PDAML Report: Parameter Estimation

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Description

This checkpoint is based around a particle decay from which you will measure a parameter related to the matter/anti-matter asymmetry of the Universe.

This relevant decay $X \to D$ has follows the following PDF:

$$P(t; V, \tau, \Delta m_s) \propto (1 + V \sin(\Delta m t)) \times \exp(-t/\tau)$$

where

- t is the observable quantity - the decay time of each decay;
- τ is a lifetime parameter
- V is a parameter which measures matter/anti-matter asymmetry and has the value zero if the universe is symmetric (which we know it isnt !)
- Δm is a mass difference parameter which leads to sinusoidal oscillations superimposed on the exponential decay

The nominal values of the parameters are

- $\tau = 1.5$
- V = 0.1
- $\Delta m = 20.0$

Part 1: Estimating statistical precision [4 marks]

Use the method of pseudo-experiments (toy Monte Carlo of week 3 semester 1) to determine the expected statistical precision with which one could measure each of the parameters with (i) 10,000 events and (ii) 100,000 events. Assume perfect detector-resolution/perfect time measurements.

Part 2: Estimating possible bias due to time resolution [3 marks]

In reality the decay time is measured with a resolution (random error) with a standard deviation of of $\sigma = f\tau$ where f is some fraction. What this means is that if the true decay time is t_{true}, then it is actually measured as t where t is distributed around t_{true} with a Gaussian probability distribution with standard deviation σ .

Determine the bias which will be introduced to the measurement of each of the parameters, when the data is subject to the resolution effect, but this is not included into the PDF used for fitting (measuring) the parameters.

You should do this for the case of 10,000 event data samples. Use both $f = 0.01$ and $f = 0.03$ and in each case compare the bias (if any) to the expected statistical precision.

Hint: this means producing monte-carlo data where you include the effect of resolution, but then fitting to it without allowing for this in the PDF

Part 3: Estimating a systematic error due to time acceptance [3 marks]

The method of measuring decay-time (i.e. in some detector) is thought to exhibit a decay-time acceptance given by

$$a(t) = (1 + st)$$

where s is only known with a precision of $s = 0 \pm 0.03$

Determine a suitable systematic error to assign to the measurement of each of the parameters due this this limited knowledge of $a(t)$ and in each case compare the this systematic error (if any) to the expected statistical precision.