mcmc

June 21, 2017

```
In [1]: from __future__ import division, print_function
        import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline
```

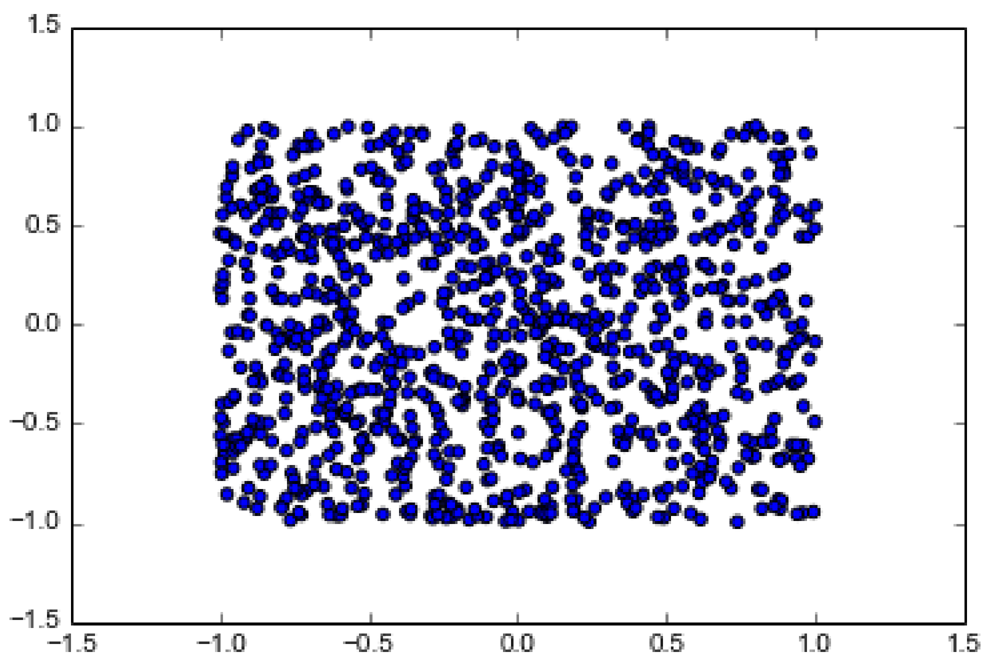
1 Monte Carlo Simulations

1.1 Calculating Pi

```
In [2]: def random_number_plusminus1(n):
            return 2*np.random.random(n) - 1

        x, y = random_number_plusminus1((2,1000))
```

```
In [3]: plt.scatter(x, y)
        plt.show()
```



```
In [4]: area_of_square = 2*2
        ratio_of_dart_inside = np.mean(x**2 + y**2 < 1)
        pi_estimate = area_of_square * ratio_of_dart_inside
        print(pi_estimate, np.pi)
```

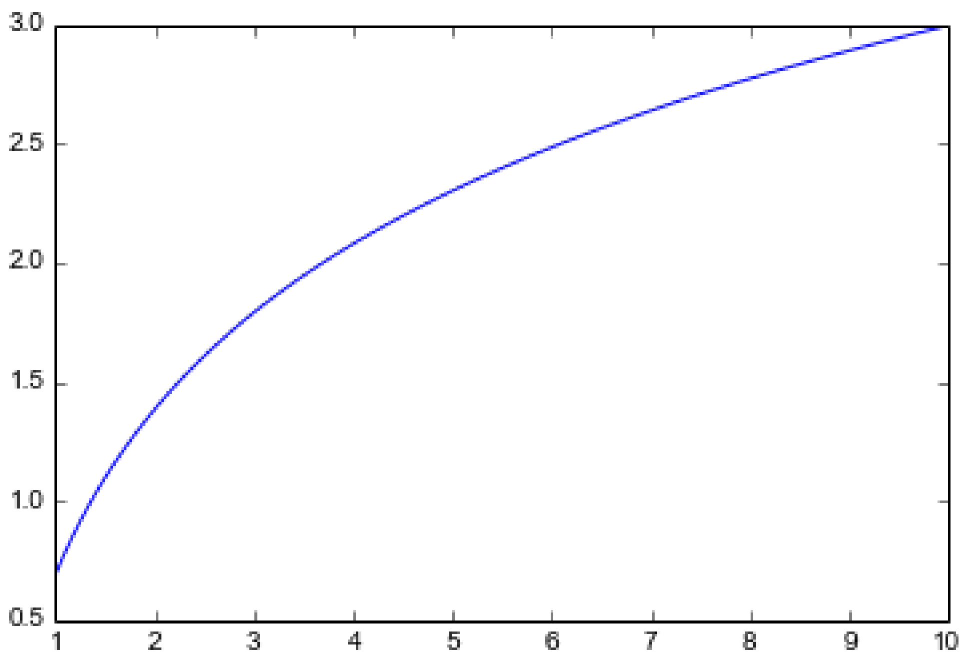
```
3.156 3.14159265359
```

```
In [5]: x, y = random_number_plusminus1((2,10000000))
        area_of_square = 2*2
        ratio_of_dart_inside = np.mean(x**2 + y**2 < 1)
        pi_estimate = area_of_square * ratio_of_dart_inside
        print(pi_estimate, np.pi)
```

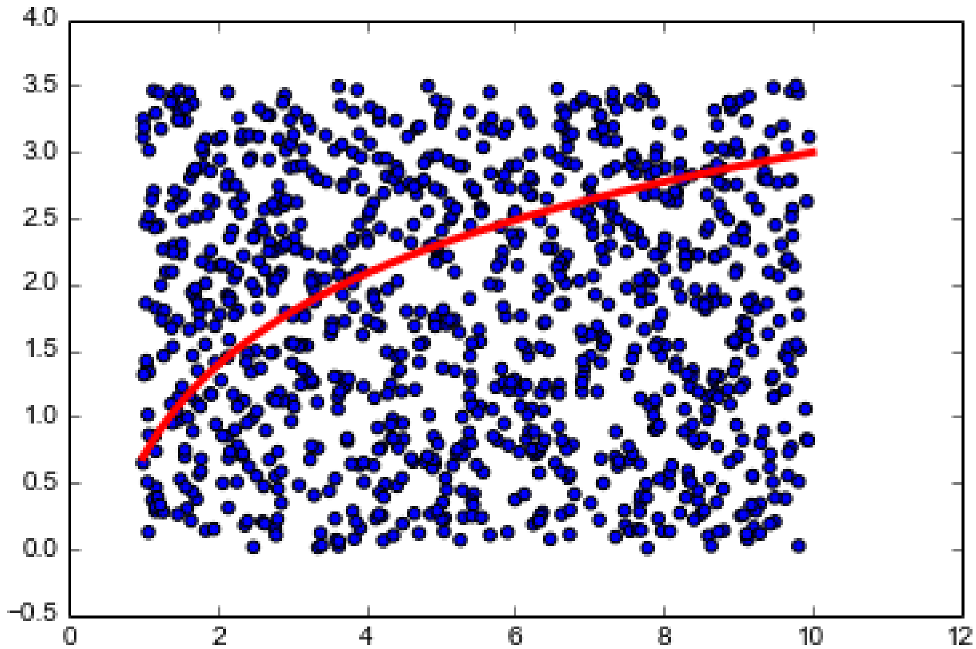
```
3.1426696 3.14159265359
```

1.2 Calculating an integral

```
In [6]: def f(x):
            return np.log(2*x) # Integral from 1 to 10 is 20.264
        x = np.linspace(1,10,1000)
        plt.plot(x, f(x))
        plt.show()
```



In [7]:
```
n = 1000
x_draw = 1 + 9*np.random.random(n)
y_draw = 3.5 * np.random.random(n)
plt.scatter(x_draw, y_draw)
plt.plot(x, f(x), 'r', lw=3)
plt.show()
```



In [8]:
```
area_square = 3.5*9
ratio_inside = np.mean(y_draw < f(x_draw))
integral = area_square * ratio_inside
print(integral)
```

```
19.719
```

In [9]:
```
def calc_intergal(n):
    x_draw = 1 + 9*np.random.random(n)
    y_draw = 3.5 * np.random.random(n)
    ratio_inside = np.mean(y_draw < f(x_draw))
    return area_square * ratio_inside

estimates = [calc_intergal(100000) for i in range(100)]
print(np.mean(estimates), '+-', np.std(estimates)/np.sqrt(100))
```

```
20.26510605 +- 0.00501253800948
```

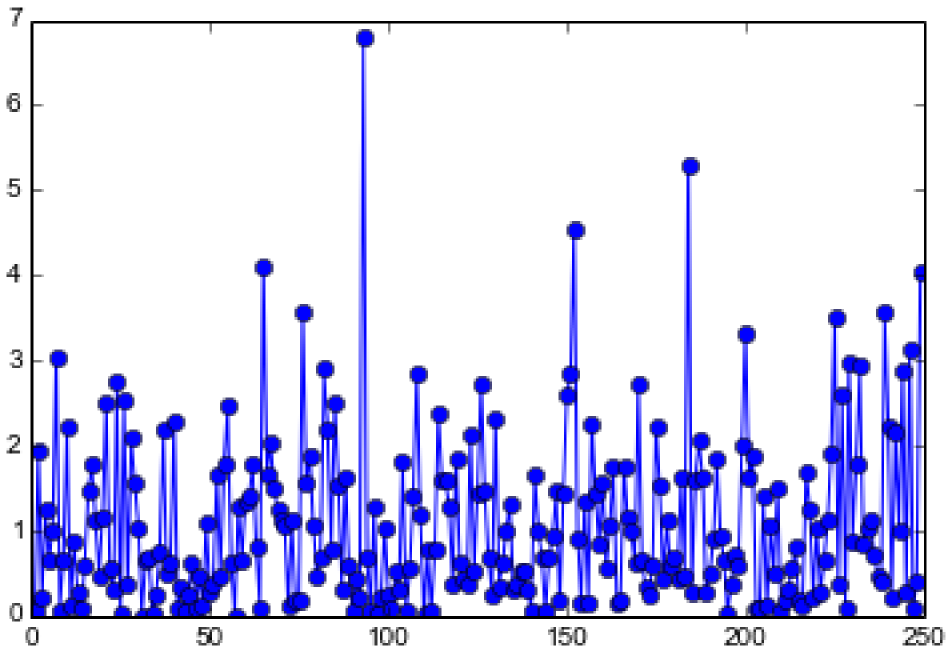
1.3 Drawing random numbers

Numpy has tons of random number functions. See https://docs.scipy.org/doc/numpy/reference/routines.rand
But it doesn't have everything.

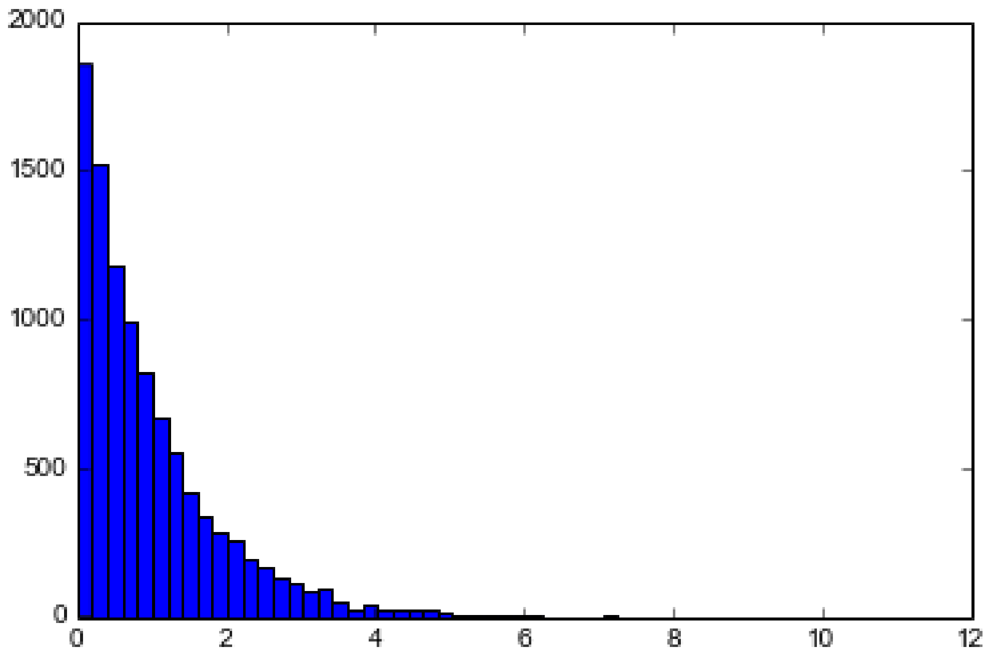
It does have the exponential distributions, but let's try and make it ourselves.

```
In [10]: def exponential_numbers(a, n):
             u = np.random.random(n)
             return -1/a * np.log(u) # inverse method
```

```
In [11]: x = exponential_numbers(1, 250)
         plt.plot(x, '-o')
         plt.show()
```



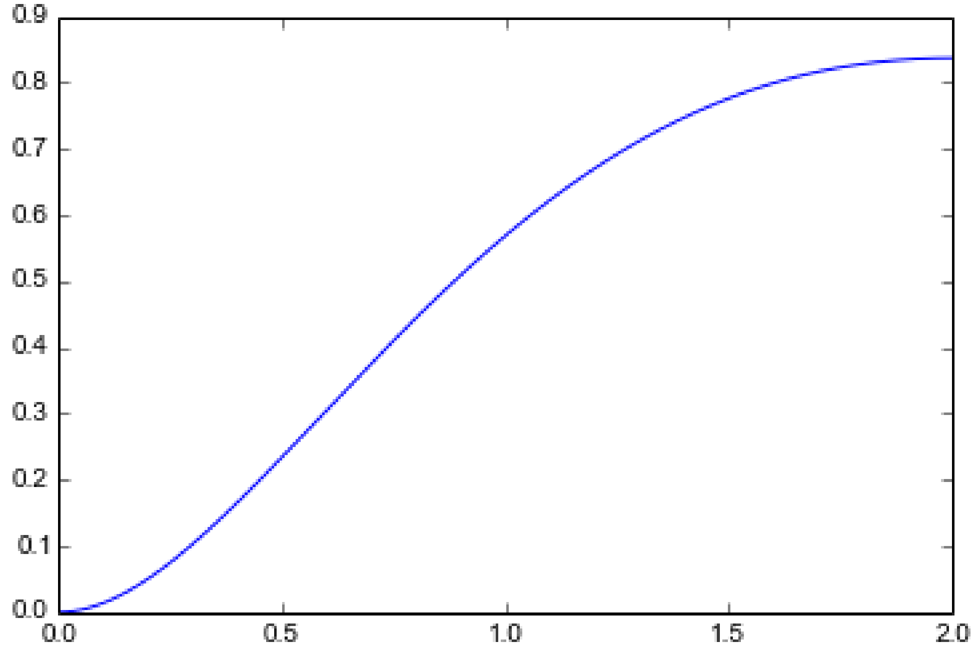

```
In [12]: x = exponential_numbers(1, 10000)
         plt.hist(x, bins=50)
         plt.show()
```



1.4 Hit-miss

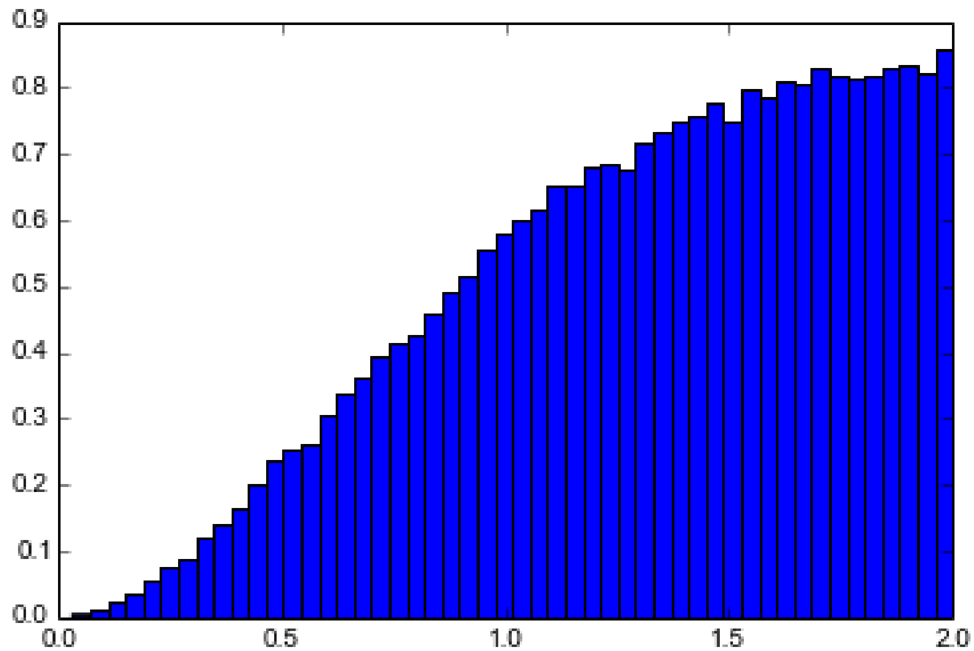
Now let draw from a weird distrubtion:

```
In [13]: def f(x):
             return np.exp(-x) * x**2/(2-10*np.exp(-2))
         x = np.linspace(0, 2, 10000)
         plt.plot(x, f(x))
         plt.show()
```



In [14]:
```
def draw_random_number(f, minx, maxx, maxy):
    while True:
        x = minx + (maxx - minx) * np.random.random()
        y = maxy * np.random.random()
        if f(x) > y:
            return x
```

In [15]:
```
x = [draw_random_number(f, 0, 2, 1) for i in range(100000)]
plt.hist(x, bins=50, normed=True)
plt.show()
```



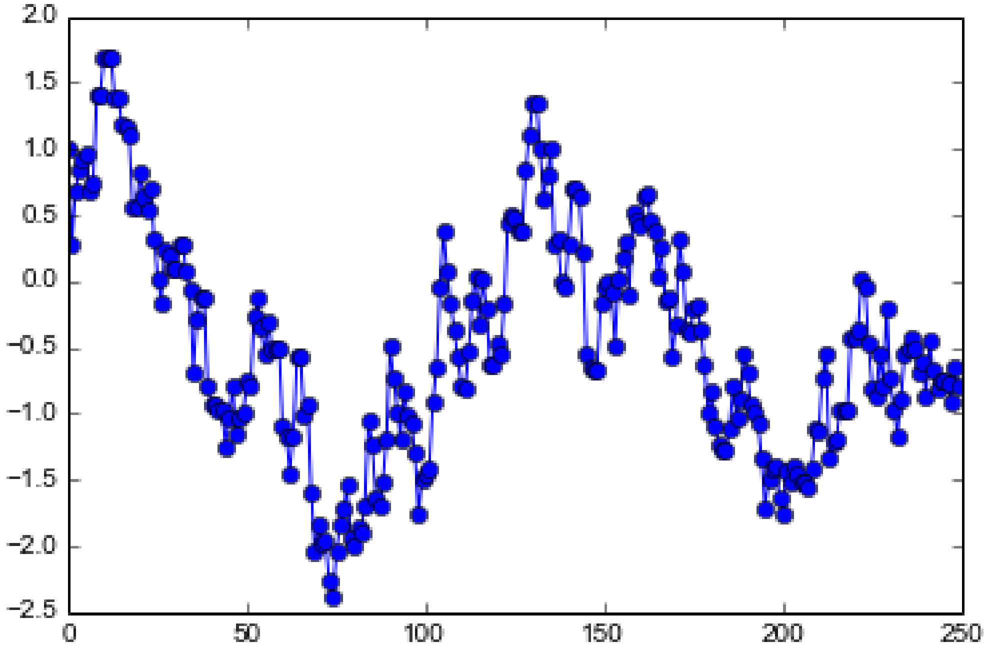
1.5 Markov Chain Monte Carlo: Metropolis hastings

We give up the requirement that samples are independent.

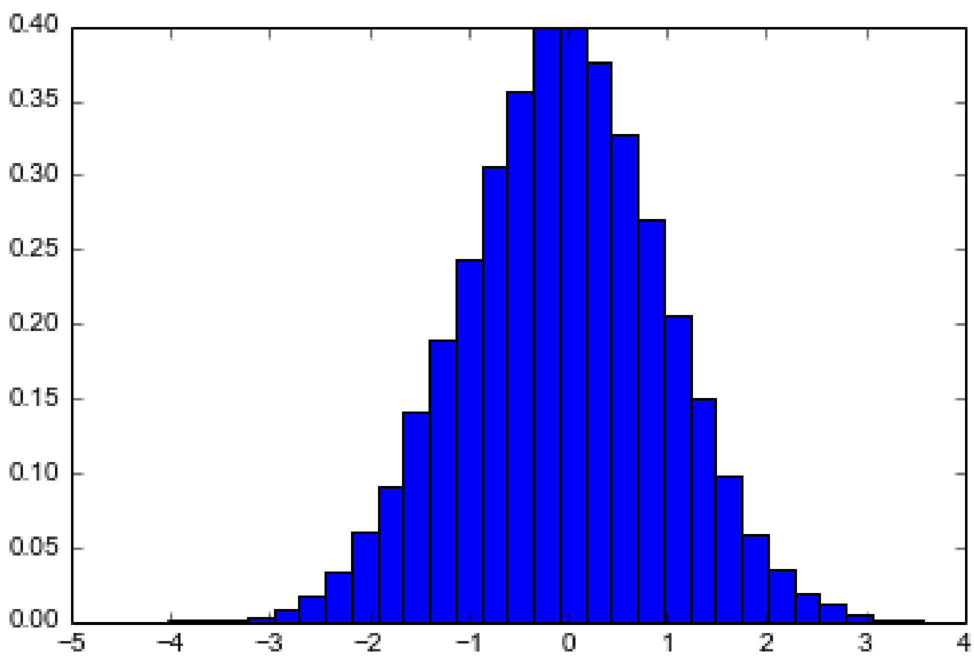
```
In [16]: def metropolis(f, x0, n=1000, std=0.3):
             current = f(x0)
             x = [x0]
             for i in range(1, n):
                 xn = x0 + std * np.random.randn()
                 new = f(xn)
                 if np.random.random() < new/current:
                     x0 = xn
                     current = new
                 x.append(x0)
             return x
```

```
In [17]: gauss = lambda x : np.exp(-x**2/2)
         exp = lambda x : np.exp(-x) * (x>=0)
```

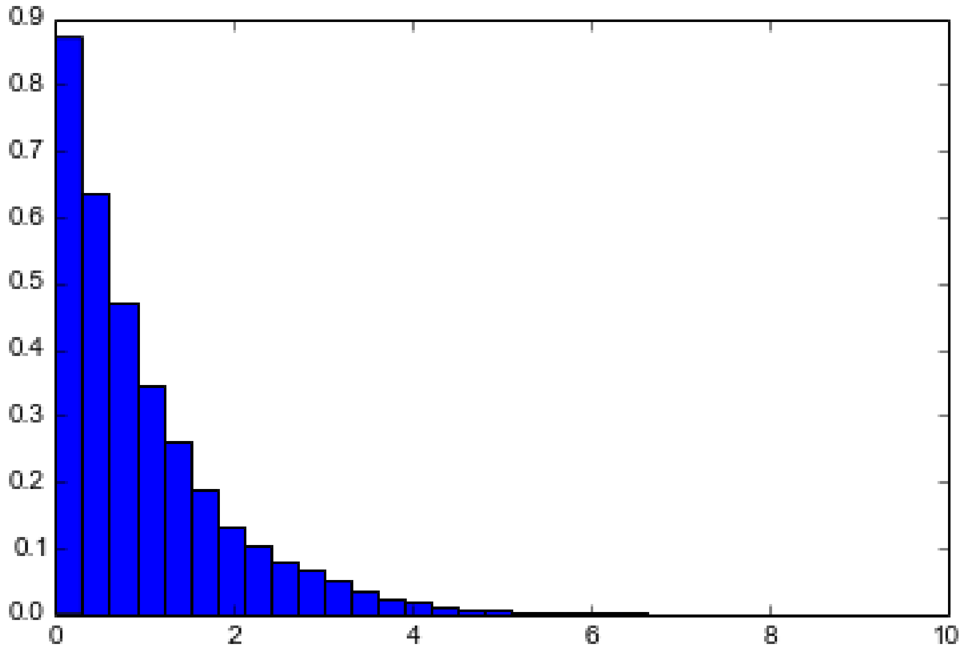
```
In [18]: x = metropolis(gauss, 1, 250)
         plt.plot(x, '-o')
         plt.show()
```




```
In [19]: x = metropolis(gauss, 1, 100000)
         plt.hist(x, bins=30, normed=True)
         plt.show()
```



```
In [20]: x = metropolis(exp, 1, 100000)
         plt.hist(x, bins=30, normed=True)
         plt.show()
```



1.6 2D MCMC Visualised

```
In [ ]: def some_2d_distribution(x, y): # doesn't have to be normalised
            return x**2*np.exp(-y**2) * (x>=0) * (x<=10) * (y>=-5) * (y<=5)

        X, Y = np.meshgrid(np.linspace(0,10,50), np.linspace(-5,5,50))
        d = some_2d_distribution(X, Y)
        plt.imshow(d, extent=(np.min(X),np.max(X),np.max(Y),np.min(Y)))
        plt.show()
```

```
In [ ]: def metropolis(f, x0, y0, n=1000, std=1.0):
            current = f(x0, y0)
            x = [x0]
            y = [y0]
            plt.ion()
            for i in range(1, n):
                xn = x0 + std * np.random.randn()
                yn = y0 + std * np.random.randn()
                new = f(xn, yn)
                if np.random.random() < new/current:
                    x0 = xn
```

```
                y0 = yn
                current = new
            x.append(x0)
            y.append(y0)
            plt.clf()
            plt.plot(x, y)
            plt.axis([0,10,-5,5])
            plt.pause(0.001)
        return x
```

In [ ]:
```
%matplotlib
metropolis(some_2d_distribution, 0, 0)
```

In [ ]: