COMP5121 Data Mining and Data Warehousing Applications

Week 6: Classification – Basic Concepts

(Chapter 8 in textbook)

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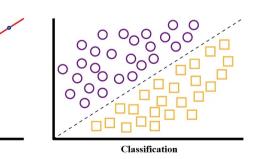
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Outline

- ☐ Classification: Basic Concepts
- □ Decision Tree Induction Entropy-based ID3 Algorithm
- □ Bayes Classification Methods
- Model Evaluation and Selection

Prediction Analysis: Classification vs. Regression

- □ Prediction: a general process of forecasting or estimating an unknown outcome
 Regression
- Classification: categorize data objects into predefined classes
 - ☐ Output: categorical/discrete labels
 - □ Applications: spam detection, medical diagnosis, credit/loan approval
- Classfication
 Will tomorrow be a cold or hot day?
- Regression: predict continuous numerical values values
 - ☐ Output: numbers on a continuous scale
 - □ Applications: stock price, temperature forecasting, traffic flow control



What temperature will there be tomorrow?

Supervised vs. Unsupervised Learning

- □ Supervised Learning: Classification
 - **Supervision**: The training data are accompanied by labels, indicating classes of observations.
- Unsupervised Learning: Clustering
 - The class labels of training data is unknown.
 - Given a set of observations, it aims to establish clusters in the data through a self-discovery process.
 - Applications: customer segmentation, topic modeling

Classification – A Two-Step Process

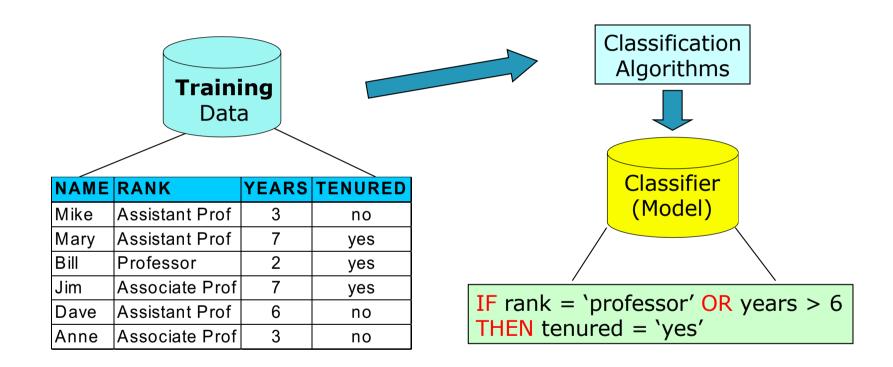
1. Model Construction

- To learn a model using a training set (data with known labels), assuming each tuple belongs to a predefined class
- Model: represented as classification rules, decision trees, or mathematical formulas

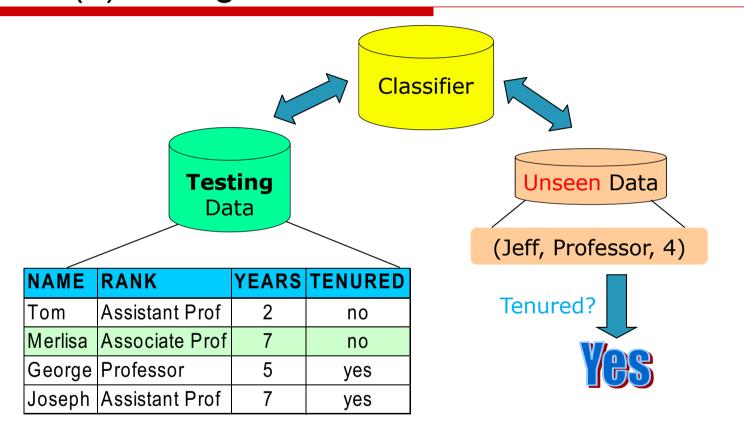
2. Model Usage

- To classify future or unknown objects
- Evaluation by comparing predicted labels with actual ones
 - ☐ **Accuracy**: % of samples correctly classified by the models
 - Avoid overfitting by testing on independent data (testing/validating)
 - ☐ If the accuracy is acceptable, use the model for future predictions

Process (1): Model Construction



Process (2): Using the Model in Prediction



Issues in Classification and Prediction

□ Data Preparation

- Data cleaning: reduce noises and handle missing values
- Correlation analysis: remove irrelevant or redundant attributes
- Data transformation: generalize and normalize data

Model Evaluation

- Accuracy: how well the model performs
- Speed: time to construct and use the model
- Scalability: efficiency when handling large-scale DBs
- Robustness: ability to handle noise and missing values
- Interpretability: how easily the model's insights can be understood
- Goodness of rules: 1) decision tree size; 2) compactness of rules

Iterative Dichotomiser (ID3), CART, C4.5, ...

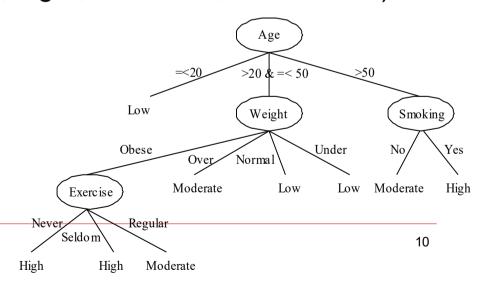
CLASSIFICATION BY DECISION TREE INDUCTION

Decision Tree Structure

- A flow-chart-like structure used for classification
 - Internal node: a test on an attribute (e.g., age, exercise, weight, smoking)
 - Branch: an outcome of the test
 - Leaf nodes: class labels (e.g., high-, moderate-, and low-risk)

How it works:

An object is classified by traversing the tree from its root to a leaf.



Decision Tree Induction

□ Decision Tree Generation

- 1. Tree Construction
 - Initially, all training examples are at the root (a single node).
 - ➤ The data is partitioned recursively based on selected attributes.
 - Data-driven: requires NO domain knowledge or parameter settings.
- 2. Tree Pruning
 - ➤ Identify and remove branches that reflect noise or outliers.
 - Improve accuracy on unseen data by preventing overfitting.
- Use of Decision Tree: Classify an unknown sample by testing its attribute values against the decision tree

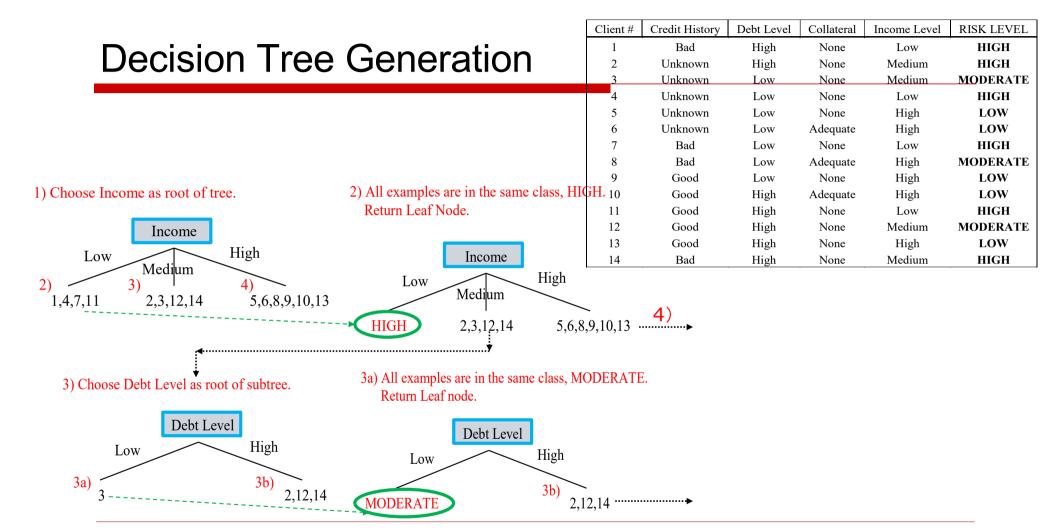
Decision Tree Construction

- ☐ A decision tree is said to represent the classification if it correctly classifies all training instances → consistent with the training data.
- □ Simple Idea, Complex Problem
 - Can be built in many possible ways to split the data and represent the classification → finding the best one is a challenge!
 - Does a tree consistent with training data have the highest likelihood of accurately classifying unseen instances of the population?
- The goal is not just consistency with the training data, but also generalization to unseen new data.

Example: Training Set

Risk Assessment for Loan Applications

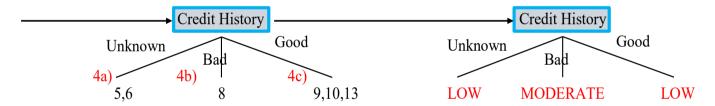
Client #	Credit History	Debt Level	Collateral	Income Level	RISK LEVEL
1	Bad	High	None	Low	HIGH
2	Unknown	High	None	Medium	HIGH
3	Unknown	Low	None	Medium	MODERATE
4	Unknown	Low	None	Low	HIGH
5	Unknown	Low	None	High	LOW
6	Unknown	Low	Adequate	High	LOW
7	Bad	Low	None	Low	HIGH
8	Bad	Low	Adequate	High	MODERATE
9	Good	Low	None	High	LOW
10	Good	High	Adequate	High	LOW
11	Good	High	None	Low	HIGH
12	Good	High	None	Medium	MODERATE
13	Good	High	None	High	LOW
14	Bad	High	None	Medium	HIGH



Decision Tree Generation

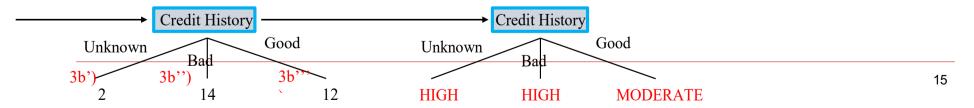
RISK LEVEL Client # Credit History Debt Level Collateral Income Level Bad High None Low HIGH 2 HIGH Unknown High None Medium Unknown Low None Medium MODERATE Unknown Low None Low HIGH 5 Unknown None High LOW Low 6 Unknown Low Adequate High LOW 7 Bad None HIGH Low Low 8 Bad High **MODERATE** Low Adequate 9 Good Low None High LOW 10 Good LOW High Adequate High 11 Good High None Low HIGH 12 Good High None Medium **MODERATE** 4a-4c) All examples are in the same classed LOW High None High Return Leaf nodes. 14 Bad High None Medium HIGH

4) Choose Credit History as root of subtree.

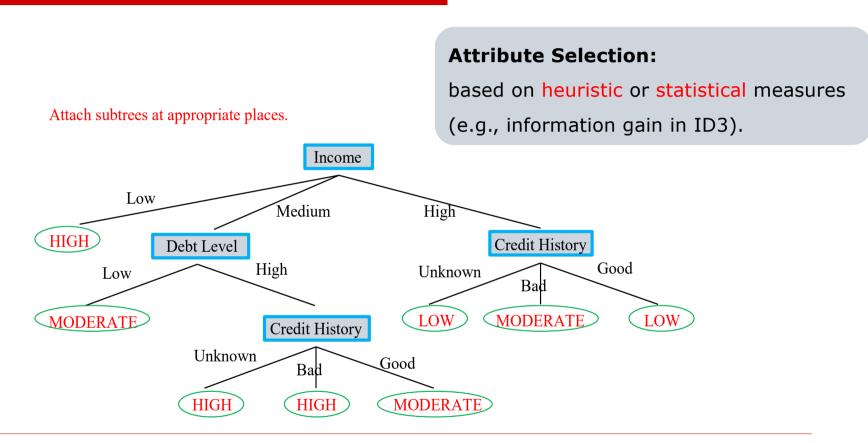


3b'-3b''') All examples are in the same class. Return Leaf nodes.

3b) Choose Credit History as root of subtree.



Final Decision Tree



Entropy

- □ A measure of randomness, uncertainty, and disorder in a system with probability distributions of outcome.
- ☐ Entropy is formulated as a *function* that measures disorder.
 - "The higher the entropy, the greater the disorder."
 - For classification, it tells how diverse the classes are in a set.
- \square Let D be a set of examples from m classes.

$$Info(D) = -\sum_{i=1}^{m} p_i \cdot \log_2(p_i)$$

- Input: Distribution of outcomes
- Output: A value indicating how disordered the outcomes are
- p_i : The proportion of examples observed in D that belong to i-th class within [0,1].

Example: Tossing Coins in Casino

☐ Casino A with real coins (50/50 chances):

$$Info(Coin Toss) = -p(head) \log_2 p(head) - p(tail) \log_2 p(tail)$$

$$= -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = 1$$





☐ Casino B with fake coins (75/25 chances):

HEAD

TAIL

$$Info(Coin \, Toss) = -p(head) \log_2 p(head) - p(tail) \log_2 p(tail)$$

$$= -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) = 0.811$$

Entropy is a measure of randomness and disorder.

Higher entropy means higher uncertainty.

Information Gain and Iterative Dichotomiser (ID3)

- ☐ Classification Goal: To split the dataset in a way that reduces entropy the most.
- □ Information Gain: To measure the reduction in entropy after splitting the dataset on an attribute A

$$Gain(D, A) = Info(D) - Info_A(D)$$

- Weighted entropy after split: $Info_A(D) = \sum_{j=1}^n p(D_j|A)Info(D_j)$
 - \square D_i : subsets of D created by splitting on A

ID3 Algorithm: Repeatedly selects the attribute with the highest information gain at each step to build the decision tree.

ID3 Example (Decision: buy computer or not)

□ Class P: buys_computer = 'yes' → 9

Class N: buys_computer = 'no' → 5

• Info(D) = $\sum -p_i \times \log_2 p_i$

• $\operatorname{Info}_A(D) = \sum [p(D_i|A) \times \operatorname{Info}(D_i)]$

• $Gain(D, A) = Info(D) - Info_A(D)$

age	income	student	credit rating	buys computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
3140	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
3140	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
3140	medium	no	excellent	yes
3140	high	yes	fair	yes
>40	medium	no	excellent	no

$$Info(D) = I(9,5) = -\frac{9}{14}\log_2(\frac{9}{14}) - \frac{5}{14}\log_2(\frac{5}{14}) = 0.940$$

age	p _i	n _i	I(p _i , n _i)	Info
<=30	2	3	0.971	Inju
3140	4	0	0	
>40	3	2	0.971	
	_			

$$Info_{age}(D) = \frac{5}{14}I(2,3) + \frac{4}{14}I(4,0) + \frac{5}{14}I(3,2) = 0.694$$

 $\frac{5}{14}I(2,3)$ means 'age <=30' has 5 out of 14 samples, with 2 'yes' and 3 'no'.

Hence,

$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

Similarly,
$$Gain(income) = 0.029$$

 $Gain(student) = 0.151$
 $Gain(credit_rating) = 0.048$

ID3 Example (Decision: buy computer or not)

Detailed Calculations

income	total	р	n	I(p,n)
low	4	3	1	0.8113
medium	6	4	2	0.918
high	4	2	2	1

 $INFO(D) = -9/14 \log_2(9/14) - 5/14 \log_2(5/14) = 0.940 \text{ bits}$

 $Info_{INCOME}(D)=4/14 I(3,1)+6/14 I(4,2)+4/14I(2,2)$

Info_{INCOME}(D)=4/14 *0.811+6/14 *0.918)+4/14*1= 0.231+0.393+0.285 =0.909

 $GAIN(INCOME) = INFO(D) - INFO_{income}(D) = 0.940 - 0.909 = 0.029$

student	total	р	n	I(p,n)
yes	7	6	1	0.5917
no	7	3	4	0.9852

 $INFO(D) = -9/14 \log_2(9/14) - 5/14 \log_2(5/14) = 0.940 bits$

INFO(student)= 7/14 *I(6,1)+7/14* I(3,4)

INFO(student)= 7/14 *0.597 +7/14* 0.9852 = 0.791

GAIN(STUDENT) = INFO(D)- INFO_{STUDENT}(D)= 0.940-0.791= 0.151 bits

creditrating	total	р	n	I(p,n)
fair	8	6	2	0.8113
excellent	6	3	3	1

 $INFO(D) = -9/14 \log_2(9/14) - 5/14 \log_2(5/14) = 0.940 \text{ bits}$

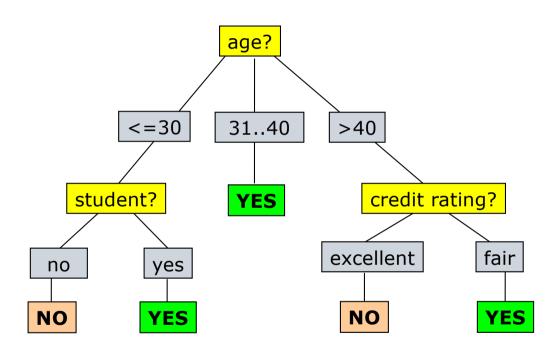
INFO(creditrating)= 8/14 *I(6,2)+7/14* I(3,3)

INFO(creditrating)= 7/14 *0.811 +7/14* 1 =0.9055

GAIN(credit rating) = INFO(D)- INFO_{creditrating}(D)= 0.940-0. 9055= 0.048 bits

Final Decision Tree based on ID3

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
3140	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
3140	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
3140	medium	no	excellent	yes
3140	high	yes	fair	yes
>40	medium	no	excellent	no



ID3 Example (Decision: play outside or not)

Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

- $\mathbf{Info}(D) = \sum -p_i \times \log_2 p_i$ $\mathbf{Info}_A(D) = \sum [p(D_j|A) \times \mathbf{Info}(D_j)]$
- $Gain(D, A) = Info(D) Info_A(D)$

Step 1: Entropy based on "Decision"

Info(Decision) =
$$-p(Yes) \times \log_2 p(Yes) - p(No) \times \log_2 p(No)$$

= $-\frac{9}{14} \times \log_2 \frac{9}{14} - \frac{5}{14} \times \log_2 \frac{5}{14} = 0.940$

ID3 Example (Decision: play outside or not)

			Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
13	Overcast	Hot	Normal	Weak	Yes
2	Sunny	Hot	High	Strong	No
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
14	Rain	Mild	High	Strong	No

Wind on Decision

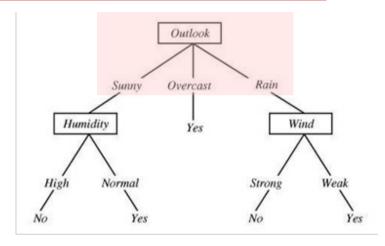
Gain(Decision, Wind)

- = Info(Decision)
 - [p(Decision | Wind=Weak) x Info(Decision | Wind=Weak)]
 - [p(Decision | Wind=Strong) x **Info**(Decision | Wind=Strong)]

- Info(Decision | Wind=Weak) = $-\frac{2}{8} \times \log_2 \frac{2}{8} \frac{6}{8} \times \log_2 \frac{6}{8} = 0.811$
- Info(Decision | Wind=Strong) = $-\frac{3}{6} \times \log_2 \frac{3}{6} \frac{3}{6} \times \log_2 \frac{3}{6} = 1$
- **Gain**(Decision, Wind) = 0.940 - (8/14) x 0.811 - (6/14) x 1 = 0.048

ID3 Example (Decision: play outside or not)

Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No



Outlook, Temperature and Humidity on Decision

- **Gain**(*Decision*, *Outlook*) = 0.246 (highest gain)
- **Gain**(*Decision*, *Humidity*) = 0.151
- Gain(Decision, Wind) = 0.048
- Gain(Decision, Temperature) = 0.029

Day	Outlook	Temp.	Humidity	Wind	Decision
3	Overcast	Hot	High	Weak	Yes
7	Overcast	Cool	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

(1) Outlook = Overcast on Decision

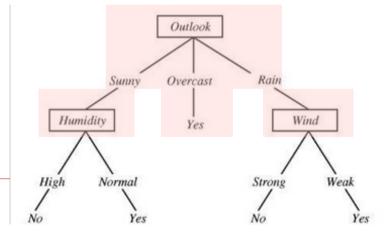
Decision will always be 'Yes', no need to calculate this branch, it is leaf node already.

(2) Outlook = Sunny on Decision

- For 5 sunny instances: 3/5 'No', and 2/5 'Yes'.
 - Gain(Outlook = Sunny, Temp.) = 0.570
 - Gain(Outlook = Sunny, Humidity) = 0.971
 - Gain (Outlook = Sunny, Wind) = 0.020

(3) Outlook = Rain on Decision

- For 5 rain instances: 2/5 "No", and 3/5 "Yes".
 - Gain(Outlook = *Rain*, Temp.) = 0.020
 - Gain(Outlook = Rain, Humidity) = 0.020
 - Gain (Outlook = Rain, Wind) = 0.971



Overfitting and Tree Pruning

- Overfitting: An induced tree may overfit the training data.
 - **Too many branches**: some may reflect noise or outliers, rather than meaningful patterns → Poor accuracy for unseen samples
- Avoid Overfitting
 - Pre-pruning: stop tree construction early do not split a node if it would result in the goodness measure falling below a threshold
 - ☐ Challenge: choosing an appropriate threshold can be difficult.
 - Post-pruning: start with a fully grown tree, then remove branches progressively to simplify the tree
 - ☐ Use a validation dataset different from training sets to decide the best-pruned tree

statistical, probabilistic, efficient

BAYES CLASSIFICATION METHODS

Bayesian Classifier

- Decision Tree: data-driven, rule-based reasoning; generating highly interpretable and explainable decisions
- Real-world problems involve uncertainty.
 - How likely is this email a spam based on the words it contains?
 - What is the likelihood of a patient having flu based on symptoms?
- □ Probability-based Bayesian Classifiers:
 - prior knowledge (what we already know from training data) + observed evidence (new data) + Bayesian Theorem
 - → Make informed decisions

Bayesian Theorem

- \square P(H|E): Posterior probability, the probability of H holds given E
 - E: Evidences (e.g., a data tuple) with attribute description
 - \blacksquare H: Hypothesis to be verified (e.g., a class label that E belongs to)

$$P(H|E) = \frac{P(E|H) P(H)}{P(E)}$$

- \square P(H): prior probability, i.e., the initial probability of hypothesis H before observing evidence E
- \square P(E): marginal probability, i.e., the total probability of observing evidence E under all possible hypotheses
- \square P(E|H): likelihood, i.e., the probability of observing evidence E given that the hypothesis H = true

Bayesian Classification

- \square A data tuple: $X = (A_1 = x_1, A_2 = x_2, A_3 = x_3, ..., A_n = x_n)$
- \square To classify X, we need to estimate $P(C_i \mid X)$
 - \blacksquare C_i represents the **hypothesis** that X belongs to C_i .
 - We say X belongs to C_i iff: $P(C_i|X) > P(C_j|X)$, for all $j \neq i$
- \square How to estimate $P(C_i \mid X)$ for classifying X?
 - Bayesian theorem: $P(C_i|X) = \frac{P(X|C_i)P(C_i)}{P(X)}$
 - The problem becomes \rightarrow estimating $P(X|C_i)$ and $P(C_i)$

Bayesian Classification

- □ Estimate the priori probability of the i-th class C_i from the training set D: $P(C_i) = \frac{|C_i|}{|D|}$
- □ Independence Assumption: For $P(X \mid C_i)$, we assume that the effect of each attribute A_i is independent to others:

$$P(X = (A_1 = x_1, A_2 = x_2, ..., A_n = x_n) | C_i)$$

= $P(A_1 = x_1 | C_i) \times P(A_2 = x_2 | C_i) \times ... \times P(A_n = x_n | C_i)$

where $P(A_i = x_i | C_i)$ can also be estimated from the training set D.

Example

Bayesian: $P(C_i|X) = \frac{P(X|C_i)P(C_i)}{P(X)}$

- ☐ Given a training set, predict if a person *X* will buy a computer
 - X: {age = youth, income = medium, student = yes, credit_rating = fair}
 - \blacksquare Yes or No? $P(buy_computer|X)$

Priori Probability in training Data:

- $P(buy_computer = yes) = 9/14 = 0.643$
- $P(buy_computer = no) = 5/14 = 0.357$

	buys_computer	
age	yes	no
youth middle_aged senior	2 4 3	3 0 2

	buys_computer	
income	yes	no
low medium high	3 4 2	1 2 2

To calculate $P(X \mid buy_computer = yes)$:

- $P(age = youth \mid yes) = 2/9 = 0.222$
- $P(income = medium \mid yes) = 4/9 = 0.444$
- $P(student = yes \mid yes) = 6/9 = 0.667$
- $P(credit_rating = fair | yes) = 6/9 = 0.667$
- $\rightarrow P(X \mid buy_computer = yes) = 0.044$
- \rightarrow Similarly, $P(X \mid buy_computer = no) = 0.019$

	buys_computer		
student	yes	no	
yes no	6 3	1 4	

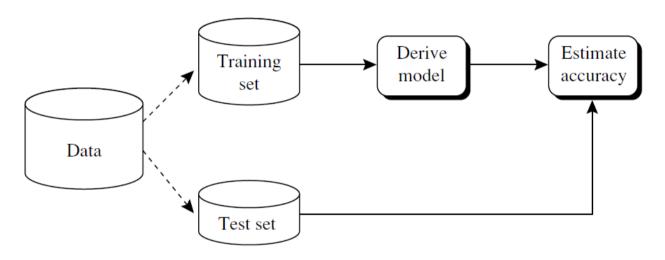
	buys_computer	
credit_ratting	yes	no
fair	6	2
excellent	3	3



Through Bayesian:

- $P(X \mid yes) \times P(buy_computer = yes) = 0.028$
- $P(X \mid no) \times P(buy_computer = no) = 0.007$

Conclusion: X will buy a computer.



MODEL EVALUATION AND SELECTION

Evaluation Measures

- ☐ To assess how "accurate" your classifier is at predicting the class label of tuples compared to actual labels
 - True Positives TP: positive tuples that were correctly labeled
 □ Positive tuples: tuples of the main class of interest
 - True Negatives TN: negative tuples that were correctly labeled
 - False Positives FP: negative tuples that were incorrectly labeled as positive (e.g., people who do not buy computers but are labeled as buys_computer = yes)

 Predicted class
 - False Negatives FN: positive tuples that were mislabeled as negative (e.g., people who really buy computers but are labeled as buys_computer = no)

Actual class

_	_		
	yes	no	Total
yes	TP	FN	P
no	FP	TN	N
Total	P'	N'	$^{35}P+N$

Evaluation Measures

$ \begin{array}{c cccc} & no & FP & TN \\ \hline & Total & P'_1 & N' \end{array} $	N
Total D' M'	
Total F N /	P + N
Recall =	e negatives O Ositives
How many retrieved items are relevant?	0
Precision =	0

Predicted class

yes

Total

Measure	Formula
accuracy, recognition rate	$\frac{TP + TN}{P + N}$
error rate, misclassification rate	$\frac{FP + FN}{P + N}$
sensitivity, true positive rate, recall	TP P
specificity, true negative rate	$\frac{TN}{N}$
precision	$\frac{TP}{TP + FP}$
F, F ₁ , F -score, harmonic mean of precision and recall	$\frac{2 \times precision \times recall}{precision + recall}$
F_{β} , where β is a non-negative real number	$\frac{(1+\beta^2) \times precision \times recall}{\beta^2 \times precision + recall}$

K-Fold Cross-Validation

☐ Key Concepts

- Partitioning: splits data into k equal-sized folds
- Rotation: each fold is a test set once, others form the training set
- Iteration: k rounds of training/testing

Process

- 1st Iteration: Train on D2 to Dk, test on D1.
- 2nd Iteration: Train on D1, D3 to Dk, test on D2.
- •
- kth Iteration: Train on D1 to D(k-1), test on Dk.

□ Advantages

- Bias Reduction: Each data point is used for training and testing.
- Robustness: Accuracy is averaged over k iterations.

Summary

- □ Classification: a form of data analysis that extracts models describing data classes. A classifier predicts categorical labels.
- □ Decision tree induction: a top-down recursive tree induction model, using an attribute selection measure to select the attribute tested for each non-leaf node in the tree ID3 as the example algorithm
 - Tree pruning: to improve accuracy by removing tree branches reflecting noise in the data.
- ☐ Bayesian classifier: based on Bayes' theorem of posterior probability
 - the effect of an attribute value on a given class is independent of the values of the other attributes
- □ Evaluation: accuracy, precision, recall, F1, ...

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THANK YOU!

