Study on the Optimal Order for the Auto-Regressive Time-Frequency Analysis of Heart Rate Variability

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Abstract—**Time-frequency analysis of heart rate variability (HRV) makes it easier to evaluate how the balance between the sympathetic and parasympathetic influences on heart rhythm varies with time. The auto-regressive model can be used to calculate the Power Spectrum Density of HRV and to create an auto-regressive spectrogram. This work presents these techniques and describes a series of tests performed with the goal of determining the AR order that is more adequate for the calculation of the AR spectrogram. As a result, ranges of optimal orders for different interpolation rates of the HRV signal are presented.**

Keywords—**AR model order, auto-regressive spectrogram, heart period, HRV, RR interval, time-frequency analysis**

I. Introduction

This work presents the auto-regressive (AR) spectrogram, which is an alternate technique to the Fourier spectrogram [1][2] for time-frequency analysis. In order to minimize possible distortions caused by the approximations inherent to the auto-regressive model and avoid unnecessary computational load, some tests were performed in order to determine what AR model order is more appropriate for the calculation of the AR spectrogram.

The analysis of heart rate variability (HRV) is important when studying the autonomic nervous system because it helps in evaluating the equilibrium between the sympathetic and parasympathetic influences on the heart rhythm. Classical techniques for analysis of HRV are unable to provide information on how the sympathetic-parasympathetic balance change with time. The time-frequency analysis of HRV makes it easy to observe how the sympathetic-parasympathetic activations alternate with time. The spectrogram analysis provides many graphs and indexes that help in evaluating how the sympathetic and parasympathetic influences vary with time.

The AR spectrogram, calculated with an appropriated order, yields time-frequency indexes [1][2] that are very close to the ones obtained from the Fourier spectrogram. However, the plot obtained with AR modeling is clearer than the one obtained with the Fourier spectrogram.

Moreover, the frequency resolution in the AR spectrogram is affected mostly by the order of the model, instead of the window length [3]. This is another advantage of this technique, since one can use a shorter window and thus increase time resolution without losing much frequency resolution. This cannot be accomplished with the Fourier spectrogram.

Since the order of the AR model affects the numerical calculation of the time-frequency indexes, as well as the frequency resolution of the spectrogram, a study on the optimal order to be used for the auto-regressive modeling seemed to be needed. Thus, some tests were performed, based on criteria for the determination of the optimal order of AR modeling and based on the order influence in the time-frequency indexes.

These tests were performed for different interpolation rates of the HRV signal. Thus, ranges of optimal orders are proposed for each interpolation rate.

II. Spectral Analysis

The spectral analysis of the HRV signal allows one to separate the energies related to the sympathetic and parasympathetic activities of the nervous system in different frequency bands. The most popular techniques for spectral analysis of HRV are the Discrete Fourier Transform (DFT) and auto-regressive (AR) modeling. The power spectral density is obtained as follows:

- The series of RR intervals is interpolated by cubic splines and the interpolated signal is re-sampled at a higher, uniform rate (usually 2 or 4 Hz);
- The reconstructed signal is multiplied by a 5-minute length window (Hamming and Hanning are the most popular ones);
- The DC component of the windowed signal is removed;
- The Fourier power spectrum is calculated by the squared absolute value of the DFT, which is multiplied by the sampling period and divided by the number of samples in the window.
- The AR power spectrum is calculated by the squared absolute value of the transfer function, which is multiplied by the sampling period and by the variance of the prediction error of the model.

Fig. 1 shows a comparison between the power spectrum obtained through DFT and its approximation by AR modeling. Many cardiologists prefer to use the AR model, since it is easier to see the concentrations of the sympathetic and parasympathetic components with this representation. The AR approximation gets closer to the Fourier spectrum as the model order increases.

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The frequency spectrum is divided in 3 bands: VLF (0 to 0.04 Hz), LF (0.04 to 0.15 Hz) and HF (0.15 to 0.5 Hz). Some authors use slightly different divisions, but the important fact is that the energy contained in the LF band is related to the sympathetic component of the signal, and the HF is related to the parasympathetic component. The power in these bands is calculated based on the area under the curve, and the ratio between them indicates the sympathetic-parasympathetic balance in the segment of signal.

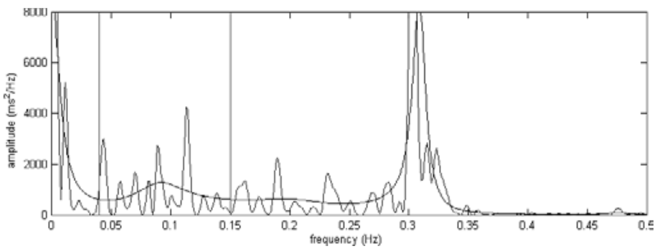

Fig. 1. HRV Power Spectrum Density. The auto-regressive approximation is smoother than the Fourier PSD.

The absolute power contained in the LF and HF bands and the LF/HF ratio are good indicators for detecting alterations in the nervous system behavior. These alterations have been associated with pathological conditions, such as high blood pressure, HIV, Chagas' disease and ischemic attacks.

In some experiments, the researcher wants to observe the reaction of the nervous system of a subject to specific physical and mental stimuli. In this case, it is usual to obtain the power spectra of two segments of the signal, one before the stimulus, and the other after the stimulus. The two spectra are analyzed separately and the comparison between the obtained indexes for each segment can be used to evaluate the response of the sympathetic and parasympathetic branches of the nervous system to the stimulus. But this technique does not reveal how the components respond as a function of time. This can be accomplished with time-frequency analysis.

III. Time-Frequency Analysis

The spectrogram is a traditional technique for time-frequency analysis. In this technique, the 5-minute-long window is replaced by a short-time window (e.g., 30 seconds). This window is shifted sample by sample in time, and for each shift, a new power spectrum is calculated.

When the spectrogram is applied to the HRV signal, it allows the observation of how the sympathetic and parasympathetic branches of the nervous system behave as a function of time. The spectrogram can be calculated using the Fourier transform or the AR model.

The major shortcoming of the Fourier spectrogram is the tradeoff between spectral and time resolution. For short windows, the time resolution is good, but the spectral resolution is poor. On the other hand, with long windows, the spectral resolution is good, but the time resolution is poor.

The frequency resolution in the AR spectrogram is affected mostly by the model order [3]. This is an advantage of this technique, since one can use a shorter window and thus increase time resolution without losing much frequency resolution. This cannot be accomplished with the Fourier spectrogram. But the main advantage of the AR spectrogram is that it is more straightforward for visual inspection than the Fourier spectrogram (Fig. 2).

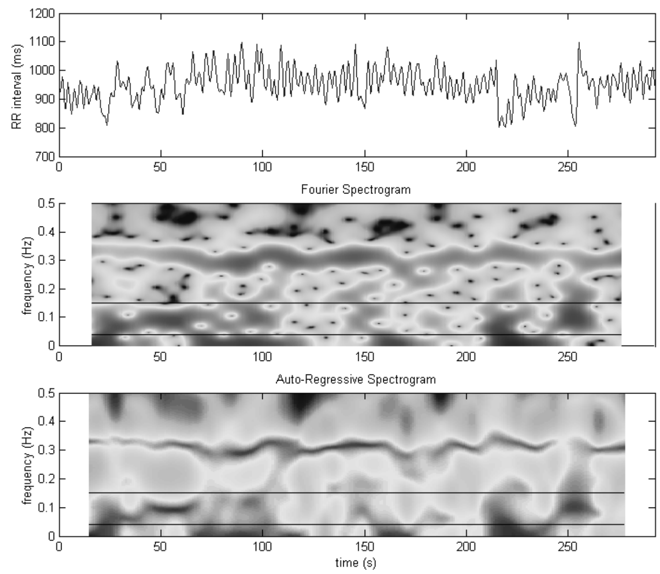

Fig. 2. Fourier and auto-regressive spectrograms of an HRV signal from a subject with no autonomic dysfunction. The AR spectrogram is clearer than the Fourier spectrogram.

With the spectrogram, one can obtain plots that show how the instantaneous absolute power in the VLF, LF and HF bands vary with time by calculating the power in each band for each power spectrum, and plotting the obtained indexes as a function of time. In a similar way, it is possible to obtain a plot that shows the variation of the LF/HF ratio with time.

However, the choice of the order for the AR model can affect these indexes. Therefore, it is important to determine which order must be used for the calculation of the spectrogram.

IV. Criteria for AR Order Selection

Among the criteria for selection of the order of the AR model found in literature, five were evaluated in this work: FPE, AIC, MDL, CAT e BIC. These criteria weight the variance of the prediction error of the model in order to determine the order that minimizes the error without leading to unnecessary computational load. The number of samples of the input signal is also taken into account.

The final prediction error criterion (FPE) [4], selects the order of the AR model in a way that the variance of the prediction error is minimized without leading to an unnecessarily high order. In (1), ρ_p is the variance of the prediction error for the order p, and N is the number of samples.

$$FPE[p] = \rho_p \left(\frac{N + (p+1)}{N - (p+1)} \right) \tag{1}$$

Using Akaike information criterion (AIC) [5], the optimal order for the autoregressive model is the one that minimizes (2). If the number of samples is big, these two criteria yield similar results.

$$AIC[p] = N \ln(\rho_p) + 2p \tag{2}$$

The minimum description length (MDL) [6] defined in (3) was developed in order to correct the FPE and AIC, which overestimate the optimal order when signals with a large sample set are used [3].

$$MDL[p] = N \ln(\rho_p) + p \ln(N) \tag{3}$$

The criterion autoregressive transfer function (CAT) [7] defined in (4) provides results similar to those attained with FPE and AIC.

$$CAT[p] = \left(\frac{1}{N} \sum_{j=1}^{p} \frac{1 - j/N}{\rho_j} \right) - \frac{1 - p/N}{\rho_p} \tag{4}$$

The bayesian information criterion (BIC) [8][9][10] is described in (5), where ρ_x is the variance of the output signal (in this particular case, the HRV signal).

$$BIC[p] = N \ln(\rho_p) - (N-p) \ln\left(1 - \frac{p}{N}\right) + p \ln(N) + p \ln\left[\frac{1}{p}\left(\frac{\rho_x}{\rho_p} - 1\right)\right] \tag{5}$$

However, for some signals these criteria might not work properly. Hence, the validity of the results attained from the criteria must be subjectively analyzed [3]. In this work, this subjective test consists of an analysis of how the order choice affects the spectral indexes.

V. Results

Preliminary tests using the criteria discussed previously show that FPE, AIC and CAT tend to overestimate the AR model order for HRV signals. This happens because the curves generated from the equation related to those criteria do not have a sharp valley at the optimal order, as noticed with MDL and BIC. In fact, the FPE/AIC/CAT curve diminishes rapidly as it approaches the optimal order, and then it diminishes slowly as the order increase (Fig. 3). Thus, the order that minimizes the curve is not precisely defined and definitely does not represent the optimal solution.

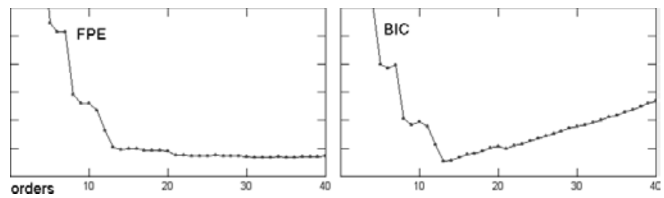

Fig. 3. FPE and BIC curves. Using BIC or MDL criteria there is a well-defined valley at the optimal order. With FPE, AIC or CAT, the minimum of the function is not well defined. In this case, there is a plateau around the optimal order.

Since FPE, AIC and CAT have poor performance for HRV signals, they will not be used further. The remaining tests will be performed only for MDL e BIC criteria.

HRV signals from thirty two healthy subjects [11] [12], thirteen subjects with organic Chagas' disease (both digestive and cardiac forms) [12], and thirteen subjects with high blood pressure [13] were used for the subsequent tests. The signals were recorded for approximately 5 minutes. The aim is to analyze signals with different spectral characteristics so that the results can be applied to different types of diseases.

Tables 1 and 2 show the mean and standard deviation for the optimal order estimated according BIC and MDL criteria for the sampling rates of 4 Hz and 2 Hz, respectively.

TABLE I
OPTIMAL ORDER: MEAN AND STANDARD DEVIATION (4 HZ)

HRV signal length	BIC			MDL		
	healthy subjects	Chagas' disease	high blood pressure	healthy subjects	Chagas' Disease	high blood pressure
300 s	16.6±4.5	17.4±3.0	17.0±4.2	19.8±5.9	23.5±8.5	19.5±6.3
100 s	12.5±3.4	15.6±3.0	13.7±2.1	15.7±4.4	18.1±4.1	17.0±2.9
60 s	10.6±2.8	13.7±2.9	11.8±2.3	14.2±3.9	16.7±3.3	15.6±3.4
30 s	8.1±2.5	10.3±2.4	8.8±2.3	12.3±3.4	15.3±3.3	13.2±3.5
15 s	4.9±1.3	6.0±2.1	5.9±1.9	9.9±3.1	12.6±3.5	10.8±3.4

TABLE II
OPTIMAL ORDER: MEAN AND STANDARD DEVIATION (2 HZ)

HRV signal length	BIC			MDL		
	healthy subjects	Chagas' disease	high blood pressure	healthy subjects	Chagas' disease	high blood pressure
300 s	11.2±2.7	14.1±3.3	13.5±3.4	12.6±3.1	16.0±4.3	14.7±5.1
100 s	9.4±2.0	11.4±2.7	10.5±2.8	10.3±2.4	12.7±3.0	11.3±3.2
60 s	8.6±2.0	9.9±2.5	8.5±2.5	9.7±2.3	11.4±2.7	10.7±3.7
30 s	7.3±1.7	7.6±2.2	7.1±1.9	8.6±2.1	9.2±2.5	8.3±2.8
15 s	5.3±1.5	5.3±1.7	5.3±2	8.0±3.3	9.6±4.5	7.8±4.2

The results show that the BIC criterion suggests lower order than MDL, and it has a smaller standard deviation. Nevertheless, both criteria tend to underestimate the optimal order as the window length decreases. In order to test the validity of the results, we decided to observe the effect of the choice of the order on the spectral indexes to be obtained.

VI. The Influence of the AR Order in the Spectral Indexes

The AR order selection criteria provide estimates of the optimal orders based on the prediction error and on the number of samples, but it does not take specific frequency bands into consideration. However, for the time-frequency analysis of HRV, the AR approximation must provide reliable power spectrum estimates in the frequency bands that are important for diagnostic purposes. Therefore, the evaluation of the effect of the choice of the AR model order in the computed spectral indexes is imperative.

The indexes used in the spectral analysis of HRV are calculated using the power in the VLF, LF and HF bands. Hence, when one evaluates how the order choice affects the power in these bands, he is also evaluating the distortion of the spectral indexes.

The experiment is designed as follows. The sampling rate and the window length are fixed, and the AR model order is varied. Since the auto-regressive power spectrum is used as an approximation to the Fourier power spectrum, the Fourier results will be used as a reference for the estimation of the mean error for each AR order.

The performance analysis shows that the relative mean error decreases when the order increases. Above a certain order, a plateau is reached and the error does not change much. There is no advantage in increasing the AR model order after the plateau is reached. However, for very high orders the error increases.

The tests also show that the point where the error reaches a plateau does not depend on the window length [1]. In fact, this is the expected result, as these short-length HRV signals are stationary.

Based on the results, we concluded that one should use AR orders between 12 and 15 for a 2 Hz sampling rate and orders between 15 and 20 for a 4 Hz sampling rate. Tests were performed for the 1 Hz sampling rate as well, and the results pointed to orders between 9 and 13. However, sampling at 1 Hz is not recommended, since the HRV signal may have components above 0.5 Hz in some cases.

VII. Conclusion

This work presented a study on what is the proper AR model order to be used for the auto-regressive time-frequency analysis of HRV.

Fourier and AR spectrograms are equivalent in terms of time resolution. But the AR spectrogram is simpler to visualize than the Fourier spectrogram and it is very objective. Using an appropriate order, the time-frequency indexes attained from the AR spectrogram are very close to those obtained with the DFT.

Optimization criteria were used in order to determine what is the proper order for the calculation of the AR spectrogram,. The FPE, AIC and CAT did not seem proper for this problem, since they overestimate the optimal AR model order. With MDL and BIC, we observed that the optimal order decreases with the window length.

However, based on the influence of the AR model order on the time-frequency indexes, the tests have shown that the range where the mean error gets stable depends very little on the window length.

Based on these results, we determined that AR model orders between 12 and 14 should be used with a 2 Hz sampling rate and orders between 15 and 20 should be used with a 4 Hz sampling rate. These orders guarantee that the AR spectrogram to be a good alternative to the Fourier spectrogram.

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