Parallel Algorithms

Patrick Cozzi
University of Pennsylvania
CIS 565 - Fall 2012

Announcements

- Project 1 due Sunday 09/30
  - Email grade to Karl
  - Include blog link in README.md
  - Be ready to present on Wednesday, 10/01

Agenda

- Parallel Algorithms
  - Parallel Reduction
  - Scan
  - Stream Compression
  - Summed Area Tables
  - Radix Sort

Parallel Reduction

- Given an array of numbers, design a parallel algorithm to find the sum.
- Consider:
  - *Arithmetic intensity*: compute to memory access ratio

Parallel Reduction

- Given an array of numbers, design a parallel algorithm to find:
  - The sum
  - The maximum value
  - The product of values
  - The average value
- How different are these algorithms?

5

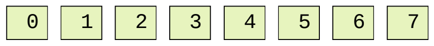
Parallel Reduction

- *Reduction*: An operation that computes a single result from a set of data
- Examples:
  - Minimum/maximum value
  - Average, sum, product, etc.
- *Parallel Reduction*: Do it in parallel. Obviously

6

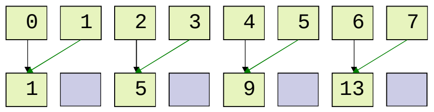
Parallel Reduction

- Example. Find the sum:



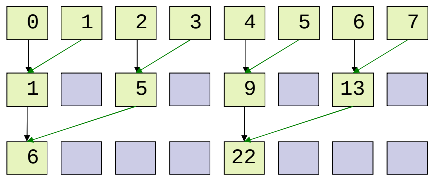
7

Parallel Reduction

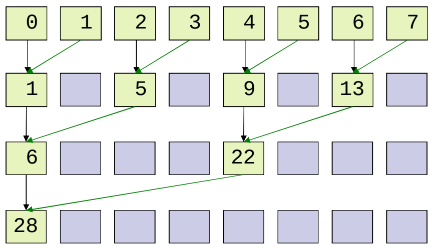


8

Parallel Reduction

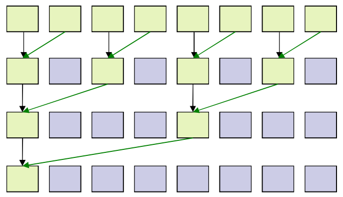


Parallel Reduction



Parallel Reduction

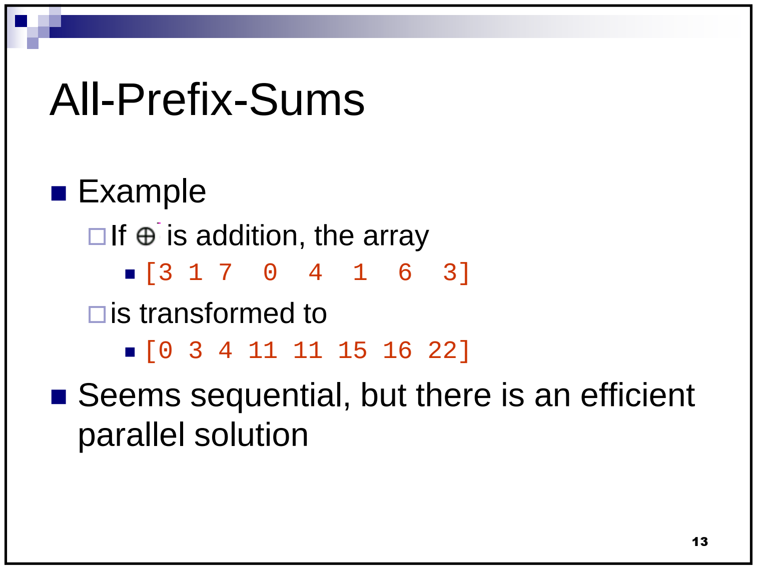
- Similar to brackets for a basketball tournament
- log(n) passes for n elements



All-Prefix-Sums

- *All-Prefix-Sums*
  - Input
    - Array of *n* elements: $[a_0, a_1, \ldots, a_{n-1}]$
    - Binary associate operator: $\oplus$
    - Identity: *I*
  - Outputs the array: $[I, a_0, (a_0 \oplus a_1), \ldots, (a_0 \oplus a_1 \oplus \ldots \oplus a_{n-2})]$

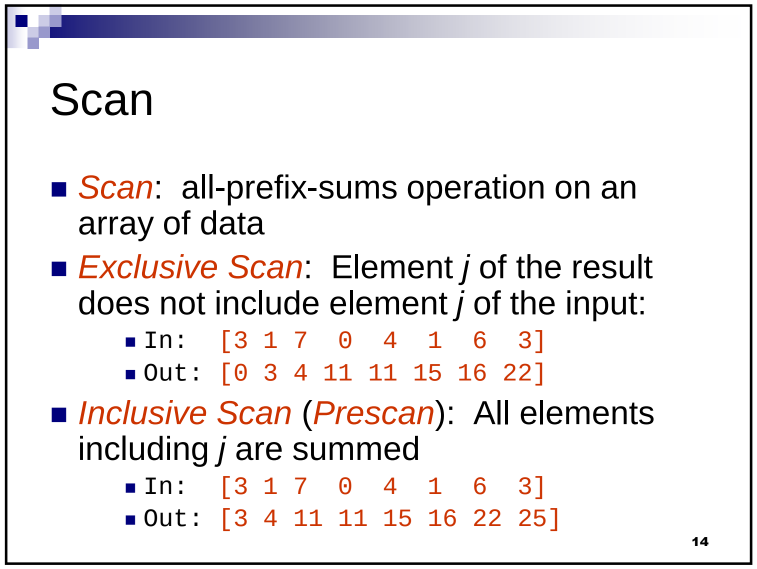
Images from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html



All-Prefix-Sums

- Example
  - If ⊕ is addition, the array
    - `[3 1 7  0  4  1  6  3]`
  - is transformed to
    - `[0 3 4 11 11 15 16 22]`
- Seems sequential, but there is an efficient parallel solution

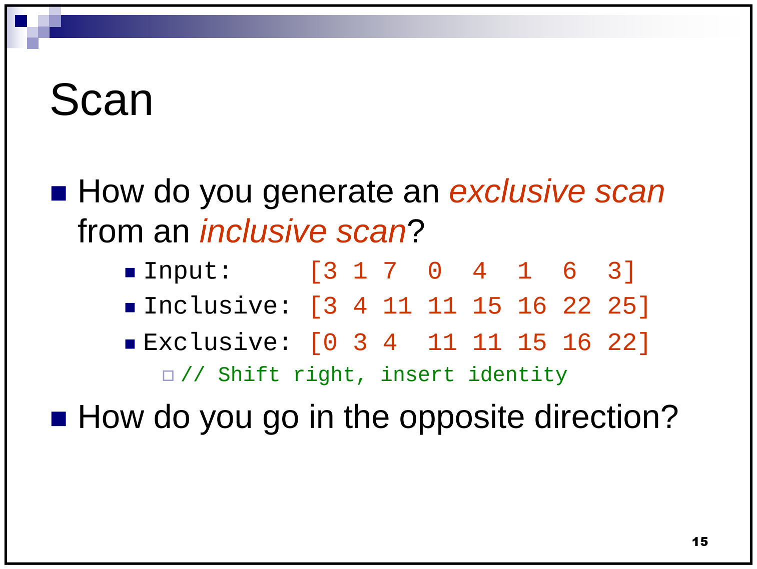
13



Scan

- *Scan*: all-prefix-sums operation on an array of data
- *Exclusive Scan*: Element *j* of the result does not include element *j* of the input:
  - `In:  [3 1 7  0  4  1  6  3]`
  - `Out: [0 3 4 11 11 15 16 22]`
- *Inclusive Scan* (*Prescan*): All elements including *j* are summed
  - `In:  [3 1 7  0  4  1  6  3]`
  - `Out: [3 4 11 11 15 16 22 25]`

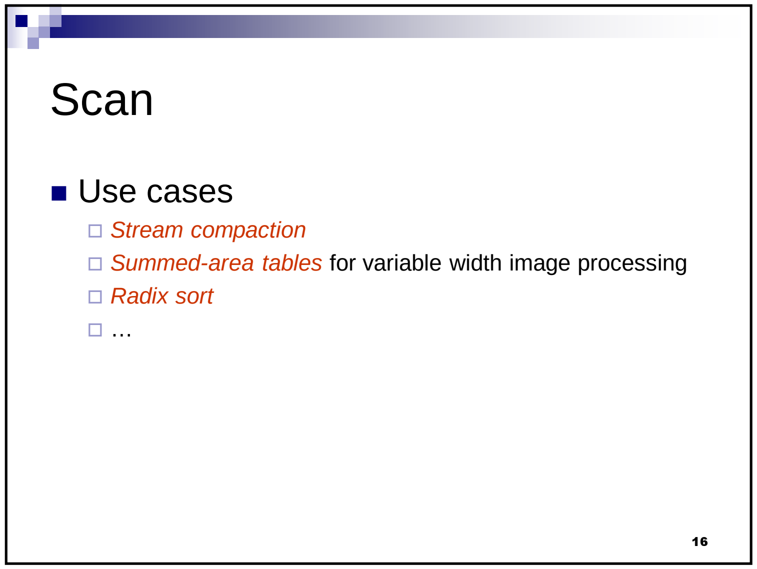
14



Scan

- How do you generate an *exclusive scan* from an *inclusive scan*?
  - `Input:     [3 1 7  0  4  1  6  3]`
  - `Inclusive: [3 4 11 11 15 16 22 25]`
  - `Exclusive: [0 3 4  11 11 15 16 22]`
    - `// Shift right, insert identity`
- How do you go in the opposite direction?

15



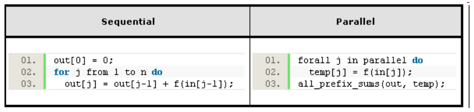
Scan

- Use cases
  - *Stream compaction*
  - *Summed-area tables* for variable width image processing
  - *Radix sort*
  - …

16

Scan

- Used to convert certain sequential computation into equivalent parallel computation



Sequential	Parallel

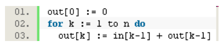
`out[0] = 0;`
`for j from 1 to n do`
`  out[j] = out[j-1] + f(in[j-1]);`
	`forall j in parallel do`
	`  temp[j] = f(in[j]);`
	`all_prefix_sums(out, temp);`

Scan

- Design a parallel algorithm for exclusive scan
  - In: [3 1 7 0 4 1 6 3]
  - Out: [0 3 4 11 11 15 16 22]
- Consider:
  - Total number of additions

Scan

- *Sequential Scan*: single thread, trivial



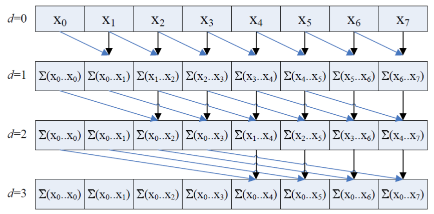
```
out[0] := 0
for k := 1 to n do
  out[k] := in[k-1] + out[k-1]
```

- *n* adds for an array of length *n*
- *Work complexity*: O(n)
- How many adds will our parallel version have?

Scan

- *Naive Parallel Scan*

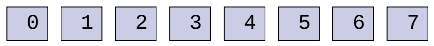
```
for d = 1 to log₂n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```


d=0
x0 x1 x2 x3 x4 x5 x6 x7
d=1
Σ(x0..x0) Σ(x0..x1) Σ(x1..x2) Σ(x2..x3) Σ(x3..x4) Σ(x4..x5) Σ(x5..x6) Σ(x6..x7)
d=2
Σ(x0..x0) Σ(x0..x1) Σ(x0..x2) Σ(x0..x3) Σ(x1..x4) Σ(x2..x5) Σ(x3..x6) Σ(x4..x7)
d=3
Σ(x0..x0) Σ(x0..x1) Σ(x0..x2) Σ(x0..x3) Σ(x0..x4) Σ(x0..x5) Σ(x0..x6) Σ(x0..x7)

- Is this exclusive or inclusive?
- Each thread
  - Writes one sum
  - Reads two values

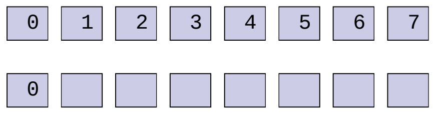
Scan

- *Naive Parallel Scan*: Input



Scan

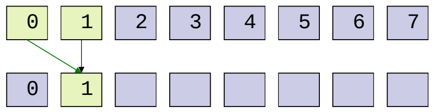
- *Naive Parallel Scan*: d = 1, 2^{d-1} = 1



```
for d = 1 to log₂n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

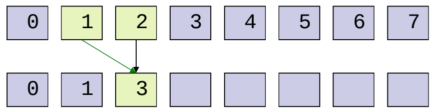
- *Naive Parallel Scan*: d = 1, 2^{d-1} = 1



```
for d = 1 to log₂n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

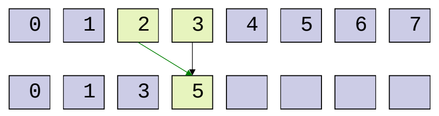
- *Naive Parallel Scan*: d = 1, 2^{d-1} = 1



```
for d = 1 to log₂n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

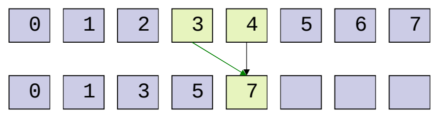
- *Naive Parallel Scan*: $d = 1,\ 2^{d-1} = 1$



```
for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= $2^{d-1}$)
      x[k] = x[k – $2^{d-1}$] + x[k];
```

Scan

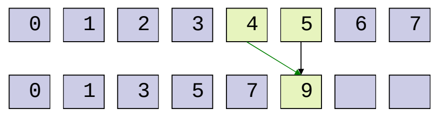
- *Naive Parallel Scan*: $d = 1,\ 2^{d-1} = 1$



```
for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= $2^{d-1}$)
      x[k] = x[k – $2^{d-1}$] + x[k];
```

Scan

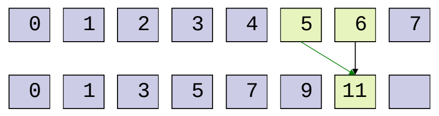
- *Naive Parallel Scan*: $d = 1,\ 2^{d-1} = 1$



```
for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= $2^{d-1}$)
      x[k] = x[k – $2^{d-1}$] + x[k];
```

Scan

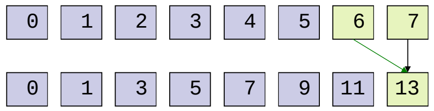
- *Naive Parallel Scan*: $d = 1,\ 2^{d-1} = 1$



```
for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= $2^{d-1}$)
      x[k] = x[k – $2^{d-1}$] + x[k];
```

Scan

- *Naive Parallel Scan*: $d = 1, 2^{d-1} = 1$

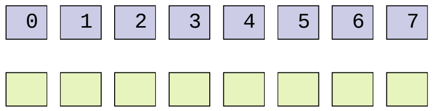


```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

29

Scan

- *Naive Parallel Scan*: $d = 1, 2^{d-1} = 1$



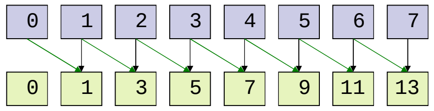
- Recall, it runs in parallel!

```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

30

Scan

- *Naive Parallel Scan*: $d = 1, 2^{d-1} = 1$



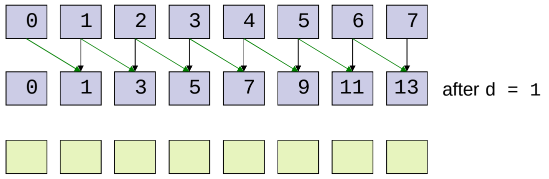
- Recall, it runs in parallel!

```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

31

Scan

- *Naive Parallel Scan*: $d = 2, 2^{d-1} = 2$

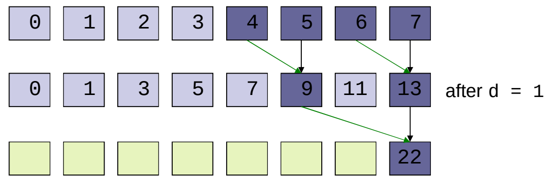


```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

32

Scan

- *Naive Parallel Scan*: d = 2, 2^{d-1} = 2

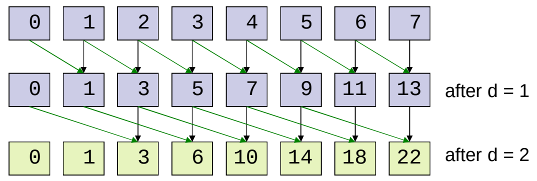


- Consider only k = 7

```
for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= $2^{d-1}$)
      x[k] = x[k – $2^{d-1}$] + x[k];
```

Scan

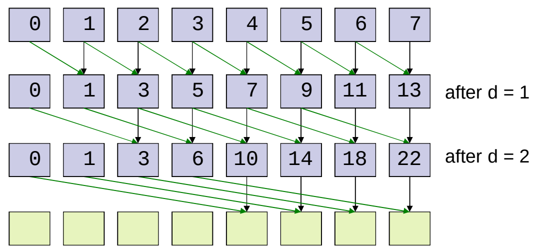
- *Naive Parallel Scan*: d = 2, 2^{d-1} = 2



```
for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= $2^{d-1}$)
      x[k] = x[k – $2^{d-1}$] + x[k];
```

Scan

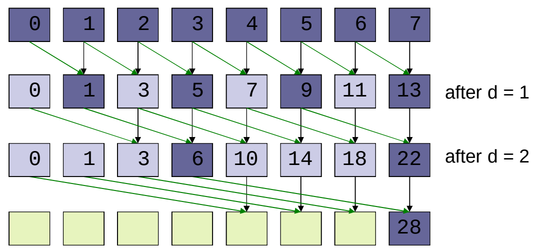
- *Naive Parallel Scan*: d = 3, 2^{d-1} = 4



```
for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= $2^{d-1}$)
      x[k] = x[k – $2^{d-1}$] + x[k];
```

Scan

- *Naive Parallel Scan*: d = 3, 2^{d-1} = 4

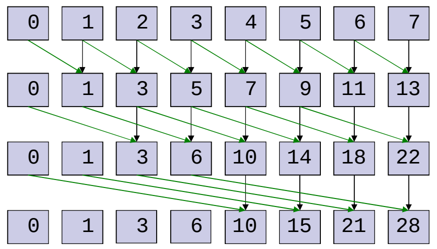


- Consider only k = 7

```
for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= $2^{d-1}$)
      x[k] = x[k – $2^{d-1}$] + x[k];
```

Scan

- *Naive Parallel Scan*: Final

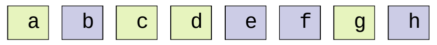


Scan

- *Naive Parallel Scan*
  - What is naive about this algorithm?
    - What was the work complexity for sequential scan?
    - What is the work complexity for this?

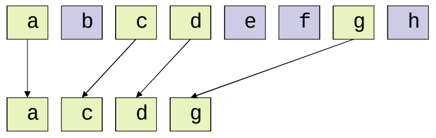
Stream Compaction

- *Stream Compaction*
  - Given an array of elements
    - Create a new array with elements that meet a certain criteria, e.g. non null
    - Preserve order



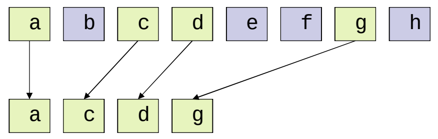
Stream Compaction

- Stream Compaction
  - Given an array of elements
    - Create a new array with elements that meet a certain criteria, e.g. non null
    - Preserve order



Stream Compaction

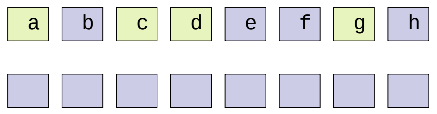
- Stream Compaction
  - Used in path tracing, collision detection, sparse matrix compression, etc.
  - Can reduce bandwidth from GPU to CPU



41

Stream Compaction

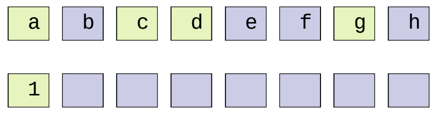
- Stream Compaction
  - *Step 1*: Compute temporary array containing
    - 1 if corresponding element meets criteria
    - 0 if element does not meet criteria



42

Stream Compaction

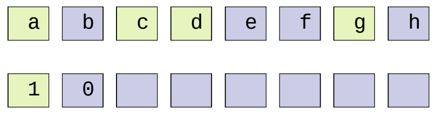
- Stream Compaction
  - *Step 1*: Compute temporary array



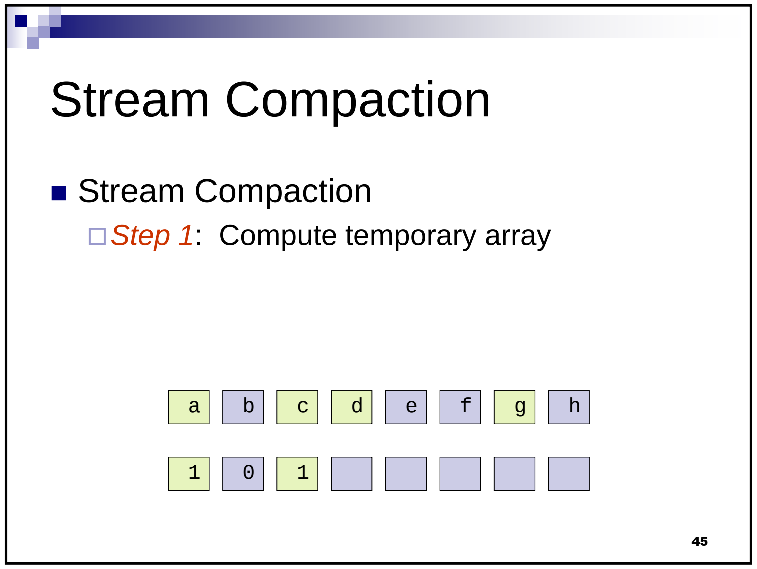
43

Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array



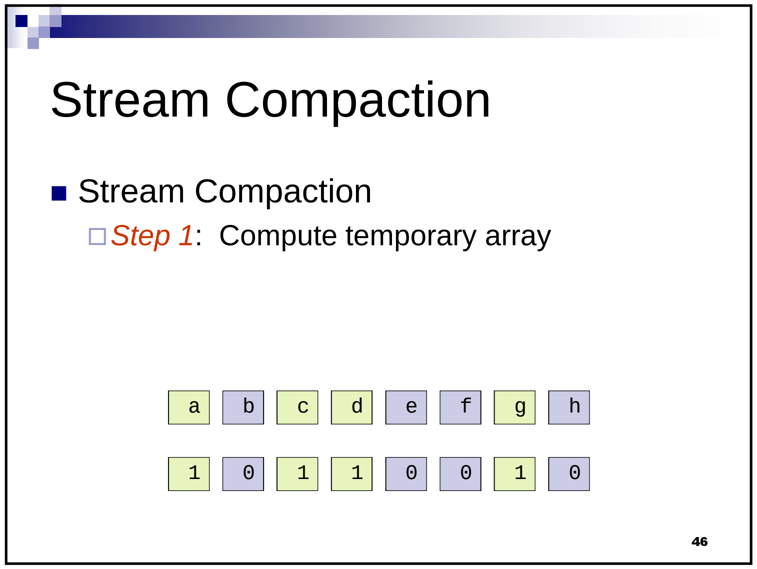
44



Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

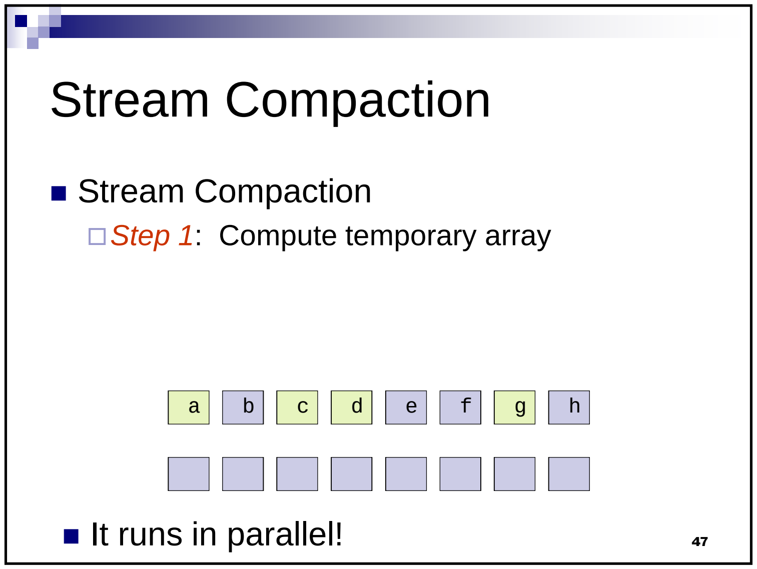
a	b	c	d	e	f	g	h
1	0	1					



Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

a	b	c	d	e	f	g	h
1	0	1	1	0	0	1	0

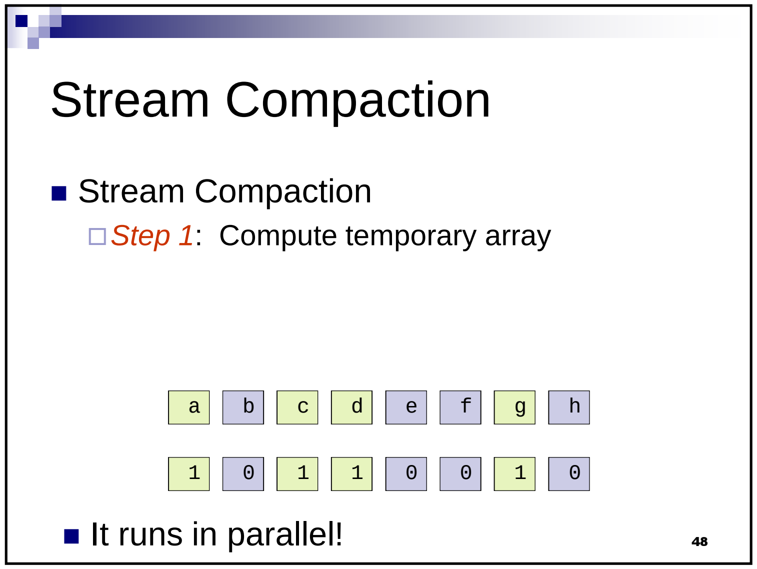


Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

a	b	c	d	e	f	g	h

- It runs in parallel!

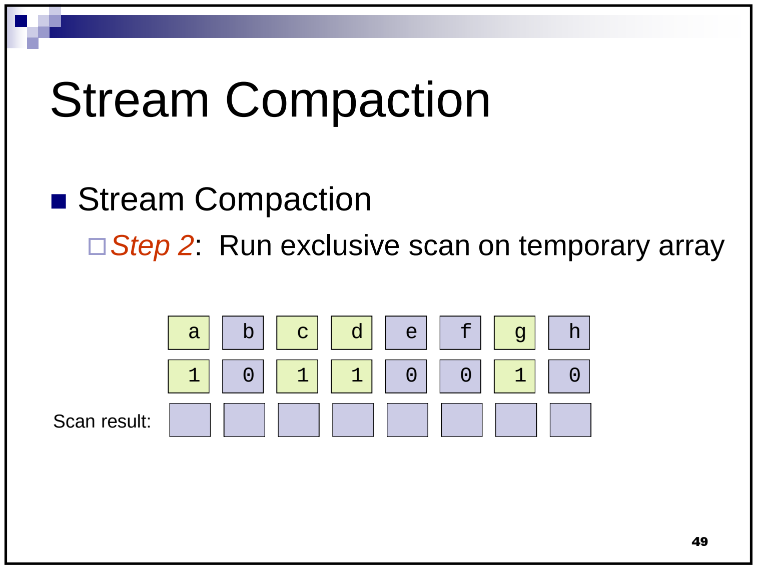


Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

a	b	c	d	e	f	g	h
1	0	1	1	0	0	1	0

- It runs in parallel!

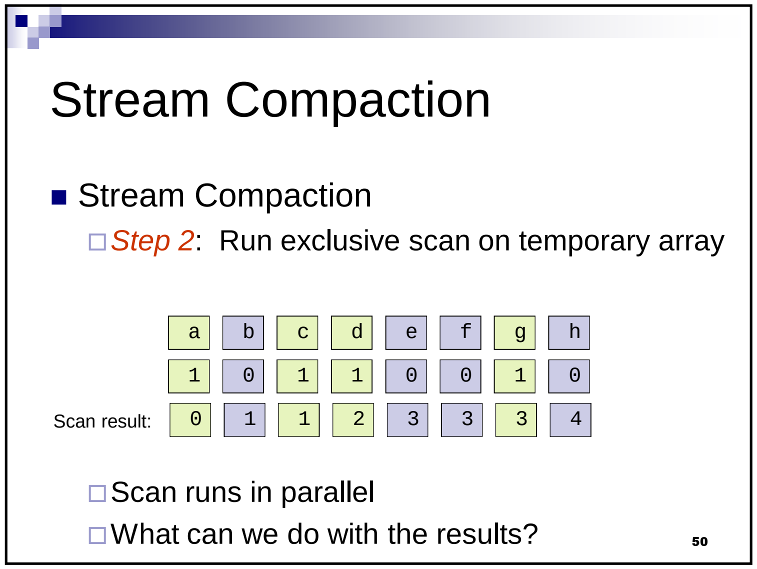


Stream Compaction

- Stream Compaction
  - *Step 2*: Run exclusive scan on temporary array

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:								

49



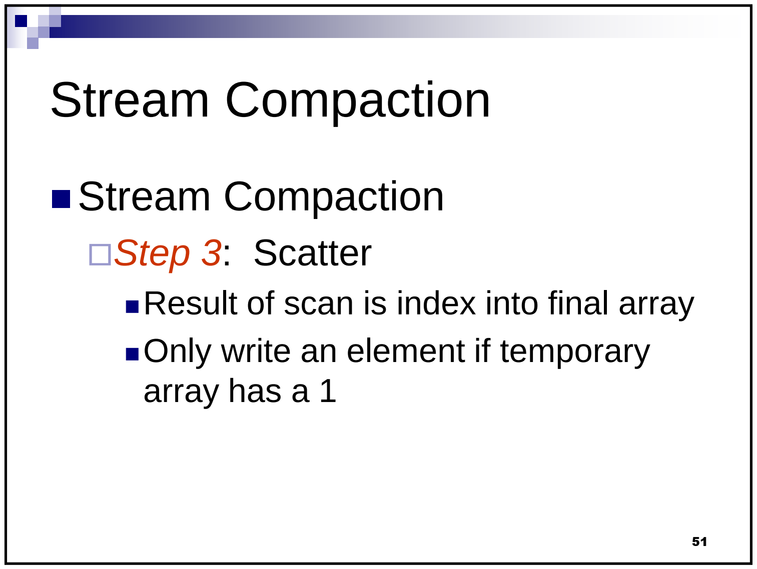
Stream Compaction

- Stream Compaction
  - *Step 2*: Run exclusive scan on temporary array

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

  - Scan runs in parallel
  - What can we do with the results?

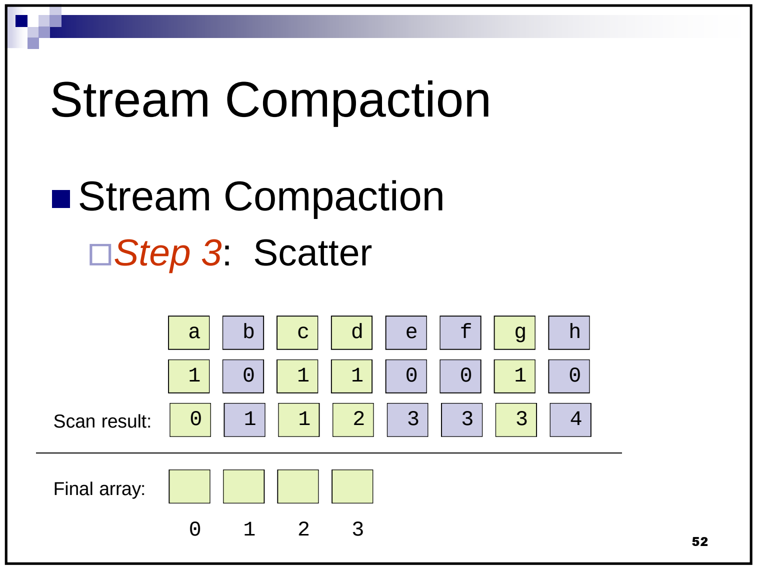
50



Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter
    - Result of scan is index into final array
    - Only write an element if temporary array has a 1

51



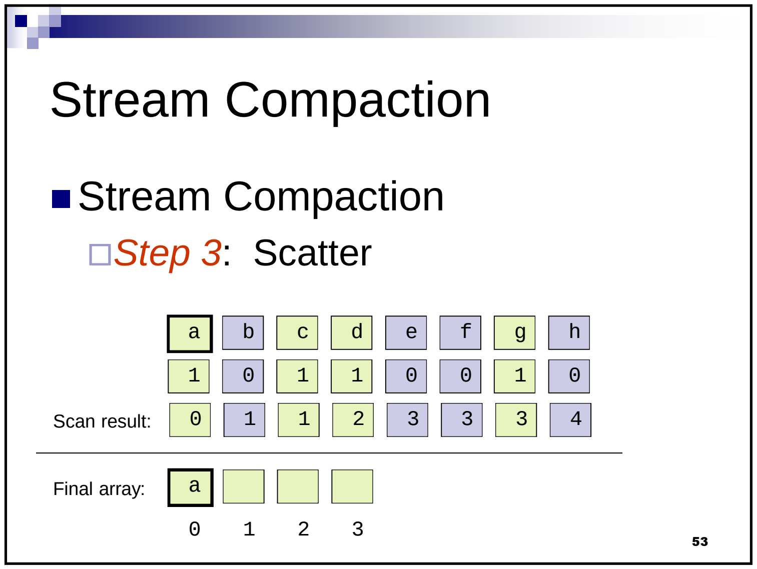
Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:				
	0	1	2	3

52

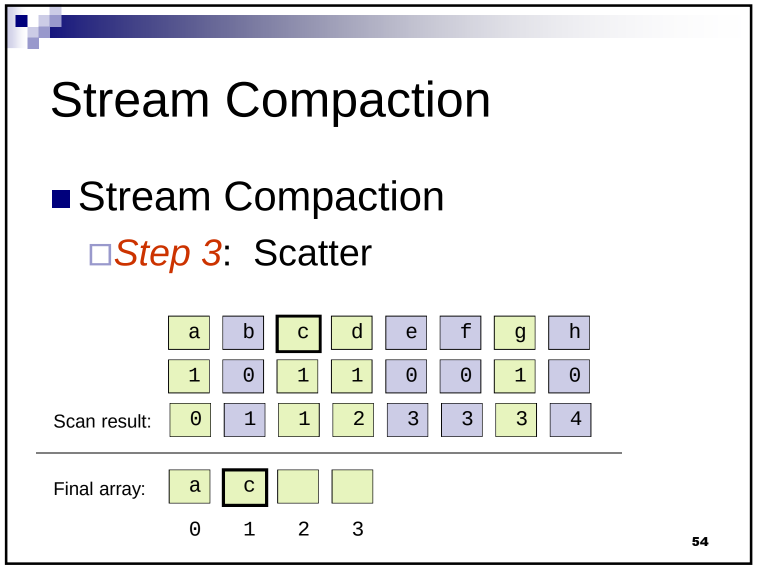


Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:	a			
	0	1	2	3

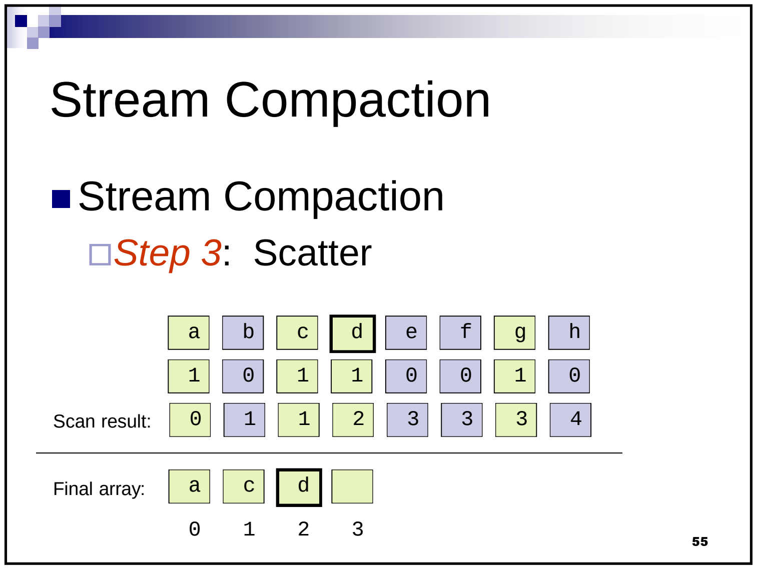


Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:	a	c		
	0	1	2	3

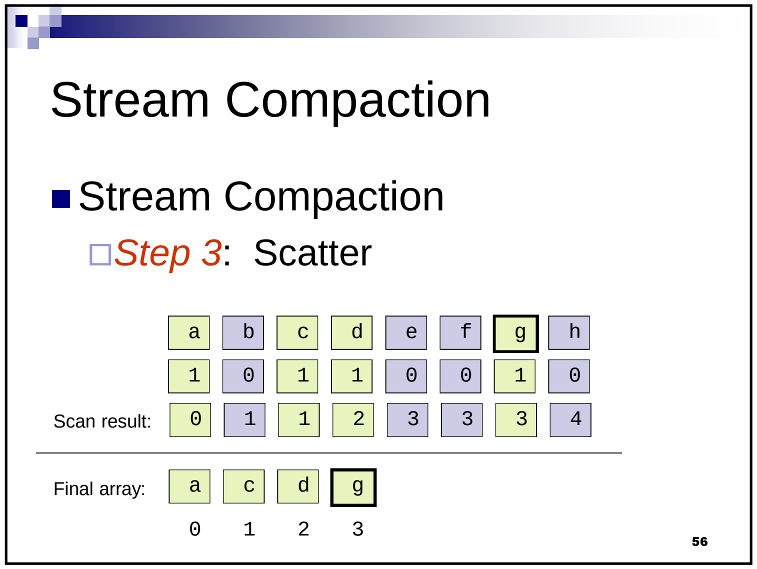


Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:	a	c	d	
	0	1	2	3

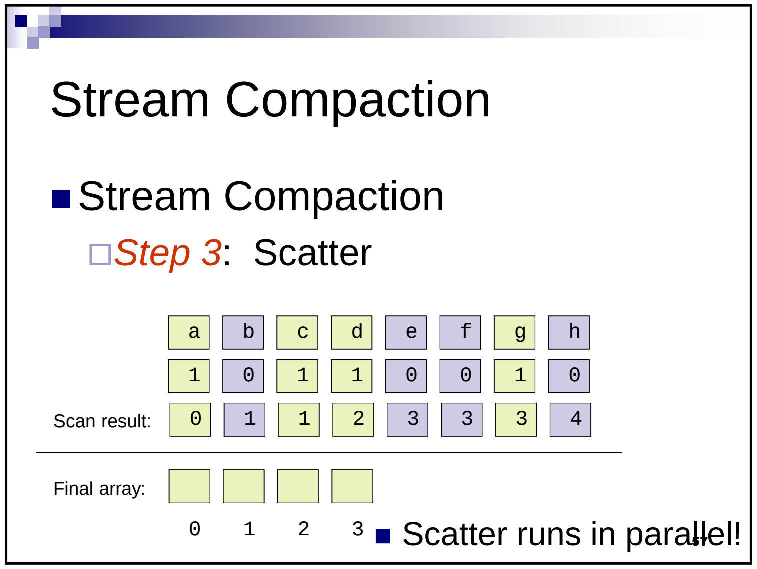


Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:	a	c	d	g
	0	1	2	3



Stream Compaction

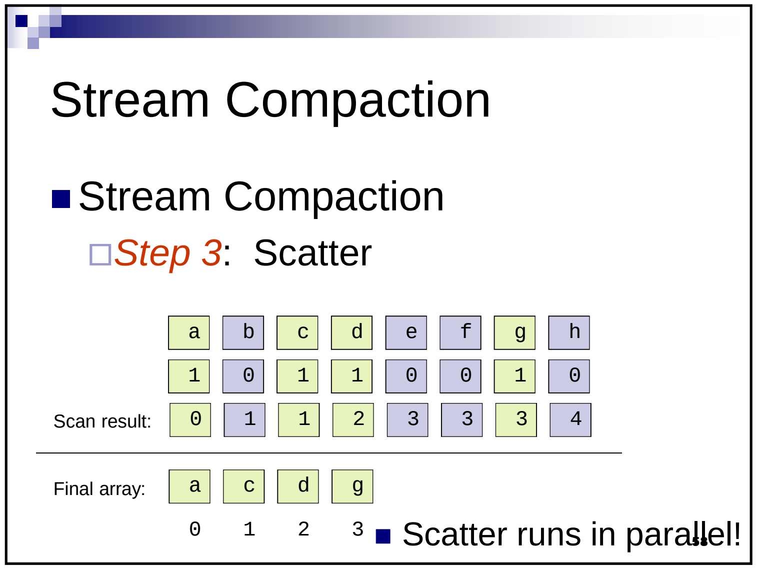
- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:				
	0	1	2	3

- Scatter runs in parallel!

57



Stream Compaction

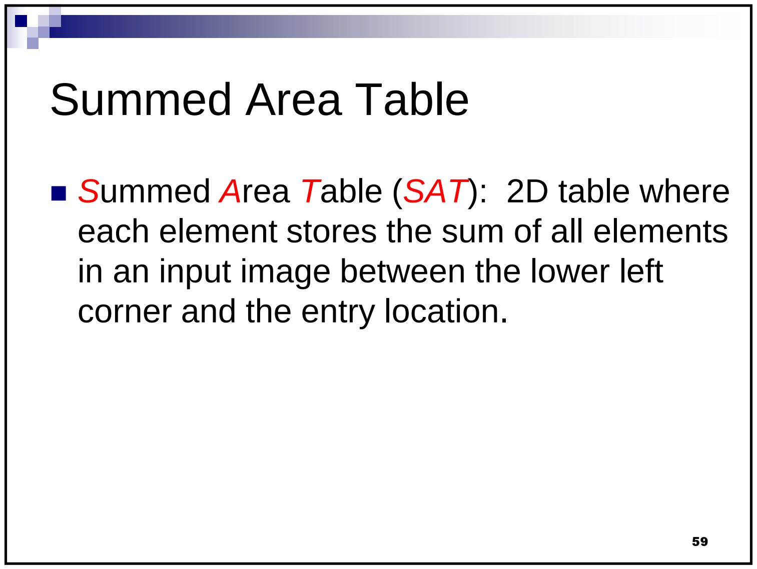
- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:	a	c	d	g
	0	1	2	3

- Scatter runs in parallel!

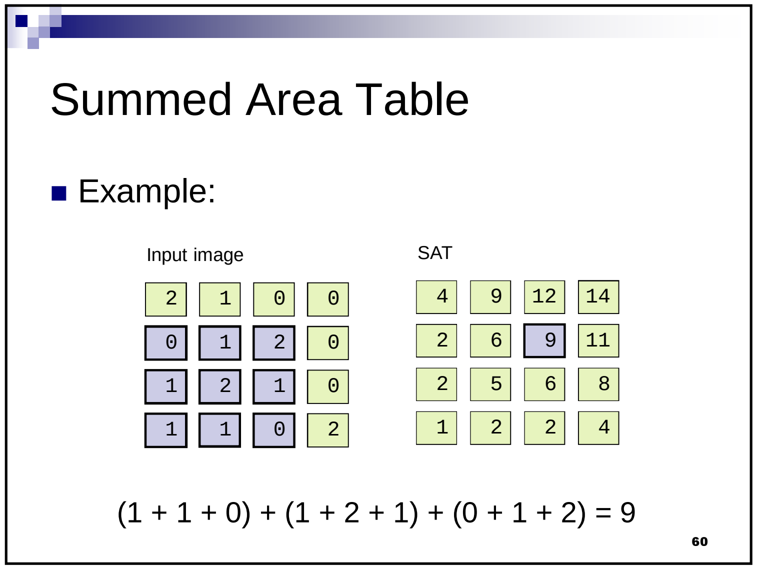
58



Summed Area Table

- *S*ummed *A*rea *T*able (*SAT*): 2D table where each element stores the sum of all elements in an input image between the lower left corner and the entry location.

59



Summed Area Table

- Example:

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

4	9	12	14
2	6	9	11
2	5	6	8
1	2	2	4

(1 + 1 + 0) + (1 + 2 + 1) + (0 + 1 + 2) = 9

60

Summed Area Table

- Benefit
  - Used to perform different width filters at every pixel in the image in constant time per pixel
  - Just sample four pixels in SAT:

$$s_{filter} = \frac{s_{ur} - s_{ul} - s_{lr} + s_{ll}}{w \times h},$$

Summed Area Table

- Uses
  - Approximate depth of field
  - Glossy environment reflections and refractions

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

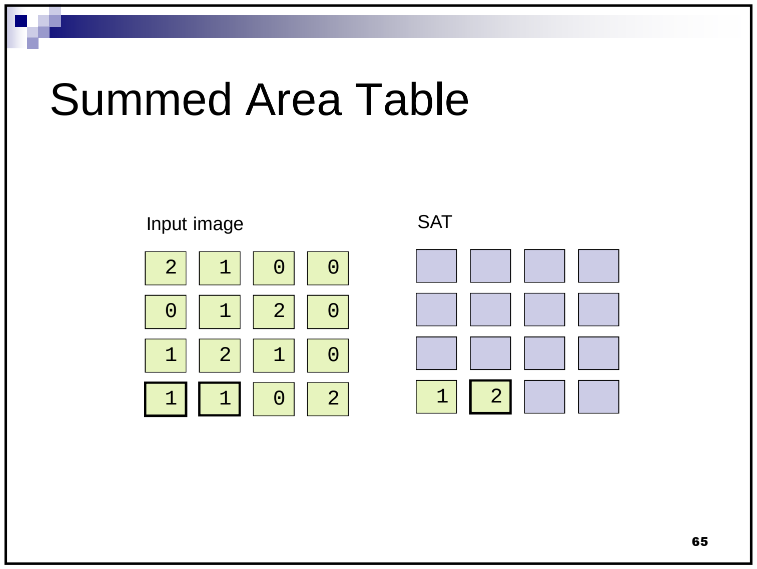
Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

1			



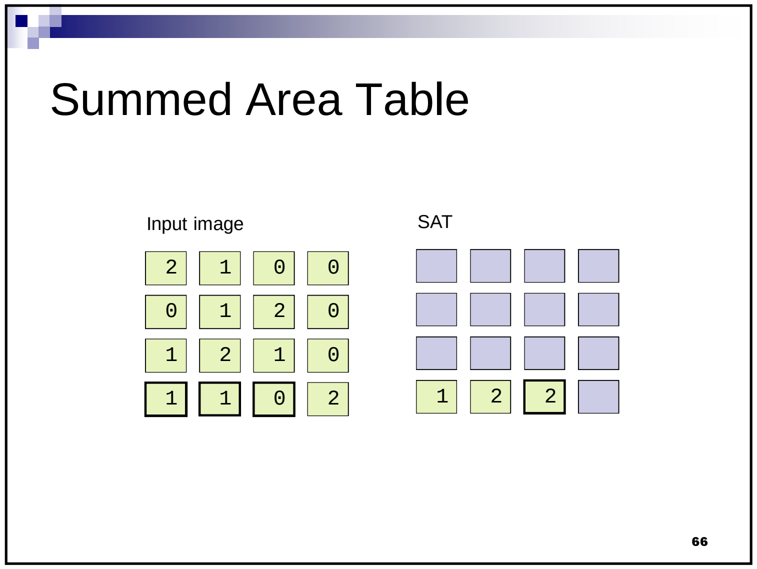
Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

1	2		



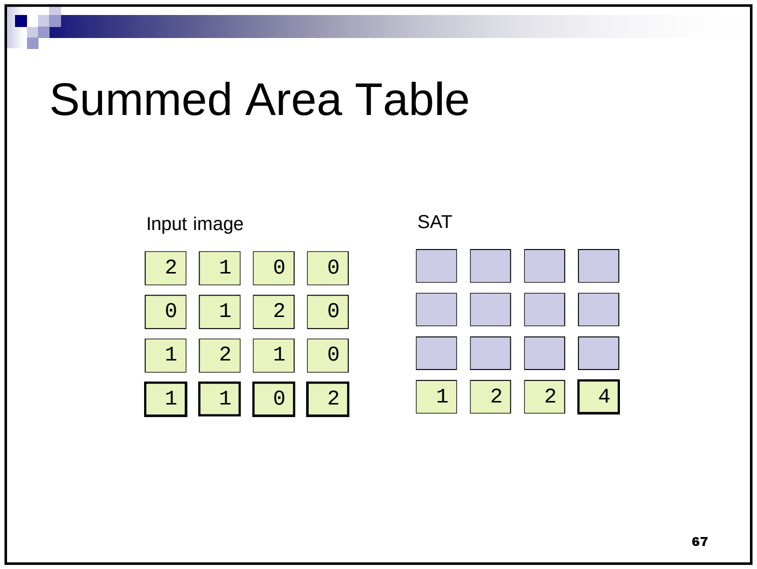
Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

1	2	2	



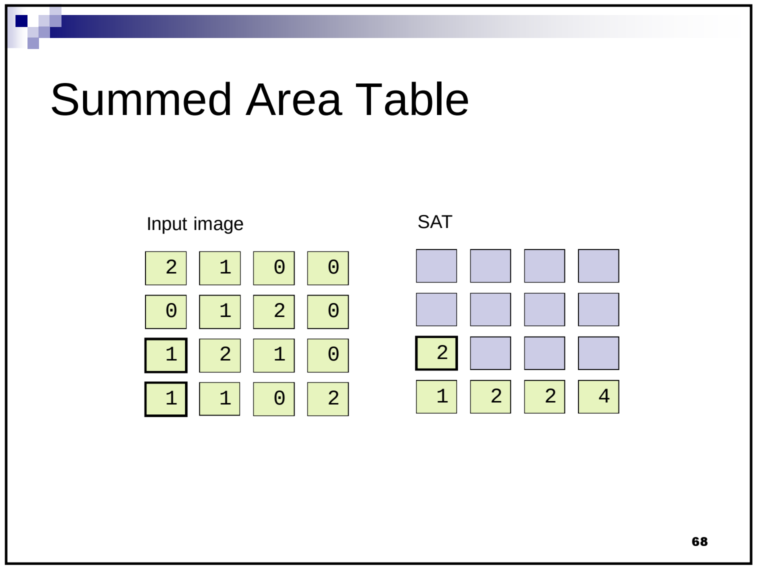
Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

1	2	2	4



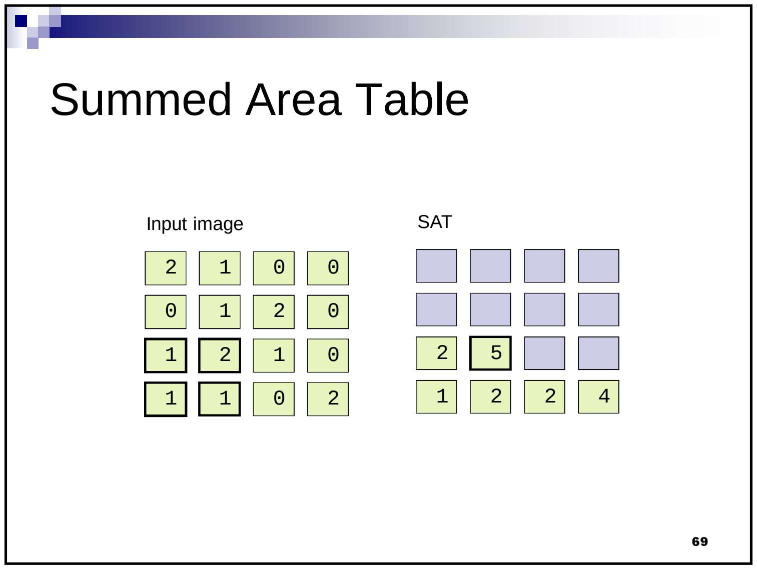
Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

2			
1	2	2	4



Summed Area Table

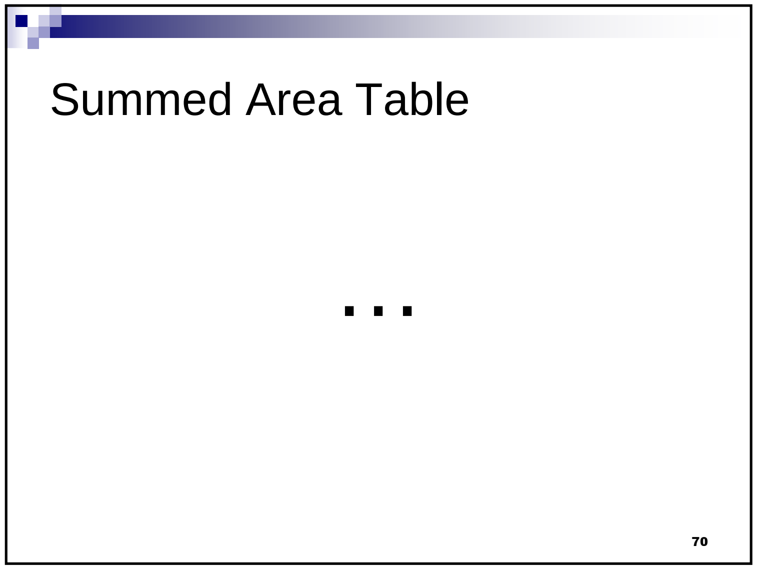
Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

2	5		
1	2	2	4

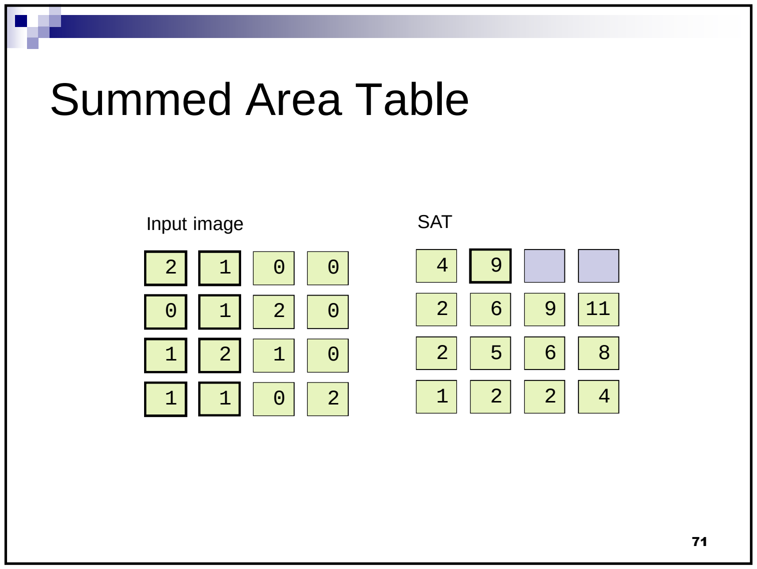
69



Summed Area Table

. . .

70



Summed Area Table

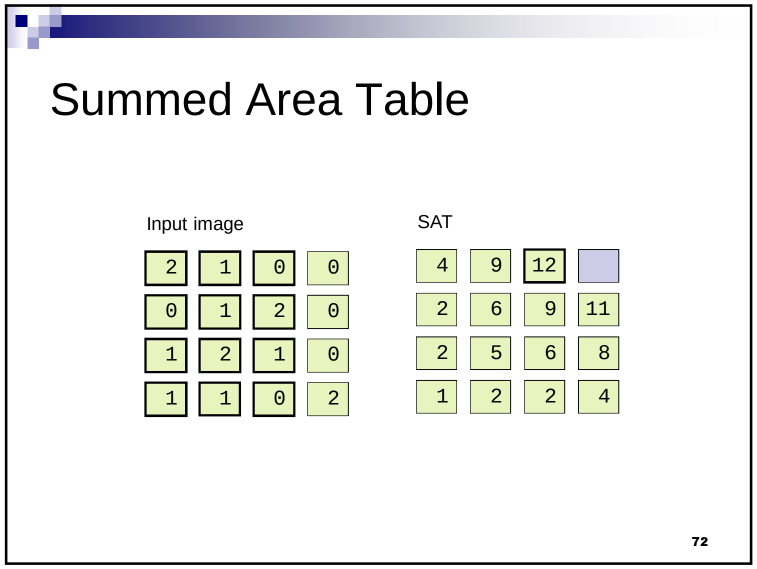
Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

4	9		
2	6	9	11
2	5	6	8
1	2	2	4

71



Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

4	9	12	
2	6	9	11
2	5	6	8
1	2	2	4

72

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

4	9	12	14
2	6	9	11
2	5	6	8
1	2	2	4

73

Summed Area Table

How would implement this on the GPU?

74

Summed Area Table

How would compute a SAT on the GPU using inclusive scan?

75

Summed Area Table

- Step 1 of 2:

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

Partial SAT

2	3	3	3
0	1	3	3
1	3	4	4
1	2	2	4

$\longrightarrow$

One inclusive scan for each row

76

Summed Area Table

- Step 2 of 2:

Partial SAT

2	3	3	3
0	1	3	3
1	3	4	4
1	2	2	4

Final SAT

4	9	12	14
2	6	9	11
2	5	6	8
1	2	2	4

One inclusive scan for each column, bottom to top

77

Radix Sort

- Efficient for small sort keys
    - k-bit keys require k passes

78

Radix Sort

- Each radix sort pass partitions its input based on one bit
- First pass starts with the *l*east *s*ignificant *b*it (*LSB*). Subsequent passes move towards the *m*ost *s*ignificant *b*it (*MSB*)

MSB 010 *LSB*

79

Radix Sort

- Example input:

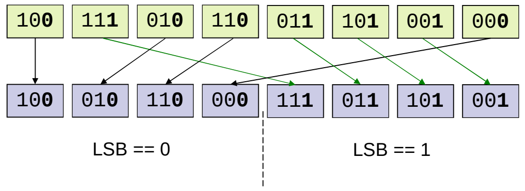
100	111	010	110	011	101	001	000

80

Example from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

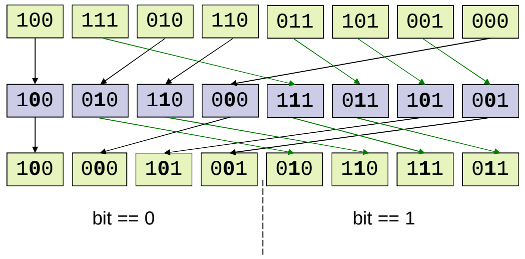
Radix Sort

- First pass: partition based on LSB



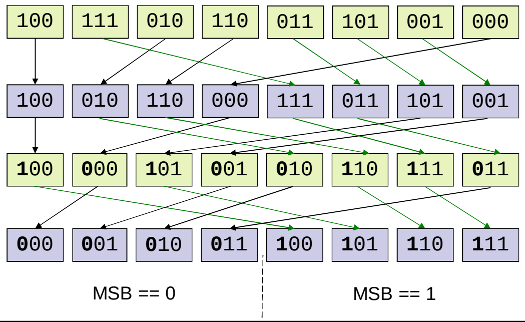
Radix Sort

- Second pass: partition based on middle bit



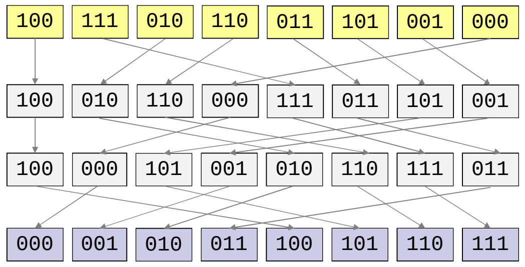
Radix Sort

- Final pass: partition based on MSB



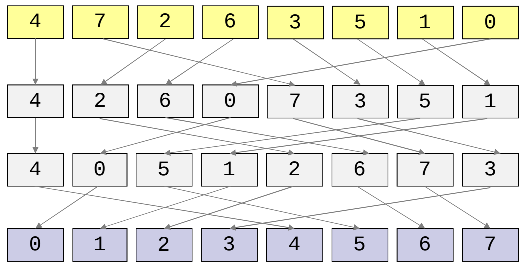
Radix Sort

- Completed:



Radix Sort

- Completed:



85

Parallel Radix Sort

- Where is the parallelism?

86

Parallel Radix Sort

1. Break input arrays into tiles
   - Each tile fits into shared memory for an SM
2. Sort tiles in *parallel* with *radix sort*
3. Merge pairs of tiles using a *parallel bitonic merge* until all tiles are merged.

Our focus is on Step 2

87

Parallel Radix Sort

- Where is the parallelism?
  - Each tile is sorted in parallel
  - Where is the parallelism within a tile?

88

Parallel Radix Sort

- Where is the parallelism?
  - Each tile is sorted in parallel
  - Where is the parallelism within a tile?
    - Each pass is done in sequence after the previous pass. No parallelism
    - Can we parallelize an individual pass? How?
  - Merge also has parallelism

Parallel Radix Sort

- Implement *spilt*. Given:
  - Array, *i*, at pass *n*:

100	111	010	110	011	101	001	000

  - Array, *b*, which is true/false for bit *n*:

0	1	0	0	1	1	1	0

- Output array with false keys before true keys:

100	010	110	000	111	011	101	001

Parallel Radix Sort

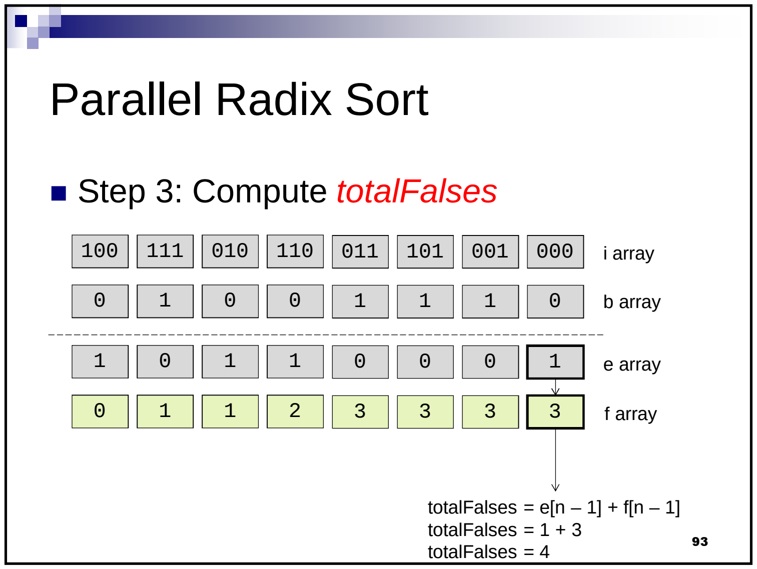
- Step 1: Compute *e* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array

Parallel Radix Sort

- Step 2: Exclusive Scan *e*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array



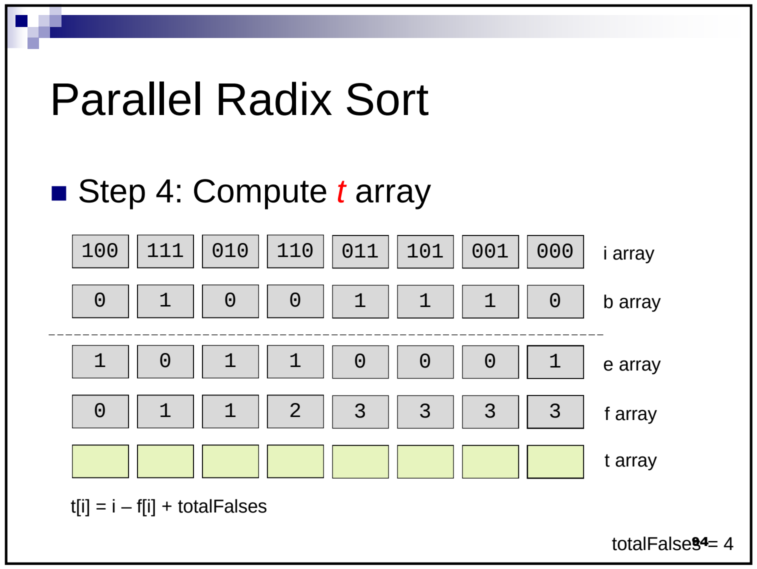
Parallel Radix Sort

- Step 3: Compute *totalFalses*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array

totalFalses = e[n – 1] + f[n – 1]
totalFalses = 1 + 3
totalFalses = 4

93



Parallel Radix Sort

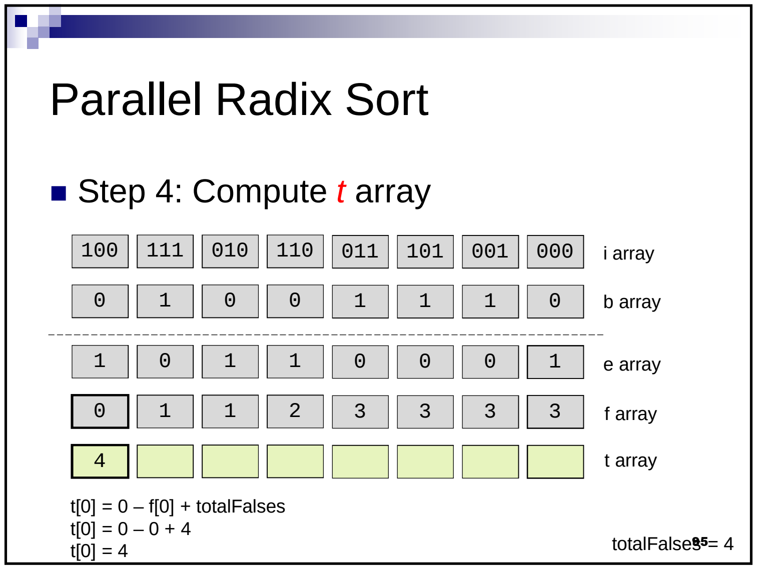
- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
								t array

t[i] = i – f[i] + totalFalses

totalFalses = 4

94



Parallel Radix Sort

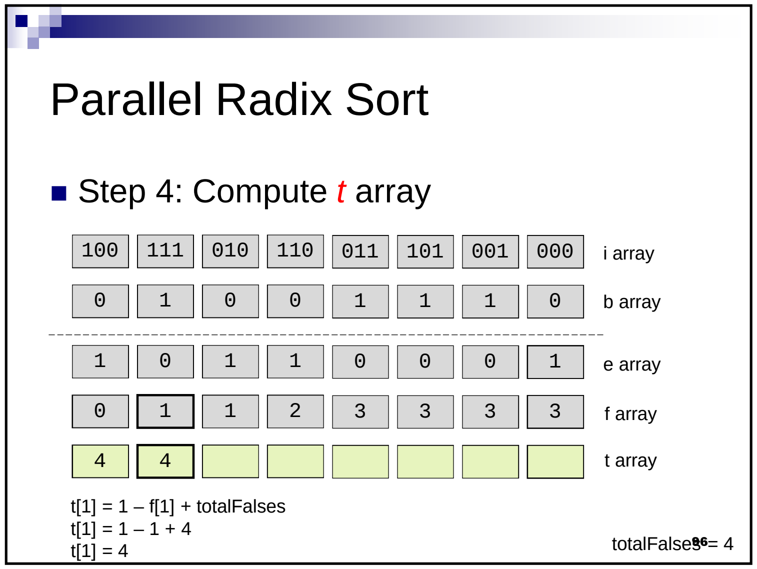
- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4								t array

t[0] = 0 – f[0] + totalFalses
t[0] = 0 – 0 + 4
t[0] = 4

totalFalses = 4

95



Parallel Radix Sort

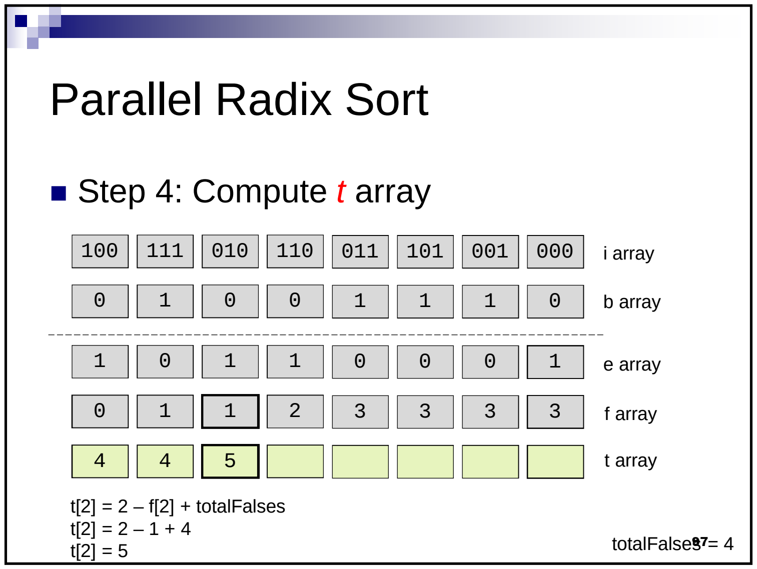
- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4							t array

t[1] = 1 – f[1] + totalFalses
t[1] = 1 – 1 + 4
t[1] = 4

totalFalses = 4

96



Parallel Radix Sort

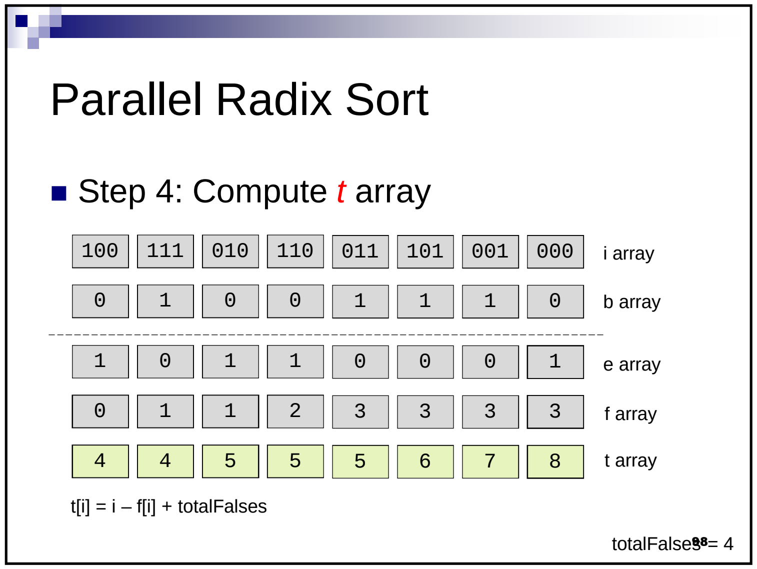
- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5						t array

t[2] = 2 – f[2] + totalFalses
t[2] = 2 – 1 + 4
t[2] = 5

totalFalses = 4

97



Parallel Radix Sort

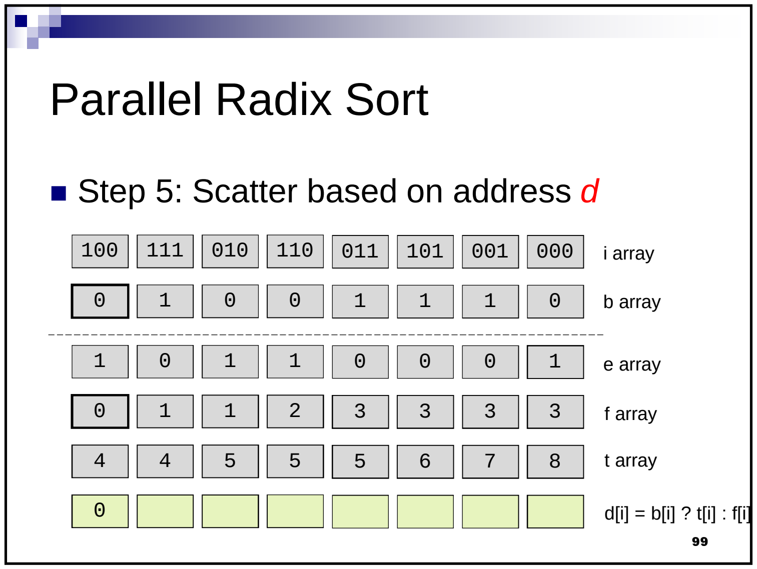
- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array

t[i] = i – f[i] + totalFalses

totalFalses = 4

98

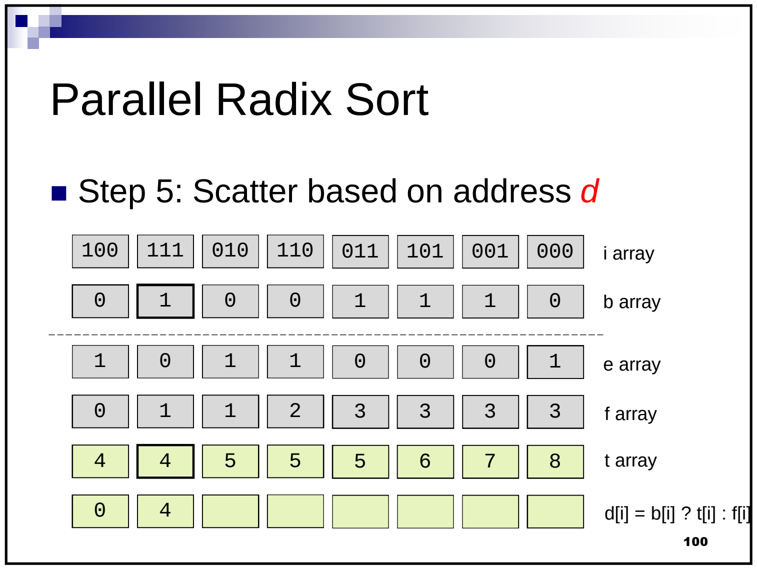


Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0								d[i] = b[i] ? t[i] : f[i]

99

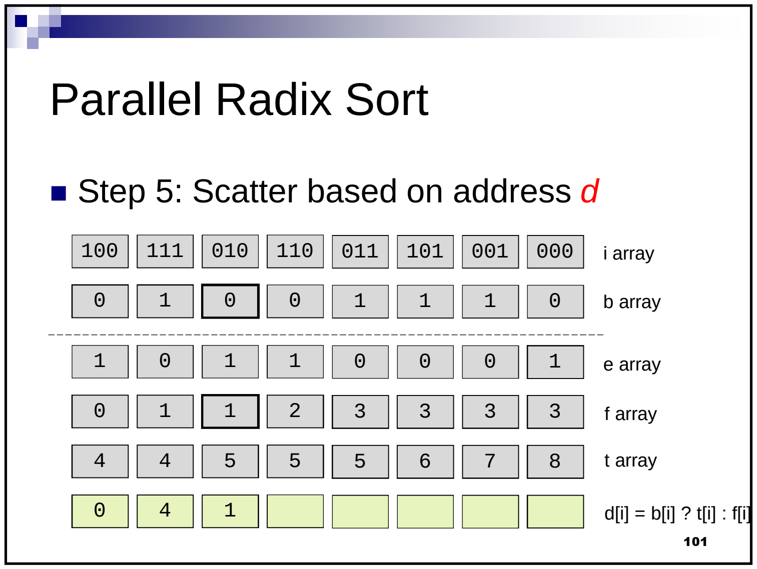


Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4							d[i] = b[i] ? t[i] : f[i]

100

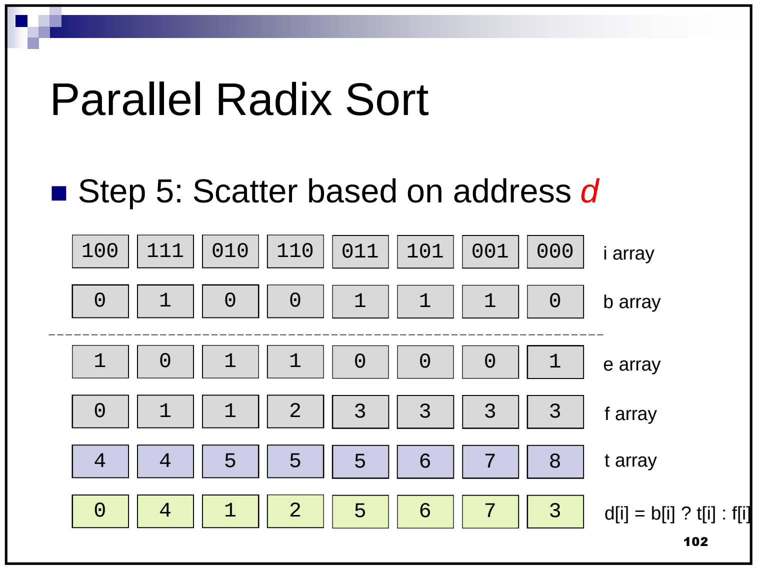


Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4	1						d[i] = b[i] ? t[i] : f[i]

101

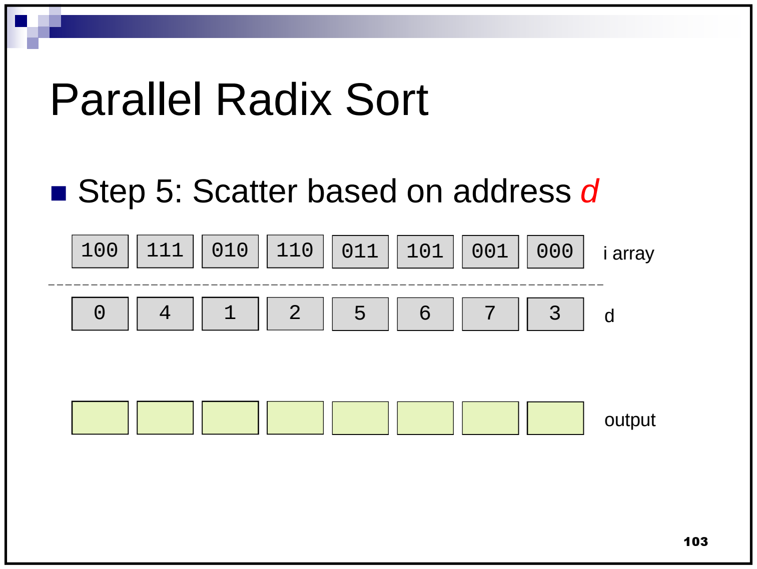


Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4	1	2	5	6	7	3	d[i] = b[i] ? t[i] : f[i]

102

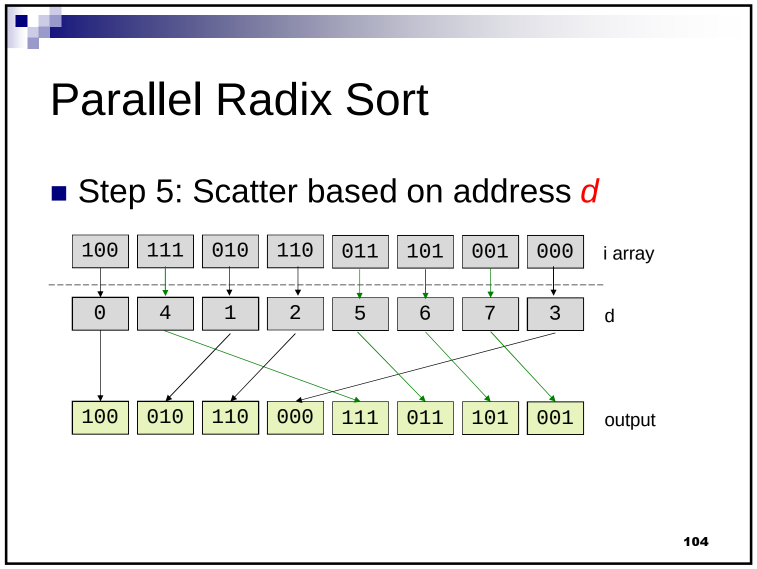


Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	4	1	2	5	6	7	3	d
								output

103

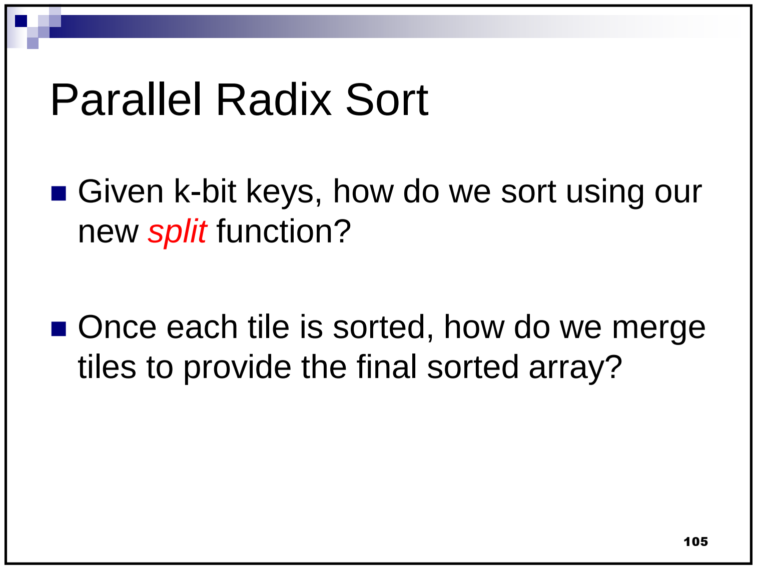


Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	4	1	2	5	6	7	3	d
100	010	110	000	111	011	101	001	output

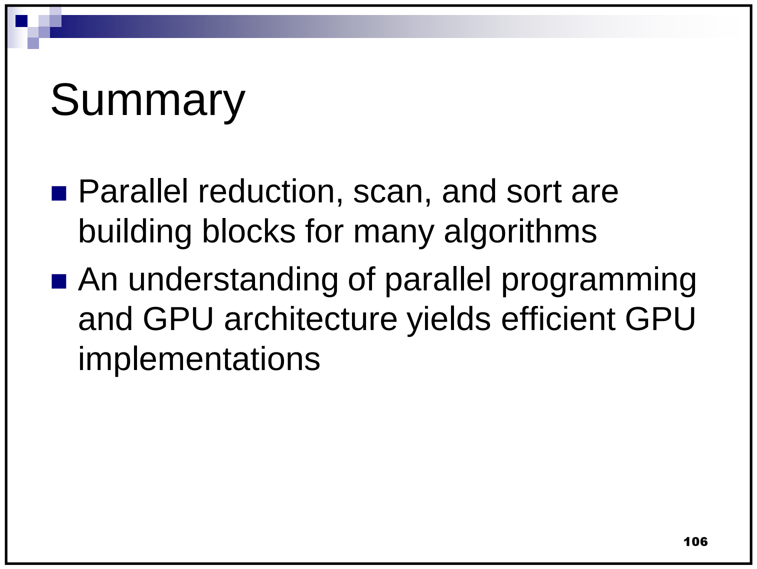
104



Parallel Radix Sort

- Given k-bit keys, how do we sort using our new *split* function?
- Once each tile is sorted, how do we merge tiles to provide the final sorted array?

105



Summary

- Parallel reduction, scan, and sort are building blocks for many algorithms
- An understanding of parallel programming and GPU architecture yields efficient GPU implementations

106