# Factor Momentum

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Factors display strong cross-sectional momentum that subsumes momentum in industries and other portfolio characteristics. The profits of all these momentum strategies—based on factors, industries, and other characteristics—significantly correlate with each other and therefore likely emanate from the same source. If factors display momentum, so will any set of portfolios with cross-sectional variation in factor loadings. Consistent with factors being at the root of momentum, we find that momentum in industry-neutral factors explains industry momentum, but industry momentum explains none of the factor momentum. Cross-sectional factor momentum concentrates in the first few highest-eigenvalue factors and is distinct from time-series factor momentum. (JEL G12, G14)

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Industries exhibit momentum similar to that found in stock returns (Moskowitz and Grinblatt 1999). However, unlike stock momentum,

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Asness, Porter, and Stevens (2000), Grundy and Martin (2001), and Hoberg and Phillips (2018), among others, also study industry momentum.

industry momentum is at its strongest at the one-month horizon; at this horizon individual stocks display reversals. Lewellen (2002) shows that portfolios sorted by size and book-to-market ratio display similar short-term momentum. In this paper we first show that industry momentum is not about industries, but about the factors against which these portfolios load. Factor momentum transmits into industry returns through the differences in industries' factor loadings. A strategy that buys and sells industries sorted by their past returns implicitly invests in factors that have done well and shuns those that have done poorly. We then show that this same mechanism drives the profits of all other characteristic momentum strategies as well, such as those of Lewellen (2002).

Factor momentum strategies select factors based on their prior returns. We study a cross-sectional strategy that conditions on factors' prior one-month returns and is long and short an equal number of factors. This effect is distinct from both the time-series factor momentum of Ehsani and Linnainmaa (2022), which is long all factors with positive returns over the prior year and short those with negative returns, and the individual stock momentum effect of Jegadeesh and Titman (1993), which is a cross-sectional strategy that chooses stocks based on their prior one-year returns. The Lo and MacKinlay (1990) decomposition shows that the cross-sectional factor momentum strategy derives its profits from the positive autocorrelation in factor returns, not from persistent differences in factor means or from negative cross-serial correlations.

The hypothesis we test in this paper is that factor, industry, and characteristic momentum strategies' profits all stem from the same source. If factors display momentum, this momentum transmits into the cross-section of portfolio returns through variation in the portfolios' factor loadings. We start with industry momentum because it is the most well-studied momentum effect after individual stock momentum. The challenge in testing the hypothesis that industry momentum stems from factor momentum is in demonstrating the direction of this effect: Does factor momentum give rise to industry momentum, or vice versa? If factors have incidental industry exposures, industry return shocks affect factor returns via factors' industry bets (Asness, Frazzini, and Pedersen 2014). Factor momentum could thus be an expression of industry momentum. In the case of industries, we can resolve this identification problem by using industry-neutral factors. We show that industry-neutral factors display at least as much momentum as the standard factors and that they, too, subsume industry momentum. This result is consistent with industry momentum being incidental to factor momentum.

An analysis of other characteristic momentum strategies also shows that industry momentum is unlikely about industries *per se*. We extend Lewellen's (2002) analysis and show that portfolios formed by clustering on multiple characteristics display amounts of momentum comparable to industry momentum. These strategies significantly correlate with each other and with industry momentum, suggesting that a common source likely drives their profits. Indeed, when we control for factor momentum, all characteristic momentum strategies' alphas fall toward zero and lose statistical significance. By contrast, factor momentum remains highly profitable net of all characteristic momentum strategies.

We use industries to show how the transmission mechanism works. The requirement necessary for factor momentum to transmit into portfolio returns is cross-sectional variation in factor loadings. We show that factor loadings vary significantly from one industry and one quarter to the next. The Fama and French (2015) five-factor model, for example, explains just 2% of the variation in market-adjusted industry returns when we assume that all industries have the same fixed factor loadings. This explanatory power, however, increases to 17% when the loadings vary across industries and to 36% when they also vary from quarter to quarter.

We construct "systematic industries" to show that the variation in factor loadings alone generates all of industry momentum. We use short-window regressions to estimate industries' factor loadings. Systematic industries are the combinations of the factors implied by these factor loadings. Systematic industries retain only factor-level variation in returns, discarding any industry-specific shocks. We show that these systematic industries exhibit more and more momentum as we increase the number of factors from which we build them. At the same time, the amount of industry momentum that remains net of the momentum in systematic industries falls to zero. That is, no industry-specific information is required to capture industry momentum, a strategy based on the factor-level replications of the industries does even better.

Our explanation for characteristic momentum, up to this point, lacks some economic substance. Although it is indisputable that factor momentum in the data subsumes industry and other forms of characteristic momentum, what are "factors"? Factors, just as industries, are combinations of individual assets. If factors have no more or less economic weight than industries, moving momentum from industries to factors would represent but a small step in our understanding of momentum.<sup>2</sup> We first address this issue by following in the footsteps of Kozak, Nagel, and Santosh (2018, 2020), Haddad, Kozak, and Santosh (2020), and Ehsani and Linnainmaa (2022). These

Moskowitz and Grinblatt (1999, p. 1287), for example, note that "the results beg the question: Why industries? This paper presents a great deal of evidence documenting a strong and robust industry momentum phenomenon, but we do not state why such an effect might or should exist."

studies note that, in the absence of near-arbitrage opportunities, only the factors that explain the most variation in returns should carry any pricing effects. The mispricing model of Kozak, Nagel, and Santosh (2018) predicts the existence of factor momentum when sentiment is sufficiently persistent (Ehsani and Linnainmaa 2022).

We extract principal component (PC) factors from 43 industry-neutral factors and show that momentum dramatically concentrates in the high-eigenvalue PC factors. Factor momentum found among the first five highest-eigenvalue PC factors has the highest Sharpe ratio and, controlling for momentum found within this subset of PC factors, there is neither economically nor statistically significant momentum in any of the other subsets. Moreover, high-eigenvalue PC factor momentum, and this form of momentum alone, subsumes all forms of characteristic momentum, including industry momentum.

Our results are consistent with what we expect to find if momentum resides in systematic factors. It resides exactly in those factors Kozak, Nagel, and Santosh (2018) and others argue are the economically important factors. In a world absent of near-arbitrage opportunities, nonsystematic returns cannot display momentum because arbitrageurs could profit from such mispricings without assuming any risk. The finding that industries do not display momentum net of factor momentum is perhaps not surprising; Moskowitz and Grinblatt (1999, p. 1251) themselves found the existence of industry momentum surprising because "previous literature [had] shown relatively little impact of industries on asset prices...."

If the blocks from which the factors are built—individual stocksdisplay short-term reversals, how is it that factors display momentum? We reconcile factor momentum with short-term reversals by decomposing stock returns into systematic and idiosyncratic components. We show that both components are positively serially correlated, but that the systematic and idiosyncratic components exhibit strong negative cross-serial correlation. This pattern is similar to the "excess covariance" pattern described by Lewellen (2002). Using Lewellen's behavioral interpretation, the data are consistent with an economy in which stocks move "too much" with factors. For example, when a group of small growth stocks does well, other small growth stocks also do well because investors overestimate the relevance of non-stock-specific information. The systematic component then negatively predicts the idiosyncratic component when this excess comovement later corrects itself. In short, short-term reversals in stock returns occur because (1) momentum is present in factor returns and (2) stocks comove too strongly with similar stocks (Lewellen 2002). We show that systematic stocks, ones that replicate each stock using the factors and thereby strip out the idiosyncratic components, display short-term momentum.

In addition to Moskowitz and Grinblatt (1999) and Lewellen (2002), our results also relate to, for example, the findings of McLean and Pontiff (2016) and Avramov et al. (2017) who show that momentum also works for combinations of many well-diversified portfolios or predictors. We view these results as immediate precursors for the finding that factors display momentum.

### 1. Data

We use monthly and daily CRSP return data for stocks listed on the NYSE, AMEX, and Nasdaq. We include ordinary common shares (share codes 10 and 11) and use CRSP delisting returns. If a stock's delisting return is missing and the delisting is performance-related, we impute a return of -30% for NYSE and AMEX stocks (Shumway 1997) and -55% for Nasdaq stocks (Shumway and Warther 1999). We use accounting data from annual Compustat files. We lag accounting information by 6 months (Fama and French 1993). For example, if a firm's fiscal year ends in December in year t, we assume this information is available to investors at the end of June in year t+1. We compute returns on all factors except for two from July 1963 through December 2021. For the two exceptions, the debt issuance and total external financing factors, the returns start in July 1972.

We construct 43 factors using price, return, volume, and accounting information. We list these factors in Table 1 along with their monthly Fama and French (1993) three-factor model alphas and  $t(\hat{\alpha})s$ .<sup>3</sup> We divide the factors into eight groups: common factors, nonfundamental, profitability, earnings quality, investment and growth, financing, distress, and composite. This set of factors is the same as those used by Linnainmaa and Roberts (2018) with two modifications. First, because we study factor *momentum*, we do not include factors that relate to stock returns over the prior month or year, that is, we exclude all versions of short-term reversals or momentum. Second, because we study industry effects, we exclude the industry concentration factor of Hou and Robinson (2006).

We construct the SMB factor using the same procedure as Fama and French (1993); all other factors are similar to the HML factor, constructed by sorting stocks into six portfolios by size and the return predictor. We use the NYSE breakpoints—median for size and the 30th and 70th percentiles for the return predictor—to assign stocks to

<sup>&</sup>lt;sup>3</sup> We report three-factor model alphas in Table 1, where many of the factors on the list were identified as anomalies (or additional factors) because they earn alphas with respect to the three-factor model. We report five-factor model alphas in Table A3 in the Internet Appendix.

Table 1 Standard and industry-adjusted factors: Three-factor model alphas

This table reports monthly Fama and French (1993) three-factor model alphas for 43 factors. We construct the SMB factor using the same procedure as Fama and French (1993); all other factors are similar to their HML factor, constructed by sorting stocks into six portfolios by firm size and the return predictor using the NYSE breakpoints, namely, median for size and the 30th and 70th percentiles for the return predictor. The exceptions to this rule are the factors based on discrete signals. The high and low portfolios of the debt issuance factor, for example, include firms that did not issue (high portfolio) or did issue (low portfolio) debt during the prior fiscal year. A factor's return is the value-weighted average return on the two high portfolios minus that on the two low portfolios. We sign the return predictors so that the high portfolios contain those stocks that the original study identifies as earning higher average returns. The standard factor sorts stocks into portfolios by the raw return predictors and the industry-neutral factor sorts them by the industry-adjusted predictors. An industry-adjusted predictor subtracts the predictor's cross-sectional industry average. Returns begin in the month indicated in the start-date column and end in December 2021.

				idard tors	Industry-neutral factors	
Category	Factor	date	$\hat{\alpha}_{\mathrm{ff3}}$	$t(\hat{\alpha}_{\mathrm{ff3}})$	$\hat{\alpha}_{\mathrm{ff3}}$	$t(\hat{\alpha}_{\mathrm{ff3}})$
Common	Size, SMB	7/63		,,	-0.03	-0.93
factors	Book to market, HML	7/63			0.14	2.67
	Operating profitability, RMW	7/63	0.34	4.34	0.28	5.76
	Asset growth, CMA	7/63	0.16	3.03	0.11	2.23
	Long-term reversals, LTREV	7/63	-0.01	-0.12	-0.04	-0.70
	Residual variance, RVAR	7/63	0.53	5.16	0.59	8.02
	Quality minus junk, QMJ	7/63	0.27	4.95	0.25	5.56
	Low beta, BAB	7/63	0.35	3.61	0.31	4.29
Non-	Amihud's illiquidity	7/63	0.34	1.95	0.03	0.33
fundamental	Firm age	7/63	-0.14	-1.91	-0.09	-1.50
	Nominal price	7/63	-0.45	-3.96	-0.28	-3.95
	High-volume premium	7/63	0.47	7.74	0.27	5.35
Profitability	Gross profitability	7/63	0.39	5.77	0.37	7.76
v	Return on equity	7/63	0.26	3.59	0.12	1.98
	Return on assets	7/63	0.35	4.88	0.40	7.47
	Profit margin	7/63	0.12	1.80	0.15	2.34
	Change in asset turnover	7/63	0.21	4.13	0.16	3.76
Earnings	Accruals	7/63	0.19	3.26	0.23	5.43
quality	Net operating assets	7/63	0.27	4.64	0.11	2.36
	Net working capital changes	7/63	0.22	4.19	0.21	4.91
	Cash-flow to price	7/63	0.33	4.76	0.21	3.54
	Earnings to price	7/63	0.22	3.23	0.28	4.50
	Enterprise multiple	7/63	0.17	2.96	0.13	2.59
	Sales to price	7/63	0.11	1.64	0.16	2.71
Investment	Growth in inventory	7/63	0.18	3.16	0.19	4.99
and growth	Sales growth	7/63	-0.04	-0.71	0.03	0.63
	Sustainable growth	7/63	0.08	1.54	0.08	1.50
	Growth in sales — inventory	7/63	0.22	4.42	0.12	2.18
	Investment growth rate	7/63	0.16	3.40	0.11	2.46
	Abnormal investment	7/63	0.11	2.11	0.12	2.68
	CAPX growth rate	7/63	0.15	3.44	0.16	3.62
	Investment to capital	7/63	0.06	0.93	0.11	2.28
	Investment to assets	7/63	0.11	1.97	0.05	1.05

 $({\rm continued})$ 

Financing	Debt issuance Leverage One-year share issuance Five-year share issuance Total external financing	7/72 7/63 7/63 7/63 7/72	0.19 $-0.17$ $0.23$ $0.21$ $0.42$	3.52 $-3.28$ $5.05$ $4.57$ $6.82$	0.19 $-0.02$ $0.23$ $0.21$ $0.43$	3.52 $-0.33$ $5.05$ $4.57$ $8.48$
Distress	Ohlson's O-score Altman's Z-score Distress risk	7/63 7/63 7/63	0.17 $0.18$ $0.58$	2.97 $2.66$ $6.72$	0.11 $0.09$ $0.48$	2.08 $1.46$ $7.72$
Composite	Piotroski's F-score M/B and accruals	7/63 $7/63$	$0.33 \\ 0.17$	$5.25 \\ 2.82$	$0.31 \\ 0.21$	$6.39 \\ 3.98$

portfolios.<sup>4</sup> A factor's return is the value-weighted average return for the two high portfolios minus the average return for the two low portfolios. We rebalance the factors either monthly or annually, depending on the choices made in the original studies.

We construct standard and industry-neutral versions of each factor. The standard factors in Table 1 sort stocks by the unadjusted return predictors. The standard value factor, for example, assigns stocks into portfolios by their raw book-to-market ratios. With the industry-neutral factors, we first demean the predictors each month by subtracting the predictor's cross-sectional industry average using the 49 Fama-French industry classification (Novy-Marx 2013). The industry-neutral value factor, for example, assigns stocks into portfolios based on the stocks' book-to-market ratios relative to their same-industry peers. This demeaning step ensures that each factor's long and short legs are approximately evenly diversified across industries, that is, they are "industry neutral."

We sign the return predictors so that the long portfolios contain those stocks that the original study identifies as earning higher average returns.<sup>5</sup> This signing convention, however, is inconsequential from a momentum investor's viewpoint. For example, if value stocks significantly outperform growth stocks over the momentum strategy's formation period, the factor momentum strategy is long value stocks and short growth stocks. It does not matter whether we express the value factor as being long value and short growth, or vice versa. Factors' signs would matter for the purposes of earning the *unconditional* premiums associated with each factor, but not for strategies that condition on past returns.

<sup>&</sup>lt;sup>4</sup> The exceptions to this rule are factors that use discrete signals. The high and low portfolios of the debt issuance factor, for example, include firms that did not issue (high portfolio) or did issue (low portfolio) debt during the prior fiscal year. Because the industry adjustment, which subtracts the industry mean from each firm's predictor, breaks the discreteness, we use the standard 30/70 rule for all industry-neutral factors.

This signing rule does not ensure that all factors' three-factor models should be positive in Table 1; the sign may flip because of the additional data that have accrued since the original study's sample period, because the original study did not adjust for the threefactor model or because of corrections made to the CRSP and Compustat data since the original study.

#### Table 2 Industry momentum

This table reports average monthly returns and five- and six-factor model alphas for four industry momentum strategies. These strategies trade the 20 Moskowitz and Grinblatt (1999) industries. The six-factor model augments the Fama and French (2015) model with the stock momentum factor (UMD) of Carhart (1997). The strategies use 1-, 6-, or 12-month formation periods and rebalance either monthly or biannually. The strategies sort industries by their returns over the formation period and take long and short positions in the above- and below-median industries. When the holding period is 6 months, we use the Jegadeesh and Titman (1993) methodology to account for overlapping observations. The sample begins in August 1963 for the strategy with the 1-month formation period and ends in December 2021. The samples for the strategies with 6- and 12-month formation periods begin in January 1964 and July 1964.

Formation	Holding		Asset pricing model					
period,	period,		FF5	FF5 + U	JMD			
months	months	$ar{r}$	$\hat{\alpha}_{\mathrm{ff5}}$	$\hat{\alpha}_{\text{ff5+umd}}$	$\hat{b}_{\mathrm{umd}}$			
1	1	0.36	0.44	0.37	0.10			
		(4.41)	(5.21)	(4.36)	(5.16)			
6	1	0.14	0.16	-0.09	0.36			
		(1.54)	(1.76)	(-1.21)	(20.43)			
12	1	0.26	0.34	0.03	0.44			
		(2.78)	(3.56)	(0.43)	(29.59)			
6	6	0.13	0.16	-0.09	0.36			
		(1.70)	(2.06)	(-1.70)	(28.58)			

### 2. Industry versus Factor Momentum

### 2.1 Industry momentum is a short-term effect

Table 2 reports average monthly returns and Fama and French (2015) five- and six-factor model alphas for four industry momentum strategies. These strategies have 1- or 6-month holding periods and choose positions based on industry returns over the prior month, the prior 6 months, or the prior year. These four strategies are the same as those examined by Moskowitz and Grinblatt (1999), constructed here for a longer sample period. Each strategy is long the industries with above-median returns and short the below-median industries.<sup>6</sup> We follow Moskowitz and Grinblatt (1999) as closely as possible and report the main results

Moskowitz and Grinblatt (1999) consider strategies that are long the top-three and short the bottom-three industries. Table A4 in the Internet Appendix replicates Table 2 using this alternative definition. The top-three/bottom-three strategies are less diversified than the above-median/below-median strategies and, as a consequence, the t-values associated with the alphas are lower. Because t-values are proportional to information ratios, this difference implies that investors would benefit less, from the perspective of expanding the investment opportunity set, by trading these less-diversified versions. The t-value associated with the short-term industry momentum's six-factor model alpha, for example, falls from 4.36 to 3.86 when we move from the 10/10 portfolio rule in Table 2 to the 3/3 rule in Table A4 in the Internet Appendix. We examine the well-diversified industry momentum strategies throughout this study because they set a higher bar for explaining industry momentum.

for their 20-industry classification scheme. We compute value-weighted returns for each industry.

All four versions of industry momentum earn positive average returns and five-factor model alphas. However, only the short-term strategy, which selects industries based on their prior one-month returns and rebalances monthly, earns a statistically significant six-factor model alpha of 37 basis points per month (t-value = 4.36). The alphas of the three other strategies range from -9 to 3 basis points. Individual stock momentum, therefore, subsumes everything but short-term industry momentum. Short-term industry momentum is largely unrelated to individual stock momentum; the addition of the UMD factor lowers its alpha only by 7 basis points.

The result that individual stock momentum subsumes all forms of industry momentum except for its short-term version is known. Asness, Porter, and Stevens (2000) and Grundy and Martin (2001) replicate the key parts of the Moskowitz and Grinblatt (1999) research and reach the same conclusion: individual stock momentum is almost entirely independent of industry momentum and industry-adjusted momentum is more profitable than standard momentum.<sup>8</sup> This result is not due to differences in sample periods. Table A4 in the Internet Appendix shows that the long-term industry momentum strategy, based on 6-month formation and holding periods, has a five-factor model alpha of 31 basis points (t-value = 1.67) over the Moskowitz and Grinblatt (1999) sample period. This alpha falls to -32 basis points (t-value = -2.58) in the six-factor model.

We henceforth use the term *industry momentum* to refer to the strategy with the one-month formation and holding periods. It is this part, and only this part, of industry momentum that is distinct from individual stock momentum and represents a separate anomaly.

#### 2.2 Cross-sectional factor momentum

Table 3 reports average monthly returns and alphas for a cross-sectional factor momentum strategy. This strategy is identical to the industry momentum strategy except that it takes positions in factors rather than industries. It rebalances monthly and is long the factors with above-median returns in month t-1 and short those with below-median returns. This strategy trades the 43 factors listed in Table 1; we later

We show, in Table 6, that the results based on the Fama and French 49-industry classification, which is the preferred definition in Asness, Porter, and Stevens (2000), are both qualitatively and quantitatively similar.

<sup>8</sup> Asness, Porter, and Stevens (2000, p. 14), for example, write: "Moskowitz and Grinblatt (1999) find that once adjusted for industry effects, momentum profits from individual equities are significantly weaker and for the most part are statistically insignificant. We disagree and find significant profits to within-industry momentum."

Table 3 Cross-sectional factor momentum

This table reports monthly average returns and alphas for a cross-sectional factor momentum strategy that trades the 43 factors listed in Table 1. The strategy rebalances monthly and is long the factors with above-median returns in month t-1 and short those with below-median returns. Column 1 reports the average return; column 2 the monthly five-factor model alpha; column 3 the Hou, Xue, and Zhang (2015) q-factor model alpha; column 4 the six-factor model alpha that adds the UMD factor of Carhart (1997); and column 5 the seven-factor model alpha that also controls for the time-series factor momentum. The samples in columns 1 through 4 begin in August 1963 and end in December 2021; the sample in column 5 begins in July 1964 because of the one-year formation period.

		Regression							
	(1)	(2)	(3)	(4)	(5)				
$\hat{\hat{lpha}}$	0.62 (5.95)	0.67 (6.30)	0.70 (6.19)	0.63 (5.82)	0.50 (4.75)				
UMD				$0.06 \\ (2.53)$	-0.14 $(-4.21)$				
Time-series factor momentum					0.83 $(8.87)$				
FF5 factors $q$ factors	N N	Y N	$_{\rm Y}^{\rm N}$	Y N	Y N				
$N$ Adjusted $R^2$	689	$689 \\ 5.6\%$	$648 \\ 3.7\%$	$689 \\ 6.4\%$	678 $16.2%$				

consider strategies that trade smaller subsets of, or principal components extracted from, these factors.  $^9$ 

The factor momentum strategy earns an average monthly return of 62 basis points (t-value = 5.95). The five-factor model explains none of its profits; the five-factor model alpha is 67 basis points (t-value = 6.30). This result is not specific to the use of the Fama-French model. When we swap this model for the q-factor model of Hou, Xue, and Zhang (2015), the alpha is 70 basis points (t-value = 6.19). Individual stock momentum also does not explain the profitability of the factor momentum strategy. The six-factor model alpha is 63 basis points per month (t-value = 5.82). The five-factor, q-factor, and six-factor models are largely unrelated to the cross-sectional momentum effect: they explain at most  $R^2 = 6.4\%$  of the variation in the monthly factor momentum profits.

<sup>&</sup>lt;sup>9</sup> In Table A5 in the Internet Appendix we examine cross-sectional factor momentum strategies that use longer formation and holding periods. Although these alternative cross-sectional factor momentum strategies also earn statistically significant five-factor model alphas, all of them, except for the short-term strategy analyzed in Table 3, are almost completely subsumed by individual stock momentum. The cross-sectional factor momentum strategies therefore behave similarly to the industry momentum strategies: only the strategy with the one-month formation and holding periods is distinct from individual stock momentum.

In the analyses that follow, we use the Fama-French model augmented with UMD as the benchmark model. The results with the q-factor model as the benchmark are almost identical. Although the q-factor model explains individual stock momentum (Hou, Xue, and Zhang 2015), this model does not subsume the cross-sectional factor momentum.

The cross-sectional factor momentum strategy is similar to industry momentum. This strategy is different from the time-series factor momentum strategy of Ehsani and Linnainmaa (2022) that is long the factors with positive returns over the prior year and short those with negative returns. The differences between these two strategies is that the cross-sectional strategy is about factors' relative prior one-month returns. The rightmost column in Table 3 shows that the cross-sectional factor momentum strategy is largely unrelated to time-series factor momentum. The seven-factor model alpha is 50 basis points (t-value = 4.75). Short-term cross-sectional factor momentum therefore represents an anomaly from the viewpoint of the five-factor model, individual stock momentum, and time-series factor momentum. 11 Table A6 in the Internet Appendix shows that the opposite is true as well: time-series factor momentum is a puzzle from the viewpoint of the five-factor model, individual stock momentum, and cross-sectional factor momentum, although time-series factor momentum's alpha associates with a t-value of 1.93 when confronted with both the UMD factor and cross-sectional factor momentum.

Tables 2 and 3 indicate that both industry and factor momentum strategies are short-term effects that grow *stronger* when we control for the five-factor model. The *t*-values associated with industry momentum's average return and its five-factor model alpha are 4.41 and 5.21; those of factor momentum are 5.95 and 6.30.

# 2.3 Industry momentum, factor momentum, and momentum in industry-neutral factors

Table 4 reports estimates from spanning regressions between industry momentum, factor momentum, and momentum in *industry-neutral* factors. We use these regressions to show that a significant connection exists between the factor and industry momentum. They set the stage for our later tests that propose and show *why* they are connected. The momentum strategy that trades industry-neutral factors is identical to the one that trades standard factors: it allocates to industry-neutral factors based on their past returns.

The differences in the sizes of the premiums of the cross-sectional and time-series factor momentum effects largely explains the relatively small reduction in the cross-sectional strategy's alpha. The cross-sectional strategy's six-factor model alpha is 63 basis points (column 4). The slope on the time-series factor momentum strategy in column 5 is 0.83. Because the time-series factor momentum strategy's average return is 31 basis points (column 1 in Table A6 in the Internet Appendix), we would expect the cross-sectional strategy's alpha to decrease from 63 to 63-0.83×31=36 basis points. However, because time-series factor momentum positively correlates with UMD (Ehsani and Linnainmaa 2022), UMD's slope also decreases from 0.06 to -0.14, which explains why the alpha in column 5 is still 50 basis points. Put differently, adding time-series factor momentum to a regression that already includes UMD does not do as much as it would if we started from a model without UMD.

 ${\bf Table~4} \\ {\bf Industry~momentum,~factor~momentum,~and~momentum~in~industry-neutral~factors}$ 

This table reports monthly alphas and selected factor loadings from time-series regressions in which the dependent variable is the monthly return on one of three strategies: momentum in standard factors, momentum in industry-neutral factors, and industry momentum. The model is either the Fama-French five-factor model or a six-factor model that also includes one of the other momentum strategies. We report factor loadings for factor momentum ( $\hat{b}_{\rm in-fmom}$ ), industry-neutral factor momentum ( $\hat{b}_{\rm in-fmom}$ ), and industry momentum ( $\hat{b}_{\rm imom}$ ). The two factor momentum strategies trade the 43 factors of Table 1. Industry momentum trades the 20 industries of Moskowitz and Grinblatt (1999). All strategies are cross-sectional strategies that rebalance monthly and are long factors (or industries) with above-median returns in month t-1 and short those with below-median returns. The sample begins in August 1963 and ends in December 2021.

Dependent			Coefficients			
variable	$\hat{lpha}$	$\hat{b}_{ ext{fmom}}$	$\hat{b}_{ ext{in-fmom}}$	$\hat{b}_{\mathrm{imom}}$	FF5	$R^2$
Factor momentum	0.67				Y	5.6%
in standard factors	(6.30)					
	0.08		1.00		Y	71.4%
	(1.34)		(39.69)			
	0.33			0.78	Y	43.5%
	(3.88)			(21.42)		
Factor momentum	0.59				Y	4.4%
in industry-neutral factors	(6.63)					
	0.12	0.70			Y	71.1%
	(2.41)	(39.69)				
	0.31			0.65	Y	41.7%
	(4.32)			(20.92)		
Industry momentum	0.44				Y	2.6%
industry mementum	(5.10)				-	2.070
	0.10	0.51			Y	41.7%
	(1.40)	(21.42)			-	11.170
	0.08		0.60		Y	40.6%
	(1.19)		(20.92)		-	10.070

Industry-neutral factors display as much momentum as the standard factors. Although the industry-neutral strategy has a lower monthly alpha—59 versus 67 basis points—it is also less volatile because the factors it trades remove industry-level variation in returns (Asness, Porter, and Stevens 2000). This decrease in volatility more than makes up for the decrease in alpha: the two strategies' five-factor model alphas have t-values of 6.30 (standard factors) and 6.63 (industry-neutral factors). When controlling for each other, the momentum in standard factors does not reach conventional levels of statistical significance (t-value = 1.34), while the industry-neutral version retains a t-value of 2.41. Industry momentum subsumes neither version of factor momentum.<sup>12</sup>

Factor momentum, by contrast, subsumes industry momentum. Industry momentum's alpha falls from 44 basis points (t-value = 5.10) in

<sup>&</sup>lt;sup>12</sup> Section I in the Internet Appendix shows that the two factor momentum strategies are profitable throughout the 1963–2021 sample period.

the five-factor model to 10 and 8 basis points, net of momentum, in the standard and industry-neutral factors. Neither estimate is statistically significantly different from zero. These regressions demonstrate that, whatever the source of industry momentum's profits, factor momentum strategies derive their profits from the same source. Industry momentum's loadings against the two factor momentum strategies are significant with t-values of 21.4 and 20.9, and the model's explanatory power rises by an order of magnitude, from under 3% to 41%, when we augment the five-factor model with one of the factor momentum strategies.

Table 4 indicates, first, that factor momentum subsumes industry momentum and, second, that industry momentum cannot be responsible for factor momentum. If factor momentum stemmed from industry momentum, we would not expect to see meaningful momentum in the industry-neutral factors; we would expect industry momentum to subsume factor momentum; and we would expect standard factor momentum to outperform the industry-neutral version. The data, however, run counter to all of these predictions.

# 2.4 Does factor momentum profit from persistent differences in factor premiums?

Momentum strategies can be profitable even if the underlying assets or factors exert no "momentum." Because momentum strategies sort assets into portfolios based on past returns, these strategies may profit by leaning toward assets with (persistently) high expected returns and away from those with low expected returns (Conrad and Kaul 1998). For example, if asset A's expected return is fixed at 0% and that of asset B is 10% and realized returns equal these expectations plus noise, a momentum strategy is typically long asset A and short asset B.

This mechanism, which is also described by Heston and Sadka (2008) in the context of the seasonality in stock returns, does not appear to contribute meaningfully to the factor momentum strategy's profits. We first illustrate this result in Table 5 with an alternative factor momentum strategy that only trades the five factors of the Fama-French model. The strategy is long the two factors with the above-median returns and short the two with below-median returns. Column 1 shows that this factor momentum strategy is also profitable; earning an average return of 66 basis points per month (t-value = 4.47).

The Conrad and Kaul (1998) hypothesis is that this strategy might profit by leaning toward factors with persistently higher premiums. Over our sample, the five factors' means are 59 (MKTRF), 20 (SMB), 27 (HML), 28 (RMW), and 27 (CMA) basis points per month. A strategy that is long the market factor and short the size factor through the sample period would earn an average return of 59-20=39 basis points

Table 5
Five-factor model momentum

This table reports the average return (column 1) and five-factor model alpha (column 2) for a strategy that trades cross-sectional momentum in the five Fama-French factors: market, size, value, profitability, and investment. The strategy rebalances monthly and is long the two factors with above-median returns in the prior month and short the two below-median factors. The sample begins in August 1963 and ends in December 2021.

	Regression			
	(1)	(2)		
$\hat{\alpha}$	0.66	0.76		
	(4.47)	(5.17)		
MKTRF		-0.21		
		(-5.77)		
SMB		-0.01		
		(-0.16)		
HML		-0.12		
		(-1.81)		
RMW		-0.11		
		(-1.61)		
CMA		0.30		
		(2.89)		
N	701	701		
Adjusted $R^2$	_	8.1%		

per month. However, if we were to regress this strategy against the five factors, its alpha would be identically zero because the regression would adjust for these unconditional tilts. How much of the factor momentum strategy's alpha comes from unconditional factor exposures? Column 2 shows that only two of the loadings—on the market (-) and investment factors (+)—are statistically significant. Because these tilts are in the "wrong" direction, the strategy's five-factor model alpha of 76 basis points is higher than its average return.

An alternative approach also suggests that the Conrad-Kaul mechanism contributes little, if at all, to the profits of the factor momentum strategy. In this approach we require at least 5 years of factor returns, estimate factors' average premiums using all historical data up to month t-2, and then subtract these averages from factors' month t-1 returns. Implicit in the Conrad-Kaul hypothesis is the presence of persistent and detectable differences in premiums; under this hypothesis, the long-term average is a viable estimate of the factors' risk premiums and the demeaning step removes the (best estimate of the) persistent risk premiums from the month t-1 returns. We construct an alternative factor momentum strategy by assigning factors to portfolios using these adjusted factor returns. This adjustment does not affect the profitability of the factor momentum strategies. In the last column of Table 3, for example, the "original" factor momentum's alpha in the sample that skips the first 5 years is 47 basis points (t-value = 4.32). In untabulated

results, we find that, over this same sample period, the alternative strategy that sorts factors by their mean-adjusted return has an alpha of 44 basis points (t-value = 4.10).<sup>13</sup>

The finding that the "risk premium" mechanism does not contribute to factor momentum profits is not entirely unexpected. First, because the strategy selects factors based on their prior *one-month* returns, a single month's return would have to be a meaningfully precise signal of a factor's premium. Monthly factor returns are, however, very noisy. For example, while the size factor's average monthly return is 20 basis points, the volatility of these monthly returns is 305 basis points. Second, prior studies have studied the importance of this mechanism in the context of other momentum strategies and reached a similar result. Jegadeesh and Titman (2002, p. 143), for example, note that their "empirical tests indicate that cross-sectional differences in expected returns explain very little, if any, of the momentum profits."

### 2.5 Is industry momentum about industries?

Our explanation for industry momentum is about factor loadings: industries inherit factor momentum through the variation in factor loadings. We test this mechanism later. However, an alternative approach for assessing the plausibility of this mechanism is to measure momentum in other (i.e., nonindustry) portfolios of stocks. If we create portfolios that plausibly display comparable variation in factor loadings, we would expect to observe (1) comparable amounts of momentum and (2) high correlation between this momentum effect and industry momentum. Under our hypothesis, both forms of momentums would emanate from the same source, that is, factor momentum.

In Table 6 we construct strategies that trade cross-sectional momentum in nine sets of portfolios:

- 1-2. **Industries.** In addition to the 20 Moskowitz and Grinblatt industries, we use the 49 industry portfolios of Fama and French.
- 3-5. **Size and book-to-market portfolios.** Following Lewellen (2002), we take the size and book-to-market decile portfolios and the 25 portfolios double-sorted by size and book-to-market ratio.
- 6-9. **Fundamental clusters.** We assign stocks into 25 clusters using either the four characteristics of the Fama-French five-factor model—size, book-to-market ratio, profitability, and investment—or an extended set of characteristics that also includes the

The estimates in Table 11 also quantify the importance of the risk premium mechanism using the Lo and MacKinlay (1990) decomposition. Although this factor momentum strategy is slightly different from the one considered here—specifically, the Lo-MacKinlay strategy's investment weights are proportional to factors' prior returns—the conclusion is again the same: the mean effect is negative, economically small, and not statistically significantly different from zero.

characteristics of the other popular factors: 5-year return skipping a year (i.e., the long-term reversal effect), residual variance, quality minus junk, and market beta. We form the clusters once a year for the basic set of characteristics and monthly for the full set. We normalize all characteristics each month to percentile ranks using NYSE stocks as the reference distribution. The standardization step ensures that the algorithms give the same weight to each characteristic. We form the clusters using both the k-means and agglomerative (Ward Jr. 1963) clustering algorithms.

Fundamental clusters represent an extension of the standard characteristic-sorted portfolios, such as book-to-market deciles. Instead of hardcoding the breakpoints, which would lead to a large number of empty or poorly diversified portfolios when the number of characteristics is large, the clustering algorithms return 25 portfolios that most closely resemble each other in multiple dimensions. We compute value-weighted returns on all characteristic portfolios. <sup>15</sup> We call the momentum present in the industry and characteristic portfolios and the fundamental clusters characteristic momentum.

In panel A of Table 6, we first report six-factor model alphas for each of the nine characteristic-momentum strategies and their equal-weighted portfolio ("diversified"). All characteristics momentum strategies have positive five-factor model alphas and, except for size deciles, all alphas associate with t-values greater than 3.0. An investor would benefit the most by trading the diversified portfolio of characteristic momentum strategies in addition to the six-factor model; this strategy's information ratio (IR)—IRs are proportional to t-values—is higher than any of the individual strategies.

Many of the nonindustry characteristic momentum strategies significantly correlate with industry momentum. Three of the seven nonindustry strategies lose their statistical significance when we add industry momentum to the six-factor model. Some strategies, such as those that trade the clusters formed using the basic set of characteristics, however, retain statistically highly significant alphas. Although this remaining significance could indicate that these strategies profit from

Each month we rank all NYSE stocks from 1 to N based on the characteristics and convert these ranks to run from 0 to 1. We then rank all non-NYSE stocks and, based on their characteristics relative to NYSE stocks, linearly interpolate to assign them percentile ranks. This method is a continuous-variables version of the Fama-French approach. For example, stocks with a book-to-market percentile rank less than 0.3 are the stocks that Fama and French assign to the growth portfolio.

When we form the portfolios at the end of month t, we compute these portfolios' returns in both months t (which is the formation period) and month t+1 (which is the holding period) to construct the cross-sectional momentum strategy. That is, we do not stitch together the cluster portfolio returns into one time series because, say, cluster 8 in month t is not the same as cluster 8 in month t+1 if we reform the clusters at the end of month t+1 if we reform the clusters a

#### Table 6 Characteristic momentum

This table reports alphas and correlations for different characteristic momentum strategies and factor momentum. All momentum strategies rebalance monthly and are long portfolios (or factors) with above-median returns in month t-1 and short those with belowmedian returns. The characteristic momentum strategies trade industry portfolios based on either the 20-industry (Moskowitz and Grinblatt) or 49-industry (Fama and French) definitions: decile portfolios based on size or book-to-market ratio: 25 portfolios sorted by size and book-to-market ratio; fundamental clusters based on size, book-to-market ratio, profitability, and investment ("basic") formed using either the k-means or agglomerative algorithms; and fundamental clusters based on a broader set of characteristics that also include market beta, idiosyncratic volatility, 5-year return, and quality minus junk ("full"). The factor momentum strategy trades the 43 factors listed in Table 1. Panel A reports alphas for the characteristic momentum strategies from the five-factor model augmented with UMD, Moskowitz-Grinblatt industry momentum, or factor momentum. Panel B reports correlations between different characteristic momentum strategies and factor momentum. Panel C reports alphas for the factor momentum strategy from the sixfactor model augmented with different characteristic momentum strategies. The diversified strategy in panels A and C is an equal-weighted portfolio of all characteristic momentum strategies. The sample begins in October 1963 and ends in December 2021.

A. Characteristic mon	entum	ı								
		Fama-French model augmented with:								
		UMD + Ind.			UN.	ID + Fa	ctor			
Characteristic		UMD		n	oment	um	n	nomentu	m	
momentum	$\hat{\alpha}$	$t(\hat{\alpha})$	$R^2$	$\hat{\alpha}$	$t(\hat{\alpha})$	$R^2$	$\hat{\alpha}$	$t(\hat{\alpha})$	$R^2$	
Industry										
GM	0.37	4.36	6.0%				0.05	0.78	43.3%	
Fama and French	0.33	4.16	7.6%	0.09	1.54	52.2%	0.03	0.40	46.2%	
Lewellen (2002)										
Size	0.13	1.75	3.8%	0.06	0.78	8.6%	-0.02	-0.25	15.3%	
Book-to-market	0.27	4.14	3.6%	0.14	2.38	23.6%	0.09	1.53	26.2%	
$Size \times B/M$	0.32	3.55	8.3%	0.16	1.89	24.1%	-0.03	-0.50	50.1%	
Fundamental clusters										
k-means, basic	0.38	4.51	9.4%	0.21	2.79	28.1%	0.04	0.66	53.0%	
Agglomerative, basic	0.40	4.60	7.3%	0.22	2.80	29.0%	0.04	0.64	54.1%	
k-means, full	0.37	3.52	7.5%	0.10	1.15	39.6%	-0.06	-0.82	54.7%	
Agglomerative, full	0.32	3.30	6.1%	0.07	0.88	38.9%	-0.08	-1.15	53.5%	
Diversified	0.32	4.75	8.4%	0.12	2.35	52.8%	0.01	0.15	67.1%	

B. Correlations								Funda	mental	
		Industry		I	Lewellen			clusters		
Definition		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Industry										
GM	(1)	1								
Fama and French	(2)	0.71	1							
Lewellen (2002)										
Size	(3)	0.25	0.33	1						
Book-to-market	(4)	0.47	0.45	0.23	1					
$Size \times B/M$	(5)	0.44	0.51	0.71	0.49	1				
Fundamental clusters										
k-means, basic	(6)	0.47	0.50	0.62	0.43	0.80	1			
Agglomerative, basic	(7)	0.48	0.54	0.59	0.40	0.78	0.90	1		
k-means, full	(8)	0.59	0.62	0.48	0.39	0.67	0.81	0.80	1	
Agglomerative, full	(9)	0.59	0.60	0.46	0.42	0.65	0.79	0.77	0.88	1
Factor momentum	(10)	0.64	0.66	0.38	0.51	0.70	0.70	0.72	0.72	0.72

C. Factor momentum when controlling for characteristic momentum

			Chara	cteristic			
Control for	Alı	Alpha		$_{ m entum}$	FF5 + UMD		
momentum in:	$\hat{\alpha}$	$t(\hat{\alpha})$	β	$t(\hat{\beta})$	factors	$R^2$	
Industry							
Grinblatt and Moskowitz	0.33	3.96	0.79	21.18	Y	43.5%	
Fama and French	0.34	4.11	0.87	22.15	Y	45.5%	
Lewellen (2002)							
Size	0.56	5.53	0.49	9.65	Y	17.5%	
Book-to-market	0.41	4.27	0.79	14.51	Y	28.4%	
$Size \times B/M$	0.37	4.59	0.80	23.91	Y	49.0%	
Fundamental clusters							
k-means, basic	0.29	3.66	0.88	25.15	Y	51.4%	
Agglomerative, basic	0.27	3.57	0.87	26.38	Y	53.6%	
k-means, full	0.35	4.63	0.73	26.64	Y	54.1%	
Agglomerative, full	0.37	4.89	0.78	26.38	Y	53.6%	
Diversified	0.22	3.31	1.26	34.88	Y	66.4%	

some other mechanism than what drives industry momentum, they also can be sufficiently different expressions of factor momentum. The estimates in the last set of columns support this view: a six-factor model augmented with factor momentum (almost always) explains more of the return variation and lowers all alphas below the conventional levels of statistical significance. These results are consistent with the mechanism we propose for industry momentum: nonindustry portfolios display momentum very similar to industry momentum because they, too, inherit factor momentum through the variation in factor loadings.

Panel B shows that all characteristic momentum strategies highly positively correlate with each other and factor momentum. These correlations suggest that, whatever the source of all these momentum profits, they likely stem from the same source. An investor who trades momentum in the 25 portfolios sorted on size and book-to-market ratio, for example, earns returns that are 0.7 correlated with factor momentum. The estimates in panels A and B suggest that industry momentum is just one form of a broader set of momentum strategies; by extension, these results suggest that industry momentum is not really about industries per se. Any scheme that partitions stocks into portfolios that have sufficiently different characteristics (and therefore factor loadings) is likely to produce a momentum effect comparable, in both magnitude and statistical significance, to industry momentum.

Panel C of Table 6 shows that none of the characteristic momentum strategies subsumes factor momentum. A regression of factor momentum against the five-factor model augmented with the diversified characteristic momentum, for example, returns a monthly alpha of 22 basis points (t-value = 3.31). We do not interpret these regressions as suggesting that factor momentum is unrelated characteristic momentum. Rather, the results in Table 6 suggest that industry, other characteristic, and factor momentum strategies are all about the same mechanism,

which is about factors and factor loadings, and that factor momentum, as the source, is the cleanest expression of this effect.

# 3. Transmission of Factor Momentum into the Cross-Section of Industries

Momentum in factors transmits to the cross-section of industries if industries' factor loadings differ. Even if industry-specific returns—that is, returns unrelated to industries' factor exposures—are serially uncorrelated, past industry returns positively predict future industry returns as long as past factor returns predict future factor returns. Why? A winning industry, on average, loads positively on winning factors and negatively on losing factors. If we sort industries by their past returns, we therefore indirectly sort on the past factor returns. The extent to which factor momentum generates industry momentum depends on the extent to which factor loadings vary across industries.

In the Internet Appendix, we illustrate this transmission mechanism with simulations. In these simulations only factor returns display momentum, but this momentum carries over to industry returns through variation in stocks' betas, which have industry-wide components. In these simulations, industry momentum is profitable only *if* factor betas vary across industries, but no industry momentum exists separately from factor momentum.

In Table 7 we measure the amount of cross-industry variation in factor loadings. We estimate panel regressions in which the dependent variable is the daily market-adjusted return on the 20 Moskowitz and Grinblatt (1999) industries. The explanatory variables are the daily factor returns from five asset pricing models, starting with the capital asset pricing model (CAPM) and ending with a nine-factor model that includes the four other popular factors from Table 1: long-term reversals, residual variance, quality minus junk, and betting against beta. We first estimate static regressions which constrain industries' factor loadings to be equal across industries and over time. We then introduce 20 dummy variables for the industries and 234 dummy variables for all quarters starting from the third quarter of 1963 and ending in the fourth quarter of 2021. By interacting these dummy variables or their products with factor returns, we let the regressions capture variation in factor loadings across industries, over time, or in both dimensions. <sup>16</sup>

As we add more and more interactions, these panel regressions build up toward a model in which we, in effect, estimate a different model for each industry and quarter. That is, when the model has both the time and industry fixed effects, the panel regression gives the same coefficient estimates as what we would get by estimating 4,680 separate regressions, one for each industry-quarter.

Table 7 Cross-sectional and time-series variation in industries' factor loadings

This table reports adjusted  $R^2$ s from models in which the dependent variable is the daily market-adjusted industry return. The bottom part indicates which factors are included in each model. We estimate four specifications to capture cross-sectional and time-series variation in factor loadings: (a) factor loadings are identical across industries and over time (static); (b) factor loadings vary from quarter to quarter; (c) factor loadings vary across industries; and (d) factor loadings vary in both dimensions. We capture these variations by interacting the right-hand-side factors with 20 dummy variables for the Moskowitz and Grinblatt (1999) industries, with 228 dummy variables that represent quarters, or with the product of these dummy variables. We compute standard errors using a delete-k jackknife procedure (Shao and Wu 1989) that estimates adjusted  $R^2$ s for 63 samples: the first sample deletes the first day of every quarter, the second day of every quarter, and so forth. The sample begins in July 1963 and ends in December 2021.

		Asse	et pricing r	nodel	
Specification	(1)	(2)	(3)	(4)	(5)
		$\mathbf{Adj}$	usted $R^2$ s	s (%)	
Static	_	1.6	2.7	2.8	3.0
Variation across time	1.7	3.8	4.3	4.6	4.8
Variation across industries	4.0	13.0	16.8	18.5	21.8
Variation across time and industries	15.2	29.1	35.5	40.7	45.2
		Stand	lard erro	rs (%)	
Static	_	0.1	0.2	0.2	0.2
Variation across time	0.2	0.2	0.2	0.2	0.2
Variation across industries	0.2	0.4	0.6	0.5	0.6
Variation across time and industries	0.7	0.8	0.7	0.7	0.7
Factors included in the model:					
MKTRF	×	×	×	×	×
SMB		×	×	×	×
HML		×	×	×	×
RMW			×	×	×
CMA			×	×	×
LTREV				×	×
RVAR				×	×
$_{ m QMJ}$					×
BAB					×

Table 7 reports adjusted  $R^2$ s from these panel regressions. Because the dependent variable is market adjusted, the static CAPM is the benchmark model. When the model permits market betas to vary across industries, CAPM explains 4.0% of the daily return variation. When the model permits for variation across industries and over time, it explains 15.2% of the daily return variation;<sup>17</sup> that is, exposures to

Because we compute adjusted  $R^2$ s, the model's explanatory power does not increase mechanically in the number of regressors. That is, if we generate an equal number of random indicator variables that have no rhyme or reason, the adjusted  $R^2$  is close to zero because we account for the degrees of freedom lost. We also verify this result in our specific context with a bootstrap simulation. In this simulation we first compute daily residual industry returns from the model that permits variation in slopes across both time and industries. We then draw one set of factor loadings for each industry from normal distributions whose means and variances match the sample moments of the betas estimated in the first step. These simulated data, by construction, exhibit no across-time variation in loadings. When we estimate the regressions in Table 7 using these data, we find that the adjusted  $R^2$ s are close to zero for the static and across-time specifications, the highest for the across-industry specification (which is the true model), and slightly lower

market risk vary significantly from one industry and quarter to the next. Because industry momentum is a short-term effect, time-series variation in exposures is important. For factor momentum to transmit to the cross-section of industry returns, industries' factor loadings do not have to be fixed over the entire 58-year sample; this transmission occurs as long as industry loadings persist from one month to the next.<sup>18</sup>

Adjusted  $R^2$ s increase as we add more factors. A three-factor model explains up to 29.1% of the return variation, a five-factor model up to 35.5%, and the nine-factor model up to 45.2%. The ordering of the models and factors is somewhat arbitrary. We will address this point later by ordering principal components extracted from a large set of factors. But this arbitrariness is beside the point. The results in Table 7 show is that, at every point in time, industries have significantly different loadings against many common factors. Any momentum found in the factors will therefore transmit to industry returns.

## 4. Momentum in Systematic Industries

## 4.1 Defining industry-mimicking portfolios

Industries are exposed to multiple factors which, in turn, display momentum. In this section we measure the extent to which a parsimonious combination of factors, tailored for each industry, can account for a meaningful proportion of industry momentum. If an industry has loadings  $\beta_1, \beta_2, \ldots, \beta_K$  on factors 1, 2, ..., K, we can use these betas together with the factors to construct a mimicking portfolio for the industry. What we call a systematic industry is a linear combination of the K factors; systematic industry j earns a return of  $r_{i,t}^s = \beta_{j,1} F_{1,t} + \cdots + \beta_{j,K} F_{K,t}$  in month t, where the  $F_t$ s are the factor returns. This definition of systematic industries strips out any industry-specific information. Based on the estimates in Table 7, a systematic industry based on the five-factor model, for example, would miss approximately two-thirds of the variation in daily industry returns, either because of omitted factors or because this remaining variation represents industry-specific return shocks. We call this remainder, the difference between the actual and systematic industry, residual industry.

than that for the across time-and-industry specification (because the additional degrees of freedom go to waste). Moreover, the average adjusted  $R^2$  from the delete-k jackknife samples, which we use to compute the standard errors in Table 7, are close to the adjusted  $R^2$ s of the panel regressions, suggesting that these statistics are not biased.

The results in Table 7 are consistent with the conclusions of Fama and French (1997), who measure cross-sectional and time-series variation in industries' three-factor model loadings. They conclude that "the variation through time in the true HML slopes of many industries is almost as large as the cross-sectional standard deviation of the long-term average HML slopes of the 48 industries" (Fama and French, 1997, p. 160).

### 4.2 Systematic industry momentum and spanning regressions

We measure the amount of momentum in systematic industries and the connection between this form of momentum and industry momentum in Table 8. We use the same models and methods as in Table 7; columns in Table 7 are rows in Table 8. We estimate each industry's factor loadings at the end of month t from daily industry and factor returns over the prior 3 months. <sup>19</sup> We compute each systematic industry's returns for months t and t+1 using the end-of-month-t estimated loadings. We define the systematic industry momentum strategy in the same way we define the standard industry momentum strategy: we sort systematic industries by their month-t returns, buy and sell those with above-and below-median returns, and rebalance monthly. Importantly, because we estimate factor loadings using information only up to month t, the systematic industry momentum strategy is tradeable as well. That is, instead of investing in industry j at the end of month t, an investor could alternatively invest in its mimicking portfolio.

Panel A of Table 8 reports monthly alphas for strategies that trade momentum in systematic industries. On the FF3 row, for example, each systematic industry is a combination of the three factors in the Fama and French (1993) model. This systematic industry momentum strategy has a monthly five-factor model alpha of 53 basis points (t-value = 5.70). This strategy, by definition, only trades momentum found in the underlying model's three factors. It does so by constructing 20 different portfolios of these factors. When the strategy takes a long position in the systematic industry j, it does so because the factors against which the actual industry j has positive betas must have performed well or the factors against which it has negative betas must have performed poorly.

Systematic industry momentum grows stronger when we build the mimicking portfolios from more factors. From the first model to the last, systematic industry momentum's five-factor model alpha increases from 25 basis points (t-value = 3.20) to 64 basis points (t-value = 6.66) per month. Controlling also for industry momentum, the last form of systematic industry momentum retains an alpha of 34 basis points (t-value = 4.34). That is, similar to the results in Table 4, there is momentum in factors not found in industry returns.

In Table A7 in the Internet Appendix we use 5 years of data to estimate the factor loadings. The results based on this longer estimation window are quantitatively close to those in Table 8. The choice of the estimation window length balances two competing considerations: (1) we prefer to use as much data as possible to estimate loadings more precisely and (2) we prefer to use as little recent data as possible to capture time variation in loadings. Lewellen and Nagel (2006) and Ang et al. (2006), among other studies, use windows as short as one month to estimate betas. We choose the 3-month window length to err on the side of capturing more time variation. Our rationale is that even if the resultant beta estimates are noisy, such noise would create bias against finding momentum in the systematic industries.

#### Table 8 Momentum in systematic industries

This table reports monthly alphas for systematic industry (panel A) and industry (panel B) momentum strategies. We define systematic industries as linear combinations of the factors against which the actual industries load. We use the 3 months of daily data up to the end of month t to estimate each industry's factor loadings for five asset pricing models. Industry j's systematic return in month t' is  $r_{j,t'}^s = \sum_{k=1}^K \hat{\beta}_{j,k,t} F_{k,t'}$ , where t' is either t or t+1; and the industry's residual return is  $r_{j,t'}^r = r_{j,t'} - r_{j,t'}^s$ . Month t+1 returns are out-of-sample from the viewpoint of the beta estimates. Industry (imom), systematic industry (smom), and residual industry (rmom) momentum strategies rebalance monthly and are long industries with above-median returns in month t and short those with below-median returns. Panel A reports alphas for the industry momentum strategy from the five-factor model augmented with either the systematic or residual industry momentum, except for the first row, which is the standard five-factor model. Panel B reports alphas for systematic industry momentum strategies from the five-factor model or from this model augmented with industry momentum. The sample begins in October 1963 and ends in December 2021.

A. Momentum in systematic industries

		Regression						
Model for systematic	(1)	(2)						
industries	$\frac{\hat{\alpha}_{\text{ff5}}}{\hat{\alpha}_{\text{ff5}}}$	$\hat{\alpha}_{\text{ff5+imom}}$	$\hat{b}_{\mathrm{imom}}$	$R^2$				
CAPM	0.25	0.15	0.21	7.5%				
	(3.20)	(2.01)	(6.28)					
FF3	0.53	0.32	0.46	23.5%				
	(5.70)	(3.75)	(12.55)					
FF5	0.51	0.28	0.52	29.6%				
	(5.67)	(3.49)	(14.87)					
FF5 + LTREV + RVAR	0.58	0.31	0.60	35.7%				
	(6.30)	(4.00)	(17.74)					
FF5 + LTREV + RVAR	0.64	0.34	0.67	39.2%				
+  QMJ + BAB	(6.66)	(4.34)	(19.51)					

 $B. \ Explaining \ industry \ momentum \ with \ momentum \ in \ systematic \ and \ residual \ industries$ 

	Control for momentum in:									
Model for systematic	Systema	tic indust	ries	Residual industries						
industries	$\hat{\alpha}_{\text{ff5+smom}}$ $\hat{b}_{\text{smom}}$ $R^2$		$R^2$	$\hat{\alpha}_{\text{ff5+rmom}}$	$\hat{b}_{ ext{rmom}}$	$R^2$				
None	0.45 (5.21)		2.5%							
CAPM	$0.38 \\ (4.55)$	0.26 $(6.28)$	7.6%	$0.03 \\ (0.47)$	0.88 (34.14)	63.7%				
FF3	0.24 (2.99)	$0.40 \\ (12.55)$	20.5%	0.14 (1.87)	0.62 $(16.91)$	30.9%				
FF5	0.21 $(2.72)$	0.47 (14.87)	26.0%	0.19 $(2.35)$	$0.55 \\ (13.32)$	22.3%				
FF5 + LTREV + RVAR	0.14 $(1.98)$	$0.52 \\ (17.74)$	32.9%	0.22 $(2.73)$	0.51 $(11.72)$	18.5%				
$\begin{array}{c} \text{FF5} + \text{LTREV} + \text{RVAR} \\ + \text{QMJ} + \text{BAB} \end{array}$	0.11 $(1.51)$	0.53 $(19.51)$	37.0%	0.25 $(3.08)$	0.50 (11.63)	18.3%				

In the first columns of panel B, we report industry momentum's alphas from the five-factor model augmented with systematic industry momentum. On the FF3 row, for example, we measure how much industry momentum remains when controlling for momentum found in just the three factors of the Fama and French (1993) model. Although industry momentum's alpha remains significant at 24 basis points (t-value = 2.99), it is approximately half of its five-factor model alpha of 45 basis points (t-value = 5.21). That is, the momentum found in the three Fama-French factors alone account for a meaningful amount of industry momentum.

As systematic momentum grows stronger, less and less remains of industry momentum. Momentum in the CAPM-based systematic industries remove just 14% of industry momentum's alpha; the three-factor model removes 47%; the five-factor model removes 53%; the seven-factor model removes 68%; and the nine-factor model removes 76%. By the last two models, industry momentum's alpha falls to 14 and 11 basis points and loses statistical significance. At the same time as industry momentum's alpha decreases, the model's explanatory power increases. This increase again suggests that industry momentum profits derive from the same source as the factor momentum profits.

The other columns in panel B illustrate the same conclusion from the opposite direction. Instead of controlling for momentum in the systematic industries, we control for momentum in the residual industries. When the model for systematic industries has just a few factors, the residual industries represent a combination of true industryspecific shocks and momentum in the factors omitted from the model. The CAPM row, for example, shows that when we control for residual industry momentum, industry momentum has no alpha—and the  $R^2$ is 64%—there is no meaningful distinction between "industries" and "residual industries" at this point; residual industries are just industries without market risk. However, as we add more factors, residual industry momentum loses much of its correlation with industry momentum— $R^2$ decreases to 18% by the last row—and industry momentum regains an alpha of 25 basis points (t-value = 3.08). Both approaches in panel B indicate that industry momentum stems from momentum in the factors against which the industries load.

## 5. Momentum in Principal Component Factors

### 5.1 Momentum in high- and low-eigenvalue PC factors

What are factors and why would factors display momentum? Economic theory provides guidance on these questions by identifying which factors should associate with pricing effects. Kozak, Nagel, and Santosh (2018, 2020), building on Ross (1976), note that the absence of near-arbitrage

opportunities alone indicates that the only factors that should have pricing effects are those that explain systematic variation in returns. Whether pricing is rational or behavioral, differences in expected returns should align with covariances. In rational models the argument is that of Merton (1973); in mispricing-based explanations, it is based on the optimal behavior of arbitrageurs. When mispricings do not align with covariances, arbitrageurs can trade the mispricings aggressively without assuming any factor risk; only those mispricings that align with covariances remain (Kozak, Nagel, and Santosh 2018).

Thus, from the perspective of economic theory, stocks, characteristic portfolios, and factors therefore *all* lack substance; the only element that matters is the risk that arbitrageurs would bear if they were to trade a factor. The extent to which factors can display any pricing effects, such as momentum, therefore depends on their exposures to undiversifiable risk. The theory guides us to look for those combinations of factors that explain the most variation in returns.

Kozak, Nagel, and Santosh (2018) extract principal component factors from a large set of factors and show that factor premiums concentrate in a small set of the highest-eigenvalue factors; their result therefore supports their conjecture regarding the lack of near-arbitrage opportunities. We use the same methods to examine factor momentum. The difference is that, instead of studying PC factors' unconditional average returns, we measure the amount of momentum in these factors' returns. We expect factor momentum to concentrate in the higheigenvalue factors; the existence of significant momentum among the low-eigenvalue PC factors would be inconsistent with the absence of near-arbitrage opportunities. It would indicate that arbitrageurs are leaving (or at least historically were leaving) money on the table. Factors can display momentum without violating the absence of near-arbitrage opportunities. Ehsani and Linnainmaa (2022) note that factors in the sentiment-based mispricing model of Kozak, Nagel, and Santosh (2018) have momentum when sentiment is highly persistent.

We extract principal component factors from the 43 industry-neutral factors listed in Table 1.<sup>20</sup> We use a rolling-window approach similar to that of Ehsani and Linnainmaa (2022) to render the returns on the PC factors in month t+1 out of sample relative to the estimation of the eigenvectors. We use 10 years of daily returns up to the end of month t, normalize all factors to 10% volatility, and then compute the first 41

Similar to, for example, Kozak, Nagel, and Santosh (2018), we use the standard PCA. Other methods, such as sparse and robust PCA, could be used as well. In the context of factors, Lettau and Pelger (2020), for example, create a method that extracts "weak factors" that have higher Sharper ratios and that price test assets better than the standard factors. A comparison of these alternative methods is left for future research.

Table 9 Momentum in high- and low-eigenvalue PC factors

This table reports monthly alphas for momentum strategies that trade subsets of PC factors ordered by their eigenvalues. We extract 41 principal components from the 43 industry-neutral factors listed in Table 1; we stop at 41 because the returns on two of the factors on this list begin 9 years after the start of the sample. We use 10 years of daily returns up to the end of month t, normalize all factors to 10% volatility, compute all eigenvectors, and compute month t and t+1 returns on the PC factors from these eigenvectors. We order the PC factors by their eigenvalues and assign them into groups. A PC factor momentum strategy sorts the factors by their month t returns, takes long and short positions in the factors with above- and below-median returns, and rebalances monthly. We report five-factor models alphas for the subsets of PC factors and alphas from the five-factor model augmented with the momentum found in the high-eigenvalue PC factors (factors 1–5). Factor returns begin in July 1973 and end in December 2021.

	Momentum in subsets of PC factors ordered by eigenvalues							
	1-5	6-10	11-15	16-20	21-25	26-30	31–35	36-41
			1	Five-facto	or model			
$\hat{lpha}$	$0.71 \\ (5.80)$	$0.15 \\ (1.20)$	$0.36 \\ (2.85)$	0.14 $(1.11)$	$0.22 \\ (1.75)$	$0.32 \\ (2.58)$	$0.36 \\ (2.89)$	0.28 $(2.28)$
Adj. $\mathbb{R}^2$	5.9%	0.5%	1.2%	1.8%	1.0%	2.0%	3.7%	3.1%
			Five-facto omentun					
$\hat{lpha}$		-0.11 $(-0.89)$	0.15 $(1.22)$	$0.04 \\ (0.29)$	0.07 $(0.53)$	0.12 $(0.98)$	0.14 (1.13)	0.23 $(1.79)$
$\hat{b}_{\mathrm{fmom}1-5}$		0.36 (9.10)	0.29 (7.11)	0.14 $(3.42)$	$0.22 \\ (5.15)$	0.28 $(6.93)$	0.31 $(7.74)$	$0.08 \\ (1.85)$
Adj. $R^2$		12.8%	9.0%	3.6%	5.1%	9.4%	12.6%	3.5%

eigenvectors, ordered by their eigenvalues.  $^{21}$  We stop at 41 because the returns on two factors on the list begin in July 1972. We compute month t and month t+1 returns on the PC factors from these eigenvectors. The month t+1 returns are out of sample relative to the estimation step. Because we compute both month t and t+1 returns using the same set of eigenvectors, the rotation of the factors is locally the same. That is, when we sort PC factors by their month t returns to create the momentum strategy, the month t+1 returns correspond to the t+1 returns of factors.

In Table 9 we examine the performance of momentum strategies that trade eight different subsets of PC factors. The first subset contains the five highest-eigenvalue PC factors; the second subset contains the five

We use 10 years of daily returns to account for the possibility that the 43 factors' loadings on the latent factors that we are trying to extract are time varying (Su and Wang 2017) instead of constant over time (Bai and Ng 2002). In Table A8 in the Internet Appendix we use an expanding window instead and find that the estimates are not very sensitive to this choice. Factors based on the "spot covariance matrix" display marginally more momentum, consistent with the presence of some time-variation in the factor loadings. An alternative approach for capturing time-variation in factor loadings, proposed by Pelger and Xiong (2022), would be to model factor loadings as being state dependent.

next-highest-eigenvalue factors, and so forth. The first row of Table 9 shows that the highest-eigenvalue set of PC factors has a five-factor model alpha of 71 basis points per month (t-value = 5.80). The second set has an alpha of 15 basis points per month (t-value = 1.20).

The significance of the alphas beyond the set of the highest-eigenvalue factors—the alpha of the third set, for example, is significant with a t-value of 2.85—does not imply that these momentums are incrementally informative about future returns. Although the PC factors are, by definition, orthogonal in the estimation period, the strategies that time them need not be. That is, if the momentum profits that the PC factors earn derive from a common source, timed portfolios of the orthogonal factors can correlate. The bottom part of Table 9 augments the five-factor model with a strategy that trades momentum in the five highest-eigenvalue PC factors. This regression shows that all PC factor momentum strategies indeed significantly correlate with each other. In the regression for the strategy that trades the second set of PC factors, for example, the slope on the high-eigenvalue strategy is 0.36 (t-value = 9.10), suggesting a high degree of commonality in the momentum profits.

The alphas from these six-factor model regressions measure the extent to which the other momentum strategies would expand the investment opportunity set of an investor who already trades high-eigenvalue factor momentum (Huberman and Kandel 1987). The economically small and statistically insignificant alphas indicate that the other strategies are not incrementally informative about future returns. From an asset pricing perspective, the results imply that a model that includes only the high-eigenvalue momentum suffices to describe the cross-section of average returns (Barillas and Shanken 2016).<sup>22</sup>

Figure 1 illustrates the striking difference in the amount of momentum found in the high- and low-eigenvalue factors. We start with a strategy that trades momentum in just two PC factors; in this case the strategy is always long the better-performing factor and short the worse-performing one. We start either from the two highest-eigenvalue factors (black line) or from the two lowest-eigenvalue factors (red line). We then increase the number of factors included in these strategies, one factor at a time, until they both include all 41 PC factors. We plot the  $t(\hat{\alpha})$ s from the five-factor model for each of the resultant strategies.

By construction, the two lines converge at the same point when both strategies include all 41 PC factors. These two lines, however, start

<sup>&</sup>lt;sup>22</sup> In Table A9 in the Internet Appendix, we regress the high-eigenvalue momentum strategy against the Fama-French model augmented with each (or all) of the other PC factor momentum strategies. The highest-eigenvalue strategy's alpha is always high and statistically significant. It is at its lowest, 50 basis points per month (t-value = 4.53), in the model that controls for all other PC momentum strategies at the same time.

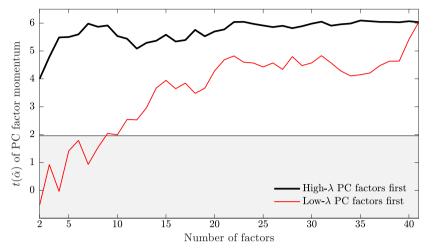


Figure 1 Factor momentum  $t(\hat{\alpha})$ s as a function of the number of PC factors We extract 41 principal component factors from the factors listed in Table 1 using 10 years of daily returns up to month t, compute monthly returns for months t and t+1, and order the resultant PC factors by eigenvalues. We construct strategies that trade factor momentum in an increasing number of these PC factors with the number of factors in the strategy shown on the x-axis. The black line starts from the two highest-eigenvalue factors and works down the list toward the lowest-eigenvalue factor; the red line works through the factors in the reverse order. We also plot the t-value associated with the Fama-French five-factor model alpha for each strategy. The sample begins in July 1973 and ends in December 2021.

from very different levels and follow different paths before converging. Consistent with Table 9, momentum strategies that trade the highest-eigenvalue factors are more profitable than those that trade any of the low-eigenvalue factors. Strategies that trade just a few of the highest-eigenvalue factors are just as profitable as the one that trades the full set. The reason is that these additional factors, as shown in the bottom part of Table 9, are not incrementally informative about future returns.

The strategies that start from the lowest-eigenvalue PC factors, by contrast, benefit from adding more and more factors, with the final three PC factors—those with the highest eigenvalues—being decisive. The profitability of these strategies jumps higher at the end because the few highest-eigenvalue PC factors contain information not found in the earlier factors.<sup>23</sup>

In Table A7 in the Internet Appendix, we revisit the systematic industries analysis of Table 8, constructing systematic industries from the PC factors. A strategy that trades systematic industries constructed from three or more highest-eigenvalue PC factors subsumes industry momentum.

Table 10 Industry and other forms of characteristic momentum versus momentum in PC factors

This table reports monthly alphas for the characteristic momentum strategies from Table 6. The first set of columns report alphas and adjusted  $R^2$ s from the five-factor model. The second set of columns reports alphas from this model augmented with a strategy that trades momentum in the five highest-eigenvalue PC factors. The sample begins in July 1973 and ends in December 2021.

					FF5 +	
Characteristic	FF5			PC factor momentum		
momentum	$\hat{\alpha}$	$t(\hat{\alpha})$	$R^2$	$\hat{\alpha}$	$t(\hat{\alpha})$	$R^2$
Industry						
GM	0.37	3.81	2.9%	0.09	0.99	27.5%
Fama and French	0.29	3.31	4.6%	0.03	0.43	28.7%
Lewellen (2002)						
Size	0.06	0.75	3.5%	-0.09	-1.08	12.8%
Book-to-market	0.31	4.27	2.4%	0.12	1.74	23.0%
$Size \times B/M$	0.26	2.61	8.8%	-0.07	-0.82	38.3%
Fundamental clusters						
k-means, basic	0.35	3.74	9.6%	0.04	0.49	39.1%
Agglomerative, basic	0.34	3.50	7.2%	0.01	0.16	38.3%
k-means, full	0.34	2.87	8.0%	-0.06	-0.58	38.7%
Agglomerative, full	0.27	2.56	7.2%	-0.07	-0.72	34.8%
Diversified	0.29	3.85	7.9%	0.00	0.02	47.9%

# 5.2 Industry and other forms of characteristic momentum vis-à-vis high-eigenvalue PC factor momentum

Table 10 shows that the momentum found in the high-eigenvalue PC factors fully subsumes all forms of characteristic momentum. We price these momentum strategies either with the five-factor model or with this model augmented with the strategy that trades momentum in the five highest-eigenvalue PC factors.

Momentum found among the highest-eigenvalue PC factors explains both forms of industry momentum. The Grinblatt-Moskowitz industry momentum's five-factor model alpha in the post-1973 sample is 37 basis points per month (t-value = 3.81). This alpha falls to 9 basis points per month (t-value = 0.99) when we add to the model the momentum in the five highest-eigenvalue PC factors. None of the other characteristic momentum strategies retains statistically significant alphas. The alpha of the diversified characteristic momentum strategy, for example, falls from 29 basis points (t-value = 3.85) to zero (t-value = 0.02).

Table A10 in the Internet Appendix shows that this spanning result is specific to trading momentum in the highest-eigenvalue PC factors. All alphas associated with the diversified strategy remain all significant in regressions that augment the five-factor model with one of the lower-eigenvalue PC factor momentum strategies.

Figure 2 illustrates the connection between characteristic and PC factor momentum using the same technique as in Figure 1. We construct the same high- or low-eigenvalue-factors-first strategies and augment the Fama-French five-factor model with each of them in

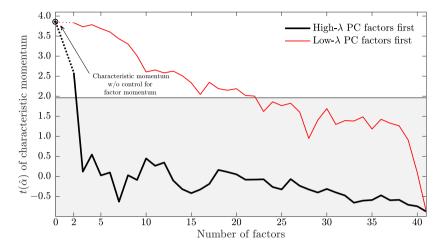


Figure 2 Characteristic momentum  $t(\hat{\alpha})$ s when controlling for momentum in PC factors We extract 41 principal component factors from the factors listed in Table 1 using 10 years of daily returns up to month t, compute monthly returns for months t and t+1, and order the resultant PC factors by eigenvalues. We construct strategies that trade factor momentum in an increasing number of these PC factors with the number of factors in the strategy shown on the x-axis. The black line starts from the two highest-eigenvalue factors and works down the list toward the lowest-eigenvalue factor; the red line works through the factors in the reverse order. We compute diversified characteristic momentum's alpha from all of the different six-factor models that augment the five-factor model with the resultant strategies. The diversified characteristic momentum strategy is an equal-weighted portfolio of industry momentum and strategies that trade momentum in different characteristics-based portfolios, such as book-to-market deciles. This figure shows the t-values associated with these alphas. The sample begins in July 1973 and ends in December 2021.

turn. We compute diversified characteristic momentum's alphas from the resultant six-factor models and plot the t-values associated with these alphas as a function of the number of PC factors included in the momentum strategies. The point on top of the y-axis represents characteristic momentum's  $t(\hat{\alpha})$  from the five-factor model without any factor momentum.

Characteristic momentum's alpha falls quickly to statistical insignificance when we control for momentum found among the first few high-eigenvalue factors. By the time we consider a strategy with the first three PC factors, characteristic momentum's alpha is no longer statistically significant (t-value = 1.33). The pattern in the t-values and the contrast between this pattern and that for the strategy which starts from the low-eigenvalue factors (red line) is even more striking. As we add more and more factors, the black line still declines, but at a decreasing pace. The red line, which starts from the "wrong" factors, has the opposite shape. Controlling for momentum in the first few low-eigenvalue factors does very little to characteristic momentum's alpha; this alpha does not breach the 5% significance level until we consider

a strategy that includes more than 22 factors. And, similar to the pattern in Figure 1, the last few factors, which are now the highest-eigenvalue PC factors, carry a disproportionate weight in explaining industry momentum's profits.

# 6. Reconciling Factor Momentum with Short-Term Reversals in Stock Returns

# 6.1 What are the sources of momentum profits and short-term reversals?

The profits earned by industry and factor momentum strategies must ultimately derive from some feature of stock returns because industries and factors are, after all, nothing more than portfolios of individual stocks. Stocks, however, display short-term reversals, not momentum (Jegadeesh 1990). In this section we first decompose profits of the industry and factor momentum strategies and (individual stock) short-term reversals using the Lo and MacKinlay (1990) method. This method constructs trading strategies in which the weights are proportional to assets' month t-1 returns. The investment in asset i—which for us is an individual stock, industry, or factor—is proportional to its performance in the prior month relative to the other assets in the cross-section,

$$w_{i,t} = \frac{1}{N_t} (r_{i,t-1} - \bar{r}_{t-1}), \tag{1}$$

where  $\bar{r}_{t-1}$  is the cross-sectional average of the  $N_t$  assets in the cross-section at time t. (We track the size of the cross-section to accommodate the analysis of individual stocks.) Because the month-t return on the position in asset i is  $w_{i,t}r_{i,t}$ , which, given the definition of the weights, is a product of past and current returns, the *expectation* of this product equals the product of means plus a covariance term. The Lo-MacKinlay approach decomposes this expected return,  $\mathrm{E}(r_t^{\mathrm{LM}})$ , into three terms,

$$\mathbf{E}(r_t^{\mathrm{LM}}) = \underbrace{\frac{N_t - 1}{N_t^2} \mathrm{Tr}(\Omega_t)}_{\text{autocovariance}} - \underbrace{\frac{1}{N_t^2} (1'\Omega_t 1 - \mathrm{Tr}(\Omega_t))}_{\text{cross-serial covariance}} + \sigma_{\mu,t}^2, \tag{2}$$

where  $\Omega_t = \mathcal{E}_t \left[ (r_{i,t} - \mu)(r_{i,t+1} - \mu)' \right]$  is the autocovariance matrix of asset returns at time t,  $\text{Tr}(\Omega_t)$  is the trace of this matrix, and  $\sigma_{\mu,t}^2$  is the cross-sectional variance of mean asset returns at time t. This decomposition separates cross-sectional momentum profits into three sources:

- 1. Positive autocovariances in asset returns: a past asset return signals high future return.
- Negative cross-serial covariances: a past asset return signals low returns on other assets.

Table 11 Lo-MacKinlay decomposition of short-term momentum profits in industry, factor, and individual stock returns

This table decomposes profits of one-month cross-sectional momentum strategies using the Lo and MacKinlay (1990) methodology. The strategies' weights are proportional to assets' cross-sectionally demeaned month t-1 returns. The first two columns report average autocovariances and cross-serial covariances for the 20 Moskowitz and Grinblatt (1999) industry portfolios, the 43 factors listed in Table 1, and individual stocks. The remaining columns decompose momentum profits into three terms: profits due to asset autocovariances, cross-serial covariances, and the mean effect. The mean effect is the difference between the total return to the Lo-MacKinlay strategy minus the sum of the first two components. The total return on the individual stock strategy is negative because stocks display short-term reversals, not momentum, at the one-month horizon. Standard errors are reported in parentheses. We compute standard errors by bootstrapping the data by calendar month. In the individual stock analysis, we randomly select 2,000 stocks into each sample. The three components of the return composition and the total return are scaled so the total strategy realizes an annualized volatility of 10% over the full sample. Returns are expressed in percentages per month. The sample begins in July 1963 and ends  $\frac{1}{2}$  and  $\frac{1}{2}$  are  $\frac{1}{2}$  and  $\frac{1}{2}$  are  $\frac{1}{2}$  and  $\frac{1}{$ 

	Average el	ements of $\Omega$	Return decomposition				
	Auto-	Cross-serial	Auto-	Cross-serial	Mean		
Assets	covariance	covariance	covariance	covariance	effect	Total	
Industries	1.95	1.37	0.86	-0.60	0.01	0.26	
	[1.15]	[1.05]	[0.51]	[0.46]	[0.00]	[0.12]	
Factors	1.13	0.16	0.38	-0.05	-0.02	0.31	
	[0.36]	[0.06]	[0.16]	[0.02]	[0.04]	[0.15]	
Individual	-8.07	6.34	-0.46	-0.37	0.19	-0.64	
stocks	[3.38]	[1.73]	[0.16]	[0.12]	[0.04]	[0.11]	

3. Cross-sectional variance of mean returns: some assets earn persistently high or low returns.

The intuition for the first two terms is that the bets that the cross-sectional momentum strategy makes can be viewed from two directions because of the adding-up constraint: a bet for asset i is always a bet against all other assets in the cross-section. The momentum strategy can be profitable if assets with high returns continue to earn high returns, or if a high return on an asset signals low returns on all other assets. The last term is the Conrad and Kaul (1998) effect from Section 2.4: momentum strategies may profit by leaning toward assets with (persistently) high expected returns and away from those with low expected returns.

Table 11 reports average autocovariances and cross-serial covariances of industry, factor, and individual stock returns. Factors and industries are similar: the returns on both are positively autocorrelated and cross-serially correlated. These patterns differ from those reported in Lewellen (2002)—he reports negative autocovariances and cross-serial covariances for industries—because we examine one-month lead-lag relationships; Lewellen's estimates are between month t returns and average returns over the prior year, averaged using lags up to 18 months. That is, his estimates are about lead-lag patterns in long-horizon portfolio returns. Individual stock returns are also positively cross-serially correlated at

one-month horizons but, in contrast to the industries and factors, they are significantly negatively autocorrelated.

In the other columns of Table 11 we decompose the momentum strategy returns using equation (2). For the factor and industry strategies, we compute the first two terms in equation (2) from the full-sample autocovariance matrix. We compute the last term as the difference between the strategy return and the sum of the first terms. Individual stocks represent a complication because stocks enter and exit the sample and we do not want to condition on stocks surviving over any sample period. We resolve this issue by estimating the decomposition separately each month—to keep track of which stocks exist at time t—and then averaging the estimates over the full sample period. We compute the first two terms in equation (2) using the pairwise autocovariance matrix each month t for all stocks that exist that month. The third term in month t is the difference between the momentum strategy's total return that month minus the sum of the first two components. We estimate standard errors for all three decompositions by bootstrapping the data by calendar month. In bootstrapping the individual stock data, we also randomly select 2,000 stocks into each sample for computational reasons. We scale the three return components so that each strategy's annualized volatility is 10%.

The signs of the return decomposition estimates follow directly from the estimates in the first two columns. Cross-sectional momentum strategies in industries and factors profit from positive autocorrelations, with the positive cross-serial covariances detracting from these returns. The mean effect is economically small in both cases. The source of momentum returns, for both industries and factors, is therefore the positive short-term return autocorrelation.

Individual stock returns display short-term reversals because both the autocovariances and cross-serial covariances work against momentum: a high return on a stock predicts low return on the same stock and high returns on other stocks. These two components contribute equally to the momentum strategy's losses. The mean effect, which works against short-term reversals, is economically small but positive and statistically significant.

## 6.2 Decomposing stock returns

The differences in the autocorrelations in Table 11 must stem from the dynamics of stocks' systematic and idiosyncratic components. If the building blocks (individual stocks) of the portfolios are negatively autocorrelated while the well-diversified portfolios are not, the leadlag relationships associated with the idiosyncratic components drive the negative autocovariances. We now measure these lead-lag effects by decomposing stock returns into systematic and idiosyncratic components.

We use the same approach as in Section 4.1 to decompose stock returns. We regress each stock on alternative sets of factors using 3 months of daily data. The systematic stock component in month t is the sum of the products of the estimated betas and factor returns. The return on this systematic portfolio is in sample to the end of month t, but out of sample in month t+1. That is, investors could estimate betas using data available by the end of month t and construct either (1) a factor portfolio that earns the systematic component in month t+1 or (2) combine this factor portfolio with the stock itself to earn the idiosyncratic component.

In panel A of Table 12, we measure the amount of factor risk in the cross-sectional variance of stock returns. We regress firms' total stock returns on the out-of-sample (i.e., month t+1) estimate of the systematic component. We report the average adjusted  $R^2$  from cross-sectional regressions (first column) or from a pooled regression with month fixed effects (second column). These two sets of estimates are similar to each other. The results show that variation in stocks' market betas explain approximately 5% of the cross-sectional return variation out of sample. The estimated share of factor risk increases, but at a decreasing rate, as we add more factors to the model. The nine-factor model explains 15% to 17% of the cross-sectional variation in stock returns.

In panel B of Table 12, we report estimates from cross-sectional regressions that predict total, systematic, and idiosyncratic stock returns with lagged values of these components. Column 1 shows the baseline result that stock returns display reversals. Column 2 shows that, when we decompose month t return into the systematic and idiosyncratic components, the systematic component negatively predicts returns. In fact, the systematic component (t-value = -7.88) is a far more powerful predictor of stock returns than the stock return itself (t-value = -4.87).

In the other columns of panel B, we have either the systematic or residual component of stock returns as the dependent variable.<sup>24</sup> The two component positively predict themselves and negatively each other. That is, a high value of the systematic component of a stock in month t

We should emphasize, given the magnitudes of some of the t-values in Table 12, there is no lookahead bias: an investor can earn the systematic component by trading an appropriate factor portfolio or they can earn the residual component by combining this factor portfolio with the stock itself. Some of the estimates are statistically very significant, but not entirely surprisingly so. The largest t-value of -29.8 translates to an annualized Sharpe ratio of 3.9. Detzel, Novy-Marx, and Velikov (2021) note that the one-month low-volatility industry-relative reversals factor has a squared Sharpe ratio of 4.81, the highest among their set of factors. The concerns Detzel, Novy-Marx, and Velikov (2021) express about the tradeability of that particular factor apply to types of effects measured in panel B.

Table 12 Lead-lag relationships between systematic and idiosyncratic stock returns

This table reports estimates of the share of factor risk in stock returns (panel A) and on the lead-lag relationships between the total, systematic, and idiosyncratic stock returns (panel B). Systematic stocks are linear combinations of the factors against which each stock loads. We use the 3 months of daily data to the end of month t to estimate each stock's factor loadings for five asset pricing models, starting with the CAPM and ending with a nine-factor model. Systematic stock i's return in month t' is  $r_{i,t'}^s = \sum_{k=1}^K \hat{\beta}_{i,k,t} F_{k,t'}$ , where t' is either t or t+1. Month t+1 return is out-of-sample from the viewpoint of the beta estimates. Panel A reports adjusted  $R^2$ s from regressions of total stock returns on the out-of-sample (i.e., month t+1) estimate of the systematic component from different asset pricing models. We report the average adjusted  $R^2$  from cross-sectional regressions (FMB) and the average adjusted  $R^2$  from a pooled regression with month fixed effects ("Pooled"). Panel B reports average coefficients and t-values associated with these coefficients from cross-sectional regressions that predict the total, systematic, and idiosyncratic stock returns in month t+1 with lagged values of these components. The asset pricing model for the systematic and idiosyncratic components in panel B is the nine-factor model. The sample begins in July 1963 and ends in December 2021.

A. Estimated share of factor risk in individual stock returns

Model for	Estimated share				
systematic	of fac	of factor risk			
stocks	FMB	Pooled			
CAPM	5.3%	4.6%			
FF3	9.4%	11.1%			
FF5	11.5%	13.7%			
FF5 + LTREV + RVAR	13.5%	15.7%			
FF5 + LTREV + RVAR + QMJ + BAB	14.8%	16.8%			

B. Lead-lag relationships between systematic and idiosyncratic stock returns

			Depender	it variable		
				matic	Idiosyncratic	
Independent	Stock return, $r_{t+1}$		component, $\hat{r}_{t+1}^s$		component, $\hat{r}_{t+1}^e$	
variable	(1)	(2)	(3)	(4)	(5)	(6)
$\overline{r_t}$	-0.02		-0.04		0.00	
	(-4.87)		(-7.54)		(1.55)	
$\hat{r}_t^s$		-0.10		0.22		-0.19
·		(-7.88)		(13.41)		(-25.65)
$\hat{r}_t^e$		0.00		-0.10		0.06
·		(1.39)		(-29.75)		(19.70)
Avg. $N$	1,584.0	1,584.0	1,584.0	1,584.0	1,584.0	1,584.0
Avg. $R^2$	1.3%	4.2%	3.8%	20.3%	0.7%	3.9%

predicts (1) a high value of the systematic component also in month t+1 and (2) a low value of the residual component. What do these estimates tell us about stock returns and, given these values, why do *total* stock returns display reversals? Consider two cases:

1. If the total stock return in month t is high because of the systematic component, the stock's systematic component in month t+1 is, on average, high and its residual component low. Now, because most of the variation in stock returns is due to the residual component—this is the point of panel A—the total return in month t+1 is, on average, negative. Column 2 shows this net

- effect: the systematic component negatively predicts stock returns with a t-value of -7.88.
- 2. If the total stock return in month t is high because of the residual component, the stock's residual component in month t+1 is, on average, high and its systematic component low. These two effects approximately offset each other: most of the variation is due to the residual component but the residual component correlates more with the systematic component than itself. Because of these offsetting effects, the t-value associated with the residual component in column 2 is just 1.39.

The average of these two effects is negative, which is why the lagged stock return in column 1 predicts stock returns with a t-value of -4.87.

Table 12 indicates that the negative serial correlation in stock returns is about the systematic components negatively predicting the idiosyncratic components. That is, short-term reversals are not about reversals in the idiosyncratic component itself; the idiosyncratic component, if anything, is positively autocorrelated as well. This pattern is consistent with Lewellen's 2002 notion of excess covariance in stock returns. Adapting Lewellen's behavioral explanation for the pattern in panel B, the estimates suggest that stocks move "too much" with the factors. If we express this pattern in terms of characteristics, it is about, for example, small growth stocks moving too much in lockstep with other small growth stocks, perhaps because investors place too much weight on news about similar stocks (Lewellen 2002). This excess comovement must later correct itself, which means that the systematic component negatively predicts the idiosyncratic component. In short, short-term reversals in stock returns occur because (1) momentum is present in factor returns and (2) stocks comove too much (Lewellen 2002).<sup>25</sup>

The estimates in Table 12 explain why industries and factors display short-term momentum even though their building blocks, individual stocks, display short-term reversals. The reversals emanate from the dynamics between the stocks' idiosyncratic components and factor returns. Because industries and factors are well-diversified portfolios, they are clear of the idiosyncratic components. Only the positive autocovariances in the stocks' systematic components remain, and so both industries and factors display momentum. In the Internet Appendix, we construct momentum strategies that isolate and trade the systematic components of stock returns. This strategy is very profitable

Lewellen (2002) notes that his behavioral explanation for excess comovement is observationally equivalent to a rational model in which stocks' discount-rate and cashflow betas correlate positively. Put differently, the patterns in Table 12 by themselves do not circumnavigate the joint hypothesis problem; they cannot be construed as evidence that either factor momentum or short-term reversals are due to mispricing.

and, similar to the systematic-industries result in Table 8, also spans industry momentum.

### 7. Conclusions

Momentum is present not only in equities (Jegadeesh and Titman 1993) but also in other asset classes as well (Asness, Moskowitz, and Pedersen 2013). Moskowitz and Grinblatt (1999) show that industries also have momentum. This momentum, unlike that in stock returns, is particularly strong at the one-month horizon, and only this one-month effect is distinct from stock momentum (Asness, Porter, and Stevens 2000).

In this paper we first show that factors exhibit short-term momentum similar to industry momentum: both are fully unrelated to individual stock momentum and, if anything, both grow stronger when controlling for the five-factor model. This form of factor momentum is also distinct from the time-series factor momentum of Ehsani and Linnainmaa (2022).

We show that the similarities of industry and factor momentum are not coincidental: industries appear to exhibit momentum because of factor momentum. The transmission mechanism is the variation in industries' factor loadings. An industry with a high past return, on average, loads positively on factors with positive past returns and negatively on those with negative returns. By buying industries with high returns and selling those with low returns, industry momentum strategy implicitly bets on the continuation in factor returns.

We first show that factor momentum subsumes industry momentum and does so because of the transmission mechanism we posit. We demonstrate this connection in four different ways:

- 1. Momentum found in *industry-neutral* factors subsumes industry momentum. This result suggests that the momentum in factors drives industry momentum and not vice versa.
- 2. Systematic industries—mimicking portfolios we build from small sets of factors, thereby removing industry-specific returns—exhibit similar amounts of momentum as actual industries. Systematic industry momentum also subsumes industry momentum, which suggests that investors can capture industry momentum without using any industry-specific information.
- 3. Momentum found in the first three highest-eigenvalue PC factors subsumes industry momentum. Because we extract these PC factors from industry-neutral factor returns, the principal components cannot possibly recover any industry-specific information from the factor data.

4. An entirely different approach suggests that industry momentum is unlikely to have anything to do with industries. Many other nonindustry portfolios exhibit similar levels of momentum; all of these characteristic momentum strategies significantly correlate with each other, industry momentum, and factor momentum. And factor momentum subsumes them all.

Why do factors display momentum? Kozak, Nagel, and Santosh (2018) suggest that the absence of near-arbitrage opportunities would guarantee that only "systematic factors" can have pricing effects. The fact that factor momentum subsumes industry momentum and other forms of characteristic momentum would be void of economic content if the factors necessary for doing so were as arbitrary and economically unimportant as individual stocks or industries. The PC factor results, however, show that momentum concentrates, in its entirety, precisely where the lack of near-arbitrage opportunities argument says it should: among the highest-eigenvalue PC factors. Controlling for the momentum found among these, by definition, the most important factors, the other PC factors display no momentum of their own. Momentum therefore exists only where it can exist: in places where arbitrageurs find it risky to trade it. One explanation for factor momentum is the mispricing model of Kozak, Nagel, and Santosh (2018) coupled with highly persistent sentiment (Ehsani and Linnainmaa 2022).

We reconcile the existence of factor momentum with the presence of short-term reversals by decomposing stock returns into systematic and idiosyncratic components. Stocks do not display reversals because the idiosyncratic components reverse; rather, the reversals emanate from the negative cross-serial correlation between stocks' systematic and idiosyncratic components. This pattern in the data is consistent with an excess comovement mechanism: stocks comove too strongly with similar stocks (i.e., those with similar characteristics and factor loadings), and when this mispricing corrects, it registers as a reversal through the idiosyncratic component. Although this pattern is consistent with a behavioral mechanism—investors overweight information about similar stocks—it is observationally equivalent to a rational pricing model. A further study of the lead-lag relationship between stocks' systematic and idiosyncratic components could provide additional clues about the economic mechanism that drives it.

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