

Soal 1 : $A = \begin{pmatrix} 6 & -8 \\ 0 & 8 \end{pmatrix} \Rightarrow \det A = \begin{vmatrix} 6 & -8 \\ 0 & 8 \end{vmatrix} = 6 \cdot 8 - 0 \cdot (-8) = 48$

Zadanie 2: $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 4 & 1 \\ 1 & -1 & 6 \end{pmatrix} \Rightarrow \det A = \begin{vmatrix} 2 & 1 & 3 & 2 & 1 \\ 4 & 4 & 1 & 4 & 4 \\ 1 & -1 & 6 & 1 & -2 \end{vmatrix} = \begin{matrix} 2 \cdot 4 \cdot 6 + 1 \cdot 1 \cdot 1 + 3 \cdot 4 \cdot (-1) \\ - 1 \cdot 4 \cdot 3 - (-1) \cdot 1 \cdot 2 \\ - 6 \cdot 4 \cdot 1 \end{matrix} = 3$

Zagata 3:

$$\begin{vmatrix} -2 & 1 & -2 & 2 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & -3 & -2 \\ 0 & 0 & 0 & 2 \end{vmatrix} = (-2) \cdot (-2) \cdot (-3) \cdot 2 = -24$$

Задача 4:

$$\begin{vmatrix} 0 & 0 & 4 & 4 \\ 0 & 2 & 0 & 3 \\ 2 & -5 & 2 & 0 \\ 0 & 0 & -4 & 1 \end{vmatrix} = - \begin{vmatrix} 2 & -5 & 2 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & -4 & 1 \end{vmatrix} = - \begin{vmatrix} 2 & -5 & 2 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 \end{vmatrix} = -(2 \cdot 2 \cdot 4 \cdot 5) = -80$$

Jagoras:

$$\begin{vmatrix} 2 & 0 & 1 & -3 \\ 6 & -2 & 3 & -10 \\ -12 & 6 & -4 & 21 \\ -60 & 30 & -22 & 107 \end{vmatrix} = \sum_{j=0}^4 (-1)^{1+j} \cdot a_{1j} \cdot M_{1j} = (-1)^{1+1} \cdot 2 \cdot M_{11} + (-1)^{1+2} \cdot 0 \cdot M_{12} + (-1)^{1+3} \cdot 1 \cdot M_{13} + (-1)^{1+4} \cdot (-3) \cdot M_{14}$$

$$M_{11} = \begin{vmatrix} \cancel{2} & \cancel{0} & \cancel{1} & \cancel{-3} \\ 6 & -2 & 3 & -10 \\ -12 & 6 & -4 & 21 \\ -60 & 30 & -22 & 107 \end{vmatrix} = \begin{vmatrix} -2 & 3 & -10 & -2 & 3 \\ 6 & -4 & 21 & 6 & -4 \\ 30 & -22 & 107 & 30 & -22 \end{vmatrix} = (-2)(-4) \cdot 107 + 3 \cdot 21 \cdot 30 + (-10) \cdot 6 \cdot (-22) - 30 \cdot (-4) \cdot (-10) - (-22) \cdot 21 \cdot (-62) - 107 \cdot 6 \cdot 3 = 16$$

$$M_{13} = \begin{vmatrix} \cancel{2} & \cancel{0} & \cancel{1} & \cancel{-3} \\ 6 & -2 & 3 & -10 \\ -12 & 6 & -4 & 21 \\ -60 & 30 & -22 & 107 \end{vmatrix} = \begin{vmatrix} 6 & -2 & -10 & 6 & -2 \\ -12 & 6 & 21 & -12 & 6 \\ -60 & 30 & 107 & -60 & 30 \end{vmatrix} = 6 \cdot 6 \cdot 107 + (-2) \cdot 21 \cdot (-60) + (-10) \cdot (-12) \cdot 30 - (-12) \cdot 30 - (-60) \cdot 6 \cdot (-10) - 30 \cdot 21 \cdot 6 - 107 \cdot (-12) \cdot (-2) = 24$$

$$M_{14} = \begin{vmatrix} \cancel{2} & \cancel{0} & \cancel{1} & \cancel{-3} \\ 6 & -2 & 3 & -10 \\ -12 & 6 & -4 & 21 \\ -60 & 30 & -22 & 107 \end{vmatrix} = \begin{vmatrix} 6 & -2 & 3 & 6 & -2 \\ -12 & 6 & -4 & -12 & 6 \\ -60 & 30 & -22 & -60 & 30 \end{vmatrix} = 6 \cdot 6 \cdot (-22) + (-2) \cdot (-4) \cdot (-60) + 3 \cdot (-12) \cdot 30 - (-60 \cdot 6 \cdot 3) - 30 \cdot (-4) \cdot 6 - (-22) \cdot (-12) \cdot (-2) = -24$$

$$\Rightarrow \det A = (-1)^2 \cdot 2 \cdot 16 + 0 + (-1)^4 \cdot 1 \cdot 24 + (-1)^5 \cdot (-3) \cdot (-24) = -16$$

Задача 6:

$$\begin{vmatrix} \mu & -5 & -3 \\ -2 & \mu & 3 \\ 5 & 5 & 5 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} \mu & -5 & -3 & \mu & -5 \\ -2 & \mu & 3 & -2 & \mu \\ 5 & 5 & 3 & 5 & 5 \end{vmatrix} = 3\mu^2 - 75 + 10 + 15\mu - 15\mu - 30 = 3\mu^2 - 75.$$

$$\Leftrightarrow \mu_1 = 5$$

$$\mu_2 = -5$$

Sagawa 7.

$$\det A_{11} = \begin{vmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & -1 \\ 2 & 1 & -3 & 5 \\ 4 & -2 & 6 & -5 \end{vmatrix} = \begin{vmatrix} 0 & 1 & -2 & 6 & 1 \\ 1 & -2 & -1 & 1 & -2 \\ -2 & 6 & -5 & -2 & 6 \end{vmatrix} = \begin{matrix} 2 & -12 & 18 & -5 \\ 2 & 3 \end{matrix}$$

Soal 8:

$$\begin{cases} x_1 = -1 \\ x_1 + x_2 + x_3 = -3 \\ -2x_1 - x_2 = 3 \end{cases} ; A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -2 & -1 & 0 \end{pmatrix} \quad b = \begin{pmatrix} -1 \\ -3 \\ 3 \end{pmatrix}$$

$$A \cdot X = b \Rightarrow X = A^{-1} \cdot b$$

$$A \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ -2 & -1 & 0 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 2 & 0 & 0 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 3 & -1 & 0 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 3 & -1 & 0 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -3 & 1 & 0 \end{array} \right) \Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ -3 & 1 & 0 \end{pmatrix}$$

$$\therefore X = A^{-1} \cdot b = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ -3 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -3 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

Задача 9:

$$\begin{cases} x_1 + x_3 = 0 \\ -x_1 + x_2 - 3x_3 = -1 \\ -2x_1 + x_2 - 3x_3 = -1 \end{cases} ; A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -3 \\ -2 & 1 & -3 \end{pmatrix} ; b = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$$

$$\Delta = \det A = \begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ -1 & 1 & -3 & -1 & 1 \\ -2 & 1 & -3 & -2 & 1 \end{vmatrix} = -3 - 1 + 2 + 3 = 1$$

$$\Delta_1 = \begin{vmatrix} 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & -3 & -1 & 1 \\ -1 & 1 & -3 & -1 & 1 \end{vmatrix} = 0 + 0 - 1 - 1 - 0 - 0 = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ -1 & 1 & -3 & -1 & 1 \\ -2 & 1 & -3 & -2 & 1 \end{vmatrix} = 3 + 1 + 2 - 3 = 1$$

$$\Delta_3 = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 \\ -1 & 1 & -1 & -1 & 1 \\ -2 & 1 & -1 & -2 & 1 \end{vmatrix} = -1 + 0 + 0 + 0 + 1 = 0$$

$$\Rightarrow x_1 = \frac{\Delta_1}{\Delta} = \frac{0}{1} = 0 ; x_2 = \frac{\Delta_2}{\Delta} = \frac{-1}{1} = -1 ; x_3 = \frac{\Delta_3}{\Delta} = \frac{0}{1} = 0$$

Задача 10.

$$\begin{cases} x_1 + 2x_2 - 2x_3 = 2 \\ 2x_1 + 5x_2 + 5x_3 = -5 \\ x_1 - 2x_2 = 2x_3 = 1 \end{cases}; A = \begin{pmatrix} 1 & -2 & -2 \\ -2 & 5 & 5 \\ 1 & -2 & -1 \end{pmatrix}; b = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$$

$$\tilde{A} = \left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ -2 & 5 & 5 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & -1 & 0 & 2 \\ 0 & 1 & 0 & 3 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 0 & 0 \\ 0 & 1 & 0 & 3 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right)$$

$$\rightarrow A^{-1} = \begin{pmatrix} 5 & 0 & 0 \\ 3 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \Rightarrow X = A^{-1} \cdot b =$$

$$= \begin{pmatrix} 5 & 0 & 0 \\ 3 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ -1 \end{pmatrix}$$