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DPN: Decoupling Partition and Navigation for Neural Solvers of Min-max Vehicle Routing Problems

(/pdf?id=ar174skI9u)

Zhi Zheng (/profile?id=~Zhi_Zheng2), Shunyu Yao (/profile?id=~Shunyu_Yao3), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Ke Tang (/profile?id=~Ke_Tang2)

Published: 02 May 2024, Last Modified: 25 Jun 2024 ICML 2024 Poster Everyone Revisions (/revisions?id=ar174skI9u) BibTeX

Verify Author List: I have double-checked the author list and understand that additions and removals will not be allowed after the submission deadline.

Keywords: Neural Combinatorial Optimization, Vehicle Routing Problems, Min-max Vehicle Routing Problems, Reinforcement Learning

TL;DR: DPN leverages problem-specific properties and significantly improves the representation ability of neural solvers of min-max vehicle routing problems.

Abstract:
The min-max vehicle routing problem (min-max VRP) traverses all given customers by assigning several routes and aims to minimize the length of the longest route. Recently, reinforcement learning (RL)-based sequential planning methods have exhibited advantages in solving efficiency and optimality. However, these methods fail to exploit the problem-specific properties in learning representations, resulting in less effective features for decoding optimal routes. This paper considers the sequential planning process of min-max VRPs as two coupled optimization tasks: customer partition for different routes and customer navigation in each route (i.e., partition and navigation). To effectively process min-max VRP instances, we present a novel attention-based Partition-and-Navigation encoder (P&N Encoder) that learns distinct embeddings for partition and navigation. Furthermore, we utilize an inherent symmetry in decoding routes and develop an effective agent-permutation-symmetric (APS) loss function. Experimental results demonstrate that the proposed Decoupling-Partition-Navigation (DPN) method significantly surpasses existing learning-based methods in both single-depot and multi-depot min-max VRPs. Our code is available at

Primary Area: Applications (computational biology, crowdsourcing, healthcare, neuroscience, social good, climate science, etc.)

Position Paper Track: No

Paper Checklist Guidelines: I certify that all co-authors of this work have read and commit to adhering to the Paper Checklist Guidelines, Call for Papers and Publication Ethics.

Verify Author Names: My co-authors have confirmed that their names are spelled correctly both on OpenReview and in the camera-ready PDF. (If needed, please update 'Preferred Name' in OpenReview to match the PDF.)

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Submission Number: 3742

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38 / 38 replies shown

Add: **Withdrawal**

Paper Decision

Decision Program Chairs 02 May 2024, 07:40 (modified: 06 Jun 2024, 23:53) Program Chairs, Authors
Revisions (/revisions?id=ahbrm8aSny)

Decision: Accept (Poster)
Comment:
There is a divergence among the four reviewers. While one reviewer expressed significant concerns regarding the applicability of the methodology to other types of Vehicle Routing Problems (VRPs), the other three reviewers strongly support this paper, citing the novelty of the methodology and the superiority of its performance. Overall, I believe this submission is of good quality and worthy of acceptance. However, I also suggest that the authors include a discussion about the broader applicability of their methodology in the final version of the paper.

Message to AC

Official Comment
Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))
05 Apr 2024, 19:17 (modified: 05 Apr 2024, 19:23)
Program Chairs, Senior Area Chairs, Area Chairs, Reviewers, Reviewers Submitted, Authors
Revisions (/revisions?id=E6ZjoSPGqP)

Comment:
Dear AC,

Thank you very much for your time and effort in organizing the review of our work. Here is our summary of the author-reviewers discussion.

This work proposes a new method called DPN to deal with min-max VRPs. As an RL-based sequential planning-based constructive solver, DPN presents P&N Encoder, APS-Loss, and Rotation-based PE for efficient representations. It achieves outstanding experimental results in five min-max VRPs (min-max mTSP, min-max mPDP, min-max MDVRP, min-max FMDVRP, and min-max mCVRP in the Common Concern 1).

Reviewer iPLg and **Reviewer 4ZsZ** found our work appears promising on min-max VRPs, and the paper is easy to follow. **Reviewer 4ZsZ** raised the rating from **Borderline Accept (5)** to **Accept (7)** after the discussion, regarding all the concerns being well addressed. **Reviewer iPLg** also raised the rating from **Weak Accept (6)** to **Accept (7)** and emphasized the importance of min-max combinatorial optimization tasks `I believe the min-max combinatorial optimization tasks are significant , This work focuses on a topic that has been underemphasized by the community, which I find important. .`

Reviewer P1eu found the proposed DPN performs well with various techniques, and the APS-enabled strategy works efficiently. By addressing the main concerns about the presentation of contributions and main ideas, **Reviewer P1eu** raised the rating from **Borderline Reject(4)** to **Week Accept (6)** after the discussion and believes that `the proposed concept is important` and `the proposed method clearly works well for various settings (i.e., various min-max problems)`.

Reviewer wVP5 initially misunderstood our work to be similar to previous learn-to-divide approaches [1,2,3] and gave a rating of **Strong Reject(2)**. After the author-reviewer discussion and a closer examination, **Reviewer wVP5** recognized that our method has notable disparities from existing learn-to-divide approaches and raised the rating to **Borderline Reject(4)**. **Reviewer wVP5**'s remained concerns are: **1)** the semblance of similarity between our method and existing ones in the learn-to-divide aspect and **2)** the limitation to extending our method to mainstream VRPs.

- Regarding concern **1)**: We believe that our method is significantly different from existing ones in the learn-to-divide aspect. Unlike these methods which adopt a separate network as well as an individual loss function for the divide, DPN adopts a unified framework and loss function for both divide and conquer (i.e., partition and navigation) tasks. Compared with learning the divide separately, DPN can consider the conquer-related features in learning the divide, thus avoiding the conflict between navigation and partitioning.
- Regarding concern **2)**: In this work, the proposed DPN is specific to min-max VRPs. We would like to clarify that min-max VRPs are significantly different from mainstream VRPs such as CVRP and CVRPTW. The number of vehicles (i.e., agents or routes) in the min-max VRP is limited, which leads to a completely different problem property and solving difficulty [4,5]. To the best of our knowledge, there is no constructive NCO method to date that can effectively solve both min-max VRPs and mainstream VRPs. Therefore, we believe that this limitation is unfairly criticized.

Best Regards,

Paper3742 Authors

Reference

[1] Kaempfer, Y. , et al. Learning the multiple traveling salesmen problem with permutation invariant pooling networks. arXiv preprint arXiv:1803.09621, 2018.

[2] Liang, H., et al. Splitnet: a reinforcement learning based sequence splitting method for the minmax multiple travelling salesman problem. In Proceedings of the AAAI Conference on Artificial Intelligence, 2023.

[3] Hu, Y., et al. A reinforcement learning approach for optimizing multiple traveling salesman problems over graphs. Knowledge-Based Systems, 2020.

[4] Bertazzi, L. et al. Min–max vs. min–sum vehicle routing: A worst-case analysis. European Journal of Operational Research, 2015.

[5] Son, Jiwoo, et al. Solving np-hard min-max routing problems as sequential generation with equity context. arXiv preprint arXiv:2306.02689, 2023.

Official Comment by Area Chair VmF6

Official Comment Area Chair VmF6 02 Apr 2024, 07:56

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:
Dear reviewers of #3742,

The authors have responded to your comments. Please advise whether your concerns have been addressed or not, if you have not done so. Thanks.

Best, AC of #3742

Response to author's rebuttal

Official Comment Area Chair VmF6 03 Apr 2024, 13:00

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:

Dear reviewer, given the deadline for author-reviewer discussion is approaching, please make sure you have responded to the authors' rebuttal if you have not done so. Thanks.

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 00:35 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=8gPsK9jDss)

Rebuttal:

(1/3)

We sincerely thank all reviewers for their constructive comments and valuable suggestions. These suggestions effectively help us to improve our manuscript. The current manuscript has received positive evaluations regarding its novelty, clarity, and impact:

- **Reviewer iPLg:** The paper is well-structured and easy to comprehend. The performance of the proposed DPN appears promising.
- **Reviewer P1eu:** The proposed DPN methods with augmentations and permutation tricks perform better than existing heuristic and neural solvers with various techniques.
- **Reviewer 4ZsZ:** The paper is clear and well-written, providing clear explanations and motivations backing up the observations. DPN demonstrates significant advantages on most min-max VRP datasets.

We address some common concerns shared by different reviewers in this response.

> **Common Concern 1: Adaptation to min-max VRPs with Contexts / Complex Constraints** (Reviewer iPLg, Reviewer wVP5)

By changing the feasibility mask and adding problem-specific contexts to the attention-based decoder, DPN achieves outstanding performance on the min-max capacitated multi-vehicle routing problem (min-max mCVRP), which has a contextual capacity requirement. The settings of the capacity of min-max mCVRP are followed in [1] and the experiment results on 128 20-node and 30-node instances are listed as follows. We use the report results of OR-Tools, AM, HM, and NCE in [2] and the report results of Equity-Transformer in [3]. Results in the table below preliminarily validate the ability of DPN to handle more complex constraints and process contextual information.

	min-max	**mCVRP**	**(128 instances)**			
N=		20			30	
M=		3			3	
Methods	Obj,	Gap.	Time	Obj,	Gap.	Time
OR-Tools*	2.04	0.63%	1s	2.44	9.67%	1s
AM* [1]	2.2	8.52%	11.8s	2.47	11.02%	11.7s
HM* [1]	2.28	12.47%	<1s	2.39	7.43%	<1s
NCE* [2]	2.06	1.62%	<1s	2.25	1.13%	<1s
Equity-Transformer* [3]	-	-	<1s	2.23	0.23%	<1s
DPN	**2.0272**	-	<1s	**2.2248**	-	<1s

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 00:35 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=AkBI3aVG8e)

Rebuttal:

(2/3)

> **Concern 2: Omni Generalization Ability (Reviewer iPLg, Reviewer 4ZsZ)**: Need more experiments on different distribution (Reviewer 4ZsZ), benchmarks (Reviewer 4ZsZ), and very large-scale (Reviewer iPLg).

We conduct more experiments to evaluate the performance of DPN on different distributions by evaluating instances of different distributions (Reviewer 4ZsZ), and the mTSPLib benchmark (Reviewer 4ZsZ). For min-max mTSP with different distributions (Using the TSP datasets provided in [4]), as shown in the table below, DPN achieves significant advantages over Equity-Transformer and outperforms an advanced heuristic algorithms HGA in the Rotaion and Gaussian distributions.

		min-max mTSP	200-Gaussian			
M=	10		15		20	
Methods	Obj.	Gap.	Obj.	Gap.	Obj.	Gap.
HGA	**1.5063**	-	1.5031	0.00%	1.5031	0.00%
Equity-Transformer-F-$\times$8aug	1.6426	9.05%	1.5458	2.84%	1.5265	1.56%
DPN-F-$\times$8aug	1.5204	0.93%	1.5032	0.00%	1.5031	0.00%
DPN-F-$\times$8aug-$\times$16per	1.5181	0.78%	**1.5031**	-	**1.5031**	-
		min-max mTSP	200-Explosion			
M=	10		15		20	
Methods	Obj.	Gap.	Obj.	Gap.	Obj.	Gap.
HGA	**1.7651**	-	**1.7488**	-	**1.7486**	-
Equity-Transformer-F-$\times$8aug	1.9269	9.17%	1.8277	4.51%	1.8062	3.30%
DPN-F-$\times$8aug	1.7924	1.55%	1.7668	1.03%	1.7663	1.01%
DPN-F-$\times$8aug-$\times$16per	1.7894	1.37%	1.7667	1.02%	1.7663	1.01%
		min-max mTSP	200-Rotation			
M=	10		15		20	
Methods	Obj.	Gap.	Obj.	Gap.	Obj.	Gap.
HGA	1.7492	2.79%	1.7393	4.23%	1.7391	4.28%
Equity-Transformer-F-$\times$8aug	1.8871	10.90%	1.7524	5.01%	1.7151	2.84%
DPN-F-$\times$8aug	1.7064	0.28%	1.6697	0.05%	1.6678	0.00%
DPN-F-$\times$8aug-$\times$16per	**1.7017**	-	**1.6688**	-	**1.6677**	-

The mTSPLib is a widely used and well-known dataset for min-max mTSP. As shown in the table below, compared to the best known solution, DPN with $\times$16 permutation achieves an average gap of 2.81%

Methods	**Gap to Best Known Solution (BKS)**
HGA	0.06%

Methods	Gap to Best Known Solution (BKS)
ScheduleNet	5.99%
DAN	5.97%
Equity-Transformer	5.06%
DPN	2.87%
DPN-$\times$16per	**2.81%**

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 00:34 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=hSePOZmVsU)

Rebuttal:
(3/3)

To solve very large scales (N=100,000) (Reviewer iPLg), we further propose a variant of DPN. By removing the navigation part in each P&N Encoder layer and then training from scratch, this variant has a linear space complexity regarding N (i.e., $\mathcal{O}(NM)$) and can solve very large min-max mTSP instances (i.e., N=100,000) on a single GPU with considerable time. All the heuristic baselines (like HGA and LKH) with default settings cannot generate results within 5 days and other solvers are out of memory on a single GPU.

min-max mTSP (10 instances)

	N=50,000		N=70,000		N=100,000	
M=	500		500		500	
Methods	Obj.	Time	Obj.	Time	Obj.	Time
DPN-w/o Navigation Part	3.4183	34m	3.7832	72m	4.2078	2h

Concern 3: Reproducibility (Reviewer wVP5, Reviewer iPLg)

We will publish the code, datasets, and pre-trained models upon publication.

Contribution of DPN:

DPN is an RL-based sequential planning method, it effectively utilizes problem-specific properties of min-max VRPs. DPN presents a P&N Encoder to process decoupled features for the requirement of the customer partition and the customer navigation within each route. DPN also contains an APS-Loss, leveraging the agent permutation symmetry for training efficiency. In experiments, DPN significantly outperforms existing neural solvers on several min-max VRPs.

Point-to-point responses can be found below to each reviewer. We are also glad to continually improve our work to address any further concerns.

Best Regards,

Paper3742 Authors

Reference [1] Aigerim Bogyrbayeva, et al. A deep reinforcement learning approach for solving the traveling salesman problem with drone. arXiv preprint arXiv:2112.12545, 2021.

[2] Kim, Minjun, et al. Learning to CROSS exchange to solve min-max vehicle routing problems. The Eleventh International Conference on Learning Representations. 2022.

[3] Son, Jiwoo, et al. Solving np-hard min-max routing problems as sequential generation with equity context. arXiv preprint arXiv:2306.02689, 2023.

[4] Zhou, Jianan, et al. Towards omni-generalizable neural methods for vehicle routing problems. International Conference on Machine Learning. PMLR, 2023.

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 00:14 (modified: 29 Mar 2024, 00:36) Program Chairs, Authors

Revisions (/revisions?id=DEs2m32dND)

[Deleted]

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 00:02 (modified: 29 Mar 2024, 00:35) Program Chairs, Authors

Revisions (/revisions?id=ambptCO2HP)

[Deleted]

Official Review of Submission3742 by Reviewer iPLg

Official Review Reviewer iPLg 17 Mar 2024, 21:31 (modified: 02 Apr 2024, 06:10)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors, Reviewer iPLg

Revisions (/revisions?id=5iW6cjhHqE)

Summary:
This paper introduces a new approach to the multi-agent min-max routing problem by presenting a partition and navigation encoder, along with an agent permutation symmetric loss (APS) scheme. Building on the sequential planning framework proposed by Son et al. (2023), our method incorporates additional context and symmetry considerations specific to min-max problems, resulting in improved performance.

Son, Jiwoo, et al. "Solving np-hard min-max routing problems as sequential generation with equity context." arXiv preprint arXiv:2306.02689 (2023).

Strengths And Weaknesses:
Strengths:

- The paper is well-structured and easy to comprehend.
- The performance of the proposed method appears promising.
- The literature review is thorough, with clear articulation of the relationships to existing work.

Weaknesses:

-There is no guarantee that the proposed method can be extended to similar min-max Vehicle Routing Problems (VRPs) that require problem-specific contexts.

Minor Comments:

-In equation (5), "arg min" should not be italicized.

-In the tables, use mathematical notation, such as M and N, instead of plain letters.

-The tables need improvement in aesthetics; consider enhancing clarity and visual appeal by adjusting line spacing and employing \toprule, \midrule, and \bottomrule for better structure

Questions:

1. Is there a plan to release the source code for this paper?
2. Does the method have the capability to manage very large problems, for example, with N=100,000? If it does not, could you discuss potential adaptations or considerations for handling such large-scale problems in future work?

Limitations:
No

Ethics Flag: No
Soundness: 3: good
Presentation: 2: fair
Contribution: 3: good
Rating: 7: Accept: Technically solid paper, with high impact on at least one sub-area, or moderate-to-high impact on more than one areas, with good-to-excellent evaluation, resources, reproducibility, and no unaddressed ethical considerations.
Confidence: 5: You are absolutely certain about your assessment. You are very familiar with the related work and checked the math/other details carefully.
Code Of Conduct: Yes

- = ≡

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

28 Mar 2024, 23:40 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=e5uydNXhqE)

Rebuttal:
(1/3)

We sincerely thank all reviewers for their constructive comments and valuable suggestions. These suggestions effectively help us to improve our manuscript. The current manuscript has received positive evaluations regarding its novelty, clarity, and impact:

- **Reviewer iPLg:** The paper is well-structured and easy to comprehend. The performance of the proposed DPN appears promising.
- **Reviewer P1eu:** The proposed DPN methods with augmentations and permutation tricks perform better than existing heuristic and neural solvers with various techniques.
- **Reviewer 4ZsZ:** The paper is clear and well-written, providing clear explanations and motivations backing up the observations. DPN demonstrates significant advantages on most min-max VRP datasets.

We address some common concerns shared by different reviewers in this response.

> **Common Concern 1: Adaptation to min-max VRPs with Contexts / Complex Constraints** (Reviewer iPLg, Reviewer wVP5)

By changing the feasibility mask and adding problem-specific contexts to the attention-based decoder, DPN achieves outstanding performance on the min-max capacitated multi-vehicle routing problem (min-max mCVRP), which has a contextual capacity requirement. The settings of the capacity of min-max mCVRP are followed in [1] and the experiment results on 128 20-node and 30-node instances are listed in the table below. It preliminarily validates the ability of DPN to handle more complex constraints and process contextual information. Obj. values of OR-Tools, AM, HM, and NCE are reported in [2] and the performance of Equity-Transformer is reported in [3].

min-max mCVRP (128 instances)

N=	20	30

min-max mCVRP (128 instances)

M=		3			3	
Methods	Obj,	Gap.	Time	Obj,	Gap.	Time
OR-Tools*	2.04	0.63%	1s	2.44	9.67%	1s
AM* [1]	2.2	8.52%	11.8s	2.47	11.02%	11.7s
HM* [1]	2.28	12.47%	<1s	2.39	7.43%	<1s
NCE* [2]	2.06	1.62%	<1s	2.25	1.13%	<1s
Equity-Transformer* [3]	-	-	<1s	2.23	0.23%	<1s
DPN	**2.0272**	-	<1s	**2.2248**	-	<1s

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

28 Mar 2024, 23:44 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=wFI4s5nzob)

Rebuttal:
(2/3)

> **Minor Comments**: Typos and Formats. The tables need improvement in aesthetics.

Thank you for pointing this out. We will correct all typos and adjust the layout of the tables.

> **Question 1. Release Source Codes.**

Yes, we will publish the code, datasets, and pre-trained models upon publication.

> **Question 2. Capability to Very Large Problems (N=100,000).**

The original P&N encoder of DPN cannot manage large-scale problems such as $N=100,000$. When generating a single solution, the space complexity of DPN is $\mathcal{O}(N(N+M))$, so the proposed DPN can only solve up to 10,000-scale problems on a single Tesla V-100S GPU.

DPN-w/o Navigation Part for N=100,000: To handle large-scale data such as N=100,000, we further propose a variant of DPN. We design a variant of DPN by removing the navigation part in each P&N Encoder layer (only preserving the partition part, marked as DPN-w/o Navigation Part in the table below) and training this variant from scratch. The space complexity of the DPN-w/o Navigation Part reduces to $\mathcal{O}(NM)$ so this model can derive feasible solutions on 100,000 scale problems. Results in the tables below show that this model can solve very large min-max mTSP(N=100,000) on a single GPU with considerable time.

min-max mTSP (10 instances)

	N=50,000		N=70,000		N=100,000	
M=	500		500		500	
Methods	Obj.	Time	Obj.	Time	Obj.	Time
DPN-w/o Navigation Part	3.4183	34m	3.7832	72m	4.2078	2h

All the heuristic baselines (like HGA and LKH) with default settings cannot generate results within 5 days on 100,000-scale problems. However, the performance of solutions can be partially reflected by the comparison to DPN and Equity-Transformer on min-max mTSP10,000 shown in the table below. It is worth noting that although this version performs well on the min-max mTSP10,000, it is not competitive on the 100-scale to 1000-scale instances. We will continue to supplement the results of this experiment and present them in the final version of the paper.

min-max mTSP 10000(N=9,999 ,D=1, 10 instances)

M=	500		750		1000	
Methods	Obj.	Time	Obj.	Time	Obj.	Time
Equity-Transformer	4.5645	3m	2.9245	3m	2.8524	3m
DPN	2.4640	2.8m	2.4724	2.8m	2.3853	2.8m
DPN-w/o Navigation Part	**2.3487**	2.8m	**2.3333**	2.8m	**2.2568**	2.8m

- = ≡

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

28 Mar 2024, 23:57 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=zhzBLKEUmV)

Rebuttal:
(3/3)

References

[1] Aigerim Bogyrbayeva, et al. A deep reinforcement learning approach for solving the traveling salesman problem with drone. arXiv preprint arXiv:2112.12545, 2021.

[2] Kim, Minjun, et al. Learning to CROSS exchange to solve min-max vehicle routing problems. The Eleventh International Conference on Learning Representations. 2022.

[3] Son, Jiwoo, et al. Solving np-hard min-max routing problems as sequential generation with equity context. arXiv preprint arXiv:2306.02689, 2023.

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Replying to Rebuttal by Authors

Official Comment by Reviewer iPLg

Official Comment Reviewer iPLg 02 Apr 2024, 06:10

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:
Thank you for the thoughtful rebuttal; it resolved most of my concerns. I believe the min-max combinatorial optimization tasks are significant. This work focuses on a topic that has been underemphasized by the community, which I find important. Therefore, I have raised my score to a 7.

- = ≡

Replying to Official Comment by Reviewer iPLg

Official Comment by Authors

Official Comment

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

02 Apr 2024, 10:06

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:
Thanks very much for your appreciation of our response, and we are glad to hear that most of my concerns have been resolved. Furthermore, we sincerely thank you for your recognition of the significance of the min-max combinatorial optimization.

Official Review of Submission3742 by Reviewer P1eu

Official Review Reviewer P1eu 15 Mar 2024, 00:08 (modified: 02 Apr 2024, 17:17)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors, Reviewer P1eu

Revisions (/revisions?id=eQDAvev6EU)

Summary:
The class of min-max VRP involves various important combinatorial problems (e.g., TSP, VRP, PDP, etc.), and the paper tackles such problems by a new concept named decoupling partition and navigation (DPN); the key idea is P+N encoder, aims at generating tours in parallel together with Transformer block and trained the policy through REINFORCE; the standard methodology for learning-based solvers. The experimental results suggest that DPN methods with augmentations and permutation tricks perform better than existing heuristic and neural solvers.

Strengths And Weaknesses:
Strengths:

- [S1] Although the min-max VRP is one of the most known classes, the proposed learning-based neural solvers performed well with various techniques.
- [S2] Precisely, a simple strategy using augmentation and permutation-based methods works efficiently.

Weaknesses:

- [W1] The presentation of the key component (i.e., the permutation-based technique and loss, named APS-Loss) needs to be improved.

Questions:

- After reading the results, the main contribution seems to be the discussion of APS-Loss, in my opinion (e.g., in small mTSP, naive DPN was weaker than LKH3. In mPDP, DPN seems good. In larger mTSP, naive DPN is not included in the main text). Comparing mTSP and mPDP, I imagined that symmetry has a much greater effect in mPDP than mTSP as it contains P and D in different locations. So, I think Sec. 3.2 should be improved to clarify the main contribution. The computational trick of using K-sampled permutations for baseline (i.e., b(G)) seems to be a straightforward (Monte-Carlo) estimator. However, this works efficiently in the experiments. So, I recommend the authors to update this section (e.g., shrinking Sec 2.3 of Transformer block). I could be positive if I clearly understood the concept of the proposed method of the paper.

Limitations:
Well described in the paper/not related to such a negative societal impact in my opinion.

Ethics Flag: No
Soundness: 4: excellent
Presentation: 2: fair
Contribution: 2: fair
Rating: 6: Weak Accept: Technically solid, moderate-to-high impact paper, with no major concerns with respect to evaluation, resources, reproducibility, ethical considerations.
Confidence: 4: You are confident in your assessment, but not absolutely certain. It is unlikely, but not impossible, that you did not understand some parts of the submission or that you are unfamiliar with some pieces of related work.
Code Of Conduct: Yes

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Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 01:36 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=5nQa3itCva)

Rebuttal:

(1/5)

Thank you very much for your time and effort in reviewing our work. We are glad to know that you find the proposed DPN performs well with various techniques.

Thank you for your suggestions on presentation and we address your concerns as follows.

> **Weakness 1. The presentation of the APS-Loss:** The presentation of the key component (i.e., the permutation-based technique and loss, named APS-Loss) needs to be improved.

Thank you for your valuable suggestion. We will modify the presentations of key components to improve clarity. For APS-Loss, based on the suggestions in Questions 2 to 5, we will clarify the contributions and add a sketch map to facilitate comprehension.

> **Question 1. Naive DPN in large-scale problems**.

Thanks for your comment. We complement the Naive DPN results on larger-scale min-max mTSP instances and larger-scale min-max mPDP instances in our paper now. The results are as follows.

		min-max mTSP200			**min-max mTSP500**	
M=	10	15	20	30	40	50
Methods	Obj.	Obj.	Obj.	Obj.	Obj.	Obj.
HGA	1.9861	**1.9628**	**1.9627**	**2.0061**	2.0061	2.0061
LKH3	**1.9817**	1.9628	1.9628	2.0061	2.0061	2.0061
Equity-Transformer	2.0867	1.9774	1.9638	2.0401	2.0162	2.0091
Equity-Transformer-F-$\times$8aug	2.0500	1.9688	1.9631	2.0165	2.0084	2.0068
DPN	2.0175	1.9661	1.9630	2.0074	2.0062	2.0064
DPN-F-$\times$8aug	2.0030	1.9647	1.9628	2.0065	2.0061	2.0061
DPN-F-$\times$8aug-$\times$16per	1.9993	1.9640	1.9628	2.0061	**2.0061**	**2.0061**
		min-max mPDP200			**min-max mPDP500**	
M=	10	15	20	30	40	50
Methods	Obj.	Obj.	Obj.	Obj.	Obj.	Obj.
Equity-Transformer	5.4465	4.2575	3.6921	5.3181	4.1140	3.7566
Equity-Transformer-F-$\times$8aug	4.9143	3.8186	3.3417	4.4619	3.7723	3.4455
DPN	3.4455	2.9519	2.7405	3.3433	3.2080	3.4034

	min-max mTSP200			min-max mTSP500		
DPN-F-$\times$8aug	3.3227	2.8630	2.6735	3.1615	3.0264	2.9379
DPN-F-$\times$8aug-$\times$16per	**3.2959**	**2.8363**	**2.6519**	**3.0878**	**2.9510**	**2.8690**

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 01:51 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=A5epPFG2hd)

Rebuttal:
(2/5)

> **Question 2. Computation method of the APS-equipped baseline.** The computational trick of using K-sampled permutations for baseline (i.e., $b(\mathcal{G})$) seems to be a straightforward (Monte-Carlo) estimator.

Thank you for your insightful comment. Just as you understand, the APS-equipped baseline serves as a Monte-Carlo estimator in training.

As you mentioned in the Summary part, REINFORCE is the standard methodology for learning-based solvers. Using critic networks may cause training instability [1], so solvers generally use problem properties for baseline estimation [1,2,3]. Based on the specific properties of min-max VRPs, there is an agent-permutation-symmetry (APS), which preserves the same optimal solution in sequentially decoding routes according to different given orders. So, we get a baseline estimation method based on APS by random sampling K (K=60 in training) permutations of M routes and then generating K solutions. The average reward of these K solutions serves as the baseline in APS-Loss.

> **Question 3. Effectiveness of the APS-equipped baseline.** The APS-equipped baseline seems straightforward but works efficiently in the experiments.

Thanks for your comments. Compared to DPN, Equity-Transformer [4] utilizes the "problem symmetry" property proposed in Sym-NCO [3] which generates 8 equivalent instances (with the same optimal solution) based on rotation and inputs these instances into the Equity-Transformer to estimate a baseline. Problem symmetry is general for all VRPs and the APS proposed in DPN is specific for min-max VRPs. Compared to the "problem symmetry", APS has advantages in the following **four** aspects.

APS-Loss is more efficient in reducing variance: Firstly, the APS-equipped baseline reduces the variance of the baseline estimation. We test the reward variance of the DPN and the ablation model (DPN-w/o APS-Loss) with 16 sampled permutations on min-max mTSP and min-max mPDP. As shown in the table below (**next part**), APS-Loss plays a role in reducing the variance of the baseline estimation. Moreover, though the DPN-w/o APS-Loss ablation model leverages the "problem symmetry" for baseline estimation, its variance over 8 augmentations is even higher. So it means that the effect of problem symmetry is not significant in min-max VRPs and APS-Loss is more efficient in reducing variance.

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:05 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=BrzLV6ywlo)

Rebuttal:
(3/5)

mean variance over 16 sampled permutations (100 instances)

	min-max mTSP100			min-max mPDP100		
M=	5	7	10	5	7	10
DPN	0.00018	0.00007	0.00004	0.00141	0.00169	0.00209
DPN-w/o APS-Loss	0.82153	0.79956	0.51874	1.86095	1.17915	0.81210

mean variance over 16 sampled permutations (100 instances)

	min-max mTSP100			min-max mPDP100		
M=	5	7	10	5	7	10
DPN	0.00108	0.00041	0.00007	0.01119	0.00770	0.00431
DPN-w/o APS-Loss	0.00312	0.00167	0.00037	1.13495	0.11982	0.01565

APS-Loss is more efficient in reducing bias. We test the performance of the DPN-w/o APS Loss variant in min-max mTSP100 (left half of the table below). Results on permutation $\boldsymbol{o}=(1,2,3,\ldots, M)$ exhibit a significant advantage compared to other permutations. The huge bias represents that the representation of M agents (i.e., routes) retains a large amount of order-related information, which does not align with optimality. The right half table shows the results of the same experiment results of DPN. DPN significantly reduces the bias caused by different agent permutations, which not only makes agent representations more likely to the optimal but also **enables the $\times$K permutation augmentation**.

Obj. of min-max mTSP100	**DPN-w/o APS-Loss**			**DPN**		
M=	5	7	10	5	7	10
$\boldsymbol{o}=(1,2,3,\ldots,M)$	2.2625	2.0347	1.9571	2.2693	2.0340	1.9590
Average of random o	3.4260	3.2069	2.8477	2.2696	2.0344	1.9590
Bias	1.1634	1.1722	0.8906	0.0004	0.0004	0.0001

APS-Loss helps convergence. In addition, the right half of the table above demonstrates that the APS-equipped baseline is almost unbiased. In policy gradient works, it is generally recognized that for unbiased estimation, baselines with lower variations are outstanding for conversion [5,6]. Therefore, by combining the above two advantages (i.e., lower bias and variance), the effect of APS-Loss can be preliminarily explained.

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:11 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=ntAVLRbOSq)

Rebuttal:
(4/5)

APS-Loss is of space efficiency. As the last advantage we want to show, APS-Loss is of space efficiency. Decoding the K sampled solutions can share the same embeddings (output of encoder). Compared to the "problem symmetry" (w/o APS-Loss version in the ablation study) which should process rotated instances through encoders as well, solutions for the APS-equipped baseline can be processed in parallel only in the single-layer decoder, greatly improving the overall training efficiency. The table below shows that APS-Loss significantly reduces the GPU memory consumption which allows a larger batch size.

Memory consumption on training batch=1

	APS-Loss	w/o APS-Loss(leveraging problem symmetry)
min-max mTSP100	118.59MB	236.49MB
min-max mPDP100	146.03MB	318.09MB

> **Question 4. Effect of the agent-permutation-symmetry on mTSP and mPDP.** Comparing mTSP and mPDP, I imagined that symmetry has a much greater effect in mPDP than mTSP as it contains P and D in different locations.

Thank you for your insightful comment. As you have observed, the ablation study indicates that APS-Loss is more effective in min-max mPDP compared to its performance in min-max mTSP. The different Pickups and Deliveries in mPDP indeed result in higher solutions for each path, so your understanding is reasonable and inspiring.

Based on the current results, the first table in Question 3 shows that DPN-w/o APS Loss has a very high variance over 8 equivalent instances in min-max mPDP, so the clearer advantage of APS-Loss in min-max mPDP may be due to **the worse efficiency of problem symmetry in min-max mPDP**. Before publication, we will further experiment on the significant effect of APS-Loss on min-max mPDP, to explain the different effects in different min-max VRPs.

> **Question 5. APS-Loss's contributions.** Sec. 3.2 should be improved to clarify the main contribution.

Thanks for your valuable suggestion. We should regard the P&N and APS-Loss as the two main contributions of DPN. We will adjust the narrative preferences based on your suggestions to improve clarity.

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Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:12 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=BrzmATdicL)

Rebuttal:
(5/5)

> **References**

[1] Kool, et al. Attention, learn to solve routing problems!. arXiv preprint arXiv:1803.08475, 2018.

[2] Kwon, Yeong-Dae, et al. Pomo: Policy optimization with multiple optima for reinforcement learning. Advances in Neural Information Processing Systems 33, 2020.

[3] Kim, M., et al. Sym-nco: Leveraging symmetricity for neural combinatorial optimization. Advances in Neural Information Processing Systems, 35, 2022.

[4] Son, Jiwoo, et al. Solving np-hard min-max routing problems as sequential generation with equity context. arXiv preprint arXiv:2306.02689, 2023.

[5] Wu, Cathy, et al. Variance reduction for policy gradient with action-dependent factorized baselines. arXiv preprint arXiv:1803.07246, 2018.

[6] Schulman, John, et al. High-dimensional continuous control using generalized advantage estimation. arXiv preprint arXiv:1506.02438, 2015.

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Replying to Rebuttal by Authors

Thank you for your comments.

Official Comment Reviewer P1eu 02 Apr 2024, 17:16

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:
I appreciate all comments from the authors (not limited to those to my review), and I basically have a positive feeling, although my score was not high (>= 5) in the first status. This was because I could not clearly follow the contribution and its main idea (i.e., I mentioned some points related to APS-Loss).

After rebuttal and carefully reading the comments, I feel the proposed concept is important. Further, as I noted in the first round, the proposed method clearly works well for various settings (i.e., various min-max problems). To wrap up them, I increased my score from 4 to 6.

- = ≡

Replying to Thank you for your comments.

Official Comment by Authors

Official Comment

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

02 Apr 2024, 18:45

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:
We are immensely grateful for your recognition of our responses and the value of the DPN concept, and glad to hear that you are more clear with our contribution and the main idea. We will improve the presentation of our manuscript accordingly.

- = ≡

Official Review of Submission3742 by Reviewer 4ZsZ

Official Review Reviewer 4ZsZ 14 Mar 2024, 09:10 (modified: 02 Apr 2024, 08:27)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors, Reviewer 4ZsZ

Revisions (/revisions?id=5FuOdHZuvt)

Summary:
This paper presents DPN, a new sequential planning-based method for min-max vehicle routing problems (min-max VRP). Highlighting problem-specific properties of min-max VRP, DPN designs a new network architecture (P&N Encoder), a new representation method (Rotation-based PE), and a new loss function (APS-Loss). It achieves significant improvements on four min-max VRPs.

Strengths And Weaknesses:

Strengths: ·DPN demonstrates significant advantages on most min-max VRP datasets. ·Provided clear explanations and motivations backing up the observations. ·The paper is clear and well-written. Weaknesses: ·Need more detailed data for the training time of DPN. ·For the instances with different distributions discussed in [1], more robustness experiments are needed. ·Need more experiments on benchmark datasets (like TSPLib). ·APS-Loss clearly develops from the common baseline calculation method in NCO work. Need to review and introduce previous work to enhance clarity. ·Minor suggestions are listed in Questions. [1] Zhou, Jianan, et al. "Towards omni-generalizable neural methods for vehicle routing problems." International Conference on Machine Learning. PMLR, 2023.

Questions:

1. Tables 1 and 2 are too dense in content.
2. Lack of ablation experiments about the setting of the base (1,000) in Eq (19). Is the algorithm sensitive to this parameter?
3. Need consistent style in the three subgraphs of Figure 1.

Limitations:
N/A

Ethics Flag: No
Soundness: 3: good
Presentation: 2: fair
Contribution: 3: good
Rating: 7: Accept: Technically solid paper, with high impact on at least one sub-area, or moderate-to-high impact on more than one areas, with good-to-excellent evaluation, resources, reproducibility, and no unaddressed ethical considerations.
Confidence: 5: You are absolutely certain about your assessment. You are very familiar with the related work and checked the math/other details carefully.
Code Of Conduct: Yes

- = ≡

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:22 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=INsZ8Csesr)

Rebuttal:
(1/3)

Thank you very much for your time and effort in reviewing our work. We are glad to know that you find the proposed DPN demonstrates significant advantages and this article provides clear explanations and motivations.

We address your concerns as follows.

> **Weakness 1. Detailed Training Time of DPN.** Need more detailed data for the training time of DPN.

Thanks a lot for the suggestion. The detailed training time of DPN on four involved min-max VRPs are listed as follows and we will include this table in our modified paper.

Problem	min-max mTSP100	min-max mPDP100	min-max MDVRP100	min-max FMDVRP100
Epoch Time	11.0min	11.2min	12.6min	12.6min
Total Time	91.7h	93.3h	105h	105h

> **Weakness 2. Robustness Experiments.** For the instances with different distributions discussed in [1], more robustness experiments are needed.

We agree that the cross-distribution generalization ability of neural combinatorial optimization solvers is necessary. So we evaluate the DPN on the Gaussian distribution, the Rotation distribution, and the Explosion distribution provided in [1] on min-max mTSP200. The result is shown in the table below and the DPN demonstrates outstanding robustness on different distributions.

			min-max mTSP 200-Gaussian			
M=	10		15		20	
Methods	Obj.	Gap.	Obj.	Gap.	Obj.	Gap.
HGA	**1.5063**	-	1.5031	0.00%	1.5031	0.00%
Equity-Transformer-F-$\times$8aug	1.6426	9.05%	1.5458	2.84%	1.5265	1.56%
DPN-F-$\times$8aug	1.5204	0.93%	1.5032	0.00%	1.5031	0.00%
DPN-F-$\times$8aug-$\times$16per	1.5181	0.78%	**1.5031**	-	**1.5031**	-
			min-max mTSP 200-Explosion			
M=	10		15		20	
Methods	Obj.	Gap.	Obj.	Gap.	Obj.	Gap.
HGA	**1.7651**	-	**1.7488**	-	**1.7486**	-
Equity-Transformer-F-$\times$8aug	1.9269	9.17%	1.8277	4.51%	1.8062	3.30%
DPN-F-$\times$8aug	1.7924	1.55%	1.7668	1.03%	1.7663	1.01%
DPN-F-$\times$8aug-$\times$16per	1.7894	1.37%	1.7667	1.02%	1.7663	1.01%
			min-max mTSP 200-Rotation			
M=	10		15		20	
Methods	Obj.	Gap.	Obj.	Gap.	Obj.	Gap.
HGA	1.7492	2.79%	1.7393	4.23%	1.7391	4.28%
Equity-Transformer-F-$\times$8aug	1.8871	10.90%	1.7524	5.01%	1.7151	2.84%
DPN-F-$\times$8aug	1.7064	0.28%	1.6697	0.05%	1.6678	0.00%
DPN-F-$\times$8aug-$\times$16per	**1.7017**	-	**1.6688**	-	**1.6677**	-

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:25 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=Fw3VxZQMmG)

Rebuttal:
(2/3)

> **Weakness 3. More Benchmark Experiments.**

Thanks a lot for the suggestion. We further test our method in mTSPLib[2], including eil51, belin52, eil76, rat99, M=2,3,5,7). The average gap to the best-known solution (reported in [3]) is listed as follows and we will include this experiment in our paper.

Method	Gap to Best Krown Solution (BKS)
HGA	0.06%
ScheduleNet	5.99%
DAN	5.97%
Equity-Transformer	5.06%
DPN	2.87%
DPN-$\times$16per	**2.81%**

Weakness 4. Literature Reviews about APS-Loss.

Thank you for your comment. APS-Loss uses APS-equipped baseline $b(\mathcal{G})$ as an unbiased estimate for the current reward. The use of unbiased estimation as the form of baseline was first proposed in POMO [4], which takes into account the symmetry of the solutions (called solution symmetry in [5]) starting from each node in TSP, CVRP, and KP. Using these solutions with the same optimal cost, the solution symmetry in POMO greatly improves the convergence speed in training. Sym-NCO [5] develops the problem symmetry, it rotates the original instances by different rotation matrices for baselines. The problem symmetry improves the robustness of POMO but it causes more space consumption in training. MatNet [6] proposes the symmetry of pipeline decoding order and extends this baseline form to job scheduling problems in solving the flexible flow shop scheduling problem (FFSP).

We will discuss these contents in detail in the article to enhance the clarity of the presentation.

Question 1. Formats.

Thank you for pointing this out. We will adjust the font size of Table I and Table II.

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:27 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=haxAJFOAPV)

Rebuttal:
(3/3)

Question 2. Ablation on the base of Rotation-based PE.

Thank you for your valuable suggestion. We complement a parameter analysis on the base of Rotation-based PE. We train 250 epochs (half of the total training) from scratch using models with the base being 10,000 and 1,000, respectively. The results shown in the table below indicate that the model performance is not sensitive to the setting of the base on both min-max mTSP and min-max mPDP.

		min-max mTSP100	
	5	7	10
HGA	2.1893	1.9963	1.9507
base=1,000(half training)	2.2384	2.0177	1.9530
base=10,000(half training)	2.2363	2.0155	1.9528
		min-max mPDP100	
	5	7	10

min-max mTSP100			
base=1,000(half training)	3.5520	3.0582	2.7105
base=10,000(half training)	3.5705	3.0723	2.7207

References

[1] Zhou, Jianan, et al. Towards omni-generalizable neural methods for vehicle routing problems. International Conference on Machine Learning. PMLR, 2023.

[2] mTSPLib. mTSPLib. URL https://profs.info.uaic.ro/˜mtsplib/MinMaxMTSP/ (https://profs.info.uaic.ro/%CB%9Cmtsplib/MinMaxMTSP/).

[3] Mahmoudinazlou, S., et al. A hybrid genetic algorithm for the min–max Multiple Traveling Salesman Problem. Computers & Operations Research, 162, p.106455, 2024.

[4] Kwon, Yeong-Dae, et al. Pomo: Policy optimization with multiple optima for reinforcement learning. Advances in Neural Information Processing Systems 33, 21188-21198, 2020.

[5] Kim, M., et al. Sym-nco: Leveraging symmetricity for neural combinatorial optimization. Advances in Neural Information Processing Systems, 35, 1936-1949, 2022.

[6] Kwon Y D, et al. Matrix encoding networks for neural combinatorial optimization[J]. Advances in Neural Information Processing Systems, 34: 5138-5149, 2021.

Replying to Rebuttal by Authors

Thanks for the rebuttal.

Official Comment Reviewer 4ZsZ 02 Apr 2024, 08:26

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:
Thanks for the rebuttal. All my concerns have been well addressed. I raise my score to 7.

Replying to Thanks for the rebuttal.

Official Comment by Authors

Official Comment

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

02 Apr 2024, 10:14

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:
Thank you very much for increasing your rating to 7, and we are glad to know all your concerns have been addressed. Thank you for your time and effort in reviewing our work.

Official Review of Submission3742 by Reviewer wVP5

Official Review Reviewer wVP5 12 Mar 2024, 10:42 (modified: 05 Apr 2024, 10:57)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors, Reviewer wVP5

Revisions (/revisions?id=pXwLoLbN1e)

Summary:

This paper first partitions VRP to TSPs and then optimizes the TSPs to solve min-max VRP. For the symmetry of the sub-routes, the paper uses the APS-Loss to make the sub-routes invariants to the possible permutations. It introduces a new Encoder to encode partition and customer information using MHA. However, a similar neural partition idea has been proposed by TAM [1] to solve large-scale VRPs in real-time by dividing large-scale VRP to TSPs with the Attention Model. The TAM is neither cited in the paper nor discussed/compared with the proposed DPN. In addition, the symmetry idea is borrowed from the POMO. Therefore, although the topic is interesting, I think the method contribution is a bit marginal.

[1] Hou, Qingchun, et al. "Generalize learned heuristics to solve large-scale vehicle routing problems in real-time." The Eleventh International Conference on Learning Representations. 2022.

Strengths And Weaknesses:
The paper proposes a new Encoder to encode partition and customer information using MHA.

For the symmetry of the sub-routes, the paper borrows the ideas from POMO to use the APS-Loss to make the sub-routes invariants to the possible permutations.

The idea of partition and navigation with Attention model is not new. It's true that the TAM focuses on VRP while DPN focuses on min-max VRP. That does not make a big difference considering that the min-max VRP is a sub-set of VRP. The difference between DPN and TAM should be discussed and compared in terms of partition.

The details of the decoder are missing in the main part. Some related works are missing in the reviews. The reviewer encourages the authors to improve the reviews.

The code is not provided. It's hard to evaluate the reproducibility.

The method is only evaluated in min-max VRP. To make the method significant, the method or encoder should be evaluated on more general VRPs, such as CVRP or CVRPTW, which will have more benchmarks to make the work more convincing.

Questions:
What's the performance of the paper on main-stream VRP variants, such as CVRP, and CVRPTW?

How to consider more complex constraints in the DPN?

Limitations:
It would be good to summarize the limitations of DPN.

Ethics Flag: No
Soundness: 2: fair
Presentation: 2: fair
Contribution: 1: poor
Rating: 4: Borderline reject: Technically solid paper where reasons to reject, e.g., limited evaluation, outweigh reasons to accept, e.g., good evaluation. Please use sparingly.
Confidence: 4: You are confident in your assessment, but not absolutely certain. It is unlikely, but not impossible, that you did not understand some parts of the submission or that you are unfamiliar with some pieces of related work.
Code Of Conduct: Yes

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Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:33 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=M3ca6NKrpt)

Rebuttal:
(1/5)

Thank you for your comments. Unfortunately, we cannot agree with your partial evaluation of our article but we appreciate the opportunity to explain them. We especially make rebuttals to your Summary and address your raised concerns point-by-point as follows:

> **Rebuttal to Summary 1: Method classification of DPN**. This paper first partitions VRP to TSPs and then optimizes the TSPs to solve min-max VRP. **Weakness 1: Novelty & Differences from TAM [1]**. TAM focuses on VRP while DPN focuses on min-max VRP. The difference between DPN and TAM should be discussed and compared in terms of partition.

Thank you for your comment. **DPN does not conduct** such a process as **'first partitions VRP to TSPs and then optimizing the TSPs''**.

DPN is actually a single-stage end-to-end constructive solver: On the contrary, DPN is an end-to-end sequential planning method that directly proposes the min-max VRPs by the P&N Encoder and then directly decodes the total solution of min-max VRPs. As the first work focusing on problem-specific properties of min-max VRPs, DPN gets decoupled representations (output of the encoder) of partition and navigation through P&N encoder and APS-Loss.

Like the idea of TAM [1], models that partition min-max VRP to TSPs and then optimize the TSPs have been proposed in [2,3,4]: Decomposing VRP to TSPs and solving the TSP sub-problem is a compelling method, and a series of efficient papers such as TAM[1] have achieved success in CVRP. It's right that "The idea of partition and navigation with Attention model is not new", so there have been several papers adopting such a two-stage (or two-step occurred in TAM) solving framework in TAM to min-max VRPs like GNN-DisPN[2] and SplitNet[3], etc[4] (Discussed in the second paragraph of Introduction and Appendix A.2.). However, due to the difficulty in coordinating the two stages, these works are not outstanding in min-max VRP (We compare their report results to DPN in the table below) so we focus on constructive methods especially constructive methods with sequential planning to enhance performance.

min-max mTSP

	\$N\$=100,\$M\$=5		\$N\$=100,\$M\$=10		\$N\$=200,\$M\$=10	
Method	Cost	Time	Cost	Time	Cost	Time
GNN-DisPN	2.56	-	2.22	-	2.97	-
SplitNet(s.64)	2.42	8.6s	2.01	14.3s	2.27	8.6s
DPN	2.2314	1s	1.9532	1s	1.9993	1s

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Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:41 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=GsIDPj7eEn)

Rebuttal:
(2/5)

Partition and navigation requirement in min-max VRP. The frequent use of the words `Partition-and-Navigation` may cause misunderstandings. However, unlike the CVRP, min-max VRPs always limit the number of routes or vehicles (to \$M\$). So, the Partition-and-Navigation in our article represents a problem-specific requirement for min-max VRP. As a single-stage constructive method (similar to POMO [5] in solving CVRP), we design the P&N Encoder to handle this requirement, allowing the DPN model to learn effective representations of \$N\$ customers and the limited number of \$M\$ routes (vehicles or agents).

> **Rebuttal to Summary 2: Compare to TAM.** A similar neural partition idea has been proposed by TAM [1]. The TAM is neither cited in the paper nor discussed/compared with the proposed DPN.

Thank you for your comments. **The idea of TAM is not similar to DPN**. Firstly, as mentioned in Rebuttal to Summary 1, the proposed DPN uses a one-stage constructive decoding method which is different from the two-step solving method of TAM [1]. Moreover, TAM uses different networks to solve the problem in two stages but there is only one model in DPN.

The framework of TAM cannot directly apply to min-max VRP. Next, we would like to clarify that min-max VRP is a special variant of VRP, which incorporates a constraint on the number of routes, which is not considered in typical VRP tasks such as CVRP. The TAM and most other solvers do not take the number of routes (vehicles) as input, therefore, to handle various M requirements on the same min-max VRP instances, TAM needs additional designs. We will add a discussion about the difference between DPN and TAM in terms of partition in our paper. Meanwhile, we will also add the comparison results with existing two-stage methods to the article (Results in Rebuttal to Summary 1).

Weakness 2. Details of Decoder

The decoder of DPN adopts the common structure of constructive NCO solvers, containing a single layer of multi-head attention (Eq. (76)) and a final probability calculation function (Eq. (78)). The decoder constructs customer one by one or chooses to end the current vehicle (route or agent). It introduces a dynamic query (Eq. (75)) to represent the current partial solution. To obtain the baselines in APS-Loss effectively, the decoder processes K solutions together in the query. The details are in Appendix C.2, C.3, and C.4 and we will modify these parts.

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:44 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=jlmuXATlkp)

Rebuttal:
(3/5)

Weakness 3. Reviews about two-stage solvers for VRP.

Review of two-stage methods in VRPs. In traditional VRP, customer partition between agents is not a mandatory requirement, the similar idea of partition and navigation appears to be compelling. To divide and conquer large-scale problems, some works such as TAM [5], RBG [6], H-TSP [7], and GLOP [8] use two sets of independent neural networks (or heuristics) to generate a large-scale solution, which are respectively responsible for problem splitting and subproblem solving. These two-step solving methods have successfully achieved efficient solutions for large-scale problems such as TSP with 10,000 nodes.

We will add these contents to Appendix A.1. **For the review of the min-max VRP solvers**, we conduct further research and found it enough. **Reviewer iPLg also stated**, `The literature review is thorough, with a clear articulation of the relationships to existing work.`

Weakness 4. Reproducibility

Thank you for your comments. We will publish the code, datasets, and pre-trained models upon publication.

Question 1. How to consider more complex constraints in the DPN?

As a constructive NCO method, DPN satisfies complex constraints by changing the current query and modifying the attention mask. To further demonstrate the ability of DPN to handle more complex constraints, we apply the proposed DPN to the min-max Capacity Vehicle Routing Problem (CVRP) It has load-carrying capacity constraints that require contexts in coding The min-max mCVRP is defined in [9]. Each customer has a specified demand that our vehicle must fulfill while minimizing the maximum route length among all vehicles. Compared to the reported result of the advanced learning-based method AM (proposed in [9]), HM [9], NCE [10], and Equity-Transformer [11]. The proposed DPN outperforms other methods on min-max mCVRP20 and min-max mCVRP30. It shows the ability of DPN to handle min-max VRPs with more complex constraints.

	min-max	**mCVRP**	**(128 instances)**			
N=		20			30	
M=		3			3	
Methods	Obj,	Gap.	Time	Obj,	Gap.	Time
OR-Tools*	2.04	0.63%	1s	2.44	9.67%	1s
AM* [9]	2.2	8.52%	11.8s	2.47	11.02%	11.7s
HM* [9]	2.28	12.47%	<1s	2.39	7.43%	<1s
NCE* [10]	2.06	1.62%	<1s	2.25	1.13%	<1s
Equity-Transformer* [11]	-	-	<1s	2.23	0.23%	<1s
DPN	**2.0272**	-	<1s	**2.2248**	-	<1s

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:50 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=ruEeKqsaPX)

Rebuttal:
(4/5)

> **Question 2. What's the performance of the paper on main-stream VRP variants, such as CVRP, and CVRPTW?**

Different from CVRP or CVRPTW, min-max VRPs limit the total number of vehicles. DPN adopts the P&N Encoder and the APS-Loss to effectively process this constraint so **the DPN is unable for CVRP or CVRPTW (VRPs without this constraint)**. Due to the existence of this constraint, together with min-max VRP emphasizing balancing the lengths between different routes, powerful heuristic approaches for min-sum problems are not well-generalized to the min-max case [12,13], and all the existing constructive min-max VRP solving frameworks can not effectively generalize to CVRP or CVRPTW [14].

We acknowledge the enormous value of a universal and effective framework for both VRPs and min-max VRPs. Therefore, as future work, we are eager to adopt the idea of DPN to design a universal constructive solver to process decoupled representations between routes and effectively handle both VRPs and min-max VRPs.

> **Limitation. Summarize the Limitations of DPN**

Thank you for your comments. DPN performs relatively badly on min-max VRP with large decision spaces such as 100-scale FMDVRP, and the ability to adapt to larger decision spaces may be the main limitation of DPN.

Rebuttal by Authors

Rebuttal

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

29 Mar 2024, 02:51 (modified: 29 Mar 2024, 20:44)

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Revisions (/revisions?id=RUd4TJw71N)

Rebuttal:

(5/5)

References

[1] Hou, Qingchun, et al. Generalize learned heuristics to solve large-scale vehicle routing problems in real-time. The Eleventh International Conference on Learning Representations. 2022.

[2] Kaempfer, Y. , et al. Learning the multiple traveling salesmen problem with permutation invariant pooling networks. arXiv preprint arXiv:1803.09621, 2018.

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[5] Qingchun Hou, et al. Generalize learned heuristics to solve large-scale vehicle routing problems in real-time. International Conference on Learning Representations, 2022.

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[10] Kim, Minjun, et al. Learning to CROSS exchange to solve min-max vehicle routing problems. The Eleventh International Conference on Learning Representations. 2022.

[11] Son, Jiwoo, et al. Solving np-hard min-max routing problems as sequential generation with equity context. arXiv preprint arXiv:2306.02689, 2023.

[12] Bertazzi, L. et al. Min–max vs. min–sum vehicle routing: A worst-case analysis. European Journal of Operational Research, 2015.

[13] Son, Jiwoo, et al. Solving np-hard min-max routing problems as sequential generation with equity context. arXiv preprint arXiv:2306.02689, 2023.

[14] Kim, Minjun, et al. Learning to CROSS exchange to solve min-max vehicle routing problems. The Eleventh International Conference on Learning Representations. 2022.

Official Comment by Authors

Official Comment

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

05 Apr 2024, 02:26

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:

Dear reviewer wVP5,

Thank you very much for your time and effort in reviewing our work. We really appreciate your comments.

There are less than 1 day left to the rebuttal deadline, and we sincerely want to know whether our responses can successfully address all your concerns. Please also let us know if you have further concerns. We are glad to continually improve our work to address them.

Best Regards, Paper3742 Authors

- = ≡

Replying to Rebuttal by Authors

Official Comment by Reviewer wVP5

Official Comment Reviewer wVP5 05 Apr 2024, 10:56

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:

Thank you for your responses. I appreciate the comparisons and reviews regarding the previous learn-to-divide and conquer methods. It's encouraging to see such engagement.

Upon closer examination, it's evident that while there are notable disparities between the proposed method and previous learn-to-divide approaches, particularly in the conquering phase, there remains a semblance of similarity in the learn-to-divide aspect. Undoubtedly, this presents its own set of challenges.

Furthermore, it's worth noting that the proposed method is tailored specifically for min-max VRP, which may pose limitations when attempting to apply it to more mainstream VRP variants such as CVRP and CVRPTW.

Taking all of these factors into account, I have revised my evaluation and now rate the proposal a 4, acknowledging the advancements made while also considering the challenges and constraints inherent in its application.

- = ≡

Replying to Official Comment by Reviewer wVP5

Official Comment by Authors

Official Comment

Authors (Tong Xialiang (/profile?id=~Tong_Xialiang2), Mingxuan Yuan (/profile?id=~Mingxuan_Yuan1), Zhenkun Wang (/profile?id=~Zhenkun_Wang1), Zhi Zheng (/profile?id=~Zhi_Zheng2), +2 more (/group/info?id=ICML.cc/2024/Conference/Submission3742/Authors))

05 Apr 2024, 15:04

Program Chairs, Senior Area Chairs, Area Chairs, Reviewers Submitted, Authors

Comment:

Thank you very much for appreciating our previous response and recognizing that our method has notable disparities from previous learn-to-divide approaches. We would like to provide the following discussion to address your remaining concerns further.

> **Concern 1: Semblance of similarity in the learn-to-divide aspect**

Although DPN and previous learning partitioning methods adopt similar divide-and-conquer (i.e., partition and navigate) ideas, they have the following essential differences. Unlike previous methods that deal with partitioning and navigation separately or in two stages, DPN handles the representation of partitioning and navigation in the P&N encoder and generates solutions in a single-stage construction manner. Moreover, previous approaches adopt different losses for partitioning and navigation while DPN employs a unified loss function (i.e., APS-Loss) to learn both representations. This unified framework enables DPN to collaboratively learn representations of partitioning and navigation, thus avoiding potential conflicts between them in dealing with min-max VRPs.

> **Concern 2: Extend to mainstream VRPs**

We must clarify that min-max VRPs have essential differences from mainstream VRPs, such as CVRP and CVRPTW. Min-max VRP limits the number of vehicles (i.e., agents or routes) and plays an important role in real-world applications [2]. Our DPN is designed explicitly for min-max VRP.

We agree that the ultimate goal is to develop a general method for both mainstream VRPs and min-max VRPs, yet existing heuristics [3] and learning-constructive-based solvers [4] cannot achieve it. Extending the advantages of DPN to other mainstream VRPs and moving towards realizing a universal framework is important future work for us.

We sincerely hope that the above discussion can address your remaining concerns. Please let us know if you have any further concerns.

Reference

[1] Qingchun Hou, et al. Generalize learned heuristics to solve large-scale vehicle routing problems in real-time. International Conference on Learning Representations, 2022.

[2] Cheikhrouhou, O., et al. A comprehensive survey on the Multiple Traveling Salesman Problem: Applications, approaches and taxonomy. *Computer Science Review*, *2021*.

[3] Bertazzi, L. et al. Min–max vs. min–sum vehicle routing: A worst-case analysis. European Journal of Operational Research, 2015.

[4] Son, Jiwoo, et al. Solving np-hard min-max routing problems as sequential generation with equity context. arXiv preprint arXiv:2306.02689, 2023.