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Securities Markets in Which Some Investors Receive Information About Cash Flow Betas

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Abstract. We analyze a single-factor setting in which there is private information regarding cash flows as well as their betas. Private information about betas, together with market makers' risk aversion and mean betas' nonnegativity, implies a nonlinear price schedule whose stochastic slope covaries positively with order flow when the expected factor payoff is positive and vice versa. We predict a negative relation between the covariance and expected returns and an attenuation of the beta anomaly in asset returns after accounting for this relation. Empirical tests confirm these predictions.

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1. Introduction

A large literature in theoretical finance considers how agents with private information trade in financial markets and, in turn, how their actions affect market liquidity and the informational efficiency of prices.[1] From a standard factor model, the components of cash flows that are unique to the firm encompass both the idiosyncratic component and the coefficient on the common factor (beta). Thus, an agent who specializes in firm-specific information acquisition has incentives to acquire signals about each of the components provided that both have stochastic elements.

There is empirical evidence that betas do shift over time.[2] From an economic standpoint, one such scenario arises when the firm expands into new product lines. For example, Nokia started out as a paper manufacturer and moved into personal electronics. Such a transition would naturally affect its beta. In general, shifting real investment policies (such as taking projects across different industries or sectors) can alter betas. Forecasting such beta shifts is of value to agents in financial markets, and indeed, estimating the cyclicality of firms' revenues is a component of the chartered financial analysts' education on security analysis techniques.[3]

To understand the equilibrium consequences of speculating on beta information, an asymmetric information model with private beta signals is needed, and our paper fills this gap. Indeed, to our knowledge, our paper is the first to consider private beta information within the multiplicative setting of a factor model. In our framework, the cash flows of a security follow a single-factor structure: $v = \beta\theta + \epsilon$. That is, cash flows emanate from three random sources: ϵ, θ, and β. Consistent with evidence (e.g., Fama and French 1992), we assume that betas are positive on average. We model private information about both firm-specific components of value so that each informed agent has a private signal about either ϵ or β. In our base model, each informed trader can buy or sell up to a maximum quantity, and noise trades also have bounded support. A risk-averse, competitive market maker absorbs the net order flow of the other traders.[4]

The nature of equilibrium depends on whether the mean of the factor θ (denoted $\overline{\theta}$) is positive or negative. For expositional clarity, consider first the case of $\overline{\theta} > 0$. We consider an intuitive equilibrium in which the β and ϵ informed both buy on high signals and sell on low signals. The market maker learns about both the mean and variance of the asset from the order flow. Because each risk-neutral informed trades up to a discrete maximum in equilibrium, the pricing function increases in steps, and we also find that it is linear between steps. The steps of the pricing function reflect the market maker's conditional mean updates, whereas the slopes are proportional to the conditional variance.

We show that the conditional mean increases in the signed order flow, which is intuitive. We then develop two intriguing results about the conditional variance (alternatively, the slope), which, as we show, plays a key role in asset risk premia. First, we show that the slope is higher for large buy orders than for large sell orders. Intuitively, a large buy order indicates that informed traders have observed a high β with high probability, which implies a high cash flow variance and, hence, a high price impact, whereas a large sell order implies a low β and a low conditional variance and, therefore, a low price impact.[5] Second, the slope is nonmonotonic in the order flow and, under reasonable conditions, takes on a hump shape. This is because extreme values of the order flow resolve more uncertainty relative to intermediate values. Specifically, a very high (low) order flow implies that ϵ and β are likely very high (low), and this belief update lowers conditional variance related to intermediate order flow values.

We next develop implications for asset returns. When the market maker observes a large buy order, the higher beta estimate implies a higher conditional variance. Thus, there is a positive covariance between the price schedule slope and order flow.[6] A high order flow also implies a high price and a lower expected return because of the high premium demanded by the market maker for shorting the risky asset. The bigger the covariance between order flow and the slope, the higher the price and the lower the expected return. Further, the covariance is larger (and the expected return smaller) if the firm's beta has a higher ex ante mean. We, thus, obtain a beta anomaly that high beta stocks underperform low beta ones (e.g., Baker et al. 2011, Frazzini and Pedersen 2014).

Let us turn now to the case in which the factor mean is negative. In this case, β-informed traders sell on high beta, which causes a higher price impact on the sell side relative to the buy side. Therefore, the covariance between the slope of the price schedule and order flow is smaller (i.e., more negative) and expected returns are larger with higher mean beta. Thus, our model yields the implication that the beta anomaly is more likely to obtain in boom states (states of positive expected factor payoffs) than in recessions.

With bounded support of noise trades under an assumed uniform distribution, we can solve the model analytically; however, the price schedule is stepwise linear. We consider a variant of the main model in which the noise trades follow a normal distribution. In this case, the price schedule is smooth; however, the model cannot be solved in closed form. Our numerical analysis of this model variant yields similar results as in the main model however.

We use transactions data in conjunction with the standard Center for Research in Security Prices (CRSP) database to test key predictions from our theoretical model. Because some of our implications are conditional on the signs of factor means, we consider proxies for periods when these signs are likely to be positive or negative. Specifically, we assume that factor means are negative and positive during National Bureau of Economic Research (NBER) recession and nonrecession months, respectively. We validate these proxies by documenting that increments to aggregate earnings are, on average, indeed negative during recessions and positive during nonrecessions with the difference being statistically significant.[7]

Our tests concern the novel covariance between order flows and the slope of the pricing schedule (*Cov*). First, our analysis predicts a negative relation between *Cov* and future stock returns. Second, we predict that *Cov* should be positive in nonrecessionary periods and negative in recessionary periods. Third, our model suggests a positive relation between *Cov* and betas in nonrecession periods and vice versa. Finally, we predict that accounting for the cross-sectional relation between *Cov* and expected returns should attenuate the beta anomaly in asset returns. Our evidence generally confirms these predictions.

Our analysis contributes to the extensive work on informed trading in financial markets (e.g., Grossman and Stiglitz 1980, Admati 1985, Glosten and Milgrom 1985, Kyle 1985, Subrahmanyam 1991b, Hughes et al. 2007, Kacperczyk et al. 2016). Some of these papers assume factor structures but with constant betas. More recent work by Albagli et al. (2021) makes an effort to use information aggregation to explain asset-pricing puzzles, which shares similarity to our approach that ties expected asset returns to the market's information structure. In an interesting paper, Andrei et al. (2023) show that, when aggregate macroeconomic uncertainty is high, firms with higher betas attract more attention and react more to news because the benefit of acquiring and trading on factor-related information about high-beta firms is high. We complement all of these papers by explicitly considering private information about asset betas.

Some existing models explore multiple sources of cash-flow information, which are related to our ϵ and β information (e.g., Goldman 2005, Yuan 2005, Kondor 2012, Goldstein and Yang 2015). In those models, the multiple pieces of cash-flow information are additive, and so the results in those papers are distinct from those in our paper. Specifically, our results on the asymmetric price responses to order flows as well as the asset pricing implications of the covariance between order flows and the slope of the price schedule are unique to the multiplicative feature of β information. In Ganguli and Yang (2009), traders' learning about payoff and supply information can lead to complementarities in information acquisition and multiple equilibria. Another related paper is Smith (2019), in which traders with information about the first moment trade in a spot market, whereas those with second moment information trade in a derivative. Our paper differs from the preceding papers by capturing $\epsilon-$ and β-informed trading in the same asset market and isolating the novel role of the covariance between order flow and the slope of the pricing schedule in asset pricing.

There is also a growing literature that explores the implications of beta uncertainty. Armstrong et al. (2013) find that the risk premium increases in beta uncertainty. Barahona et al. (2021) discuss the effect of beta uncertainty on asset prices under ambiguity aversion.[8] In other related work, Heinle et al. (2018) and Beyer and Smith (2021) impose a factor structure on asset payoffs and explore the impact of disclosures about risk factors on expected returns. In these models, there is no information asymmetry, and the asset price is typically determined by a representative agent. We add to these studies by introducing information asymmetry about cash-flow betas into financial markets, which allows us to speak to key financial market variables, such as price impact and its variability. In terms of asset pricing, our asymmetric information framework connects the expected returns to the degree of information asymmetry about cash-flow betas.

2. A Model of Asymmetric Information About Cash-Flow Betas

We now describe the structure of our model (Section 2.1) followed by a characterization of the equilibrium (Section 2.2).

2.1. The Economic Setting

2.1.1. Environment. There is a risky asset traded at $t = 0$ and liquidated at $t = 1$. The liquidation value of the asset is

$$v = \beta\theta + \epsilon. \tag{1}$$

The payoff v in Equation (1) is exposed to three dimensions of uncertainty. First, the market factor, θ, is normally distributed with mean $\overline{\theta}$ and volatility $\sigma_\theta > 0$; that is, $\theta \sim N(\overline{\theta}, \sigma_\theta^2)$.[9] Second, the factor loading β takes on two values, $\overline{\beta} - \Delta_\beta$ and $\overline{\beta} + \Delta_\beta$, with equal probability. Here, $\overline{\beta} > 0$ and $\Delta_\beta > 0$ are the mean and volatility of β.[10] The third component ϵ takes on two values, $\Delta_\epsilon > 0$ and $-\Delta_\epsilon < 0$, with equal probability. That is, ϵ has a mean of zero and volatility of Δ_ϵ. All three payoff components $(\theta, \epsilon, \beta)$ are mutually independent. Note that, if $\sigma_\theta = 0$, beta information becomes similar to ϵ information, affecting asset cash flow in an additive way.

At date $t = 0$, the risky asset is traded among three types of traders: (i) a unit mass of informed traders within which different traders alternatively observe signals about the ϵ and β components of the payoff; (ii) noise traders (or liquidity traders) with exogenous demand; and (iii) a risk-averse, competitive market maker who sets the price after seeing the total order flow from informed traders and noise traders. We describe these agents in more detail below.

2.1.2. Informed Traders and Noise Traders. Informed traders are risk-neutral and belong to one of two groups. The first group of traders, labeled as ϵ-informed traders, observe the realization of ϵ perfectly (and have no information about β). The second group of traders observe the realization of β perfectly (and have no information about ϵ). This dichotomy makes the analysis simpler, but the thrust of our economic arguments is unaltered when we allow traders to be informed about both components. We use χ_ϵ and χ_β to denote the masses of the two groups informed about ϵ and β, respectively, where $\chi_\epsilon \in [0,1], \chi_\beta \in [0,1]$, and $\chi_\epsilon + \chi_\beta = 1$. We denote an individual informed trader by $i \in [0,1]$ and restrict the asset position of the investor to lie on the interval $[-1, +1]$.[11] Each informed trader solves the following optimization problem:

$$\max_{y_i \in [-1,+1]} \mathbb{E}[y_i(v - p)|\Omega_i], \tag{2}$$

where $\Omega_i = \{\epsilon\}$ for ϵ-informed and $\Omega_i = \{\beta\}$ for β-informed traders. Hence, informed traders buy one unit of the asset when $\mathbb{E}[v - p|\Omega_i] > 0$, sell one unit when $\mathbb{E}[v - p|\Omega_i] < 0$, and are indifferent when $\mathbb{E}[v - p\,|\Omega_i] = 0$.[12] We use y_ϵ and y_β to denote the equilibrium demands by ϵ- and β-informed traders, respectively.

Noise traders trade for exogenous reasons and collectively demand z. As in Glosten and Milgrom (1985), we assume for tractability that the noise trader positions are bounded. Given that these traders are likely to be retail investors who likely face wealth constraints, this assumption is not unreasonable. For ease of analytical presentation, we assume that z is drawn from a uniform distribution on the interval $[-1,1]$ and that it is independent of all other random variables. The total order flow is, thus, given by

$$X = \chi_\epsilon y_\epsilon + \chi_\beta y_\beta + z. \tag{3}$$

2.1.3. Market Maker and Asset Prices. The price is set by the market maker. Because the new feature of β information is that it affects cash-flow volatility in a multiplicative way, we assume that the market maker is risk-averse, so this volatility is reflected in the price. We follow Subrahmanyam (1991a) and assume that the risk-averse market maker observes total order flow X and earns the "autarky" utility, which is normalized to zero for convenience. We write the market maker's expected utility as[13]

$$\mathbb{E}[U_{MM}] = \mathbb{E}[X(p - v)|X] - \frac{\gamma}{2}\mathrm{Var}[X(p - v)|X], \tag{4}$$

where $\gamma > 0$ represents the market maker's coefficient of risk aversion. Imposing $\mathbb{E}[U_{MM}] = 0$ and solving for p yields

$$p = \underbrace{\mathbb{E}(v|X)}_{\text{adverse selection}} + \underbrace{\frac{\gamma}{2}X\mathrm{Var}(v|X)}_{\text{inventory concerns}}. \tag{5}$$

The first term, $\mathbb{E}(v|X)$, in the price function captures the usual adverse selection intuition that the market maker will set a higher stock price in response to a high signal about the asset payoff based on total order flow X. The second term, $\frac{\gamma}{2}X\mathrm{Var}(v|X)$, reflects the inventory concerns of the market maker. We can view the market maker in our setting as representing risk-averse uninformed traders in the Grossman and Stiglitz (1980) setting. To see this point, let $d = \frac{\mathbb{E}[v|X]-p}{(\gamma/2)\mathrm{Var}(v|X)}$ denote the demand function of an uninformed, risk-averse investor as in Grossman and Stiglitz (1980) (see their equation (8)). Then, the market-clearing condition, $d + X = 0$, leads to the same pricing function as that in Equation (5).

The above assumptions allow analytic solutions and transparent intuition. We leave extensions for the Online Appendix. (All sections of this appendix are prefixed with "OA.") For example, in Online Appendix OA.1, we analyze conditions under which the equilibrium with risk-averse informed is observationally equivalent to that under risk-neutral informed. In Online Appendix OA.2, we consider a setting with multiple assets and multiple market makers and show that our main comparative statics results are qualitatively unaltered in this extension. In a sense, our base model can be viewed as a limit of a model in which market makers have portfolio considerations. Thus, raising cognitive costs of making a market in all stocks other than a particular one would lead to our model in the limit.

2.2. Equilibrium Characterization

We seek an equilibrium in which informed traders are active and trade in the financial market, that is, $y_i \neq 0$. The total order flow X aggregates the traders' private signals and reveals information about the payoff components ϵ and β. The market maker then utilizes this information in X to set the asset price. In our model, expected profit maximization ensures that the informed trade up to -1 or $+1$, that is, the extremes of the support of the distribution for noise traders' orders. Therefore, the total order flow falls in the interval $[-2, +2]$, and the estimate of the asset's value changes discretely for specific order-flow values at discrete intervals. Within these intervals, the slope of the pricing schedule captures the market maker's estimate of the conditional variance of the asset's value for that interval. To ensure prices for every order-flow realization, we also postulate that, at the points of discrete changes in the pricing function (which occur with measure zero), the market maker chooses the higher price for nonnegative order flow and the lower price for negative order flow.[14] The following theorem characterizes the equilibria of the model. The proof of this theorem appears in Appendix A.1 (all equations in this appendix are prefixed with "A").

Theorem 1 (Equilibrium). *A unique pure-strategy (type* 1*) equilibrium in which both types of informed traders trade on their information exists if and only if market maker risk aversion, γ, is below a maximum threshold, and the proportion of β-informed traders, χ_β, lies in a range defined in the appendix. The equilibrium is characterized as follows:*

1(a). *If the factor mean $\overline{\theta} > 0$, both the ϵ and β informed buy (sell) on high (low) signals.*

1(b). *If $\overline{\theta} < 0$, whereas the ϵ informed behave as in* 1(a), *the β informed buy (sell) on a low (high) signal.*

In the only other pure-strategy equilibrium (type 2*), the trading behavior of β informed is the same as in type* 1 *above. Further, the ϵ informed trade as follows:*

2(a). *If the factor mean $\overline{\theta} > 0$, then ϵ informed always sell.*

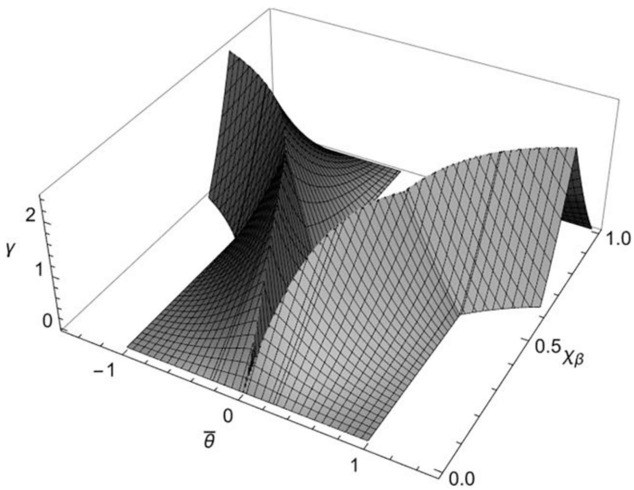
2(b). *If the factor mean $\overline{\theta} < 0$, then ϵ informed always buy.*

This equilibrium exists in ranges for γ and χ_β defined in the appendix and is mutually exclusive with type 1 *because the minimum threshold required for γ in type* 2 *exceeds the maximum threshold in type* 1.

In the type 1 equilibrium, order flows and prices reflect two-dimensional information; this equilibrium is, thus, the focus of our ensuing analysis. The equilibrium's parameter restrictions ensure positive expected profits for both types of informed traders. First, if market maker risk aversion γ is too high, informed trading is unprofitable, so the equilibrium exists for sufficiently low γ. The restriction on χ_β is related to the adverse selection component in the pricing function. Note that the market maker prices in a probability-weighted average of ϵ and β. Thus, there is a crowding-out effect between different types of informed trading. Specifically, if one dimension is much more payoff-relevant (e.g., if Δ_ϵ is very large relative to $\Delta_\beta|\overline{\theta}|$), trading on the less relevant dimension might become unprofitable and, therefore, cease. This effect can be offset if the order flow reveals more information about the more relevant dimension and helps camouflage trading on the other dimension; that is, if Δ_ϵ is large, then χ_β must be small. As a result, the upper bound on χ_β increases with the payoff relevance of β information (i.e., $\Delta_\beta|\overline{\theta}|$). Figure 1 illustrates the existence conditions for the type 1 equilibrium and confirms that its existence requires a sufficiently low γ.[15] Further, as argued above, the bigger is $|\overline{\theta}|$, the larger is the required upper bound on χ_β (for the fixed value of Δ_β).

In the type 2 equilibrium, ϵ-informed traders always buy or always sell, depending on the sign of $\overline{\theta}$, so that total order flow only reveals information about β. To

Figure 1. Using a Three-Dimensional Plot, This Figure Plots the Existence Conditions for the Type 1 Equilibria in Theorem 1 Across Ranges of γ, χ_β, and $\overline{\theta}$



Notes. Other parameter values are $\overline{\beta}=1, \Delta_\beta=\frac{1}{2}, \Delta_\epsilon=\frac{1}{4}$, and $\sigma_\theta=\frac{1}{4}$. Type 1 equilibria exist in the parameter ranges below the gray surfaces to the left and right of $\overline{\theta}=0$.

see the economic intuition for this equilibrium, note that a low χ_β reveals too much information about ϵ and makes trading on ϵ information unprofitable. In this case, ϵ informed always buy or always sell. However, this behavior also requires a sufficiently high χ_β because, then, β informed generate a large covariance between X and $\text{Var}(v|X)$, and the ϵ-informed profit from trading on this covariance via their always-buy or always-sell strategies.[16] Because this covariance scales with the market maker's degree of risk aversion, a necessary condition for this equilibrium is that γ not be too low. However, we further require γ not to be too high to allow β informed to profit on their private signals. Thus, we find that the type 2 equilibrium occurs when γ and χ_β fall in an intermediate range. Further, just as in type 1, if $|\overline{\theta}|$ is small, then trading on β information is not profitable, and the type 2 equilibrium does not exist. We also find that type 2 never exists together with the type 1 because the required γ ranges for the two types do not overlap. In Figure A.1, we illustrate these results and confirm that the two types of equilibria do not coexist.

From Figures 1 and A.1, we also observe the range of existence of either type of equilibrium. In particular, the equilibria obtain for large but not small values of the absolute factor mean ($|\overline{\theta}|$). To see why, consider the case when the factor mean is exactly zero. In this case β-informed traders do not find it worthwhile to trade because, regardless of what they learn about β, their expected profits are zero. Given the existence of a bid–ask spread, the same observation holds in a neighborhood of $\overline{\theta}=0$. Thus, the absolute value of the factor mean has to be sufficiently high for either equilibrium type to exist.

3. Implications of Private Information About Betas

Henceforth, we focus on the equilibrium in which both the ϵ- and β-informed trade on their signal (i.e., on the type 1 equilibrium of Theorem 1). In this section, we discuss implications of this equilibrium for the mean and variance of market liquidity. In the next section, we show that the covariance between the stochastic slope of the pricing function and order flows drives a wedge between the expected asset price and its expected value, and the size of this wedge (and, hence, the expected return) is determined by the degree of β-information asymmetry.

3.1. Characteristics of the Pricing Schedule

Because each risk-neutral informed trades up to a discrete maximum in equilibrium, the price schedule is a piecewise linear function. From Equation (5), the schedule can be defined as $p(X)=\alpha(X)+\delta(X)\cdot X$, where $\alpha(X)=\mathbb{E}[v|X]$ and $\delta(X)=(\gamma/2)\text{Var}(v|X)$. For convenience, let $\epsilon_H\equiv\Delta_\epsilon$ and $\epsilon_L\equiv-\Delta_\epsilon$. The appendix shows that, for $\overline{\theta}>0$ and $\chi_\epsilon>\chi_\beta$, $\alpha(X)$ is the following discrete function of X (with analogous expressions for the complementary ranges of $\overline{\theta}$ and χ_ϵ):

$$\alpha(X) = \begin{cases} \epsilon_H+(\overline{\beta}+\Delta_\beta)\overline{\theta} & \forall X\in[1+\chi_\epsilon-\chi_\beta, 2], \\ \epsilon_H+\overline{\beta}\overline{\theta} & \forall X\in[1-\chi_\epsilon+\chi_\beta, 1+\chi_\epsilon-\chi_\beta), \\ \dfrac{\Delta_\epsilon}{3}+\left(\overline{\beta}+\dfrac{\Delta_\beta}{3}\right)\overline{\theta} & \forall X\in[0, 1-\chi_\epsilon+\chi_\beta), \\ -\dfrac{\Delta_\epsilon}{3}+\left(\overline{\beta}-\dfrac{\Delta_\beta}{3}\right)\overline{\theta} & \forall X\in[-1+\chi_\epsilon-\chi_\beta, 0), \\ \epsilon_L+\overline{\beta}\overline{\theta} & \forall X\in[-1-\chi_\epsilon+\chi_\beta, -1+\chi_\epsilon-\chi_\beta), \\ \epsilon_L+(\overline{\beta}-\Delta_\beta)\overline{\theta} & \forall X\in[-2, -1-\chi_\epsilon+\chi_\beta). \end{cases}$$

As can be seen, $\alpha(X)$ is characterized by jumps at threshold points of X;[17] these are points at which the market maker updates the conditional expectation of the asset's value. Note that the conditional expectation is increasing in X, which is intuitive.

To consider $\alpha(X)$ further, suppose the order flow is particularly high, say, close to two. In this case, the market maker can infer that both ϵ- and β-informed traders have observed good news and, thus, sets the price based on $\epsilon=\Delta_\epsilon$ and $\beta=\overline{\beta}+\Delta_\beta$. In contrast, if the total order flow is particularly low, say, close to -2, then the market maker is confident that both ϵ- and β-informed traders have observed bad news. The price is, thus, set based on $\epsilon=-\Delta_\epsilon$ and $\beta=\overline{\beta}-\Delta_\beta$. For intermediate cases of the order flow, the market maker sets the price taking into account the assessment of the asset's value given the exogenous parameters χ_ϵ and χ_β.

An easy way to further obtain intuition about $\alpha(X)$ is to consider

$$\alpha(2) - \alpha(-2) = 2[\Delta_\epsilon + \Delta_\beta \overline{\theta}].$$

Thus, at the extreme ends of the order flow range, the market maker knows that informed traders have observed extreme values of ϵ and β, and the α difference accordingly spans the full range of ϵ and β. In general, the difference in $\alpha(X)$ for equal magnitudes of buying and selling increases in Δ_ϵ and Δ_β as these quantities indicate the profit potential to informed traders. Note that $\alpha(X)$ does not depend on factor variance, which has no role in the conditional expectation.

We now turn to the slope of the price function $\delta(X)$. Because this slope is related to the conditional variance of the asset's value, it plays a key role in asset pricing via the risk premium. Hence, we analyze it in some detail. For convenience, let $\beta_H = \overline{\beta} + \Delta_\beta$ and $\beta_L = \overline{\beta} - \Delta_\beta$. The appendix shows that, for $\overline{\theta} > 0$ and $\chi_\epsilon > \chi_\beta$, the slope can be characterized as follows (with analogous expressions for the complementary ranges):

$$2\gamma^{-1}\delta(X) = \begin{cases} \beta_H^2 \sigma_\theta^2 & \forall X \in [1 + \chi_\epsilon - \chi_\beta, 2], \\ \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2 \overline{\theta}^2 & \forall X \in [1 - \chi_\epsilon + \chi_\beta, \\ & \quad 1 + \chi_\epsilon - \chi_\beta), \\ \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2 \overline{\theta}^2 & \forall X \in [0, 1 - \chi_\epsilon + \chi_\beta), \\ \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2 \overline{\theta}^2 & \forall X \in [-1 + \chi_\epsilon - \chi_\beta, 0), \\ \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2 \overline{\theta}^2 & \forall X \in [-1 - \chi_\epsilon + \chi_\beta, \\ & \quad -1 + \chi_\epsilon - \chi_\beta), \\ \beta_L^2 \sigma_\theta^2 & \forall X \in [-2, -1 - \chi_\epsilon \\ & \quad + \chi_\beta). \end{cases}$$

Note that the slope also changes at the same discrete points as does $\alpha(X)$ because any updates to the conditional expectation also imply updates to the conditional variance. At the extreme ends of the order flow range +2 and −2, the conditional variance does not depend on Δ_ϵ as there is no uncertainty about ϵ in this case. Because β is also known, the uncertainty arises solely from the variance of the factor. For intermediate ranges of X, the conditional variance depends on the relative likelihood of dealing with the two types of informed traders.

We now introduce a measure that helps succinctly characterize $\delta(X)$:

$$\text{DiffE}_\delta \equiv \delta(2) - \delta(-2). \tag{6}$$

Thus, DiffE_δ is defined as the difference in δ across the extremes of the order-flow range. We summarize the properties of δ in Corollary 1 as follows.

Corollary 1 (Properties of the Slope of the Pricing Function $\delta(X)$). *In the equilibrium of Theorem 1, we have the following:*

1. *In general, $\delta(X)$ is nonmonotonic in the order flow X. If the factor volatility $\sigma_\theta < \overline{\sigma}_\theta$, where the positive constant $\overline{\sigma}_\theta$ is given in* Equation (A.48), *and the factor mean $\overline{\theta}^2 \in \left(\frac{\Delta_\epsilon^2}{8\Delta_\beta^2}, \frac{8\Delta_\epsilon^2}{\Delta_\beta^2}\right)$, where Δ_β and Δ_ϵ are, respectively, the ranges of the factor and the firm-specific terms in the asset's payoff, then $\delta(X)$ is hump shaped and reaches an interior maximum.*
2. *The difference in the slope of the pricing function across the extremes of the order-flow range, DiffE_δ, is positive (negative) only if the factor volatility $\sigma_\theta > 0$ and the mean factor $\overline{\theta} > 0$ ($\overline{\theta} < 0$). It is zero if $\sigma_\theta = 0$; If $\overline{\theta} > 0$ ($\overline{\theta} < 0$), and then DiffE_δ increases (decreases) with the mean beta, $\overline{\beta}$; the range of beta, Δ_β; and the factor volatility, σ_θ.*

Proof. See Appendix A.2.

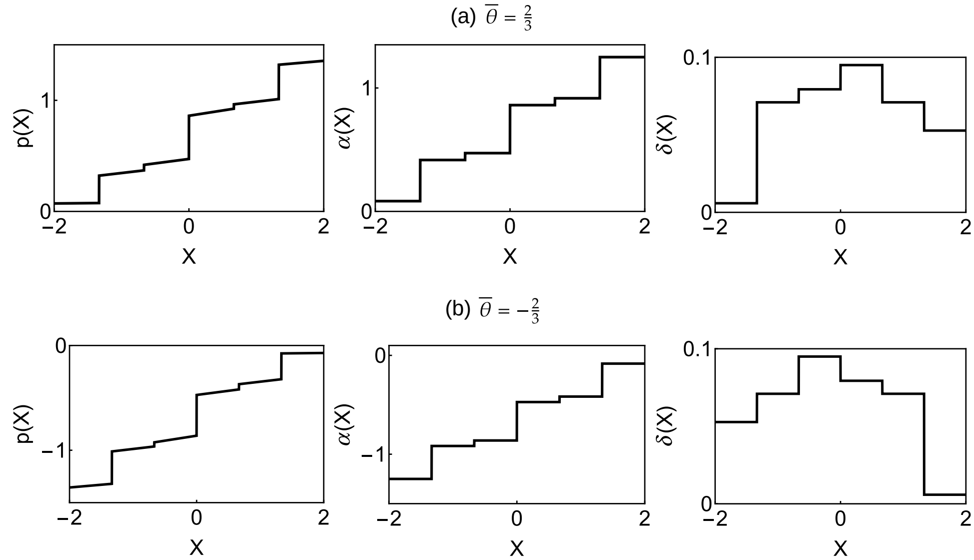
Figure 2 graphically illustrates Corollary 1. The left panels plot $p(X)$ for the parameter configuration $\overline{\beta} = 1, \Delta_\beta = \frac{1}{2}, \Delta_\epsilon = \frac{1}{4}, \chi_\epsilon = \frac{2}{3}, \sigma_\theta = \frac{1}{4}$, and $\gamma = \frac{3}{4}$. The middle panels plot $\alpha(X)$, which is monotonic in X because of the usual reason of adverse selection (Kyle 1985). The right panels depict $\delta(X)$. We see that, if $\overline{\theta}$ is positive, $\text{DiffE}_\delta > 0$ and vice versa, which is part 2 of Corollary 1. The intuition behind this result is the following. Recall that the slope $\delta(X)$ reflects the conditional variance of holding the asset. Now, consider the case of $\overline{\theta} > 0$. In this case, the marginal price impact of an extreme buy order is bigger than that of an extreme sell order because the conditional variance estimate is high when X is at its maximum (when the β estimate is high) but low as X drops to its minimum (when the β estimate is low). For a negative $\overline{\theta}$, we obtain the result that the slope is greater for sell orders because β-informed traders optimally sell the asset if β is high (see Theorem 1). We show in Section 4 that this behavior of $\delta(X)$ has important implications for cross-sectional asset returns.

Turning now to part 1, as demonstrated in Corollary 1 and Figure 2, $\delta(X)$ or, equivalently, $\text{Var}(v|X)$ is nonmonotonically related to the order flow X. To better understand the intuition behind the nonmonotonicity result, we decompose the conditional payoff variance into the following three components:

$$\text{Var}(v|X) = \text{Var}(\epsilon|X) + (\overline{\theta}^2 + \sigma_\theta^2)\text{Var}(\beta|X) + \sigma_\theta^2 \mathbb{E}(\beta|X)^2. \tag{7}$$

The first two components represent uncertainty about ϵ and β, respectively, whereas the last component corresponds to the expected loading on the unknown factor θ. Higher order flows indicate that the level of β is high (low) if $\overline{\theta}$ is positive (negative). As a result, $\mathbb{E}[\beta|X]$ is a monotonic function of total order flow. However, the same intuition does not apply to uncertainty about ϵ and β. On the one hand, particularly

Figure 2. This Figure Plots the Equilibrium Price Function and the Intercept and Slope of the Price Schedule as Functions of Total Order Flow X



Note. Parameters: $\overline{\beta} = 1, \Delta_\beta = \frac{1}{2}, \Delta_\epsilon = \frac{1}{4}, \chi_\epsilon = \frac{2}{3}, \sigma_\theta = \frac{1}{4}$, and $\gamma = \frac{3}{4}$.

high and low order flows are very informative about the two-dimensional (ϵ and β) signal to the market maker and minimize these conditional variances. On the other hand, intermediate values of X are less informative, which leads to an increase in $\mathrm{Var}(\epsilon|X)$ and $\mathrm{Var}(\beta|X)$ relative to their values when the order flow X is extreme. We formally derive in Appendix A.2 the condition under which this nonmonotonicity leads to $\delta(X)$ being hump shaped in X. Empirically, the larger marginal price impact for buy versus sell orders is consistent with the evidence in Holthausen et al. (1987) and Chan and Lakonishok (1993). Further, Barclay and Warner (1993) suggest that midsize orders have the highest marginal price impact, supporting part 1 of Corollary 1.

As in previous papers on two-dimensional additive information, for example, Goldstein and Yang (2015), the payoff of the risky asset in our setting has two components of risks: θ and ϵ. However, whereas prior studies assume a deterministic relation between the asset payoff and the risk factor, we assume that both the single factor and the beta are uncertain, and investors have information about beta. In fact, in our model, when $\sigma_\theta = 0$, β plays the same role as ϵ, and the risky asset has two additive components of risks as in prior studies. However, as shown in Corollary 1, δ is symmetric when $\sigma_\theta = 0$, which highlights that the asymmetric pattern of δ is unique when investors have β information.

3.2. Expectation and Volatility of the Stochastic Slope

Because $\delta(X)$ is determined by the payoff variance conditional on the order flow, let us first compute how the total order flow or, equivalently, the price reflects private information about ϵ and β. We follow the existing literature (Peress 2010, Ozsoylev and Walden 2011, Bai et al. 2016, Edmans et al. 2016) and define price informativeness as the relative reduction in variance that can be achieved by conditioning on the price:

$$PI_\epsilon \equiv 1 - \mathbb{E}\left[\frac{\mathrm{Var}(\epsilon|p)}{\mathrm{Var}(\epsilon)}\right] \quad \text{and} \quad PI_\beta \equiv 1 - \mathbb{E}\left[\frac{\mathrm{Var}(\beta|p)}{\mathrm{Var}(\beta)}\right]. \tag{8}$$

Using Theorem 1, we can compute PI_ϵ and PI_β as follows:

$$PI_\epsilon = \begin{cases} \chi_\epsilon - \dfrac{\chi_\beta}{3}, & \text{if } \chi_\epsilon > \dfrac{1}{2} \\ \dfrac{2\chi_\epsilon}{3}, & \text{if } \chi_\epsilon < \dfrac{1}{2} \end{cases} \quad \text{and}$$
$$PI_\beta = \begin{cases} \dfrac{2\chi_\beta}{3}, & \text{if } \chi_\epsilon > \dfrac{1}{2} \\ \chi_\beta - \dfrac{\chi_\epsilon}{3}, & \text{if } \chi_\epsilon < \dfrac{1}{2}. \end{cases} \tag{9}$$

The component-wise price informativeness measures depend only on $(\chi_\epsilon, \chi_\beta)$. Because $\chi_\beta = 1 - \chi_\epsilon$, increasing χ_ϵ also simultaneously decreases or crowds out χ_β. This means that the crowding-out effect actually strengthens the direct effect; that is, an increase in χ_ϵ directly increases PI_ϵ by injecting more ϵ information into the price, and at the same time, an increase in χ_ϵ means a decrease in χ_β, which can further increase PI_ϵ via the crowding-out effect. Taken together, increasing the mass χ_ϵ of ϵ-informed traders improves price informativeness

PI_ϵ. A similar argument shows that increasing χ_ϵ reduces PI_β.

Equipped with these price-informativeness measures, we can compute $E[\delta(X)]$ as follows:

$$\mathbb{E}[\delta(X)] = \frac{\gamma}{2}\mathbb{E}[\mathrm{Var}(v|X)] = \frac{\gamma}{2}[\mathrm{Var}(\beta\theta) + \Delta_\epsilon^2(1 - PI_\epsilon) + \Delta_\beta^2\overline{\theta}^2(1 - PI_\beta)]. \tag{10}$$

The bracketed term of the right-hand side of Equation (10), which is $\mathbb{E}[\mathrm{Var}(v|X)]$, has three components. The first component is simply the unconditional variance $\mathrm{Var}(\beta\theta)$ in the computation of $\mathrm{Var}(v|X)$. The second and third components reflect how the conditional variance is affected by ϵ and β components of the asset payoff, respectively. We find that, as price informativeness decreases, the conditional variance increases, which is intuitive. We also see that $\mathbb{E}[\delta(X)]$ is increasing in $\overline{\theta}^2, \sigma_\theta, \Delta_\epsilon, \overline{\beta}$, and Δ_β. As an empirical implication, our model predicts that the expected price response to a marginal change in order flow is high under high factor volatility (high σ_θ) and/or when firms have high and volatile betas (high $\overline{\beta}$ and Δ_β). Intuitively, when σ_θ, $\overline{\beta}$, or Δ_β is high, cash-flow volatility is high, and so the risk-averse market maker faces severe inventory concerns, on average, which, in turn, implies a high expected price move per unit order flow.

In addition, $\mathbb{E}[\delta(X)]$ is generally nonmonotonic in χ_ϵ. This is due to the dependence of price informativeness on the composition of traders. Because PI_ϵ increases with χ_ϵ but PI_β decreases with χ_ϵ, $\mathbb{E}[\delta(X)]$ can either increase or decrease with χ_ϵ. When Δ_β is sufficiently high, the effect through PI_β dominates so that $\mathbb{E}[\delta(X)]$ increases with χ_ϵ. We summarize the above discussions in the first part of Corollary 2.

Corollary 2. *In the equilibrium of Theorem 1, the following results hold:*

1. *The expected slope of the pricing function, $\mathbb{E}(\delta)$, increases with the square of the factor mean, $\overline{\theta}^2$; the factor volatility, σ_θ; the range of ϵ, Δ_ϵ; the mean beta, $\overline{\beta}$; and the range of β, Δ_β. If the proportion of ϵ-informed traders $\chi_\epsilon > \frac{1}{2}$, then $\mathbb{E}(\delta)$ increases with χ_ϵ if and only if $\Delta_\beta^2 > \frac{2\Delta_\epsilon^2}{\overline{\theta}^2}$; if $\chi_\epsilon < \frac{1}{2}$, then $\mathbb{E}(\delta)$ increases with χ_ϵ if and only if $\Delta_\beta^2 > \frac{\Delta_\epsilon^2}{2\overline{\theta}^2}$.*

2. *The volatility of the pricing function's slope, $\sigma(\delta)$, increases with the factor volatility, σ_θ, and the mean beta, $\overline{\beta}$, and is minimized when the proportion of ϵ-informed traders $\chi_\epsilon = 1$.*

Proof. See Appendix A.3.

We summarize the comparative statics of the variation in the conditional risk of holding the asset, $\sigma(\delta)$, in the second part of Corollary 2. Recall that $\sigma(\delta) = \frac{\gamma}{2}\sigma[\mathrm{Var}(v|X)]$ by Equation (5). So the comparative statics of $\sigma(\delta)$ can be understood via the comparative statics of $\sigma[\mathrm{Var}(v|X)]$. For instance, as σ_θ or $\overline{\beta}$ increase, the market maker faces higher uncertainty about v, which tends to increase the variance of δ.

We also show that $\sigma(\delta)$ is minimized at $\chi_\epsilon = 1$. This relationship between χ_ϵ and $\sigma(\delta)$ arises from the market maker's learning behavior about β. Because the market maker updates beliefs about β using the total order flow X, $\mathrm{Var}(\beta\theta|X)$ is stochastic and varies with X. When there is more β information (i.e., χ_ϵ is smaller), $\mathrm{Var}(v|X)$ is more sensitive to the order flow. To see this point, consider two extreme cases. First, when there is no β information (i.e., $\chi_\beta = 0$ and $\chi_\epsilon = 1$), from the market maker's perspective, the conditional variance of $\beta\theta$, which is an important component of δ, is constant without any variation. At the other extreme, when all informed investors are β informed, the market maker's belief update about β is extremely sensitive to the order flow, leading to a large variation in $\mathrm{Var}(v|X)$ as well as in δ. In some cases, δ is most volatile for intermediate values of χ_ϵ. In this case, it is particularly difficult for the market maker to disentangle ϵ from β information because the order flow reveals information about both components. Overall, $\mathrm{Var}(v|X)$ is lowest when there are no β-informed traders ($\chi_\epsilon = 1$).

4. Asset Prices and Returns

In this section, we investigate the mean and volatility of asset returns $v - p$.[18] The capital asset pricing model (CAPM) does not hold in our asymmetric-information setting as traders have heterogeneous beliefs and the informed are risk-neutral. We follow Albagli et al. (2021) and assume that the risk-averse market maker only trades with the risk-neutral informed and noise traders and, thus, ends up holding zero inventory on average (i.e., $\mathbb{E}(-X) = 0$). Taking unconditional expectations on Equation (5) leads to the following expression for the expected asset price:

$$\mathbb{E}(p) = \mathbb{E}(v) + \mathbb{E}(\delta X) = \mathbb{E}(v) + \mathrm{Cov}(\delta, X), \tag{11}$$

where the second equation follows from $\mathbb{E}(X) = 0$. So the stochastic feature of δ drives a wedge, $\mathrm{Cov}(\delta, X)$, between the expected price $\mathbb{E}(p)$ and the expected payoff $\mathbb{E}(v)$.

Examining the right panels of Figure 2, we can see that, if $\overline{\theta} > 0$, δ and X are positively correlated (although δ is not monotonically increasing in X) and negatively correlated if $\overline{\theta} < 0$. To see this point, recall from Corollary 1 that $\delta(2) > \delta(-2)$ when $\overline{\theta} > 0$ and vice versa. Hence, X and δ covary positively if $\overline{\theta} > 0$ and vice versa. This argument also suggests that $\mathrm{Cov}(\delta, X)$ is determined by parameters governing β and θ. The following corollary presents the main results related to $\mathrm{Cov}(\delta, X)$.

Corollary 3 (Covariance Between the Pricing Function's Slope, $\delta(X)$, and the Order Flow, X). *In the equilibrium of Theorem 1, $\mathrm{Cov}(\delta, X)$, the covariance between the pricing function's slope δ and the order flow X is negatively related* *to the expected return and given by*

$$\mathrm{Cov}(\delta, X) = \begin{cases} +\gamma\Delta_\beta\overline{\beta}\sigma_\theta^2\chi_\beta & \textit{if } \overline{\theta} > 0 \\ -\gamma\Delta_\beta\overline{\beta}\sigma_\theta^2\chi_\beta & \textit{if } \overline{\theta} < 0, \end{cases} \tag{12}$$

where γ, Δ_β, $\overline{\beta}$, σ_θ, χ_β, and $\overline{\theta}$ represent market maker risk aversion, mean beta, the factor volatility, the proportion of β-informed traders, and the factor mean, respectively. Moreover, $\mathrm{Cov}(\delta, X)$ *increases (decreases) in σ_θ, $\overline{\beta}$, Δ_β, and χ_β if $\overline{\theta} > 0$ ($\overline{\theta} < 0$).*

Proof. See Appendix A.4.

Note from Equations (11) and (12) that the expected return $\mathbb{E}(v-p)$ is decreasing in $\mathrm{Cov}(\delta, X)$. The intuition is the following. For $\overline{\theta} > 0$, a high X implies a high conditional variance (owing to the high inferred β estimate) and a high price, which reflects the premium demanded by the market maker for shorting the risky asset. This high price, in turn, implies a lower expected return. The higher the covariance between conditional variance (δ) and X, the lower the expected return. An analogous reasoning applies to the case of $\overline{\theta} < 0$.

The above asset-pricing results are derived under the assumption of positive mean betas and zero average inventory of the market maker. In Online Appendix OA.3, we consider the case of negative mean betas and show that our reasoning is largely robust to that scenario. Specifically, with $\overline{\theta} > 0$, in equilibrium, informed traders sell (buy) on low (high) betas, and the slope of the pricing schedule in this case is higher for sell (buy) orders as the absolute value of low beta is higher than that of high beta when the mean beta is negative and conversely for $\overline{\theta} < 0$. So the qualitative behavior of $\mathrm{Cov}(\delta, X)$ and its relation to asset prices remains unchanged. We also analyze an extension with positive asset supply in Online Appendix OA.4 and show that the channel driven by $\mathrm{Cov}(\delta, X)$ is robust. The major difference is that the expected return $\mathbb{E}(v-p)$ includes an additional term representing the standard risk premium, and our asset-pricing predictions continue to hold for a robust set of parameter values.

We can also show that return volatility $\sigma(v-p)$ increases with $\overline{\theta}^2$, σ_θ, $\overline{\beta}$, Δ_β, and Δ_ϵ. As these parameters increase, the ex ante uncertainty about v also increases, which increases $\sigma(v-p)$. The impact of χ_ϵ on return volatility is ambiguous. This is due to the effect of χ_ϵ on the informational content of the asset price. As in the standard information literature, when this informativeness increases, the asset price incorporates more fundamental information and becomes closer to the fundamental value, decreasing return volatility. Because there is a dichotomy between price informativeness about ϵ and β, we can obtain a nonmonotone relation between χ_ϵ and $\sigma(v-p)$.

The following corollary summarizes the above comparative statics results.

Corollary 4 (Asset Returns). *In the equilibrium of Theorem 1, the following results hold:*

1. *The equilibrium expected return $\mathbb{E}(v-p)$ decreases in covariance between the pricing function's slope and the order flow,* $\mathrm{Cov}(\delta, X)$.
2. *In turn, the equilibrium expected return decreases (increases) in the factor volatility, σ_θ; the mean beta, $\overline{\beta}$; the range of beta, Δ_β; and the proportion of β-informed traders, χ_β so long as the factor mean $\overline{\theta} > 0$ ($\overline{\theta} < 0$).*
3. *Equilibrium return volatility $\sigma(v-p)$ increases in the squared factor mean, $\overline{\theta}^2$; the factor volatility, σ_θ; the mean beta, $\overline{\beta}$; the range of beta, Δ_β; and the proportion of ϵ-informed traders, Δ_ϵ; it can increase or decrease in the proportion of ϵ-informed traders, χ_ϵ.*

Proof. See Appendix A.5.

It is worth comparing our work with Heinle et al. (2018) and Beyer and Smith (2021). Heinle et al. (2018) show that beta uncertainty (when both beta and factor realizations are random) leads to prices exhibiting nonzero higher order moments, such as skewness and kurtosis, and disclosure about beta leads to updating of these moments in addition to means and variances. In Beyer and Smith (2021), earnings follow a factor structure, and earnings releases convey information about the firm's beta, thus moderating the response of price to earnings via the demanded beta premium. Specifically, in nonrecession times, higher earnings imply higher betas and higher risk premia, which attenuates the normal positive stock price response to earnings. In contrast to both of these papers, within our model, order flows convey information to the market maker about betas. Under positive expected beta, a large order flow implies a high beta and higher risk and, thus, a lower price. The covariation between prices and order flows is, thus, the unique contribution of our setting.

Note that our equilibrium pricing function is stepwise linear, and the steps in the price schedule follow from binary payoff components and a uniform distribution of noise trading. In Online Appendix OA.5, we consider an alternative specification that smooths out the stepwise pricing function by assuming a normal distribution for noise trades. Unfortunately, this extension can only be solved numerically, but we show that it gives similar results as in the main model. In Online Appendix OA.6, we present another extension in which we endogenize traders' information acquisition of ϵ and β information within the model of Section 2. This setting can be interpreted as capturing long-run responses to changes in primitive model parameters after traders are able to adjust how they gather information. In that setting, we assume that traders are ex ante identical. They can decide to acquire either ϵ information or β information but not both.[19] We show that our results are robust to this consideration as well.

5. Empirical Analysis

In this section, we provide empirical content to our key asset-pricing results. Our focus is on testing implications unique to our model.

5.1. Model Implications

Our theory operates through the novel covariance between order flow X and the marginal price impact δ—$\mathrm{Cov}(X, \delta)$. Three predictions arise immediately from Equations (11) and (12). Based on Equation (11), $\mathrm{Cov}(X, \delta)$ is inversely related to expected returns $\mathbb{E}(v - p)$ (prediction 1). From Equation (12), $\mathrm{Cov}(X, \delta)$ is positive (negative) when $\overline{\theta}$ is positive (negative) (prediction 2), and it is positively related to expected equity betas $\overline{\beta}$ if $\overline{\theta}$ is positive and vice versa (prediction 3). In addition, part 2 of Corollary 4 connects our theory to the beta anomaly documented by Frazzini and Pedersen (2014). That is, we predict that the beta anomaly should be more evident in periods in which $\overline{\theta} > 0$ and that accounting for this channel should attenuate the beta anomaly in asset returns, and this forms our last prediction (prediction 4).[20]

5.2. Model Interpretation and Empirical Implementation

To test the asset-pricing implications, we need a reasonable proxy for the factor mean $\overline{\theta}$. We proceed as follows. We begin by proposing that $\overline{\theta}$ is positive during nonrecession times and negative during recessions. That is, our model repeats periodically during nonrecession periods with $\overline{\theta} > 0$ and repeats during recessions with $\overline{\theta} < 0$. The common factor θ can be interpreted as periodic cash flows on a single macrofactor so that θ is the (signed) increment to the existing stock of market-wide assets. The assumption is that the mean of this increment ($\overline{\theta}$) is public knowledge.

As it is difficult to obtain a proxy for the ex ante factor mean (i.e., $\overline{\theta}$), we use NBER recessions to proxy for the parameter space in which $\overline{\theta} < 0$ and nonrecession (boom) periods for the space in which $\overline{\theta} > 0$. We validate this procedure by documenting as part of our empirical tests that quarterly innovations to aggregate earnings are, on average, positive (negative) in nonrecession (recession) periods. We hasten to add that we are not claiming that these innovations are a perfect proxy for $\overline{\theta}$, which is common knowledge in our model. Instead, we propose that the mean value of the innovations behaves in a manner consistent with the notion that the mean of the common component to earnings is likely to be positive in booms and negative in recessions. Note that, because our theoretical predictions are contingent on the sign of the ex ante factor mean, it is challenging from an interpretational standpoint to conduct our empirical exercises directly based on the realized earnings innovations (i.e., realized values of θ).

We perform our asset pricing tests using monthly returns, which allows us to relate our results to the extensive literature on cross-sectional asset pricing. The interpretation is that the information in quarterly earnings innovations is released periodically every month. The alternative to monthly returns is to use quarterly returns and match them with quarterly earnings innovations. However, given the need to use transactions data, which are only available since the mid-1980s, this would cause a loss of statistical power in our asset-pricing tests. Our choice of horizon can also be motivated by Hasbrouck and Sofianos (1993), Madhavan and Smidt (1993), Pástor and Stambaugh (2003), and Hameed and Mian (2015), who all argue that inventory effects on stock prices last for one to two months.

5.3. Data Description and Empirical Design

We first describe our data and the construction of our microstructure measures. We use the CRSP, Compustat, and Trade and Quote (TAQ) databases on Wharton Research Data Services (WRDS). The sample consists of common stocks traded on the New York Stock Exchange (NYSE), American Stock Exchange, or Nasdaq. To ensure our results are not driven by extremely illiquid securities, we exclude stocks with share prices below \$5 and market capitalizations below the NYSE 10th percentile as of the end of the month.

The key asset pricing variable in our study is the covariance between order flow (or order imbalance) and the price schedule's slope, which is denoted as *Cov* for brevity. For each stock, we compute *Cov* on a three-month rolling basis.[21] Specifically, at the end of month t, we compute *Cov* as the covariance between the daily order imbalance and daily slope from month $t-2$ to month t. Data for the calculation of order imbalance and slope is from the intraday indicators product within the TAQ database on WRDS. The sample period is from March 1993 to December 2019.[22]

We construct two measures of *Cov* based on different measures of order imbalance and the estimate of the price schedule's slope. Order imbalance 1 (*OIB1*) is defined as the difference between dollar buy volume (TAQ item: *BuyDollar_LR1*) and dollar sell volume (TAQ item: *SellDollar_LR1*) scaled by the sum of buy and sell dollar volumes on a day. Order imbalance 2 (*OIB2*) is defined as the difference between share buy volume (TAQ item: *BuyVol_LR1*) and share sell volume (TAQ item: *SellVol_LR1*) scaled by the sum of buy and sell share volumes on a day.[23] The slope (δ) is estimated using signed dollar volume aggregated over five-minute intervals (TAQ item: *TSignSqrtDVol2*).[24] We define *Cov1* as the covariance between *OIB1* and δ and *Cov2* as the covariance between *OIB2* and δ.

We construct other variables as follows: the market beta for a stock in a month is estimated following the method of Frazzini and Pedersen (2014) (see their section 3.1 for details). *Size* is the natural logarithm of market capitalization as of end of the month. Book-to-market ratio (*BM*) is the ratio of book equity to market value of equity computed as in Fama and French (1992). Past return (*PRET*) is the cumulative past six-month return, for which the most recent month is skipped (Jegadeesh and Titman 1993). Asset growth (*AG*) is the annual percentage change in total assets. Gross profitability (*GP*) is sales minus cost of goods sold scaled by total assets at current year end. The variables *Size*, *BM*, *PRET*, *AG*, and *GP* form the controls in our asset pricing regressions.

5.4. Empirical Results

We now present our empirical results relating *Cov* to expected returns $\mathbb{E}(v-p)$, the expected factor payoff $\overline{\theta}$, and mean beta $\overline{\beta}$ (i.e., predictions 1–3). We then assess the role of *Cov* in understanding the beta anomaly (i.e., prediction 4).

5.4.1. *Cov* and Returns. We use a portfolio-sorting approach as well as Fama–MacBeth (FM) regressions. In the portfolio sorting, at each month t, we sort all stocks into deciles based on *Cov* and construct equally weighted portfolios of stocks within each decile. We hold the portfolio over the month $t+1$, and the portfolio is rebalanced monthly. Table 1 reports the portfolio sorting results.

There are two observations. First, as shown in panel A, both the mean and the median of *Cov* are positive. Second, we find that *Cov* is indeed negatively and significantly associated with average returns for both measures of *Cov*. Specifically, as shown in panel B of Table 1, excess returns are almost monotonically decreasing in *Cov*. The lowest decile of *Cov* outperforms the highest decile by about 23 basis points (bps) per month. In panels C and D, we show that the monotonic association between *Cov* and future returns is preserved when we measure alphas relative to the Fama–French–Carhart four-factor (FFC4) model or the five-factor model augmented with momentum (FF5+UMD).

We next use FM regressions. We measure returns in month $t+1$, and our control variables are as of month t. As mentioned above, these variables include market capitalization (*Size*), the book-to-market ratio (*BM*), past six-month returns (*PRET*), asset growth (*AG*), and gross profitability (*GP*). The time-series averages, together with the associated t-statistics, of the coefficients from monthly cross-sectional regressions are presented in Table 2.[25]

We again find that *Cov* is negatively and significantly associated with average returns. In columns (2) and (4), we use the decile rank of *Cov* instead of the value of *Cov* to mitigate the influence of outliers and find that the coefficient on the rank is also negative and more strongly significant than that on the actual value. For our sample of stocks, whereas *AG* and *GP* are significant with a sign consistent with prior literature, the other three control variables are insignificant. The latter finding accords with the notion that well-known anomalies have decayed in recent years (Chordia et al. 2014, Hou et al. 2020).

5.4.2. *Cov* and $\overline{\boldsymbol{\theta}}$. To compare *Cov* between periods with positive and negative $\overline{\theta}$, we distinguish nonrecession periods from recession periods, in which the recession periods include those from March of 2001 to November of 2001 and those from December of 2007 to June of 2009 (defined by NBER). We first use the means of aggregate earnings innovations as a proxy for factor means to show that nonrecession periods indeed have positive $\overline{\theta}$ and vice versa. We follow Glosten et al. (2021) to calculate aggregate earnings innovations. Specifically, for each firm in each quarter q, we compute the earnings innovation as $(EPS_{i,q}-EPS_{i,q-4})/PRC_{i,q-1}$, where $EPS_{i,q}$ is the earnings per share (excluding extraordinary items) for firm i in quarter q and $PRC_{i,q-1}$ is the share price of firm i at the end of quarter $q-1$. Then, we take a value-weighted average of firm-level earnings innovations in a quarter as the aggregate earnings innovation in the quarter.

As shown in panel A of Table 3, whereas the average earnings innovation in nonrecessions is positive (0.001), that in recessions is negative (−0.008).[26] Furthermore, we compare *Cov* between the nonrecession and recession periods and find that the nonrecession periods have positive *Cov1* and *Cov2* on average, whereas the opposite is true in recession periods. Panel B of Table 3 tests the difference in *Cov* across nonrecession and recession periods and shows that it is statistically significant even after controlling for macrovariables, including the Baker and Wurgler sentiment index, inflation, investment-to-capital ratio (I/K), and the TED spread (the difference between 3-Month LIBOR based on US dollars and 3-Month Treasury Bill). Overall, Table 3 provides supporting evidence for our Corollary 3, which predicts that *Cov* has the same sign as $\overline{\theta}$.

5.4.3. *Cov* and $\overline{\boldsymbol{\beta}}$. To examine how *Cov* relates to $\overline{\beta}$, we run Fama–MacBeth regressions (with Newey–West corrected t-statistics) of *Cov* on $\overline{\beta}$ as this method facilitates controls for other firm characteristics (specifically, the ones in Table 2). Table 4 reports the results. We find that *Cov* is indeed positively associated with $\overline{\beta}$ only during nonrecession periods (see panel B), and the coefficients of $\overline{\beta}$ in the regressions are all significant at the 1% level. During recessions, *Cov* is negatively associated with $\overline{\beta}$, but it is statistically insignificant probably because of a small sample during the recession

Table 1. Performance of *Cov*-Sorted Portfolios

Panel A. Summary statistics of *Cov*

Variable	Mean	Median	Standard deviation
Cov1 ($\times 10^8$)	0.57	0.08	37.57
Cov2 ($\times 10^8$)	0.44	0.07	37.57

Panel B. Monthly excess returns

Cov measured by	*Cov1*		*Cov2*	
Cov deciles	ExRet	*t*-statistic	ExRet	*t*-statistic
1	1.08	(3.37)	1.07	(3.37)
2	1.03	(3.19)	1.03	(3.18)
3	0.94	(3.14)	0.95	(3.19)
4	1.01	(3.55)	1.00	(3.52)
5	0.97	(3.59)	0.99	(3.64)
6	0.93	(3.36)	0.92	(3.35)
7	1.03	(3.49)	1.03	(3.44)
8	0.85	(2.69)	0.86	(2.76)
9	0.89	(2.81)	0.86	(2.72)
10	0.85	(2.60)	0.86	(2.63)
1–10	0.23	(2.93)	0.22	(2.86)

Panel C. Monthly FFC4 alphas

Cov measured by	*Cov1*		*Cov2*	
Cov deciles	FFC4	*t*-statistic	FFC4	*t*-statistic
1	0.25	(2.57)	0.25	(2.59)
2	0.17	(3.07)	0.17	(2.95)
3	0.14	(2.21)	0.14	(2.41)
4	0.22	(3.60)	0.22	(3.47)
5	0.22	(3.87)	0.24	(3.96)
6	0.19	(3.03)	0.19	(3.02)
7	0.26	(4.24)	0.24	(4.05)
8	0.03	(0.54)	0.05	(0.81)
9	0.06	(0.90)	0.04	(0.60)
10	0.06	(0.73)	0.07	(0.86)
1–10	0.19	(2.61)	0.18	(2.51)

Panel D. Monthly FF5+UMD alphas

Cov measured by	*Cov1*		*Cov2*	
Cov deciles	FFC4	*t*-statistic	FFC4	*t*-statistic
1	0.28	(3.11)	0.28	(3.10)
2	0.22	(4.03)	0.22	(4.09)
3	0.13	(2.37)	0.14	(2.62)
4	0.21	(3.32)	0.20	(3.18)
5	0.17	(2.97)	0.19	(3.33)
6	0.20	(3.12)	0.20	(3.15)
7	0.30	(4.87)	0.28	(4.74)
8	0.09	(1.52)	0.11	(1.69)
9	0.13	(1.87)	0.11	(1.64)
10	0.12	(1.52)	0.12	(1.60)
1–10	0.17	(2.19)	0.15	(2.09)

Notes. This table reports the performance of stock portfolios sorted by *Cov* measures. At each month end, we sort stocks into quintiles by *Cov* and hold the equal-weighted portfolios over the next month. Stocks with share prices below \$5 or market capitalizations below the NYSE 10th percentile value at the end of the portfolio formation month *t* are excluded. Panel A reports summary statistics for the *Cov* variables. Panel B reports average monthly excess returns. Panel C reports the average monthly FFC4 alpha. Panel D reports the average monthly alpha relative to the FF5+UMD. The sorting variable *Cov* is the covariance between order imbalance and the estimate of marginal price impact estimated at each month end. *Cov* at month *t* end is defined as the covariance between daily order imbalance and daily estimate of marginal price impact from month $t-2$ to month *t*. We construct two *Cov* measures (*Cov1*, *Cov2*) based on different measures of order imbalance (*OIB1* and *OIB2*). *OIB1* is defined as the buy dollar volume minus the sell dollar volume scaled by the sum of buy and sell dollar volumes during a day. *OIB2* is defined as the buy share volume minus the sell share volume scaled by the sum of buy and sell share volumes on a day. The pricing schedule's slope estimate (δ) is the coefficient corresponding to TAQ item *TSignSqrtDVol2*. We define *Cov1* as the covariance between *OIB1* and δ and *Cov2* as the covariance between *OIB2* and δ. *t*-statistics in parentheses are computed based on standard errors with a Newey–West correction of five lags.

Table 2. Fama–MacBeth Predictive Regressions

	(1)	(2)	(3)	(4)
Cov measured by	*Cov1*		*Cov2*	
Cov	-0.157^{**}		-0.153^{**}	
	(−2.58)		(−2.52)	
Decile_Rank_Cov		-0.207^{***}		-0.207^{***}
		(−3.77)		(−3.81)
Size	−0.000	−0.000	−0.000	−0.000
	(−0.41)	(−0.42)	(−0.40)	(−0.42)
B/M	0.002	0.002	0.002	0.002
	(1.19)	(1.17)	(1.19)	(1.17)
PRET	0.004	0.004	0.004	0.004
	(1.63)	(1.63)	(1.62)	(1.63)
AG	-0.003^{***}	-0.003^{***}	-0.003^{***}	-0.003^{***}
	(−4.05)	(−4.06)	(−4.05)	(−4.06)
GP	0.006^{***}	0.006^{***}	0.006^{***}	0.006^{***}
	(2.79)	(2.79)	(2.79)	(2.79)
Adjusted R^2	0.044	0.044	0.044	0.044

Notes. This table reports results from Fama–MacBeth predictive regressions of stock returns. The dependent variable is the stock return (in percentage) in month $t+1$. The key independent variables are the covariance between order imbalance and price schedule's slope (*Cov*) estimated at month t end and the decile ranking of *Cov* (*Decile_Rank_Cov*) at month t end. *Cov* in month t is defined as the covariance between daily order imbalance and daily estimate of the price schedule's slope during month $t-2$ to month t. We construct two *Cov* measures (*Cov1*, *Cov2*) based on different measures of order imbalance (*OIB1* and *OIB2*). *OIB1* is defined as buy dollar volume minus sell dollar volume scaled by the sum of buy and sell dollar volume on a day. *OIB2* is defined as buy share volume minus sell share volume scaled by the sum of buy and sell share volume on a day. The slope estimate (δ) is the price impact coefficient corresponding to TAQ item *TSignSqrtDVol2*. We define *Cov1* as the covariance between *OIB1* and the slope and define *Cov2* as the covariance between *OIB2* and the slope. Control variables include natural logarithm of market capitalization (*Size*) at month t end, book-to-market ratio at month t end, cumulative stock returns from month $t-6$ to month $t-1$ (*PRET*), asset growth (*AG*) as of month t end, and gross profitability (*GP*) as of month t end. Stocks with share price below \$5 or market capitalization below the NYSE 10th percentile value at month t end are excluded. t-statistics in parentheses are computed based on standard errors with Newey–West correction of five lags. Coefficient estimates for *Cov* and *Decile_Rank_Cov* reported in this table are multiplied by 10^{-4} and 10^4, respectively.

period (with only 26 months). Overall, the findings accord with our model.

5.4.4. *Cov* and the Beta Anomaly in Asset Returns.

We follow Bali et al. (2017) and employ a portfolio approach to assess the role of *Cov* in the beta anomaly documented by Frazzini and Pedersen (2014). In particular, we examine the relation between market beta and future returns after controlling for *Cov*. Our analysis predicts that the beta anomaly should be present primarily in nonrecession periods, and accounting for the relation between expected returns and *Cov* should attenuate the effect of beta on asset returns.

We conduct our test in two steps. In the first step, we confirm the beta anomaly in our sample of stocks. At the end of each month, we sort our sample stocks into deciles by their market betas. We hold the decile portfolios over the next month and compute equal-weighted average portfolio returns. Panel A of Table 5 confirms that the lowest beta decile portfolio outperforms the highest decile by 59 bps per month for the FF5+UMD alphas.

To further test our theory, we compare the performance of the portfolio that goes long (short) stocks in the lowest (highest) beta decile across nonrecession and recession periods. Consistent with part 2 of Corollary 4, we find that the long–short portfolio only generates a positive FF5+UMD alpha (70 bps per month with a t-statistic of 2.63) during the nonrecession period. Interestingly, during the recession period, the long–short portfolio generates a negative FF5+UMD alpha of −58 bps, but it is statistically insignificant probably because of the smaller recession sample.

In our second step, we study the profitability of the beta anomaly after controlling for *Cov*. We first test for the relation between *Cov* and the beta anomaly for the full sample. Specifically, at the end of each month t, we group all sample stocks into quintiles based on *Cov*. We then sort all sample stocks in each *Cov* quintile into deciles based on market beta. By employing this bivariate portfolio approach, we effectively control for the role of *Cov* in the beta anomaly. Further, comparing the profitability of the beta anomaly after controlling for *Cov* to panel A of Table 5 demonstrates how *Cov* affects the beta anomaly.

Table 3. Aggregate Earnings and *Cov* in Nonrecession and Recession Periods

Panel A. Average *Cov* and earnings

	Cov		Aggregate
	Cov1 (in 10^8)	*Cov2* (in 10^8)	Earnings innovation
Nonrecession	0.666	0.562	0.001
Recession	−1.480	−1.714	−0.008
Difference	2.146	2.276	0.009
t-statistic	(3.58)	(3.77)	(5.17)

Panel B. Time-series regression

	(1)	(2)	(3)	(4)
Dependent variable	*Cov1* ($\times 10^8$)		*Cov2* ($\times 10^8$)	
Dummy_Recession	−2.15**	−3.94***	−2.28**	−3.99***
	(−2.54)	(−3.80)	(−2.51)	(−3.85)
Sentiment		0.53		0.52
		(0.73)		(0.70)
Inflation		105.71***		109.54***
		(2.65)		(2.76)
I/K		−1.71		−43.45
		(−0.02)		(−0.47)
Ted_Spead		284.03**		291.84**
		(2.49)		(2.55)
Intercept	0.67*	−0.74	0.56	0.61
	(1.73)	(−0.25)	(1.46)	(0.20)
Adjusted R^2	0.036	0.138	0.040	0.132
Number of observations	322	310	322	310

Notes. This table compares earnings/*Cov* in nonrecession and recession periods. Panel A reports the average *Cov* and means of aggregate earnings innovations in recession and nonrecession periods separately. For *Cov* measures, we compute the cross-sectional average *Cov* measured at month end across sample stocks and then report the time-series average of *Cov* during the NBER-defined recession and nonrecession periods. For aggregate earnings innovations, we follow Glosten et al. (2021). Specifically, for each firm in each quarter q, we compute the earnings innovation as $(EPS_{i,q} - EPS_{i,q-4})/PRC_{i,q-1}$, where $EPS_{i,q}$ is the earnings per share (excluding extraordinary items) for firm i in quarter q and $PRC_{i,q-1}$ is the share price of firm i at the end of quarter $q-1$. Then, we take value-weighted averages of firm-level earnings innovations in a quarter as the aggregate earnings innovation in the quarter. Panel B reports results from time-series regressions for *Cov1* (*Cov2*) measured at each month end on a dummy variable indicating whether the current month is in NBER-defined recession period (*Dummy_Recession*). We control for a set of macrovariables including the Baker and Wurgler sentiment index, inflation, investment-to-capital ratio (I/K), and TED spread.

Panel B of Table 5 presents the monthly FF5+UMD alphas for each of the long–short beta-based portfolios when *Cov* is used as the first sorting variable. The results in panel B indicate that, after controlling for the effect of *Cov*, the profitability of the beta anomaly is reduced from an FF5+UMD alpha of 59 bps per month (from panel A of Table 5) to about 31 bps per month. These results suggest that *Cov* subsumes about half of the beta anomaly. In panel C, we split the sample into nonrecession and recession periods and consider the effect of *Cov* in the beta anomaly in nonrecession periods. Consistent with the results in panel B, we again find that *Cov* attenuates the beta anomaly by about 50%.

In a further cross-sectional study, we try to identify stocks for which inventory issues may be more relevant. Whereas finding variables that solely reflect such issues is challenging, we follow Chordia and Subrahmanyam (2004) to propose that the risk-bearing capacity of market makers would be lower in small stocks. Thus, we expect (i) the cross-sectional association between $Cov(\delta, X)$ and stock returns to be more pronounced among small stocks and (ii) the cross-sectional association between $Cov(\delta, X)$ and β to be more pronounced among small stocks. The findings in Online Appendix OA.7 confirm this conjecture.

Overall, we find the empirical results encouraging. They support the central implications of our theoretical analysis. We particularly reiterate that the covariance between order flows and marginal price impact of order flows behaves in the expected way as per our theory (it has a negative sign during recessions and a positive sign otherwise) and is indeed negatively related to average returns, also as predicted by our theory. The analysis, thus, introduces a novel focus on the covariance between order flows and the price schedule's slope as a determinant of equity expected returns.

Table 4. Fama–MacBeth Regressions of *Cov* on Beta

Panel A. Full sample				
Dependent variable: *Cov* measured by	*Cov1* (×10^8)		*Cov2* (×10^8)	
Beta	1.515***		1.540***	
	(3.29)		(3.32)	
Decile_Rank_Beta		0.189***		0.190***
		(3.49)		(3.50)
Size	−0.042	−0.046	0.013	0.010
	(−0.25)	(−0.27)	(0.08)	(0.06)
BM	−0.364	−0.319	−0.368	−0.323
	(−1.28)	(−1.16)	(−1.29)	(−1.17)
PRET	−2.903***	−2.908***	−2.901***	−2.908***
	(−5.97)	(−6.01)	(−5.96)	(−6.00)
AG	−0.171	−0.171	−0.167	−0.168
	(−1.07)	(−1.07)	(−1.05)	(−1.05)
GP	0.574*	0.541*	0.613**	0.580*
	(1.93)	(1.82)	(2.05)	(1.95)
Adjusted R^2	0.009	0.009	0.009	0.009
Panel B. Nonrecession subsample				
Dependent variable: *Cov* measured by	*Cov1* (×10^8)		*Cov2* (×10^8)	
Beta	1.674***		1.701***	
	(3.47)		(3.51)	
Decile_Rank_Beta		0.208***		0.210***
		(3.67)		(3.69)
Size	−0.115	−0.119	−0.066	−0.069
	(−0.65)	(−0.67)	(−0.37)	(−0.39)
BM	−0.431	−0.383	−0.436	−0.388
	(−1.41)	(−1.31)	(−1.43)	(−1.32)
PRET	−2.766***	−2.754***	−2.759***	−2.749***
	(−5.39)	(−5.42)	(−5.37)	(−5.41)
AG	−0.158	−0.159	−0.155	−0.156
	(−0.91)	(−0.92)	(−0.89)	(−0.90)
GP	0.612**	0.586*	0.640**	0.614**
	(2.02)	(1.94)	(2.10)	(2.02)
Adjusted R^2	0.009	0.010	0.010	0.010
Panel C. Recession subsample				
Dependent variable: *Cov* measured by	*Cov1* (× 10^8)		*Cov2* (× 10^8)	
Beta	−0.213		−0.217	
	(−0.18)		(−0.18)	
Decile_Rank_Beta		−0.024		−0.027
		(−0.15)		(−0.19)
Size	0.759*	0.758*	0.877**	0.877**
	(1.97)	(1.94)	(2.14)	(2.11)
BM	0.365	0.383	0.365	0.383
	(0.70)	(0.73)	(0.70)	(0.73)
PRET	−4.394***	−4.583***	−4.451***	−4.648***
	(−3.36)	(−3.29)	(−3.37)	(−3.31)
AG	−0.311*	−0.303	−0.302*	−0.294
	(−1.74)	(−1.66)	(−1.71)	(−1.63)
GP	0.157	0.044	0.323	0.210
	(0.15)	(0.04)	(0.31)	(0.21)
Adjusted R^2	0.006	0.006	0.006	0.007

Table 4. (Continued)

Panel D. Difference in coefficient between nonrecession and recession subsamples		
Dependent variable	*Cov1* (×10^8)	*Cov2* (×10^8)
Difference in *Beta*	2.203***	2.246***
	(2.67)	(2.71)
Difference in *Decile_Rank_Beta*	0.244***	0.250***
	(2.67)	(2.73)

Notes. This table reports results from Fama–MacBeth regressions of *Cov* on contemporaneous market beta. The dependent variable is the covariance between order imbalance and the price schedule's slope (*Cov*) measured at month *t* end. The key independent variables are market beta estimated at month *t* end following the methodology of Frazzini and Pedersen (2014) and the decile ranking of market beta at month *t* end. Control variables include natural logarithm of market capitalization (*Size*) at month *t* end, book-to-market ratio at month *t* end, cumulative stock returns from month $t-6$ to month $t-1$ (*PRET*), asset growth (*AG*) as of month *t* end, and gross profitability (*GP*) as of month *t* end. Stocks with share prices below $5 or market capitalizations below NYSE 10th percentile value at month $t-1$ end are excluded. *t*-statistics in parentheses are computed based on standard errors with Newey–West correction of five lags. Panel A reports the regression results in the full sample. Panels B and C report the regression results within the nonrecession and recession subsamples, respectively. Panel D compares the coefficient estimates between recession and nonrecession subsamples. Specifically, for a given independent variable, we estimate Fama–MacBeth regressions over the full sample period and obtain the time series of coefficient estimates. Then, we regress the time series of coefficient estimates on a dummy variable indicating whether a month belongs to a nonrecession period. We report the coefficient estimates and *t*-statistics associated with the nonrecession dummy variable in panel D.

6. Summary and Concluding Remarks

We study a setting in which cash flows follow a factor structure $v=\beta\theta+\epsilon$ and there is private information about both β and ϵ. Under a positive factor mean, a large buy signals that beta is likely high and, thus, raises the conditional variance of the asset's value, whereas a large sell signals that beta is closer to zero and lowers this variance. Thus, marginal price impacts are higher for buy relative to sell orders, which implies a positive covariance between prices and order flow. When factor means are negative, a reverse intuition applies, and this covariance turns negative.

In our framework, the covariance (*Cov*) between the price schedule's slope and order flow is associated with the equilibrium risk premium. We predict that *Cov* should be inversely related to average stock returns. Further, we predict a positive *Cov* and a positive relation between *Cov* and beta during booms (periods of positive factor means) and vice versa during recessions. We also predict a beta anomaly in asset returns, particularly during boom periods, and predict that accounting for the effect of *Cov* attenuates the beta anomaly in asset returns. We provide empirical evidence supportive of these key implications.

The recognition of ϵ and β information also opens new areas for theoretical research. For example, it would be useful to see how the incentives to acquire different types of information change in a dynamic setting in which the volatility of beta changes over time. Changes in asset liquidity as the firm's cyclicality (or

Table 5. *Cov* and the Beta Anomaly

Panel A. Beta anomaly decile portfolio						
	Full sample		Nonrecession		Recession	
Beta deciles	Alpha	*t*-statistic	Alpha	*t*-statistic	Alpha	*t*-statistic
1	0.43	(5.04)	0.44	(4.93)	0.27	(0.84)
10	−0.16	(−0.78)	−0.25	(−1.20)	0.85	(1.03)
1–10	0.59	(2.31)	0.70	(2.63)	−0.58	(−0.59)

Panel B. Beta anomaly (controlling for *Cov*)) in full sample				
Cov measured by	*Cov1*		*Cov2*	
Cov quintiles	Alpha	*t*-statistic	Alpha	*t*-statistic
1	0.36	(2.02)	0.38	(2.19)
2	0.41	(2.52)	0.37	(2.21)
3	0.26	(1.46)	0.27	(1.55)
4	0.16	(0.91)	0.17	(0.93)
5	0.36	(2.26)	0.35	(2.22)
Average	0.31	(2.26)	0.31	(2.27)

Table 5. (Continued)

Panel C. Beta anomaly (controlling for *Cov*) in nonrecessions

Cov measured by	*Cov1*		*Cov2*	
Cov quintiles	Alpha	*t*-statistic	Alpha	*t*-statistic
1	0.35	(1.94)	0.38	(2.08)
2	0.49	(2.92)	0.44	(2.50)
3	0.35	(1.86)	0.35	(1.89)
4	0.16	(0.90)	0.18	(0.97)
5	0.43	(2.69)	0.42	(2.64)
Average	0.36	(2.52)	0.35	(2.49)

Notes. This table examines the performance of the beta anomaly after controlling for the covariance between order imbalance and the price schedule's slope (*Cov*). Panel A shows the performance of the beta-sorted stock portfolios. We estimate the market beta of stocks following Frazzini and Pedersen (2014). At each month end, we sort all stocks with share price no less than \$5 and market capitalization above NYSE 10th percentile value into deciles by their market beta at the month end. We hold the decile portfolios over the next month and compute equal-weighted average portfolio returns. Then we report the average monthly FF5+UMD alphas of the decile portfolios during the holding period, in which the FF5+UMD alphas are computed relative to Fama–French (2015) five-factor model augmented with momentum factor. The performance in the full sample period, the nonrecession periods, and the recession periods, are reported separately. Panel B shows the profitability of the beta anomaly after controlling for *Cov*. Specifically, at each month end, we first sort stocks into quintiles by *Cov* estimated at the month end. *Cov* in month *t* is defined as the covariance between daily order imbalance and the pricing schedule's slope during month $t-2$ to month *t*. Within each *Cov* quintile, we further sort stocks into deciles by market beta, and we construct a beta low-minus-high portfolio by buying (selling short) the equal-weighted portfolio of stocks in the lowest (highest) beta decile. We hold the beta low-minus-high portfolios within each *Cov* quintile over the next month and compute equal-weighted portfolio returns. Average monthly FF5+UMD alphas of the beta low-minus-high portfolios within each *Cov* quintiles are reported. At the bottom of panel B (labeled "Average"), we report the performance of an average beta anomaly low-minus-high portfolio, which is the equal-weighted combination of the five beta low-minus-high portfolios across *Cov* quintiles. In panel C, we classify the sample period into nonrecession and recession subperiods based on NBER-defined recessions. We report the profitability of beta anomaly after controlling for *Cov* in the nonrecession periods. *t*-statistics in parentheses are based on standard errors with Newey–West correction of five lags.

average beta level) shifts over time also form an interesting area for investigation. In addition, capital structure shifts affect equity betas, and how this affects asset liquidity would be worth investigating. Further, how the dynamics of factor volatility affect the dynamics of liquidity and risk premia via the information channel is also an intriguing issue.

As a final point, the method we use to solve for the equilibrium can potentially prove useful in other settings. One potential application is the framework of Gabaix and Landier (2008), in which CEO talent and firm size interact via their product (analogous to beta and the factor) in determining firm outcomes. Considering private information about talent would be an intriguing application. This and other investigations are left for future research.

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Appendix. Proofs

A.1. Proof of Theorem 1

We start with the characterization of equilibria 1(a) and 1(b) and define $\epsilon_H \equiv \Delta_\epsilon$, $\epsilon_L \equiv -\Delta_\epsilon$, $\beta_H \equiv \overline{\beta} + \Delta_\beta$, and $\beta_L \equiv \overline{\beta} - \Delta_\beta$ for ease of exposition. In Sections A.1.1–A.1.3, we discuss the three cases, $\overline{\theta} > 0$, $\overline{\theta} < 0$, and $\overline{\theta} = 0$ separately. Section A.1.4 summarizes the existence conditions for equilibria 1(a) and 1(b). Equilibria 2(a) and 2(b) are analyzed and discussed in Section A.1.5. The corresponding existence conditions are stated in Equations (A.21) and (A.26). Finally, Section A.1.6 shows that there exist no other pure-strategy equilibria.

A.1.1. Positive $\overline{\theta}$. This proof has two steps. In the first step, we take the trading strategies of informed investors as given and pin down the pricing rule used by the market maker. In the second step, we characterize the conditions to ensure informed investors will buy on high signals and sell on low signals.

Step 1: Because the pricing rule is $p = \mathbb{E}[v|X] + \frac{\gamma}{2}X\mathrm{Var}(v|X)$, in this step, we first compute $\mathbb{E}[v|X]$ and $\mathrm{Var}(v|X)$. Moreover, we distinguish between the two scenarios (i) $\chi_\epsilon > \chi_\beta$ and (ii) $\chi_\epsilon \le \chi_\beta$. To ensure prices for every order-flow realization, we also postulate that, at the points of discrete changes in the pricing function (which occur with measure zero), the market maker chooses the higher price for nonnegative order flow and the lower price for negative order flow.

The distribution of total order flow depends on the signals received by informed investors as follows:

1. When $\epsilon = \epsilon_H$ and $\beta = \beta_H$, total order flow X is uniformly distributed in $[0, 2]$.
2. When $\epsilon = \epsilon_H$ and $\beta = \beta_L$, total order flow X is uniformly distributed in $[-1 + \chi_\epsilon - \chi_\beta, 1 + \chi_\epsilon - \chi_\beta]$.
3. When $\epsilon = \epsilon_L$ and $\beta = \beta_H$, total order flow X is uniformly distributed in $[-1 - \chi_\epsilon + \chi_\beta, 1 - \chi_\epsilon + \chi_\beta]$.
4. When $\epsilon = \epsilon_L$ and $\beta = \beta_L$, total order flow X is uniformly distributed in $[-2, 0]$.

Scenario (i): $\chi_\epsilon > \chi_\beta$. To compute the conditional mean and the conditional variance, we consider the following intervals of X:

1. If $X \in [1 + \chi_\epsilon - \chi_\beta, 2]$, the market maker knows that $\epsilon = \epsilon_H$ and $\beta = \beta_H$. It follows that

$$\mathbb{E}[v|X] = \epsilon_H + \beta_H\overline{\theta}$$
$$\mathrm{Var}(v|X) = \beta_H^2\sigma_\theta^2.$$

2. If $X \in [1 - \chi_\epsilon + \chi_\beta, 1 + \chi_\epsilon - \chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_H$. The market maker also knows that $\beta = \beta_H$ and $\beta = \beta_L$ are equally likely. It follows that

$$\mathbb{E}[v|X] = \epsilon_H + \overline{\beta\theta}$$
$$\mathrm{Var}(v|X) = \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2.$$

3. If $X \in [0, 1 - \chi_\epsilon + \chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_H$ with probability $2/3$ and $\epsilon = \epsilon_L$ with probability $1/3$. The market maker also knows that $\beta = \beta_H$ with probability $2/3$ and $\beta = \beta_L$ with probability $1/3$. It follows that

$$\mathbb{E}[v|X] = \frac{\Delta_\epsilon}{3} + \left(\overline{\beta} + \frac{\Delta_\beta}{3}\right)\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2.$$

4. If $X \in [-1 + \chi_\epsilon - \chi_\beta, 0)$, then the market maker knows that $\epsilon = \epsilon_H$ with probability $1/3$ and $\epsilon = \epsilon_L$ with probability $2/3$. The market maker also knows that $\beta = \beta_H$ with probability $1/3$ and $\beta = \beta_L$ with probability $2/3$. It follows that

$$\mathbb{E}[v|X] = -\frac{\Delta_\epsilon}{3} + \left(\overline{\beta} - \frac{\Delta_\beta}{3}\right)\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2.$$

5. If $X \in [-1 - \chi_\epsilon + \chi_\beta, -1 + \chi_\epsilon - \chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_L$. The market maker also knows that $\beta = \beta_H$ and $\beta = \beta_L$ are equally likely. It follows that

$$\mathbb{E}[v|X] = \epsilon_L + \overline{\beta\theta}$$
$$\mathrm{Var}(v|X) = \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2.$$

6. If $X \in [-2, -1 - \chi_\epsilon + \chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_L$ and that $\beta = \beta_L$. It follows that

$$\mathbb{E}[v|X] = \epsilon_L + \beta_L\overline{\theta}$$
$$\mathrm{Var}(v|X) = \beta_L^2\sigma_\theta^2.$$

Inserting the expressions for $\mathbb{E}[v|X]$ and $\mathrm{Var}(v|X)$ into the price function yields a step function for the equilibrium price.

Scenario (ii): $\chi_\epsilon \le \chi_\beta$. To compute the conditional mean and the conditional variance, we consider the following intervals of X.

1. If $X \in [2, 1 + \chi_\beta - \chi_\epsilon]$, then the market maker knows that $\epsilon = \epsilon_H$ and $\beta = \beta_H$. It follows that

$$\mathbb{E}[v|X] = \epsilon_H + \beta_H\overline{\theta}$$
$$\mathrm{Var}(v|X) = \beta_H^2\sigma_\theta^2.$$

2. If $X \in [1 + \chi_\epsilon - \chi_\beta, 1 + \chi_\beta - \chi_\epsilon)$, then the market maker knows that $\epsilon = \epsilon_H$ and $\epsilon = \epsilon_L$ are equally likely. The market maker also knows that $\beta = \beta_H$. It follows that

$$\mathbb{E}[v|X] = \beta_H\overline{\theta}$$
$$\mathrm{Var}(v|X) = \Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2.$$

3. If $X \in [0, 1 + \chi_\epsilon - \chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_H$ with probability $2/3$ and $\epsilon = \epsilon_L$ with probability $1/3$. The market maker also knows that $\beta = \beta_H$ with probability $2/3$ and $\beta = \beta_L$ with probability $1/3$. It follows that

$$\mathbb{E}[v|X] = \frac{\Delta_\epsilon}{3} + \left(\overline{\beta} + \frac{\Delta_\beta}{3}\right)\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2.$$

4. If $X \in [-1 + \chi_\beta - \chi_\epsilon, 0)$, then the market maker knows that $\epsilon = \epsilon_H$ with probability $1/3$ and $\epsilon = \epsilon_L$ with probability $2/3$. The market maker also knows that $\beta = \beta_H$ with probability $1/3$ and $\beta = \beta_L$ with probability $2/3$. It follows that

$$\mathbb{E}[v|X] = -\frac{\Delta_\epsilon}{3} + \left(\overline{\beta} - \frac{\Delta_\beta}{3}\right)\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2.$$

5. If $X \in [-1 + \chi_\epsilon - \chi_\beta, -1 + \chi_\beta - \chi_\epsilon)$, then the market maker knows that $\epsilon = \epsilon_H$ and $\epsilon = \epsilon_L$ are equally likely. The market maker also knows that $\beta = \beta_L$. It follows that

$$\mathbb{E}[v|X] = \beta_L\overline{\theta}$$
$$\mathrm{Var}(v|X) = \Delta_\epsilon^2 + \beta_L^2\sigma_\theta^2.$$

6. If $X \in [-2, -1 + \chi_\epsilon - \chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_L$ and $\beta = \beta_L$. It follows that

$$\mathbb{E}[v|X] = \epsilon_L + \beta_L\overline{\theta}$$
$$\mathrm{Var}(v|X) = \beta_L^2\sigma_\theta^2.$$

Again, inserting the expressions for $\mathbb{E}[v|X]$ and $\mathrm{Var}(v|X)$ into the price function yields a step function for the equilibrium price.

Step 2: Next, we characterize the conditions under which it is optimal for informed investors to buy on high signals and sell on low signals. To this end, we calculate the expected trading profits of ϵ- and β-informed investors when they receive high and low signals. Let $\pi_{\epsilon_H}(\pi_{\epsilon_L})$ and $\pi_{\beta_H}(\pi_{\beta_L})$ be the expected trading profits of ϵ-informed investors and β-informed investors when they receive high (low) signals, respectively. Simple algebraic calculations yield the following.

Scenario (i): $\chi_\epsilon > \chi_\beta$.

$$\pi = \begin{cases} \frac{2}{3}(1-\chi_\epsilon)(2\Delta_\epsilon - \overline{\theta}\Delta_\beta) \\ \quad + \frac{\gamma}{18}(9\sigma_\theta^2(-2\overline{\beta}\Delta_\beta - \chi_\epsilon(\overline{\beta}-\Delta_\beta)^2) + \overline{\theta}^2\Delta_\beta^2(5-2\chi_\epsilon(2\chi_\epsilon+5)) \\ \quad - 4\Delta_\epsilon^2(\chi_\epsilon-1)^2) & \text{if } \pi = \pi_{\epsilon_H} \\ \frac{2}{3}(1-\chi_\epsilon)(2\Delta_\epsilon - \overline{\theta}\Delta_\beta) \\ \frac{1}{18}\gamma(9\sigma_\theta^2(2\overline{\beta}\Delta_\beta - \chi_\epsilon(\overline{\beta}+\Delta_\beta)^2) + \overline{\theta}^2\Delta_\beta^2(5-2\chi_\epsilon(2\chi_\epsilon+5)) \\ \quad - 4\Delta_\epsilon^2(\chi_\epsilon-1)^2) & \text{if } \pi = \pi_{\epsilon_L}, \end{cases} \tag{A.1}$$

$$\pi = \begin{cases} \frac{1}{3}(\overline{\theta}\Delta_\beta(2\chi_\epsilon+1) - 2\Delta_\epsilon(1-\chi_\epsilon)) \\ \quad + \frac{\gamma}{18}(1-\chi_\epsilon)(-9\sigma_\theta^2(\overline{\beta}+\Delta_\beta)^2 + 4\overline{\theta}^2\Delta_\beta^2(\chi_\epsilon-1) \\ \quad + 4\Delta_\epsilon^2(\chi_\epsilon-1)) & \text{if } \pi = \pi_{\beta_H} \\ \frac{1}{3}(\overline{\theta}\Delta_\beta(2\chi_\epsilon+1) - 2\Delta_\epsilon(1-\chi_\epsilon)) \\ \quad + \frac{\gamma}{18}(1-\chi_\epsilon)(-9\sigma_\theta^2(\overline{\beta}-\Delta_\beta)^2 + 4\overline{\theta}^2\Delta_\beta^2(\chi_\epsilon-1) \\ \quad + 4\Delta_\epsilon^2(\chi_\epsilon-1)) & \text{if } \pi = \pi_{\beta_L}. \end{cases} \tag{A.2}$$

Scenario (ii): $\chi_\epsilon \le \chi_\beta$.

$$\pi = \begin{cases} \Delta_\epsilon - \frac{2}{3}\chi_\epsilon(\overline{\theta}\Delta_\beta + \Delta_\epsilon) \\ \quad + \frac{\gamma}{18}(-18\overline{\beta}\Delta_\beta\sigma_\theta^2 - 9\sigma_\theta^2\chi_\epsilon(\overline{\beta}-\Delta_\beta)^2 \\ \quad - 4\chi_\epsilon^2(\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)) & \text{if } \pi = \pi_{\epsilon_H} \\ \Delta_\epsilon - \frac{2}{3}\chi_\epsilon(\overline{\theta}\Delta_\beta + \Delta_\epsilon) \\ \quad + \frac{\gamma}{18}(18\overline{\beta}\Delta_\beta\sigma_\theta^2 - 9\sigma_\theta^2\chi_\epsilon(\overline{\beta}+\Delta_\beta)^2 \\ \quad - 4\chi_\epsilon^2(\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)) & \text{if } \pi = \pi_{\epsilon_L}, \end{cases} \tag{A.3}$$

$$\pi = \begin{cases} \frac{2}{3}\chi_\epsilon(2\overline{\theta}\Delta_\beta - \Delta_\epsilon) \\ \quad + \frac{\gamma}{18}(-9\sigma_\theta^2(\overline{\beta}+\Delta_\beta)^2 + 9\sigma_\theta^2\chi_\epsilon(\overline{\beta}+\Delta_\beta)^2 \\ \quad - 4\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2 + \Delta_\epsilon^2(2(9-2\chi_\epsilon)\chi_\epsilon - 9)) & \text{if } \pi = \pi_{\beta_H} \\ \frac{2}{3}\chi_\epsilon(2\overline{\theta}\Delta_\beta - \Delta_\epsilon) \\ \quad + \frac{\gamma}{18}(-9\sigma_\theta^2(\overline{\beta}-\Delta_\beta)^2 + 9\sigma_\theta^2\chi_\epsilon(\overline{\beta}-\Delta_\beta)^2 \\ \quad - 4\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2 + \Delta_\epsilon^2(2(9-2\chi_\epsilon)\chi_\epsilon - 9)) & \text{if } \pi = \pi_{\beta_L}. \end{cases} \tag{A.4}$$

The expressions for π above imply that $\pi_{\epsilon_H} < \pi_{\epsilon_L}$ and $\pi_{\beta_H} < \pi_{\beta_L}$. Next, we find conditions such that expected profits are positive, that is, $\pi_{\epsilon_H} > 0$ and $\pi_{\beta_H} > 0$, which implies that $\pi_{\epsilon_L} > 0$ and $\pi_{\beta_L} > 0$.

1. If $\overline{\theta} < \frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta}$, then we require that $\chi_\beta \le \underline{\chi}_\beta \in (0, \frac{1}{2})$ and $\gamma < \min(\overline{\Gamma}_1, \overline{\Gamma}_2)$ with

$$\begin{aligned} \underline{\chi}_\beta &\equiv \frac{3}{2}\frac{\Delta_\beta|\overline{\theta}|}{\Delta_\epsilon + \Delta_\beta|\overline{\theta}|} \\ \overline{\Gamma}_1 &\equiv \max\left\{\frac{12(|\overline{\theta}|\Delta_\beta + \Delta_\epsilon)(\underline{\chi}_\beta - \chi_\beta)}{\chi_\beta(9\sigma_\theta^2(\overline{\beta}-\Delta_\beta)^2 + 4\chi_\beta(\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2))}, 0\right\} \\ \overline{\Gamma}_2 &\equiv \max\left\{\frac{12\chi_\beta(2\Delta_\epsilon - |\overline{\theta}|\Delta_\beta)}{\begin{array}{c}9\sigma_\theta^2((1-\chi_\beta)(\overline{\beta}-\Delta_\beta)^2 + 2\overline{\beta}\Delta_\beta(1-2\chi_\beta)) \\ + 4\overline{\theta}^2\Delta_\beta^2\chi_\beta^2 + 9\overline{\theta}^2\Delta_\beta^2(1-2\chi_\beta) + 4\chi_\beta^2\Delta_\epsilon^2\end{array}}, 0\right\}, \end{aligned} \tag{A.5}$$

where $\chi_\beta \le \underline{\chi}_\beta$ and $\gamma < \overline{\Gamma}_1$ follow from $\pi_{\beta_H} > 0$, and $\gamma < \overline{\Gamma}_2$ follows from $\pi_{\epsilon_H} > 0$.

2. If $\overline{\theta} > 2\frac{\Delta_\epsilon}{\Delta_\beta}$, then we require that $\chi_\beta \ge \overline{\chi}_\beta \in (\frac{1}{2}, 1)$ and $\gamma < \min(\overline{\Gamma}_3, \overline{\Gamma}_4)$ with

$$\begin{aligned} \overline{\chi}_\beta &\equiv 1 - \frac{3}{2}\frac{\Delta_\epsilon}{\Delta_\epsilon + \Delta_\beta|\overline{\theta}|} \\ \overline{\Gamma}_3 &= \max\left\{\frac{6(2\Delta_\epsilon - |\overline{\theta}|\Delta_\beta + (2\chi_\beta - 1)(|\overline{\theta}|\Delta_\beta + \Delta_\epsilon))}{\begin{array}{c}9(1-\chi_\beta)\sigma_\theta^2(\overline{\beta}-\Delta_\beta)^2 + 18\overline{\beta}\Delta_\beta\sigma_\theta^2 \\ + 4(1-\chi_\beta)^2(\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)\end{array}}, 0\right\} \\ \overline{\Gamma}_4 &= \max\left\{\frac{12(1-\chi_\beta)(2|\overline{\theta}|\Delta_\beta - \Delta_\epsilon)}{\begin{array}{c}9\chi_\beta\sigma_\theta^2(\overline{\beta}+\Delta_\beta)^2 + 4\overline{\theta}^2\Delta_\beta^2(1-\chi_\beta)^2 \\ + (4\chi_\beta^2 + 5(2\chi_\beta - 1))\Delta_\epsilon^2\end{array}}, 0\right\}, \end{aligned} \tag{A.6}$$

where $\chi_\beta \ge \overline{\chi}_\beta$ and $\gamma < \overline{\Gamma}_3$ follow from $\pi_{\epsilon_H} > 0$, and $\gamma < \overline{\Gamma}_4$ follows from $\pi_{\beta_H} > 0$.

3. If $\frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta} < \overline{\theta} < 2\frac{\Delta_\epsilon}{\Delta_\beta}$ and $\chi_\beta \in (0, \frac{1}{2})$, we require that $\gamma < \min(\overline{\Gamma}_1, \overline{\Gamma}_2)$, and for $\chi_\beta \in [\frac{1}{2}, 1)$, we require that $\gamma < \min(\overline{\Gamma}_3, \overline{\Gamma}_4)$.

Under these conditions, informed investors always submit a buy order upon high signals and a sell order upon low signals.

A.1.2. Negative $\overline{\theta}$. For $\overline{\theta} < 0$, we conjecture that $\beta-$informed traders buy if $\beta = \beta_L$ and that they sell if $\beta = \beta_H$. We follow the same steps as above and obtain the following distribution for total order flow.

1. When $\epsilon = \epsilon_H$ and $\beta = \beta_L$, total order flow X is uniformly distributed in $[-1 + \chi_\epsilon + \chi_\beta, 2]$.
2. When $\epsilon = \epsilon_H$ and $\beta = \beta_H$, total order flow X is uniformly distributed in $[-1 + \chi_\epsilon - \chi_\beta, 1 + \chi_\epsilon - \chi_\beta]$.
3. When $\epsilon = \epsilon_L$ and $\beta = \beta_L$, total order flow X is uniformly distributed in $[-1 - \chi_\epsilon + \chi_\beta, 1 - \chi_\epsilon + \chi_\beta]$.
4. When $\epsilon = \epsilon_L$ and $\beta = \beta_H$, total order flow X is uniformly distributed in $[-2, 1 - \chi_\epsilon - \chi_\beta]$.

Scenario (i): $\chi_\epsilon > \chi_\beta$. To compute the conditional mean and the conditional variance, we proceed as before:

1. If $X \in [1+\chi_\epsilon-\chi_\beta, 2]$, the market maker knows that $\epsilon = \epsilon_H$ and $\beta = \beta_L$. It follows that

$$\mathbb{E}[v|X] = \epsilon_H + \beta_L\overline{\theta}$$
$$\mathrm{Var}(v|X) = \beta_L^2\sigma_\theta^2.$$

2. If $X \in [1-\chi_\epsilon+\chi_\beta, 1+\chi_\epsilon-\chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_H$. The market maker also knows that $\beta = \beta_H$ and $\beta = \beta_L$ are equally likely. It follows that

$$\mathbb{E}[v|X] = \epsilon_H + \overline{\beta}\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2.$$

3. If $X \in [1-\chi_\epsilon-\chi_\beta, 1-\chi_\epsilon+\chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_H$ with probability 2/3 and $\epsilon = \epsilon_L$ with probability 1/3. The market maker also knows that $\beta = \beta_L$ with probability 2/3 and $\beta = \beta_H$ with probability 1/3. It follows that

$$\mathbb{E}[v|X] = \frac{\Delta_\epsilon}{3} + \left(\overline{\beta} - \frac{\Delta_\beta}{3}\right)\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_L^2 + \frac{1}{3}\beta_H^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2.$$

4. If $X \in [-1+\chi_\epsilon-\chi_\beta, -1+\chi_\epsilon+\chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_H$ with probability 1/3 and $\epsilon = \epsilon_L$ with probability 2/3. The market maker also knows that $\beta = \beta_L$ with probability 1/3 and $\beta = \beta_H$ with probability 2/3. It follows that

$$\mathbb{E}[v|X] = -\frac{\Delta_\epsilon}{3} + \left(\overline{\beta} + \frac{\Delta_\beta}{3}\right)\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_L^2 + \frac{2}{3}\beta_H^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2.$$

5. If $X \in [-1-\chi_\epsilon+\chi_\beta, -1+\chi_\epsilon-\chi)$, then the market maker knows that $\epsilon = \epsilon_L$. The market maker also knows that $\beta = \beta_H$ and $\beta = \beta_L$ are equally likely. It follows that

$$\mathbb{E}[v|X] = \epsilon_L + \overline{\beta}\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2.$$

6. If $X \in [-2, -1-\chi_\epsilon+\chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_L$ and that $\beta = \beta_H$. It follows that

$$\mathbb{E}[v|X] = \epsilon_L + \beta_H\overline{\theta}$$
$$\mathrm{Var}(v|X) = \beta_H^2\sigma_\theta^2.$$

Inserting the expressions for $\mathbb{E}[v|X]$ and $\mathrm{Var}(v|X)$ into the price function yields a step function for the equilibrium price.

Scenario (ii): $\chi_\epsilon \le \chi_\beta$. In this scenario, we find that

1. If $X \in [1+\chi_\beta-\chi_\epsilon, 1+\chi_\epsilon+\chi_\beta]$, then the market maker knows that $\epsilon = \epsilon_H$ and $\beta = \beta_L$. It follows that

$$\mathbb{E}[v|X] = \epsilon_H + \beta_L\overline{\theta}$$
$$\mathrm{Var}(v|X) = \beta_L^2\sigma_\theta^2.$$

2. If $X \in [1+\chi_\epsilon-\chi_\beta, 1+\chi_\beta-\chi_\epsilon)$, then the market maker knows that $\epsilon = \epsilon_H$ and $\epsilon = \epsilon_L$ are equally likely. The market maker also knows that $\beta = \beta_L$. It follows that

$$\mathbb{E}[v|X] = \beta_L\overline{\theta}$$
$$\mathrm{Var}(v|X) = \Delta_\epsilon^2 + \beta_L^2\sigma_\theta^2.$$

3. If $X \in [1-\chi_\epsilon-\chi_\beta, 1+\chi_\epsilon-\chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_H$ with probability 2/3 and $\epsilon = \epsilon_L$ with probability 1/3. The market maker also knows that $\beta = \beta_L$ with probability 2/3 and $\beta = \beta_H$ with probability 1/3. It follows that

$$\mathbb{E}[v|X] = \frac{\Delta_\epsilon}{3} + \left(\overline{\beta} - \frac{\Delta_\beta}{3}\right)\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_L^2 + \frac{1}{3}\beta_H^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2.$$

4. If $X \in [-1+\chi_\beta-\chi_\epsilon, -1+\chi_\epsilon+\chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_H$ with probability 1/3 and $\epsilon = \epsilon_L$ with probability 2/3. The market maker also knows that $\beta = \beta_L$ with probability 1/3 and $\beta = \beta_H$ with probability 2/3. It follows that

$$\mathbb{E}[v|X] = -\frac{\Delta_\epsilon}{3} + \left(\overline{\beta} + \frac{\Delta_\beta}{3}\right)\overline{\theta}$$
$$\mathrm{Var}(v|X) = \frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_L^2 + \frac{2}{3}\beta_H^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2.$$

5. If $X \in [-1+\chi_\epsilon-\chi_\beta, -1+\chi_\beta-\chi_\epsilon)$, then the market maker knows that $\epsilon = \epsilon_H$ and $\epsilon = \epsilon_L$ are equally likely. He also knows that $\beta = \beta_L$. It follows that

$$\mathbb{E}[v|X] = \beta_H\overline{\theta}$$
$$\mathrm{Var}(v|X) = \Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2.$$

6. If $X \in [-1-\chi_\epsilon-\chi_\beta, -1+\chi_\epsilon-\chi_\beta)$, then the market maker knows that $\epsilon = \epsilon_L$ and $\beta = \beta_H$. It follows that

$$\mathbb{E}[v|X] = \epsilon_L + \beta_H\overline{\theta}$$
$$\mathrm{Var}(v|X) = \beta_H^2\sigma_\theta^2.$$

Again, inserting the expressions for $\mathbb{E}[v|X]$ and $\mathrm{Var}(v|X)$ into the price function yields a step function for the equilibrium price.

Step 2: Next, we characterize the conditions under which the conjectured trading strategies are optimal. To this end, we calculate the expected trading profits of ϵ- and β-informed investors when they receive high and low signals. Let $\pi_{\epsilon_H}(\pi_{\epsilon_L})$ and $\pi_{\beta_H}(\pi_{\beta_L})$ be the expected trading profits of ϵ-informed investors and β-informed investors when they receive high (low) signals, respectively. Simple algebraic calculations yield the following.

Scenario (i): $\chi_\epsilon > \chi_\beta$.

$$\pi = \begin{cases} \frac{2}{3}(1-\chi_\epsilon)(\overline{\theta}\Delta_\beta + 2\Delta_\epsilon) \\ \quad + \frac{\gamma}{18}(9\sigma_\theta^2(2\overline{\beta}\Delta_\beta - \chi_\epsilon(\overline{\beta}+\Delta_\beta)^2) + \overline{\theta}^2\Delta_\beta^2(5-2\chi_\epsilon(2\chi_\epsilon+5)) \\ \quad - 4\Delta_\epsilon^2(\chi_\epsilon-1)^2) & \text{if } \pi = \pi_{\epsilon_H} \\ \frac{2}{3}(1-\chi_\epsilon)(\overline{\theta}\Delta_\beta + 2\Delta_\epsilon) \\ \quad + \frac{\gamma}{18}(9\sigma_\theta^2(-\chi_\epsilon(\overline{\beta}-\Delta_\beta)^2 - 2\overline{\beta}\Delta_\beta) + \overline{\theta}^2\Delta_\beta^2(5-2\chi_\epsilon(2\chi_\epsilon+5)) \\ \quad - 4\Delta_\epsilon^2(\chi_\epsilon-1)^2) & \text{if } \pi = \pi_{\epsilon_L}, \end{cases} \tag{A.7}$$

$$\pi = \begin{cases} \frac{1}{3}(2\Delta_\epsilon(\chi_\epsilon - 1) - \overline{\theta}\Delta_\beta(2\chi_\epsilon + 1)) \\ + \frac{\gamma}{18}(1-\chi_\epsilon)(-9\sigma_\theta^2(\overline{\beta} - \Delta_\beta)^2 + 4\overline{\theta}^2\Delta_\beta^2(\chi_\epsilon - 1) \\ \quad + 4\Delta_\epsilon^2(\chi_\epsilon - 1)) & \text{if } \pi = \pi_{\beta_L} \\ \frac{1}{3}(2\Delta_\epsilon(\chi_\epsilon - 1) - \overline{\theta}\Delta_\beta(2\chi_\epsilon + 1)) \\ + \frac{\gamma}{18}(1-\chi_\epsilon)(-9\sigma_\theta^2(\overline{\beta} + \Delta_\beta)^2 + 4\overline{\theta}^2\Delta_\beta^2(\chi_\epsilon - 1) \\ \quad + 4\Delta_\epsilon^2(\chi_\epsilon - 1)) & \text{if } \pi = \pi_{\beta_H}. \end{cases} \tag{A.8}$$

Scenario (ii): $\chi_\epsilon \le \chi_\beta$.

$$\pi = \begin{cases} \frac{2}{3}\chi_\epsilon(\overline{\theta}\Delta_\beta - \Delta_\epsilon) + \Delta_\epsilon \\ \frac{1}{18}\gamma(18\overline{\beta}\Delta_\beta\sigma_\theta^2 - 9\sigma_\theta^2\chi_\epsilon(\overline{\beta} + \Delta_\beta)^2 \\ \quad - 4\chi_\epsilon^2(\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)) & \text{if } \pi = \pi_{\epsilon_H} \\ \frac{2}{3}\chi_\epsilon(\overline{\theta}\Delta_\beta - \Delta_\epsilon) + \Delta_\epsilon \\ + \frac{\gamma}{18}(-18\overline{\beta}\Delta_\beta\sigma_\theta^2 - 9\sigma_\theta^2\chi_\epsilon(\overline{\beta} - \Delta_\beta)^2 \\ \quad - 4\chi_\epsilon^2(\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)) & \text{if } \pi = \pi_{\epsilon_L}, \end{cases} \tag{A.9}$$

$$\pi = \begin{cases} \frac{2}{3}\chi_\epsilon(-2\overline{\theta}\Delta_\beta - \Delta_\epsilon) \\ + \frac{\gamma}{18}(-9\sigma_\theta^2(\overline{\beta} - \Delta_\beta)^2 + 9\sigma_\theta^2\chi_\epsilon(\overline{\beta} - \Delta_\beta)^2 \\ \quad - 4\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2 + \Delta_\epsilon^2(2(9-2\chi_\epsilon)\chi_\epsilon - 9)) & \text{if } \pi = \pi_{\beta_L} \\ \frac{2}{3}\chi_\epsilon(-2\overline{\theta}\Delta_\beta - \Delta_\epsilon) \\ + \frac{\gamma}{18}(-9\sigma_\theta^2(\overline{\beta} + \Delta_\beta)^2 + 9\sigma_\theta^2\chi_\epsilon(\overline{\beta} + \Delta_\beta)^2 \\ \quad - 4\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2 + \Delta_\epsilon^2(2(9-2\chi_\epsilon)\chi_\epsilon - 9)) & \text{if } \pi = \pi_{\beta_H}. \end{cases} \tag{A.10}$$

Next, we use the expressions for π_{ϵ_H}, π_{ϵ_L}, π_{β_H}, and π_{β_L} and find conditions such that trading profits are positive in expectation. Following the same steps as before, we find that

1. If $-\overline{\theta} < \frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta}$, then we require that $\chi_\beta \le \underline{\chi}_\beta \in (0, \frac{1}{2})$ and $\gamma < \min(\overline{\Gamma}_1, \overline{\Gamma}_2)$.
2. If $\frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta} < -\overline{\theta} < 2\frac{\Delta_\epsilon}{\Delta_\beta}$ and $\chi_\beta \in (0, \frac{1}{2})$, we require that $\gamma < \min(\overline{\Gamma}_1, \overline{\Gamma}_2)$, and for $\chi_\beta \in [\frac{1}{2}, 1)$, we require that $\gamma < \min(\overline{\Gamma}_3, \overline{\Gamma}_4)$.
3. If $-\overline{\theta} > 2\frac{\Delta_\epsilon}{\Delta_\beta}$, then we require that $\chi_\beta \ge \overline{\chi}_\beta \in (\frac{1}{2}, 1)$ and $\gamma < \min(\overline{\Gamma}_3, \overline{\Gamma}_4)$.

Under these conditions, ϵ-informed (β-informed) investors always submit a buy (sell) order upon high signals and a sell (buy) order upon low signals.

A.1.3. $\overline{\theta}$=0. For $\overline{\theta} = 0$, the expressions above imply that β-informed traders make strictly negative expected profits for any $\chi_\beta > 0$. If $\chi_\beta = 0$ so that only ϵ-informed traders are present, then expected profits for ϵ-informed traders are equal to zero. Hence, we conclude that this equilibrium does not exist for $\overline{\theta} = 0$.

A.1.4. Existence Conditions for Equilibria 1(a) and 1(b). Based on the derivations in steps 2 of Sections A.1.1 and A.1.2, we compactly describe the existence conditions for equilibria 1(a) and 1(b) below. Equilibrium 1(a) (1(b)) exists if and only if $\overline{\theta} > 0$ ($\overline{\theta} < 0$), $\gamma < \overline{\gamma}_1$, and $\chi_\beta \in \mathcal{X}_1$, where

$$\overline{\gamma}_1 \equiv \begin{cases} \min(\overline{\Gamma}_1, \overline{\Gamma}_2) & \text{if } |\overline{\theta}| < \frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta} \text{ or if } \frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta} < |\overline{\theta}| < 2\frac{\Delta_\epsilon}{\Delta_\beta} \\ & \qquad \text{and } \chi_\beta < \frac{1}{2} \\ \min(\overline{\Gamma}_3, \overline{\Gamma}_4) & \text{otherwise,} \end{cases} \tag{A.11}$$

and

$$\mathcal{X}_1 \equiv \begin{cases} (0, \underline{\chi}_\beta) & \text{if } |\overline{\theta}| < \frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta} \\ (0,1) & \text{if } \frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta} < |\overline{\theta}| < 2\frac{\Delta_\epsilon}{\Delta_\beta} \\ (\overline{\chi}_\beta, 1) & \text{if } |\overline{\theta}| > 2\frac{\Delta_\epsilon}{\Delta_\beta}. \end{cases} \tag{A.12}$$

From Equations (A.5) and (A.6), the constants ($\overline{\Gamma}_1, \overline{\Gamma}_2, \overline{\Gamma}_3, \overline{\Gamma}_4, \underline{\chi}_\beta, \overline{\chi}_\beta$) are defined as follows:

$$\overline{\Gamma}_1 \equiv \max\left\{\frac{12(|\overline{\theta}|\Delta_\beta + \Delta_\epsilon)(\underline{\chi}_\beta - \chi_\beta)}{\chi_\beta\left(9\sigma_\theta^2(\overline{\beta} - \Delta_\beta)^2 + 4\chi_\beta(\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)\right)}, 0\right\}, \tag{A.13}$$

$$\overline{\Gamma}_2 \equiv \max\left\{\frac{\frac{4\chi_\beta}{3\sigma_\theta^2}(2\Delta_\epsilon - |\overline{\theta}|\Delta_\beta)}{\begin{array}{c}((1-\chi_\beta)(\overline{\beta} - \Delta_\beta)^2 + 2\overline{\beta}\Delta_\beta(1 - 2\chi_\beta)) \\ + \overline{\theta}^2\Delta_\beta^2((2\chi_\beta - 3)^2 - 6\chi_\beta) + 4\chi_\beta^2\Delta_\epsilon^2\end{array}}, 0\right\}, \tag{A.14}$$

$$\overline{\Gamma}_3 \equiv \max\left\{\frac{6(2\Delta_\epsilon - |\overline{\theta}|\Delta_\beta + (2\chi_\beta - 1)(|\overline{\theta}|\Delta_\beta + \Delta_\epsilon))}{\begin{array}{c}9(1-\chi_\beta)\sigma_\theta^2(\overline{\beta} - \Delta_\beta)^2 + 18\overline{\beta}\Delta_\beta\sigma_\theta^2 \\ + 4(1-\chi_\beta)^2(\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)\end{array}}, 0\right\}, \tag{A.15}$$

$$\overline{\Gamma}_4 \equiv \max\left\{\frac{12(1-\chi_\beta)(2|\overline{\theta}|\Delta_\beta - \Delta_\epsilon)}{\begin{array}{c}9\chi_\beta\sigma_\theta^2(\overline{\beta} + \Delta_\beta)^2 + 4\overline{\theta}^2\Delta_\beta^2(1-\chi_\beta)^2 \\ + (4\chi_\beta^2 + 5(2\chi_\beta - 1))\Delta_\epsilon^2\end{array}}, 0\right\}, \tag{A.16}$$

$$\underline{\chi}_\beta \equiv \frac{3}{2}\frac{\Delta_\beta|\overline{\theta}|}{\Delta_\epsilon + \Delta_\beta|\overline{\theta}|}, \tag{A.17}$$

$$\overline{\chi}_\beta \equiv 1 - \frac{3}{2}\frac{\Delta_\epsilon}{\Delta_\epsilon + \Delta_\beta|\overline{\theta}|}. \tag{A.18}$$

Note that $\underline{\chi}_\beta \in (0,1)$ if $|\overline{\theta}| < \frac{1}{2}\frac{\Delta_\epsilon}{\Delta_\beta}$ and $\overline{\chi}_\beta \in (0,1)$ if $|\overline{\theta}| > 2\frac{\Delta_\epsilon}{\Delta_\beta}$ so that $\mathcal{X}_1 \subseteq (0,1)$. Figure 1 (discussed in Section 2.2) provides a graphical illustration of the equilibrium conditions. These equilibria exist in the parameter space below the gray surface.

A.1.5. Equilibria Without Informed Trading by ϵ-Informed Investors. Next, we characterize equilibria 2(a) and 2(b), starting with the former. In this equilibrium, ϵ-informed investors always sell and β-informed buy (sell) on a high (low) signal. Total order flow is equal to

$$X = \begin{cases} 2\chi_\beta - 1 + z & \text{if } \beta = \beta_H \\ -1 + z & \text{if } \beta = \beta_L. \end{cases} \tag{A.19}$$

The equilibrium price is equal to $p = E[v|X] + \frac{\gamma}{2} X\mathrm{Var}(v|X)$, which is given by

$$p = \begin{cases} \beta_H \overline{\theta} + \frac{\gamma}{2} X(\Delta_\epsilon^2 + \beta_H^2 \sigma_\theta^2) & \text{if } 2\chi_\beta > X > 0 \\ \overline{\beta}\,\overline{\theta} + \frac{\gamma}{2} X(\Delta_\epsilon^2 + \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2 \overline{\theta}^2) & \text{if } 0 > X > -2(1-\chi_\beta) \\ \beta_L \overline{\theta} + \frac{\gamma}{2} X(\Delta_\epsilon^2 + \beta_L^2 \sigma_\theta^2) & \text{if } -2(1-\chi_\beta) > X > -2. \end{cases}$$

We can now compute expected trading profits by taking conditional expectations over $y_\epsilon(v-p)$ and $y_\beta(v-p)$. We then derive parameter restrictions such that trading profits (conditional on private signals) are positive. After some algebraic simplifications, we find that expected trading profits for an ϵ-informed investor are equal to

$$\Pi_i(\tilde{\epsilon}) = \frac{-\gamma\overline{\theta}^2\Delta_\beta^2}{2}\chi_\beta^2 + \frac{\gamma(\sigma_\theta^2(\overline{\beta}+\Delta_\beta)^2 + 2\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)}{2}\chi_\beta - \frac{1}{2}(\gamma\sigma_\theta^2(\overline{\beta}^2 + \Delta_\beta^2) + \gamma\overline{\theta}^2\Delta_\beta^2 + \gamma\Delta_\epsilon^2 + 2\tilde{\epsilon}) \tag{A.20}$$

with $\tilde{\epsilon} \in \{\epsilon_L, \epsilon_H\}$ and $\Pi_i(\epsilon_L) > \Pi_i(\epsilon_H)$. Note that $\Pi_i(\epsilon_H)$ is a quadratic function of χ_β, denoted by $f(\chi_\beta) = a_1\chi_\beta^2 + a_2\chi_\beta + a_3$, with the following coefficients:

$$a_1 = \frac{-\gamma\overline{\theta}^2\Delta_\beta^2}{2}$$

$$a_2 = \frac{\gamma(\sigma_\theta^2(\overline{\beta}+\Delta_\beta)^2 + 2\overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2)}{2}$$

$$a_3 = -\frac{1}{2}(\gamma\sigma_\theta^2(\overline{\beta}^2 + \Delta_\beta^2) + \gamma\overline{\theta}^2\Delta_\beta^2 + \gamma\Delta_\epsilon^2 + 2\Delta_\epsilon).$$

This function has the following properties: (i) $f(0) = a_3 < 0$, (ii) $f'' = 2a_1 < 0$, and (iii) $f'(1) = 2a_1 + a2 > 0$. It follows that $\Pi_i(\epsilon_H)$ is positive if and only if $f(1) = a_1 + a_2 + a_3 = \gamma\overline{\beta}\Delta_\beta\sigma_\theta^2 - \Delta_\epsilon > 0$ and $\chi_\beta > \underline{\chi}_{\beta,2}$, where $\underline{\chi}_{\beta,2} \in (0,1)$ is defined as the unique root in [0, 1] that sets $\Pi_i(\epsilon_H) = 0$. These conditions are equivalent to

$$\gamma > \frac{\Delta_\epsilon}{\Delta_\beta\overline{\beta}\sigma_\theta^2} \equiv \underline{\gamma}_2 \tag{A.21}$$

$$\chi_\beta > \frac{-a_2 + \sqrt{a_2^2 - a_1a_3}}{2a_1} \equiv \underline{\chi}_{\beta,2} \in (0,1). \tag{A.22}$$

Proceeding as before, expected profits for a β-informed investor equal

$$\Pi_i(\beta_H) = |\overline{\theta}|\Delta_\beta(1-\chi_\beta) + \frac{\gamma}{2}[\overline{\theta}^2\Delta_\beta^2(1-\chi_\beta)^2 + (1-2\chi_\beta)\Delta_\epsilon^2 - \sigma_\theta^2(2\overline{\beta}\Delta_\beta\chi_\beta^2 + (2\chi_\beta - 1)(\overline{\beta}^2 + \Delta_\beta^2))]$$

$$\Pi_i(\beta_L) = |\overline{\theta}|\Delta_\beta(1-\chi_\beta) - \frac{\gamma}{2}[\overline{\theta}^2\Delta_\beta^2(1-\chi_\beta)^2 + \Delta_\epsilon^2 + \sigma_\theta^2(\overline{\beta}^2 + 2\Delta_\beta\overline{\beta}(\chi_\beta - 2)\chi_\beta + \Delta_\beta^2)]. \tag{A.23}$$

We can show that $\Pi_i(\epsilon_H) > 0$ implies that $\Pi_i(\beta_L) > \Pi_i(\beta_H)$ so that we can focus on the parameter conditions such that $\Pi_i(\beta_H) > 0$. Using the expression for expected trading profits above, we can rewrite $\Pi_i(\beta_H)$ as a quadratic function of χ_β as follows: $g(\chi_\beta) = b_1\chi_\beta^2 + b_2\chi_\beta + b_3$ with

$$b_1 = \frac{1}{2}\gamma\Delta_\beta(\overline{\theta}^2\Delta_\beta - 2\overline{\beta}\sigma_\theta^2)$$

$$b_2 = -\gamma(\sigma_\theta^2(\overline{\beta}^2 + \Delta_\beta^2) + \overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2) - |\overline{\theta}|\Delta_\beta$$

$$b_3 = \frac{1}{2}\gamma(\sigma_\theta^2(\overline{\beta}^2 + \Delta_\beta^2) + \overline{\theta}^2\Delta_\beta^2 + \Delta_\epsilon^2) + |\overline{\theta}|\Delta_\beta.$$

Hence, we have $g(0) = b_3 > 0$ and $g(1) = b_1 + b_2 + b_3 = -\frac{1}{2}\gamma(\sigma_\theta^2(\overline{\beta} + \Delta_\beta)^2 + \Delta_\epsilon^2) < 0$. Because g is quadratic, it follows that $\Pi_i(\beta_H) > 0$ if and only if $\chi_\beta < \overline{\chi}_{\beta,2}$, where $\overline{\chi}_{\beta,2} \in (0,1)$ is the unique root in [0, 1] that sets $\Pi_i(\beta_H) = 0$. Thus, the threshold $\overline{\chi}_{\beta,2}$ is given by

$$\overline{\chi}_{\beta,2} \equiv \begin{cases} \frac{-b_2 - \sqrt{b_2^2 - 4b_1b_3}}{2b_1} & \text{if } b_1 \neq 0 \\ -\frac{b_3}{b_2} & \text{if } b_1 = 0. \end{cases} \tag{A.24}$$

Therefore, equilibrium 2(a) exists if and only if the following conditions hold jointly: (i) $\gamma > \underline{\gamma}_2$, (ii) $\underline{\chi}_{\beta,2} < \overline{\chi}_{\beta,2}$, (iii) $\chi_\beta > \underline{\chi}_{\beta,2}$, and (iv) $\chi_\beta < \overline{\chi}_{\beta,2}$.

In equilibrium 2(b), the total order flow equals

$$X = \begin{cases} 1 + z & \text{if } \beta = \beta_L \\ 1 - 2\chi_\beta + z & \text{if } \beta = \beta_H. \end{cases} \tag{A.25}$$

The equilibrium price is equal to $p = E[v|X] + \frac{\gamma}{2}X\mathrm{Var}(v|X)$. Proceeding as before, we find the same expression for expected profits $\Pi_i(\epsilon_H)$, $\Pi_i(\epsilon_L)$, $\Pi_i(\beta_H)$, and $\Pi_i(\beta_L)$. Hence, we obtain the same existence conditions as before. Equilibria 2(a) and 2(b) exist if and only if $\gamma > \underline{\gamma}_2$ and $\chi_\beta \in \mathcal{X}_2$ with

$$\mathcal{X}_2 \equiv \begin{cases} (\underline{\chi}_{\beta,2}, \overline{\chi}_{\beta,2}) & \text{if } \underline{\chi}_{\beta,2} < \overline{\chi}_{\beta,2} \\ \emptyset & \text{otherwise.} \end{cases} \tag{A.26}$$

The thresholds $\underline{\gamma}_2$, $\underline{\chi}_{\beta,2}$, and $\overline{\chi}_{\beta,2}$ are defined in Equations (A.21), (A.22), and (A.24). Note that $\mathcal{X}_2$ can be empty because, for a subset of the parameter space, $\underline{\chi}_{\beta,2} \geq \overline{\chi}_{\beta,2}$. In particular, note that Equation (A.23) implies that $\Pi_i(\beta_L)$ strictly decreases in γ with $\lim_{\gamma\to\infty}\Pi_i(\beta_L) = -\infty$. Hence, if γ is sufficiently large, the equilibrium ceases to exist because $\Pi_i(\beta_L) < 0$, and we have that $\mathcal{X}_2 = \emptyset$.

For $\overline{\theta} = 0$, the expressions above imply that β-informed traders make strictly negative expected profits for any $\chi_\beta > 0$. If $\chi_\beta = 0$ so that only ϵ-informed traders are present, then expected profits for ϵ-informed traders are negative. Hence, we conclude that equilibria 2(a) and 2(b) do not exist for $\overline{\theta}=0$.

Finally, we show that the four equilibria are mutually exclusive. For $\overline{\theta}>0$, only equilibria 1(a) and 2(a) are possible. To show that they do not occur for the same parameter space, it is sufficient to focus on the profits of ϵ-informed investors in each case. We have shown above that equilibrium 2(a) requires that

$$\gamma>\overline{\gamma}_2=\frac{\Delta_\epsilon}{\overline{\beta}\Delta_\beta\sigma_\theta^2}. \tag{A.27}$$

In equilibrium 1(a), the expected profits of ϵ-informed investors (given $\epsilon=\epsilon_H$) are decreasing in γ. We can, thus, evaluate π_{ϵ_H} at $\gamma=\frac{\Delta_\epsilon}{\overline{\beta}\Delta_\beta\sigma_\theta^2}$, which yields

$$\frac{1}{18}\chi_\epsilon\left(-12\overline{\theta}\Delta_\beta-\frac{4\chi_\epsilon(\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon+\Delta_\epsilon^3)}{\overline{\beta}\Delta_\beta\sigma_\theta^2}+\Delta_\epsilon\left(-\frac{9\overline{\beta}}{\Delta_\beta}-\frac{9\Delta_\beta}{\overline{\beta}}+6\right)\right)<0, \tag{A.28}$$

where we have used that $\frac{\overline{\beta}}{\Delta_\beta}+\frac{\Delta_\beta}{\overline{\beta}}>1$ if $\overline{\beta}$ and Δ_β are positive. As a result, equilibrium 1(a) cannot coexist with equilibrium 2(a) because ϵ-informed investors make negative profits for all $\gamma>\underline{\gamma}_2$.

Similarly, for $\overline{\theta}<0$, only equilibria 1(b) and 2(b) are possible. To show that they do not occur for the same parameter space, it is again sufficient to focus on the profits of ϵ-informed investors in each case. We have shown above that equilibrium 2(b) requires that $\gamma>\frac{\Delta_\epsilon}{\overline{\beta}\Delta_\beta\sigma_\theta^2}$. As before, the expected profits for ϵ-informed investors in equilibrium 1(b) (given $\epsilon=\epsilon_H$) are decreasing in γ. We can, thus, again evaluate π_{ϵ_H} at $\gamma=\frac{\Delta_\epsilon}{\overline{\beta}\Delta_\beta\sigma_\theta^2}$, which yields

$$\frac{1}{18}\chi_\epsilon\left(12\overline{\theta}\Delta_\beta-\frac{4\Delta_\epsilon\chi_\epsilon(\overline{\theta}^2\Delta_\beta^2+\Delta_\epsilon^2)}{\overline{\beta}\Delta_\beta\sigma_\theta^2}-\frac{9\Delta_\epsilon(\overline{\beta}-\Delta_\beta)^2}{\overline{\beta}\Delta_\beta}-12\Delta_\epsilon\right)<0, \tag{A.29}$$

where we have used $\overline{\theta}<0$. Hence, equilibrium 1(b) cannot coexist with equilibrium 2(b) because ϵ-informed investors make negative profits for all $\gamma>\underline{\gamma}_2$. Note that $\overline{\gamma}_1$ depends on χ_β (see Figure 1), whereas $\underline{\gamma}_2=\frac{\Delta_\epsilon}{\overline{\beta}\Delta_\beta\sigma_\theta^2}$ does not. We can, therefore, consider $\chi_\beta^*\in\mathcal{X}_1$ that maximizes $\overline{\gamma}_1$ and leads to $\overline{\gamma}_1^*$. Because we have shown that equilibria type 1 and 2 never coexist, it follows that $\overline{\gamma}_1^*$ must be less than $\underline{\gamma}_2$ in the entire parameter space for which equilibrium type 1 exists. It follows that $\underline{\gamma}_2>\overline{\gamma}_1$.

A.1.6. Equilibrium Uniqueness. Next, we rule out alternative, pure-strategy equilibria in which informed investors trade (i.e., $y_i\neq 0$) to prove uniqueness for this type of equilibrium.

1. Equilibria in which β-informed always buy, that is, $y_i(\beta_H)=y_i(\beta_L)=+1$.

a. ϵ-informed always buy. In this case, total order flow is equal to $X=1+z$ and, thus, orthogonal to the asset payoff. The equilibrium price is equal to $p=E[v]+\frac{\gamma}{2}(1+z)$ $\mathrm{Var}(v)$. To prove that this equilibrium cannot exist, we consider trading profits given $\epsilon=\epsilon_L$:

$$\Pi_i(\epsilon=\epsilon_L)=-\Delta_\epsilon-\frac{\gamma}{2}\mathrm{Var}(v)<0. \tag{A.30}$$

b. ϵ-informed always sell. In this case, total order flow is equal to $X=\chi_\beta-\chi_\epsilon+z=2\chi_\beta-1+z$ and, thus, orthogonal to the asset payoff. The equilibrium price is equal to $p=E[v]+\frac{\gamma}{2}(2\chi_\beta-1+z)\mathrm{Var}(v)$. We consider trading profits for an ϵ-informed investor given $\epsilon=\epsilon_H$:

$$\Pi_i(\epsilon=\epsilon_H)=-\Delta_\epsilon+\gamma\left(\chi_\beta-\frac{1}{2}\right)\mathrm{Var}(v); \tag{A.31}$$

and expected trading profits for a β-informed investor given $\tilde{\beta}\in\{\beta_L,\beta_H\}$:

$$\Pi_i(\tilde{\beta})=(\tilde{\beta}-\overline{\beta})\overline{\theta}-\gamma\left(\chi_\beta-\frac{1}{2}\right)\mathrm{Var}(v). \tag{A.32}$$

Hence, $\Pi_i(\epsilon=\epsilon_H)<0$ or $\Pi_i(\tilde{\beta})<0$ (or both).

c. ϵ-informed buy on a high signal and sell on a low signal. In this case, the total order flow is equal to

$$X=\begin{cases}1+z & \text{if } \epsilon=\epsilon_H\\ 2\chi_\beta-1+z & \text{if } \epsilon=\epsilon_L.\end{cases} \tag{A.33}$$

The equilibrium price is equal to $p=E[v|X]+\frac{\gamma}{2}X\mathrm{Var}(v|X)$. Trading profits for a β-informed investor are equal to

$$\Pi_i(\tilde{\beta})=(\tilde{\beta}-\overline{\beta})\overline{\theta}-\frac{\gamma}{4}\left[\chi_\epsilon\frac{2-\chi_\epsilon}{2}\mathrm{Var}(\beta\theta)+2(1-\chi_\epsilon)^2\mathrm{Var}(v)-\chi_\epsilon^2\mathrm{Var}(\beta\theta)\right], \tag{A.34}$$

which is negative for $\tilde{\beta}=\beta_L$ (if $\overline{\theta}>0$) or for $\tilde{\beta}=\beta_H$ (if $\overline{\theta}<0$).

d. ϵ-informed buy on a low signal and sell on a high signal. In this case, the total order flow is equal to

$$X=\begin{cases}1+z & \text{if } \epsilon=\epsilon_L\\ 2\chi_\beta-1+z & \text{if } \epsilon=\epsilon_H.\end{cases} \tag{A.35}$$

The equilibrium price is equal to $p=E[v|X]+\frac{\gamma}{2}X\mathrm{Var}(v|X)$. Trading profits for a β-informed investor are equal to

$$\Pi_i(\tilde{\beta})=(\tilde{\beta}-\overline{\beta})\overline{\theta}-\frac{\gamma}{4}[\chi_\epsilon(2-\chi_\epsilon)\mathrm{Var}(\beta\theta)+2(1-\chi_\epsilon)^2\mathrm{Var}(v)-\chi_\epsilon^2\mathrm{Var}(\beta\theta)], \tag{A.36}$$

which is negative for $\tilde{\beta}=\beta_L$ (if $\overline{\theta}>0$) or for $\tilde{\beta}=\beta_H$ (if $\overline{\theta}<0$).

2. Equilibria in which β-informed always sell, that is, $y_i(\beta_H)=y_i(\beta_L)=-1$.

a. ϵ-informed always buy. In this case, total order flow is equal to $X=\chi_\epsilon-\chi_\beta+z=1-2\chi_\beta+z$ and, thus, orthogonal to the asset payoff. The equilibrium price is equal to $p=E[v]+\frac{\gamma}{2}(1-2\chi_\beta+z)\mathrm{Var}(v)$. Consider the expected trading profits for an ϵ-informed investor given $\epsilon=\epsilon_L$:

$$\Pi_i(\epsilon=\epsilon_L)=-\Delta_\epsilon-\gamma\left(\frac{1}{2}-\chi_\beta\right)\mathrm{Var}(v); \tag{A.37}$$

and expected trading profits for a β-informed investor given $\tilde{\beta}\in\{\beta_L,\beta_H\}$:

$$\Pi_i(\tilde{\beta})=-(\tilde{\beta}-\overline{\beta})\overline{\theta}+\gamma\left(\frac{1}{2}-\chi_\beta\right)\mathrm{Var}(v). \tag{A.38}$$

Hence, $\Pi_i(\epsilon=\epsilon_L)<0$ or $\Pi_i(\tilde{\beta})<0$ (or both).

b. ϵ-informed always sell. In this case, total order flow is equal to $X = -1 + z$ and, thus, orthogonal to the asset payoff. The equilibrium price is equal to $p = E[v] + \frac{\gamma}{2}(-1+z)\mathrm{Var}(v)$. To prove that this equilibrium cannot exist, we can consider the trading profits given $\epsilon = \epsilon_H$:

$$\Pi_i(\epsilon = \epsilon_H) = -\Delta_\epsilon - \frac{\gamma}{2}\mathrm{Var}(v) < 0. \tag{A.39}$$

c. ϵ-informed buy on a high signal and sell on a low signal. In this case, the total order flow is equal to

$$X = \begin{cases} 1 - 2\chi_\beta + z & \text{if } \epsilon = \epsilon_H \\ -1 + z & \text{if } \epsilon = \epsilon_L. \end{cases} \tag{A.40}$$

The equilibrium price is equal to $p = E[v|X] + \frac{\gamma}{2}X\mathrm{Var}(v|X)$. Trading profits for a β-informed investor are equal to

$$\Pi_i(\tilde{\beta}) = -(\tilde{\beta} - \overline{\beta})\overline{\theta} + \frac{\gamma}{4}[\chi_\epsilon^2\mathrm{Var}(\beta\theta) - 2(1-\chi_\epsilon)^2\mathrm{Var}(v) - \chi_\epsilon(2-\chi_\epsilon)\mathrm{Var}(\beta\theta)], \tag{A.41}$$

which is negative for $\tilde{\beta} = \beta_H$ (if $\overline{\theta} > 0$) or for $\tilde{\beta} = \beta_L$ (if $\overline{\theta} < 0$).

d. ϵ-informed buy on a low signal and sell on a high signal. In this case, the total order flow is equal to

$$X = \begin{cases} 1 - 2\chi_\beta + z & \text{if } \epsilon = \epsilon_L \\ -1 + z & \text{if } \epsilon = \epsilon_H. \end{cases} \tag{A.42}$$

The equilibrium price is equal to $p = E[v|X] + \frac{\gamma}{2}X\mathrm{Var}(v|X)$. Trading profits for a β-informed investor are equal to

$$\Pi_i(\tilde{\beta}) = -(\tilde{\beta} - \overline{\beta})\overline{\theta} + \frac{\gamma}{4}[\chi_\epsilon^2\mathrm{Var}(\beta\theta) - 2(1-\chi_\epsilon)^2\mathrm{Var}(v) - \chi_\epsilon(2-\chi_\epsilon)\mathrm{Var}(\beta\theta)], \tag{A.43}$$

which is negative for $\tilde{\beta} = \beta_H$ (if $\overline{\theta} > 0$) or for $\tilde{\beta} = \beta_L$ (if $\overline{\theta} < 0$).

3. Equilibria in which β-informed buy on a high signal and sell on a low signal, that is, $y_i(\beta_H) = +1$ and $y_i(\beta_L) = -1$.

a. ϵ-informed always buy. In this case, the total order flow is equal to

$$X = \begin{cases} 1 + z & \text{if } \beta = \beta_H \\ 1 - 2\chi_\beta + z & \text{if } \beta = \beta_L. \end{cases} \tag{A.44}$$

The equilibrium price is equal to $p = E[v|X] + \frac{\gamma}{2}X\mathrm{Var}(v|X)$. Trading profits for an ϵ-informed investor are equal to

$$\Pi_i(\epsilon_L) = -\Delta_\epsilon - \frac{\gamma}{4}[2(1-\chi_\beta)\Delta_\epsilon + \chi_\beta(2-\chi_\beta)\beta_H^2\sigma_\theta^2 + 2(1-\chi_\beta)^2\mathrm{Var}(\beta\theta) - \chi_\beta^2\beta_L^2\sigma_\theta^2], \tag{A.45}$$

which is negative because $\beta_H^2 > \beta_L^2$.

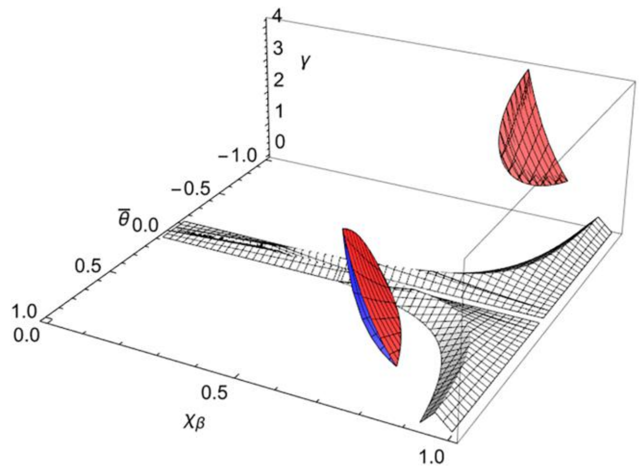
b. ϵ-informed always sell. This case corresponds to equilibrium 2(a).

c. ϵ-informed buy on a high signal and sell on a low signal. If $\overline{\theta} > 0$, this case corresponds to equilibrium 1(a). If $\overline{\theta} < 0$, expected trading profits for β-informed investors are negative.

d. ϵ-informed buy on a low signal and sell on a high signal. Following the analogous steps as before, at the beginning of the proof, we can show that this strategy always leads to negative profits for ϵ-informed investors.

4. Equilibria in which β-informed sell on a high signal and buy on a low signal, that is, $y_i(\beta_H) = -1$ and $y_i(\beta_L) = +1$.

Figure A.1. Using a Three-Dimensional Plot, This Figure Plots the Existence Conditions for the Type 1 and Type 2 Equilibria in Theorem 1 Across Ranges of γ, χ_β, and $\overline{\theta}$



Notes. Other parameter values are $\overline{\beta} = 1, \Delta_\beta = \frac{1}{2}, \Delta_\epsilon = \frac{1}{40}$, and $\sigma_\theta = \frac{1}{4}$. Type 1 equilibria exist in the parameter range below the gray surface; Type 2 equilibria exist in the region between the red and the blue surface. The blue surface is concealed in the existence region for $\overline{\theta} < 0$; however, the regions are symmetric about $\overline{\theta} = 0$.

a. ϵ-informed always buy. This case corresponds to equilibrium 2(b).

b. ϵ-informed always sell. In this case, the total order flow is equal to

$$X = \begin{cases} 2\chi_\beta - 1 + z & \text{if } \beta = \beta_L \\ -1 + z & \text{if } \beta = \beta_H. \end{cases} \tag{A.46}$$

The equilibrium price is equal to $p = E[v] + \frac{\gamma}{2}X\mathrm{Var}(v)$. Trading profits for an ϵ-informed investor are equal to

$$\Pi_i(\epsilon_H) = -\Delta_\epsilon + \frac{\gamma}{4}[-2\chi_\epsilon\Delta_\epsilon + \chi_\beta^2\beta_L^2\sigma_\theta^2 - 2(1-\chi_\beta)^2\mathrm{Var}(\beta\theta) - \chi_\beta(2-\chi_\beta)\beta_H^2\sigma_\theta^2], \tag{A.47}$$

which is negative because $\beta_H^2 > \beta_L^2$.

c. ϵ-informed buy on a high signal and sell on a low signal. If $\overline{\theta} < 0$, this case corresponds to equilibrium 1(b). If $\overline{\theta} > 0$, expected trading profits for β-informed investors are negative.

d. ϵ-informed buy on a low signal and sell on a high signal. Following the analogous steps as before, at the beginning of the proof, we can show that this strategy always leads to negative profits for ϵ-informed investors.

A.1.7. Existence of Both Type 1 and Type 2 Equilibria: Numerical Illustration. Figure 1 illustrates the existence conditions for type 1 equilibria in a parameter range for which type 2 equilibria do not exist. Figure A.1 considers a different set of parameters. In this setting, both type 1 and type 2 equilibria exist. The figure confirms that the two equilibrium types do not coexist because the existence regions are disjoint.

A.2. Proof of Corollary 1

This proof contains two parts. Part 1 focuses on the nonmonotonic relation between X and $\delta(X)$ and on the sufficient condition that ensures that $\delta(X)$ is hump shaped (unimodal) in X. As before, we distinguish between $\chi_\epsilon > \chi_\beta$ and $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} > 0$ and $\overline{\theta} < 0$. We also denote the six distinct intervals for order flow by $X_1 > X_2 > X_3 > X_4 > X_5 > X_6$ (defined in Appendix A.1). Part 2 analyzes the differences in δ at the extreme ranges for order flow.

A.2.1. Part 1: Nonmonotonic Relation Between X and $\delta(X)$.

Scenario (i): $\chi_\epsilon > \chi_\beta$ and $\overline{\theta} > 0$. We have shown in the proof of Theorem 1 that $\delta(X_1) = \frac{\gamma}{2}\beta_H^2\sigma_\theta^2 > \delta(X_6) = \frac{\gamma}{2}\beta_L^2\sigma_\theta^2$. Moreover, we have shown that $\delta(X_2) = \delta(X_5) = \frac{\gamma}{2}\left(\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right)$. As a result, $\delta(X)$ is monotone if and only if $\delta(X_2) = \delta(X_3) = \delta(X_4) = \delta(X_5)$. However, we have shown that $\delta(X_3) = \frac{\gamma}{2}\left(\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right) > \delta(X_4) = \frac{\gamma}{2}\left(\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right)$ so that the relation between X and $\delta(X)$ is nonmonotone.

Scenario (ii): $\chi_\epsilon > \chi_\beta$ and $\overline{\theta} < 0$. We have shown in the proof of Theorem 1 that $\delta(X_1) = \frac{\gamma}{2}\beta_L^2\sigma_\theta^2 < \delta(X_6) = \frac{\gamma}{2}\beta_H^2\sigma_\theta^2$. Moreover, we have shown that $\delta(X_2) = \delta(X_5) = \frac{\gamma}{2}\left(\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right)$. As a result, $\delta(X)$ is monotone if and only if $\delta(X_2) = \delta(X_3) = \delta(X_4) = \delta(X_5)$. However, we have shown that $\delta(X_3) = \frac{\gamma}{2}\left(\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right) < \delta(X_4) = \frac{\gamma}{2}\left(\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right)$ so that the relation between X and $\delta(X)$ is nonmonotone.

Scenario (iii): $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} > 0$. We have shown in the proof of Theorem 1 that $\delta(X_1) = \frac{\gamma}{2}\beta_H^2\sigma_\theta^2$ and that $\delta(X_2) = \frac{\gamma}{2}(\Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2) > \delta(X_1)$. However, we also have that $\delta(X_6) = \frac{\gamma}{2}\beta_L^2\sigma_\theta^2 < \delta(X_1)$ so that the relation between X and $\delta(X)$ is nonmonotone.

Scenario (iv): $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} < 0$. We have shown in the proof of Theorem 1 that $\delta(X_6) = \frac{\gamma}{2}\beta_H^2\sigma_\theta^2$ and that $\delta(X_5) = \frac{\gamma}{2}(\Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2) > \delta(X_6)$. However, we also have that $\delta(X_2) = \frac{\gamma}{2}\beta_L^2\sigma_\theta^2 < \delta(X_6)$ so that the relation between X and $\delta(X)$ is nonmonotone.

Next, we establish sufficient conditions such that $\delta(X)$ is a unimodal function of X.

Scenario (i): $\chi_\epsilon > \chi_\beta$ and $\overline{\theta} > 0$. The function $\delta(X)$ is hump shaped with a global maximum at $X = X_3$ if $\delta(X_1) < \delta(X_2) < \delta(X_3)$ and $\delta(X_3) > \delta(X_4) > \delta(X_5) > \delta(X_6)$. Plugging in the expressions for $\delta(X)$ derived above, we obtain the sufficient conditions:

$$\overline{\theta}^2\Delta_\beta^2 < 8\Delta_\epsilon^2 \text{ and } \sigma_\theta < \min\left(\overline{\theta}\sqrt{\frac{\Delta_\beta}{2\overline{\beta}}}, \sqrt{\frac{8\Delta_\epsilon^2 - \Delta_\beta^2\overline{\theta}^2}{6\Delta_\beta\overline{\beta}}}\right).$$

Scenario (ii): $\chi_\epsilon > \chi_\beta$ and $\overline{\theta} < 0$. Following the same steps as in scenario (i), we find that $\delta(X)$ reaches a global maximum at $X = X_4$ under the same sufficient conditions as above.

Scenario (iii): $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} > 0$. The function $\delta(X)$ is hump shaped with a global maximum at $X = X_3$ if $\delta(X_1) < \delta(X_2) < \delta(X_3)$ and $\delta(X_3) > \delta(X_4) > \delta(X_5) > \delta(X_6)$. Plugging in the expressions for $\delta(X)$ derived above, we obtain the sufficient conditions:

$$\overline{\theta}^2\Delta_\beta^2 > \frac{1}{8}\Delta_\epsilon^2 \text{ and } \sigma_\theta < \sqrt{\frac{8\Delta_\beta^2\overline{\theta}^2 - \Delta_\epsilon^2}{12\Delta_\beta\overline{\beta}}}.$$

Scenario (iv): $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} < 0$. Following the same steps as in scenario (iii), we find that $\delta(X)$ reaches a global maximum at $X = X_4$ under the same sufficient conditions as above.

To summarize, we have shown that $\delta(X)$ is hump shaped and reaches an interior equilibrium if $\overline{\theta}^2 \in \left(\frac{\Delta_\epsilon^2}{8\Delta_\beta^2}, \frac{8\Delta_\epsilon^2}{\Delta_\beta^2}\right)$ and $\sigma_\theta < \overline{\sigma}_\theta$ with

$$\overline{\sigma}_\theta \equiv \begin{cases} \min\left(\overline{\theta}\sqrt{\frac{\Delta_\beta}{2\overline{\beta}}}, \sqrt{\frac{8\Delta_\epsilon^2 - \Delta_\beta^2\overline{\theta}^2}{6\Delta_\beta\overline{\beta}}}\right) & \text{if } \chi_\epsilon > \chi_\beta \\ \sqrt{\frac{8\Delta_\beta^2\overline{\theta}^2 - \Delta_\epsilon^2}{12\Delta_\beta\overline{\beta}}} & \text{otherwise.} \end{cases} \tag{A.48}$$

A.2.2. Part 2: Difference in δ at the Extremes.

We obtain from the results in Theorem 1 that

$$\mathrm{DiffE}_\delta = \begin{cases} +2\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2 & \text{if } \overline{\theta} > 0 \\ -2\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2 & \text{if } \overline{\theta} < 0, \end{cases} \tag{A.49}$$

which increases (decreases) in $\overline{\beta}$, Δ_β, and σ_θ if $\overline{\theta} > 0$ ($\overline{\theta} < 0$).

A.3. Proof of Corollary 2

This proof has two parts. Part 1 is the proof regarding the expected δ, and part 2 is the proof regarding the volatility of δ.

A.3.1. Part 1: Expected Level of δ.

The expected δ depends on the price informativeness, which is, thus, calculated first.

Price informativeness: To calculate the measures for price informativeness, we first calculate the conditional variances $\mathrm{Var}(\epsilon|X)$ and $\mathrm{Var}(\beta|X)$ using the results derived in Theorem 1. As before, we distinguish the two scenarios $\chi_\epsilon > \chi_\beta$ and $\chi_\epsilon \le \chi_\beta$.

Scenario (i): $\chi_\epsilon > \chi_\beta$. In this case, the conditional variance of ϵ is given by

$$\mathrm{Var}(\epsilon|X) = \begin{cases} 0 & \text{if } X \in [1 + \chi_\beta - \chi_\epsilon, 2] \\ 0 & \text{if } X \in [1 + \chi_\epsilon - \chi_\beta, 1 + \chi_\beta - \chi_\epsilon) \\ \frac{8}{9}\Delta_\epsilon^2 & \text{if } X \in [0, 1 + \chi_\epsilon - \chi_\beta) \\ \frac{8}{9}\Delta_\epsilon^2 & \text{if } X \in [-1 + \chi_\beta - \chi_\epsilon, 0) \\ 0 & \text{if } X \in [-1 + \chi_\epsilon - \chi_\beta, -1 + \chi_\beta - \chi_\epsilon) \\ 0 & \text{if } X \in [-2, -1 + \chi_\epsilon - \chi_\beta), \end{cases} \tag{A.50}$$

which implies that $\mathbb{E}[\mathrm{Var}(\epsilon|X)] = \Delta_\epsilon^2(1 - \chi_\epsilon + \chi_\beta/3)$. It follows that $\mathbb{E}\left[\frac{\mathrm{Var}(\epsilon|X)}{\mathrm{Var}(\epsilon)}\right] = 1 - \chi_\epsilon + \chi_\beta/3$.

Similarly, the conditional variance of β is given by

$$\mathrm{Var}(\beta|X) = \begin{cases} 0 & \text{if } X \in [1 + \chi_\beta - \chi_\epsilon, 2] \\ \Delta_\beta^2 & \text{if } X \in [1 + \chi_\epsilon - \chi_\beta, 1 + \chi_\beta - \chi_\epsilon) \\ \frac{8}{9}\Delta_\beta^2 & \text{if } X \in [0, 1 + \chi_\epsilon - \chi_\beta) \\ \frac{8}{9}\Delta_\beta^2 & \text{if } X \in [-1 + \chi_\beta - \chi_\epsilon, 0) \\ \Delta_\beta^2 & \text{if } X \in [-1 + \chi_\epsilon - \chi_\beta, -1 + \chi_\beta - \chi_\epsilon) \\ 0 & \text{if } X \in [-2, -1 + \chi_\epsilon - \chi_\beta), \end{cases} \tag{A.51}$$

which implies that $\mathbb{E}[\mathrm{Var}(\beta|X)] = \Delta_\beta^2(1 - 2\chi_\beta/3)$. It follows that $\mathbb{E}\left[\frac{\mathrm{Var}(\beta|X)}{\mathrm{Var}(\beta)}\right] = 1 - 2\chi_\beta/3$.

Scenario (ii): $\chi_\epsilon \le \chi_\beta$. In this case, the conditional variance of ϵ is given by

$$\mathrm{Var}(\epsilon|X) = \begin{cases} 0 & \text{if } X \in [1-\chi_\beta+\chi_\epsilon, 2] \\ \Delta_\epsilon^2 & \text{if } X \in [1-\chi_\epsilon+\chi_\beta, 1-\chi_\beta+\chi_\epsilon) \\ \frac{8}{9}\Delta_\epsilon^2 & \text{if } X \in [0, 1-\chi_\epsilon+\chi_\beta) \\ \frac{8}{9}\Delta_\epsilon^2 & \text{if } X \in [-1-\chi_\beta+\chi_\epsilon, 0) \\ \Delta_\epsilon^2 & \text{if } X \in [-1-\chi_\epsilon+\chi_\beta, -1-\chi_\beta+\chi_\epsilon) \\ 0 & \text{if } X \in [-2, -1-\chi_\epsilon+\chi_\beta), \end{cases} \tag{A.52}$$

which implies that $\mathbb{E}[\mathrm{Var}(\epsilon|X)] = \Delta_\epsilon^2(1-2\chi_\epsilon/3)$. It follows that $\mathbb{E}\left[\frac{\mathrm{Var}(\epsilon|X)}{\mathrm{Var}(\epsilon)}\right] = 1-2\chi_\epsilon/3$.

Similarly, the conditional variance of β is given by

$$\mathrm{Var}(\beta|X) = \begin{cases} 0 & \text{if } X \in [1-\chi_\beta+\chi_\epsilon, 2) \\ 0 & \text{if } X \in [1-\chi_\epsilon+\chi_\beta, 1-\chi_\beta+\chi_\epsilon) \\ \frac{8}{9}\Delta_\beta^2 & \text{if } X \in [0, 1-\chi_\epsilon+\chi_\beta) \\ \frac{8}{9}\Delta_\beta^2 & \text{if } X \in [-1-\chi_\beta+\chi_\epsilon, 0) \\ 0 & \text{if } X \in [-1-\chi_\epsilon+\chi_\beta, -1-\chi_\beta+\chi_\epsilon) \\ 0 & \text{if } X \in [-2, -1-\chi_\epsilon+\chi_\beta), \end{cases} \tag{A.53}$$

which implies that $\mathbb{E}[\mathrm{Var}(\beta|X)] = \Delta_\beta^2(1+\chi_\epsilon/3-\chi_\beta)$. It follows that $\mathbb{E}\left[\frac{\mathrm{Var}(\epsilon|X)}{\mathrm{Var}(\epsilon)}\right] = 1+\chi_\epsilon/3-\chi_\beta$.

Expected δ: From the price function derived in the main text, we know that $\delta(X) = \frac{\gamma}{2}\mathrm{Var}(v|X)$. We can then use the expression for the conditional variance derived in the proof of Theorem 1 to show that

$$\mathbb{E}[\mathrm{Var}(v|X)] = \begin{cases} (\overline{\beta}^2+\Delta_\beta^2)\sigma_\theta^2 + \Delta_\epsilon^2(1-\chi_\epsilon+\chi_\beta/3) & \\ \quad + \Delta_\beta^2\overline{\theta}^2(1-2\chi_\beta/3) & \text{if } \chi_\epsilon > \chi_\beta \\ (\overline{\beta}^2+\Delta_\beta^2)\sigma_\theta^2 + \Delta_\epsilon^2(1-2\chi_\epsilon/3) & \\ \quad + \Delta_\beta^2\overline{\theta}^2(1+\chi_\epsilon/3-\chi_\beta) & \text{if } \chi_\epsilon \le \chi_\beta. \end{cases} \tag{A.54}$$

From this expression, we can easily obtain the comparative statics in Corollary 2.

A.3.2. Part 2: Volatility of δ. It follows from Theorem 1 that the variance of δ can be written as follows:

$$\sigma^2(\delta) = \begin{cases} \frac{\gamma^2}{54}(1-\chi_\epsilon)(36\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 + \overline{\theta}^4\Delta_\beta^4 + 6\chi_\epsilon(\overline{\theta}^2\Delta_\beta^2 - 2\Delta_\epsilon^2)^2 & \\ \quad + 20\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon^2 - 8\Delta_\epsilon^4) & \text{if } \chi_\epsilon > \chi_\beta \\ \frac{\gamma^2}{54}(54\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 + \chi_\epsilon(16\overline{\theta}^4\Delta_\beta^4 - 72\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 & \\ \quad - 6\chi_\epsilon(\Delta_\epsilon^2 - 2\overline{\theta}^2\Delta_\beta^2)^2 - 4\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon^2 + 7\Delta_\epsilon^4)) & \text{if } \chi_\epsilon \le \chi_\beta. \end{cases}$$

1. Comparative statics with respect to σ_θ: Taking the derivative with respect to σ_θ yields

$$\frac{\partial\sigma^2(\delta)}{\partial\sigma_\theta} = \begin{cases} \frac{8}{3}\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^3(1-\chi_\epsilon) & \text{if } \chi_\epsilon > \chi_\beta \\ \frac{4}{3}\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^3(3-4\chi_\epsilon) & \text{if } \chi_\epsilon \le \chi_\beta, \end{cases} \tag{A.55}$$

which is positive.

2. Comparative statics with respect to $\overline{\beta}$: Taking the derivative with respect to $\overline{\beta}$ yields

$$\frac{\partial\sigma^2(\delta)}{\partial\overline{\beta}} = \begin{cases} \frac{4}{3}\gamma^2\overline{\beta}\Delta_\beta^2\sigma_\theta^4(1-\chi_\epsilon) & \text{if } \chi_\epsilon > \chi_\beta \\ \frac{2}{3}\gamma^2\overline{\beta}\Delta_\beta^2\sigma_\theta^4(3-4\chi_\epsilon) & \text{if } \chi_\epsilon \le \chi_\beta, \end{cases} \tag{A.56}$$

which is positive.

3. Comparative statics with respect to χ_ϵ: The second derivative is given by

$$\frac{\partial^2\sigma^2(\delta)}{\partial\chi_\epsilon^2} = \begin{cases} -\frac{2}{9}\gamma^2(\overline{\theta}^2\Delta_\beta^2 - 2\Delta_\epsilon^2)^2 & \text{if } \chi_\epsilon > \chi_\beta \\ -\frac{2}{9}\gamma^2(\Delta_\epsilon^2 - 2\overline{\theta}^2\Delta_\beta^2)^2 & \text{if } \chi_\epsilon \le \chi_\beta \end{cases}, \tag{A.57}$$

which is negative. Hence, $\sigma^2(\delta)$ is a concave function of χ_ϵ. To find the minimum for $\sigma^2(\delta)$, it suffices to compare its value at $\chi_\epsilon = 0$ and $\chi_\epsilon = 1$. Evaluating $\sigma^2(\delta)$ at these two values yields that $\sigma^2(\delta)|_{\chi_\epsilon=0} = \gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 > \sigma^2(\delta)|_{\chi_\epsilon=1} = 0$. Hence, $\sigma^2(\delta)$ reaches its minimum at $\chi_\epsilon = 1$.

A.4. Proof of Corollary 3

It follows from $\mathbb{E}[X] = 0$ and the definition of δ that we can rewrite the covariance as

$$\mathrm{Cov}(\delta, X) = \mathbb{E}[\delta X] = \frac{\gamma}{2}\mathbb{E}[X\mathrm{Var}(v|X)]. \tag{A.58}$$

To compute $\mathbb{E}[X\mathrm{Var}(v|X)]$, we use the results for $\mathrm{Var}(v|X)$ in Theorem 1, which yield

1. If $\chi_\epsilon > \chi_\beta$ and $\overline{\theta} > 0$,

$$\begin{aligned} &\mathbb{E}[X\mathrm{Var}(v|X)] \\ &= \beta_H^2\sigma_\theta^2\frac{1}{4}\chi_\beta(1+\chi_\epsilon) + \left[\frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right]\frac{1}{2}(\chi_\epsilon-\chi_\beta) \\ &\quad + \left[\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\frac{3\chi_\beta}{4}(1-\chi_\epsilon) \\ &\quad + \left[\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\frac{3\chi_\beta}{4}(-1+\chi_\epsilon) \\ &\quad + \left[\frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right]\frac{\chi_\beta-\chi_\epsilon}{2} + \beta_L^2\sigma_\theta^2\frac{\chi_\beta}{4}(-1-\chi_\epsilon) \\ &= 2\chi_\beta\overline{\beta}\Delta_\beta\sigma_\theta^2. \end{aligned}$$

If $\chi_\epsilon > \chi_\beta$ and $\overline{\theta} < 0$,

$$\begin{aligned} &\mathbb{E}[X\mathrm{Var}(v|X)] \\ &= \beta_L^2\sigma_\theta^2\frac{1}{4}\chi_\beta(1+\chi_\epsilon) + \left[\frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right]\frac{1}{2}(\chi_\epsilon-\chi_\beta) \\ &\quad + \left[\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_L^2 + \frac{1}{3}\beta_H^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\frac{3\chi_\beta}{4}(1-\chi_\epsilon) \\ &\quad + \left[\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_L^2 + \frac{2}{3}\beta_H^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\frac{3\chi_\beta}{4}(-1+\chi_\epsilon) \\ &\quad + \left[\frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right]\frac{\chi_\beta-\chi_\epsilon}{2} + \beta_H^2\sigma_\theta^2\frac{\chi_\beta}{4}(-1-\chi_\epsilon) \\ &= -2\chi_\beta\overline{\beta}\Delta_\beta\sigma_\theta^2. \end{aligned}$$

2. If $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} > 0$,

$$
\begin{aligned}
&\mathbb{E}[X\mathrm{Var}(v|X)]\\
&= \beta_H^2\sigma_\theta^2\frac{\chi_\epsilon}{4}(1+\chi_\beta) + [\Delta_\epsilon^2 + (\overline{\beta}+\Delta_\beta)^2\sigma_\theta^2]\frac{\chi_\beta-\chi_\epsilon}{2}\\
&\quad + \left[\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\frac{3\chi_\epsilon}{4}(1-\chi_\beta)\\
&\quad + \left[\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\frac{3\chi_\epsilon}{4}(-1+\chi_\beta)\\
&\quad + [\Delta_\epsilon^2 + (\overline{\beta}-\Delta_\beta)^2\sigma_\theta^2]\frac{\chi_\epsilon-\chi_\beta}{2} + \beta_L^2\sigma_\theta^2\frac{\chi_\epsilon}{4}(-1-\chi_\beta)\\
&= 2\chi_\beta\overline{\beta}\Delta_\beta\sigma_\theta^2.
\end{aligned}
$$

If $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} < 0$,

$$
\begin{aligned}
&\mathbb{E}[X\mathrm{Var}(v|X)]\\
&= \beta_L^2\sigma_\theta^2\frac{\chi_\epsilon}{4}(1+\chi_\beta) + [\Delta_\epsilon^2 + (\overline{\beta}-\Delta_\beta)^2\sigma_\theta^2]\frac{\chi_\beta-\chi_\epsilon}{2}\\
&\quad + \left[\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{2}{3}\beta_L^2 + \frac{1}{3}\beta_H^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\frac{3\chi_\epsilon}{4}(1-\chi_\beta)\\
&\quad + \left[\frac{8}{9}\Delta_\epsilon^2 + \left(\frac{1}{3}\beta_L^2 + \frac{2}{3}\beta_H^2\right)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\frac{3\chi_\epsilon}{4}(-1+\chi_\beta)\\
&\quad + [\Delta_\epsilon^2 + (\overline{\beta}+\Delta_\beta)^2\sigma_\theta^2]\frac{\chi_\epsilon-\chi_\beta}{2} + \beta_H^2\sigma_\theta^2\frac{\chi_\epsilon}{4}(-1-\chi_\beta)\\
&= -2\chi_\beta\overline{\beta}\Delta_\beta\sigma_\theta^2.
\end{aligned}
$$

The comparative statics follow directly. Moreover, we have shown in the main text that $\mathbb{E}[v-p] = -\mathrm{Cov}(\delta, X)$.

A.5. Proof of Corollary 4

A.5.1. Expected Asset Return. Part 1 of the corollary follows from Equation (11). Further, as shown above, $\mathbb{E}[v-p] = -\frac{\gamma}{2}\mathbb{E}[X\mathrm{Var}(v|X)]$. Moreover, we know that $\mathbb{E}[v] = \overline{\beta}\overline{\theta}$ because ϵ has zero mean and $\beta \perp \theta$. Inserting $\mathbb{E}[X\mathrm{Var}(v|X)]$ into $\mathbb{E}[v-p]$ yields the result in Corollary 4. The comparative statics follow directly.

A.5.2. Return Volatility. If $\chi_\epsilon > \chi_\beta$ and $\overline{\theta} > 0$, we obtain from the results in Theorem 1 that, we obtain that

$$
\begin{aligned}
\mathrm{Var}(v-p) = \frac{1}{162}(&2\overline{\theta}^2\Delta_\beta^2(2\gamma^2(9\sigma_\theta^2(\chi_\epsilon(\chi_\epsilon(2\chi_\epsilon+3)-3)+1)\\
&(\overline{\beta}^2+\Delta_\beta^2) - 32\Delta_\epsilon^2(\chi_\epsilon-1)^3) + 54\chi_\epsilon + 27)\\
&-72\Delta_\epsilon^2(\chi_\epsilon-1)(2\gamma^2\sigma_\theta^2(\chi_\epsilon-1)^2(\overline{\beta}^2+\Delta_\beta^2)+3)\\
&+9\gamma^2\sigma_\theta^4(2\overline{\beta}^2\Delta_\beta^2(\chi_\epsilon(-8\chi_\epsilon^2+6\chi_\epsilon+3)+5)\\
&+\overline{\beta}^4(9(\chi_\epsilon-1)\chi_\epsilon+6)+3\Delta_\beta^4(3(\chi_\epsilon-1)\chi_\epsilon+2))\\
&+2\gamma^2\overline{\theta}^4\Delta_\beta^4(\chi_\epsilon(\chi_\epsilon(22\chi_\epsilon+15)-15)+5)\\
&+216\overline{\theta}\Delta_\beta\Delta_\epsilon(\chi_\epsilon-1) - 64\gamma^2\Delta_\epsilon^4(\chi_\epsilon-1)^3).
\end{aligned}
$$

Similarly, if $\chi_\epsilon > \chi_\beta$ and $\overline{\theta} < 0$

$$
\begin{aligned}
\mathrm{Var}(v-p) = \frac{1}{162}\Big(&2\overline{\theta}^2\Delta_\beta^2(2\gamma^2(9\sigma_\theta^2(\chi_\epsilon(\chi_\epsilon(2\chi_\epsilon+3)-3)+1)\\
&(\overline{\beta}^2+\Delta_\beta^2) - 32\Delta_\epsilon^2(\chi_\epsilon-1)^3) + 54\chi_\epsilon + 27)\\
&-72\Delta_\epsilon^2(\chi_\epsilon-1)(2\gamma^2\sigma_\theta^2(\chi_\epsilon-1)^2(\overline{\beta}^2+\Delta_\beta^2)+3)\\
&+9\gamma^2\sigma_\theta^4(2\overline{\beta}^2\Delta_\beta^2(\chi_\epsilon(-8\chi_\epsilon^2+6\chi_\epsilon+3)+5)\\
&+\overline{\beta}^4(9(\chi_\epsilon-1)\chi_\epsilon+6)+3\Delta_\beta^4(3(\chi_\epsilon-1)\chi_\epsilon+2))\\
&+2\gamma^2\overline{\theta}^4\Delta_\beta^4(\chi_\epsilon(\chi_\epsilon(22\chi_\epsilon+15)-15)+5)\\
&-216\overline{\theta}\Delta_\beta\Delta_\epsilon(\chi_\epsilon-1) - 64\gamma^2\Delta_\epsilon^4(\chi_\epsilon-1)^3\Big).
\end{aligned}
$$

If $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} > 0$,

$$
\begin{aligned}
\mathrm{Var}(v-p) = \frac{1}{162}\Big(&54\gamma^2\sigma_\theta^4(3\overline{\beta}^2\Delta_\beta^2+\overline{\beta}^4+\Delta_\beta^4) + 2\Delta_\epsilon^2(2\gamma^2(\Delta_\beta^2(32\overline{\theta}^2\\
&\chi_\epsilon^3 - 9\sigma_\theta^2((\chi_\epsilon-3)\chi_\epsilon(2\chi_\epsilon-3)-3)) - 9\overline{\beta}^2\sigma_\theta^2((\chi_\epsilon-3)\\
&\chi_\epsilon(2\chi_\epsilon-3)-3))) - 54\chi_\epsilon + 81) + 16\gamma^2\Delta_\beta^2\chi_\epsilon^3\\
&(9\overline{\theta}^2\sigma_\theta^2(\overline{\beta}^2+\Delta_\beta^2) + 4\overline{\theta}^4\Delta_\beta^2 - 18\overline{\beta}^2\sigma_\theta^4) + 81\gamma^2\sigma_\theta^4\chi_\epsilon^2\\
&(4\overline{\beta}^2\Delta_\beta^2+\overline{\beta}^4+\Delta_\beta^4) - 27\chi_\epsilon(3\gamma^2\sigma_\theta^4(\overline{\beta}^2+\Delta_\beta^2)^2 - 8\overline{\theta}^2\Delta_\beta^2)\\
&-216\overline{\theta}\Delta_\beta\Delta_\epsilon\chi_\epsilon + 2\gamma^2\Delta_\epsilon^4(\chi_\epsilon((81-22\chi_\epsilon)\\
&\chi_\epsilon-81)+27)\Big).
\end{aligned}
$$

And, finally, if $\chi_\epsilon \le \chi_\beta$ and $\overline{\theta} < 0$

$$
\begin{aligned}
\mathrm{Var}(v-p) = \frac{1}{162}\Big(&54\gamma^2\sigma_\theta^4(3\overline{\beta}^2\Delta_\beta^2+\overline{\beta}^4+\Delta_\beta^4) + 2\Delta_\epsilon^2(2\gamma^2(\Delta_\beta^2(32\overline{\theta}^2\\
&\chi_\epsilon^3 - 9\sigma_\theta^2((\chi_\epsilon-3)\chi_\epsilon(2\chi_\epsilon-3)-3)) - 9\overline{\beta}^2\sigma_\theta^2((\chi_\epsilon-3)\\
&\chi_\epsilon(2\chi_\epsilon-3)-3))) - 54\chi_\epsilon + 81) + 16\gamma^2\Delta_\beta^2\chi_\epsilon^3\\
&(9\overline{\theta}^2\sigma_\theta^2(\overline{\beta}^2+\Delta_\beta^2) + 4\overline{\theta}^4\Delta_\beta^2 - 18\overline{\beta}^2\sigma_\theta^4) + 81\gamma^2\sigma_\theta^4\chi_\epsilon^2\\
&(4\overline{\beta}^2\Delta_\beta^2+\overline{\beta}^4+\Delta_\beta^4) - 27\chi_\epsilon(3\gamma^2\sigma_\theta^4(\overline{\beta}^2+\Delta_\beta^2)^2 - 8\overline{\theta}^2\Delta_\beta^2)\\
&+216\overline{\theta}\Delta_\beta\Delta_\epsilon\chi_\epsilon + 2\gamma^2\Delta_\epsilon^4(\chi_\epsilon((81-22\chi_\epsilon)\chi_\epsilon-81)+27)\Big).
\end{aligned}
$$

From these expressions and the equilibrium conditions in Theorem 1, we can easily confirm the comparative statics with respect to $(\Delta_\epsilon, \Delta_\beta, \overline{\beta}, \overline{\theta}, \sigma_\theta, \chi_\epsilon)$

Endnotes

[1] This literature is represented in Grossman and Stiglitz (1980), Kyle (1985), Glosten and Milgrom (1985), and the extensive work stemming from these seminal pieces.

[2] See, for example, Sunder (1980), Collins et al. (1987), and Jostova and Philipov (2005).

[3] Compare this with https://tinyurl.com/y5l7ql9n.

[4] Risk aversion of liquidity providers is the cornerstone of many prominent papers; see, for example, Ho and Stoll (1981), Grossman and Miller (1988), and Nagel (2012).

[5] Glebkin et al. (2020) show that asymmetric price impacts can be generated when payoffs are not symmetrically distributed. In contrast to their work, we explicitly separate multiplicative information about firm betas from the additive (idiosyncratic) component. In Saar (2001), asymmetric price impacts arise from diversification constraints of portfolio managers.

[6] The jumps in the price schedule do not affect the covariance as, strictly speaking, they occur with vanishingly small probability.

[7] As our theoretical predictions are regarding the (ex ante) factor means, there are interpretational challenges in working with ex post (realized) earnings innovations. So we instead condition on NBER recessions, which persist. We do validate our analysis by showing that earnings innovations are negative and positive during NBER recession and nonrecession months, respectively. Further, we use monthly returns to test our hypotheses. Hasbrouck and Sofianos (1993) and Madhavan and Smidt (1993) show that inventory effects in asset prices are long-lived and last at least for a month. Our use of monthly horizons is consistent with the notion that information about quarterly earnings innovations accumulates gradually across the intervening three months.

[8] Stochastic betas are also a feature of the conditional CAPM; see Jagannathan and Wang (1996) as well as Lewellen and Nagel (2006) for examples.

[9] The assumption that θ is normally distributed simplifies some variance calculations and is made without loss of generality. It can be relaxed at the cost of greater complexity. Of course, as per standard justification, the probability of a negative value for θ can be made arbitrarily small via parameter choices.

[10] We assume that the mean beta is positive. Indeed, Fama and French (1992) indicate that the average beta in the lowest beta group (based on a double sort by market capitalization and beta) for publicly traded U.S. companies is well above zero at 0.56.

[11] This assumption can be justified by borrowing or short-selling constraints and is common in models featuring a continuum of risk-neutral traders. See, for instance, Goldstein et al. (2013), Goldstein and Yang (2019), and Albagli et al. (2021) for recent papers with similar assumptions.

[12] Without loss of generality, we assume that traders do not trade when indifferent. The indifference state does not occur in equilibrium, however, because the expected equilibrium profits of informed are positive.

[13] The payoff on the asset is not normally distributed, but nonetheless, we assume the mean–variance objective for tractability. This specification accords with quadratic utility under the usual caveats; see, for example, Magill and Quinzii (2000) and Cochrane (2014).

[14] See Dugast and Foucault (2018) and Dow et al. (2021) for similar assumptions.

[15] For the parameter values in Figure 1, the type 2 equilibria of Theorem 1 (discussed in the next paragraph) do not exist.

[16] Note that the reverse equilibrium in which β-informed always buy or always sell does not exist. In this case, X and $\mathrm{Var}(v|X)$ are uncorrelated because the multiplicative feature of β information (that leads to the nonzero correlation between X and the conditional variance) does not operate. Therefore, always selling or buying by the β informed is never profitable. See Appendix A.1 for a formal proof.

[17] Strictly speaking, these threshold points of X happen with zero probability each.

[18] As per practice in mean–variance models, we use price changes instead of returns; see, for example, Hong and Stein (1999). The technical complication in using proportional returns is that prices can be negative in our setting.

[19] This learning technology is common in the literature (e.g., Goldstein and Yang 2015, Brunnermeier et al. 2022).

[20] Our theory also has microstructural predictions for asymmetry and nonmonotonicity in price impacts (see Corollary 1), and we find supportive evidence in the data (see Online Appendix tables). The tables are presented for the dollar versions of buys and sells but are robust to the share versions. These results update those of Holthausen et al. (1987) and Barclay and Warner (1993) that use older samples.

[21] Our results are robust to using the corresponding correlation instead of the covariance and to alternative rolling windows.

[22] From March 1993 to December 2013, we use intraday indicators from the trade and quote monthly product and, for the remainder of the sample period, from the millisecond trade and quote daily product.

[23] In the millisecond trade and quote daily product, the dollar buy volume and dollar sell volume correspond to data item *BUY_DV_LR* and data item *SELL_DV_LR*, and the share buy volume and share sell volume correspond to data item *BUYVOL_LR* and data item *SELLVOL_LR*.

[24] The manual for the WRDS intraday indicator product provides further details. The slope estimate is the coefficient when quote midpoint changes are regressed on the square root of the absolute order imbalance, prefixed by the sign of the imbalance. This follows earlier work by Hasbrouck (1991). As per Brennan and Subrahmanyam (1996) and Sadka (2006), we use the specification that allows for an intercept in the regression specification.

[25] We compute t-statistics based on standard errors with a Newey–West correction of five lags. As suggested by Newey and West (1994), the lag length is determined by the integer portion of $4(T/100)^{2/9}$, where T is the number of observations. All continuous variables except returns are winsorized at the 0.5%/99.5% levels. We also use rank-based versions of key variables to ensure robustness.

[26] For robustness, we also estimate 10-year rolling AR(4) models and compute predicted values of aggregate earnings innovation in these AR(4) models as an alternative proxy for $\overline{\theta}$. We find that this alternative proxy for $\overline{\theta}$ is also positive on average during nonrecessions and negative during recessions (0.002 versus −0.018) and the difference remains statistically significant (t-statistic = 4.82).

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