Supplement Material for "Dynamic Constrained Evolutionary Algorithm Based on Deep Q-Network"

Zhengping Liang, Ruitai Yang, Qifan Shi, Yingying Zhu, Xiyu Wang, and Zexuan Zhu, *Senior Member, IEEE*

I. TIME COMPLEXITY ANALYSIS

The time consumption of DCEA-DQN is mainly composed of the time required for change response and the time required for static optimization. The change response includes calling to DQN_d, population reinitialization, generation of guidance set for mutation and DQN_d training. The time complexity of calling to DQN_d is $O(\tau \sum_{i=1}^{\mathcal{H}}(DNc^2 o_{i-1} o_i))$, where τ is the maximum number of cycles during change response in Algorithm 2 of the main file, $\mathcal{H}$ is the depth of convolutional neural network of DQN_d, c is the side length of each convolution kernel, o_i is the number of output channels at the i-layer of the network, N is the population size, and D is the dimension of decision variables. The time complexity of population reinitialization and generation of guidance set for mutation are $O(\tau NlogN)$ and $O(Nke)$ respectively, where k is the number of clustering points, e is the maximum number of iterations of clustering. The time complexity of DQN_d, training is $O(b_d \sum_{i=1}^{\mathcal{H}}(DNc^2 o_{i-1} o_i))$, where b_d is the number of training data of DQN_d,. To sum up, the time complexity of performing a change response is $\max\left(O\left(\tau \sum_{i=1}^{\mathcal{H}}(DNc^2 o_{i-1} o_i)\right), b_d \sum_{i=1}^{\mathcal{H}}(DNc^2 o_{i-1} o_i)\right)$.

The static optimization between two changes includes penalty operation, calling to DQN_m, directed mutation, crossover, selection and DQN_m training. The time complexity of penalty operation is $O(WN^2)$, where W is the maximum number of cycles for each static optimization. The time complexity of calling to DQN_m and directed mutation are $O(W \sum_{i=1}^{\mathcal{H}}(DNc^2 o_{i-1} o_i))$ and $O(W\alpha D)$ respectively, where α is the number of mutated individuals. The time complexity of simulated binary crossover and selection operation based on fitness and constraint violation are $O(WN)$ and $O(WNlogN)$ respectively. The time complexity of DQN_m training is $O(Wb_m \sum_{i=1}^{\mathcal{H}}(DNc^2 o_{i-1} o_i)/\delta)$, where b_m is the number of training data of DQN_m, δ is the training gap of DQN_m. On the whole, the time complexity of static optimization between changes is $\max\left(O\left(W \sum_{i=1}^{\mathcal{H}}(DNc^2 o_{i-1} o_i)\right), Wb_m \sum_{i=1}^{\mathcal{H}}(DNc^2 o_{i-1} o_i)/\delta\right)$.

II. TEST SUITE

MPB[1] is a classical dynamic multimodal test suite. The objective function of each test problem of MPB is composed of multiple interacting peaks. The shape of each peak is controlled by three parameters: time-varying center, height and width. The objective of each test problem is to find the center coordinates of the peak with the highest height. Because the shape of each peak in MPB is smooth and symmetrical, and changes in a single-mode way, the decision variables are also completely separable, which makes MPB easier to be optimized.

Based on MPB, Yazdani et al. proposed a DCOPs test suite GMPB [2] by adding circular constraints to the center of some peaks, limiting the feasible region to the local region associated with the center of these peaks. When dynamic changes occur, not only the center, width and height of each peak may change, but also the center and radius of each feasible region may shift and change, which has excellent time-varying characteristics. At the same time, the severity parameter of each change can be adjusted, and the constructed test suite has certain difficulty adjustability. Due to MPB is not complex enough, GMPB is still relatively easy to be optimize, so the ability of GMPB to distinguish the performance of different DCEAs is insufficient.

In order to better verify the performance of DCEA-DQN in dealing with complex DCOPs, this paper proposes a new test suite C-GMPB, which is design more difficulties in test suite construction and perform better algorithm performance differentiation based on the GMPB and the constraint addition method in [3]. C-GMPB can be divided into three sub-test suites C-GMPB-C, C-GMPB-E, and C-GMPB-H by adding constraints at the peak center, peak edge, and at different peak centers and peak edges respectively. By setting different parameters for separability, ill-conditioning, change mode, change degree, number of feasible regions, each sub-test suite contains eight test problems with increasing difficulty in dynamic change.

The definition of C-GMPB is as follows:

$$f^{(t)}(x) = \max_{i\in\{1,\dots,m\}}\left\{h_i^{(t)} - \sqrt{\mathbb{T}((x-c_i^{(t)})^T R^{(t)T}, i) W^{(t)} \mathbb{T}(R^{(t)}(x-c_i^{(t)}), i)}\right\}$$

where,

$$\mathbb{T}(y_j, i) = \begin{cases} \exp\left(log(y_i) + \tau_i^{(t)}\left(sin(\eta_{i,1}^{(t)}\, log(y_i)) + sin(\eta_{i,2}^{(t)}\, log(y_j))\right)\right) & if\ y_i > 0 \\ 0 & if\ y_i = 0 \\ -\exp\left(log(|y_j|) + \tau_i^{(t)}\left(sin(\eta_{i,3}^{(t)}\, log(|y_j|)) + sin(\eta_{i,4}^{(t)}\, log(|y_j|))\right)\right) & if\ y_i < 0 \end{cases}$$

$$\text{Subject to:} \begin{cases} g_1(x,t) = \sum_{j=1}^{D}(x - C_{1,j}(t))^2 - r_1^2(t) \le 0 \\ \text{or } g_2(x,t) = \sum_{j=1}^{D}(x - C_{2,j}(t))^2 - r_2^2(t) \le 0 \\ \dots \\ \text{or } g_l(x,t) = \sum_{j=1}^{D}(x - C_{l,j}(t))^2 - r_l^2(t) \le 0 \\ \text{or } g_{l+1}(x,t) = \sum_{j=1}^{D}(x - C_{l+1,j}(t))^2 - r_{l+1}^2(t) \le 0 \\ \text{or } g_{l+2}(x,t) = \sum_{j=1}^{D}(x - C_{l+2,j}(t))^2 - r_{l+2}^2(t) \le 0 \\ \dots \\ \text{or } g_{l+p}(x,t) = \sum_{j=1}^{D}(x - C_{l+p,j}(t))^2 - r_{l+p}^2(t) \le 0 \end{cases} \quad (1)$$

$$\text{Dynamic change:} \begin{cases} c_{i,j}^{(t+1)} = c_{i,j}^{(t)} + \tilde{c}_i \frac{r}{||r||} \\ h_{i,j}^{(t+1)} = h_{i,j}^{(t)} + \tilde{h}_i \mathcal{N}(0,1) \\ w_{i,j}^{(t+1)} = w_{i,j}^{(t)} + \tilde{w}_i \mathcal{N}(0,1) \\ \theta_{i,j}^{(t+1)} = \theta_{i,j}^{(t)} + \tilde{\theta}_i \mathcal{N}(0,1) \\ \eta_{i,l}^{(t+1)} = \eta_{i,l}^{(t)} + \tilde{\eta}_i \mathcal{N}(0,1), l \in \{1,2,3,4\} \\ \tau_i^{(t+1)} = \tau_i^{(t)} + \tilde{\tau}_i \mathcal{N}(0,1) \\ R_i^{(t+1)} = \prod_{(p,q)\epsilon\mathcal{P}} G_{(p,q)} \times R_i^{(t)} \end{cases}$$

where, x represents decision variables, D represents the dimensions of decision variables, m represents the number of peaks. $c_i^{(t)}$, $h_i^{(t)}$, $w_i^{(t)}$ and $\theta_i^{(t)}$ respectively represent the coordinates of the center, the height, the width and the rotation angle of the ith peak at time t. $R^{(t)}$ and $W^{(t)} = w_i^{(t)} * I$ respectively represent the matrix controlling peak rotation and the matrix controlling peak stretching at time t, I is the Identity matrix. $\mathbb{T}(y_j, i)$ represents the noise control function, $\eta_{i,1}^{(t)}$, $\eta_{i,2}^{(t)}$, $\eta_{i,3}^{(t)}$, $\eta_{i,4}^{(t)}$ and $\tau_i^{(t)}$ all are irregular noise control vectors at time t.

l represents the number of peak center constraint; p represents the number of peak edge constraints. $g_i(\vec{x}, t)$ represents the ith constraint added to the peak center, $C_{i,j}^{(t)}$ and $r_i(t)$ respectively represent the jth dimension of the center and width of $g_i(\vec{x}, t)$. $g_{l+i}(\vec{x}, t)$ represents the ith constraint added to the peak edge, $C_{l+i,j}^{(t)}$ and $r_{l+i}(t)$ represent the j-dimensional of the center and width of $g_{l+i}(\vec{x},t)$ respectively.

$\tilde{c_i}$, $\tilde{h_i}$, $\tilde{w_i}$, $\tilde{\theta_i}$, $\tilde{\eta_i}$ and $\tilde{\tau_i}$ respectively represent the severity parameter of the center c, the height h, the width w, the rotation angle θ, the noise η and the noise τ of the ith peak when dynamic change occurred. $\mathcal{N}(0,1)$ is the standard normal distribution, r is a random vector subject to standard normal distribution. $\mathcal{P}$ contains all unique pairs of dimensions defining all possible planes in a D-dimensional space. $G_{(p,q)}$ is firstly initialized to an identity matrix, and then four elements of $G_{(p,q)}$ are altered as:

$$\begin{cases} G_{(p,q)}(p,p) = \cos\left(\theta^{(t)}\right) \\ G_{(p,q)}(q,q) = \cos\left(\theta^{(t)}\right) \\ G_{(p,q)}(p,q) = -\sin\left(\theta^{(t)}\right) \\ G_{(p,q)}(q,p) = \sin\left(\theta^{(t)}\right) \end{cases} \tag{2}$$

where, $G_{(p,q)}(i,j)$ represents the element of row i and column j of $G_{(p,q)}$.

The three sub-test suites C-GMPB-C, C-GMPB-E and C-GMPB-H, each of them contains eight test problems with increasing difficulty in dynamic changes. The objective function of each test problem and the parameter settings related to dynamic changes are consistent with [3]. C-GMPB-C, C-GMPB-E and C-GMPB-H adopt the same constraint method for the corresponding test problems with the same dynamic change difficulty. The constraint parameter settings of each test problem are shown in table S1.

TABLE S1 THE CONSTRAINT PAREMETERS SETTINGS OF EACH TEST PROBLEM OF C-GMPB

	P1	P2	P3	P4	P5	P6	P7	P8
number of feasible region	3		6		3		6	
radius of feasible region	6							

III. EFFECTIVENESS ANALYSIS OF THE CHANGE RESPONDSE BASED ON *DQN*$_d$

In order to analyze the effectiveness of the change response based on DQN_d separately, four representative algorithms with different change response mechanism are selected from the comparison algorithms, namely, RIR-SRES based on diversity control, GSA+Repair based on memory, and LTFR-DSPSO and DyCODE based on hybrid methods. The dynamic response mechanisms of the four algorithms are replaced by algorithm 2, and the other parts remain unchange. The resulting variant algorithms are named RIR-SRES-DQN$_d$, GSA+Repair-DQN$_d$, LTFR-DSPSO-DQN$_d$ and DyCODE-DQN$_d$ respectively.

Table S2 shows the mean and standard deviation of E_B of four representative algorithms and their DQN_d variants on 24 test problems of C-GMPB over 20 independent runs. The best results of each pair algorithms are highlighted in grey. The Wilcoxon rank sum test at the significance level of 5% is conducted on the experimental results. The symbols "+", "-" and "≈" indicate that the corresponding results of variant are better than, worse than, and comparable to that of the original algorithm, respectively.

TABLE S2 E_B OF FOUR REPRESENTATIVE ALGORITHMS AND THEIR DQN_d VARIANTS ON C-GMPB OVER 20 INDEPENDENT RUNS

C-GMPB		RIR-SRES-DQN$_d$	RIRSRES	GSA+Repair-DQN$_d$	GSA+Repair	LTFR-DSPSO-DQN$_d$	LTFR-DSPSO	DyCODE-DQN$_d$	DyCODE
P1	C1	1.8118E-01 (1.7354E-02)	6.2073E-01 (9.9921E-02)	1.9206E-01 (4.4726E-03)	6.0214E-01 (6.5338E-02)	1.2743E-03 (6.8550E-02)	3.4646E-03 (3.7849E-02)	8.6681E-03 (4.6006E-03)	8.5671E-03 (3.6715E-02)
	E1	1.7216E-01 (1.5511E-03)	6.2386E-01 (9.8964E-02)	2.8343E-01 (2.1956E-04)	5.9390E-01 (3.3184E-02)	2.6697E-03 (5.2942E-02)	2.3351E-02 (2.8141E-02)	8.7043E-02 (1.0013E-01)	8.6688E-02 (7.1642E-02)
	H1	7.4497E-01 (6.8424E-04)	6.3053E-01 (5.0686E-02)	6.9697E-01 (3.3590E-02)	6.0531E-01 (2.3754E-03)	8.8929E-03 (9.0991E-02)	8.5618E-03 (2.9425E-02)	8.8669E-02 (1.3155E-02)	8.6261E-02 (7.9680E-02)
P2	C2	3.3675E+00 (3.2824E-01)	2.9896E+00 (2.6443E-02)	3.1546E+00 (5.5462E-02)	3.0256E+00 (7.7091E-02)	1.6010E-03 (9.2796E-03)	1.6302E-02 (3.1973E-02)	3.3462E-02 (4.0588E-01)	8.2492E-01 (6.5675E-02)
	E2	2.0555E+00 (2.5602E-01)	3.0254E+00 (9.4513E-03)	2.5436E+00 (1.5987E-01)	2.9404E+00 (3.9441E-02)	8.6486E-01 (2.4956E-04)	1.7215E+00 (7.1696E-02)	8.0357E-01 (1.6456E-01)	7.7944E-01 (4.0490E-03)
	H2	2.2338E+00 (3.8363E-03)	3.4764E+00 (1.8650E-03)	2.6372E+00 (7.9686E-05)	3.1764E+00 (3.6975E-02)	1.3765E+00 (4.2430E-03)	2.1214E+00 (4.6966E-02)	2.3011E-02 (2.6146E-04)	9.9531E-02 (2.8980E-02)
P3	C3	6.3641E+00 (1.3807E-01)	1.7590E+01 (2.3399E-02)	6.1132E+00 (3.9431E-03)	1.5543E+01 (8.8042E-02)	2.2180E+00 (1.1154E-01)	9.8764E+00 (8.2489E-02)	2.1598E+00 (9.0744E-02)	4.8952E+00 (2.3935E-02)
	E3	1.0117E+01 (3.8441E-02)	1.7590E+01 (9.5381E-03)	5.9830E+00 (3.3074E-02)	1.4485E+01 (2.4053E-04)	7.8120E-01 (4.6850E-02)	1.0274E+01 (2.1411E-02)	5.1139E+00 (1.3807E-03)	4.9471E+00 (1.0349E-02)
	H3	9.3318E+00 (2.8831E-02)	1.8010E+01 (2.4010E-02)	6.3150E+00 (1.3397E-01)	1.6777E+01 (2.8203E-02)	3.6584E+00 (2.6987E-01)	9.9105E+00 (5.0490E-03)	1.8060E+00 (1.5709E-02)	2.4332E+00 (3.8047E-02)
P4	C4	9.4314E+00 (1.4270E-01)	2.2322E+01 (4.0237E+02)	6.8659E+00 (1.4742E-02)	2.2583E+01 (6.0937E-02)	5.9713E+00 (2.6527E-03)	1.3085E+01 (5.2549E-02)	3.7985E+00 (1.5363E-01)	6.3651E+00 (8.0305E-02)
	E4	1.3440E+01 (1.2037E-01)	2.2305E+01 (2.2694E+02)	7.7529E+00 (2.5067E-02)	2.2559E+01 (4.4637E-02)	5.5779E+00 (2.2223E-02)	1.3115E+01 (5.3452E-02)	4.1622E+00 (5.9208E-03)	6.3596E+00 (7.4225E-02)
	H4	1.1471E+01 (1.9009E-02)	2.2463E+01 (9.3531E+04)	8.9030E+00 (6.9309E-02)	2.3513E+01 (7.7429E-02)	7.3033E+00 (8.5150E-04)	1.3517E+01 (2.0507E-02)	4.7881E+00 (2.6049E-03)	6.7559E+00 (9.6956E-03)
P5	C5	1.3210E+01 (9.2691E-02)	2.9454E+01 (7.6301E+02)	9.3370E+00 (5.2000E-02)	3.0383E+01 (2.6606E-04)	9.7178E+00 (1.4471E-01)	1.7217E+01 (6.3252E-02)	6.1594E+00 (6.3471E-02)	8.9687E+00 (3.1877E-02)
	E5	1.5907E+01 (1.1666E-02)	3.0314E+01 (4.9039E+02)	8.6744E+00 (7.7526E-03)	2.9594E+01 (9.6831E-02)	1.1595E+01 (1.3446E-03)	1.7259E+01 (1.9685E-02)	7.2511E+00 (1.0362E-02)	9.0225E+00 (2.8619E-02)
	H5	1.5779E+01 (8.0900E-02)	3.3243E+01 (7.0870E+03)	9.4334E+00 (1.9616E-01)	3.3897E+01 (8.2182E-03)	1.0645E+01 (3.6965E-02)	1.8677E+01 (2.3162E-02)	7.7011E+00 (5.9712E-02)	9.4404E+00 (2.0616E-02)
P6	C6	1.9844E+01 (6.5199E-03)	3.4882E+01 (7.3490E+03)	1.2835E+01 (1.4293E-01)	3.1082E+01 (4.2377E-02)	1.2457E+01 (7.7259E-03)	2.0420E+01 (6.4515E-02)	8.7212E+00 (1.5109E-01)	1.0434E+01 (5.7272E-02)
	E6	2.2575E+01 (2.9304E-03)	3.7479E+01 (3.2260E+02)	1.3705E+01 (1.5155E-01)	3.2041E+01 (3.4508E-02)	1.4862E+01 (1.6694E-01)	2.0454E+01 (2.4713E-02)	8.9547E+00 (1.0063E-01)	1.0252E+01 (8.0256E-02)
	H6	2.1823E+01 (7.1768E-02)	3.9756E+01 (3.9045E+02)	1.5780E+01 (2.9730E-02)	3.5713E+01 (2.7406E-02)	1.3366E+01 (3.2235E-02)	2.2653E+01 (7.4793E-03)	9.9096E+00 (8.7392E-03)	1.1705E+01 (2.5704E-02)
P7	C7	2.6928E+01 (7.2598E-04)	4.7151E+01 (3.3542E+02)	1.7488E+01 (2.8327E-01)	4.7932E+01 (5.4931E-02)	1.3370E+01 (5.3012E-02)	2.5126E+01 (1.5844E-02)	9.7324E+00 (8.2662E-02)	1.2404E+01 (5.7858E-03)
	E7	2.8510E+01 (3.9035E-01)	3.7153E+01 (6.9226E+02)	1.4542E+01 (2.1227E-02)	3.8030E+01 (8.4164E-02)	1.4636E+01 (1.8984E-01)	2.4452E+01 (2.5349E-02)	9.5364E+00 (2.4025E-01)	1.2346E+01 (5.5239E-02)
	H7	2.6491E+01 (2.6368E-02)	4.5155E+01 (2.1380E+02)	1.8386E+01 (7.8200E-03)	4.4956E+01 (1.3094E-02)	1.4955E+01 (5.6260E-02)	2.8773E+01 (3.7345E-02)	1.0782E+01 (4.5994E-04)	1.2629E+01 (5.1401E-03)
P8	C8	3.2192E+01 (7.1452E-02)	4.5149E+01 (8.5116E+02)	1.9307E+01 (1.0988E-01)	4.4142E+01 (2.7272E+02)	1.6142E+01 (6.5886E-02)	2.6295E+01 (7.6876E-02)	9.9629E+00 (2.8324E-03)	1.3599E+01 (6.1329E-02)
	E8	3.2595E+01 (3.0813E-01)	4.7151E+01 (3.3542E+02)	2.1594E+01 (2.7456E-02)	4.7932E+01 (5.4931E-02)	1.7737E+01 (2.8952E-02)	2.5126E+01 (1.5844E-02)	1.0537E+01 (1.9632E-02)	1.2404E+01 (5.7858E-03)
	H8	3.3961E+01 (1.5636E-02)	5.0222E+01 (1.4952E+02)	2.2985E+01 (8.0527E-02)	4.8192E+01 (4.0031E-02)	2.0212E+01 (1.5743E-02)	3.2393E+01 (7.8745E-02)	1.1567E+01 (1.3291E-01)	1.9818E+01 (7.9716E-02)
+/−/≈		24/0/0		24/0/0		23/0/1		20/0/4	

As can be seen from table S2, the E_B of RIR-SRES-DQN$_d$ and GSA+Repair-DQN$_d$ on all 24 test questions are better than the original algorithm. The E_B of LTFR-DSPSO-DQN$_d$ and DyCODE-DQN$_d$ also achieve better performance than the original algorithm on 23 test problems and 20 test problems, respectively. It shows that the change response based on DQN_d can help the algorithm closer to the real optimal individual by providing a better reinitialization population.

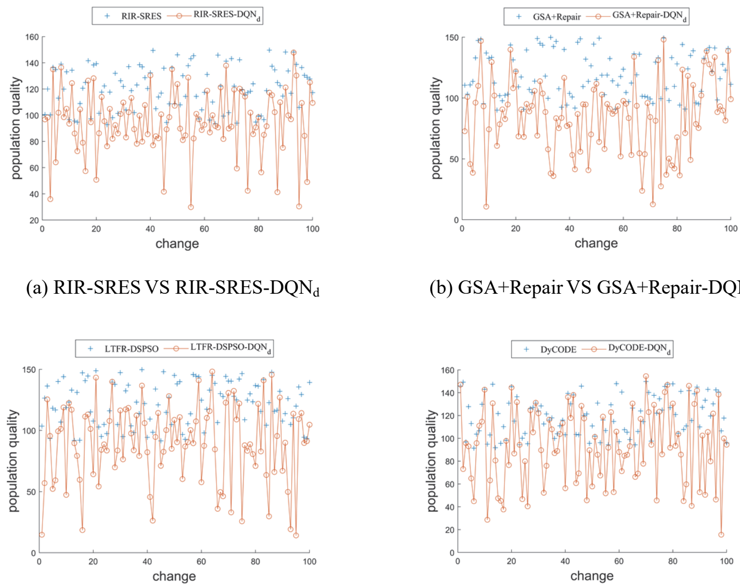
TABLE S3 E_O OF FOUR REPRESENTATIVE ALGORITHMS AND THEIR *DQN_d* VARIANTS ON C-GMPB OVER 20 INDEPENDENT RUNS

C-GMPB		RIR-SRES-DQN$_d$	RIRSRES	GSA+Repair-DQN$_d$	GSA+Repair	LTFR-DSPSO-DQN$_d$	LTFR-DSPSO	DyCODE-DQN$_d$	DyCODE
P1	C1	3.9903E+00	3.7874E+00	2.5620E+00	3.7874E+00	1.8487E+00	1.5651E+00	4.8823E-01	4.7956E-01
		(2.7713E-02)	(4.5506E-02)	(2.9998E-03)	(1.6139E-02)	(1.2505E-01)	(8.0026E-03)	(9.5470E-04)	(5.4125E-02)
	E1	5.7826E-02	6.4499E-01	2.6286E+00	5.0499E+00	2.2821E+00	2.0867E+00	6.9317E-01	6.4608E-01
		(1.1631E-02)	(8.9327E-03)	(9.8372E-02)	(1.7024E-02)	(9.7166E-02)	(2.5845E-02)	(3.2022E-03)	(3.8056E-02)
	H1	1.4439E+00	3.7645E+00	2.3598E+00	3.7367E+00	1.9546E+00	1.5633E+00	4.7105E-01	4.3544E-01
		(2.7585E-03)	(5.7735E-03)	(7.8207E-02)	(4.1502E-02)	(3.8658E-05)	(8.7211E-03)	(3.7041E-02)	(2.2721E-04)
P2	C2	2.9863E+00	8.1045E+00	3.6276E+00	9.6045E+00	7.4882E+00	7.3996E+00	1.3891E+00	1.9626E+00
		(8.1249E-04)	(1.5258E-02)	(8.8706E-03)	(1.3717E-02)	(8.5727E-02)	(7.9893E-02)	(1.4448E-01)	(3.9578E-02)
	E2	7.9792E-01	1.6806E+00	5.0044E+00	1.2806E+01	2.9440E+00	2.8661E+00	2.1254E+00	2.6168E+00
		(7.2101E-02)	(5.0947E-03)	(1.9974E-03)	(2.7682E-02)	(2.0671E-01)	(5.8230E-02)	(2.3504E-01)	(7.5611E-03)
	H2	3.9390E+00	8.0503E+00	1.3019E+01	9.8963E+00	1.6252E+00	1.5927E+00	1.4140E+00	1.9110E+00
		(3.4442E-02)	(9.9115E-03)	(5.4704E-02)	(4.7282E-02)	(6.4074E-02)	(4.8122E-04)	(2.0251E-03)	(6.6261E-02)
P3	C3	6.1979E+00	1.5786E+01	6.7271E+00	1.2786E+01	1.7857E+00	1.1044E+01	3.0099E+00	7.4673E+00
		(2.7606E-02)	(1.8290E-02)	(2.1918E-01)	(9.6810E-03)	(2.1406E-01)	(7.2453E-03)	(3.7437E-02)	(1.7648E-03)
	E3	7.1765E+00	2.1048E+01	7.3487E+00	1.7048E+01	5.7516E+00	1.4725E+01	4.5505E+00	7.9564E+00
		(2.9033E-02)	(1.4985E-02)	(1.3509E-02)	(1.0849E-03)	(2.7954E-01)	(4.5893E-02)	(6.4259E-02)	(5.7125E-03)
	H3	6.4163E+00	1.6066E+01	8.8494E+00	1.2717E+01	4.9167E+00	1.1267E+01	3.5832E+00	1.0937E+01
		(1.0193E-02)	(5.0463E-03)	(2.8961E-02)	(1.8626E-02)	(1.1976E-03)	(2.5795E-02)	(1.2841E-02)	(6.8282E-03)
P4	C4	8.8099E+00	2.8498E+01	8.8233E+00	2.8498E+01	1.1040E+01	1.8190E+01	6.4534E+00	7.5173E+00
		(6.3675E-02)	(2.4092E-02)	(7.5002E-03)	(6.6885E-02)	(8.3880E-03)	(1.7490E-03)	(3.3478E-03)	(9.8902E-02)
	E4	8.8379E+00	3.7998E+01	1.1972E+01	3.7998E+01	9.3470E+00	2.4253E+01	8.0163E+00	1.0023E+01
		(6.9834E-04)	(1.6367E-02)	(4.1522E-04)	(3.5700E-03)	(1.5155E-01)	(2.9918E-02)	(1.0305E-02)	(1.0156E-02)
	H4	9.8277E+00	2.8276E+01	1.6358E+01	2.8551E+01	1.0230E+01	1.8429E+01	5.9880E+00	7.7496E+00
		(6.2676E-02)	(6.7344E-03)	(4.9596E-01)	(3.9715E-02)	(1.6756E-01)	(1.9256E-02)	(1.0217E-03)	(1.5148E-02)
P5	C5	1.6826E+01	3.5901E+01	1.5236E+01	4.0401E+01	1.4923E+01	2.5741E+01	9.5831E+00	1.0956E+01
		(9.3174E-02)	(2.8092E-02)	(1.2052E-01)	(6.0374E-02)	(5.5559E-02)	(4.9464E-02)	(6.8703E-02)	(6.3961E-03)
	E5	1.8401E+01	4.7868E+01	1.9493E+01	5.3868E+01	1.8407E+01	3.4321E+01	1.0248E+01	1.1608E+01
		(7.2028E-03)	(4.6935E-02)	(4.9792E-03)	(7.8755E-03)	(2.7974E-02)	(5.3794E-02)	(1.9461E-02)	(1.5903E-02)
	H5	1.8995E+01	3.8415E+01	2.1451E+01	4.6404E+01	1.6793E+01	2.3609E+01	7.6156E+00	8.8778E+00
		(1.9547E-03)	(2.0457E-02)	(2.3621E-03)	(3.0659E-02)	(4.2601E-02)	(1.4752E-02)	(2.5090E-02)	(1.0690E-02)
P6	C6	1.5654E+01	5.1900E+01	2.4015E+01	4.7400E+01	2.1823E+01	3.9163E+01	1.1540E+01	2.1423E+01
		(2.1309E-02)	(8.5101E-02)	(2.8156E-02)	(1.8612E-02)	(4.9022E-02)	(2.8851E-02)	(2.4573E-02)	(2.5771E-02)
	E6	1.6339E+01	6.9200E+01	2.8580E+01	6.3200E+01	2.4675E+01	5.2217E+01	1.4173E+01	2.6230E+01
		(6.9354E-03)	(2.2295E-02)	(5.9279E-02)	(5.2723E-03)	(4.1684E-02)	(2.5847E-02)	(4.0243E-03)	(4.1841E-02)
	H6	1.8228E+01	5.4464E+01	2.9622E+01	4.9272E+01	1.9279E+01	3.9477E+01	1.4225E+01	2.2919E+01
		(1.5991E-03)	(3.0775E-02)	(3.9766E-02)	(1.4950E-02)	(6.2905E-03)	(1.8493E-03)	(2.3660E-02)	(2.2927E-02)
P7	C7	1.9369E+01	6.8505E+01	2.7874E+01	7.6005E+01	2.1592E+01	4.9800E+01	1.6475E+01	1.8722E+01
		(1.7915E-02)	(7.8051E-02)	(9.3305E-02)	(6.7119E-03)	(3.2590E-01)	(5.2375E-02)	(7.8947E-02)	(1.1867E-02)
	E7	2.1188E+01	9.1340E+01	3.1995E+01	1.0134E+02	2.6260E+01	6.6400E+01	1.8888E+01	2.4963E+01
		(8.9855E-02)	(1.4291E-02)	(8.1097E-03)	(2.8628E-02)	(1.9355E-02)	(1.1282E-03)	(3.7450E-03)	(1.2911E-03)
	H7	2.2718E+01	8.2416E+01	3.2054E+01	7.2044E+01	2.4099E+01	4.8487E+01	2.0413E+01	6.1563E+01
		(3.5259E-02)	(2.1108E-02)	(2.8902E-01)	(1.2568E-02)	(1.4995E-02)	(1.5765E-02)	(2.8967E-02)	(8.9393E-03)
P8	C8	2.6778E+01	8.9898E+01	3.7927E+01	8.2398E+01	3.1865E+01	6.6499E+01	2.0067E+01	2.7686E+01
		(2.3389E-02)	(4.3406E+02)	(6.4561E-02)	(2.9462E+03)	(3.0543E-02)	(4.4779E+02)	(1.9454E-01)	(1.5264E+02)
	E8	2.9992E+01	1.1986E+02	4.1309E+01	1.0986E+02	3.6924E+01	8.8665E+01	2.3334E+01	3.6915E+01
		(3.4537E-02)	(3.7324E-03)	(1.8834E-02)	(7.7428E-03)	(1.4613E-01)	(1.6021E-02)	(1.1605E-02)	(1.5296E-02)
	H8	3.6006E+01	9.8815E+01	4.4544E+01	7.8225E+01	3.3922E+01	7.0546E+01	1.9344E+01	2.3112E+01
		(1.5069E-02)	(2.6225E-02)	(2.1598E-03)	(2.6127E-02)	(1.9343E-01)	(3.9088E-02)	(1.4875E-02)	(3.0182E-03)
+/−/≈		24/0/0		24/0/0		18/0/6		21/0/3	

Table S3 shows the mean and standard deviation of E_O of four representative algorithms and their *DQN_d* variants on 24 test problems of C-GMPB over 20 independent runs. The best results of each pair algorithms are highlighted in grey. The Wilcoxon rank sum test at the significance level of 5% is conducted on the experimental results. The symbols "+", "-" and "≈" indicate that the corresponding results of variant are better than, worse than, and comparable to that of the original algorithm, respectively.

As can be seen from table S3, the E_O of RIR-SRES-DQN$_d$ and GSA+Repair-DQN$_d$ on all 24 test questions are better than that of the original algorithm. The E_B of LTFR-DSPSO-DQN$_d$ and DyCODE-DQN$_d$ also achieve better performance than the original algorithm on 18 test problems and 21 test problems, respectively. It shows that the change response based on DQN_d can help the algorithm approximate the feasible region of the real optimal individual faster by providing a better reinitialization population.

In order to more intuitively show the effectiveness of the change response based on DQN_d, Fig. S1 lists the comparison of the average quality of reinitialization of each change between the original algorithm and the corresponding DQN_d variant on the most complex test problem C-GMPB-H8 over 20 independent runs. The horizontal coordinates represents the number of changes, and the vertical coordinates represents the population quality. The population quality is calculated by the average of the sum of all individuals' fitness and constraint violation. It is shown that the reinitialization quality of each DQN_d variant has been significantly improved in most cases.



(a) RIR-SRES VS RIR-SRES-DQN$_d$ (b) GSA+Repair VS GSA+Repair-DQN$_d$

(c) LTFR-DSPSO VS LTFR-DSPSO-DQN$_d$ (d) DyCODE VS DyCODE-DQN$_d$

Fig. S1. The average quality of population reinitialization of the original algorithms and their DQN_d variants on C-GMPB-H8 over 20 independent runs

IV. EFFECTIVENESS ANALYSIS OF THE DIRECTED MUTATIOON BASED ON *DQN$_m$*

In order to analyze the effectiveness of the directed mutation based on DQN_m separately, three algorithms with mutation are selected from the comparison algorithms, namely, RIR-SRES, GSA+Repair and SELS. The mutation mechanisms of the three algorithms are replaced by the directed mutation based on DQN_m, and the other parts remain unchanged. The resulting variant algorithms are named RIR-SRES-DQN$_m$, GSA+Repair-DQN$_m$, and SELS-DQN$_m$ respectively.

Table S4 shows the mean and standard deviation of E_B of three algorithms with mutation and their DQN_m variants on 24 test problems of C-GMPB over 20 independent runs. The best results of each pair algorithms are highlighted in grey. The Wilcoxon rank sum test at the significance level of 5% is conducted on the experimental results. The symbols "+", "-" and "≈" indicate that the corresponding results of variant are better than, worse than, and comparable to that of the original algorithm, respectively.

TABLE S4 E_B OF THREE ALGORITHMS WITH MUTATION AND THEIR DQN_m VARIANTS ON C-GMPB OVER 20 INDEPENDENT RUNS

C-GMPB		RIR-SRES-DQN_m	RIRSRES	GSA+Repair-DQN_m	GSA+Repair	SELS-DQN_m	SELE
P1	C1	1.3226E-01	6.2073E-01	1.0426E-01	6.0214E-01	2.0102E-01	3.3809E-01
		(2.0577E-02)	(9.9921E-02)	(3.0473E-04)	(6.5338E-02)	(1.0483E-02)	(8.2102E-03)
	E1	1.5015E-01	6.2386E-01	2.5140E-01	5.9390E-01	3.1615E-01	3.3173E-01
		(6.0038E-03)	(9.8964E-02)	(1.3501E-02)	(3.3184E-02)	(3.1057E-02)	(1.5251E-02)
	H1	2.9070E-01	6.3053E-01	6.4755E-02	6.0531E-01	2.4992E-01	3.3826E-01
		(5.0139E-02)	(5.0686E-02)	(5.9914E-02)	(2.3754E-03)	(1.0448E-02)	(2.6238E-02)
P2	C2	8.8170E-01	2.9896E+00	1.5867E+00	3.0256E+00	1.5975E+00	1.6897E+00
		(3.4661E-02)	(2.6443E-02)	(8.8249E-02)	(7.7091E-02)	(1.1787E-03)	(2.7711E-02)
	E2	2.1034E+00	3.0254E+00	1.7209E+00	2.9404E+00	1.2020E+00	1.7094E+00
		(1.1932E-02)	(9.4513E-03)	(7.2679E-02)	(3.9441E-02)	(4.3465E-02)	(7.3959E-02)
	H2	1.5127E+00	3.4764E+00	2.4821E+00	3.1764E+00	1.0303E+00	1.9926E+00
		(2.5864E-02)	(1.8650E-03)	(6.1780E-02)	(3.6975E-02)	(9.5066E-03)	(2.4989E-02)
P3	C3	6.2812E+00	1.7590E+01	1.7128E+00	1.5543E+01	4.0622E+00	9.8569E+00
		(1.4654E-01)	(2.3399E-02)	(2.0789E-01)	(8.8042E-02)	(6.1485E-02)	(3.4981E-02)
	E3	7.4116E+00	1.7590E+01	4.6480E+00	1.4485E+01	6.5003E+00	9.7587E+00
		(5.0402E-02)	(9.5381E-03)	(8.0583E-03)	(2.4053E-04)	(4.0667E-02)	(8.8339E-02)
	H3	7.7061E+00	1.8010E+01	5.9692E+00	1.6777E+01	1.0060E+01	1.3209E+01
		(2.7584E-02)	(2.4010E-02)	(4.6323E-03)	(2.8203E-02)	(2.9076E-02)	(4.6731E-03)
P4	C4	1.0945E+01	2.2322E+01	6.2872E+00	2.2583E+01	9.0011E+00	1.3326E+01
		(2.1378E-01)	(4.0237E+02)	(2.1973E-02)	(6.0937E-02)	(3.6305E-03)	(5.3402E-02)
	E4	1.2403E+01	2.2305E+01	7.2077E+00	2.2559E+01	6.7118E+00	1.3255E+01
		(1.5373E-03)	(2.2694E+02)	(4.4913E-02)	(4.4637E-02)	(2.6494E-02)	(3.4612E-02)
	H4	1.3135E+01	2.2463E+01	8.5266E+00	2.3513E+01	9.8639E+00	1.3900E+01
		(1.0562E-03)	(9.3531E+04)	(1.6144E-01)	(7.7429E-02)	(6.4190E-03)	(7.1397E-03)
P5	C5	1.8423E+01	2.9454E+01	6.5468E+00	3.0383E+01	8.4430E+00	1.6931E+01
		(6.7406E-03)	(7.6301E+02)	(2.0387E-04)	(2.6606E-04)	(9.3221E-03)	(8.5897E-03)
	E5	1.7410E+01	3.0314E+01	8.8710E+00	2.9594E+01	1.2279E+01	1.6539E+01
		(1.4611E-03)	(4.9039E+02)	(4.7122E-02)	(9.6831E-02)	(4.0661E-04)	(9.4273E-02)
	H5	2.1233E+01	3.3243E+01	7.7593E+00	3.3897E+01	1.1418E+01	1.7578E+01
		(1.3482E-02)	(7.0870E+03)	(7.2145E-01)	(8.2182E-03)	(2.6492E-02)	(2.8417E-03)
P6	C6	2.2206E+01	3.4882E+01	1.1120E+01	3.1082E+01	1.3285E+01	2.3579E+01
		(1.5803E-02)	(7.3490E+03)	(2.2164E-01)	(4.2377E-02)	(1.6053E-02)	(8.6491E-02)
	E6	2.1033E+01	3.7479E+01	1.3208E+01	3.2041E+01	1.8787E+01	2.5183E+01
		(7.2118E-04)	(3.2260E+02)	(4.8878E-02)	(3.4508E-02)	(2.1223E-03)	(7.7632E-02)
	H6	2.3280E+01	3.9756E+01	1.5222E+01	3.5713E+01	2.6486E+01	2.9404E+01
		(5.3665E-02)	(3.9045E+02)	(1.6432E-02)	(2.7406E-02)	(3.9522E-02)	(3.5065E-03)
P7	C7	2.3316E+01	4.7151E+01	1.2012E+01	4.7932E+01	2.0253E+01	2.5533E+01
		(5.4636E-02)	(3.3542E+02)	(1.3032E-01)	(5.4931E-02)	(4.0319E-03)	(5.4978E-02)
	E7	2.4346E+01	3.7153E+01	1.1586E+01	3.8030E+01	1.7615E+01	2.2217E+01
		(1.2762E-01)	(6.9226E+02)	(1.5687E-01)	(8.4164E-02)	(1.0271E-03)	(4.9560E-02)
	H7	2.6000E+01	4.5155E+01	1.4464E+01	4.4956E+01	2.1233E+01	2.5656E+01
		(1.1404E-01)	(2.1380E+02)	(8.2118E-02)	(1.3094E-02)	(8.0412E-03)	(4.0260E-02)
P8	C8	2.9312E+01	4.5149E+01	1.6793E+01	4.4142E+01	1.5719E+01	2.6606E+01
		(9.8399E-02)	(8.5116E+02)	(3.3427E-01)	(2.7272E+02)	(9.1961E-03)	(7.6945E-02)
	E8	2.5769E+01	4.7151E+01	1.6472E+01	4.7932E+01	1.7945E+01	2.5533E+01
		(3.9537E-02)	(3.3542E+02)	(3.9006E-03)	(5.4931E-02)	(2.1902E-02)	(5.4978E-02)
	H8	2.6056E+01	5.0222E+01	1.7093E+01	4.8192E+01	2.1057E+01	3.3126E+01
		(5.1992E-02)	(1.4952E+02)	(1.9863E-02)	(4.0031E-02)	(7.6515E-04)	(3.1679E-03)
+/−/≈		24/0/0		24/0/0		24/0/0	

As can be seen from table S4, the E_B of RIR-SRES-DQN_m, GSA+Repair-DQN_m and SELS-DQN_m on all 24 test questions are better than of the original algorithm. It shows that the directed mutation based on DQN_m can better closer to the real optimal individual by providing an ideal mutation direction for the individuals of population.

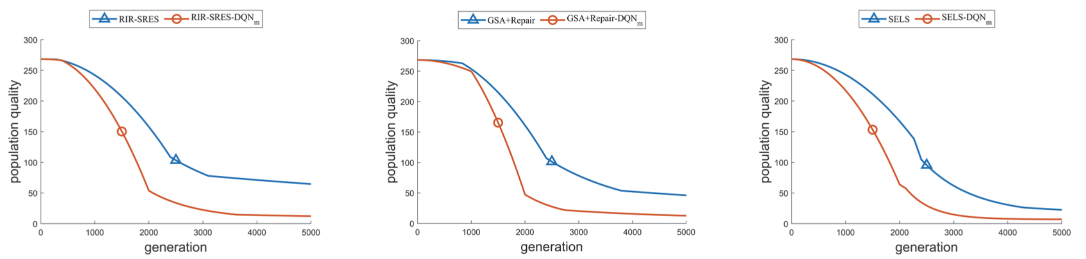
TABLE S5 E_O OF THREE ALGORITHMS WITH MUTATION AND THEIR DQN_m VARIANTS ON C-GMPB OVER 20 INDEPENDENT RUNS

C-GMPB		RIR-SRES-DQNm	RIRSRES	GSA+Repair-DQNm	GSA+Repair	SELS-DQNm	SELE
P1	C1	2.6340E-01 (1.8634E-02)	3.7874E+00 (4.5506E-02)	1.9652E+00 (5.0112E-03)	3.7874E+00 (1.6139E-02)	1.7706E+00 (5.2272E-03)	1.7417E+00 (5.4918E-02)
	E1	3.0273E-02 (6.9057E-03)	6.4499E-01 (8.9327E-03)	6.9658E-01 (1.5793E-01)	5.0499E+00 (1.7024E-02)	2.3487E+00 (3.1381E-02)	2.3223E+00 (9.7707E-02)
	H1	3.7803E-01 (2.0819E-02)	3.7645E+00 (5.7735E-03)	4.8064E+00 (5.8141E-02)	3.7367E+00 (4.1502E-02)	1.0982E+00 (4.1024E-02)	1.7048E+00 (1.4021E-02)
P2	C2	1.6670E+00 (2.2458E-03)	8.1045E+00 (1.5258E-02)	6.8200E-01 (1.1720E-02)	9.6045E+00 (1.3717E-02)	3.0440E+00 (8.8124E-04)	5.8996E+00 (1.6748E-02)
	E2	1.6653E+00 (1.5342E-01)	1.6806E+00 (5.0947E-03)	2.1006E+00 (3.0984E-03)	1.2806E+01 (2.7682E-02)	7.0717E+00 (8.5678E-03)	7.8661E+00 (6.3688E-02)
	H2	3.6020E+00 (3.7086E-02)	8.0503E+00 (9.9115E-03)	2.9497E+00 (1.5419E-03)	9.8963E+00 (4.7282E-02)	3.5251E+00 (1.2212E-02)	5.6556E+00 (2.3602E-02)
P3	C3	5.1296E+00 (8.4220E-03)	1.5786E+01 (1.8290E-02)	4.1061E+00 (1.6864E-03)	1.2786E+01 (9.6810E-03)	4.7399E+00 (4.2058E-02)	1.1044E+01 (1.2648E-03)
	E3	7.4591E+00 (8.3613E-03)	2.1048E+01 (1.4985E-02)	7.2981E+00 (7.5819E-02)	1.7048E+01 (1.0849E-03)	1.1535E+01 (3.3450E-02)	1.4725E+01 (7.8986E-02)
	H3	6.7074E+00 (5.9809E-03)	1.6066E+01 (5.0463E-03)	5.9362E+00 (1.0021E-02)	1.2717E+01 (1.8626E-02)	7.2090E+00 (4.3368E-02)	1.0847E+01 (8.7284E-02)
P4	C4	6.1429E+00 (3.5065E-05)	2.8498E+01 (2.4092E-02)	6.5382E+00 (3.5576E-04)	2.8498E+01 (6.6885E-02)	1.0960E+01 (1.9182E-02)	1.8190E+01 (1.8315E-02)
	E4	8.7790E+00 (5.3677E-03)	3.7998E+01 (1.6367E-02)	1.0749E+01 (1.3761E-01)	3.7998E+01 (3.5700E-03)	1.8948E+01 (4.3434E-03)	2.4253E+01 (1.4825E-02)
	H4	6.4179E+00 (1.1124E-01)	2.8276E+01 (6.7344E-03)	1.4306E+01 (1.1180E-02)	2.8551E+01 (3.9715E-02)	1.3629E+01 (1.2235E-02)	1.8046E+01 (4.2072E-02)
P5	C5	1.7681E+01 (3.1245E-02)	3.5901E+01 (2.8092E-02)	8.6688E+00 (5.6676E-04)	4.0401E+01 (6.0374E-02)	1.8592E+01 (5.7261E-02)	2.2741E+01 (5.7028E-03)
	E5	1.6531E+01 (2.8925E-02)	4.7868E+01 (4.6935E-02)	1.5688E+01 (1.0500E-02)	5.3868E+01 (7.8755E-03)	1.3418E+01 (4.0591E-02)	3.0321E+01 (2.8574E-02)
	H5	1.5345E+01 (7.3831E-03)	3.8415E+01 (2.0457E-02)	1.4448E+01 (2.3145E-02)	4.6404E+01 (3.0659E-02)	1.4038E+01 (6.8112E-02)	2.6431E+01 (3.0701E-02)
P6	C6	1.3877E+01 (1.1954E-02)	5.1900E+01 (8.5101E-02)	1.7715E+01 (3.7069E-03)	4.7400E+01 (1.8612E-02)	2.0351E+01 (3.0620E-02)	3.4663E+01 (1.9763E-02)
	E6	1.3495E+01 (5.9105E-02)	6.9200E+01 (2.2295E-02)	2.4964E+01 (3.7073E-01)	6.3200E+01 (5.2723E-03)	3.4821E+01 (8.0230E-03)	4.6217E+01 (6.7447E-02)
	H6	1.5644E+01 (2.1343E-02)	5.4464E+01 (3.0775E-02)	2.7713E+01 (2.9147E-02)	4.9272E+01 (1.4950E-02)	2.2691E+01 (3.1184E-02)	3.4327E+01 (4.3496E-02)
P7	C7	1.8439E+01 (4.6719E-03)	6.8505E+01 (7.8051E-02)	2.3662E+01 (8.8770E-02)	7.6005E+01 (6.7119E-03)	3.6062E+01 (3.0025E-03)	4.5300E+01 (4.7637E-02)
	E7	2.0191E+01 (3.4009E-03)	9.1340E+01 (1.4291E-02)	2.5876E+01 (4.5342E-04)	1.0134E+02 (2.8628E-02)	3.3635E+01 (1.3298E-02)	6.0400E+01 (1.6398E-02)
	H7	1.7503E+01 (8.4711E-04)	8.2416E+01 (2.1108E-02)	1.9375E+01 (5.4571E-02)	7.2044E+01 (1.2568E-02)	1.8097E+01 (2.4870E-02)	4.2614E+01 (4.5157E-02)
P8	C8	2.3454E+01 (6.4272E-02)	8.9898E+01 (4.3406E+02)	2.1648E+01 (5.0994E-04)	8.2398E+01 (2.9462E+03)	4.0186E+01 (3.4078E-02)	6.0499E+01 (7.6593E-03)
	E8	2.4193E+01 (4.1295E-02)	1.1986E+02 (3.7324E-03)	3.2897E+01 (6.0635E-02)	1.0986E+02 (7.7428E-03)	4.3057E+01 (3.3676E-03)	8.0665E+01 (4.7810E-02)
	H8	1.9921E+01 (3.3933E-04)	9.8815E+01 (2.6225E-02)	4.1077E+01 (2.5646E-01)	7.8225E+01 (2.6127E-02)	4.6548E+01 (4.6246E-02)	6.7034E+01 (2.1244E-02)
+/−/≈		24/0/0		24/0/0		22/0/2	

Table S5 shows the mean and standard deviation of E_O of three algorithms with mutation and their DQN_m variants on 24 test problems of C-GMPB over 20 independent runs. The best results of each pair algorithms are highlighted in grey. The Wilcoxon rank sum test at the significance level of 5% is conducted on the experimental results. The symbols “+”, “-” and “≈” indicate that the corresponding results of variant are better than, worse than, and comparable to that of the original algorithm, respectively.

As can be seen from table S5, the E_O of RIR-SRES-DQN$_{\mathrm{m}}$ and GSA+Repair-DQN$_{\mathrm{m}}$ on all 24 test questions are better than of the original algorithm. The E_O of SELS-DQN$_{\mathrm{m}}$ also achieve better performance than the original algorithm on 22 test problems. It shows the directed mutation based on DQN_m can help the algorithm approximate the feasible region of the real optimal individual faster by providing an ideal mutation direction for the individuals of population.

In order to more intuitively show the effectiveness of the directed mutation based on DQN_m, Fig. S2 lists the comparison of the average quality of offspring of each iteration between the original algorithm and the corresponding DQN_m variant in the 50$^{\text{th}}$ static optimization on the most complex test problem C-GMPB-H8 over 20 independent runs. The horizontal coordinates represents the number of iterations in static optimization and the vertical coordinates represents the population quality. The initial population of each experiment is set as the same randomly generated population. It is shown that the offspring quality of each DQN_m variant better than the original algorithm in each iteration.



(a) RIR-SRES VS RIR-SRES-DQN$_{\mathrm{m}}$ (b) GSA+Repair VS GSA+Repair-DQN$_{\mathrm{m}}$ (c) SELS VS SELS-DQN$_{\mathrm{m}}$

Fig. S2. The average quality of offspring of the original algorithms and their DQN_m variants in the 50$^{\text{th}}$ static optimization on C-GMPB-H8 over 20 independent runs

V. ABLATION EXPERIMENT

Compared with traditional evolutionary algorithms, DCEA-DQN introduces three new mechanisms, which are the change response based on DQN_d, the directed mutation based on DQN_m and the diversity enhancement based on penalty. In order to measure the role of these three mechanisms in DCEA-DQN, these three mechanisms are respectively replaced by using random number of archived individuals for change response, uniform mutation and non-penalty, while the other parts remain unchanged. The three ablation variants are named DCEA-RA, DCEA-UM and DCEA-NP respectively.

Table S6 shows the mean and standard deviation of E_B of three ablation variant and DCEA-DQN on 24 test problems of C-GMPB over 20 independent runs. The best results of each pair algorithms are highlighted in grey. The Wilcoxon rank sum test at the significance level of 5% is conducted on the experimental results. The symbols “+”, “-” and “≈” indicate that the corresponding results of ablation variant are better than, worse than, and comparable to that of DCEA-DQN, respectively.

Table S7 shows the mean and standard deviation of E_O of three ablation variants and DCEA-DQN on 24 test problems of C-GMPB over 20 independent runs. The best results of each pair algorithms are highlighted in grey. The Wilcoxon rank sum test at the significance level of 5% is conducted on the experimental results. The symbols “+”, “-” and “≈” indicate that the corresponding results of ablation variant are better than, worse than, and comparable to that of DCEA-DQN, respectively.

TABLE S6 E_B OF THREE ABLATION VARIANT AND DCEA-DQN ON C-GMPB OVER 20 INDEPENDENT RUNS

C-GMPB		DCEA-RA	DCEA-UM	DCEA-NP	DCEA-DQN
P1	C1	7.9168E-01 (8.3727E-02)(-)	6.8474E+00 (1.4336E-03)(-)	1.7505E+00 (1.4942E-01)(-)	3.5760E-02 (2.2508E-03)
	E1	1.2906E+00 (1.3857E-05)(-)	1.0781E+01 (2.1271E-03)(-)	1.6608E+00 (1.1226E-02)(-)	1.8454E-02 (9.5016E-04)
	H1	9.2513E-01 (3.4362E-03)(-)	1.1876E+01 (4.0627E-01)(-)	2.3371E+00 (2.3477E-01)(-)	4.1234E-03 (1.0938E-02)
P2	C2	1.3509E+00 (1.0355E-02)(-)	1.0463E+01 (4.4628E-01)(-)	2.5610E+00 (5.3896E-02)(-)	1.0728E-01 (9.3422E-04)
	E2	1.8485E+00 (6.0870E-02)(-)	1.2192E+01 (7.6956E-02)(-)	3.0940E+00 (1.2193E-02)(-)	1.2721E-01 (1.0685E-03)
	H2	1.3110E+00 (6.5805E-01)(-)	1.3513E+01 (2.4825E-01)(-)	3.2942E+00 (4.8405E-02)(-)	1.0329E-01 (8.4760E-02)
P3	C3	2.7212E+00 (4.2096E-02)(-)	1.2373E+01 (3.3899E-02)(-)	4.5829E+00 (4.4629E-02)(-)	1.4575E+00 (3.1233E-04)
	E3	2.0013E+00 (6.7131E-04)(-)	1.4723E+01 (2.6490E-01)(-)	5.3507E+00 (2.7468E-03)(-)	1.6672E+00 (3.0046E-01)
	H3	3.3889E+00 (5.2225E-02)(-)	1.3477E+01 (2.4005E-04)(-)	4.9061E+00 (3.5452E-03)(-)	2.5289E+00 (2.4852E-01)
P4	C4	2.0055E+00 (1.9675E-01)(-)	1.6743E+01 (1.6859E-01)(-)	5.5737E+00 (1.3024E-03)(-)	1.8011E+00 (9.8590E-02)
	E4	3.9109E+00 (1.6322E-01)(-)	2.0040E+01 (1.4923E-02)(-)	4.4790E+00 (1.4307E-01)(-)	2.0225E+00 (3.3022E-03)
	H4	3.6224E+00 (1.7992E-03)(-)	1.9004E+01 (1.2781E-02)(-)	6.9191E+00 (7.5300E-03)(-)	2.2768E+00 (2.8538E-03)
P5	C5	4.0912E+00 (2.8827E-03)(-)	1.9888E+01 (3.2681E-03)(-)	6.9077E+00 (1.9358E-02)(-)	2.5556E+00 (1.1842E-01)
	E5	5.1861E+00 (1.7956E-02)(-)	1.6801E+01 (6.1452E-03)(-)	5.2144E+00 (2.3863E-02)(-)	1.8113E+00 (1.8845E-02)
	H5	5.9920E+00 (1.0109E-03)(-)	1.8583E+01 (3.4680E-02)(-)	5.3728E+00 (4.8264E-02)(-)	3.6171E+00 (1.0721E-01)
P6	C6	7.7370E+00 (9.0210E-02)(-)	2.2850E+01 (3.1717E-01)(-)	7.8755E+00 (3.8781E-06)(-)	2.7753E+00 (6.8941E-02)
	E6	7.1600E+00 (4.6098E-02)(-)	2.6367E+01 (8.4347E-02)(-)	8.1774E+00 (5.0764E-02)(-)	3.1917E+00 (1.9052E-02)
	H6	7.7023E+00 (1.0426E-01)(-)	2.5873E+01 (5.3283E-02)(-)	7.2666E+00 (1.7134E-01)(-)	3.5494E+00 (3.7334E-02)
P7	C7	7.7915E+00 (1.5182E-01)(-)	2.1400E+01 (6.9541E-03)(-)	8.2354E+00 (7.5653E-03)(-)	6.2187E+00 (3.0018E-04)
	E7	8.4728E+00 (2.6140E-02)(-)	2.1083E+01 (1.0391E-03)(-)	1.0448E+01 (1.9469E-01)(-)	4.1149E+00 (1.8296E-02)
	H7	7.1179E+00 (1.4444E-02)(-)	1.9364E+01 (3.2244E-01)(-)	9.6979E+00 (1.0283E-01)(-)	4.2728E+00 (6.8715E-03)
P8	C8	8.9862E+00 (2.8973E-02)(-)	2.0865E+01 (1.8207E-03)(-)	1.1542E+01 (3.4976E-02)(-)	4.4890E+00 (1.7319E-03)
	E8	9.8030E+00 (5.3210E-02)(-)	2.2270E+01 (3.5060E-03)(-)	1.3960E+01 (4.3955E-02)(-)	6.2187E+00 (3.9716E-03)
	H8	9.5807E+00 (3.3994E-03)(-)	2.6444E+01 (5.4867E-02)(-)	1.2428E+01 (7.6169E-04)(-)	6.9106E+00 (5.6395E-02)
+/−/≈		0/24/0	0/24/0	0/24/0	

TABLE S7 E_O OF THREE ABLATION VARIANT AND DCEA-DQN ON C-GMPB OVER 20 INDEPENDENT RUNS

C-GMPB		DCEA-RA	DCEA-UM	DCEA-NP	DCEA-DQN
P1	C1	6.5642E+00 (4.0960E-02)(-)	5.6379E+00 (5.2429E-03)(-)	2.7402E+00 (7.3550E-02)(-)	5.0550E-01 (9.0378E-03)
	E1	7.7775E+00 (1.8014E-04)(-)	4.9027E+00 (1.0461E-02)(-)	2.5711E+00 (6.4711E-03)(-)	6.9856E-01 (1.4272E-02)
	H1	7.6971E+00 (2.0335E-02)(-)	6.8538E+00 (3.4094E-02)(-)	3.1072E+00 (4.2124E-01)(-)	4.9633E-01 (2.5736E-03)
P2	C2	1.6801E+01 (3.2577E-03)(-)	1.3729E+01 (2.0753E-02)(-)	6.2031E+00 (1.8371E-03)(-)	1.3435E+00 (5.3353E-02)
	E2	1.7148E+01 (1.8358E-01)(-)	1.2905E+01 (1.7349E-02)(-)	8.8272E+00 (5.1599E-02)(-)	1.7884E+00 (4.2321E-03)
	H2	1.7913E+01 (1.7452E-02)(-)	1.4430E+01 (7.8358E-02)(-)	7.4678E+00 (3.6135E-02)(-)	1.5384E+00 (4.2892E-02)
P3	C3	2.2227E+01 (1.2852E-01)(-)	2.1481E+01 (3.8908E-02)(-)	1.2241E+01 (1.2534E-02)(-)	2.0148E+00 (1.1661E-02)
	E3	2.9309E+01 (1.2953E-01)(-)	2.6161E+01 (2.0705E-01)(-)	1.4648E+01 (9.5979E-03)(-)	3.2962E+00 (1.1891E-02)
	H3	3.0806E+01 (3.0450E-02)(-)	2.4071E+01 (3.1147E-02)(-)	1.6048E+01 (4.2818E-03)(-)	2.1204E+00 (3.0681E-02)
P4	C4	3.0772E+01 (1.6740E-02)(-)	2.5461E+01 (1.0370E-02)(-)	1.3492E+01 (1.7593E-03)(-)	3.2079E+00 (5.2931E-03)
	E4	3.1837E+01 (1.2837E-01)(-)	2.7790E+01 (1.0356E-01)(-)	1.7899E+01 (1.4410E-02)(-)	4.4249E+00 (4.2615E-02)
	H4	3.4840E+01 (6.1353E-02)(-)	2.2489E+01 (2.0132E-01)(-)	1.5811E+01 (2.0771E-02)(-)	3.4376E+00 (2.2040E-02)
P5	C5	3.2196E+01 (1.0115E-03)(-)	2.3729E+01 (2.9970E-02)(-)	1.8827E+01 (3.3416E-02)(-)	9.1704E+00 (5.5669E-02)
	E5	3.4343E+01 (1.0971E-01)(-)	2.6347E+01 (1.1614E-02)(-)	2.1445E+01 (5.1152E-02)(-)	1.2245E+01 (1.0216E-02)
	H5	3.4456E+01 (2.9615E-04)(-)	2.8409E+01 (1.5671E-02)(-)	2.0500E+01 (5.6296E-02)(-)	1.2378E+01 (6.1329E-03)
P6	C6	3.6596E+01 (1.3146E-01)(-)	3.3192E+01 (4.6496E-02)(-)	2.6504E+01 (5.0596E-03)(-)	8.7070E+00 (3.7373E-02)
	E6	3.8629E+01 (8.5327E-03)(-)	3.5953E+01 (7.5938E-02)(-)	2.8778E+01 (2.3791E-02)(-)	1.1421E+01 (3.2603E-02)
	H6	4.0099E+01 (2.7926E-03)(-)	3.2313E+01 (2.1647E-02)(-)	2.7580E+01 (1.6087E-01)(-)	7.5054E+00 (7.7674E-02)
P7	C7	4.4218E+01 (3.0228E-02)(-)	3.6329E+01 (6.7720E-03)(-)	3.0309E+01 (3.0485E-02)(-)	1.0886E+01 (8.1905E-03)
	E7	4.6364E+01 (1.1101E-02)(-)	4.1196E+01 (5.9453E-03)(-)	3.2659E+01 (1.0062E-01)(-)	1.4613E+01 (1.3138E-02)
	H7	4.4510E+01 (1.4643E-01)(-)	3.9956E+01 (3.5449E-02)(-)	3.1394E+01 (4.3197E-02)(-)	1.1370E+01 (1.0758E-02)
P8	C8	5.0787E+01 (7.0922E-02)(-)	4.4696E+01 (1.0582E-04)(-)	3.5390E+01 (1.9966E-02)(-)	1.3474E+01 (9.4607E-02)
	E8	5.7076E+01 (4.3639E-03)(-)	4.6609E+01 (1.5517E-02)(-)	3.6696E+01 (7.4826E-02)(-)	1.8684E+01 (5.0924E-03)
	H8	5.0399E+01 (4.5384E-03)(-)	4.3042E+01 (4.4479E-02)(-)	3.8791E+01 (2.6725E-02)(-)	1.6281E+01 (2.6566E-02)
+/−/≈		0/24/0	0/24/0	0/24/0	

From table S6 and table S7, it can be seen that the E_B and E_O of three variants are worse than the original DCEA-DQN on all 24 test problems. Among them, there is a certain gap between the E_B of DCEA-RA and DCEA-DQN, and the gap is the smallest among the three ablation variants and DCEA-DQN. DCEA-RA has the largest difference with DCEA-DQN on E_O. On the one hand, it shows that the change response based on DQN can improve the convergence of DCEA-

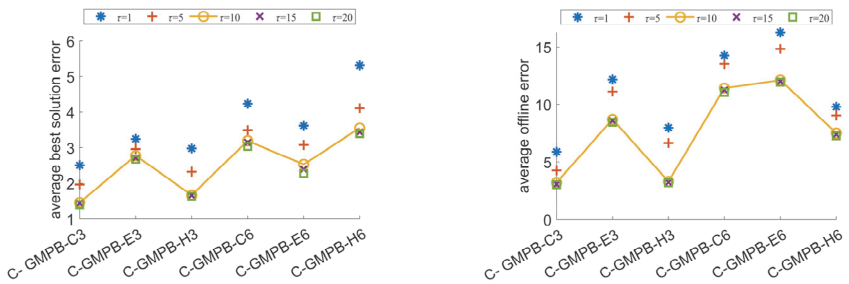
DQN to a certain extent, but the improvement is not very large. On the other hand, it shows that the change response based on DQN can greatly improve the quality of reinitialized population, thus providing a good foundation for the faster convergence of DCEA-DQN. DCEA-UM differs greatly from DCEA-DQN on E_B and E_O, with a difference of four orders of magnitude in the most serious cases, which illustrates that the directed mutation based on DQN has an important impact on the convergence ability and speed of DCEA-DQN. There are about half of the test problems shows that, the difference between the E_B of DCEA-NP and DCEA-DQN is not more than one order of magnitude. However, there is an order of magnitude gap between the E_O of DCEA-NP and DCEA-DQN on most test problems. The results shows that, the diversity enhancement mechanism based on penalty plays a certain role in improving the convergence of DCEA-DQN, but it even has a more important impact on accelerating the convergence speed of DCEA-DQN. To sum up, table S8 and table S9 show that the three newly introduced mechanisms in DCEA-DQN have played their expected roles and are important components of DCEA-DQN.

VI. SENSITIVITY ANALYSIS OF PARAMETERS

DCEA-DQN contains some unique parameters related to change response and offspring generation. In order to analyze the impact of these parameters on the performance of DCEA-DQN, this section introduces the sensitivity analysis of these parameters on the six test problems corresponding to C-GMPB-P3 and C-GMPB-P6. It is worth noting that during the sensitivity analysis of specific parameter, the values of other parameters are completely consistent with those in Table 2 of the main file.

A. Sensitivity analysis of parameter τ

The parameter τ is used to control the maximum number of cycles during change response. The larger the value of τ is, the better the quality of population reinitialization will be theoretically improve, but it will also lead to the more time consumption in change response. On the contrary, the smaller the value of τ is, the faster the response will change, but the quality of population reinitialization may be degradation. Fig. S3 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of τ is set to 1, 5, 10, 15 and 20 respectively. It is shown that DCEA-DQN has basically the same and the best E_B and E_O on all six test problems when τ is set to 10, 15 or 20. The E_B and E_O of DCEA-DQN will get worse when τ is less than 10. It indicates that when τ is set to10, DCEA-DQN will obtain the best change response effect without consuming too much computational resource.

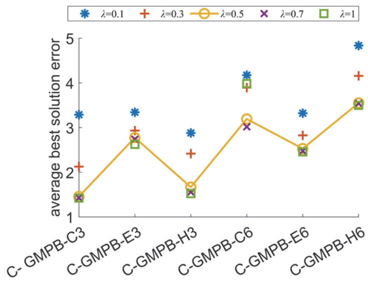


(a) E_B (b) E_O

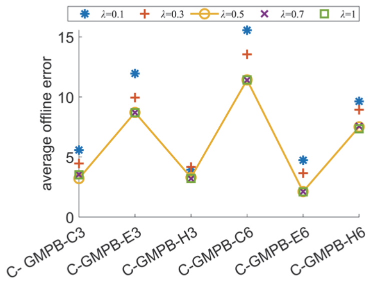
Fig. S3. Sensitivity analysis results of τ

B. Sensitivity analysis of parameter λ

The parameter λ is used to control the reward threshold of DQN_d during change response. Similar to parameter τ, the larger the value of λ is, the better the quality of population reinitialization will be theoretically improve, but it will also lead to the more time comsumption in change response. On the contrary, the smaller the value of λ is, the faster the response will change, but the quality of population reinitialization may be degradation. Fig. S4 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of λ is set to 0.1, 0.3, 0.5, 0.7 and 1 respectively. It is shown that DCEA-DQN has basically the same and the best E_B and E_O on all six test problems when λ is set to 0.5 or 0.7. The E_B and E_O of DCEA-DQN will get worse when λ is less than 0.5. Besides, when λ is set to 1, it has basically the same performance as $\lambda = 0.5$. However, the E_B of $\lambda = 1$ is poorer than $\lambda = 0.5$ on C-GMPB-C6. It indicates that when λ is set to 0.5, DCEA-DQN will obtain the best change response effect without consuming too much computational resource.



(a) E_B

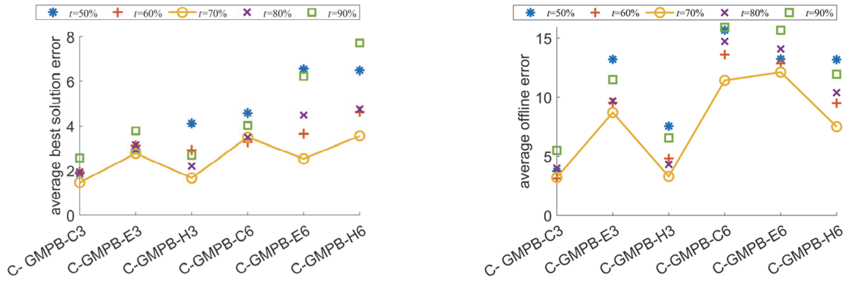


(b) E_O

Fig. S4. Sensitivity analysis results of λ

C. Sensitivity analysis of parameter t

The parameter t is used to control the period of time during which penalty are imposed on the population in static optimization. The larger the value of t is, the longer the penalty phase is. It will be more conducive to improve the diversity of the population and explore more feasible regions. However, large value of t will be more likely to lead to excessive exploration of the unknown space and ignore the fine exploitation of the feasible region with optimal individual, which will result in the decline of the algorithm performance. On the contrary, the smaller the value of t is, the more conducive the fine exploitation will be in the found feasible region, but it will be much easier to make the population fall into local optimum. Fig. S5 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of t is set to 50%, 60%, 70%, 80% and 90% of total number of iterations of a static optimization cycle, respectively. It is shown that DCEA-DQN has the best E_B and E_O on all six test problems when t is set to 70%. The E_B and E_O of DCEA-DQN will get worse when t is set to other value. It indicates that when t is set to 70%, an ideal balance can be achieved between the diversity and convergence.

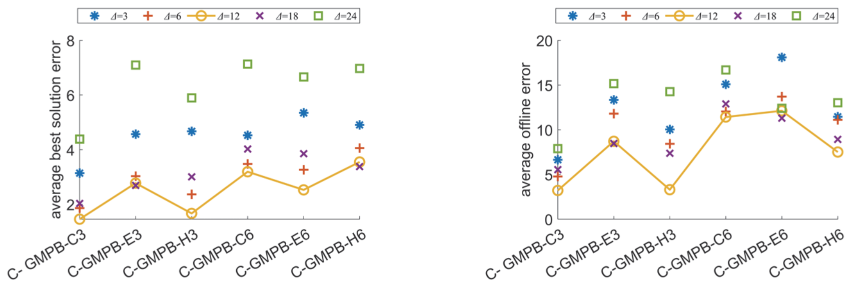


(a) E_B (b) E_O

Fig. S5. Sensitivity analysis results of t

D. *Sensitivity analysis of parameter Δ*

The parameter Δ is used to control the penalty distance in static optimization. Similar to parameter t, the larger the value of Δ is, the more beneficial it is to improve the diversity of the population and explore more feasible regions. However, the large value of Δ also will be more likely to lead to excessive exploration of the unknown space and ignore the fine exploitation of the feasible region with optimal individual, which will result in the decline of the algorithm performance. On the contrary, the smaller the value of Δ is, the more conducive the fine exploitation will be in the found feasible region, but it will be much easier to make the population fall into local optimum. Fig. S6 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of Δ is set to 3, 6, 12, 18 and 24 respectively. It is shown that DCEA-DQN has the best E_B and E_O on all six test problems when t is set to 12. The E_B and E_O of DCEA-DQN will get worse when Δ is set to other value. It indicates that when Δ is set to 12, an ideal balance can be achieved between the diversity and convergence.



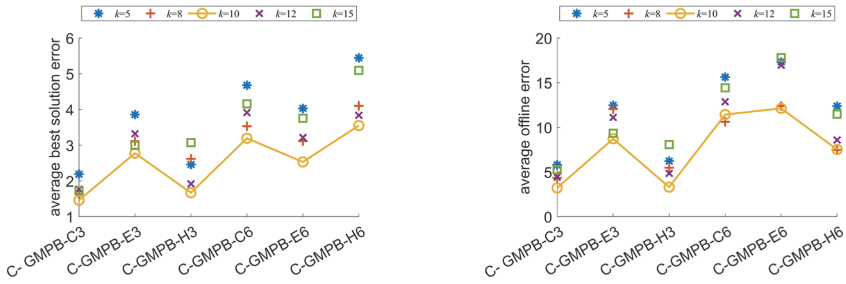
(a) E_B (b) E_O

Fig. S6. Sensitivity analysis results of Δ

E. *Sensitivity analysis of parameter k*

The parameter k is used to control the number of candidates in guidance set for mutation in static optimization. The larger the value of k is, the more beneficial it is to provide more accurate guidance direction for mutation. However, the number of alternative guidance near to the optimal individual also will increases synchronously, and the probability of "jitter" phenomenon is also greater, which is easy to cause the performance of directed mutation to decline instead of increase. On the contrary, the smaller the value of k is, the smaller the probability of "jitter" phenomenon, but the role of directed mutation is also reduced synchronously, and even the guidance deviates from the optimal individual in serious cases. Fig. S7 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of k

is set to 5, 8, 10, 12 and 15 respectively. It is shown that DCEA-DQN has the best E_B and E_O on all six test problems when k is set to 10. The E_B and E_O of DCEA-DQN will get worse when k is set to other value. It indicates that when k is set to 10, the guidance direction for mutation obtained based on DQN_m has the best guide effect.

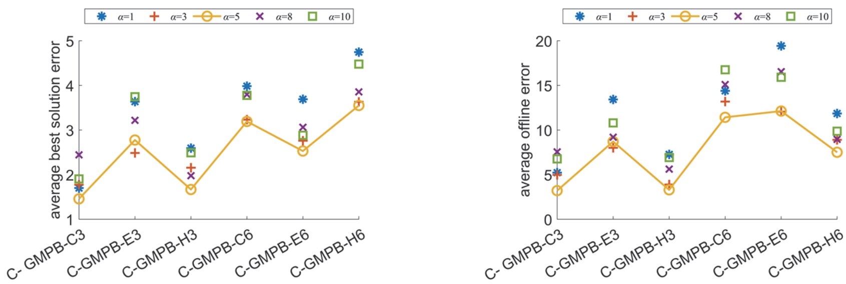


(a) E_B (b) E_O

Fig. S7. Sensitivity analysis results of k

F. *Sensitivity analysis of parameter α*

The parameter α is used to control the number of mutation individuals in each iteration of static optimization. Similar to parameter Δ, the larger the value of α is, the more beneficial it is to improve the diversity of the population and explore more feasible regions. However, the large value of α also will be more likely to lead to excessive exploration of the unknown space and ignore the fine exploitation of the feasible region with optimal individual, which will result in the decline of the algorithm performance. On the contrary, the smaller the value of α is, the more conducive the fine exploitation will be in the found feasible region, but it will be much easier to make the population fall into local optimum. Fig. S8 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of α is set to 1, 3, 5, 8 and 10 respectively. It is shown that DCEA-DQN has the best E_B and E_O on all six test problems when α is set to 5. The E_B and E_O of DCEA-DQN will get worse when α is set to other value. It indicates that when α is set to 5, an ideal balance can be achieved between the diversity and convergence.

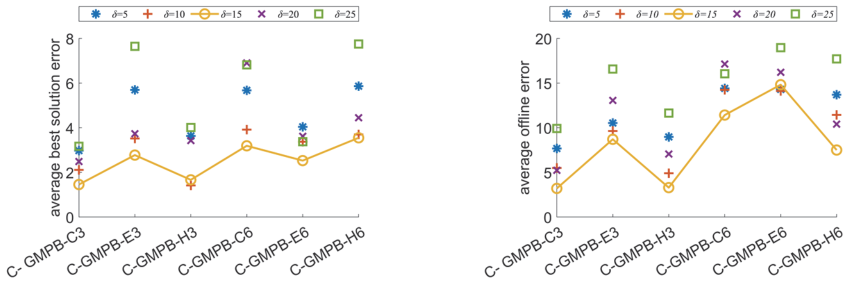


(a) E_B (b) E_O

Fig. S8. Sensitivity analysis results of α

G. *Sensitivity analysis of parameter δ*

The parameter δ is used to control the training frequency of DQN_m during static optimization. If the value of δ is very large, DQN_m can't learn the recently experience of directed mutation in time, which is easy to cause directed mutation based on DQN_m cannot select the most ideal guidance. On the contrary, If the value of δ is very small, frequent training of DQN_m will not only reduce the running speed of the algorithm, but also easily lead to over fitting of DQN_m, which also makes the directed mutation based on DQN_m unable to select the most ideal guidance. Fig. S9 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of δ is set to 5, 10, 15, 20 and 25 respectively. It is shown that DCEA-DQN has the best E_B and E_O on all six test problems when δ is set to 15. The E_B and E_O of DCEA-DQN will get worse when δ is set to other value. It indicates that when δ is set to 15, the guidance direction for mutation obtained based on DQN_m has the best guide effect.

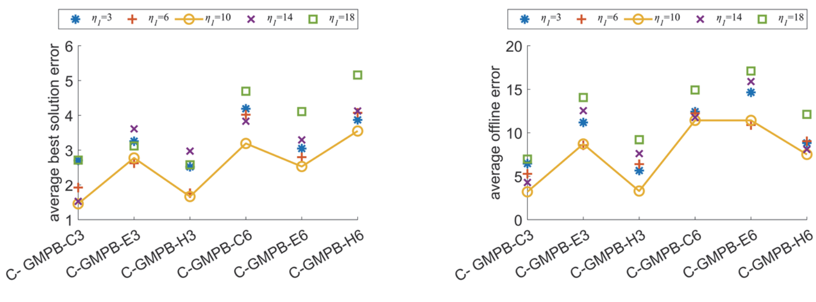


(a) E_B (b) E_O

Fig. S9. Sensitivity analysis results of δ

H. *Sensitivity analysis of parameter η_1*

The parameter η_1 is used to control how many pieces of sample data are used to train DQN_d each time. If the value of η_1 is very large, it is easy to cause over fitting of DQN_d, resulting in the failure of the change response based on DQN_d to generate ideal reinitialized population. On the contrary, if the value of η_1 is very small, it is easy for DQN_d to fail to fully learn the previous experience of change response, which also leads to the failure of the change response based on DQN_d to generate the ideal reinitialized population. Fig. S10 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of η_1 is set to 3, 6, 10, 14 and 18 respectively. It is shown that DCEA-DQN has the best E_B and E_O on all six test problems when η_1 is set to 10. The E_B and E_O of DCEA-DQN will get worse when η_1 is set to other value. It indicates that when η_1 is set to 10, the change response based on DQN_d has the best effect of reinitialization.

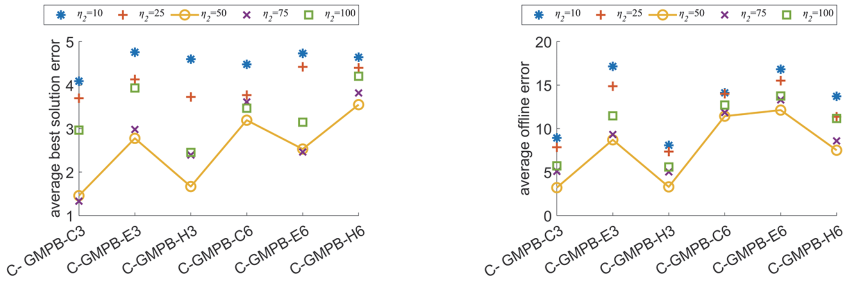


(a) E_B (b) E_O

Fig. S10. Sensitivity analysis results of η_1

I. *Sensitivity analysis of parameter η_2*

The parameter η_2 is used to control how many pieces of sample data are used to train DQN_m each time. Similar to parameter η_2, if the value of η_2 is very large, it is easy to cause over fitting of DQN_m, resulting in the failure of the directed mutation based on DQN_m to select the most ideal guidance. On the contrary, if the value of η_2 is very small, it is easy for DQN_m to fail to fully learn the previous experience of directed mutation, which also leads to the failure of the directed mutation based on DQN_m to select the most ideal guidance. Fig. S11 shows the average E_B and E_O of DCEA-DQN on six test problems of C-GMPB-P3 and C-GMPB-P6 over 20 independent runs, where the value of η_2 is set to 10, 25, 50, 75 and 100 respectively. It is shown that DCEA-DQN has the best E_B and E_O on all six test problems when η_2 is set to 50. The E_B and E_O of DCEA-DQN will get worse when η_2 is set to other value. It indicates that when η_2 is set to 50, the directed mutation based on DQN_m has the best effect of guidance.



(a) E_B (b) E_O

Fig. S11. Sensitivity analysis results of η_2

References

[1] J. Branke, "Memory enhanced evolutionary algorithms for changing optimization problems," in *Proceedings of the 1999 Congress on Evolutionary Computation-CEC99 (Cat. No. 99TH8406)*, 1999, pp. 1875-1882.

[2] D. Yazdani, M. N. Omidvar, R. Cheng, J. Branke, T. T. Nguyen, and X. Yao, "Benchmarking continuous dynamic optimization: survey and generalized test suite," *IEEE Transactions on Cybernetics*, vol. 52, no. 5, pp. 3380-3393, 2022.

[3] Y. Wang, J. Yu, S. Yang, S. Jiang, and S. Zhao, "Evolutionary dynamic constrained optimization: Test suite construction and algorithm comparisons," *Swarm and Evolutionary Computation*, vol. 50, 100559, 2019.