Lecture #5: Deep Learning and Inverse Problems in Geoscience

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Advanced course on Deep Learning and Geophysical Dynamics

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Inverse Problems in Geoscience

Mathematical formulation for inverse Problems

Inverse problems & Deep learning

Applications to geophysical dynamics

Inverse Problems in Geoscience

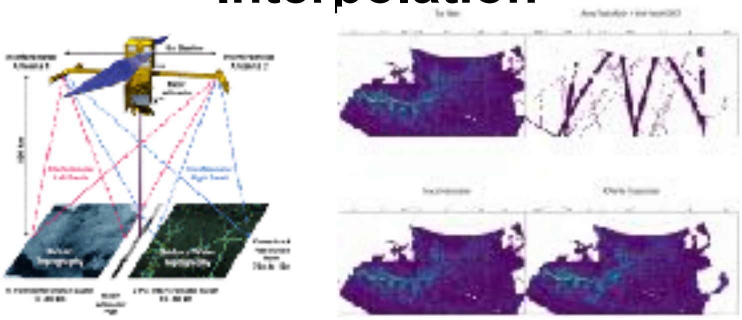
Mathematical formulations for inverse Problems

Inverse problems as learning problems

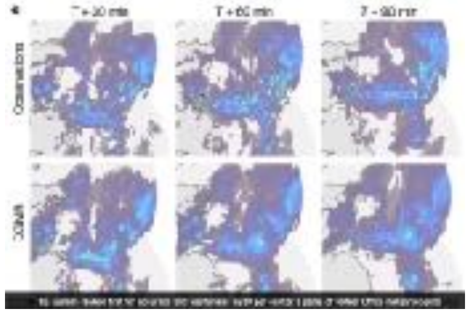
Applications to geophysical dynamics

Inverse Problems in Geoscience: some examples

Interpolation

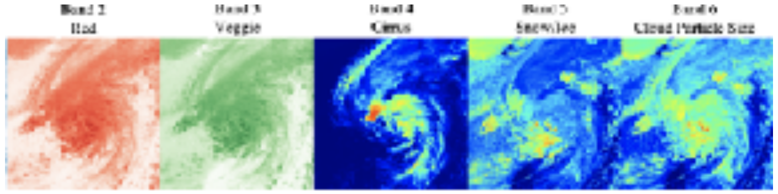


Obs.-driven Forecasting



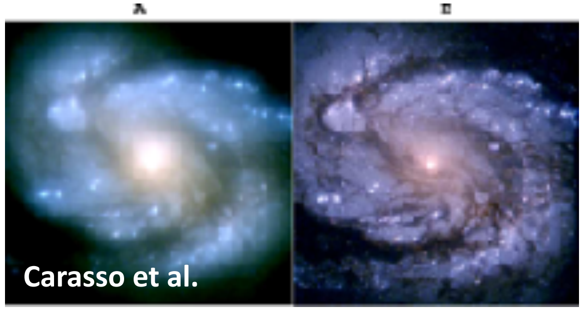
Deepmind

Multimodal fusion

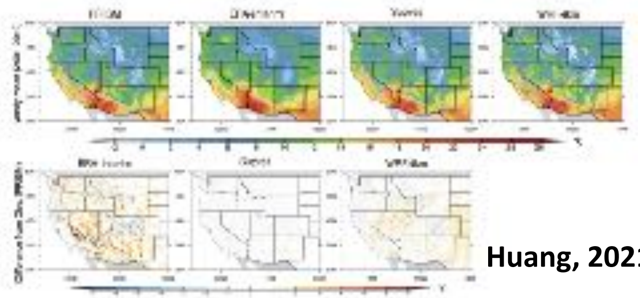


Vandal et al.

Deconvolution



Downscaling

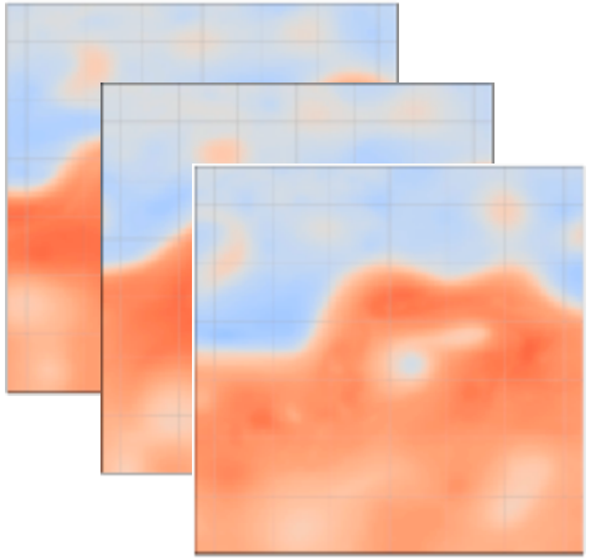


Huang, 2021

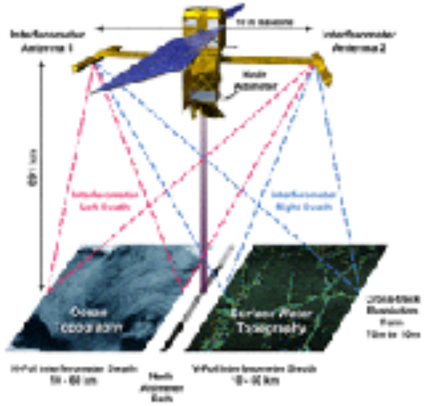
....

Inverse Problems in Geoscience: Generic formulation

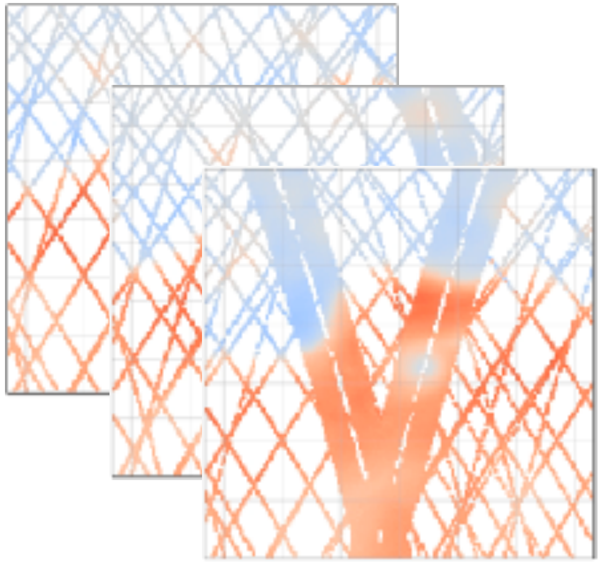
State



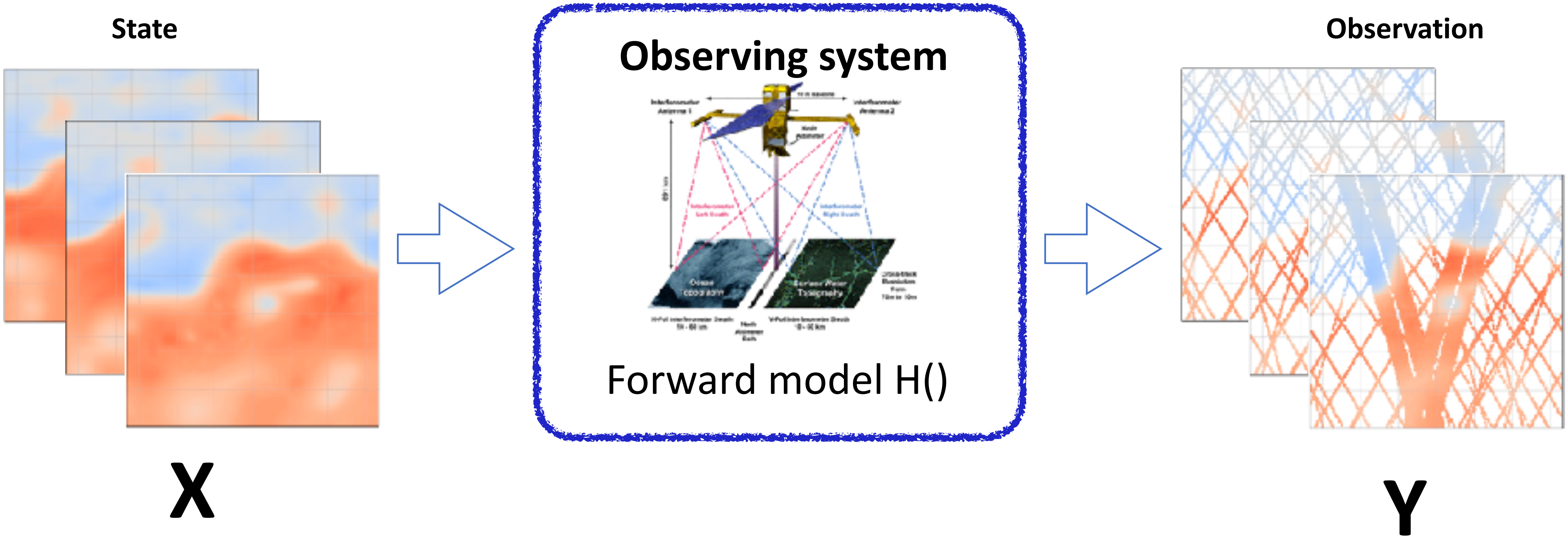
Observing system



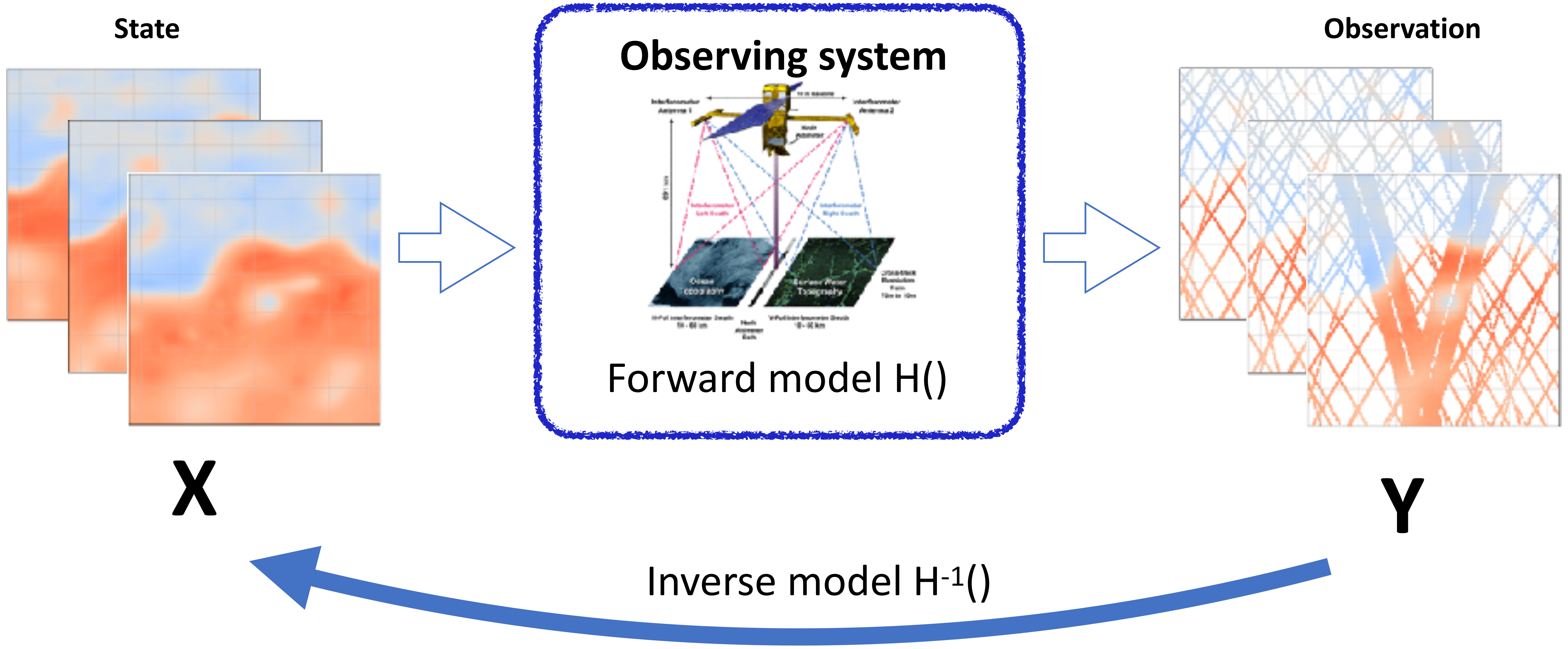
Observation



Inverse Problems in Geoscience: Generic formulation

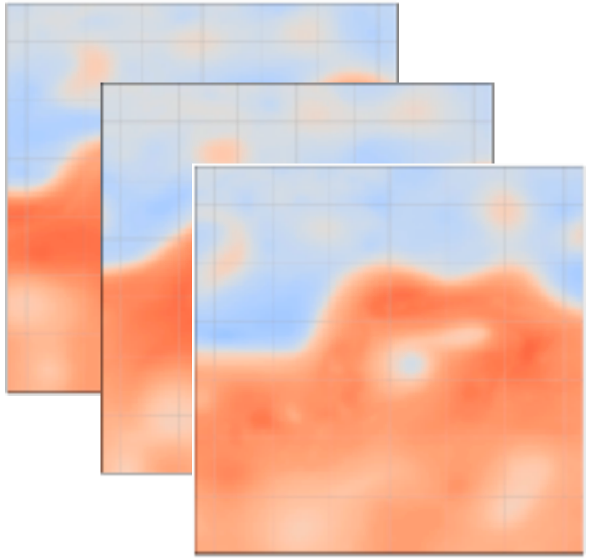


Inverse Problems in Geoscience: Generic formulation

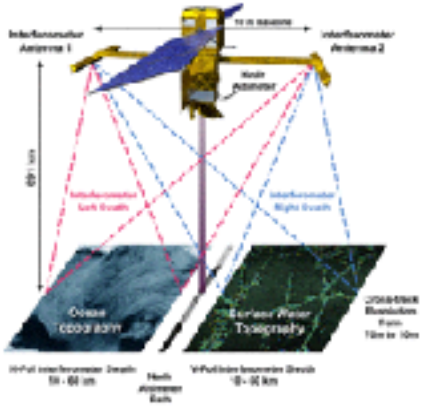


Inverse Problems in Geoscience: Examples of forward model

State

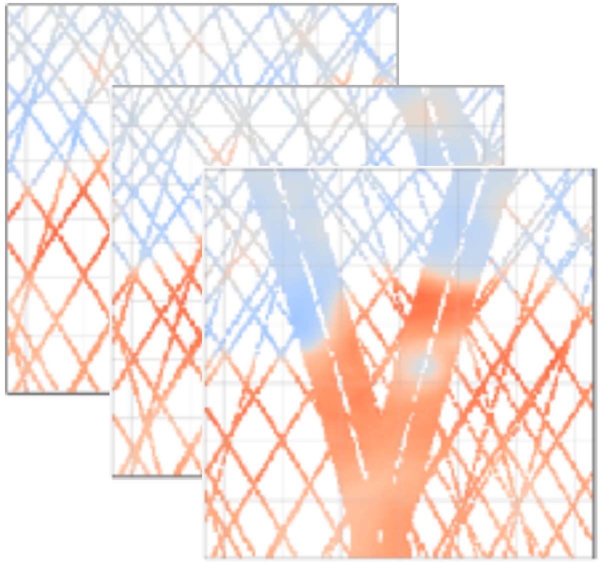


Observing system



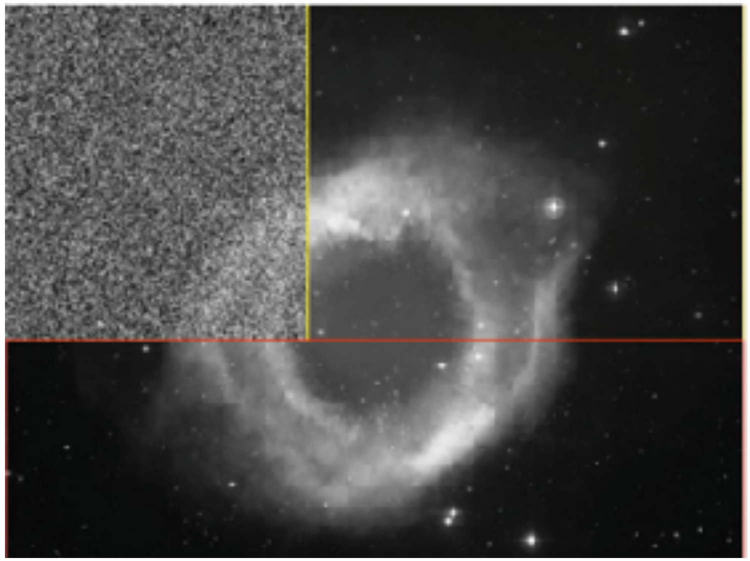
Forward model H()

Observation

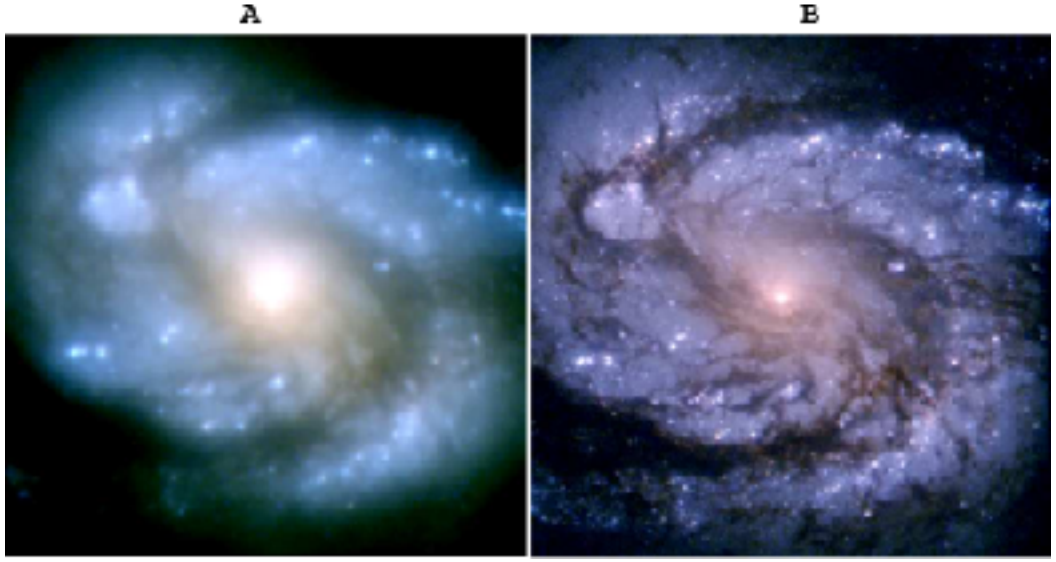


Inverse Problems in Geoscience: Examples of forward model

Denoising problem

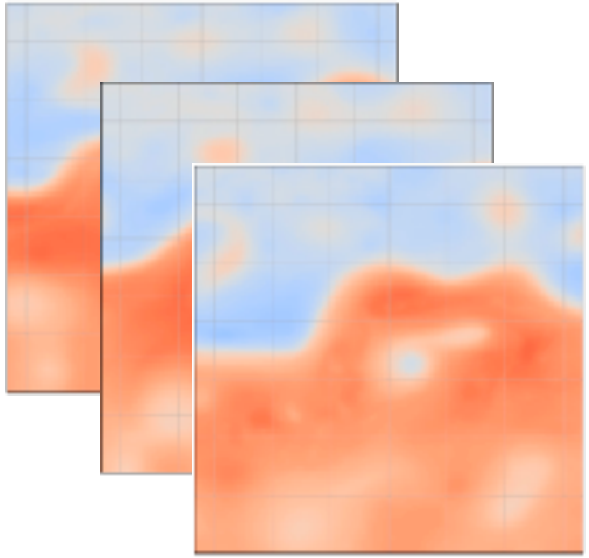


Downscaling problem

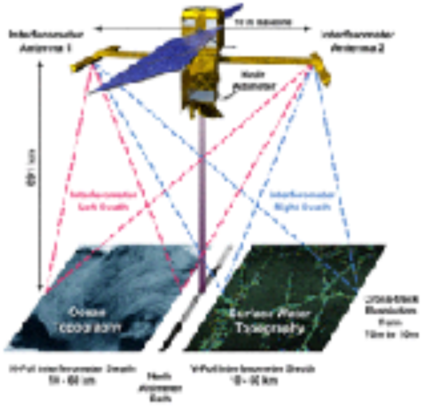


Inverse Problems and ill-posedness

State

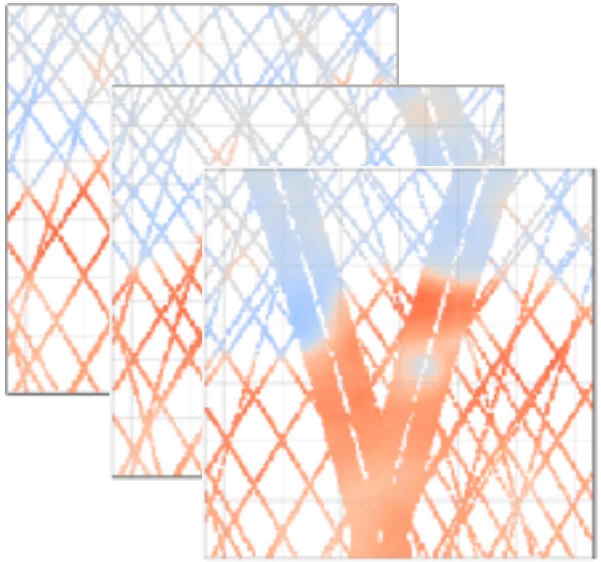


Observing system



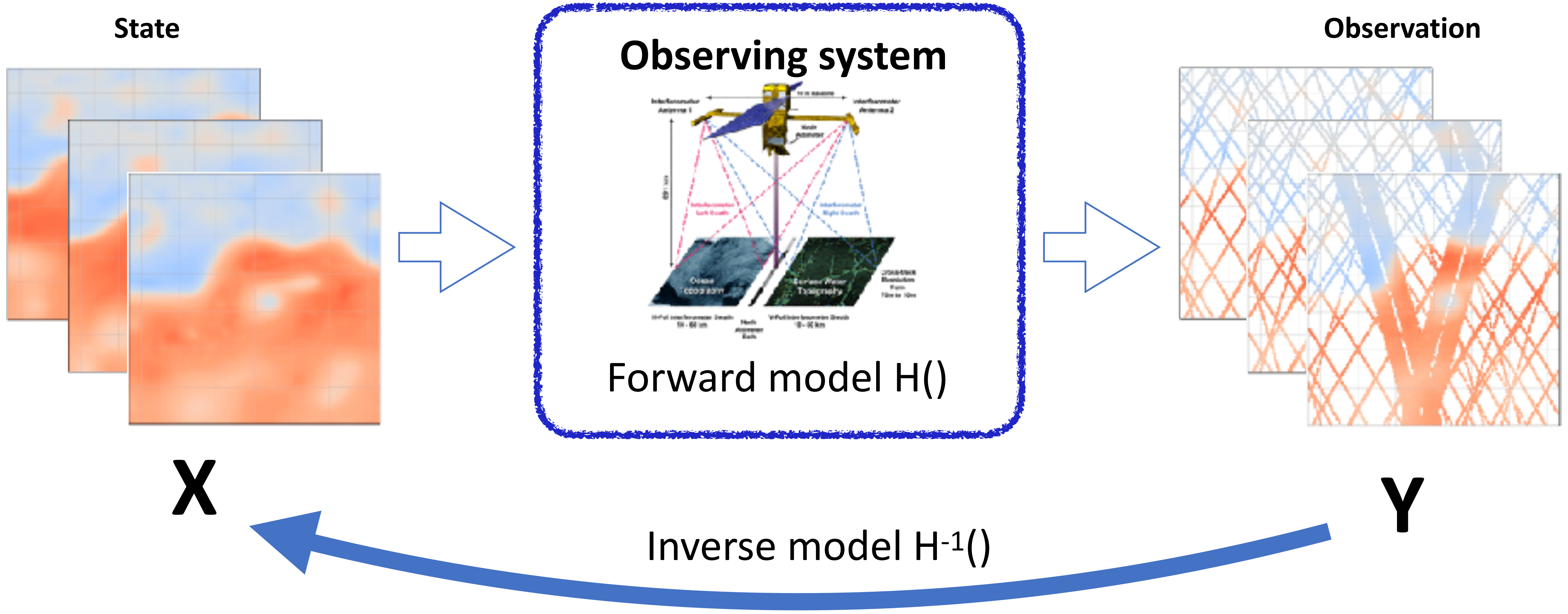
Forward model H()

Observation



Why is space-time interpolation an ill-posed problem ?

Inverse Problems and ill-posedness



We need some additional information (prior) to make the problem invertible

Inverse Problems in Geoscience

Inverse problems as learning problems

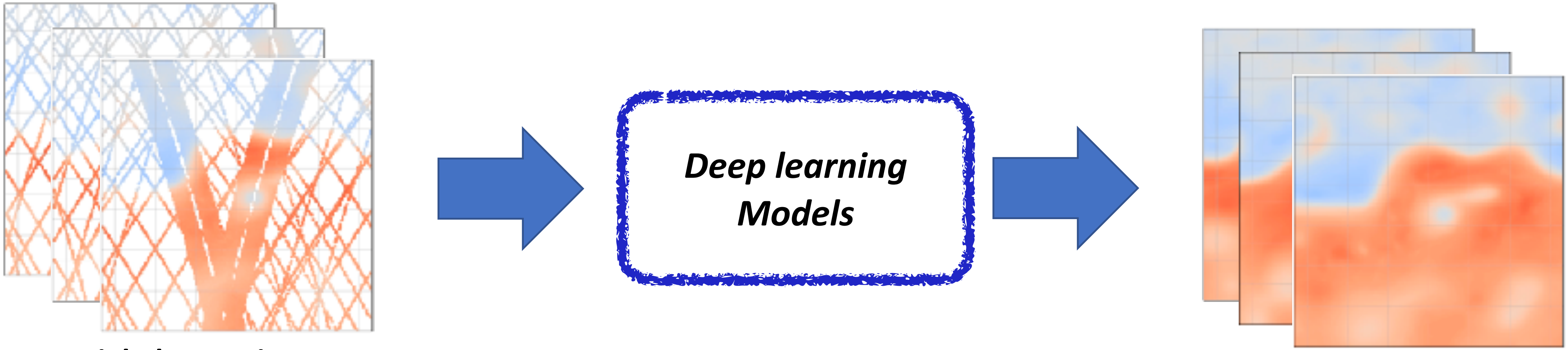
Mathematical formulations for inverse Problems

Inverse problems as learning problems

Applications to geophysical dynamics

End-to-end learning for inverse problems

End-to-end architecture



Partial observations y

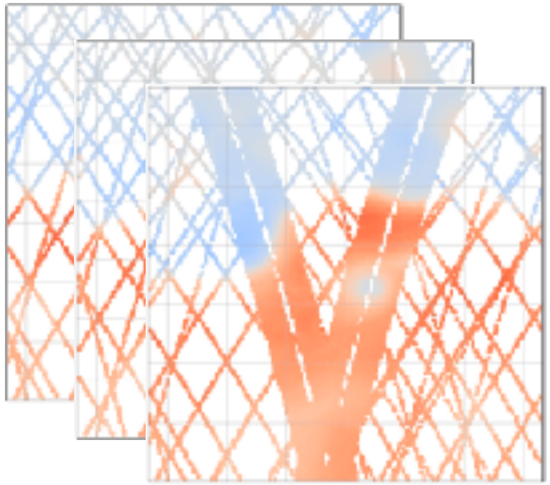
True states x

Assuming a dataset of pairs of true states and partial observations is available

Which training loss ? **Which models / architectures ?**

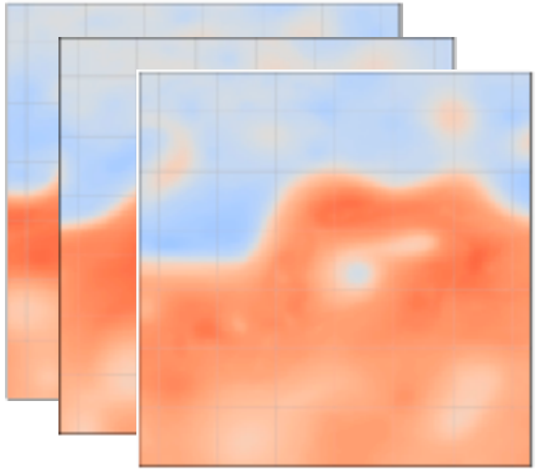
End-to-end learning for inverse problems

End-to-end architecture



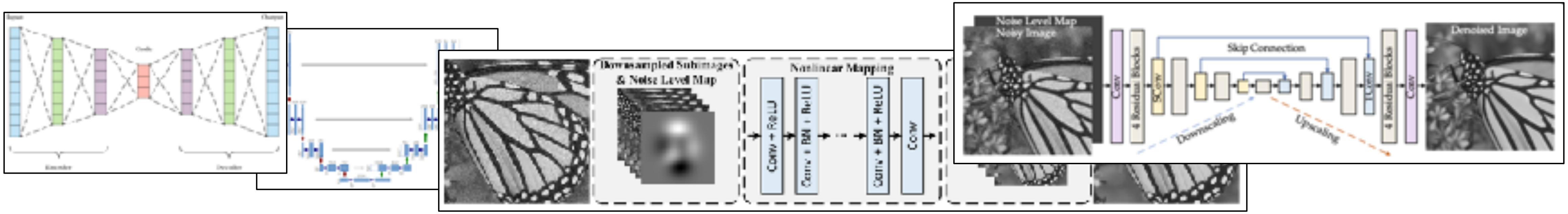
Partial observations y

Deep learning Models



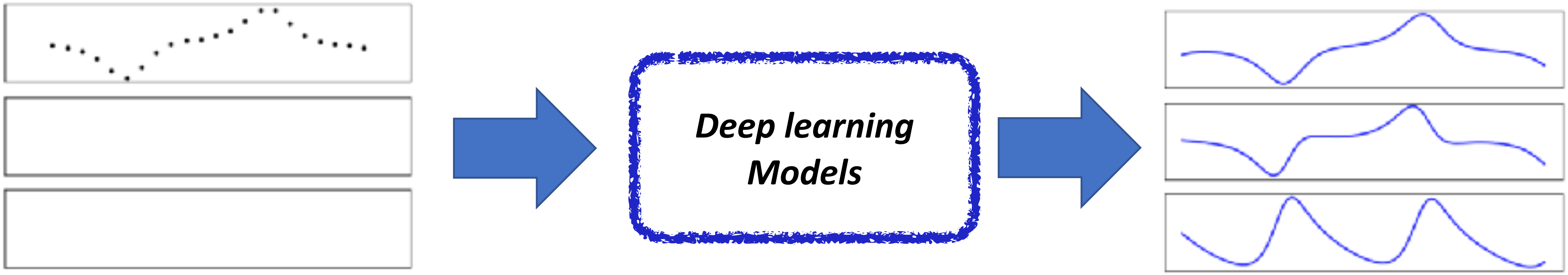
True states x

Which architectures? State-of-the-art CNN architectures?



End-to-end learning for inverse problems

An illustration for Lorenz-63 dynamics



Partial observations y

True states x

Colab notebook:

https://github.com/CIA-Oceanix/DLGD2022/blob/main/lecture-4-dl-oi-da/notebooks/notebookPyTorch_InvProb_LearningBased_UnrollingFixedPoint_L63.ipynb

Example for geoscience problems

JAMES | Journal of Advances in Modeling Earth Systems

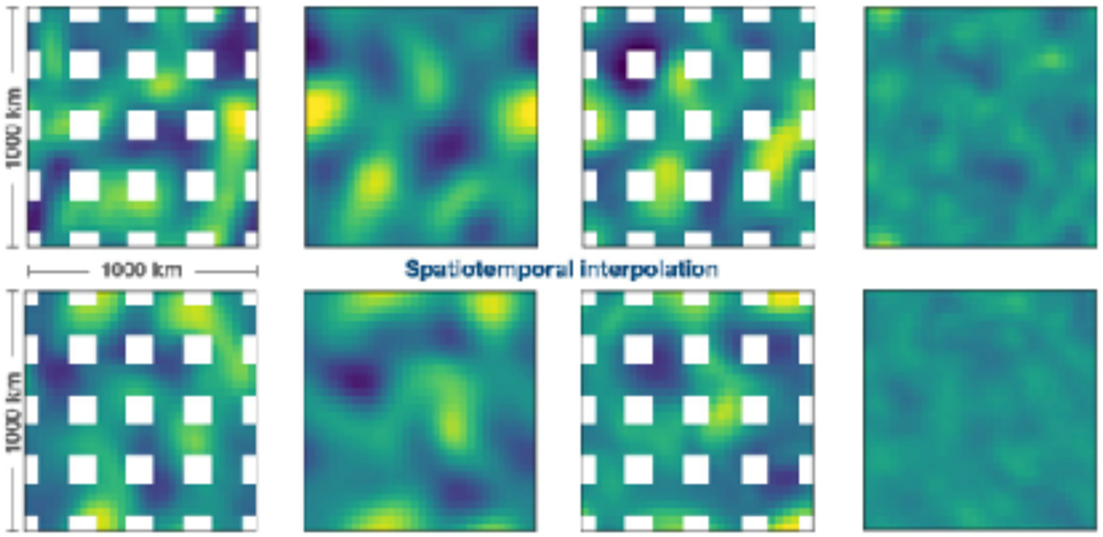
RESEARCH ARTICLE
10.1029/2019MS001965

Key Points
- The efficacy of Deep Learning in exploiting sparse sea surface height (SSH) data is demonstrated in a quasigeostrophic model
- Residual Neural Networks are superior to linear and dynamical interpolation techniques for SSH

A Deep Learning Approach to Spatiotemporal Sea Surface Height Interpolation and Estimation of Deep Currents in Geostrophic Ocean Turbulence

Georgy E. Manucharyan[1], Lia Siegelman[2], and Patrice Klein[2,3,4]

[1]School of Oceanography, University of Washington, Seattle, WA, USA, [2]Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, USA, [3]Laboratoire de Météorologie Dynamique, Ecole Normale Supérieure, CNRS, Paris, France, [4]Laboratoire d'Océanographie Physique et Spatiale, IFREMER, CNRS, Brest, France

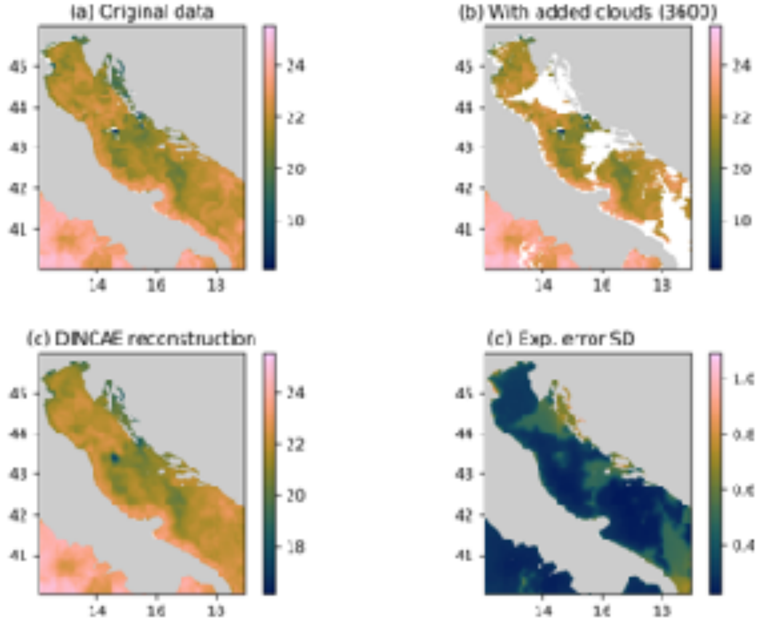


Geosci. Model Dev., 15, 2183–2196, 2022
https://doi.org/10.5194/gmd-15-2183-2022

Geoscientific Model Development
EGU

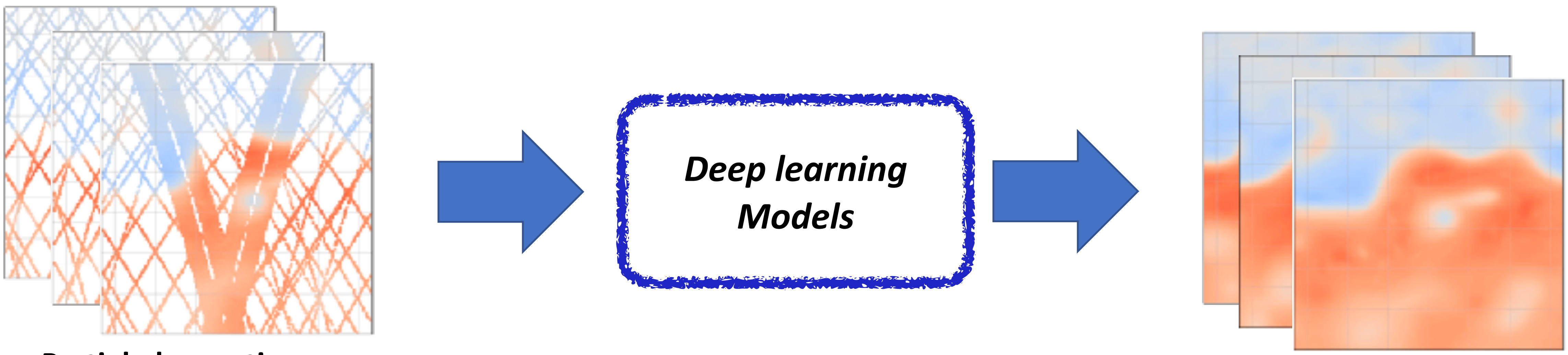
DINCAE 2.0: multivariate convolutional neural network with error estimates to reconstruct sea surface temperature satellite and altimetry observations

Alexander Barth, Aida Alvera-Azcárate, Charles Troupin, and Jean-Marie Beckers
GHER, University of Liège, Liège, Belgium



Deep learning and inverse problems

End-to-end architecture



Partial observations y

True states x

Should we reinvent the wheel ? Or can we benefit from more than 50 years of knowledge and research in signal processing, optimisation, applied math…. ?

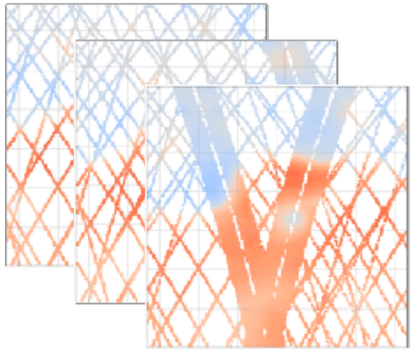
Inverse Problems in Geoscience

Mathematical formulations for inverse Problems

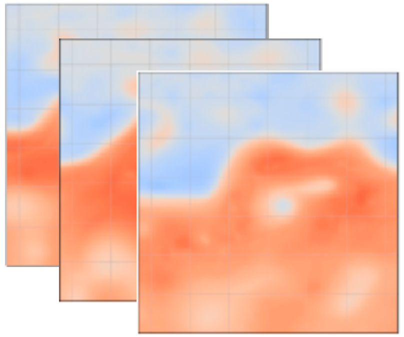
Inverse problems as learning problems

Applications to geophysical dynamics

(Variational) Data Assimilation: (Weak-Constraint) 4DVar DA



Partial observations y



True states x

State-space formulation:

$$\begin{cases} \dfrac{\partial x(t)}{\partial t} & = & \mathcal{M}\left(x(t)\right) + \eta(t) \\[2em] y(t,p) & = & x(t,p) \quad \forall t, \forall p \in \Omega_t \end{cases}$$

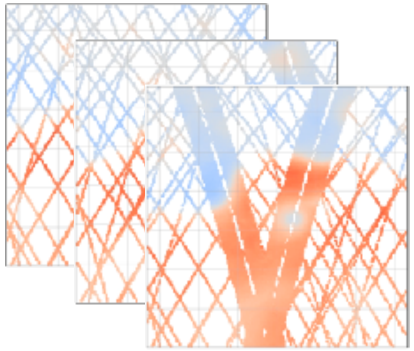
Associated variational formulation:

$$\arg\min_x \lambda_1 \sum_i \|x(t_i) - y(t_i)\|^2_{\Omega_{t_i}} + \lambda_2 \sum_n \|x(t_i) - \Phi(x)(t_i)\|^2$$

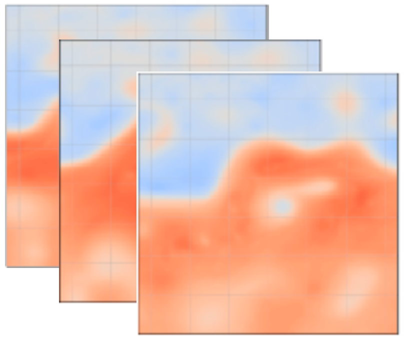
with $\quad \Phi(x)(t) = x(t-\Delta) + \displaystyle\int_{t-\Delta}^{t} \mathcal{M}\left(x(u)\right) du$

$$\boxed{\arg\min_x \lambda_1 \|x - y\|^2_\Omega + \lambda_2 \|x - \Phi(x)\|^2}$$

(Variational) Data Assimilation: (Weak-Constraint) 4DVar DA



Partial observations y

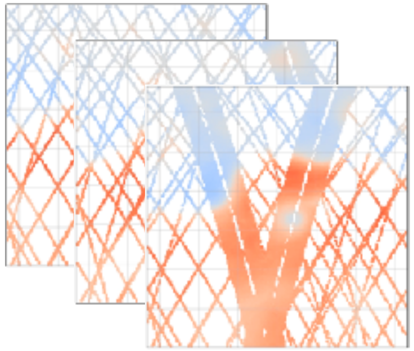


True states x

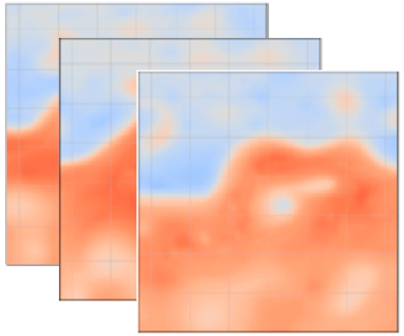
State-space formulation:

$$\begin{cases} \dfrac{\partial x(t)}{\partial t} = \mathcal{M}\left(x(t)\right) + \eta(t) \\ y(t,p) = x(t,p) \quad \forall t, \forall p \in \Omega_t \end{cases}$$

Inverse problems stated as minimisation problems



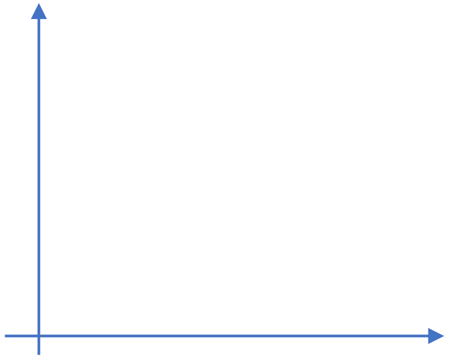
Partial observations y



True states x

Minimization problem

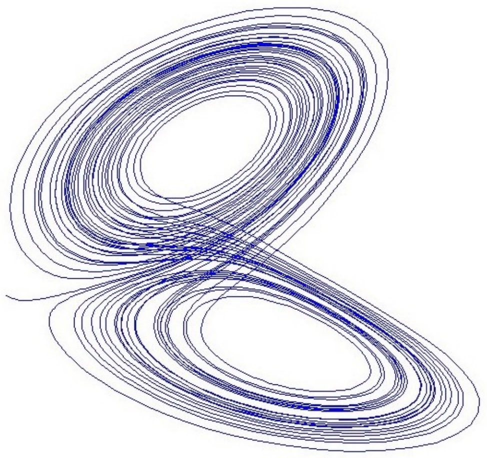
$$X = \arg\min_X \|Y - H(X)\|^2 + \lambda U_{reg}(X)$$



How to solve the minimization ?

Can we use Pytorch to implement the minimization ?

(Variational) Data Assimilation: (Weak-Constraint) 4DVar DA



$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \sigma\,(y(t) - x(t))$$
$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = x(t)\,(\rho - z(t)) - y(t)$$
$$\frac{\mathrm{d}z(t)}{\mathrm{d}t} = x(t)\,y(t) - \beta\,z(t)$$

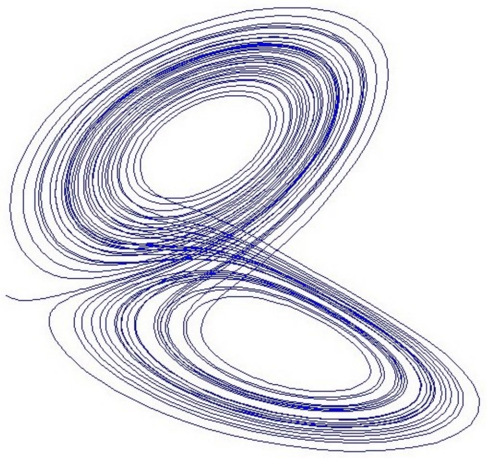
Lorenz-63 equations

Minimization problem

$$\arg\min_x \lambda_1 \|x - y\|_\Omega^2 + \lambda_2 \|x - \Phi(x)\|^2$$

with $\Phi(x)(t) = x(t-\Delta) + \int_{t-\Delta}^{t} \mathcal{M}\left(x(u)\right) du$

(Variational) Data Assimilation: (Weak-Constraint) 4DVar DA



$$\frac{dx(t)}{dt} = \sigma\,(y(t) - x(t))$$
$$\frac{dy(t)}{dt} = x(t)\,(\rho - z(t)) - y(t)$$
$$\frac{dz(t)}{dt} = x(t)\,y(t) - \beta\,z(t)$$

Lorenz-63 equations

Minimization problem

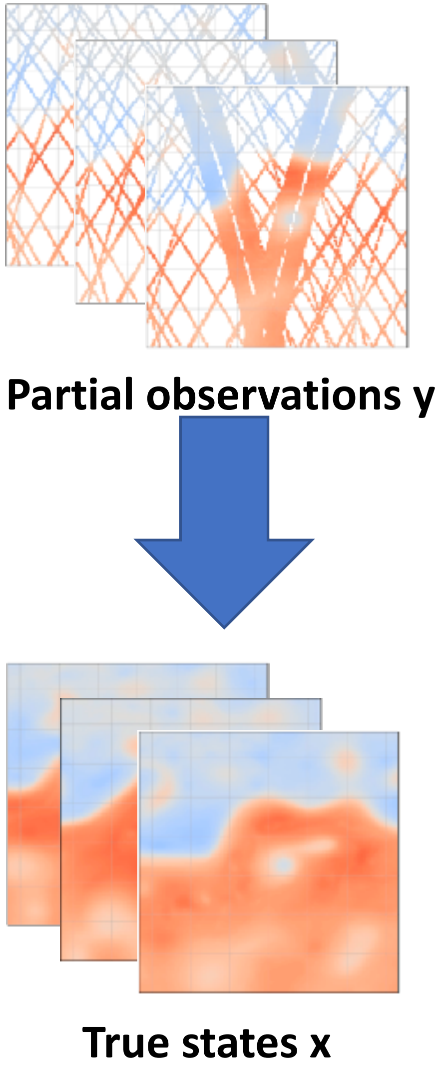
$$\arg\min_x \lambda_1 \|x - y\|_\Omega^2 + \lambda_2 \|x - \Phi(x)\|^2$$

with $\Phi(x)(t) = x(t-\Delta) + \int_{t-\Delta}^{t} \mathcal{M}\left(x(u)\right) du$

Let's try to implement this minimisation with Pytorch.

https://github.com/CIA-Oceanix/DLGD2022/blob/main/lecture-4-dl-oi-da/notebooks/notebookPyTorch_InvProb_ModelBased_L63.ipynb

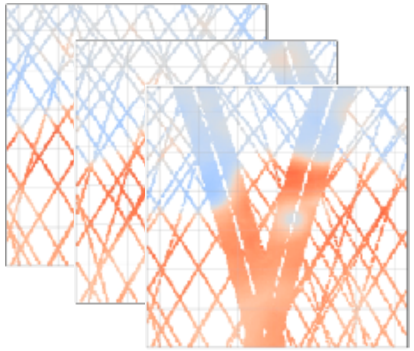
4DVar Data Assimilation and Optimal Interpolation



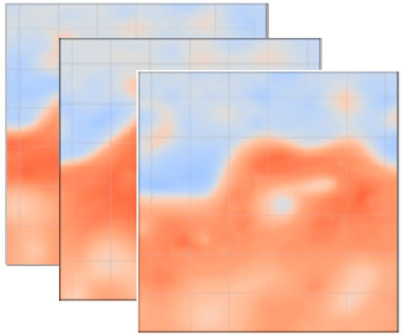
$$\left\{\begin{array}{ccl} x & \propto & \mathcal{N}\left(\mu, B\right) \\ y(p,t) & = & x(p,t) + \epsilon(t,p) \ \forall t, p \text{ with } p \in \Omega_t \\ \epsilon & \propto & \mathcal{N}\left(0, R\right) \end{array}\right.$$

$$\widehat{x} = \arg\min_{x} \log P\left(y|x\right) + \log P\left(x\right)$$

4DVar Data Assimilation and Optimal Interpolation



Partial observations y



True states x

(Weak-Constraint) 4DVar formulation

$$\arg\min_x \lambda_1 \|x - y\|_\Omega^2 + \lambda_2 \|x - \Phi(x)\|^2$$

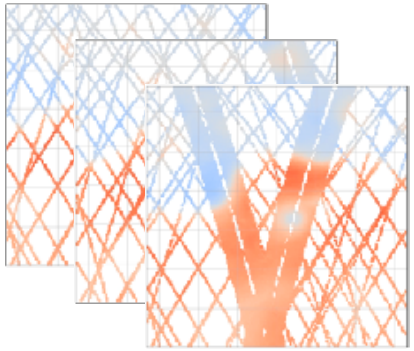
Optimal interpolation formulation

$$\arg\min_x \|y - H_\Omega \cdot x\|_R^2 + x^t B^{-1} x$$

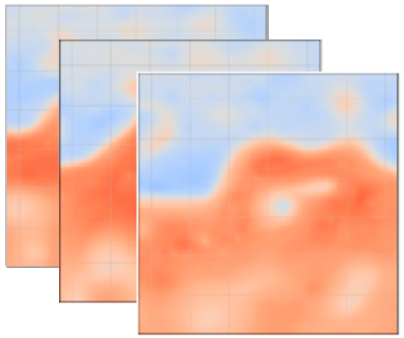
$$\arg\min_x \|y - H_\Omega \cdot x\|_R^2 + \|x - (\mathbb{I} - L) \cdot x\|^2 \text{ with } B = L^t L$$

OI solution $\widehat{x} = \mu + \mathbf{K} \cdot y \text{ with } \mathbf{K} = B H_\Omega \left(H_\Omega B H_\Omega^t + R\right)^{-1}$

Wrap-up on 4DVar DA and OI



Partial observations y



True states x

Weak-Constraint 4DVar formulation

$$\arg\min_{x} \|y - H_\Omega \cdot x\|^2 + \lambda \|x - \Phi(x)\|^2 + \nu \|x_0 - \mu_0\|^2_{B_0}$$

$$\Phi(x)(t) = x(t-\Delta) + \int_{t-\Delta}^{t} \mathcal{M}\left(x(u)\right) du$$

Strong-Constraint 4DVar formulation

$$\arg\min_{x_0} \|y - H_\Omega \cdot x\|^2 + \nu \|x_0 - \mu_0\|^2_{B_0} \text{ s.t. } \left\{ \begin{array}{lcl} x(t_0) & = & x_0 \\ \frac{dx}{dt} & = & \mathcal{M}\left(x(t)\right) \end{array} \right.$$

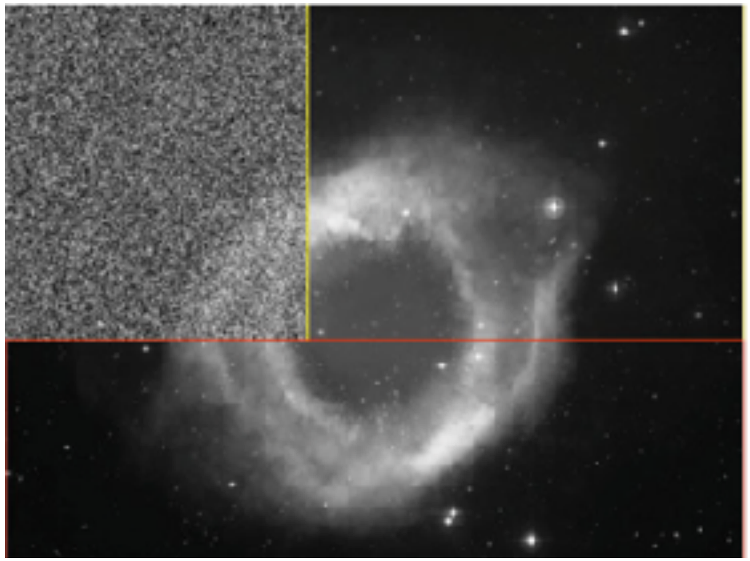
OI solution

$$\arg\min_{x} \|y - H_\Omega \cdot x\|^2_R + \|x - (\mathbb{I} - L) \cdot x\|^2 \text{ with } B = L^t L$$

Inverse problems stated as minimisation problems

Denoising problem

$$Y = X + \epsilon \text{ with } \epsilon \sim \mathcal{N}(0, \sigma^2)$$



Probablistic prior

$$X \sim P_X$$

$$X = \arg\min_X \lambda \|X - Y\|^2 - log P_X(X)$$

Dictionary-based prior

$$X = D.\alpha$$

$$\widehat{x} = \arg\min_{x,\alpha} \|y - x\|^2 + \lambda \|x - D.\alpha\|^2$$

Norm-based prior $$\widehat{x} = \arg\min_x \|y - x\|^2 + \lambda \|\nabla x\|^2$$

Generic formulation $$\widehat{x} = \arg\min_x \|y - H(x)\|^2 + \lambda U_{Reg}(x)$$

Key message for (variational) Model-based Methods

- ***Solution stated as the minimisation of a variational cost***

- ***Explicit knowledge of observation model and prior***

- ***Implementation of associated minimisation algorithm***

- ***Analytical derivation of gradient operator (**e.g., Euler-Lagrange equations, Adjoint Methods**)***

- ***Implementation using DL schemes (**eg, Pytorch**) do not require the analytical gradient operator***

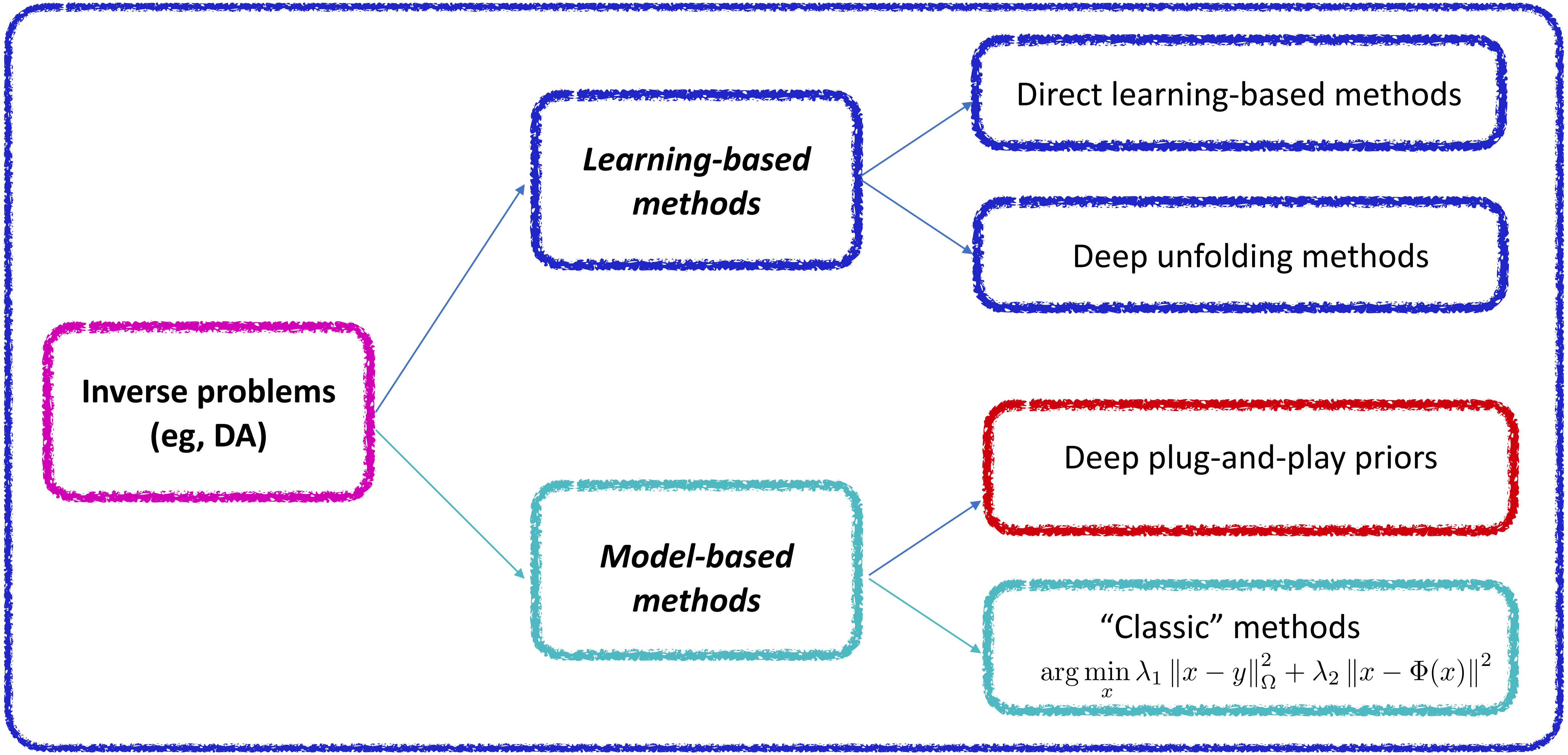
Inverse Problems in Geoscience

Mathematical formulations for inverse Problems

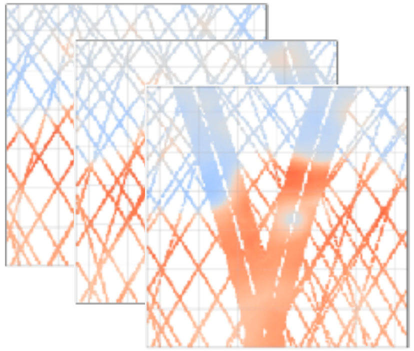
Inverse problems & Deep learning

Applications to geophysical dynamics

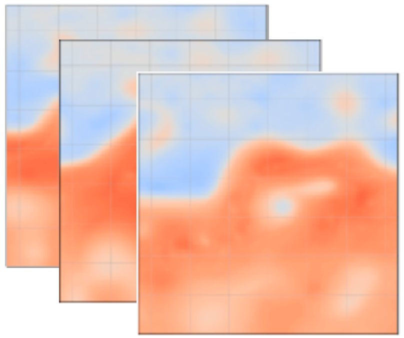
Model-driven vs. Learning-based approaches



Inverse problems using Deep plug-and-play priors



Partial observations y

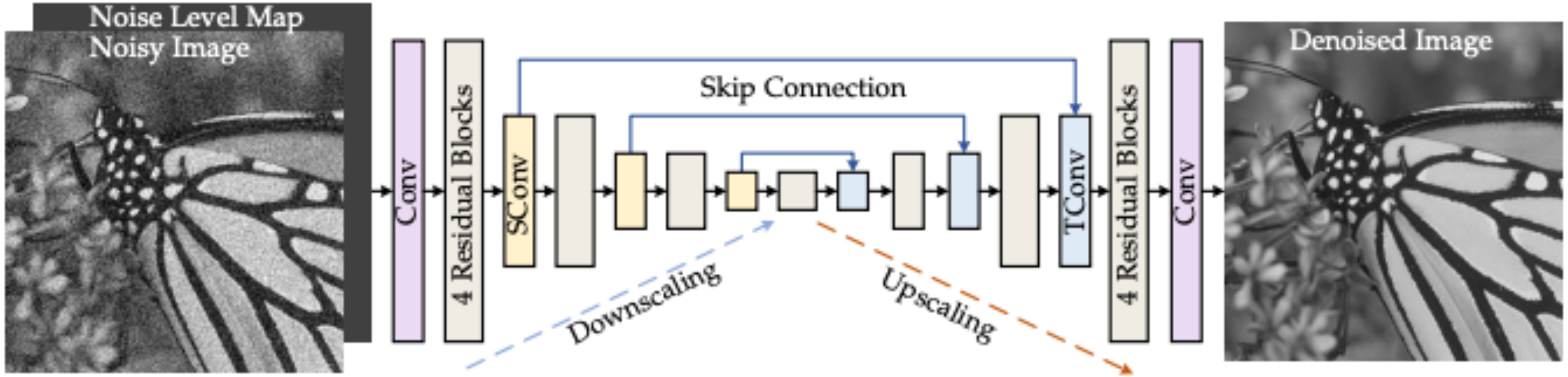


True states x

Model-based formulation with a (deep) learning-based prior

$$\arg\min_x \lambda_1 \|x - y\|_\Omega^2 + \lambda_2 \|x - \Phi(x)\|^2$$

Trainable plug-and-play prior

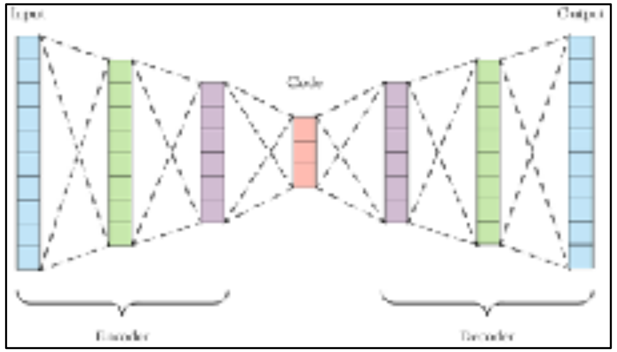


x $\Phi(x)$

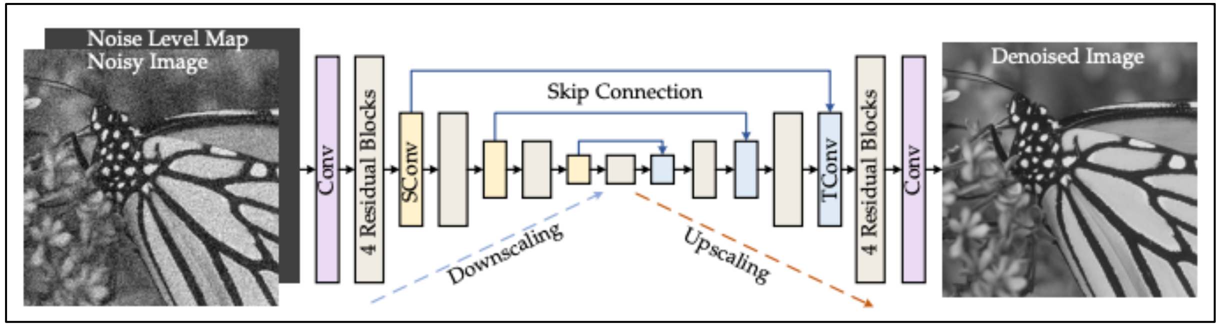
Zhang et al. Plug-and-Play Image Restoration with Deep Denoiser Prior. IEEE TPAMI, 2021. https://arxiv.org/pdf/2008.13751.pdf

Inverse problems using Deep plug-and-play priors

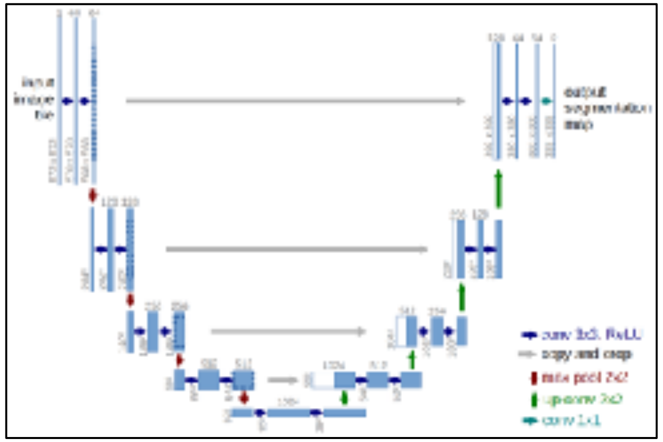
Examples of plug-and-play priors (denoiser architecture)



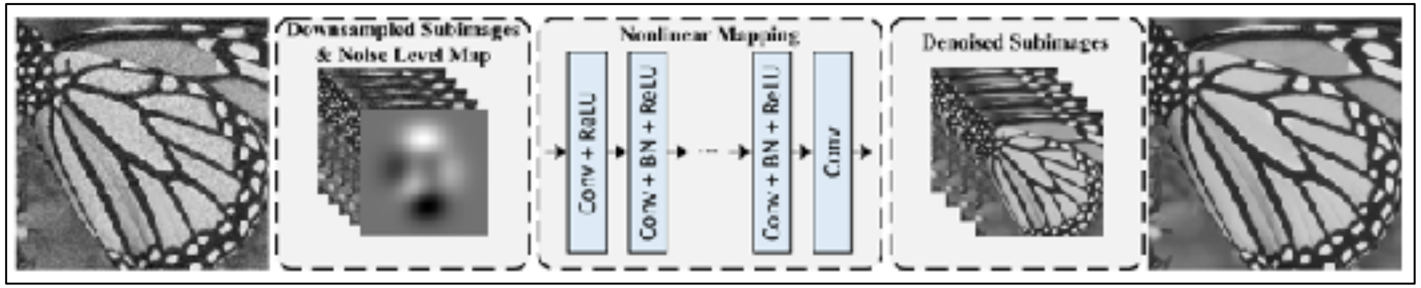
Auto-encoder



DRUNet https://arxiv.org/pdf/2008.13751.pdf

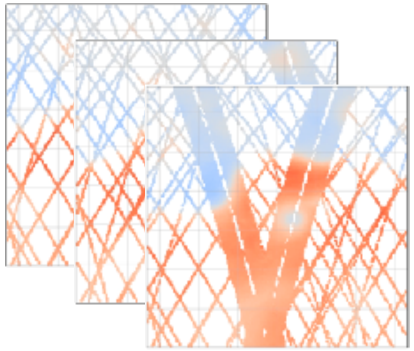


UNet https://arxiv.org/abs/1505.04597

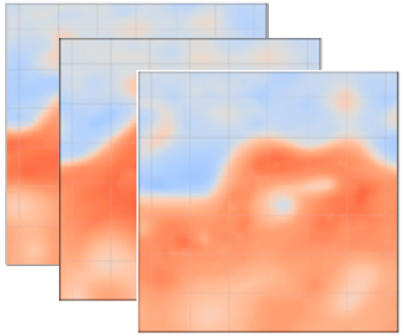


FFDNet https://arxiv.org/pdf/1710.04026.pdf

Inverse problems using Deep plug-and-play priors



Partial observations y



True states x

Model-based formulation

$$\arg\min_{x} \lambda_1 \left\| x - y \right\|_{\Omega}^{2} + \lambda_2 \left\| x - \Phi(x) \right\|^{2}$$

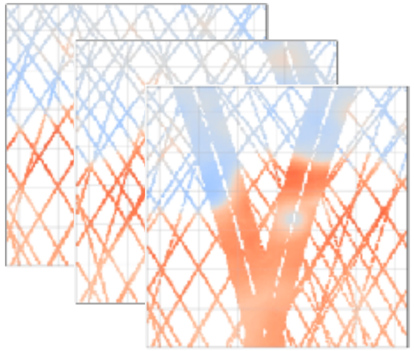
Trainable plug-and-play prior

Use of trainable priors but no actual learning specifically designed for the targeted inverse problem

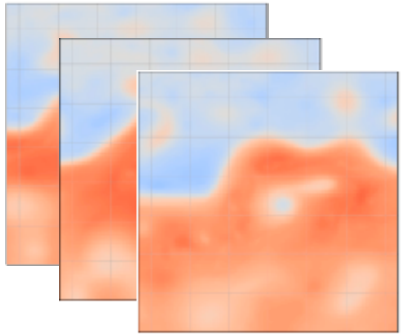
Let’s go and test it using a PCA-based prior

https://github.com/CIA-Oceanix/DLGD2022/blob/main/lecture-4-dl-oi-da/notebooks/notebookPyTorch_InvProb_ModelBased_L63.ipynb

Inverse problems: from plug-and-play to implicit priors



Partial observations y



True states x

Inverse problem with plug-and-play priors

Trainable plug-and-play prior

$$\arg\min_{x} \lambda_1 \|x - y\|_{\Omega}^2 + \lambda_2 \|x - \Phi(x)\|^2$$

Inverse problem with a prior in latent space

Pre-trained decoder

$$\arg\min_{z} \lambda_1 \|y - x\|_{\Omega}^2 \text{ s.t. } x = \mathcal{D}(z)$$

Inverse problem with deep image prior [*Ulyanov'17*]

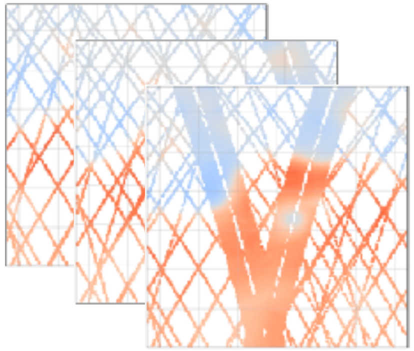
$$\arg\min_{\theta} \lambda_1 \|y - x\|_{\Omega}^2 \text{ s.t. } x = \mathcal{D}_\theta(z) \text{ and } z \sim \mathcal{N}(0, \mathbb{I})$$

https://arxiv.org/abs/1711.10925

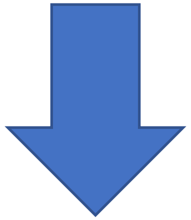
Inverse problem with implicit neural representation

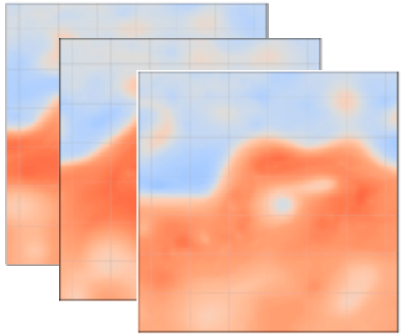
$$\arg\min_{\theta} \lambda_1 \|y - x\|_{\Omega}^2 \text{ s.t. } \forall p,\ x(p) = \mathcal{D}_\theta(p)$$

Inverse problems: from plug-and-play to implicit priors



Partial observations y





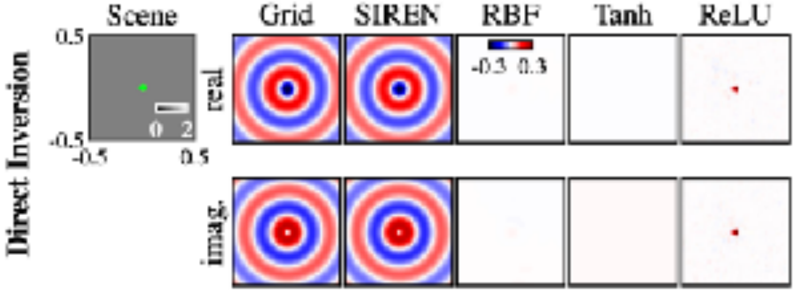
True states x

Inverse problem with implicit neural representations

$$\forall p,\ x(p) = \mathcal{D}_\theta(p)$$

Space-continuous (grid-free) representations of nD tensors

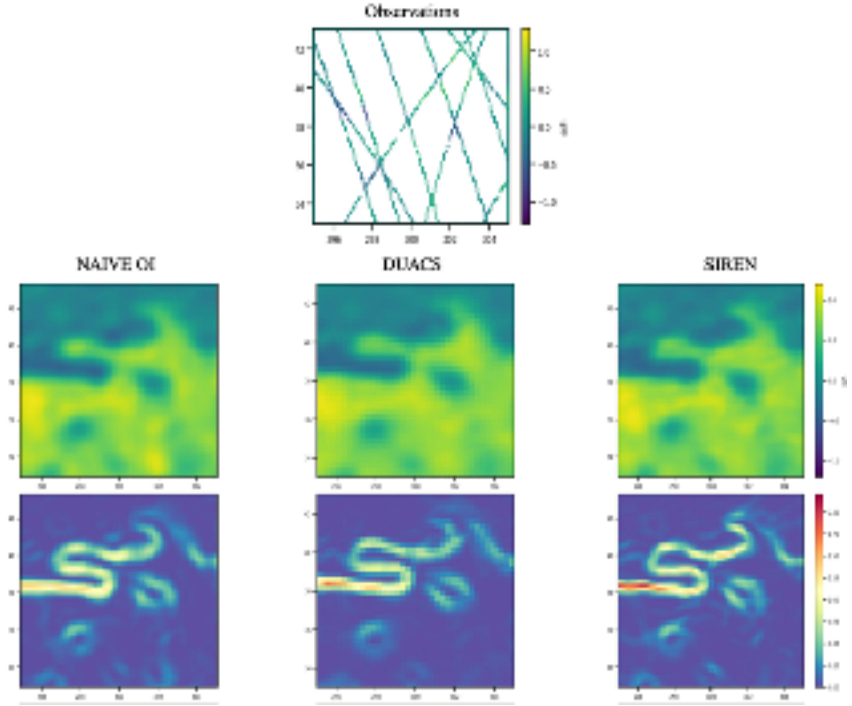
SIREN: implicit representations with periodic activations



https://www.vincentsitzmann.com/siren/

Application to SSH mapping

SIREN architecture / unsupervised learning



Jhonson et lal., ML4PS'22
https://arxiv.org/abs/2211.10444

Key messages for (variational) Model-based Methods

- ***Solution stated as the minimisation of a variational cost***
- ***Explicit knowledge of observation model and prior***
- ***Implementation of associated minimisation algorithm***
- ***Analytical derivation of gradient operator (***e.g., Euler-Lagrange equations, Adjoint Methods***)***
- ***Implementation using DL schemes (***eg, Pytorch***) do not require the analytical gradient operator***
- ***Possible extension to pre-trained plug-and-play and implicit priors***

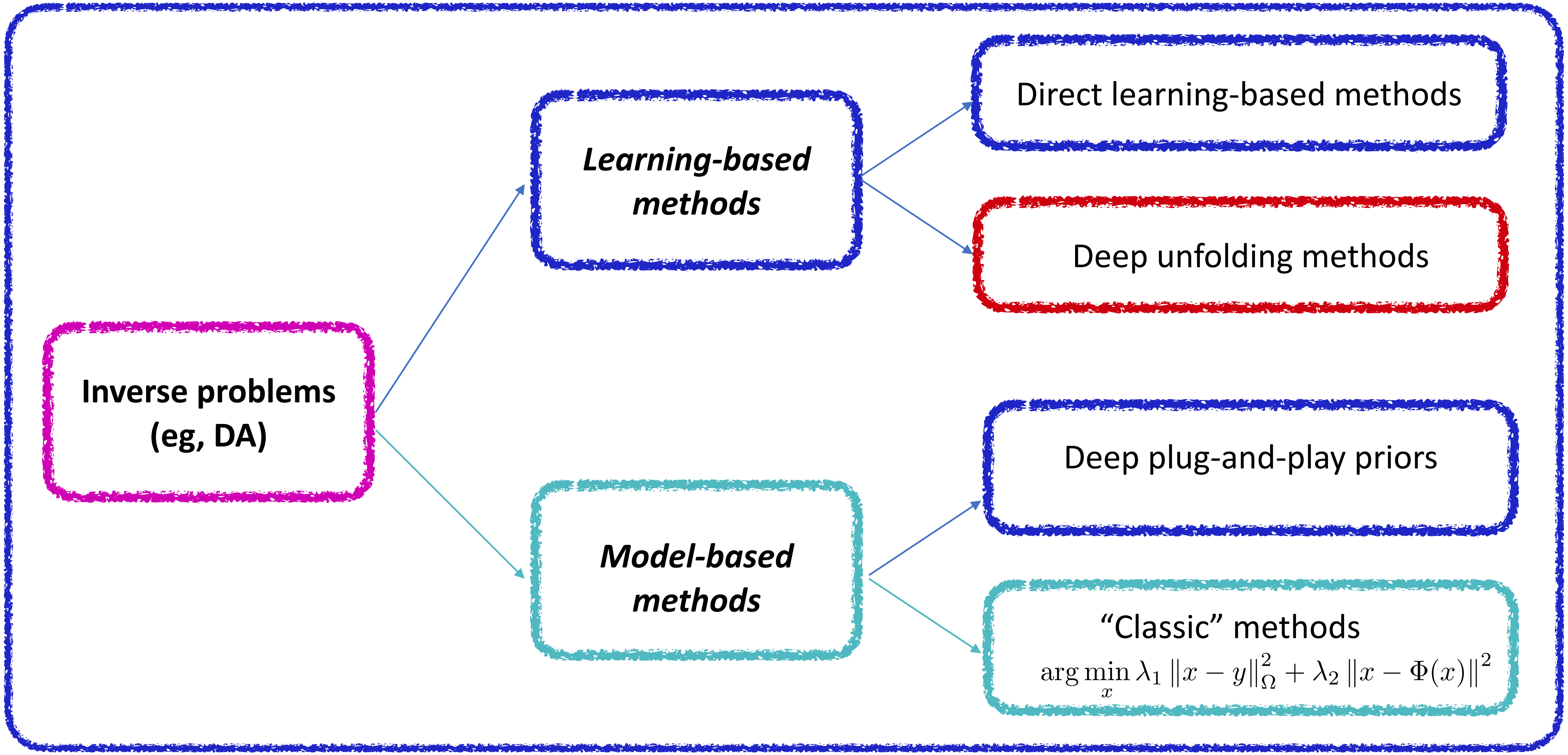
Inverse Problems in Geoscience

Mathematical formulations for inverse Problems

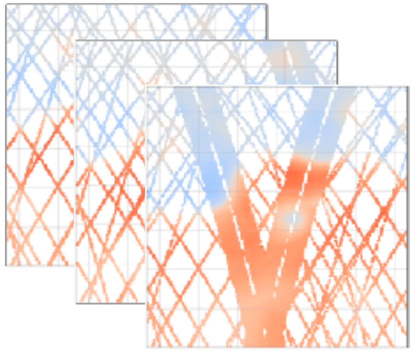
Inverse problems & Deep learning

Applications to geophysical dynamics

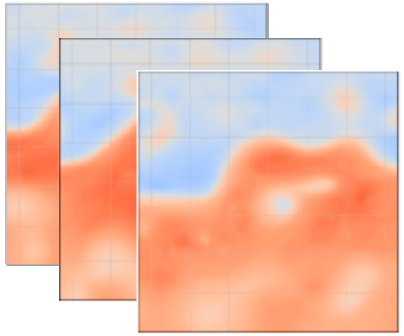
Model-driven vs. Learning-based approaches



Can we relate end-to-end learning and model-based schemes?



Partial observations y



True states x

The example of the Optimal Interpolation

$$\arg\min_x \|y - H_\Omega \cdot x\|_R^2 + \|x - (\mathbb{I} - L) \cdot x\|^2 \text{ with } B = L^t L$$

Associated optimality criterion (bi-level formulation)

$$\min_L \mathbb{E}\left(\|\widehat{x} - x^{true}\|^2\right)$$

$$\text{s.t.} \quad \widehat{x} = \arg\min_x \|y - H_\Omega \cdot x\|^2 + \|x - \Phi_L(x)\|^2$$

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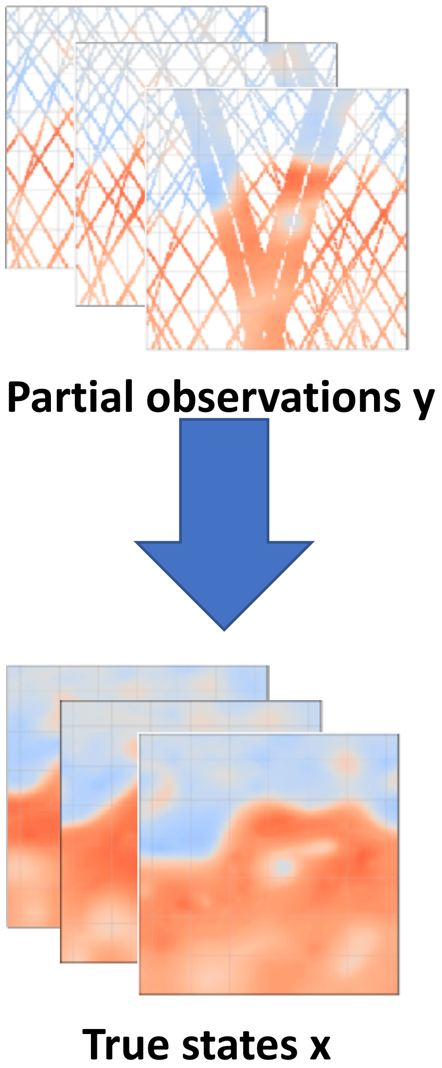

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$$\text{s.t.} \quad \widehat{x} = \arg\min_x \|y - H_\Omega \cdot x\|^2 + \|x - \Phi_L(x)\|^2$$

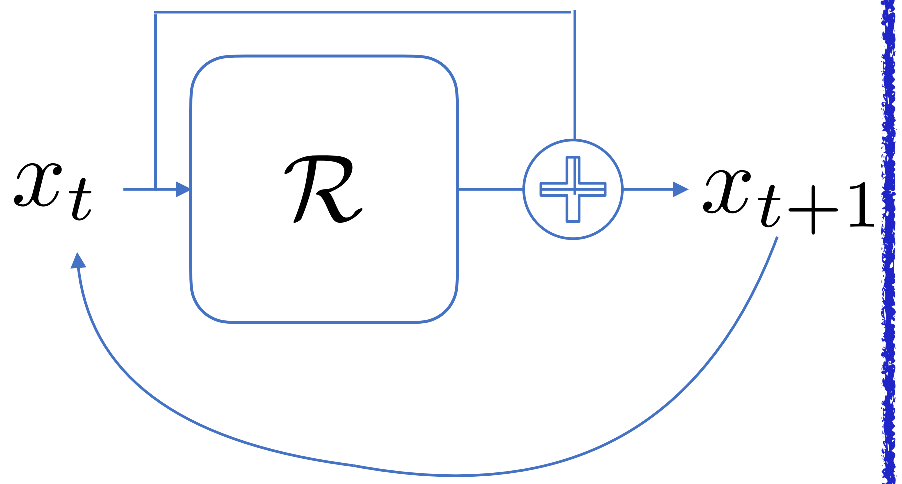
No similar property in general for non-linear/non-quadratic formulations

Folded vs. Unfolded Representations

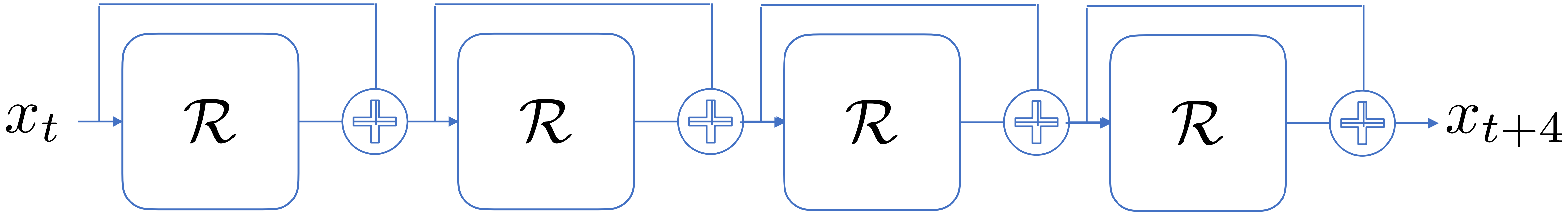
An example with a ResNet

$$x_{t+1} = x_t + \mathcal{R}(x_t)$$

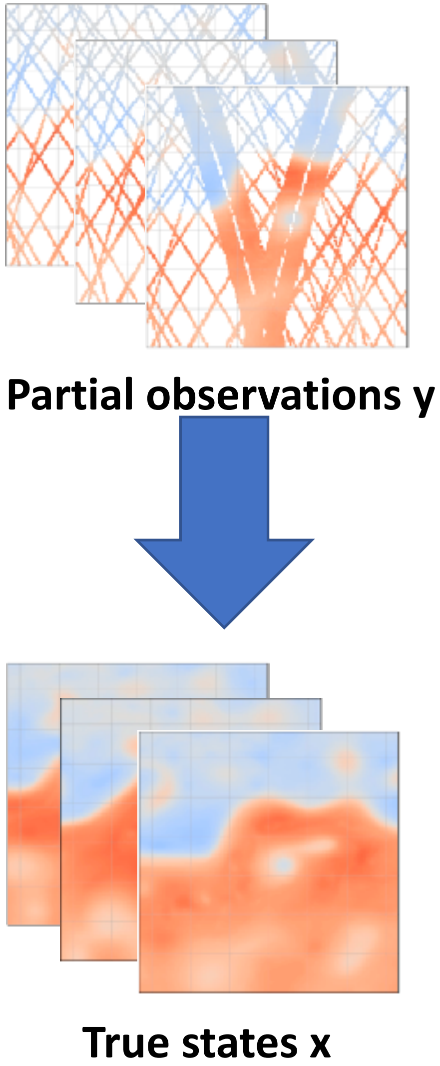
Folded Representation



Unfolded Representation



Inverse problems using Deep unfolding schemes



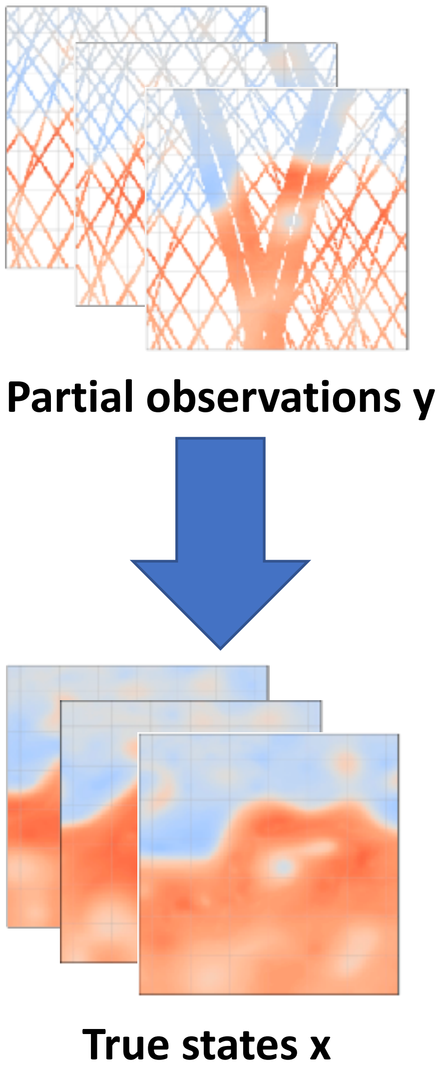
Basic idea: exploit knowledge on optimisation algorithms for inverse problems

- **Many schemes involve iterative algorithms**
- **On may unfold an Iterative procedure to define a deep learning architecture**

Examples

- **Image denoising/Unfolding of reaction-diffusion schemes** (e.g., Chen et al., 2015)
- M**edical imaging/Unfolding of ADMM schemes** (e.g., Yang et al., 2016)
- **Interpolation/Unfolding of fixed-point algorithms** (e.g., Fablet et al., 2020)
- **DA/Deep unfolding of sequential DA algorithms** (e.g., Boudier et al., 2020)

Data Assimilation using Deep unfolding schemes



Deep unfolding of Sequential DA filters (e.g., EnKF) [Boudier et al., 2020]

- **Forecasting step:** samples states at time t given the posterior at time t-1

$$x_t | y_{0:t-1} \text{ given } x_{t-1} | y_{0:t-1}$$

- **Analysis step:** update the posterior at time t given the new observations at time t

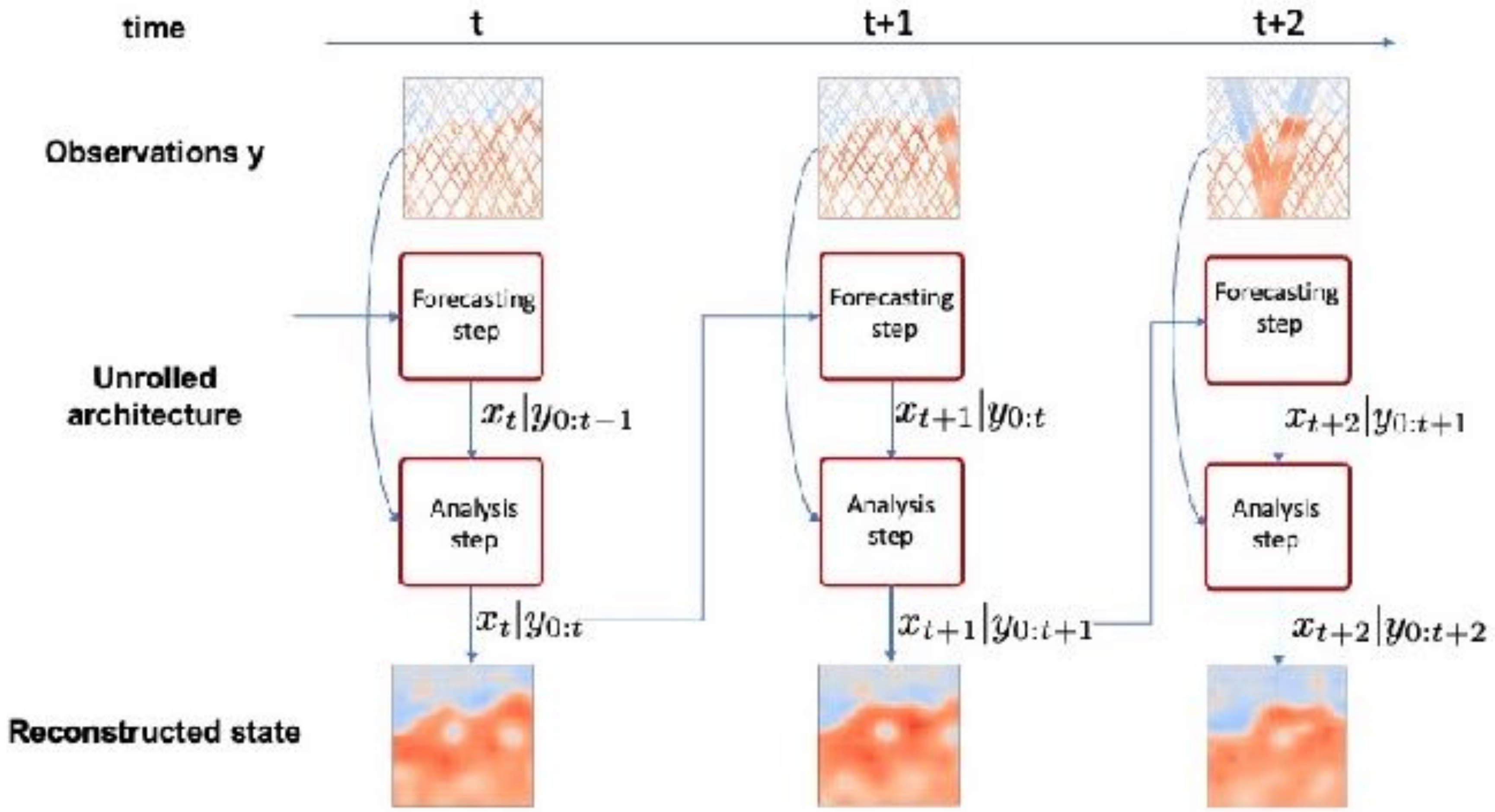
$$x_t | y_{0:t} \text{ given } y_t \text{ and } x_t | y_{0:t-1}$$

https://arxiv.org/abs/2010.09694

Data Assimilation using Deep unfolding schemes

Deep unfolding of Sequential DA filters (e.g., EnKF) [Boudier et al., 2020**]**

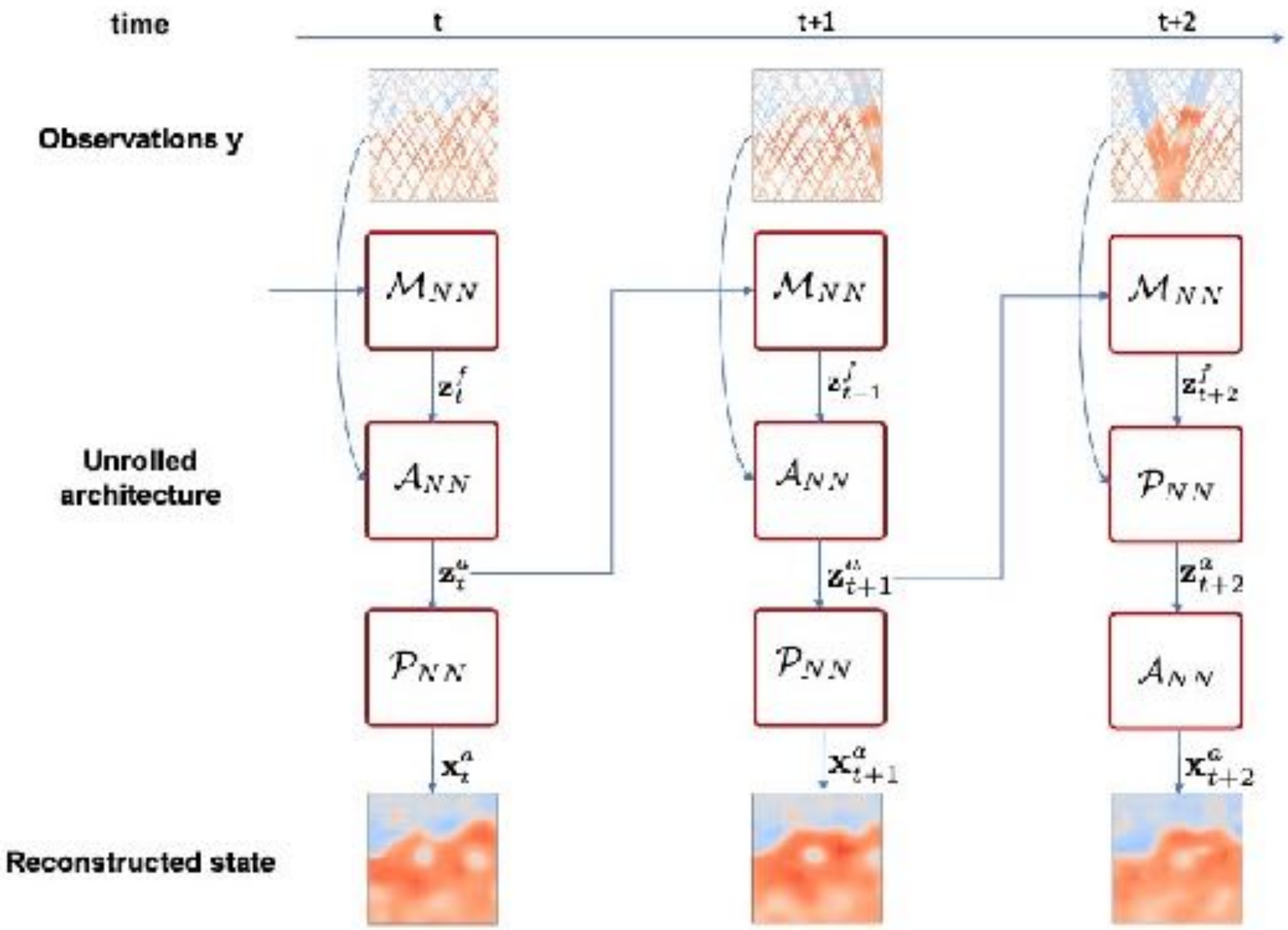
Unfolded architecture of Sequential DA schemes



https://arxiv.org/abs/2010.09694

Data Assimilation using Deep unfolding schemes

Deep unfolding of Sequential DA filters (e.g., EnKF) [Boudier et al., 2020]



Generalization of the sequential DA schemes with:

- A latent representation of the posteriors
- NNs to generalize/replace the forecasting and analysis step

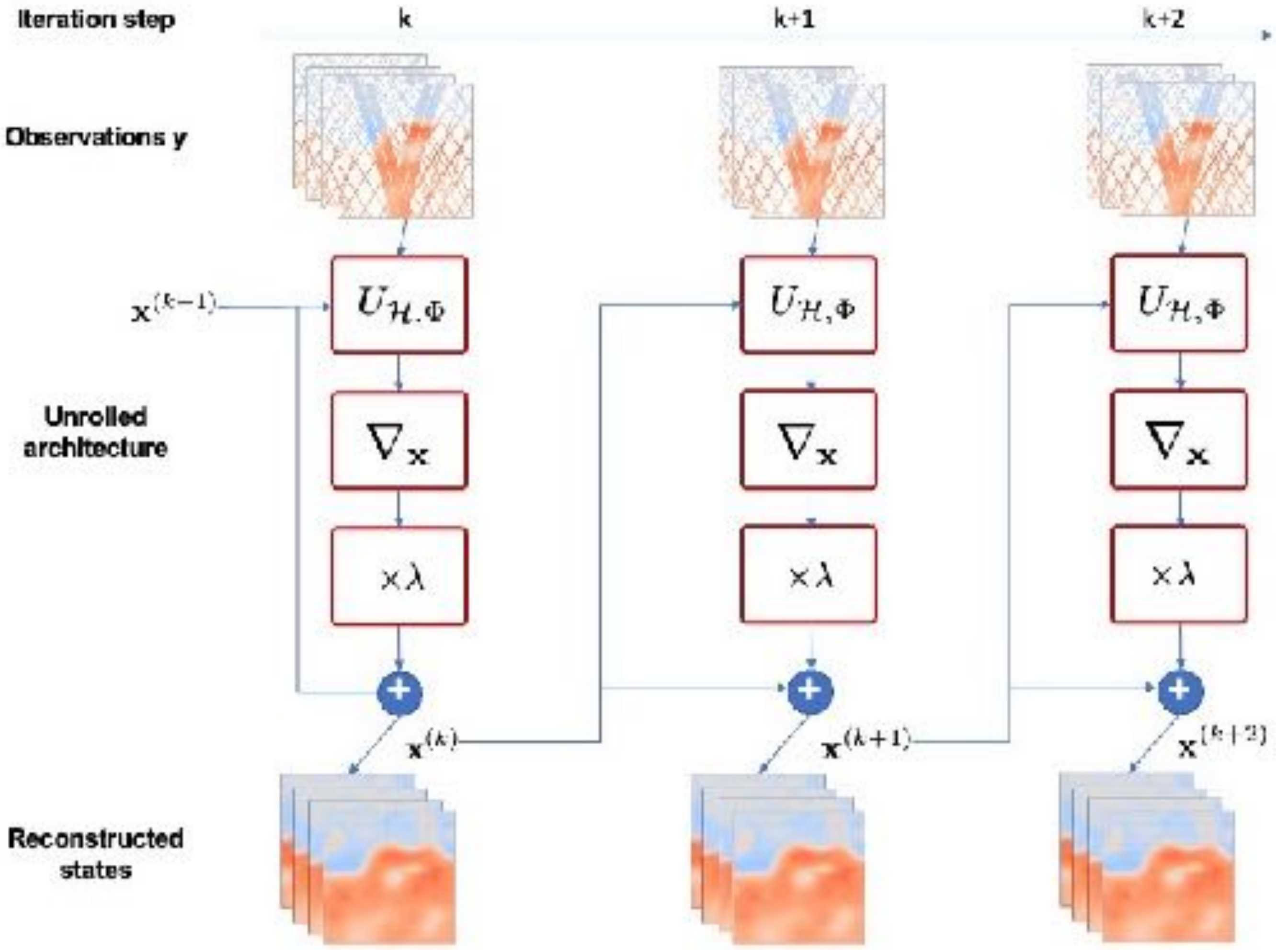
https://arxiv.org/abs/2010.09694

Data Assimilation using Deep unfolding schemes

Deep unfolding of 4DVar DA schemes (e.g., 4DVar-WC) [Fablet et al., 2021]

Unfolded architecture of a 4DVar-WC scheme

$$\min_{\mathbf{x}} \underbrace{\|\mathbf{x} - \mathcal{H}(\mathbf{x})\|^2 + \nu \|\mathbf{x} - \Phi(\mathbf{x})\|^2}_{U_{\mathcal{H},\Phi}}$$

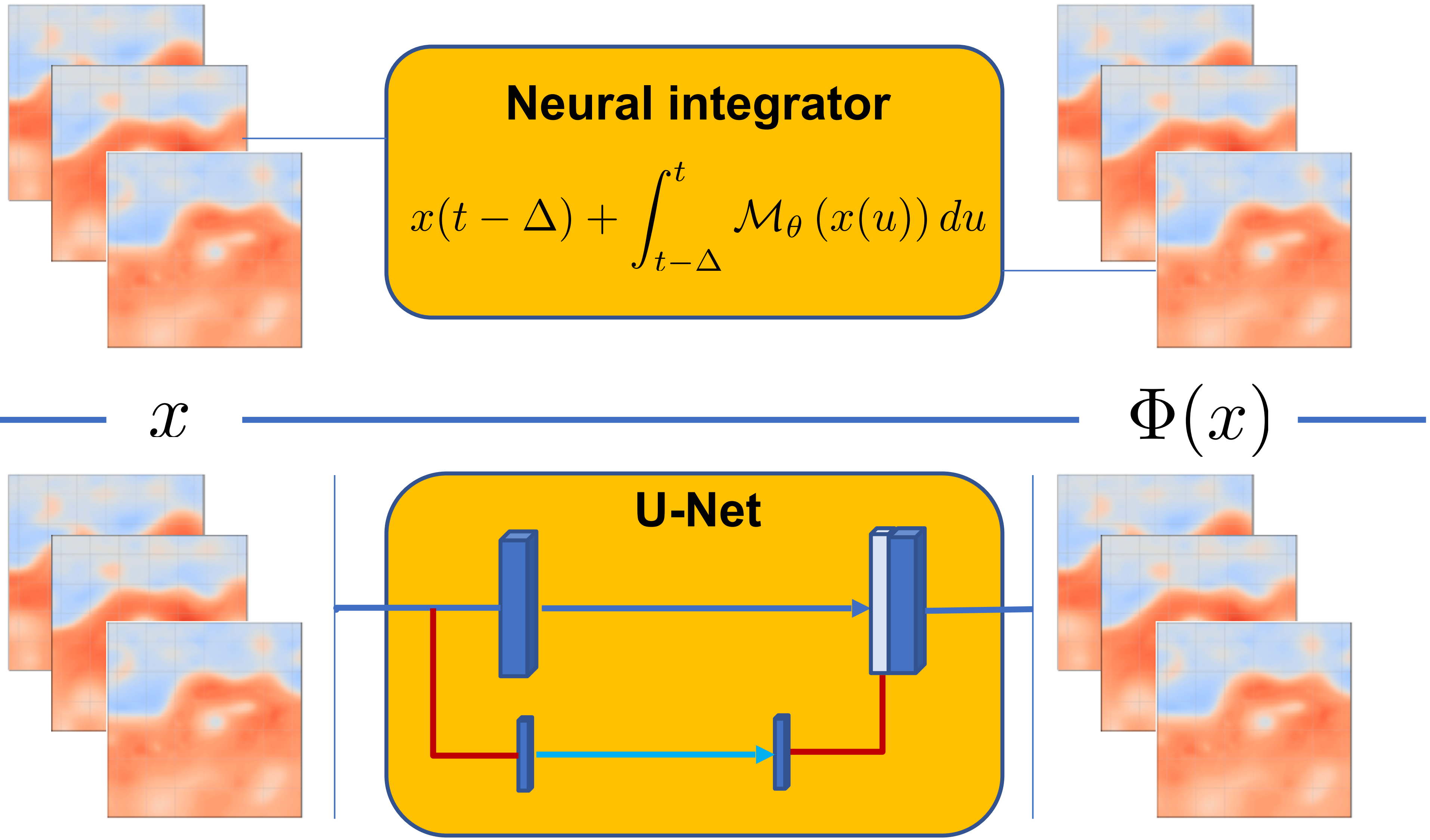


https://doi.org/10.1029/2021MS002572

End-to-end learning for 4DVar DA: projection operator Φ

Parameterization using (learnable) ODE operator

$$\frac{\partial x(t)}{\partial t} = \mathcal{M}_\theta\left(x(t)\right)$$



Two-scale U-Net-like Parameterization (Gibbs Field)

Data Assimilation using Deep unfolding schemes

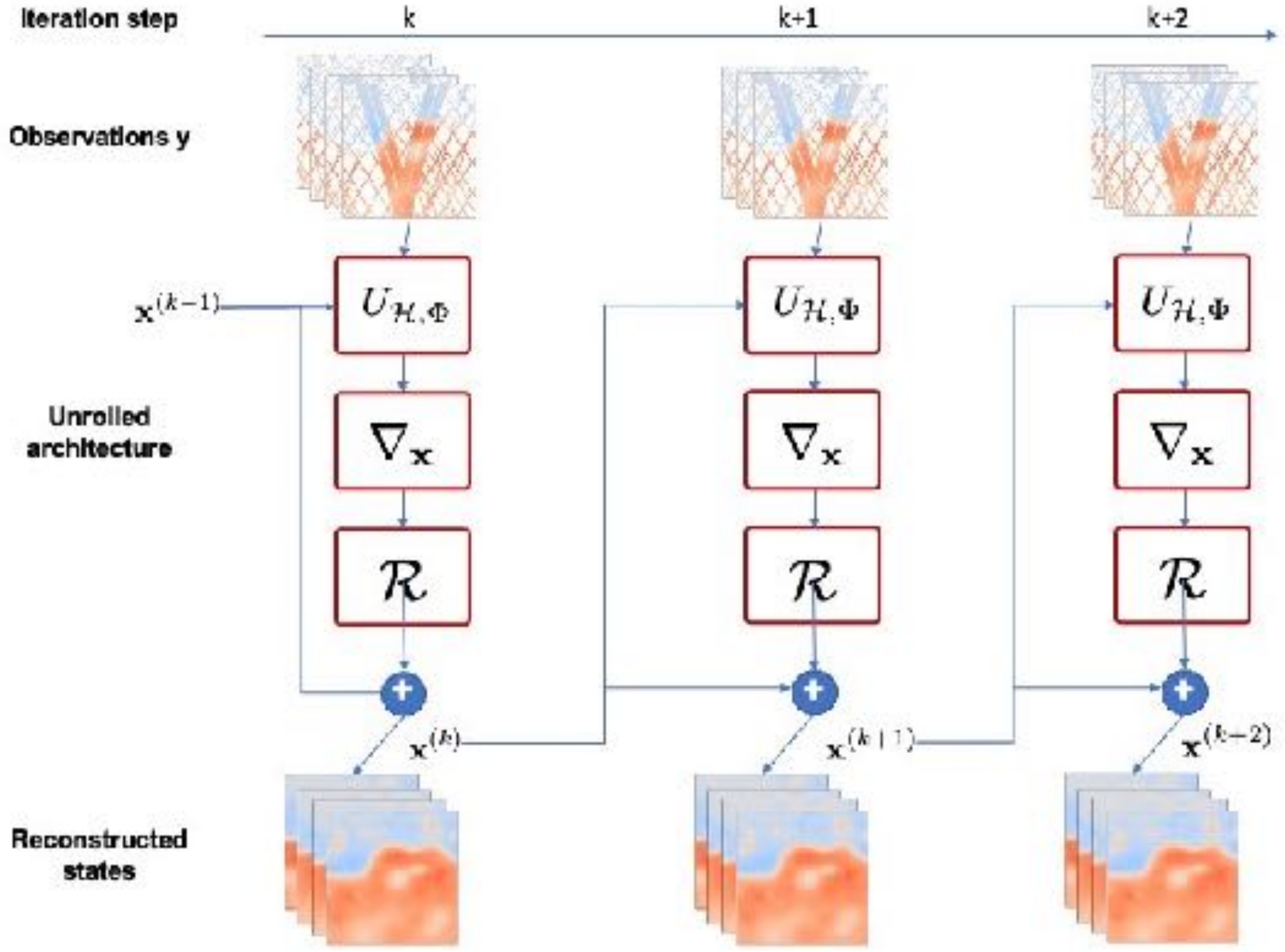
Deep unfolding of 4DVar DA schemes (e.g., 4DVar-WC) [Fablet et al., 2021]

Unfolded architecture of a 4DVarNet scheme

$$\min_{\mathbf{x}} \underbrace{\|\mathbf{x} - \mathcal{H}(\mathbf{x})\|^2 + \nu \|\mathbf{x} - \Phi(\mathbf{x})\|^2}_{U_{\mathcal{H},\Phi}}$$

Iterative gradient-based update

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k+1)} + \mathcal{R}\left(\nabla_{\mathbf{x}} U_{\mathcal{H},\Phi}(\mathbf{x},\mathbf{y})\right)$$



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Data Assimilation using Deep unfolding schemes

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Unfolded architecture of a 4DVarNet scheme

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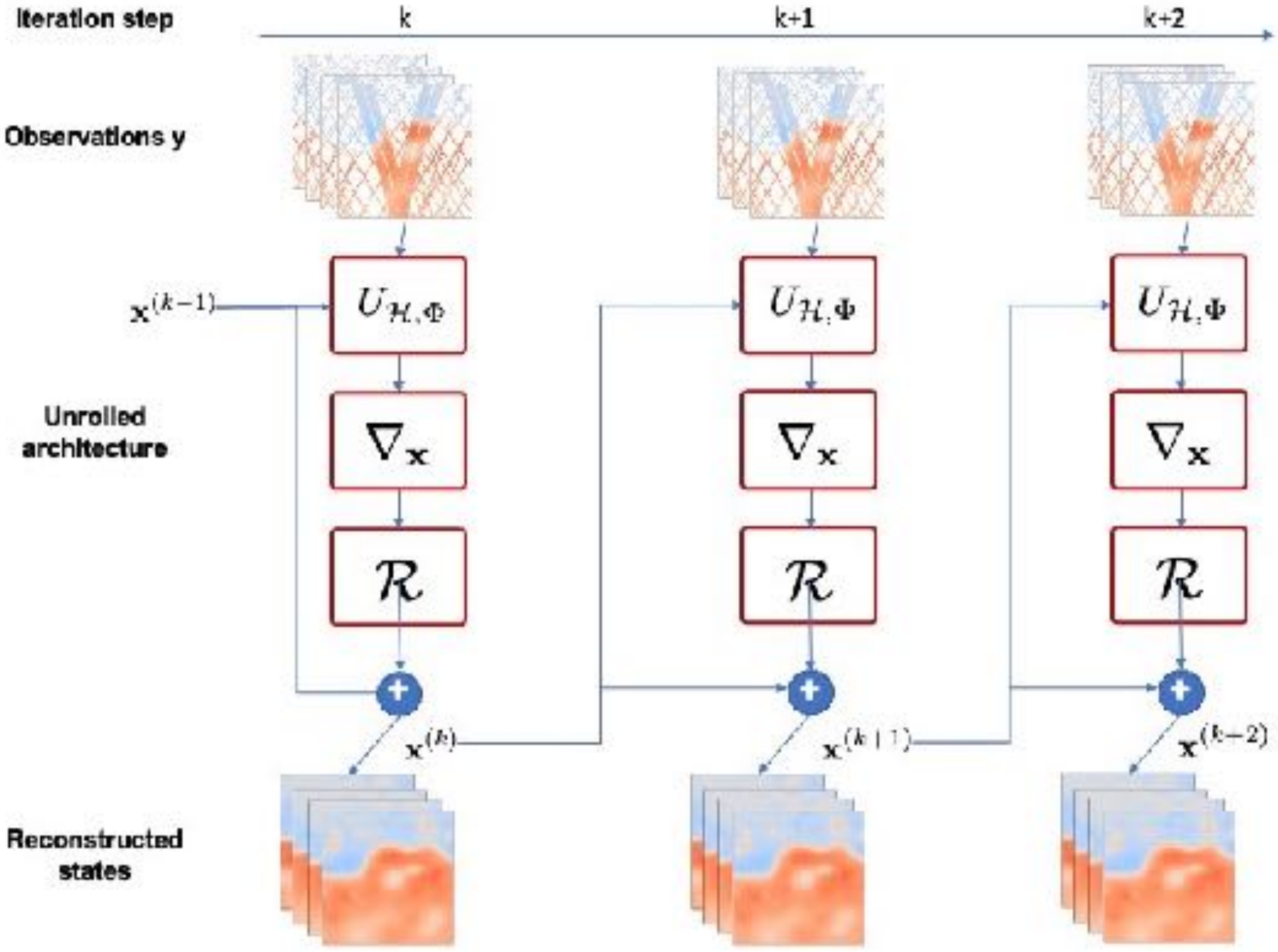
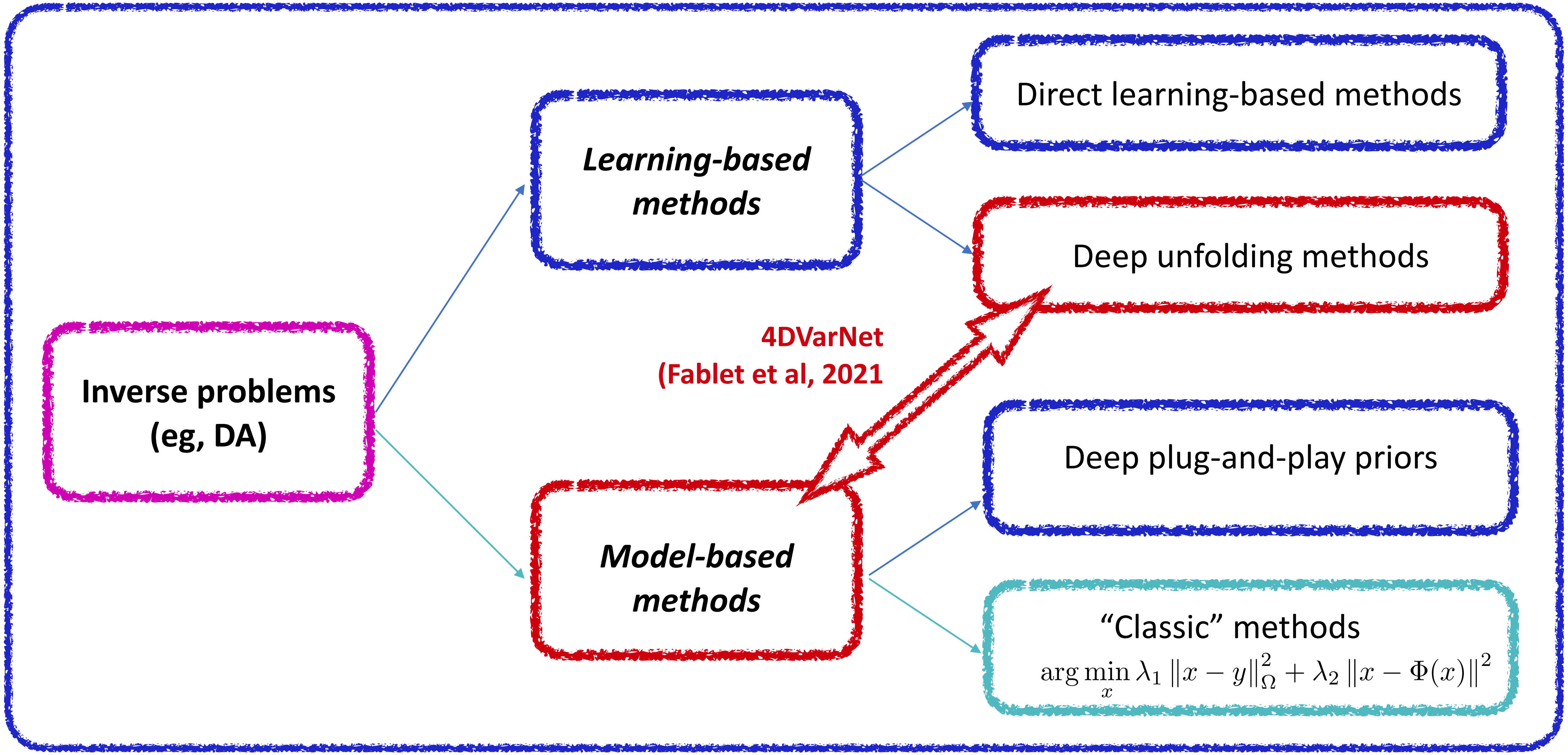
Iterative gradient-based update

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k+1)} + \mathcal{R}\left(\nabla_{\mathbf{x}} U_{\mathcal{H},\Phi}(\mathbf{x}, \mathbf{y})\right)$$



https://doi.org/10.1029/2021MS002572

Model-driven vs. Learning-based approaches



4DVarNet: Trainable 4DVar Models and Solvers

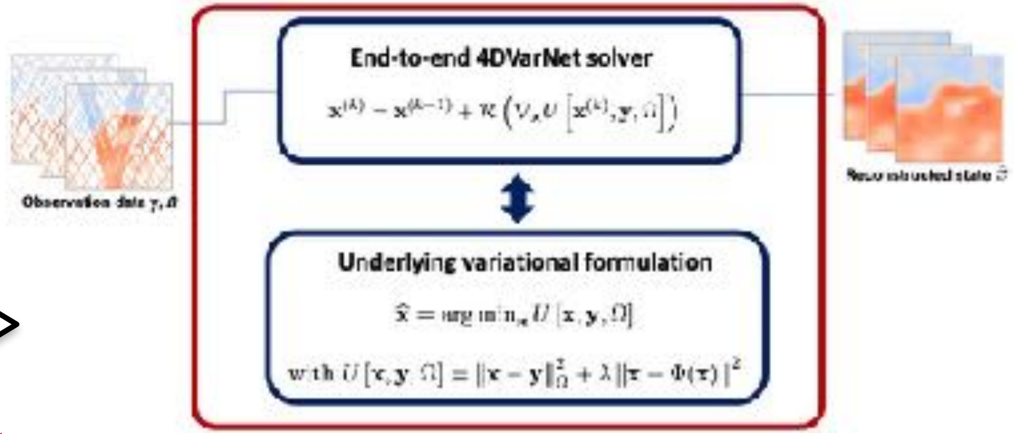
From a Variational DA formulation

$$\widehat{x} = \arg\min_{x} \|y - H(x)\|^2 + \lambda \|x - \Phi(x)\|^2$$

Trainable variational model

Trainable gradient-based solver

Associated end-to-end scheme



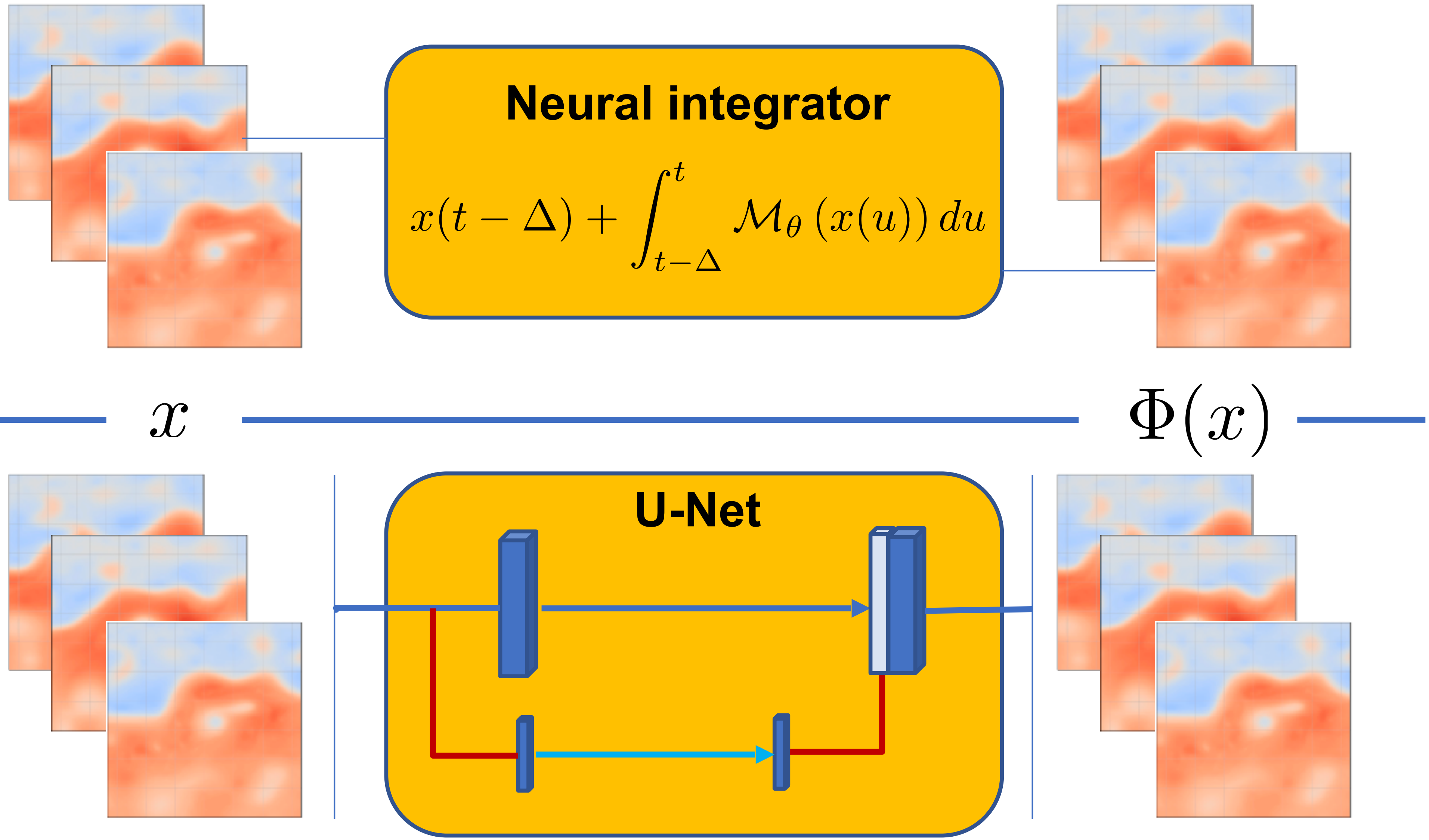
Which training loss ?

Which parameterisation for operator Φ ?

4DVarNet architecture: projection operator Φ

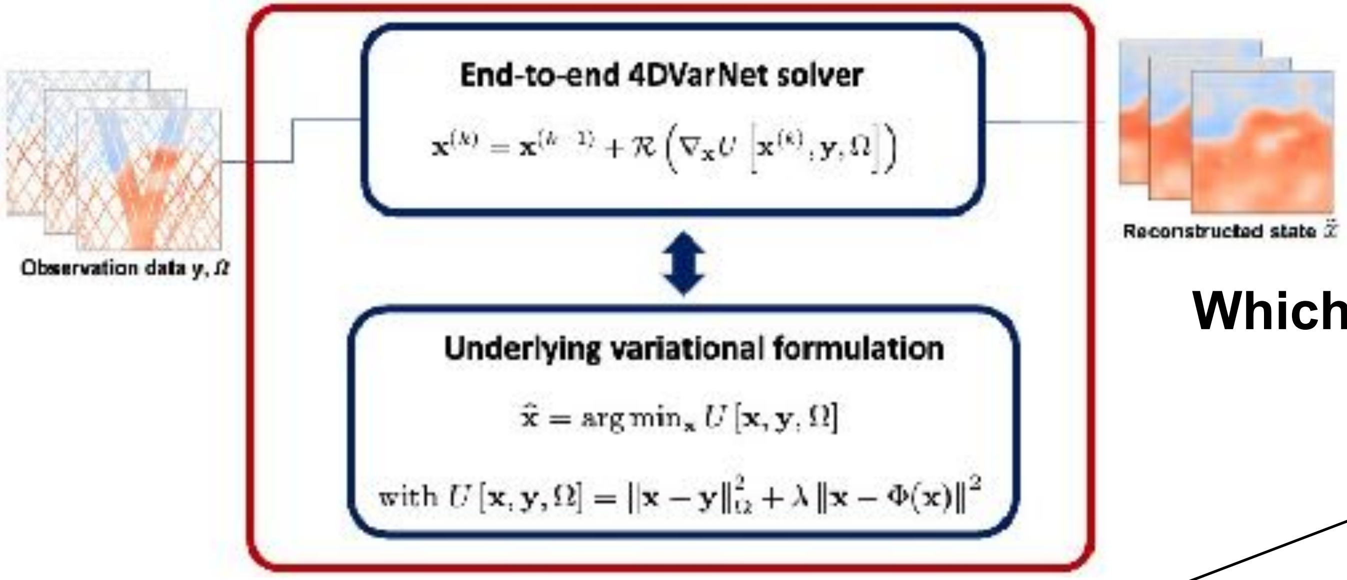
Parameterization using (learnable) ODE operator

$$\frac{\partial x(t)}{\partial t} = \mathcal{M}_\theta\left(x(t)\right)$$



Two-scale U-Net-like Parameterization (Gibbs Field)

Model-based vs. Learning-based 4DVar DA: Unsupervised vs. Supervised scheme



Which training loss for 4DVarNet scheme ?

Unsupervised loss

$$\mathcal{L}(x, y) = \|x - y\|^2 + \lambda \|x - \Phi(x)\|^2$$

Supervised loss

$$\mathcal{L}\left(x, x^{true}\right) = \left\|x - x^{true}\right\|^2$$

Regularisation loss

$$\mathcal{L}_{Reg}\left(x, x^{true}\right) = \|x - \Phi(x)\|^2 + \left\|x^{true} - \Phi(x^{true})\right\|^2$$

4DVarNet: Trainable 4DVar Models and Solvers

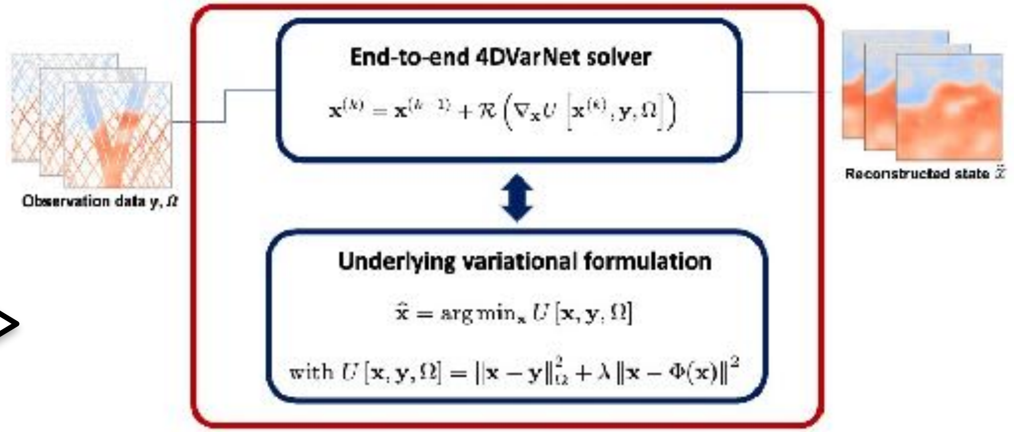
From a Variational DA formulation

$$\widehat{x} = \arg\min_x \|y - H(x)\|^2 + \lambda\|x - \Phi(x)\|^2$$

Trainable variational model

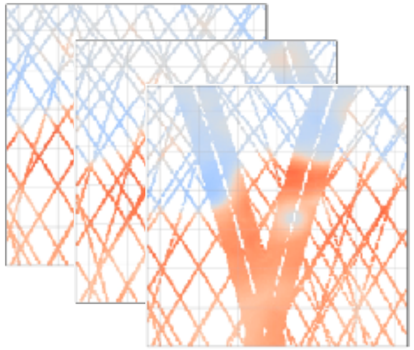
Trainable gradient-based solver

Associated end-to-end scheme

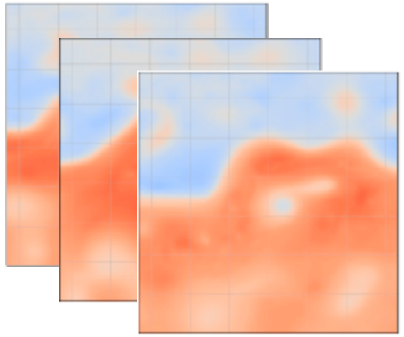


https://github.com/CIA-Oceanix/DLGD2022/blob/main/lecture-4-dl-oi-da/notebooks/notebookPyTorch_InvProb_LearningBased_4DVarNet_L63.ipynb

Model-based vs. Learning-based 4DVar DA



Partial observations y



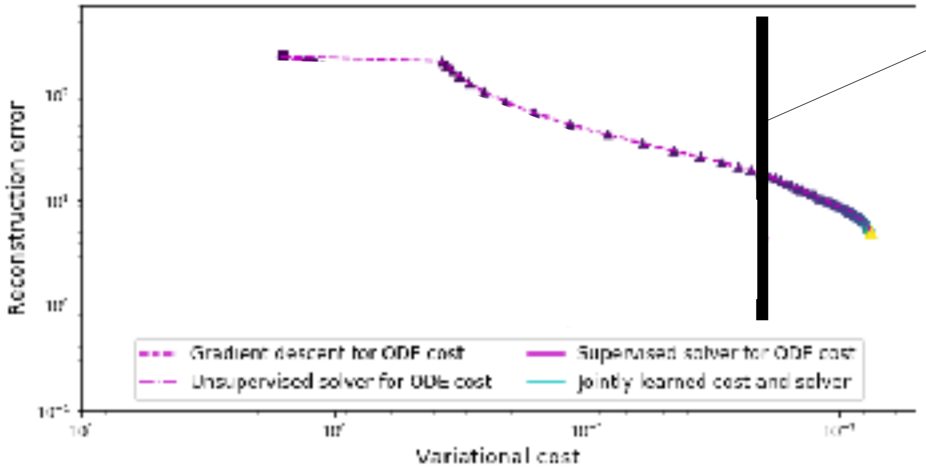
True states x

Model-driven schemes: $\widehat{x} = \arg\min_x \underbrace{\lambda_1 \left\| x - y \right\|_\Omega^2 + \lambda_2 \left\| x - \Phi(x) \right\|^2}_{U_\Phi\left(x^{(k)}, y, \Omega\right)}$

Gradient-based solver (adjoint/Euler-Lagrange method):

$$x^{(k+1)} = x^{(k)} - \alpha \nabla_x U_\Phi \left(x^{(k)}, y, \Omega\right)$$

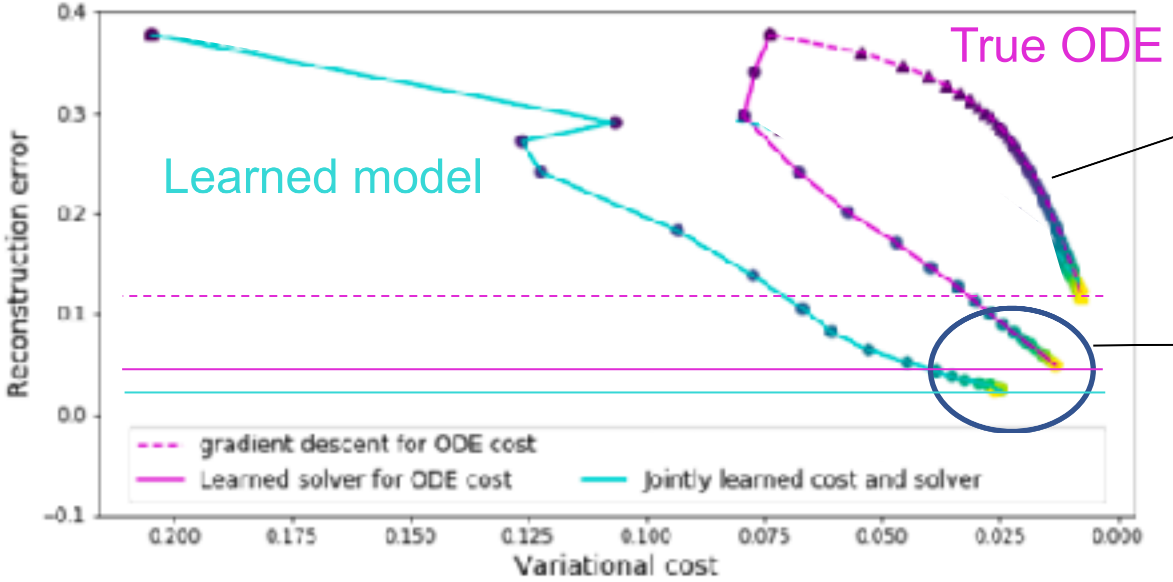
No control on the reconstruction error $x^{true} \neq \arg\min_x U_\Phi \left(x^{(k)}, y, \Omega\right)$



Variational cost
for the true state

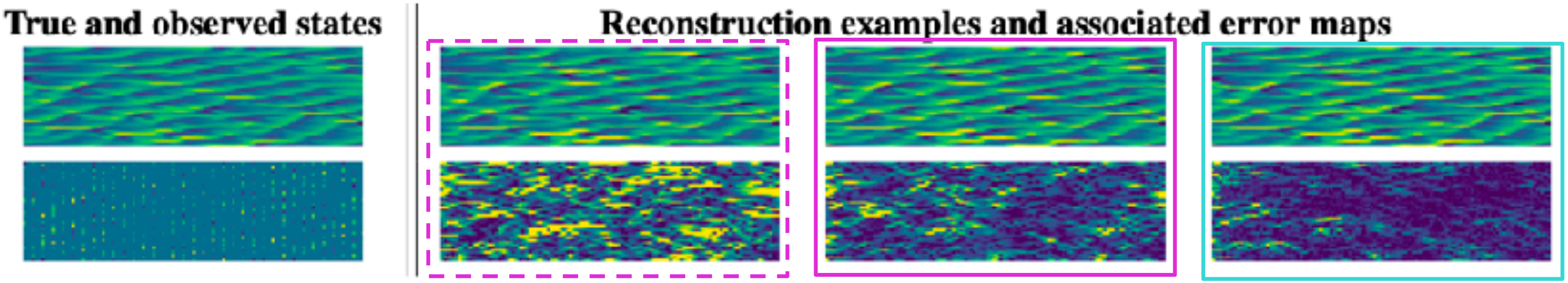
End-to-end learning for inverse problems (Fablet etal., 2020)

Illustration on Lorenz-96 dynamics (Bilinear ODE)



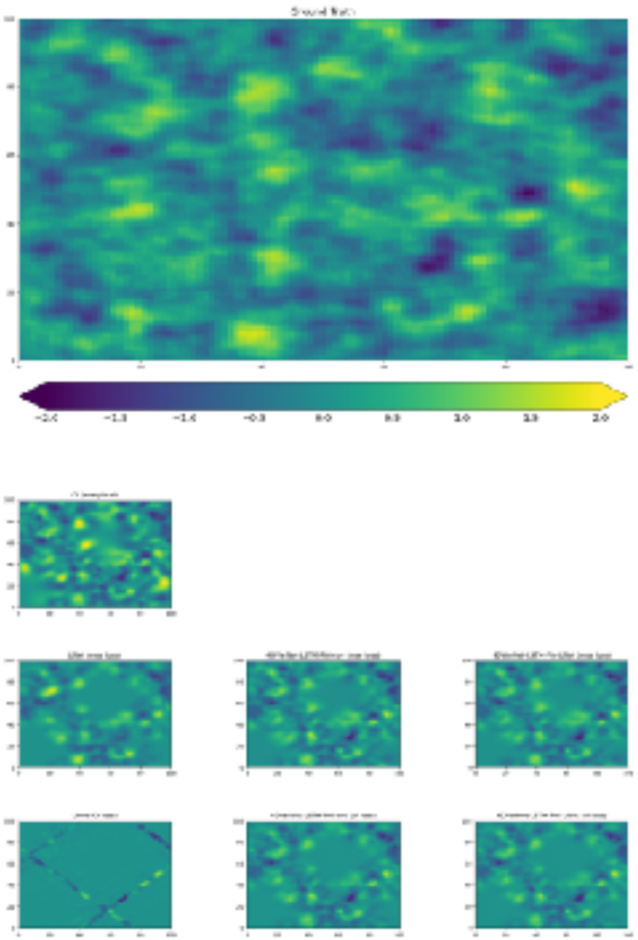
Non-supervised setting
(classic variational minimisation)

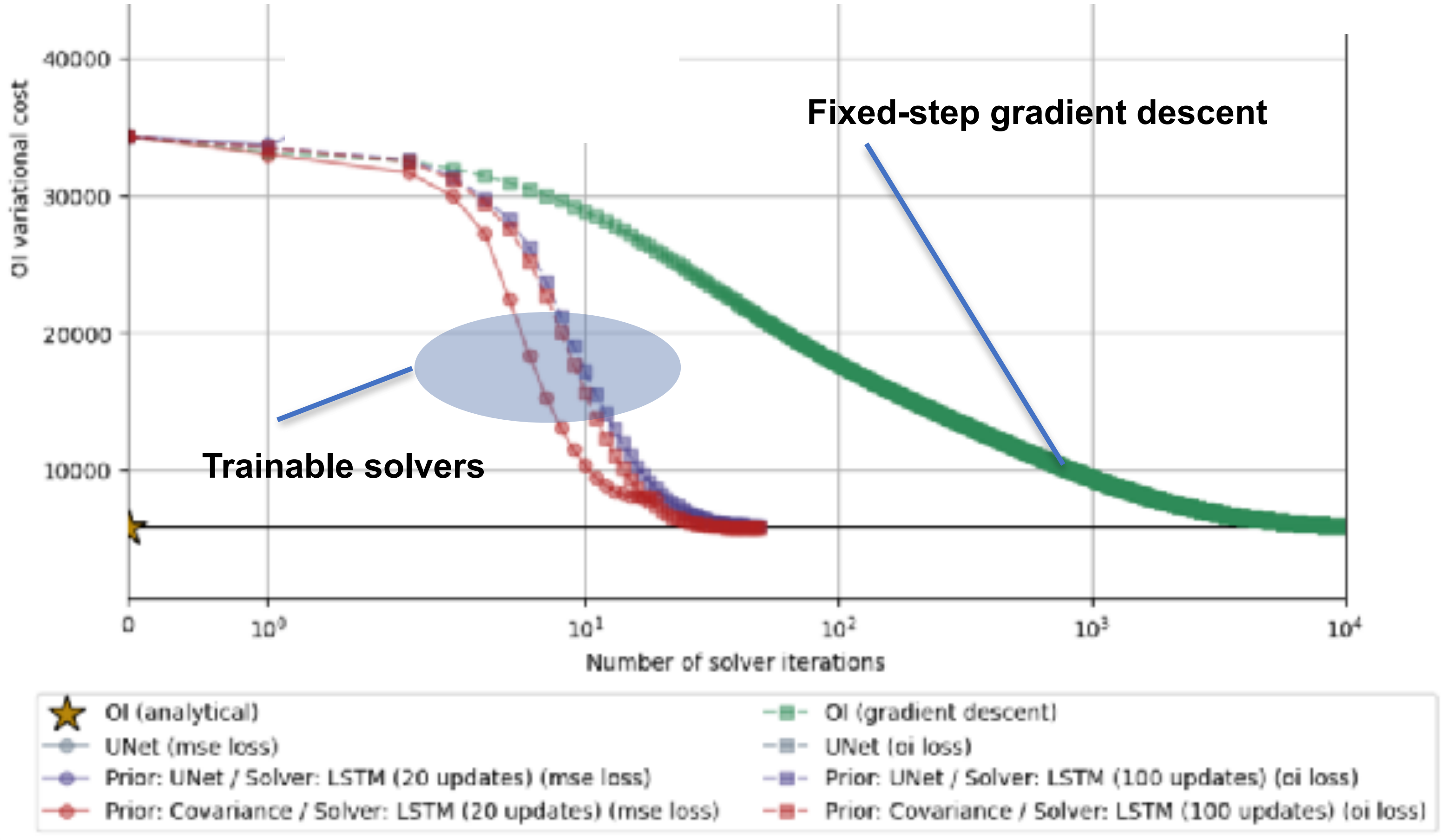
Supervised settings



4DVarNet and Optimal Interpolation

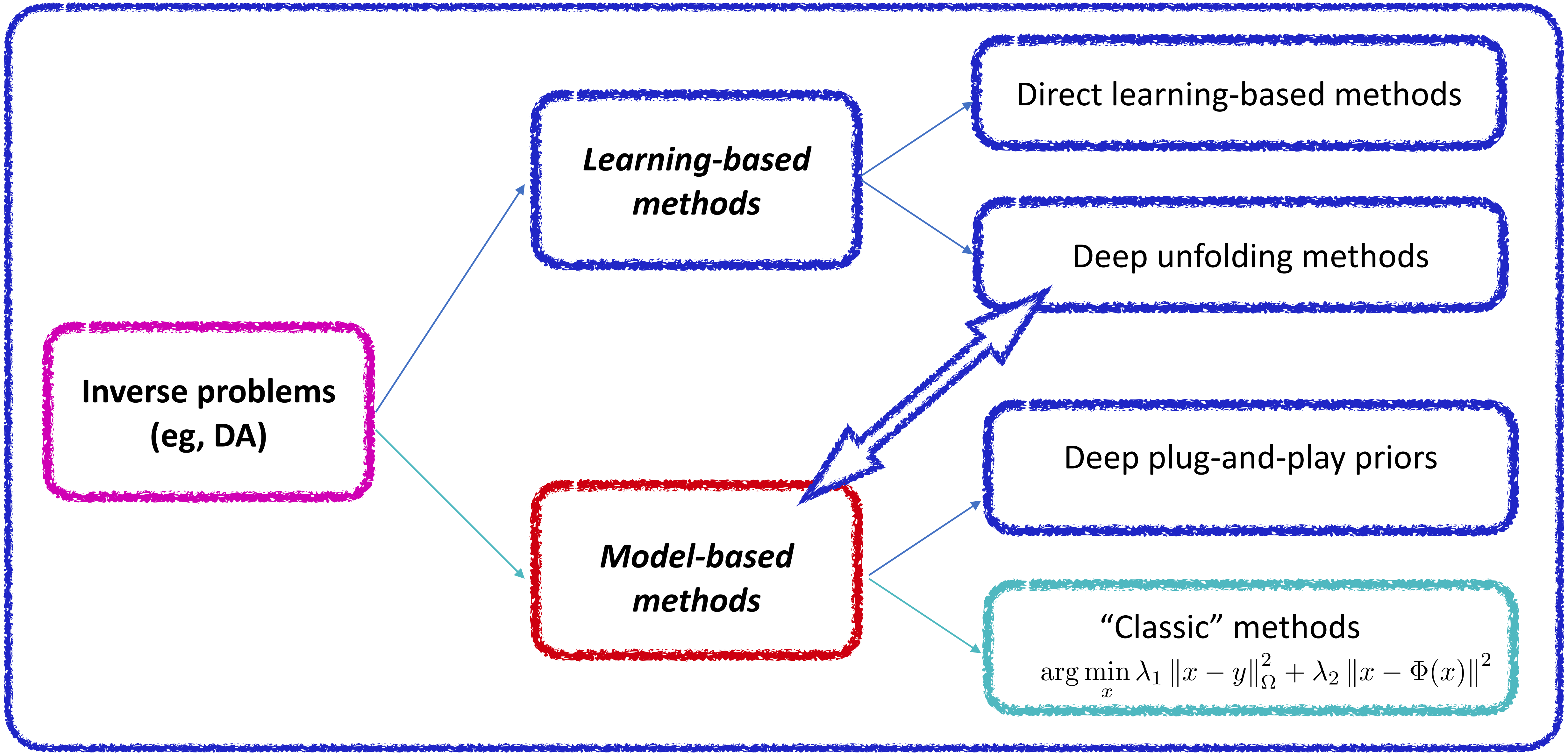
Case-study for an isotropic Gaussian Process





Summary on Inverse Problems and Deep Learning

Model-driven vs. Learning-based approaches



Summary

- ***NNs as numerical schemes for ODE/PDE/variational representations of geophysical dynamics***

- ***NN plug-and-play priors***

- ***End-to-end architecture for jointly learning a representation (**eg, ODE or NN prior**) and a solver***

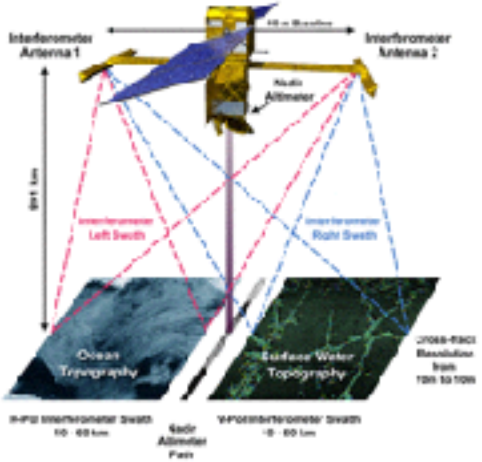
- ***Requirement for differential implementations***

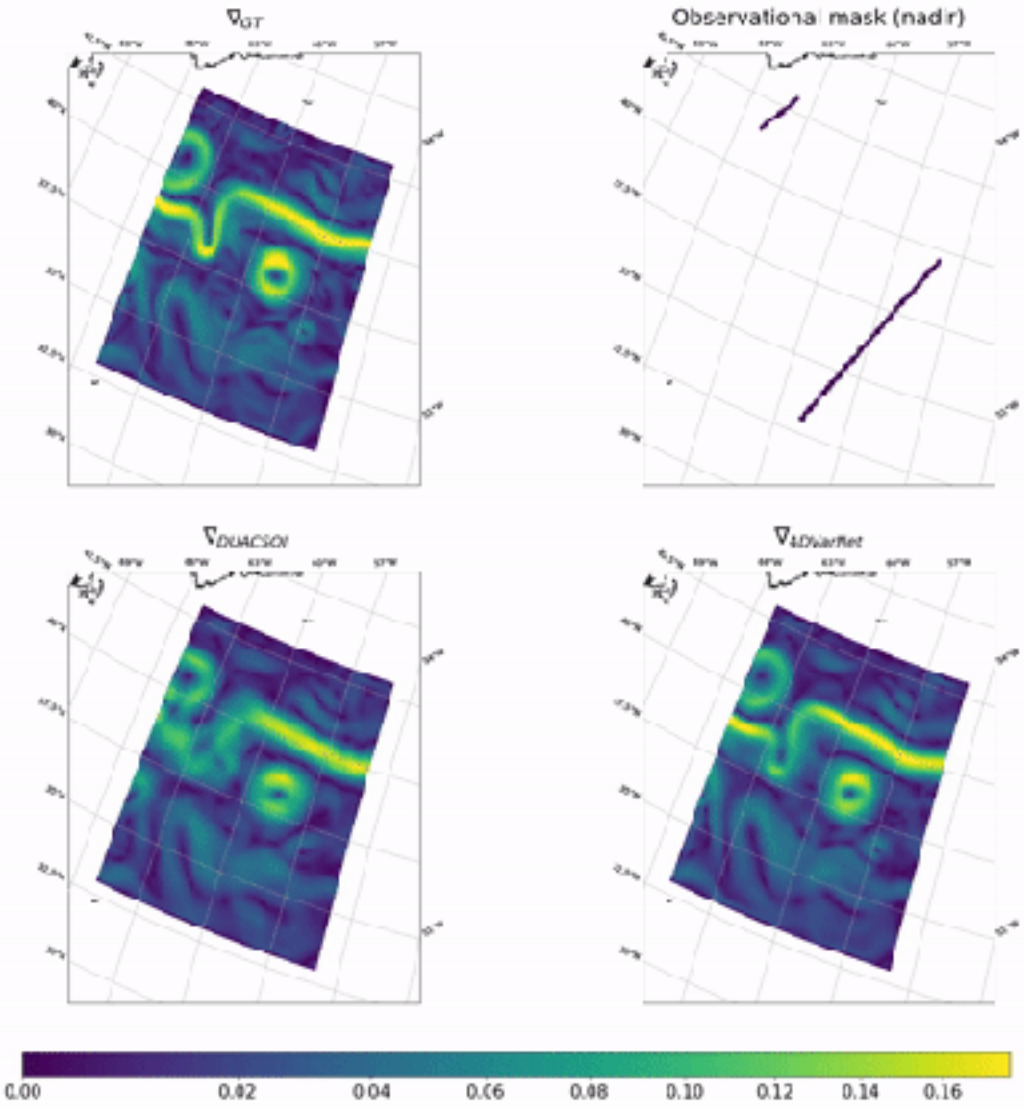
- ***The true prior might not be the optimal choice to solve inverse problems***

To which extent do neural DA schemes apply to real-world problems ?

4DVarNet as end-to-end DA schemes for space oceanography

Satellite altimetry





Best score for BOOST-SWOT SLA Data Challenge

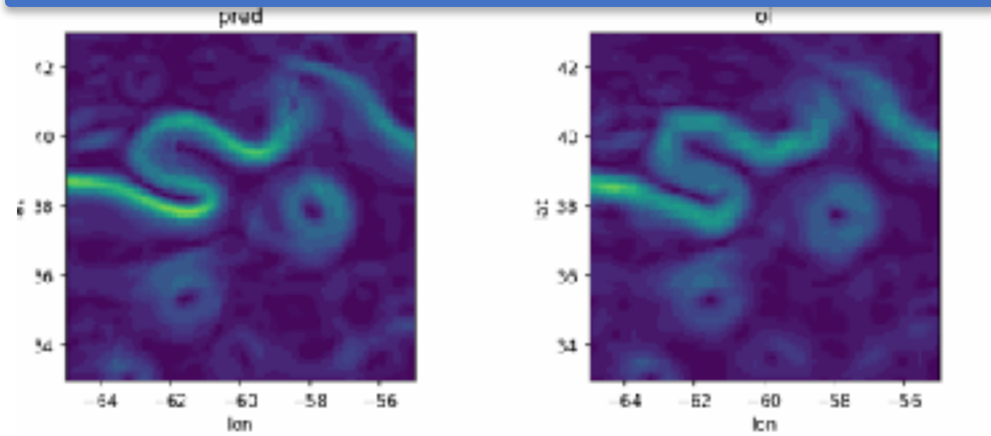
duacs 1 swot + 4 nadirs	0.92	0.02	1.22	11.15	Covariances DUACS	eval_duacs.ipynb
bfn 1 swot + 4 nadirs	0.93	0.02	0.8	10.09	QG Nudging	eval_bfn.ipynb
dymost 1 swot + 4 nadirs	0.93	0.02	1.2	10.07	Dynamic mapping	eval_dymost.ipynb
miost 1 swot + 4 nadirs	0.94	0.01	1.18	10.14	Multiscale mapping	eval_miost.ipynb
4DVarNet 1 swot + 4 nadirs 🏆	**0.96**	**0.01**	**0.70**	**4.15**	4DVarNet mapping	eval_4dvarnet.ipynb

https://github.com/ocean-data-challenges/2020a_SSH_mapping_NATL60

Application to real satellite-derived datasets

SLA mapping OSE Data Challenge

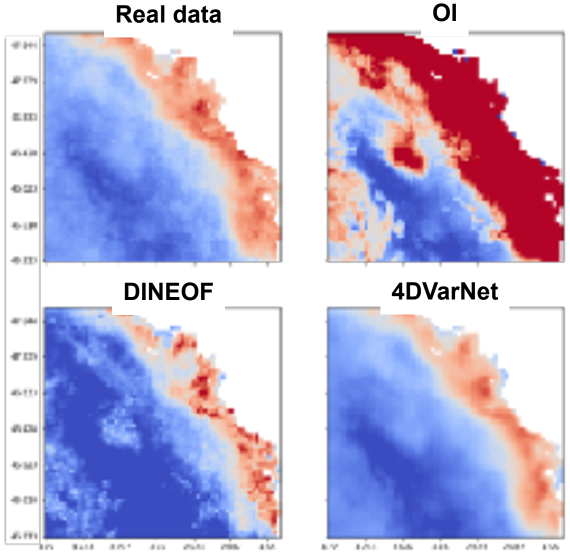
Method	µ(RMSE)	σ(RMSE)	λx (km)	Notes
BASELINE	0.85	0.09	140	Covariances BASELINE OI
DUACS	0.88	0.07	152	Covariances DUACS DT2018
MIOST	0.89	0.08	139	Multiscale mapping
DYMOST	0.89	0.06	129	Dynamic mapping
BNF	0.88	0.06	122	BFN mapping
4DVarNet	0.89	0.06	102	4DVarNet mapping



https://github.com/ocean-data-challenges/2021a_SSH_mapping_OSE

Sea surface suspended sediments

Metric	Dataset	Unit	Samp. Strat	OI	DinEOF	4DVarNet
RMSE	OSSE	$\log_{10}[g/L]/m$	-	0.176	0.167	**0.104**
	MODIS	$\log_{10}[g/L]/m$	Random	0.304	0.237	**0.156**
	MODIS	$\log_{10}[g/L]/m$	Patch	0.346	0.253	**0.168**
R-score	OSSE	%	-	90.4	91.3	**96.6**
	MODIS	%	Random	60.5	76.4	**89.5**
	MODIS	%	Patch	56.5	73.8	**87.3**



Mean gradient norm

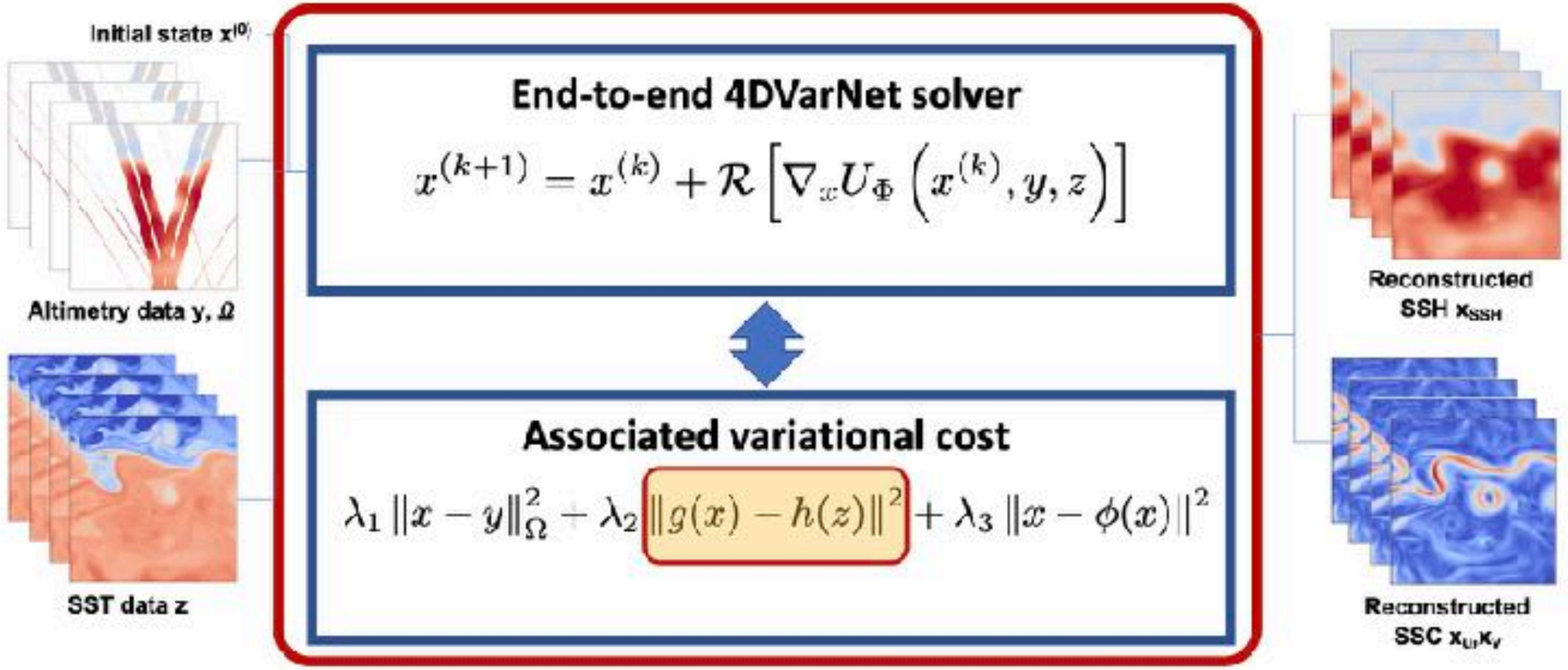
Simulation and real data

Learning from real gappy data only

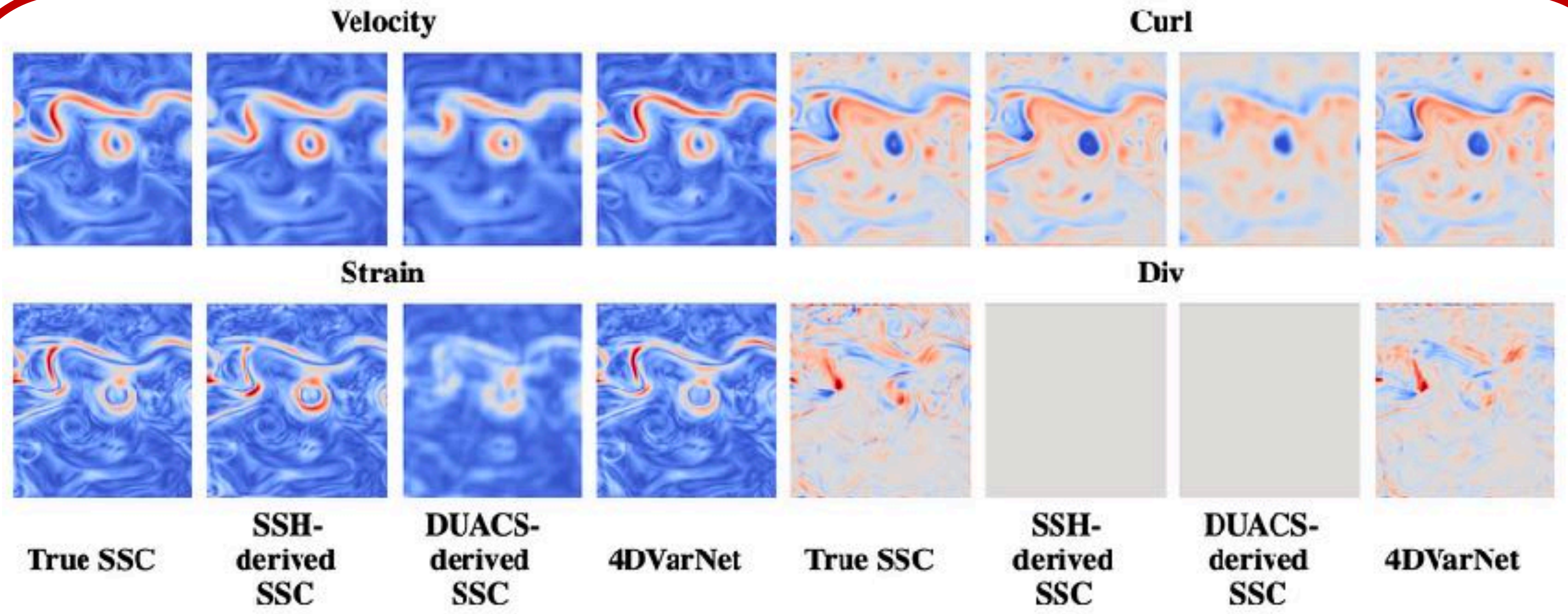
Vient et al., 2022

https://doi.org/10.3390/rs14164024

Can we inform sea surface dynamics which are never directly observed ?



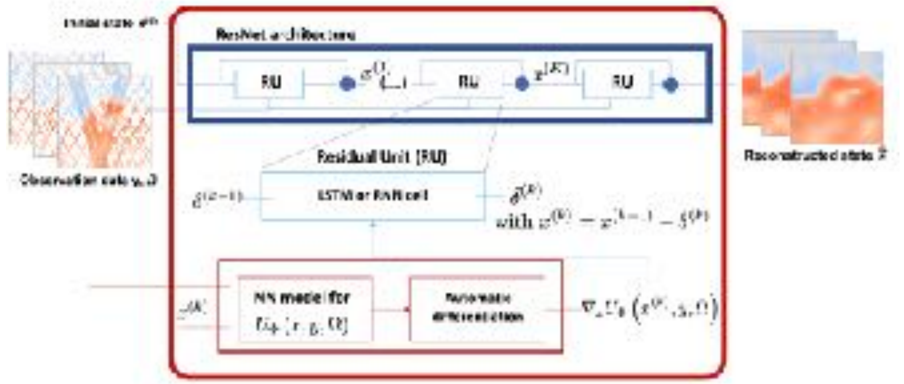
4DVarnet to inform sea surface currents (Fablet et al., arXiv 2022)

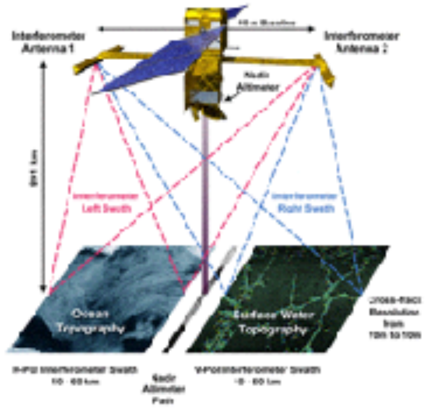


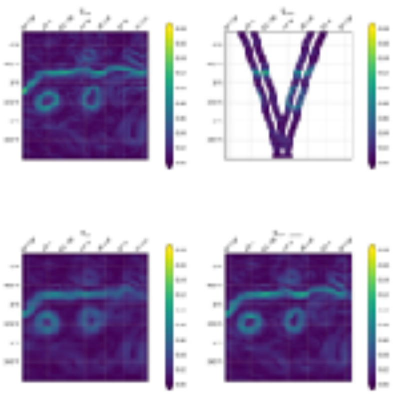
4DVarnet to recover a significant fraction of the unobserved agesotropvhic (divergent) currents

Key messages

- **Physics-informed learning for satellite ocean remote sensing**
- **Trainable variational DA models (**observation model, prior, solver**)**
- **Application to interpolation, forecasting sampling and multimodal synergies**
- **Generic framework beyond space oceanography**
- **Objective-specific vs. Generic priors and DA schemes ?**







Preprint: https://doi.org/10.1029/2021MS002572
Code: https://github.com/CIA-Oceanix/4dvarnet-core

Thank you.

AI Chair OceaniX 2020-2024

Physics-informed AI for Observation-Driven Ocean AnalytiX

PI: R. Fablet, Prof. IMT Atlantique, Brest
Web: https://cia-oceanix.github.io/

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