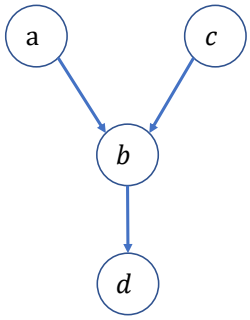
CISC 484/684: Homework 4

Due: May 15 (firm), midnight

There are two parts: individual problems and group problems. Each student should upload one submission for individual problem. Each group should upload one submission for group problems.

Individual Problem (15pt)

Bayesian Network Inference. Consider the following Bayesian network and answer the questions below.



a	c	Pr[b=1|a,c]	Pr[b=0|a,c]
0	0	0.8	0.2
0	1	0.2	0.8
1	0	γ	$1-\gamma$
1	1	0.5	0.5

b	Pr[d=1|b]	Pr[d=0|b]
0	α	$1-\alpha$
1	0.5	0.5

a=0	a=1
β	$1-\beta$

c=0	c=1
0.1	0.9

- (5pt) Express $\Pr[d=1|c=1]$
- (5pt) Express $\Pr[b=0|a=1]$
- (5pt) Express $\Pr[d=1|a=0]$

What to Turn in

- A report with your answer.

Group Problem (35+5pt)

EM Algorithm Implementation.

Consider the following EM algorithm for learning a mixture of K Gaussians with parameters $\theta_k = (\mu_k, \sigma_k^2)$ for each $k \in \{1, ..., K\}$. Let $x_1, ..., x_n$ be the data points. Introduce some hidden variables $w_{ik} \in [0, 1]$ and $\alpha_k \in [0, 1]$, for each $i \in \{1, ..., n\}$ and $k \in \{1, ..., K\}$.

- Initialize θ_k and α_k with $\sum \alpha_k = 1$
- **E step**: Compute

$$w_{ik} = \frac{\Pr[x_i|\theta_k] \cdot \alpha_k}{\sum_j \Pr_j[x_i|\theta_j] \cdot \alpha_j},$$

  where Pr[] is density of function of Gaussian.

- **M step**: Compute

$$n_k = \sum_i w_{ik},$$

$$\alpha_k = \frac{n_k}{n},$$

$$\mu_k = \frac{1}{n_k} \sum_i w_{ik} \cdot x_i,$$

$$\sigma_k^2 = \frac{1}{n_k} \sum_i w_{ik} \cdot (x_i - \mu_k)^2$$

Follow the following experiment steps:

- **Step 1.** Download the datasets on Canvas, where the points were generated from K Gaussian distributions.
- **Step 2.** Implement the above algorithm with $K \in 1, 3, 5$, with different initializations. Report the final parameters and the likelihood.
- **Step 3.** Now suppose the variance are known as 1.0. Modify the above EM algorithm to estimate μ_k, and redo the experiment in the last step.

Questions:

- What method did you use for initializations? Is the result sensitive to the initial values?
- Which K you believe is the truth? Why?
- (Extra credit 5 pt) Compare this EM algorithm with the one in our slides? What is the difference? Please explain the intuition behind such different.

What to Turn in

Please upload.

- Your **code and a Readme file** for compiling the code.
- A pdf **report** of (a) your results in step 2 and 3, and (b) your answers to the questions.