. In fact,  $e_j^{(r)}(x,s)$  and  $q_j^{(r)}(x,s)$  at  $j$ th iteration are interpreted as equivalent distributed p.u.l. virtual voltage and current sources located along the length of the line  $j$ , rather than lumped virtual sources at the end of each line or segment.



## DERIVATION OF THE PROPOSED ALGORITHM

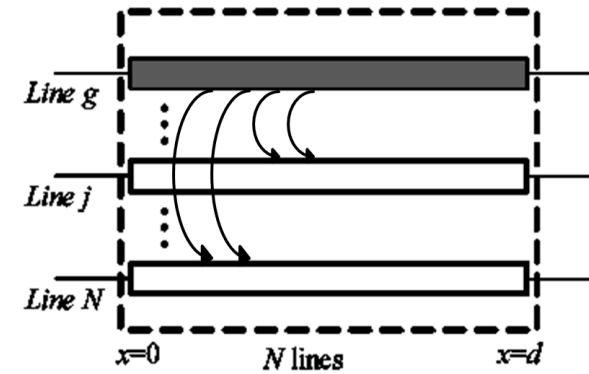


Iteration 1

9



## DERIVATION OF THE PROPOSED ALGORITHM

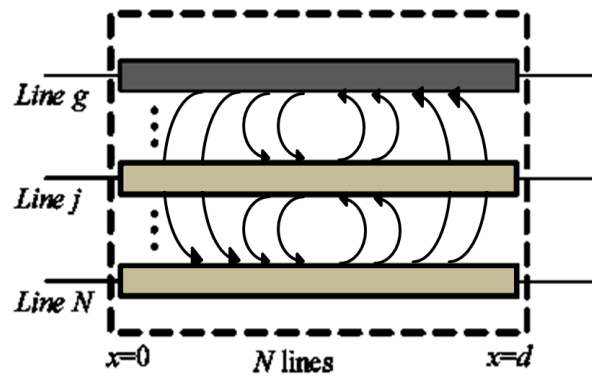


Iteration 2

10



## DERIVATION OF THE PROPOSED ALGORITHM



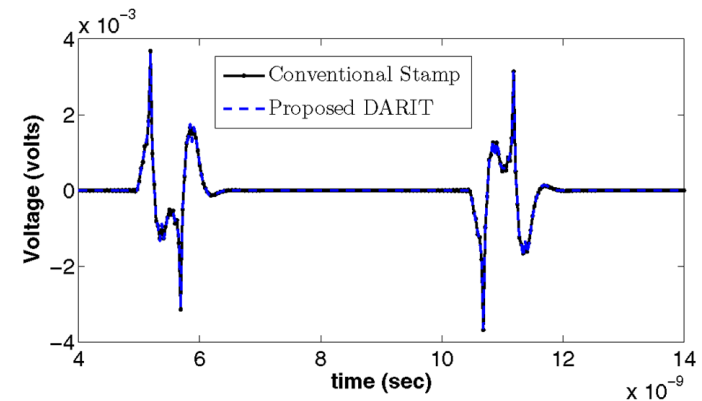
Iteration 3

11



## VALIDATION

Example 1: Symmetrical Coupled Lines

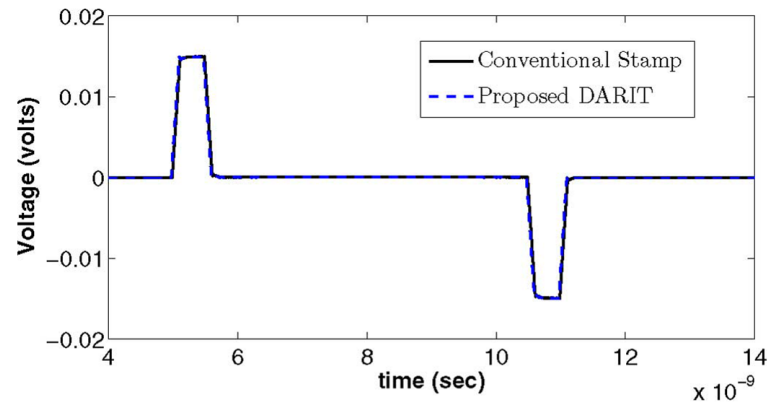


Near-end crosstalk on line #2 for Example 1 (Symmetrical lines).



## VALIDATION

Example 1: Unsymmetrical Coupled Lines

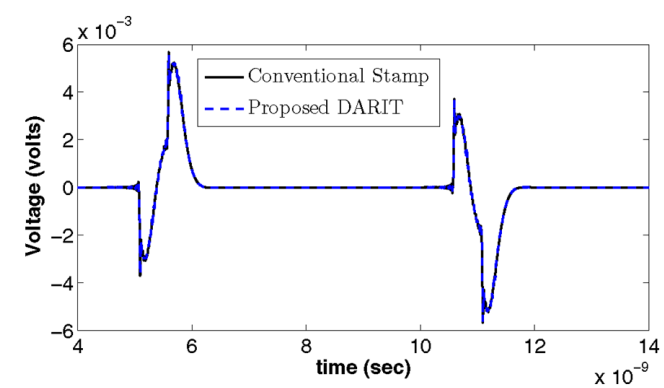


Near-end crosstalk on line #2 for Example 1 (Unsymmetrical lines).



## VALIDATION

Example 2: Loads



Far-end crosstalk of line #3 for Example 2 (series RLC Loads).



## VALIDATION

Example 3: Coupling Factor (CF)

$$CF = \max_{\substack{i,j \\ i \neq j}} \sqrt{l'_{ij}{}^2 / (l'_{ii} l'_{jj})},$$

where  $l'_{ij}$  is the  $(i, j)^{\text{th}}$  cell of the  $L'$  matrix.

Weak coupling

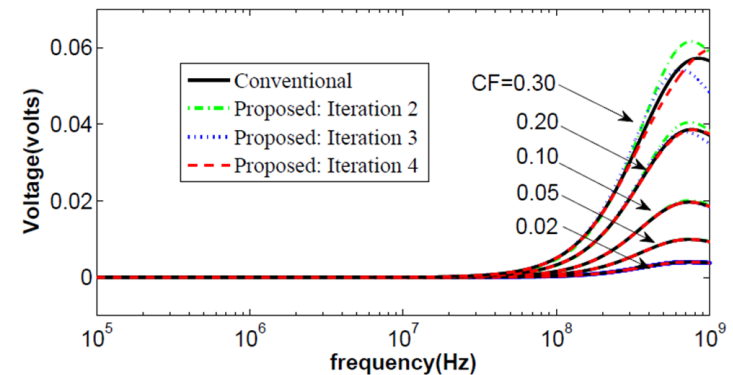
CF	Iteration 2		Iteration 3		Iteration 4	
	$\epsilon$	time(s)	$\epsilon$	time(s)	$\epsilon$	time(s)
0.02	6.58e-3	2.08	2.77e-4	4.87	4.68e-6	22.30
0.05	1.69e-2	1.99	1.80e-3	4.43	6.49e-5	22.58
0.10	3.60e-2	2.00	7.74e-3	4.46	4.10e-4	22.71
0.20	8.31e-2	2.02	3.68e-2	4.73	1.63e-3	22.80
0.30	1.42e-1	2.03	9.77e-2	4.69	1.06e-2	22.73

Strong coupling



## VALIDATION

Example 3: Coupling Factor (CF)

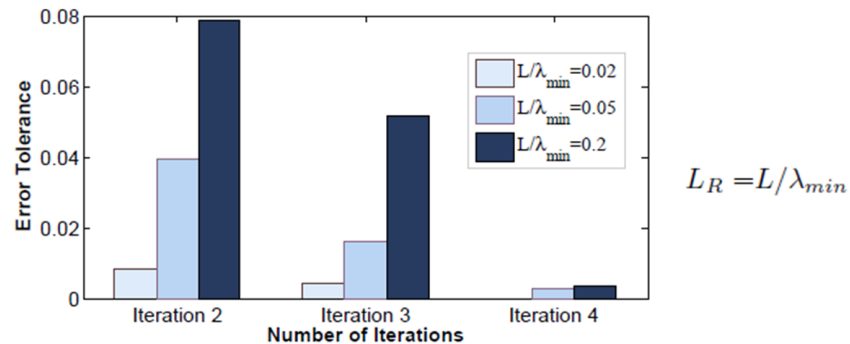


The frequency response of line #2 for an MTL with coupling factors (CF) values



## VALIDATION

Example 4: Length of the Line



Accuracy analysis for different iterations and for different values of the lines electrical length parameter  $L_R$

## Comments and Conclusions

- two major factors affecting the error tolerance or the convergence speed are the coupling coefficient and the length of the line
  - The weaker the coupling, the faster the convergence.
  - The longer the line, the slower the convergence.
- Generally, the accuracies after Iteration 4 will in fact suffice since the values of typical coupling factors are between 0.05 and 0.2 for most real interconnect cases.
- Convergence was quickly reached in 3 iterations or less for most cases, although started with initial values of zero which were far from the actual responses. This conclusion was further validated by the rigorous conceptual and theoretical explanation of the convergence speed in other article.

## Comments and Conclusions

- It significantly extends the applicability compared to WCA and provides an iterative approach to analyze crosstalk between lines in an analytical fashion.
- It has the capability of handling a very large number of lines efficiently and it is suitable for parallel implementation.
- It is flexible, allowing the user to make a compromise between computational cost and accuracy by selecting the number of iterations.

Thank you!