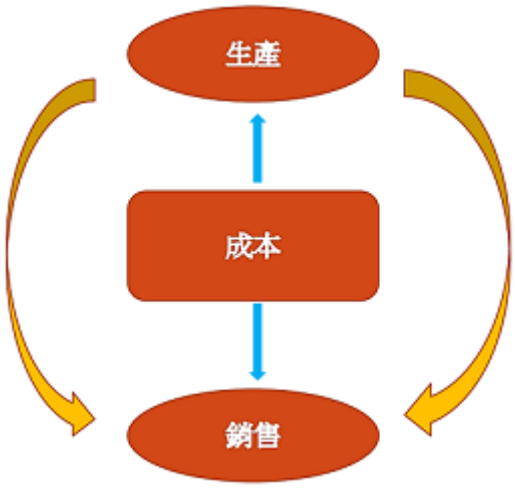
Production

Firm Theory

學習的目標：瞭解供給曲線背後所隱含的廠商決策

廠商為了獲取利潤，從生產要素市場購買生產要素，支付成本。生產出來的產品，在產品市場出售，獲得收益。



- A firm's profit is the difference between its revenue and its cost.
    - $\pi = TR - TC$
- Revenue is what it earns from selling a good.
    - TR = P(Q) * Q(L, K)
- Costs are what it pays for labor, materials, and other inputs.
    - TC = TC(Q)

廠商以追求利潤極大化為目標，成本大小影響利潤高低。

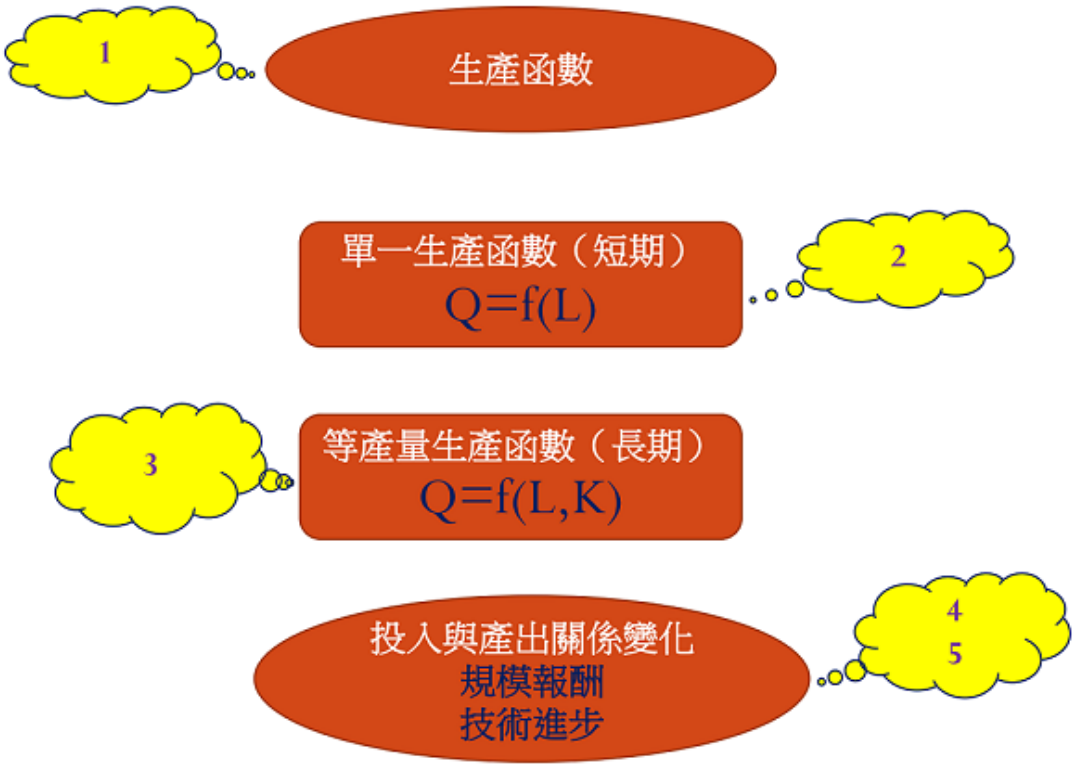
- 所有的廠商，不管規模大小，都是為了要賺錢。
- 所有的廠商，不管規模大小，當它們生產商品與服務時，都會產生成本。
- 成本是廠商生產決策與訂價決策的關鍵因素。

總收益減總成本為廠商的利潤，亦即，

- 利潤 ＝ 總收益－總成本
    - 廠商從出售產品所收到的金額為總收益
    - 總收益等於廠商的銷售量乘以銷售價格。
    - 廠商購買所有生產投入的支出為總成本。
    - 總成本包括生產商品與服務的所有機會成本。

富士康：製造業新勞工-機器人

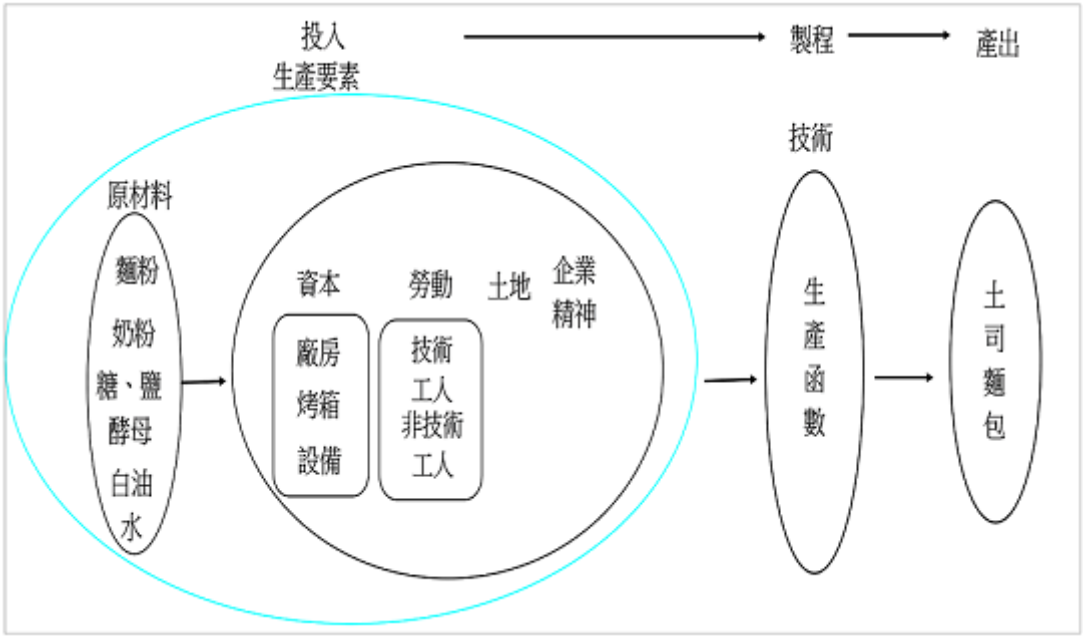
- 焦點：要用機器人取代工人嗎?我會失業嗎？
- 生產上的決策
    - 用人、用機器人？
    - 用多少數量？
- (1)生產決策要先瞭解生產函數
    - 投入和產出間的關係
    - 生產力的衡量與特性
- (2)生產決策要瞭解生產關係的變化
    - 規模報酬（生產產量多寡）
    - 技術進步（技術水準高低）
- 生產決策和成本有關
- 生產決策和銷售有關



1. Production

(1) 生產：將投入轉成產出

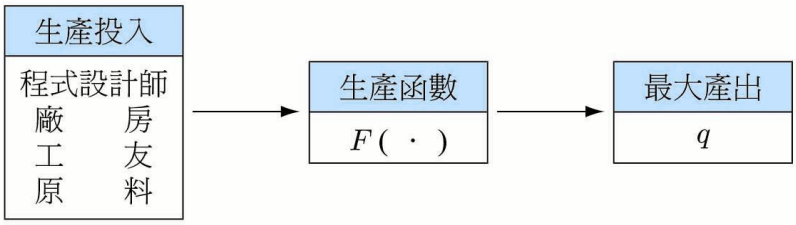
結合各種生產要素，產出最大數量的商品與勞務



(2) 生產函數：描述投入與產出之間的關係。

在某種生產技術下，運用不同數量的生產要素投入所能產出的最大商品勞務數量。 $Q = f(X_1, X_2, \ldots, X_n)$

圖 14-1　生產函數



(3) 投入與產出生產要素

a. 何謂投入(inputs) = 生產要素？

廠商用來生產產品或服務的資源。

- 勞動(labor)：工人，工資
- 資本(capital)：廠房、機器設備，資金成本（租金、利率）
- 土地、企業才能：地租、利潤
- 原料是生產上的必要投入，但經濟學不把它們視為生產要素，因為他們也是四種生產要素製造出來的產品。

b. 何謂產出 (outputs)？

生產商品或提供勞務的數量。

(4) 何謂短期、長期、固定生產要素、變動生產要素。

- 長期：在決策期間，所有生產要素的數量均能變動調整
- 短期：在決策期間，至少有一種生產要素的數量不能變動調整。

~ 短期與長期的分野不在時間的長短

- 固定要素：短期無法變動的生產要素。
- 可變動要素：短期可以變動的生產要素。

~ 短期下，生產要素有固定要素與變動要素兩類。長期下，生產要素均為變動要素。

~ 假設廠商只使用勞動與資本兩種要素。相對於勞動，資本較難調整。

- 通常短期只討論勞動變動生產要素，（資本為固定要素）。
- 長期則包含勞動與資本兩種生產要素。

1-1. **Production**: transform inputs into outputs.

- Inputs = factors of production
    - labor (L)
    - capital (K)
    - materials
- The output can be a service or physical product.

1-2. A **production function** shows how much output can be produced efficiently from various levels of inputs.

- Firms can transform inputs into outputs in many different ways.
- A production function summarizes how a firm combines inputs such as labor, capital, and materials to produce the maximum quantity of output using current state of knowledge about techniology and management.

1-3. **Varying Inputs Over Time**

A firm can vary all its inputs in the long run but only some of them in the short run. The more time a firm has to adjust its inputs, the more factors of production it can alter.

- **short run:** a period of time so brief that at least one factor of production cannot be varied practically.
- **long run:** a lengthy enough period of time that all inputs can be varied.
- **fixed input:** a factor or production that cannot be varied practically in the short run
- **variable input:** a factor of production whose quantity can be changed readily by the firm during the relevant time period.

New technologies or new forms of organization can increase the amount of output than can be produced from a given combination of inputs.

2. Short-Run Production

(1) 何謂總產量、平均產量與邊際產量。

- **總產量（Total product of labor, TP = Q）**:使用一定量的生產要素所能生產的產品總數。

總產量曲線顯示勞動數量L與商品總數量Q間的關係，通過原點、正斜率、有最高點的曲線。

- **平均產量（Average product of labor, AP = Q/L）**:平均每一單位勞動可生產的產品數量

平均產量曲線上任何一點的勞動平均產量等於總產量曲線上該點與原點連線的斜率。

AP呈現倒U型。平均產量隨著勞度雇用量的增加，一開始上升，但當雇用到某一水準後，勞動生產力變逐漸下降。

- **邊際產量（Marginal product of labor, MP=dQ/dL）**：每增加一單位勞動能增加的產量

~ 邊際產量曲線上任何一點的勞動邊際產量等於總產量曲線上該點的切線斜率。

MP呈現倒U型。勞動邊際產量先增後減。 起初因為專業化分工與熟練機器，勞動邊際產量逐漸增加，當達到某一臨界量時，總產量仍然增加，但勞動邊際產量開始下降，最後增加勞動可能越幫越忙，勞動邊際生產力為負。

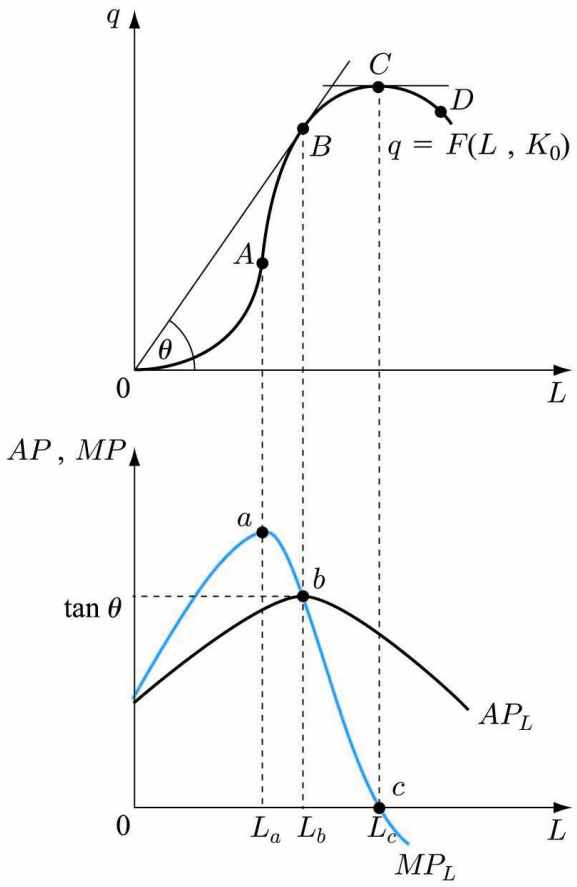
~ TP、AP、MP之關係

表 14-1　TP、AP、MP 之關係

K	L	q	AP_L	MP_L
5	0	0		
5	1	5	5	5
5	2	12	6	7
5	3	21	7	9
5	4	31	7.75	10
5	5	40	8	9
5	6	48	8	8
5	7	55	7.8	7
5	8	61	7.6	6
5	9	61	6.7	0
5	10	59	5.9	−2

~ TP、AP、MP之幾何關係 (Graphing the Product Curves)

圖 14-3　TP、AP、MP 之幾何關係



- 總產量極大時，邊際產量為0。
- MP_L 為 TP_L 曲線的斜率。
- AP在最高點時，其切線斜率為零。

- AP與MP曲線皆先升後降，MP的極大值（最高點）先出現，MP通過AP最高點，MP=AP。

邊際產量遞減法則＝邊際報酬遞減法則：勞動雇用量增加到某一程度後，勞動的邊際產量終將下降，因為有部分要素的僱用量無法調整。

2-1. One variable input and one fixed input

- In the short run, a firm cannot adjust the quantity of some inputs, such as capital.
- The firm varies its output by adjusting its variable inputs, such as labor.
- $q = f(L, \bar{K})$

2-2. Measurement of productivity

- Total Product(TP): $TP_L = q$
- Average Product(AP): $AP_L = \frac{q}{L}$
- Marginal Product(MP): $MP_L = \frac{dq}{dL}$

The change in total output, resulting from using an extra unit of labor, holding other factors constant.

The ratio of output to the number of workers used to produce that output.

2-3. The relationship between TP, AP and MP

2-4. The law of diminishing marginal returns

If a firm keeps increasing an input, holding all other inputs and technology constant, the corresponding increases in output will become smaller eventually.

- diminishing marginal product
    - if only one input is increased, the marginal product of that input will diminishing eventually.

- If all factors are fixed except labor, and a firm that was using very little labor increases its output may rise more than in proportion to the increase in labor because of greater specialization of workers. Eventually, however, as more workers are hired, the workers get in each other's way or wait to share equipment, so output increases by smaller and smaller amounts.

個案研究：生產力與工資

我們對勞動需求的分析顯示工資等於以勞動邊際產值所衡量的生產力。簡單地說，勞工的生產力愈高，其薪資就愈高；反之，勞工的生產力愈低，其薪資就愈低。

此點是了解為何今日勞工的薪資會比先前世代勞工的薪資來得高的關鍵。表2的資料顯示，美國的生產力與實質工資（亦即經物價膨脹調整後的工資）持續成長。在1960到2015年期間，以每小時產出所衡量的生產力平均每年約成長2.0%，而實質工資則平均每年約成長1.8%，這兩個成長率幾乎是一樣的。以每年2%的速度成長，生產力與實質工資每35年就會倍增一次。

期間	生產力成長率	實質工資成長率
1960-2015	2.0%	1.8%
1960-1973	2.7	2.7
1973-1995	1.4	1.2
1995-2015	2.1	1.8

資料來源： Bureau of Labor Statistics.

生產力的成長速度會隨時間變動。表2也顯示三段美國生產力表現相當不同的時期。在1973年左右，美國生產力的成長出現明顯下滑，且持續到1995年。生產力的成長率由每年2.7% 掉到每年1.4%，實質工資成長率則由每年2.7% 掉到每年1.2%。

在1995年左右，美國生產力的成長才出現回升。許多觀察家為「新經濟」（"new economy"）的來臨而喝彩。一般認為，生產力之所以會回升主要是因為電腦與資訊科技突飛猛進。如同理論所預測的，實質工資也會跟著回升；在1995-2015年間，生產力平均每年約成長2.1%，而實質工資則平均每年約成長1.8%。

總之，理論與歷史皆證實生產力與實質工資之間的密切關係。

個案研究：與日俱增的技能價值

「窮者愈窮，富者愈富。」這是一句我們常聽到的諺語。近來的實際情況就像這句諺語所講的那樣：許多研究顯示，高技能勞動與低技能勞動之間的薪資差距在過去二十年間日益擴大。

表1顯示美國大學畢業生與高中畢業生的平均薪資。在1974年，大學畢業男性的薪資比沒有大學文憑的高出42%；到了2014 年，此一數字躍升為81%。就大學畢業女性而言，上述數字從35% 躍升為71%。從這些資料可以看出，教育的報酬有升高的趨勢。

	1974	2014
男性		
高中畢業生	$52,521	$46,688
大學畢業生	$74,801	$84,567
大學畢業生高出百分比	+42%	+81%
女性		
高中畢業生	$30,185	$34,394
大學畢業生	$40,831	$58,894
大學畢業生高出百分比	+35%	+71%

為何高技能勞動與低技能勞動之間的薪資差距在近來有擴大的趨勢？經濟學提出了兩個假說來解釋這個趨勢。兩個假說的共同點是，高技能勞動之於低技能者的相對需求隨著時間上升，而使得工資差距日益擴大。

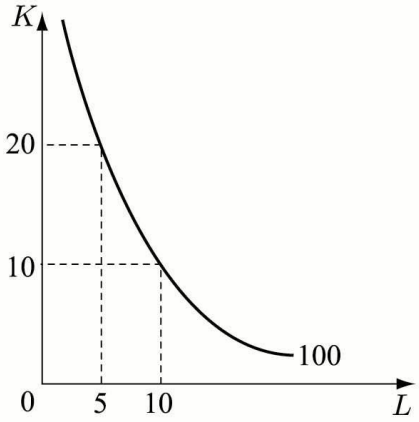
國際貿易改變了高技能勞動與低技能勞動 的相對需求。近年來，國與國之間的貿易顯著增加。以美國為例，進口占總產出的比例從1974年的8%升至2014年的17%；出口則從1974 年的8%升至2014年的14% 。在許多國家，由於低技能勞動豐沛且低廉，使得美國進口密集使用低技能勞動生產的商品，且出口密集使用高技能勞動生產的商品。因此，隨著貿易擴張，美國對高技能勞動的需求上升，對低技能勞動的需求則下滑。

科技的變化改變了高技能勞動與低技能勞動的相對需求。電腦的發明就是最好的例子。舉例來說，許多公司目前都採用電腦，而不用過去的檔案室，來儲存文書資料。這樣的改變使電腦工程師的需求增加，並使檔案人員的需求減少。因此，當愈多的公司使用電腦時，高技能勞動的需求會增加，且低技能勞動的需求會減少。經濟學家稱此一現象為偏向技能的技術變動 (skill-biased technological change)。

經濟學家對貿易、技術與其他因素在工資分配變動上的重要性有不同的看法。不過，日益增加的國際貿易與偏向技能的技術變動可能同時是過去數十年間所得不均度提高的重要原因。

3. Long-Run Production

圖 14-4　等產量線



(1) 等產量線：在特定的技術下，生產同一產量所需使用的各種勞動與資本組合之連線。

(2) 等產量曲線的特性

- 負斜率：勞動資本可以替代
- 不相交
- 凸向原點
- 越往東北產量越大
- 每一點都有一條等產量曲線通過

~ 等產量曲線的許多特性都和無異曲線類似。二者最主要的差異是：無異曲線是序列的、主觀的；等產量曲線是技術的、客觀的。

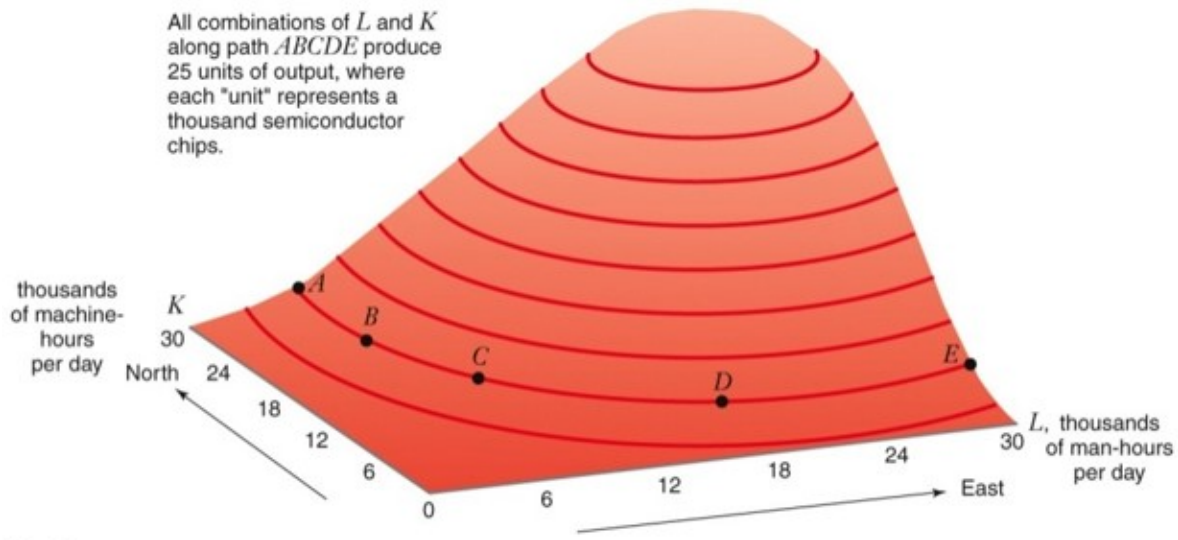


Figure 6.6

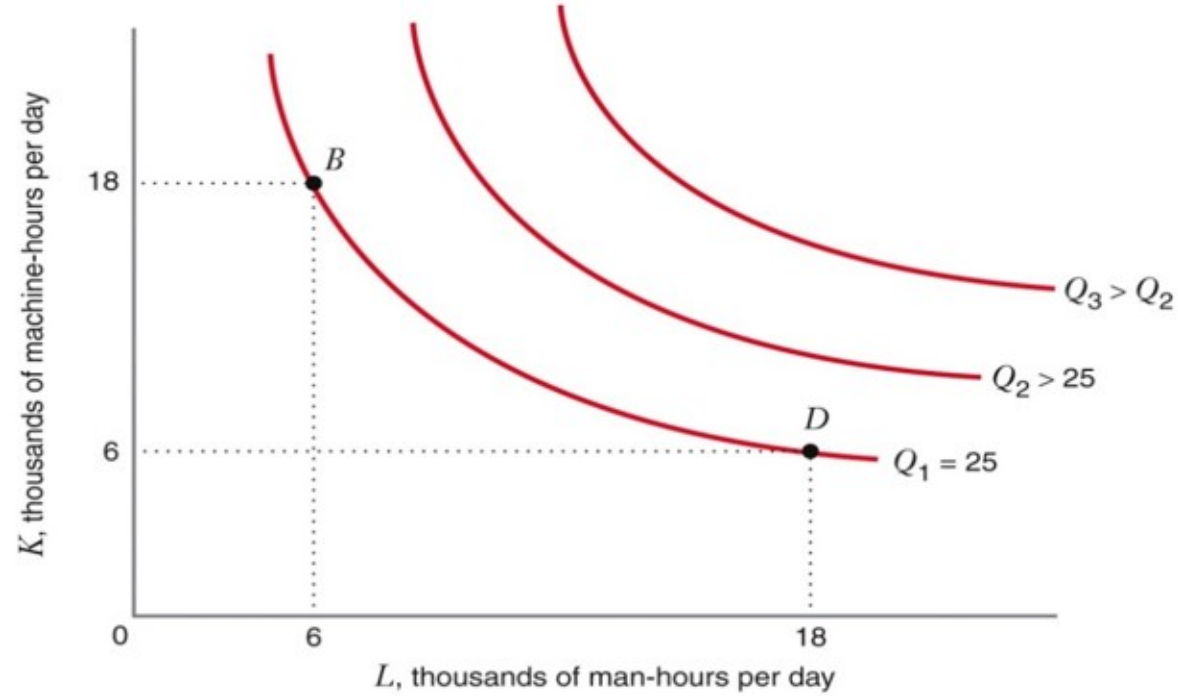
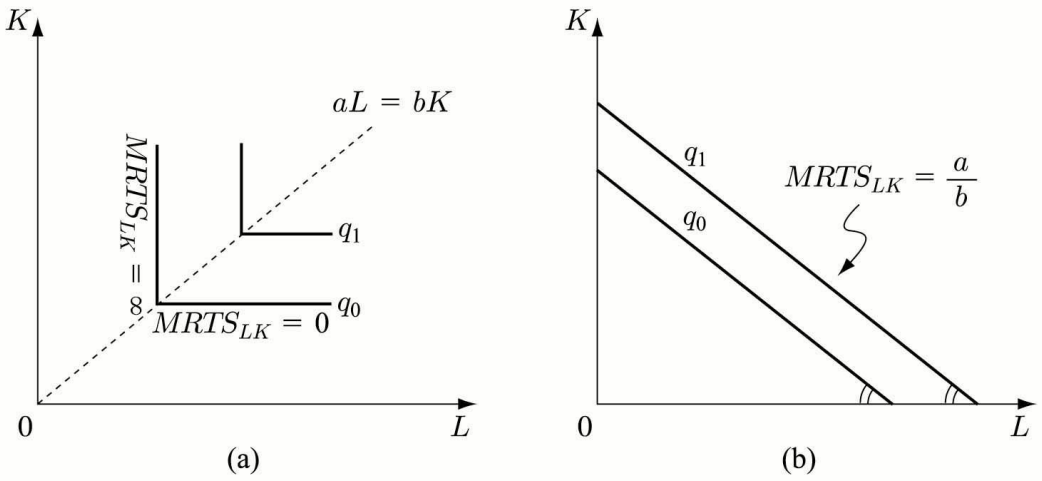


Figure 6.8

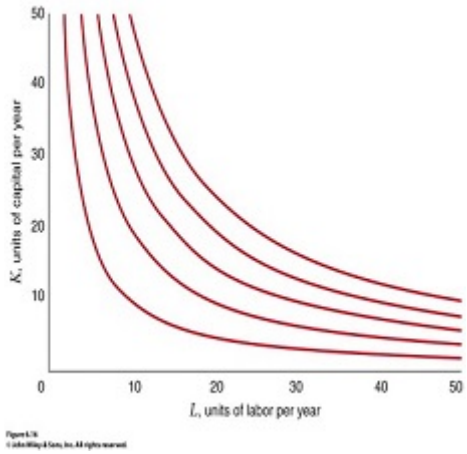
(3) 特殊生產函數

- a. 要素完全不具有替代性的Leontief生產函數
    - 兩種生產要素必須按照固定搭配比例來生產
    - 直角型
    - 一位理髮師搭配一套理髮設備，可為一人減髮。 當多一位師傅，但只有一套理髮設備，在特定期間仍只能為一人剪髮；若多一套理髮設備，但只有一位師傅，在特定期間仍只能為一人剪髮。因此，理髮這行業的產出（為多少人理髮）取決於師傅與理髮設備數量中較小值。

圖 14-6　Leontief 與 linear 生產要素



- b. 要素具有完全替代性的線性生產函數
    - 兩種生產要素間的邊際技術替代率固定
    - 負斜率直線
    - 某一果樹的種植可以用A、B兩種肥料，這兩種肥料品牌不同，但功效完全相同，因此果樹的產量取決於總共了多少肥料，而不在乎A種肥料或B種肥料用了多少。

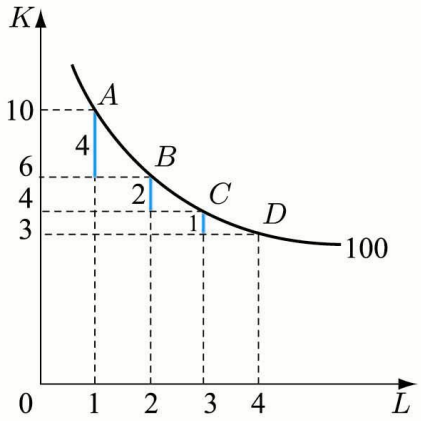


- c. 要素具有不完全替代性的Cobb-Douglas生產函數
    - 兩種要素間替代性越高，等產量曲線越平
    - 兩種生產要素間替代性越低，等產量曲線越彎
    - 可求算ＭＲＴＳ
        - ＭＲＴＳ遞減
        - 凸向原點

(4) 邊際技術替代率

- 等產量曲線斜率的絕對值。
- 在產量不變下，增加一單位勞動的雇用，所必須減少資本使用數量的比率。
- 衡量資本與勞動兩種生產要素的替代比率。

圖 14-5　邊際技術替代率



$\bar{q} = f(L, K)$
$d\bar{q} = f_L dL + f_K dK$
$|MRTS| = \frac{-dK}{dL} = \frac{MP_L}{MP_K}$

若多使用勞動(dL)，則產出量增加；若少使用資本(dK)，則產出量減少。在維持相同產量下，增加的產出等於減少的產出!

(5) **邊際技術替代率遞減法則**：維持產量不變，隨著勞動雇用料的增加，所能替代的資本越來越少的現象。

- 當勞動的數量增加時，等產量曲線斜率的絕對值越來越小。亦即勞動邊際生產力遞減，所以增加一單位勞動能替代的資本數量越來越少。
- 等產量曲線斜率的絕對值越來越小，凸向原點。

~ 邊際技術替代率遞減法則和要素的邊際產量遞減法則無關。

- 當要素的邊際產量遞減，無法確保邊際技術替代率遞減；
- 當邊際技術替代率遞減，無法確保要素的邊際產量遞減。

$$dMRTS/dL = \frac{f_K^2 f_{LL} - 2f_K f_L f_{LK} + f_L^2 f_{KK}}{(f_K)^3} < 0$$

$\Leftrightarrow MP_L > 0 \quad MP_K > 0$且$f_{LL} < 0 \quad f_{KK} < 0 \quad f_{LK} > 0$

事實上，由於f_{LK}可正可負

(1)當$f_{LL} < 0$且$f_{KK} < 0$時，無法確保（$f_K^2 f_{LL} - 2f_K f_L f_{LK} + f_L^2 f_{KK} < 0$)

(2)當($f_K^2 f_{LL} - 2f_K f_L f_{LK} + f_L^2 f_{KK} < 0$)時，無法確保$f_{LL} < 0$且$f_{KK} < 0$

3-1. Two variable inputs

In the long run, when all inputs are variable, firms can substitute between inputs.

3-2. An **isoquant** shows the efficient combinations of inputs that can produce a given level of output.

- Properties of $\bar{q} = f(L, K)$
    - the farther an isoquant is from the origin, the greater the level of output.
    - isoquant do not cross.
    - isoquant slop downward.

3-3 **Shape of Isoquants**

The curvature of an isoquant shows how readily a firm can substitute one input for another.

- Input are perfect substitutes, each isoquant is a straight line.
    - linear production funciton $q = \alpha L + \beta K$
- Inputs must be used in fixed proportions, each isoquant is L-shape .
    - fixed-proportions production function $q = min(\alpha L, \beta K)$
- Inputs are imperfect substitution, each isoquant is convex to the origion.
    - Cob-Douglas production funciton: $q = AL^{\alpha} K^{\beta}$

3-4: Substituting Inputs

Marginal rate of technical substitution, MRTS

- The absolute value of the slope of the isoquant.
- The extra units of one inputs needed to replace one unit of another input that enables a firm to keep the amount of output it products constant.
- The slope of an isoquant shows a firm's ability to replace one input with another while holding output constant.

diminishing marginal rates of technical substitution as the firm uses more of one input.

- the marginal rate of technical substitution varies along a curved isoquant.
- Usually, the more of one input the firm uses, the more difficult it is to substitute that input for another input.
- The more labor and less capital the firm has, the harder it is to replace remaining capital with labor and the faltter the isoquant becomes.

Did You Know

馬爾薩斯的人口論

- 當一個國家人口不斷地成長，而土地面積維持不變時，會越來越困難產出足夠的食物。人類必須控制人口的增長，否則貧窮是人類不可改變的命運。
- 《傲慢與偏見 (Pride and Prejudice)》
    - 「凡是有錢的單身男子必定想娶位太太，這是一條無人不曉的真理 (It is a truth universally acknowledged that a single man in possession of a good fortune must be in want of a wife)。」
- 韓賽爾與葛泰爾 (Hansel und Gretel)

        - 歐洲童話有許多因為家庭窮困導致拋棄小孩的題材，深刻地反應出當時的社會現狀。
    - 馬爾薩斯的人口解讀
        - 控制生育、防止人口增加，抑制貧窮
        - 糧食短缺、糧價上漲、人類悲劇
    - 有計劃地減緩人口增長，才能夠改善窮人的真正經濟狀況；如果因為濟貧了，讓窮人有了錢，就可能結婚而生更多的孩子，如此的社會救濟只會使得人口問題更加嚴峻。
        - 馬爾薩斯錯了嗎？

挑戰案例

Challenge Case: Labor Productivity During Recessions

Why does a measure of labor productivity—the output produced per worker—rise for many firms during recessions? During the boom years period of 2005 through November 2007, the annual average output per worker was lower in U.S. manufacturing than during the Great Recession of 2007–2009 as well during the relatively low-demand years since then through 2013. Firms produce less output during recessions as demand for their products falls. Consequently, firms typically lay off workers during recessions. Thus, **whether output per worker rises or falls depends on whether output or employment falls by more.** The labor productivity pattern over the business cycle differs across industries. If we know about a firm's production process, can we predict **whether output produced per worker will rise or fall with each additional layoff?**

Challenge Solution

We can use what we've learned to answer the question posed at the beginning of the chapter about how labor productivity, as measured by the average product of labor, changes during a recession when a firm reduces its output by reducing the number of workers it employs. How

much will the output produced per worker rise or fall with each additional layoff?

In the short run when the firm holds its capital constant, layoffs have the positive effect of freeing up machines to be used by the remaining workers. However, if layoffs mean that the remaining workers might have to "multitask" to replace departed colleagues, the firm will lose the benefits from specialization. When a firm has many workers, the advantage of freeing up machines is important and increased multitasking is unlikely to be a problem. With only a few workers, freeing up more machines does not help much—some machines might stand idle part of the time— while multitasking becomes a more serious problem. As a result, laying off a worker might raise the average product of labor if the firm has many workers relative to the available capital, but might reduce average product if it has only a few workers.

For example, in panel b of Figure 6.1, the average product of labor rises with the number of workers up to six workers and then falls as the number of workers increases. As a result, the average product of labor falls if the firm initially has two to six workers and lays one off, but rises if the firm initially has more seven or more workers and lays off a worker.

However, for some production functions, layoffs always raise labor productivity because the APL curve is downward sloping everywhere. For such a production function, the positive effect of freeing up capital always dominates any negative effect of layoffs on the average product of labor.

Consider a Cobb-Douglas production function, $q = AL^{\alpha}K^{\beta}$, where $APL = q/L = q = AL^{\alpha-1}K^{\beta}$. As Appendix 6C shows, if we increase labor slightly the change in the APL is $(\alpha - 1)AL^{\alpha-2}K^{\beta}$. Thus, if $(\alpha - 1)$ is negative (that is, α 6 1), the APL falls with extra labor. This condition holds for all of the estimated Cobb-Douglas production functions listed in the Application "Returns to Scale in Various Industries" (though not necessarily in all industries).

For example, for the beer firm's estimated Cobb-Douglas production function (Flath, 2011), q = AL0.6K0.4, α = 0.6 is less than 1, so the APL curve slopes downward at every quantity. We can illustrate how much the APL rises with a layoff for this particular production function. If A = 1 and L = K = 10 initially, then the firm's output is q = 100.6 * 100.4 = 10, and its average product of labor is APL = q/L = 10/10 = 1. If the number of workers is reduced by one, then output falls to q = 90.6 * 100.4 ≈ 9.39, and the average product of labor rises to APL ≈ 9.39/9 ≈ 1.04. That is, a 10% reduction in labor causes output to fall by 6.1%, but causes the average product of labor to rise by 4%. The firm's output falls less than 10% because each remaining worker is more productive.

This increase in labor productivity in many industries reduces the impact of the recession on output in the United States. However, this increase in labor productivity is not always observed in other countries that are less likely to lay off workers during a downturn. Until recently, most large Japanese firms did not lay off workers during recessions. Thus, in contrast to U.S. firms, their average product of labor decreased substantially during recessions because their output fell while labor remained constant.

Similarly, European firms show 30% less employment volatility over time than do U.S. firms, at least in part because European firms that fire workers are subject to a tax (Veracierto, 2008).12 Consequently, with other factors held constant in the short run, recessions might be more damaging to the profit and output of a Japanese or European firm than to the profit and output of a comparable U.S. firm. However, retaining good workers over short-run downturns might be a good long-run policy for the firm as well as for workers.

~ Question: Why does a measure of labor productivity (AP_L) rise for many firms during recessions? (Facts)

Whether output produced per worker will rise or fall with each additional layoff?

- Firms produce less output during recessions as demand for their products falls.

- Consequently, firms lay off workers during recessions.
- Thus, whether output per worker rises or falls depends on whether output or employment falls by more.

4. Returns to Scale

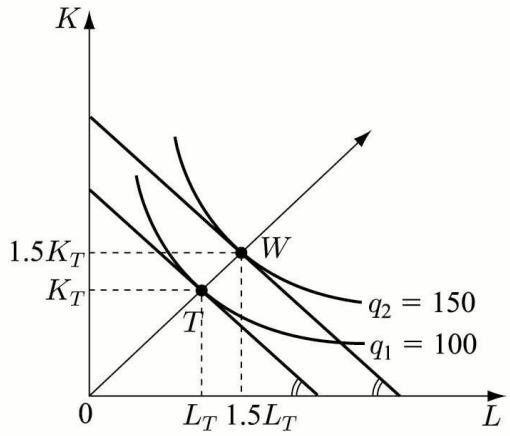
問題：假如你是老闆，當你擁有一個工廠，兩台機器與百名員工，和你擁有兩個工廠，加倍的機器與員工時，產量會如何變化？加倍嗎？

(1) 規模改變(scale change)

所有生產要素同比例變動

Question: 若所有生產要素皆按同一比例擴大，則產出會按何種比例擴大呢？

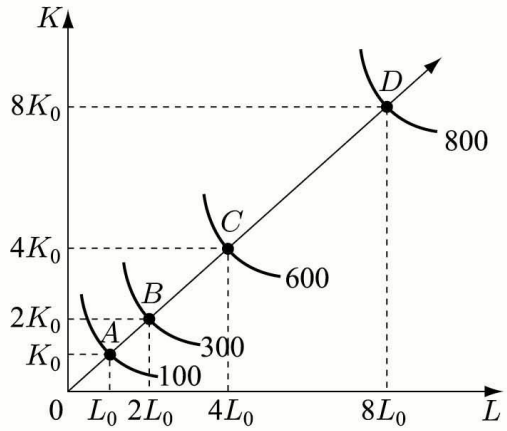
圖 14-9　CRS 等產量之特性



(2) 規模報酬

規模報酬的判定：$F(nL, nK) = n^R F(L, K) = n^R q$

圖 14-7　規模報酬



- 固定規模報酬:所有生產要素增加n倍，產量也增加n倍。$R=1$
    - 勞動與資本按同一比例擴大時，資本勞動比不變，要素平均產量與邊際產量均不變。
- 遞增規模報酬：所有生產要素增加n倍，產量增加大於n倍。$R>1$
- 遞減規模報酬：所有生產要素增加n倍，產量增加小於n倍。$R<1$

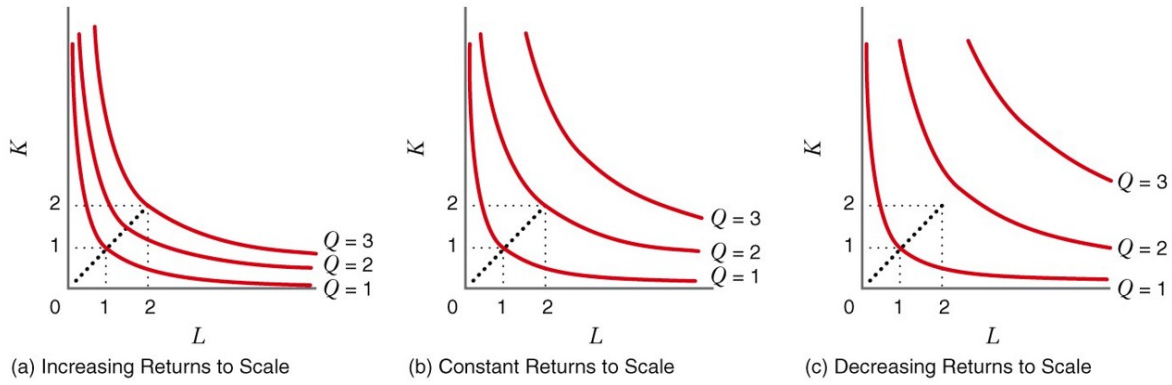


Figure 6.18

(3) 各種生產彈性

a. 產量彈性（output elasticity）:

維持其他投入不變，「特定要素」投入量變動1%所引起產量變動的百分比。

$$E_L=\frac{dlnq}{dlnL}=\frac{MP_L}{AP_L}\ E_K=\frac{dlnq}{dlnK}=\frac{MP_K}{AP_K}$$

- MP>AP，AP處於上升階段；勞動產出彈性大於1；
- MP<AP，AP處於下降階段；勞動的產出彈性小於1；

- MP=AP，AP極大；勞動的產出彈性等於1。

b. 生產力彈性**(productivity elasticity)=**函數係數＝生產函數彈性

所有要素投入變動1%產出量會變動之百分比。

$$E_n = \frac{dlnq}{dlnn} = E_L + E_K$$

- 生產力彈性為個別要素產出彈性之和。

$$\because \mathrm{Q} = \mathrm{f}(\mathrm{L},\mathrm{K}) \quad \therefore \mathrm{dQ} = \mathrm{f}_L dL + \mathrm{f}_K dK$$

$$\Rightarrow \frac{\mathrm{dQ}}{Q} = \frac{\mathrm{f}_L L}{Q}\frac{dL}{L} + \frac{\mathrm{f}_K K}{Q}\frac{dK}{K} = \left(\frac{\mathrm{f}_L L}{Q} + \frac{\mathrm{f}_K K}{Q}\right)\frac{d\lambda}{\lambda}$$

$$\Rightarrow \frac{d\ln Q}{d\ln\lambda} = \frac{\mathrm{dQ/Q}}{d\lambda/\lambda} = \frac{\mathrm{f}_L L}{Q} + \frac{\mathrm{f}_K K}{Q} = \frac{MP_L}{AP_L} + \frac{MP_K}{AP_K}$$

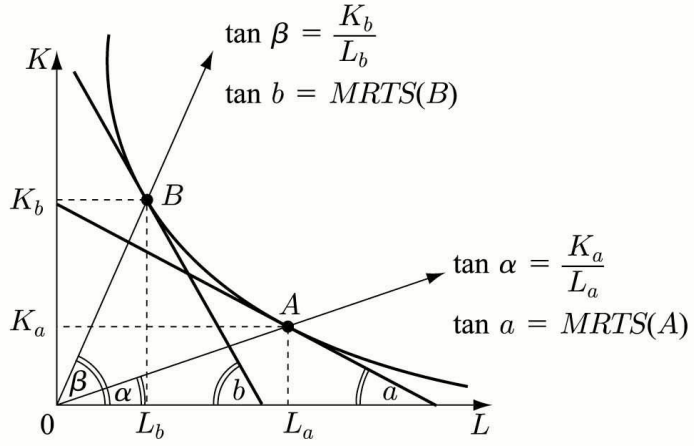
$$\Rightarrow \varepsilon = \varepsilon_{\mathrm{L}} + \varepsilon_{\mathrm{K}}$$

c. 替代彈性**(substitution elasticity)**

ＭＲＴＳ變動1%時，資本勞動比($k = \frac{K}{L}$)變動的百分比。

$$\sigma = \frac{dlnk}{dlnMRTS}$$

圖 14-8　替代彈性



- 特殊生產函數的替代性

TABLE 6.6 Characteristics of Production Functions

Production Function	Elasticity of Substitution (σ)	Other Characteristics
Linear production function	$\sigma = \infty$	Inputs are *perfect substitutes* Isoquants are straight lines
Fixed-proportions production function	$\sigma = 0$	Inputs are *perfect complements* Isoquants are L-shaped
Cobb–Douglas production function	$\sigma = 1$	Isoquants are curves
CES production function	$0 \leq \sigma \leq \infty$	Includes other three production functions as special cases Shape of isoquants varies

Cobb-Douglas Production Function

$$Q = f(K, L) = AK^{b_1}L^{b_2}$$
$$MP_L = Ab_2K^{b_1}L^{b_2-1}, AP_L = AK^{b_1}L^{b_2-1}$$
$$MRTS = MP_L / MP_K = \frac{b_2}{b_1}\frac{K}{L}$$
$$\sigma = \frac{d \ln K/L}{d \ln MRTS} = 1$$
$$\varepsilon_L = MP_L / AP_L = b_2, \quad \varepsilon_K = b_1, \quad \varepsilon = \varepsilon_L + \varepsilon_K$$
$$f(mK, mL) = A(mK)^{b_1}(mL)^{b_2} = Am^{b_1+b_2}K^{b_1}L^{b_2} = m^{a+b}f(K, L)$$
$$a + b = 1, \text{ CRTS}$$
$$a + b > 1, IRTS$$
$$a + b < 1, DRTS$$

4-1. How much output changes when a firm increases all inputs in proportion?

4-2. Constant, Increasing, and Decreasing Returns to Scale

- its output increases by the same proportion, the production process is said to exhibit **constant returns to scale, CRTS**.
    - if $q = f(L, K)$ then $f(\alpha L, \alpha K) = \alpha q$
- if output increases less than in proportion to inputs, the production process has **decreasing returns to scale, DRTS**.
    - if $q = f(L, K)$, then $f(\alpha L, \alpha K) < \alpha q$
- if output increases more than in proportion to inuts, the production process has **increasing returns to scale**.
- if $q = f(L, K)$, then $f(\alpha L, \alpha K) > \alpha q$

The answer helps a firm determine its scale in the long run.

- All three types of returns to scale are commonly seen in actual industries.
- Many production processes exhibit first increasing, then constant, and finally decreasing returns to scale as the size of the firm increases.

5. Productivity and Technical Change

問題：假如你是老闆，你擁有一個工廠，但改採更先進的科技生產，產量會有什麼變化嗎？購買機器與雇用員工的數量會不會改變？

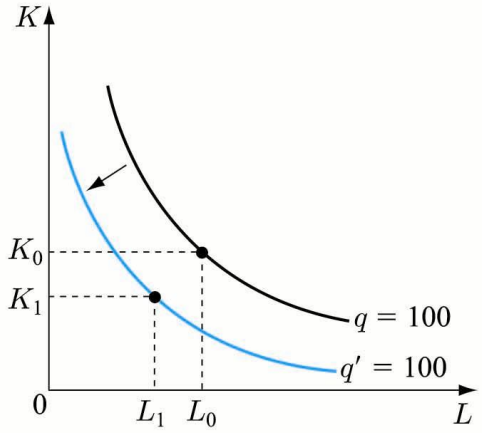
(1)技術進步 (technological progress)：生產可能集合擴大

- 用相同數量的投入，可以生產更多的產出；
- 用較少數量的投入，可以生產相同的產出。

(2) 技術進步：等產量曲線內移

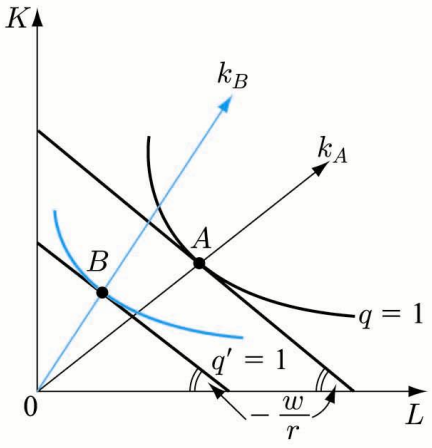
技術進步使得廠商使用較少的生產要素就能有相同的產量。

圖 15-24　技術進步



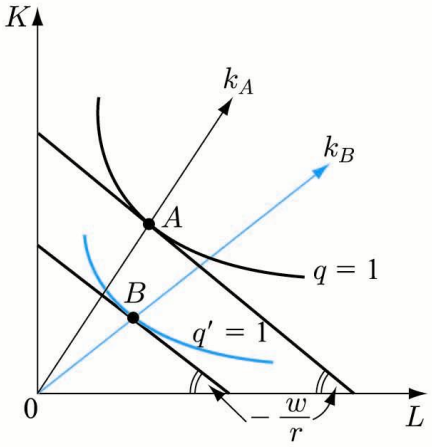
- a. 資本多用勞動節省型技術進步 (labor-saving technological progress）
    - 在資本勞動比固定下，勞動節省型技術進步，等產量曲線斜率的絕對值下降。
    - 資本多用勞動節省的技術進步，廠商傾向多用資本、發展資本密集產業。

圖 15-25　資本多用型技術進步



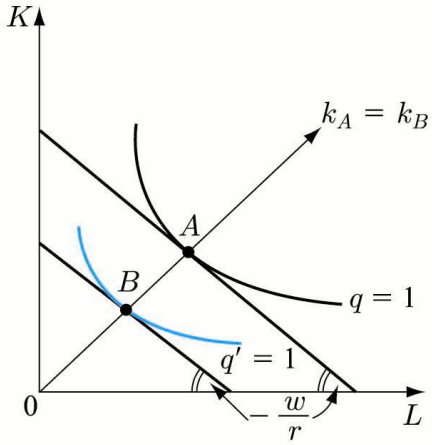
- b. 資本節省勞動多用型技術進步(capital-saving technological progress)

圖 15-26　勞動多用型技術進步



- c. 中性技術進步(neutral technological progress)

圖 15-27　中性型技術進步



5-1. Although all firms in an industry produce efficiently, given what they know and the institutional and other constraints they face, some firms may be more productive than others.

- Due to innovations such as technical progress or new means of organizing production, a firm can produce more today than it could in the past from the same bundle of inputs.
- They can produce more output from a given bundle of inputs.
- Such innovations change the production function.

5-2. **Technical Progress**

- A firm can produce more output using the same ratio of inputs.

- neutral technical change
    - the shape of isoquant unchanged
- non-neutral technical changes
    - alter the proportion in which inputs are used
        - labor saving
        - capital saving
    - the shape of isoquant change