

EST-24107: Tarea 2

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Problema 1

Problema 2

Problema 3

Problema 4

Problema 5

Tenemos $V_i \sim \mathcal{U}(-1, 1)$, $\forall i \in \{1, 2\}$, tal que *no* generamos nuevas uniformes si caemos dentro del círculo unitario — nuestra zona de aceptación — cuya área es $(0.5)^2\pi = \pi/4 \approx 0.7853982$.

Basta ver que $X \sim \text{Geo}(\pi/4)$ para conocer el número de rechazos antes de aceptar. Por lo tanto, $\mathbb{E}[X] = 1/(\pi/4) = 4/\pi \approx 1.2732395$.

Problema 6

Problema 7

Problema 8

```
set.seed(1234)

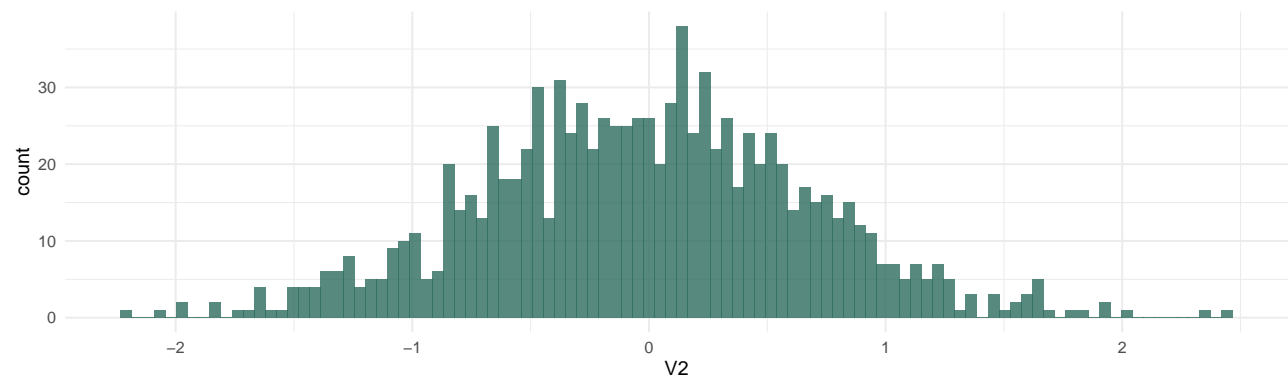
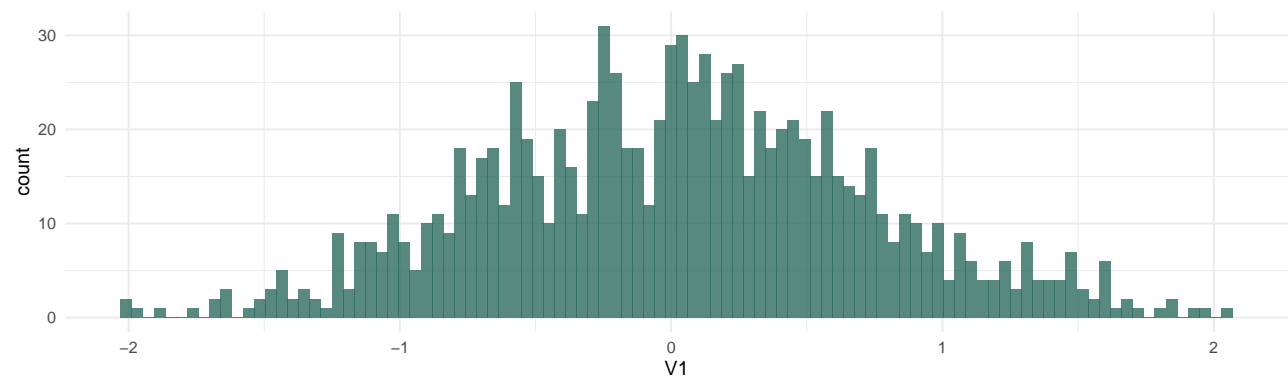
mezcla <- function(n, p, mu1, mu2, s1, s2) {
  p * rmvnorm(n, mean = mu1, sigma = s1) + (1 - p) * rmvnorm(n, mean = mu2, sigma = s2)
}

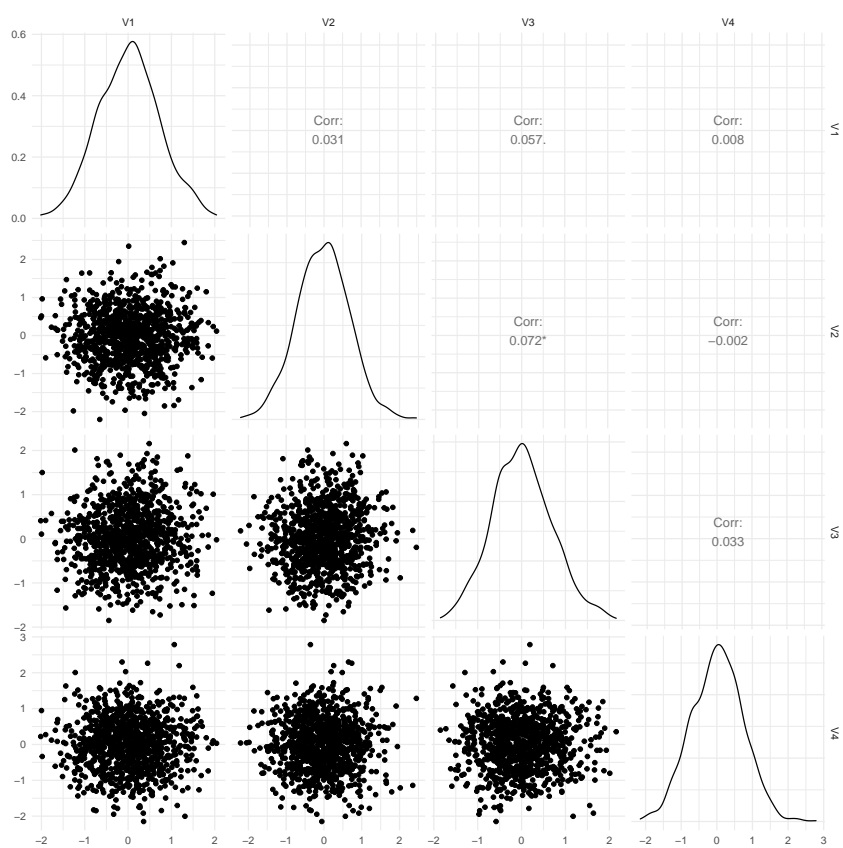
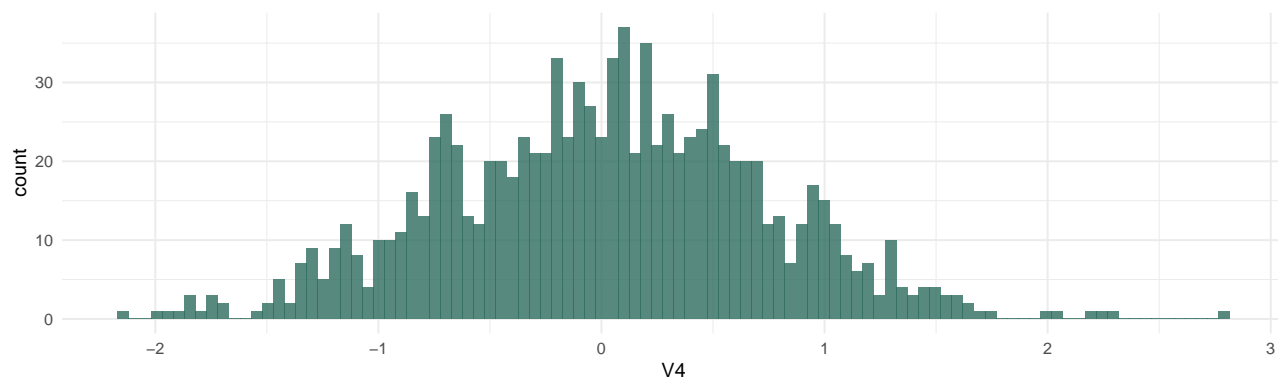
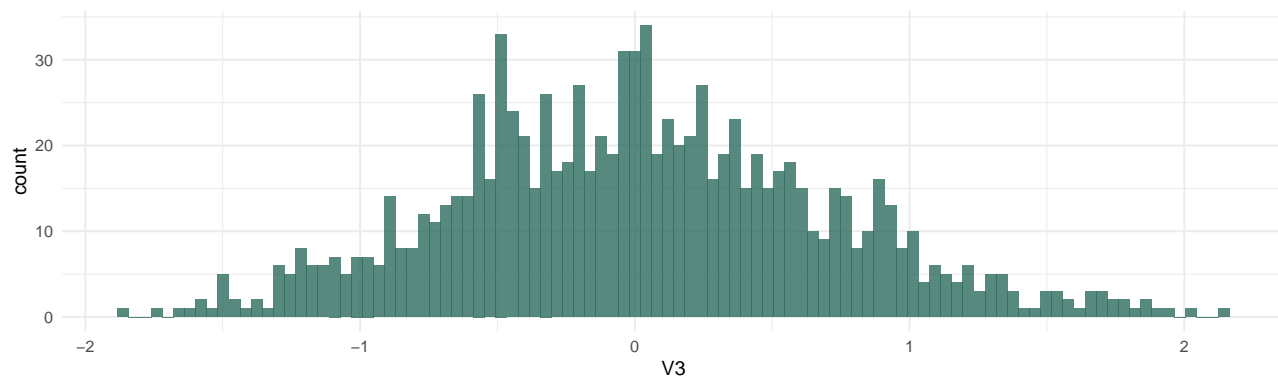
n <- 1000 # observaciones
p <- 0.5 # mezcla al 50%

mu1 <- c(0, 0, 0, 0)
mu2 <- c(2, 3, 4, 5)

s1 <- s2 <- diag(4)

z <- as.data.frame(mezcla(n, p, mu1, mu2, s1, s2))
```





Problema 9

Primer método

```
wishart1 <- function(k, n, mean, sigma) {
  require(mvtnorm)
  W <- list(NULL)
  for (i in 1:k) {
    X <- rmvnorm(n, mean = mean, sigma = sigma)
    W[[i]] <- t(X) %*% X
  }
  return(W)
}
```

Método de Bartlett

```
wishart2 <- function(k, n, mean, sigma) {
  W <- list(NULL)
  d <- length(mean)
  A <- matrix(0, nrow = d, ncol = d)
  for (i in 1:k) {
    A[lower.tri(matrix(0, nrow = d, ncol = d))] <-
      rnorm(d * (d + 1) / 2 - d)
    diag(A) <- sqrt(rchisq(d, n - (1:d) + 1))
    L <- chol(sigma)
    W[[i]] <- L %*% A %*% t(A) %*% t(L)
  }
  return(W)
}
```

Comparación

```
k <- 10000; n <- 10
mean <- c(0, 0, 0, 0, 0)
sigma <- diag(5)

tic('First method')
test1 <- wishart1(k, n, mean, sigma)
toc()

## First method: 4.15 sec elapsed

tic('Bartlett decomposition')
test2 <- wishart2(k, n, mean, sigma)
toc()

## Bartlett decomposition: 0.612 sec elapsed
```

Problema 10