Lecture 6: Reflection Coefficient

Yan-zhao XIE

Xi'an Jiaotong University

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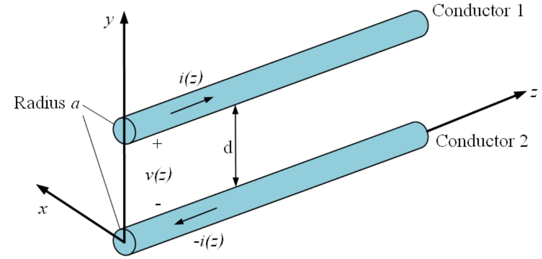
Recall: General solution



telegrapher's equation

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$
$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\,\tilde{V}(z).$$

traveling wave solutions

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

propagation constant

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \sqrt{Z' \cdot Y'}$$

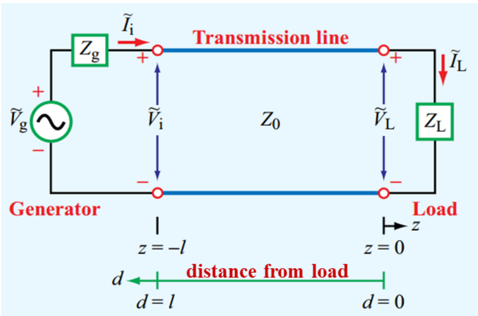
Reflection coefficient

In order to incorporate the ***source and load condition*** (boundary condition) into the general wave solution, we define a ***voltage reflection coefficient*** $\Gamma(z)$ as the ratio of the reflected and incident voltage waves

$$\Gamma(z) \stackrel{\text{def}}{=} \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^-}{V_0^+} e^{2\gamma z}$$

How to evaluate V_0^-/V_0^+ ?

One notices the direction of current: $\Gamma(z) = -\frac{I_0^-}{I_0^+} e^{2\gamma z}$



Considering the transmission line in the context of the complete circuit, including:

A generator circuit at its input terminals ($z = -l$)

A l-length TL($l \le z \le 0$)

A load terminated at the output ($z = 0$)

Reflection coefficient

The ratio of the reflected and incident voltage waves is known as the voltage reflection coefficient $\Gamma(z)$.

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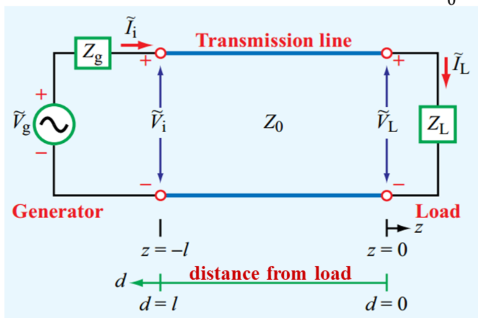
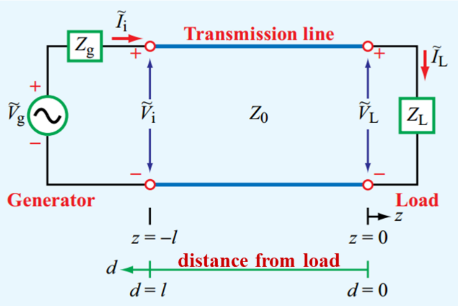


In this case, we define the coordinate:

$$-l \le z \le 0$$

generator at the input ($z = -l$);
load at the output ($z = 0$).

Reflection coefficient



load impedance

$$Z_L = \frac{\tilde{V}(0)}{\tilde{I}(0)} = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0$$

$$\frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Therefore, the reflection coefficient along the line is:

$$\Gamma(z) = \frac{V_0^-}{V_0^+} e^{2\gamma z} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{2\gamma z} = \Gamma_L e^{2\gamma z}$$

Especially, the reflection coefficient at the load :

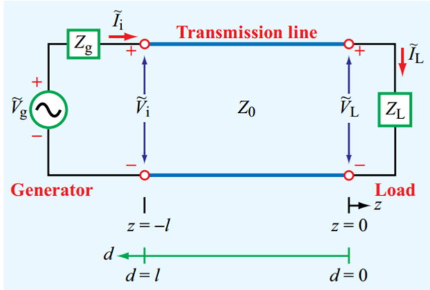
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Γ_L for matched, open, short circuit

Reflection coefficient at the load

$$\Gamma_L \overset{\text{def}}{=} \frac{Z_L - Z_0}{Z_L + Z_0}$$

A load is said to be **matched** to a transmission line if $Z_L = Z_0$ because there will be no reflection by the load ($\Gamma_L = 0$ and $V_0^- = 0$).



open circuit ($Z_L = \infty$) ,$\Gamma_L = 1$ and $V_0^- = V_0^+$

short circuit ($Z_L = 0$) ,$\Gamma_L = -1$ and $V_0^- = -V_0^+$

matched ($Z_L = Z_0$) ,$\Gamma_L = 0$ and $V_0^- = 0$

Γ_L for various types of load

In general, Z_L is a complex quantity $Z_L = R + jX$, therefore, $\Gamma_L = |\Gamma_L| e^{j\theta_r}$

Load	$\lvert\Gamma\rvert$	θ_r
Z_0 $Z_L = (r + jx)Z_0$	$\left[\frac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\frac{x}{r-1}\right) - \tan^{-1}\left(\frac{x}{r+1}\right)$
Z_0 Z_0	0 (no reflection)	irrelevant
Z_0 (short)	1	$\pm 180°$ (phase opposition)
Z_0 (open)	1	0 (in-phase)
Z_0 $jX = j\omega L$	1	$\pm 180° - 2\tan^{-1} x$
Z_0 $jX = \frac{-j}{\omega C}$	1	$\pm 180° + 2\tan^{-1} x$

$z_L = Z_L/Z_0 = (R + jX)/Z_0 = r + jx$

$r = R/Z_0$ **and** $x = X/Z_0$ **are the real and imaginary parts of** z_L**, respectively.**

Thank you!