

# Data Mining and Data Warehousing 10

## Prediction

Overfitting & Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

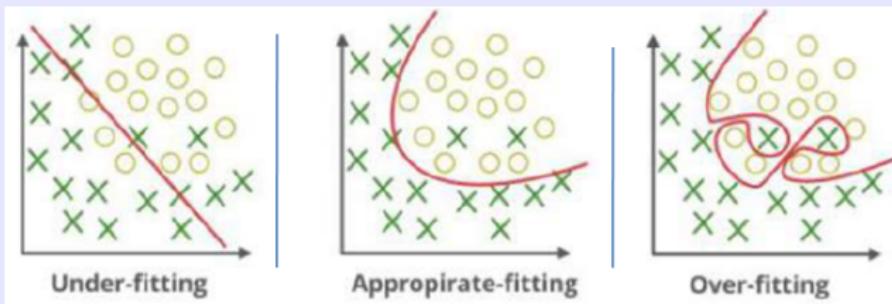
Performance Evaluation of Regression

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# Overfitting & Underfitting

## Underfitting

- When a model hasn't learned the patterns in the training data well and is unable to generalize well on the new data, it is known as underfitting
- An underfitting model has poor performance on the training data and will result in unreliable predictions
- Underfitting occurs due to high bias and low variance
- Techniques to reduce Underfitting**
  - Increase model complexity
  - Increase the number of features in the dataset
  - Reduce noise in the data
  - Increase the duration of training the data



Overfitting &amp; Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance Evaluation of Regression

# Overfitting & Underfitting

## Overfitting

- When a model performs very well for training data, but has poor performance with test data, it is known as overfitting
- Here, the machine learning model learns the details and noise in the training data such that it negatively affects the performance of the model on test data
- Overfitting can happen due to low bias and high variance
- **Techniques to reduce Overfitting**
  - Using K-fold cross-validation
  - Reduce model complexity
  - Training model with sufficient data
  - Remove unnecessary or irrelevant features
  - Early stopping during the training phase
- Overfitting is the result of using an excessively complicated model while underfitting is the result of using an excessively simple model or using few training samples
- A model that is underfit will have high training and high testing error while an overfit model will have extremely low training error but a high testing error

Overfitting &  
Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance  
Evaluation of  
Regression

## Classification vs. Prediction

- Both problems deal with the mapping of the input data to output data but in a different way
- Prediction and classification both are the supervised learning methods, where prediction is trained to predict real number outputs and classification is trained to identify/predict to which category the new values fall into
- Example (classification): Before starting of your 7th sem project, you want to predict whether it is accepted or rejected in final project defense
- Example (Prediction): Before starting 7th sem, if you want to predict how much score you want to obtain, here you use a prediction model by consulting previously obtained marks

Overfitting & Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance Evaluation of Regression

# Regression

## Regression

- Regression analysis can be used to model the relationship between one or more independent (or **predictor**) variables and a dependent (or **response**) variable
- The predictor variables are the attributes of interest describing the tuple. Generally, the values of the predictor variables are known
- The response variable is what we want to predict
- **Types of Regression**

- **Linear Regression:** A linear function of single independent variable  $x$  (Ex:  $y = a + b.x$ )
- **Multiple Linear Regression:** A linear function of multiple independent variable  $x_1, x_2, x_3, \dots$  (Ex:  
$$y = a + b_1.x_1 + b_2.x_2 + b_3.x_3\dots$$
)
- **Polynomial Regression:** A polynomial function of independent variable  $x$  (Ex:  $y = a + b_1.x + b_2.x^2 + b_3.x^3\dots$ )
- **Non-Linear Regression:** A non linear function with one or more parameters (Ex:  $y = a.e^{b.x}$ )

Overfitting &  
Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance  
Evaluation of  
Regression

# Correlation Coefficient

## Correlation Coefficient

- **Correlation** is a measure of the extent to which two variables are related
- **Positive correlation:** is a relationship between two variables in which both variables move in same direction.  
Ex: Height and Weight
- **Negative correlation:** is a relationship between two variables in which an increase in one variable is associated with a decrease in the other. Ex: Climbing mountain and getting colder
- **Zero correlation:** exists when there is no relationship between two variables. Ex: amount of tea taken and intelligence level
- Correlation can be expressed visually through **scatterplot**. It indicates the strength and direction of the correlation between the co-variables
- Correlation ranges between -1 and +1

Overfitting &amp; Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance Evaluation of Regression

# Correlation Coefficient...

## Correlation Coefficient...

- The correlation coefficient is calculated as

$$r = \frac{n \sum XY - \sum X \sum Y}{\sqrt{(n \sum X^2 - (\sum X)^2) \cdot (n \sum Y^2 - (\sum Y)^2)}}$$

where, n-> number of data points or observations

$\sum XY$  -> sum of the product of x-value and y-value for each point in the data set

$\sum X$  -> sum of the x-values in the data set

$\sum Y$  -> sum of the y-values in the data set

$\sum X^2$  ->sum of the squares of the x-values in the data set

$\sum Y^2$  ->sum of the squares of the y-values in the data set

Company	Sales in 1000s (Y)	Number of agents in 100s (X)
A	25	8
B	35	12
C	29	11
D	24	5
E	38	14
F	12	3
G	18	6
H	27	8
I	17	4
J	30	9

$$n = 10$$

$$\sum X = 80 \text{ & } \sum Y = 255$$

$$\sum XY = 2289$$

$$\sum X^2 = 756 \text{ & } \sum Y^2 = 7097$$

$$(\sum X)^2 = 6400 \text{ & } (\sum Y)^2 = 65025$$

$$r = 0.95$$

Overfitting &  
Underfitting

Prediction

Regression

Correlation Coefficient

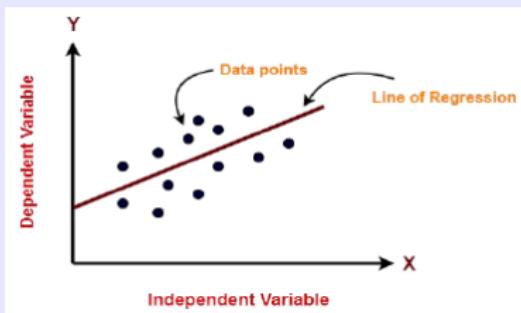
Linear Regression

Performance  
Evaluation of  
Regression

# Linear Regression

## Linear Regression

- Linear Regression is a supervised machine learning algorithm. It tries to find the best linear relationship that describes the given data
- Linear regression model represents the linear relationship between a dependent variable and independent variable(s) via a sloped straight line



- Linear regression are of two types:
  - **Simple Linear Regression:** Dependent variable depends only on a single independent variable
  - **Multiple Linear Regression:** Dependent variable depends on more than one independent variables

Overfitting &  
Underfitting

Prediction

Regression

Correlation Coefficient

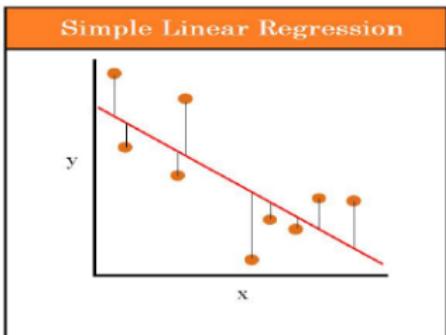
Linear Regression

Performance  
Evaluation of  
Regression

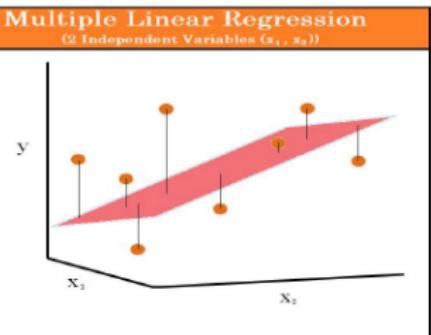
# Linear Regression...

## Linear Regression...

Fit the data with the **best line** which "goes through" the points



Fit data with the **best hyper plane** which "goes through" the points



Overfitting &  
Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance  
Evaluation of  
Regression

# Linear Regression...

## Linear Regression...

### Linear Regression cont...

A linear regression line has an equation of the form  $\mathbf{Y} = \mathbf{bX} + \mathbf{a} + \mathbf{e}$ ,

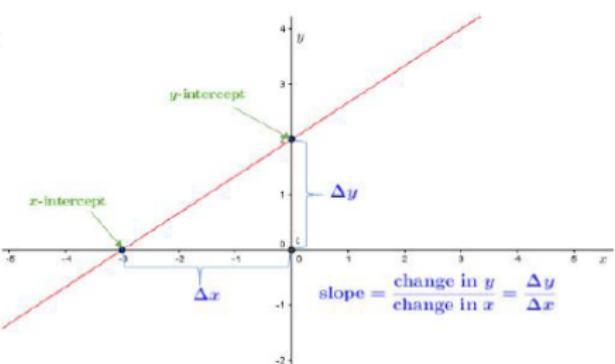
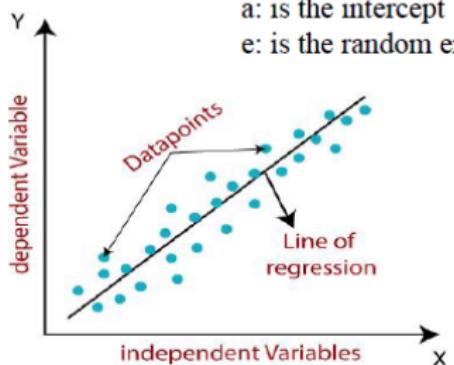
where X: explanatory variable

Y: is the dependent variable.

b: regression coefficient or slope of the line

a: is the intercept

e: is the random error



Overfitting & Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

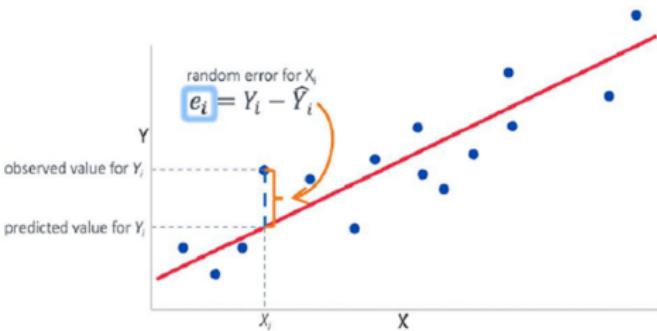
Performance Evaluation of Regression

# Linear Regression...

## Linear Regression...

The random error in the following linear equation of line:

$\hat{Y}_i$  is predicted values of  $Y_i$ .



The calculation of b and a is as follows:

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2} \quad a = \frac{\sum Y}{n} - b \frac{\sum X}{n}$$

If  $b > 0$ , then x(predictor) and y(target) have a positive relationship. That is increase in x will increase y.

If  $b < 0$ , then x(predictor) and y(target) have a negative relationship. That is increase in x will decrease y.

Overfitting &  
Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance  
Evaluation of  
Regression

# Linear Regression...

## Linear Regression...

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Overfitting &  
Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance  
Evaluation of  
Regression

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2} = \frac{10 \times 2289 - (80 \times 255)}{[10 \times 756 - (80)^2]} = 2.1466;$$

$$a = \frac{\sum Y}{n} - b \frac{\sum X}{n} = \frac{255}{10} - 2.1466 \frac{80}{10} = 8.3272$$

The linear regression will thus be Predicted  $\mathbf{Y = 2.1466 X + 8.3272}$

The above equation can be used to predict the volume of sales for an insurance company given its agent number. Thus if a company has 1000 agents (10 hundreds) the predicted value of sales will be around ?

# Performance Evaluation of Regression

## Mean Square Error (MSE)

MSE is defined as mean or average of the square of the difference between actual and estimated values.

Mathematically, it is represented as:

$$MSE = \frac{1}{n} \sum_{i=1}^n (actual\_value(i) - predicted\_value(i))^2$$

MSE is useful when large errors are particularly undesirable

Overfitting & Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance Evaluation of Regression

## Root Mean Square Error (RMSE)

It is just the square root of the mean square error.

Mathematically, it is represented as:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (actual\_value(i) - predicted\_value(i))^2}$$

RMSE provides error in the same units as the target variable

# Performance Evaluation of Regression

## Mean Absolute Percentage Error (MAPE)

The formula to calculate MAPE is as follows:

$$MAPE = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{\text{actual\_value}(i) - \text{calculated\_value}(i)}{\text{actual\_value}(i)} \right|$$

*MAPE is easy to interpret and easy to explain. The lower the value for MAPE, the better a model is able to predict values*

Since it expresses error as a percentage, making it scale-independent

Overfitting &  
Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance  
Evaluation of  
Regression

## Mean Absolute Error (MAE)

It calculates the average difference between the calculated values and actual values. It is also known as scale-dependent accuracy. The formula to calculate MAE is as follows:

$$MAE = \frac{1}{n} \sum_{i=1}^n |\text{actual\_value}(i) - \text{calculated\_value}(i)|$$

Overfitting &amp; Underfitting

Prediction

Regression

Correlation Coefficient

Linear Regression

Performance Evaluation of Regression

Month	1	2	3	4	5	6	7	8	9	10	11	12
Actual Value	42	45	49	55	57	60	62	58	54	50	44	40
Predicted Value	44	46	48	50	55	60	64	60	53	48	42	38
Error	-2	-1	1	5	2	0	-2	-2	1	2	2	2
Squared Error	4	1	1	25	4	0	4	4	1	4	4	4

Sum of Square Error=56

So,  $MSE = 56/12 = 4.6667$  $RMSE = \text{SQRT}(4.6667) = 2.2$ 

MAPE = 3.64%

MAE = 1.833

Ref: J. Han, M. Kamber and J. Pei, "Data Mining: Concepts and Techniques", Morgan Kaufmann, 3rd edition