Date / /

1-1-2 证明：$dq = \rho_0 4\pi r'^2 dr'$ 取厚度为 dr' 的带电薄壳

$$d\vec{E}_1 = \frac{\rho_0 4\pi r'^2 dr'}{4\pi\varepsilon_0 r^2}\vec{e}_r$$

$$\vec{E} = \vec{e}_r \frac{\rho_0}{\varepsilon_0 r^2}\int_0^r r'^2 dr' = \frac{\rho r}{3\varepsilon_0}\vec{e}_r$$

1-1-4 由图，取 $x=0$ 平面为电位参考点，

$$\varphi(x,y) = \frac{\tau}{2\pi\varepsilon_0}\ln\frac{\sqrt{(x+d)^2+y^2}}{\sqrt{(x-d)^2+y^2}}$$

由 $\varphi(x,y)=k$ 可知 $\frac{\sqrt{(x+d)^2+y^2}}{\sqrt{(x-d)^2+y^2}} = const$

$$\therefore \left(x-\frac{c^2+1}{c^2-1}\right)^2+y^2 = \left(\frac{2dc}{c^2-1}\right)^2$$

1-2-1. 由高斯定理 $\vec{E} = \frac{q}{4\pi r^2}\vec{e}_r$

$$\vec{P} = (\varepsilon-\varepsilon_0)\vec{E} = \frac{(\varepsilon-\varepsilon_0)q}{4\pi r^2\varepsilon}\vec{e}_r$$

$$\sigma_n = \vec{P}\cdot\vec{e}_n\Big|_{r=a} = \frac{(\varepsilon-\varepsilon_0)q}{4\pi\varepsilon r^2}$$

1-2-3. $E = \frac{\tau}{2\pi\varepsilon r}$

$$U = \int_a^b E\,dr = \frac{\tau}{2\pi\varepsilon}\ln\frac{b}{a}$$

$$\frac{dU}{da} = \frac{\tau}{2\pi\varepsilon a}\left[\ln\left(\frac{b}{a}\right)-1\right] = 0$$

即 $a = \frac{b}{e} = 0.736\text{cm}$ $U_{max} = \frac{\tau}{2\pi\varepsilon}\ln\frac{b}{a} = 1.47\times10^5\,V$

NOTES/备注