randomized_response_bayes

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.

- YES .
- .
- 100 80 YES, 20 No . theta Bayesian estimation .

1. Stan .
2. cmdstanpy .
3. mcmc sampling ,

1 Randomized Response (Bayesian) — Smoking Prevalence with Stan + cmdstanpy

- From ChatGPT 5

Design: A fair coin is flipped per respondent.
- If **Heads** ( ), they must answer **YES** (forced YES).
- If **Tails** ( ), they answer **truthfully**.

Observed: out of **N = 100**, **YES = 80**, **NO = 20**.
Forced-YES probability: **p = 0.5**.

Let the true smoking prevalence be \$ \$.

Then the probability of observing YES is

$$q = p + (1 - p)\,\theta.$$

The data model is (Y Binomial(N, q)).
We use a Beta prior on ( ), (Beta(,)).

This notebook compiles a Stan model, samples using **cmdstanpy**, and visualizes posterior summaries.

1.1 0. Environment Setup

- You need **CmdStan** installed for `cmdstanpy` to run Stan models.
- The cell below will install `cmdstanpy` (if needed) and attempt to install CmdStan to your home directory (first-time only).
- If you already have CmdStan installed, you can skip the `install_cmdstan()` step and set the `CMDSTAN` environment variable accordingly.

[5]:
```python
# If you don't have cmdstanpy installed, uncomment the next line:
# !pip install cmdstanpy

# Optional: install CmdStan (first time only). This may take several minutes.
from cmdstanpy import install_cmdstan
install_cmdstan(compiler=True)
# After installation, you can check with:
import os
print("CMDSTAN path:", os.environ.get("CMDSTAN"))
```

```
CmdStan install directory: /Users/yndk/.cmdstan
CmdStan version 2.37.0 already installed
Test model compilation
CMDSTAN path: /Users/yndk/.cmdstan/cmdstan-2.37.0
```

1.2 1. Data & Hyperparameters

We set **N=100**, **YES=80**, **p=0.5** (forced YES probability).
You can adjust the Beta prior by changing `alpha`, `beta`.

[6]:
```python
N = 100
y_yes = 80
p_forced = 0.5

# Prior: Beta(alpha, beta). Use (1,1) for uniform, (2,2) for mild shrinkage,␣
↪etc.
alpha = 1.0
beta = 1.0

print({"N": N, "y_yes": y_yes, "p_forced": p_forced, "alpha": alpha, "beta":␣
↪beta})

# Quick method-of-moments point estimate (not Bayesian): theta_hat = (q_hat -␣
↪p)/(1-p)
q_hat = y_yes / N
theta_hat_mom = max(0.0, min(1.0, (q_hat - p_forced) / (1.0 - p_forced)))
print("Method-of-moments theta_hat (for reference only):", theta_hat_mom)
```

```
{'N': 100, 'y_yes': 80, 'p_forced': 0.5, 'alpha': 1.0, 'beta': 1.0}
Method-of-moments theta_hat (for reference only): 0.6000000000000001
```

1.3 2. Stan Model

We model: - (q = p + (1-p),) - (Y Binomial(N, q)) - Prior: (Beta(,))

We also generate a posterior predictive replicate `y_rep` and record a transformed `theta_via_q`.

[8]:
```python
stan_code = r"""
data {
```

```
  int<lower=0> N;                  // total respondents
  int<lower=0, upper=N> y_yes;     // observed YES count
  real<lower=0, upper=1> p_forced;// forced-YES probability (e.g., 0.5)
  real<lower=0> alpha;             // Beta prior alpha
  real<lower=0> beta;              // Beta prior beta
}
parameters {
  real<lower=0, upper=1> theta;   // true prevalence
}
transformed parameters {
  real<lower=0, upper=1> q;       // Pr(YES observed)
  q = p_forced + (1.0 - p_forced) * theta;
}
model {
  theta ~ beta(alpha, beta);
  y_yes ~ binomial(N, q);
}
generated quantities {
  int y_rep = binomial_rng(N, q);
  real<lower=0, upper=1> theta_via_q;
  theta_via_q = fmin(fmax((q - p_forced) / (1.0 - p_forced), 0), 1);
}
"""

# Write the Stan file to disk so cmdstanpy can compile it
with open("rr_yes_model.stan", "w") as f:
    f.write(stan_code)

print("Wrote Stan model to rr_yes_model.stan")
```

```
Wrote Stan model to rr_yes_model.stan
```

1.4 3. Compile & Sample with cmdstanpy

[9]:
```
import os
from cmdstanpy import CmdStanModel

# Compile the Stan model
model = CmdStanModel(stan_file="rr_yes_model.stan")

# Prepare data dict for Stan
data = {
    "N": N,
    "y_yes": y_yes,
    "p_forced": p_forced,
    "alpha": alpha,
    "beta": beta,
```

```python
}

# Sample
fit = model.sample(
    data=data,
    chains=4,
    parallel_chains=4,
    iter_warmup=1000,
    iter_sampling=1000,
    adapt_delta=0.9,
    seed=20251008
)

print(fit.diagnose())

# Summaries
summary = fit.summary()
summary
```

```
14:13:34 - cmdstanpy - INFO - compiling stan file
/Users/yndk/Desktop/KOS6002/RRT/rr_yes_model.stan to exe file
/Users/yndk/Desktop/KOS6002/RRT/rr_yes_model
14:13:37 - cmdstanpy - INFO - compiled model executable:
/Users/yndk/Desktop/KOS6002/RRT/rr_yes_model
14:13:37 - cmdstanpy - INFO - CmdStan start processing

chain 1 |          | 00:00 Status

chain 2 |          | 00:00 Status

chain 3 |          | 00:00 Status

chain 4 |          | 00:00 Status


14:13:38 - cmdstanpy - INFO - CmdStan done processing.


Checking sampler transitions treedepth.
Treedepth satisfactory for all transitions.

Checking sampler transitions for divergences.
No divergent transitions found.

Checking E-BFMI - sampler transitions HMC potential energy.
E-BFMI satisfactory.

Rank-normalized split effective sample size satisfactory for all parameters.

Rank-normalized split R-hat values satisfactory for all parameters.
```

```
Processing complete, no problems detected.
```

[9]:

	Mean	MCSE	StdDev	MAD	5%	50%
lp__	-51.964400	0.020650	0.711218	0.306010	-53.403100	-51.692600
theta	0.589148	0.002110	0.078328	0.078435	0.456976	0.592514
q	0.794574	0.001055	0.039164	0.039218	0.728488	0.796257
y_rep	79.421300	0.123108	5.624830	5.930400	70.000000	80.000000
theta_via_q	0.589148	0.002110	0.078328	0.078435	0.456976	0.592514

	95%	ESS_bulk	ESS_tail	ESS_bulk/s	R_hat
lp__	-51.467500	1442.85	1422.25	22902.4	1.00127
theta	0.712403	1378.00	1439.14	21873.0	1.00137
q	0.856202	1378.00	1439.14	21873.0	1.00137
y_rep	88.000000	2090.69	2520.76	33185.5	1.00012
theta_via_q	0.712403	1378.00	1439.14	21873.0	1.00137

1.5 4. Extract Posterior & Summaries

[10]:
```python
import numpy as np
import pandas as pd

theta = fit.stan_variable("theta")
q = fit.stan_variable("q")
theta_via_q = fit.stan_variable("theta_via_q")
y_rep = fit.stan_variable("y_rep")

def cred_int(x, level=0.95):
    lo = (1-level)/2
    hi = 1 - lo
    return np.quantile(x, lo), np.quantile(x, hi)

theta_mean = float(np.mean(theta))
theta_lo, theta_hi = cred_int(theta, 0.95)

print(f"Posterior mean theta: {theta_mean:.3f}")
print(f"95% CrI for theta: [{theta_lo:.3f}, {theta_hi:.3f}]")

# Save posterior draws to CSV for external use if needed
post = pd.DataFrame({
    "theta": theta,
    "q": q,
    "theta_via_q": theta_via_q,
    "y_rep": y_rep
})
post.to_csv("posterior_draws.csv", index=False)
print("Saved posterior draws -> posterior_draws.csv")
```

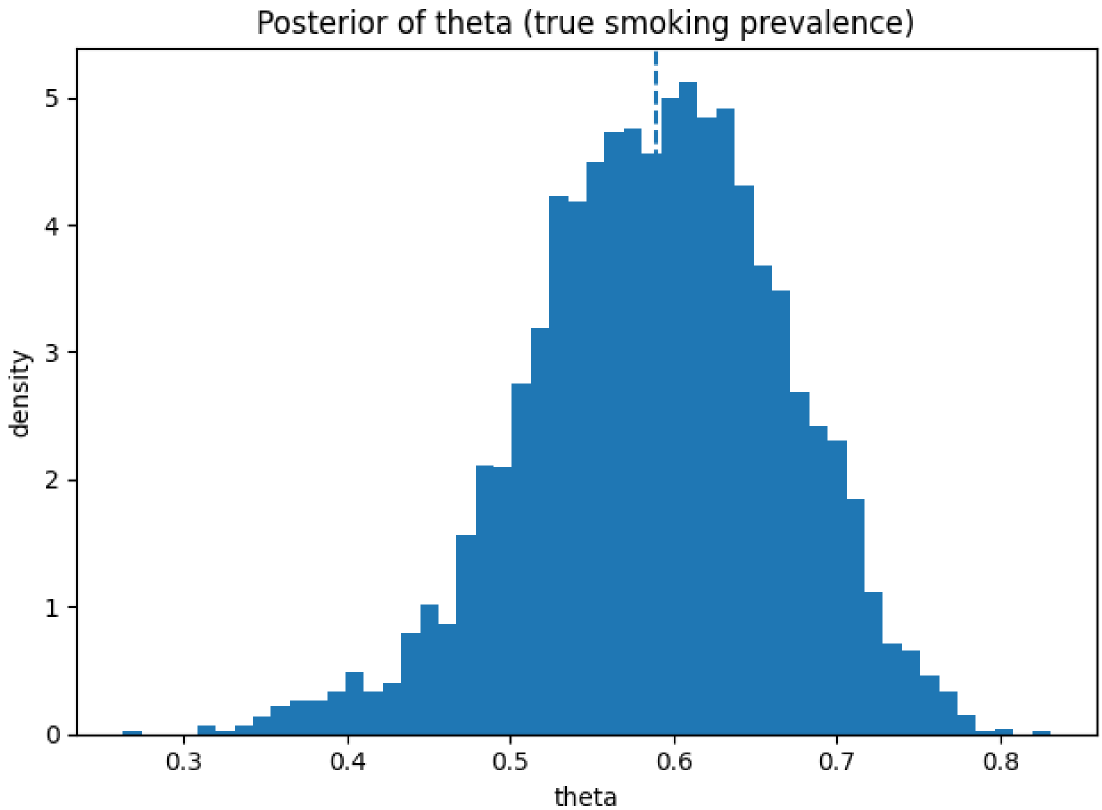
```
Posterior mean theta: 0.589
95% CrI for theta: [0.423, 0.731]
Saved posterior draws -> posterior_draws.csv
```

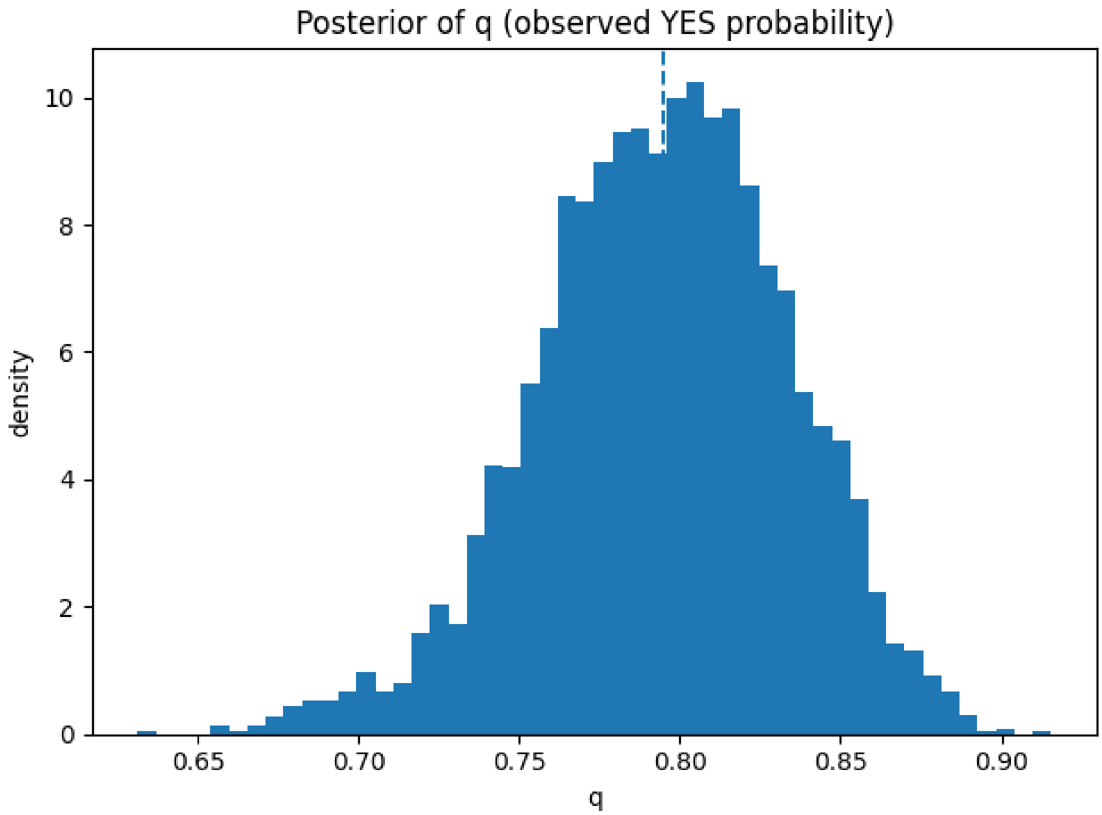
1.6 5. Visualizations

We plot: 1. Posterior density of ( )
2. Posterior density of (q)
3. Posterior predictive distribution of `y_rep`
4. ECDF of ( )

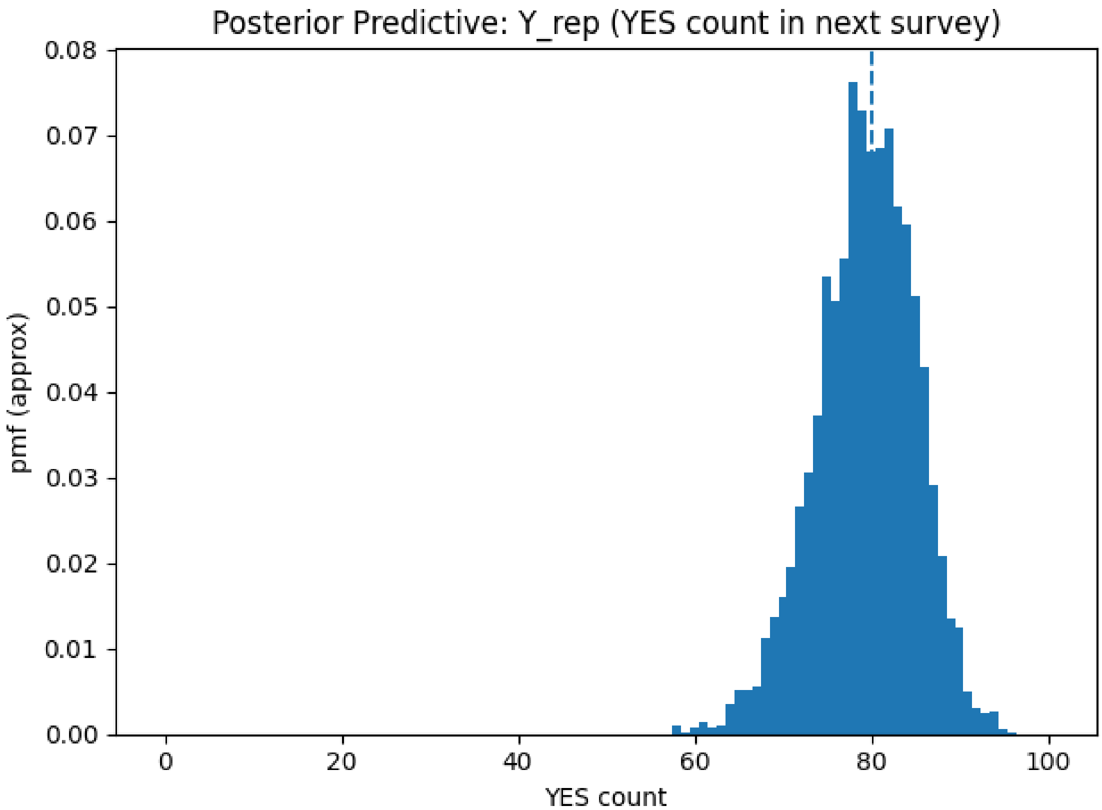
[14]:
```python
import matplotlib.pyplot as plt

# (A) Posterior of theta
plt.figure()
plt.hist(theta, bins=50, density=True)
plt.axvline(theta_mean, linestyle='--')
plt.title("Posterior of theta (true smoking prevalence)")
plt.xlabel("theta")
```

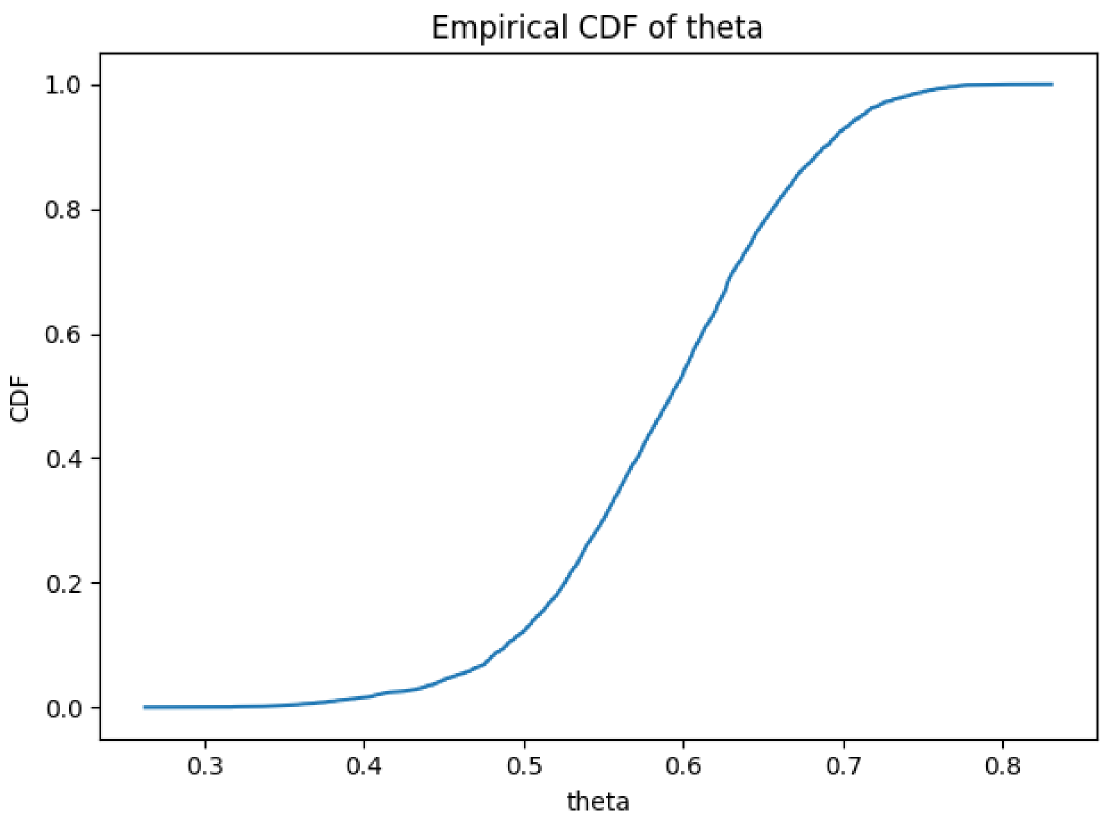
```python
plt.ylabel("density")
plt.tight_layout()
plt.show()

# (B) Posterior of q
plt.figure()
plt.hist(q, bins=50, density=True)
plt.axvline(np.mean(q), linestyle='--')
plt.title("Posterior of q (observed YES probability)")
plt.xlabel("q")
plt.ylabel("density")
plt.tight_layout()
plt.show()

# (C) Posterior predictive of y_rep
plt.figure()
plt.hist(y_rep, bins=range(0, N+2), align='left', density=True)
plt.axvline(y_yes, linestyle='--')
plt.title("Posterior Predictive: Y_rep (YES count in next survey)")
plt.xlabel("YES count")
plt.ylabel("pmf (approx)")
plt.tight_layout()
plt.show()

# (D) ECDF of theta
theta_sorted = np.sort(theta)
cdf = np.linspace(0, 1, len(theta_sorted))
plt.figure()
plt.plot(theta_sorted, cdf)
plt.title("Empirical CDF of theta")
plt.xlabel("theta")
plt.ylabel("CDF")
plt.tight_layout()
plt.show()
```


Posterior of theta (true smoking prevalence)
density
theta
0
1
2
3
4
5
0.3
0.4
0.5
0.6
0.7
0.8

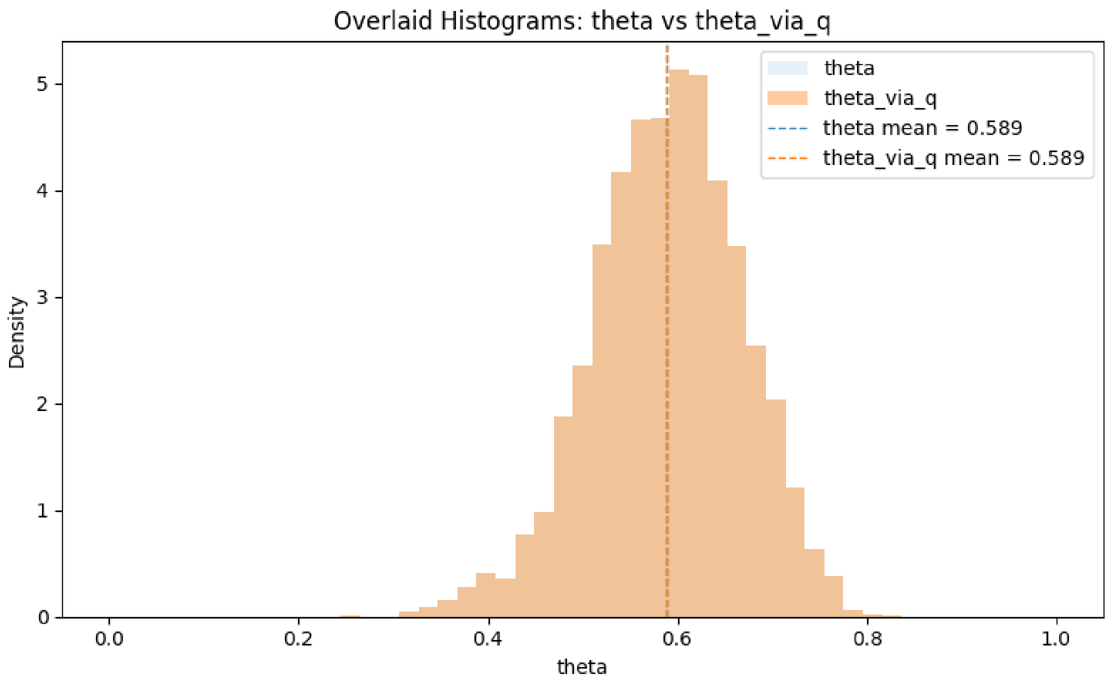


Posterior of q (observed YES probability)
density
0
2
4
6
8
10
0.65
0.70
0.75
0.80
0.85
0.90
q


Posterior Predictive: Y_rep (YES count in next survey)
pmf (approx)
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0
20
40
60
80
100
YES count

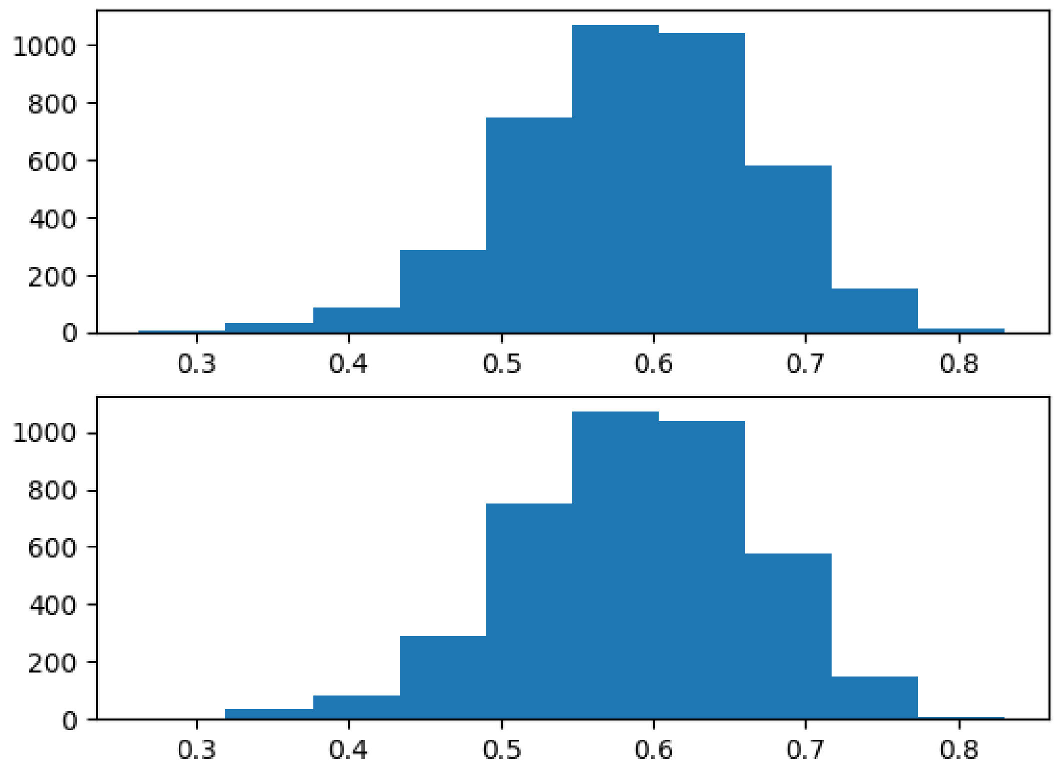


[16]:
```
plt.figure(figsize=(8, 5))
bins = np.linspace(0, 1, 50)

plt.hist(theta, bins=bins, density=True, alpha=0.1, color='C0', label='theta')
plt.hist(theta_via_q, bins=bins, density=True, alpha=0.4, color='C1',␣
    ↪label='theta_via_q')

plt.axvline(np.mean(theta), color='C0', linestyle='--', linewidth=1,␣
    ↪label=f"theta mean = {np.mean(theta):.3f}")
plt.axvline(np.mean(theta_via_q), color='C1', linestyle='--', linewidth=1,␣
    ↪label=f"theta_via_q mean = {np.mean(theta_via_q):.3f}")

plt.title("Overlaid Histograms: theta vs theta_via_q")
plt.xlabel("theta")
plt.ylabel("Density")
plt.legend()
plt.tight_layout()
plt.show()
```



[18]:
```
fig, axes = plt.subplots(2,1)
axes[0].hist(theta)
axes[1].hist(theta_via_q)
```

[18]:
```
(array([   4.,   32.,   82.,  288.,  747., 1069., 1039.,  579.,  149.,
          11.]),
 array([0.26267552, 0.31949895, 0.37632239, 0.43314582, 0.48996925,
        0.54679269, 0.60361612, 0.66043955, 0.71726298, 0.77408642,
        0.83090985]),
 <BarContainer object of 10 artists>)
```



1.7 6. Notes & Extensions

- **Misclassification**: Add small-error probabilities for forced/true responses to model careless errors.

- **Hierarchical Groups**: If multiple cohorts/years, model (_g) with a hierarchical Beta prior.

- **Different Randomizers**: If (p 0.5) or rules are asymmetric, replace `p_forced` accordingly.

Feel free to modify priors (`alpha`, `beta`) to reflect domain knowledge.

- : $ q = p + (1 - p) $
- : $\theta \sim Beta(\alpha, beta)$
- : θ , 95% Credible Interval
-
  - $p(\theta|data)$
  - $p(q|data)$
  - posterior predictive y
  - ecdf$(\theta|data)$

- / : YES No (misclassification)

- / : θ_g
- : $p = 0.5$ YES/NO

[ ]:
```

```