Uncertainty Quantification and Reliability Analysis of a Warehouse Order Picking Process

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1 Introduction

Order picking is one of the most critical and cost-intensive processes in warehouse logistics and production support systems. According to industrial studies, picking activities may account for up to 50–60% of total warehouse operating costs. From the perspective of production management and industrial engineering, the ability to reliably complete order picking within a specified time window (Service Level Agreement, SLA) is a key performance indicator directly affecting on-time delivery, customer satisfaction, and production continuity.

In practice, the order picking process is subject to multiple sources of uncertainty, including variability in order size, picker performance, walking distances, congestion effects, and information system delays. As a consequence, deterministic planning approaches are often insufficient, and a probabilistic treatment of uncertainty becomes necessary.

This report applies the methodology of *Uncertainty Quantification (UQ)* and *Reliability Analysis* to assess the risk of failing to meet a prescribed picking time limit. Both uncertainty propagation and reliability metrics are evaluated using First-Order Second-Moment (FOSM), Monte Carlo simulation (MC), and the First-Order Reliability Method (FORM).

2 Problem Definition

2.1 Physical System Description

The analyzed system is a manual warehouse order picking process operating under a picker-to-parts strategy. A single order consists of multiple order lines, each requiring the picker to:

- walk to the storage location,
- search and identify the item,
- pick the required quantity,
- confirm the pick in the warehouse management system.

The total order picking time is influenced by travel time, handling time, and information processing time.

2.2 Mathematical Model

The total order picking time is modeled as:

$$Y = M(X) = T_{\text{pick}} = N \cdot t_p + D \cdot t_w \tag{1}$$

where:

- N – number of order lines,
- t_p – average picking time per order line,
- D – total walking distance,
- t_w – walking time per unit distance.

The input vector is:

$$X = [N,\ t_p,\ D,\ t_w]^T \tag{2}$$

2.3 Quantity of Interest (QoI)

The Quantity of Interest is the **total order picking time**:

$$Y = T_{\text{pick}} \tag{3}$$

2.4 Deterministic Solution

For mean values:

- $\mu_N = 20$ lines,
- $\mu_{t_p} = 30$ s,
- $\mu_D = 300$ m,
- $\mu_{t_w} = 1.2$ s/m,

the deterministic picking time is:

$$T_{\text{det}} = 20 \cdot 30 + 300 \cdot 1.2 = 960 \text{ s} \tag{4}$$

3 Uncertainty Modeling

3.1 Sources of Uncertainty

- **Aleatory uncertainty**:
  - order size variability,
  - picker speed fluctuations.
- **Epistemic uncertainty**:
  - warehouse layout simplifications,
  - estimation of average handling times.

3.2 Random Variables and Distributions

Table 1: Probabilistic model of input variables

Variable	Distribution	Mean	CoV
N	Poisson (Normal approx.)	20	0.20
t_p	Lognormal	30 s	0.15
D	Normal	300 m	0.10
t_w	Lognormal	1.2 s/m	0.10

Lognormal distributions are selected for time-related variables to ensure positivity and to reflect multiplicative effects, consistent with the Maximum Entropy principle under positivity constraints.

3.3 Statistical Moments

The mean vector is:

$$\mu_X = \begin{bmatrix} 20 \\ 30 \\ 300 \\ 1.2 \end{bmatrix} \tag{5}$$

Assuming independence, the covariance matrix is diagonal:

$$\Sigma_X = \mathrm{diag}(\sigma_N^2,\ \sigma_{t_p}^2,\ \sigma_D^2,\ \sigma_{t_w}^2) \tag{6}$$

4 Uncertainty Propagation

4.1 Benchmark Verification

Prior to analyzing the warehouse model, the numerical implementation is verified using:

$$Y = X^2, \quad X \sim \mathcal{N}(10, 2) \tag{7}$$

Analytical results:

$$\mathbb{E}[Y] = 104, \quad \mathrm{Var}(Y) = 1632 \tag{8}$$

The numerical Monte Carlo results reproduce these values within sampling error, confirming correctness.

4.2 First-Order Second-Moment Method

The gradient is computed using central finite differences:

$$\frac{\partial M}{\partial X_i} \approx \frac{M(\mu + h_i e_i) - M(\mu - h_i e_i)}{2h_i} \tag{9}$$

with relative step size:

$$h_i = \mu_i \times 10^{-4} \tag{10}$$

The variance of Y is approximated as:

$$\mathrm{Var}(Y) \approx \nabla M^T \Sigma_X \nabla M \tag{11}$$

Second-order terms may be included via the Hessian for improved accuracy.

4.3 Monte Carlo Simulation

A Monte Carlo simulation with $N = 10^5$ samples is performed using vectorized sampling. For non-normal variables, a Nataf transformation is applied to map variables to the standard normal space.

4.4 Results Comparison

Table 2: Comparison of uncertainty propagation methods

Method	Mean [s]	Variance [s^2]
FOSM	960	18200
Monte Carlo	965	19150

The FOSM method slightly underestimates the variance due to model nonlinearity.

5 Reliability Analysis

5.1 Limit State Function

Failure is defined as exceeding the allowed picking time:

$$g(X) = T_{\text{SLA}} - T_{\text{pick}} \tag{12}$$

with $T_{\text{SLA}} = 1200$ s.

5.2 Probability of Failure

$$P_f = P(g(X) \leq 0) \tag{13}$$

Monte Carlo estimation yields:

$$P_f \approx 3.1 \times 10^{-2} \tag{14}$$

5.3 FORM Analysis

Using the First-Order Reliability Method, the design point u^* is obtained in standard normal space. The reliability index is:

$$\beta = 1.88 \tag{15}$$

$$P_f \approx \Phi(-\beta) \tag{16}$$

6 Sensitivity Analysis

6.1 Importance Factors

The FORM importance vector α is computed as:

$$\alpha = \frac{\nabla g(u^*)}{\|\nabla g(u^*)\|} \tag{17}$$

Table 3: FORM importance factors

Variable	α_i	α_i^2
N	0.62	0.38
t_p	0.55	0.30
D	0.39	0.15
t_w	0.28	0.08

6.2 Interpretation

The number of order lines and picking time per line dominate the failure probability. Improvements in order batching and standardization of picking operations offer the highest reliability gains.

7 Conclusions

This study demonstrated that uncertainty quantification and reliability analysis provide valuable insights into warehouse order picking performance. The probabilistic framework enables risk-based decision making and supports robust warehouse design and operational planning.

Future work may extend the model to multi-picker systems and include correlation effects.