

# NCTU Pattern Recognition, Homework 2

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## Part. 1, Coding (60%):

In this coding assignment, you are required to implement Fisher's linear discriminant by using only NumPy, then train your model on the provided dataset, and evaluate the performance on testing data. Find the sample code and data on the GitHub page [https://github.com/NCTU-VRDL/CS\\_ILE5065/tree/main/HW2](https://github.com/NCTU-VRDL/CS_ILE5065/tree/main/HW2)

**Please note that only NumPy can be used to implement your model, you will get 0 point by calling `sklearn.discriminant_analysis.LinearDiscriminantAnalysis`.**

1. (5%) Compute the mean vectors  $\mu_i$  ( $i=1, 2$ ) of each 2 classes on **training data**

$\mu_1 = [1.3559426 \ -1.34746216]$

$\mu_2 = [-1.29735587 \ 1.29096203]$

2. (5%) Compute the within-class scatter matrix  $S_w$  on **training data**

$S_w = \begin{bmatrix} 388.64001349 & -228.92177708 \\ -228.92177708 & 665.56910433 \end{bmatrix}$

3. (5%) Compute the between-class scatter matrix  $S_b$  on **training data**

$S_b = \begin{bmatrix} 7.03999279 & -7.00052687 \\ -7.00052687 & 6.9612822 \end{bmatrix}$

4. (5%) Compute the Fisher's linear discriminant  $W$  on **training data**

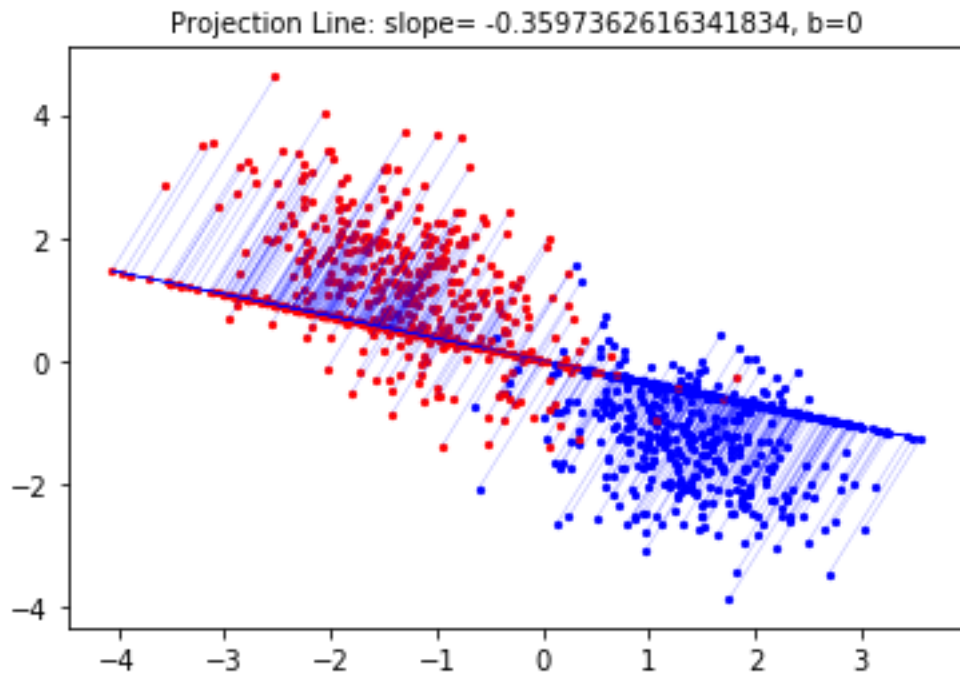
$W = \begin{bmatrix} 0.94096648 \\ -0.33849976 \end{bmatrix}$

5. (20%) Project the **testing data** by Fisher's linear discriminant to get the class prediction by nearest-neighbor rule and calculate your accuracy score on **testing data** (you should get accuracy over 0.9)

I projected the testing data and calculate Euclidean distance of all projected training data and found the nearest-neighbor and set the same class of it.

My accuracy of test-set 0.916

6. (20%) Plot the **1) best projection line** on the **training data** and show the slope and intercept on the title (you can choose any value of *intercept* for better visualization) **2) colorize the data** with each class **3) project all data points on your projection line**. Your result should look like the below image (This image is for reference, not the answer)



**Part. 2, Questions (40%):**

**The answer of Part 2 is handed written and showed in following pictures.**

1. (20%) Show that maximization of the class separation criterion given by  $J(w, b) = \frac{1}{2} (w^2 - b^2) + \frac{1}{2} (w^2 - b^2)$  with respect to  $w$ , using a Lagrange multiplier to enforce the constraint  $w^2 + b^2 = 1$ , leads to the result that  $w \propto (w^2 - b^2)$ .
2. (20%) Show that the logistic sigmoid function satisfies the property  $\sigma(-x) = 1 - \sigma(x)$  and its inverse is given by  $\sigma^{-1}(x) = \ln\{x/(1-x)\}$ .

Part 2:

1. In the case of 2-classes.

Constrain  $w$  to have unit length ( $w^T w = 1$ )

The optimization problem will be :

$$\begin{array}{ll} \text{Maximize} & w^T (m_2 - m_1) \\ \text{s.t.} & w^T w = 1 \end{array}$$

By Lagrange multiplier :

$$L(\lambda, w) = w^T (m_2 - m_1) + \lambda (w^T w - 1)$$

Differentiating with  $w$  :

$$(m_2 - m_1) + \lambda 2w = 0$$

$$\Rightarrow w = -\frac{1}{2\lambda} (m_2 - m_1)$$

$$\Rightarrow w \propto (m_2 - m_1)$$

2.

(a).

$$\text{Given: } \sigma(a) = \frac{1}{1+e^{-a}}$$

$$\text{Show: } \sigma(-a) = 1 - \sigma(a)$$

$$1 - \sigma(a) = 1 - \frac{1}{1+e^{-a}} = \frac{1+e^{-a}-1}{1+e^{-a}}$$

$$= \frac{e^{-a}}{1+e^{-a}} = \frac{1}{e^a(1+e^{-a})}$$

$$= \frac{1}{1+e^a}$$

equal

$$\text{and } \sigma(-a) = \frac{1}{1+e^a}$$

$$\Rightarrow \sigma(-a) = 1 - \sigma(a) \quad \text{✓}$$

(b)

$$\text{Set } y = \frac{1}{1 + e^{-x}} = \sigma(x)$$

$$y \times (1 + e^{-x}) = 1 \Rightarrow y + y e^{-x} = 1$$

$$e^{-x} = \frac{1-y}{y}$$

$$\text{Take natural log} \Rightarrow -x = \ln\left(\frac{1-y}{y}\right)$$

$$\Rightarrow x = -\ln\left(\frac{1-y}{y}\right) = \ln\left(\frac{1-y}{y}\right)^{-1}$$

$$= \ln\left(\frac{y}{1-y}\right)$$

$$\text{Cause } y = \sigma(x) \Rightarrow x = \sigma^{-1}(y)$$

$$\Rightarrow \sigma^{-1}(y) = \ln\left(\frac{y}{1-y}\right) \#$$