Practical Exercise: Uncertainty Propagation

Course: Uncertainty Quantification and Reliability Analysis
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Overview

This is a **formative exercise** designed to help you build the technical foundation for your final course project.

1. Objective

The goal is to apply the **Perturbation Method** and the **Monte Carlo Method** to a non-linear model to observe how uncertainty propagates from inputs to outputs. You are encouraged to choose a model relevant to your specific field of interest.

2. Implementation Freedom

- **Language:** You may use any programming language (MATLAB, Python, Julia, R, etc.).
- **Programming Style:** While vectorization is recommended for performance, it is not required for this exercise.
- **Tools:** You may use libraries for sampling (like UQLab, Chaospy, or NumPy), but the core logic of the Perturbation Method must be implemented manually.

3. Phase I: Verification

Before running your chosen model, verify your code using the quadratic benchmark:

- **Benchmark Model:** $Y = X^2$ where $X \sim \mathcal{N}(\mu = 10, \sigma = 2)$.
- **Analytical Solution:** $E[Y] = 104$ and $\text{Var}(Y) = 1632$.
- If your code cannot accurately find these values, do not proceed to the engineering model.

4. Phase II: Perturbation Method

Implement a local approximation using **Numerical Finite Differences**.

- Evaluate the response at the mean $\mathcal{M}(\boldsymbol{\mu}_X)$.
- Compute the Gradient and Hessian diagonal using **Central Differences**.

- **Numerical Stability:** Ensure your step size h is relative to the variable magnitude: $h_i = \mu_i \times 10^{-4}$. This prevents precision errors in variables with disparate scales.

5. Phase III: Monte Carlo Method

Perform a statistical analysis to validate your local approximation.

- Sample your input space ($N \approx 10^3$ to 10^5).
- Calculate the sample mean and variance.
- **Comparison:** Create a table comparing the moments (Mean and Variance) obtained by both methods.

Recommended Deliverables (For your own records)

To prepare for your final report, try to generate:

1. A table comparing $E[Y]$ and $\text{Var}(Y)$.
2. A histogram of the Monte Carlo response.
3. A brief note on why the results might differ.