Чу Ван Доан – N3247.

Домашнее задание № 3

N. 1069.

$$\begin{cases} \dot{x} = 4ty^2 & x(0) = 0 \\ \dot{y} = 1 + 5\mu x & y(0) = 0 \end{cases}$$

Найти $\left.\frac{\partial x}{\partial \mu}\right|_{\mu=0}$

*, $\frac{\partial x}{\partial \mu} = u$, $\frac{\partial y}{\partial \mu} = v$.

$$\Rightarrow \begin{cases} \dot{u}(t, \mu) = 8ty \cdot v & u(0, \mu) = 0 \\ \dot{v}(t, \mu) = 5x + 5\mu u & v(0, \mu) = 0 \end{cases}$$

при $\mu = 0$

$$\Rightarrow \begin{cases} \dot{u}(t, \mu) = 8tyv \\ \dot{v}(t, \mu) = 5x \end{cases} \Rightarrow \begin{matrix} u(0,0) = 0 \\ v(0,0) = 0 \end{matrix}$$

$\mu = 0$

$$\Rightarrow \begin{cases} \dot{x} = 4ty^2 & x(0) = 0 \\ \dot{y} = 1 & y(0) = 0 \end{cases}$$

$\frac{dy}{dt} = 1 \Rightarrow y = t + C_1$; $y(0) = C_1 = 0 \Rightarrow y = t$

$\frac{dx}{dt} = 4t^3 \Rightarrow x = t^4 + C_2$; $x(0) = C_2 = 0 \Rightarrow x = t^4$

Отсюда $\begin{cases} \dot u = 8t^2 v \\ \dot v = 5t^4 \end{cases} \Rightarrow \frac{dv}{dt} = 5t^4 \Rightarrow v = t^5 + C_3$

$v(0) = C_3 = 0 \Rightarrow v = t^5$

$\frac{du}{dt} = 8t^7 \Rightarrow u = t^8 + C_4 ;\ u(0) = C_4 = 0$

$\Rightarrow u = t^8 = \frac{\partial x}{\partial \mu}\Big|_{\mu_0} = 0$

<u>№ 1073!</u>

$\ddot x = \frac{2}{t} - \frac{2}{x},\ x(1) = 1,\ \dot x(1) = b,\ \frac{\partial x}{\partial b}\Big|_{b=1} = ?$

$\frac{\partial}{\partial b}\left(\frac{\partial x^2}{\partial t^2}\right) = \frac{\partial^2}{\partial t^2}\left(\frac{\partial x}{\partial b}\right);\ \frac{\partial x}{\partial b} = u$

$\ddot u = \frac{2}{t x^2} \cdot u \qquad u_{t,b}(1,1) = 0;\ \dot u(1,1) = 1$

$b = 1$

$\ddot x = \frac{2}{t} - \frac{2}{x};\ x(1) = 1,\ \dot x(1) = 1$

$\ddot x = 0 \Rightarrow \dot x = C_1 = 1 \Rightarrow x = t$

$\ddot u = \frac{2}{t^2} u \qquad u(1) = 0 \qquad \dot u(1) = 1$

$u = t^\nu$

$\nu(\nu - 1) t^{\nu - 2} = \frac{2}{t^2} t^\nu$

$\Leftrightarrow t^{\nu-2}(\nu^2 - \nu - 2) = 0 \Rightarrow \nu = 2,\ \nu = -1.$

$u_1 = t^2, \; u^2 = t^{-1}$

$u = C_2 u_1 + C_3 u_2 = C_2 t^2 + C_3 t^{-1}$

$\dot{u} = 2C_2 t - C_3 t^{-2}$

$\begin{cases} C_2 + C_3 = 0 \\ 2C_2 - C_3 = 1 \end{cases} \Rightarrow C_2 = \frac{1}{3}; \; C_3 = \frac{-1}{3}.$

$\Rightarrow u = \frac{\partial x}{\partial b} = \frac{1}{3} t^2 - \frac{1}{3} t^{-1}$

(№4) $\frac{\partial^2 u}{\partial y^2} = e^{x+y}$; общ. решение: $u = u(x, y)$.

$u''_{yy} = e^{x+y}$

$u'_y = e^{x+y} + \varphi_1(x)$

$u = e^{x+y} + \varphi_1(x) y + \varphi_2(x)$.

(№5). $\frac{\partial^2 u}{\partial x \partial y} + \frac{1}{x} \frac{\partial u}{\partial y} = 0$, общ. реш. уравн. $u = u(x, y)$

$u''_{xy} + \frac{1}{x} u'_y = 0$

$u''_{xy} = -\frac{1}{x} u'_y$

$(\varphi_1(y) \cdot \frac{1}{x} + \varphi_2(x))'_y = (\varphi_1(y) \cdot \frac{1}{x} + \varphi_2(x))''_{xy}$

$= (\varphi'_1(y) \cdot \frac{1}{x})'_x = \varphi_1'(y) \cdot (-\frac{1}{x^2})$

$\Rightarrow u(x,y) = \varphi_1(y)\cdot\frac{1}{x} + \varphi_2(x).$

№11.26.

$$\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x\partial y} + 13\frac{\partial^2 u}{\partial y^2} = 0$$

$A = 1,\ B = 3,\ C = 13$

$D = B^2 - AC = -4 < 0 \Rightarrow$ эллипт. тип.

$$\frac{dy}{dx} = \frac{B \pm \sqrt{D}}{A} = -3 \pm 2i$$

$dy = (-3+2i)\,dx \qquad dy = (-3-2i)\,dx$

$y = (-3+2i)x + C \qquad y = (-3-2i)x + C$

$C = y + 3x \mp 2i\,x.$

$\alpha(x,y) = y + 3x, \quad \beta(x,y) = 2x$

$\begin{cases} \xi = y + 3x \\ \eta = 2x \end{cases} \qquad u(x,y) = v(\xi, \eta)$

$u'_x = v_\xi \xi'_x + v_\eta \eta'_x = 3v'_\xi + 2v'_\eta$

$u'_y = v_\xi \xi'_y + v_\eta \eta'_y = v'_\xi$

$u''_{xx} = (3v'_\xi + 2v'_\eta)'_x = 3v''_{\xi\xi}\cdot\xi'_x + 3v''_{\xi\eta}$

$\cdot\ \eta'_x + 2V''_{\eta\xi} + \xi'_x + 2V''_{xx} = 0$

$\Rightarrow n'_x = 9V''_{\xi\xi} + 6V''_{\xi\eta} + 6V''_{\eta\xi} + 4V''_{\eta\eta}$

$= 9V''_{\xi\xi} + 12V''_{\eta\xi} + 4V''_{\eta\eta}$

$\Rightarrow u''_{xy} = u'_{yx} = (V'_{\xi})'_x = V'_{\xi\xi} \cdot \xi'_x + V''_{\xi\eta} n'_x$

$= 3V''_{\xi\xi} + 2V_{\xi\eta}$

$\Rightarrow u''_{yy} = (V'_{\xi})'_y = V''_{\xi\xi} \cdot \xi'_y + V''_{\xi\eta} \cdot \eta'_y$

$= \cancel{u_y} = V''_{\xi\xi}(9V''_{\xi\xi} + 12V''_{\eta\xi} + 4V''_{\eta\eta})$

$- 6(3V''_{\xi\xi} + 2V''_{\xi\eta}) \cdot 13V''_{\xi\xi} = 4V''_{\xi\xi} + 4V''_{\eta\eta} = 0$

$= 0$

$\Rightarrow V''_{\xi\xi} + V''_{\eta\eta} = 0.$

N. 167.

$y' = \frac{y}{x} + \mu x e^{-y}\ (x > 0),\ y(1) = 1 + 2\mu$

$y'_\mu |_{\mu=0}$!

$u = y'_\mu,\ \mu = 0$

$u' = \frac{u}{x} + x e^{-y}\ ;\ u(1) = 2$

$\mu = 0$

$$\begin{cases} y' = \frac{y}{x} \\ y(1) = 1 \end{cases} \Rightarrow \frac{dy}{dx} = \frac{y}{x} \Rightarrow \ln y = \ln x + C \Rightarrow y = Cx.$$

$$C = 1 \Rightarrow y = x.$$

$$u' = \frac{u}{x} + x e^{-x}$$

$$u = C_1(x) \cdot x \Rightarrow u' = C_1'(x) x + C_1(x)$$

$$C_1'(x) x + C_1(x) = C_1(x) + x e^{-x}$$

$$C_1'(x) = e^{-x}$$

$$C_1(x) = -e^{-x} + C_2; \quad u = (-e^{-x} + C_2) \cdot x$$

$$2 = -e^{-1} + C_2 \Rightarrow C_2 = 2 + \frac{1}{e}.$$

$$u = (-e^{-x} + e^{-1} + 2) x.$$

N 188.

$$y' = y - x + \mu x e^2 y; \quad y(1) = 2 - \mu, \quad y'_\mu|_{\mu=0} - ?$$

$$u = y'_\mu \Rightarrow u' = u + x e^{2y}, \quad u(1) = -1, \quad \mu = 0.$$

$$\begin{cases} y' = y - x \\ y(1) = 2 \end{cases} \Rightarrow \begin{aligned} v &= y - x \\ v' &= y' - 1 \end{aligned}$$

$$\Rightarrow v' + 1 = v \Rightarrow \frac{dv}{dx} = v - 1.$$

$$\Rightarrow \ln(v - 1) = x + C \Rightarrow v - 1 = e^{x + C}$$

$$y = x + 1 + Ce^{x + C} \Rightarrow C = 0, \quad y = x + 1$$

$$u' = u + x e^{x+2}$$

$$u = C_1(x) e^x \quad , \quad u' = C_1'(x) e^x + C_1(x) e^x$$

$$C_1'(x) e^x + C_1(x) e^x = C_1(x) e^x + x e^{2x+2} ;$$

$$C_1'(x) = x e^{x+2}$$

$$C_1(x) = e^{x+2} x - e^{x+2} + C_2$$

$$u = (e^{x+2} x - e^{x+2} + C_2) e^x$$

$$-1 = (e^3 - e^3 + C_2) e \qquad C_2 = -\frac{1}{e} .$$

$$\Rightarrow u = \left(e^{x+2} x - e^{x+2} - \frac{1}{e} \right) e^x$$