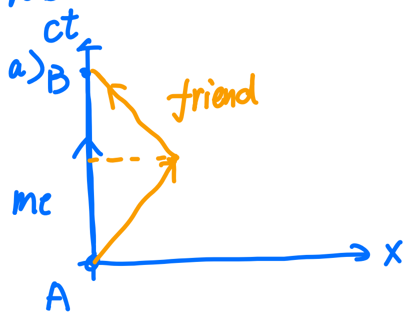
Prob 1.

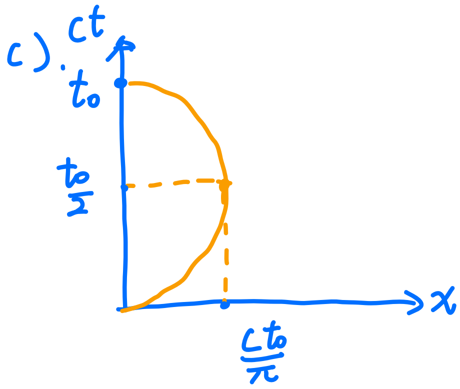
a)



b) $\tau_{me} = \int_0^{t_{end}} dt \cdot \sqrt{1-0} = t_{end} = 2 \text{ days}$

$\tau_{her} = \int_0^{t_{end}} dt \cdot \sqrt{1 - \frac{1}{c^2} \cdot \frac{99}{100} c^2} = \frac{t_{end}}{10} = 0.2 \text{ days}$

c)



d) $\tau'_{her} = \int_0^{t_0} dt \cdot \sqrt{1 - \frac{1}{c^2} \left(\frac{dx}{dt}\right)^2}$

$= \int_0^{t_0} dt \cdot \sqrt{1 - \cos^2\left(\frac{\pi t}{t_0}\right)} = \int_0^{t_0} \left|\sin\left(\frac{\pi t}{t_0}\right)\right| dt$

$= -\frac{t_0}{\pi} \cos\left(\frac{\pi t}{t_0}\right)\Big|_0^{t_0} = \frac{2t_0}{\pi}$

Prob 2.

2a) $d\theta = d\phi = dt = 0 \quad \therefore ds^2 = \frac{dr^2}{1 - \frac{r_s}{r}}$

$l = \int dl = \int_{2r_s}^{3r_s} \frac{dr}{\sqrt{1 - \frac{r_s}{r}}} = r_s \int_2^3 \frac{dx}{\sqrt{1 - x^{-1}}}, \quad x = \frac{r}{r_s}$

$= (1 + \ln 2) r_s$

2b) $l' = r_s \int_{1.1}^{2.1} \frac{dx}{\sqrt{1 - x^{-1}}} \approx 3.3979 \, r_s$

2c) let $y = \frac{r_s}{r}$, $y \ll 1$, $(1-y)^{-1} \approx 1 + (-1)(-y) + \cdots \approx 1 + y$

$\therefore l = \int_{r_1}^{r_2} \frac{dr}{\sqrt{1 - r_s/r}} \approx \int_{r_1}^{r_2} \left(1 + \frac{r_s}{r}\right) dr = r_2 - r_1 + r_s \cdot \ln \frac{r_2}{r_1}$

2d) $d\theta = dr = dt = 0$, $ds^2 = r^2 \sin^2\theta \, d\phi^2 = r^2 d\phi^2$

$C = \int_0^{2\pi} \sqrt{ds^2} = \int_0^{2\pi} r \, d\phi = 2\pi r$

Prob 3

3a). curve equation : $dx = \frac{z\,dz}{2R_c}$

$$dl^2 = dx^2 + dz^2 = dx^2 + \left(\frac{2R_c}{z}\right)^2 dx^2 = \left[1 + \frac{4R_c^2}{4R_c(x-R_c)}\right] dx^2 = \frac{x-R_c+R_c}{x-R_c}\, dx^2$$

$$= \frac{dx^2}{1-R_c/x}$$

3b). paraboloid: $dr = \frac{z\,dz}{2R_c}$

$$dl^2 = dx^2 + dy^2 + dz^2 = dr^2 + r^2(\sin^2\phi + \cos^2\phi)\,d\phi^2 + dz^2, \quad dr = \frac{z\,dz}{2R_c}$$

according to 3a),

$$dl^2 = \frac{dr^2}{1-R_c/r} + r^2 d\phi^2$$