CTA200 2021 Assignment 2

DUE: Saturday May 8th by 5:00 PM

Submit all of your code in a .ipynb file. Please clear all output before commiting your file to Github. You will also submit a .tex file, as described in Question 4. You can see the .tex file for this assignment as a sample of how to get started. I also suggest that those of you new to Latex try out the Overleaf cloud based Latex editor.

[2] Question 1

The numerical approximation of the derivative of a mathematical function f evaluated as x_0 can be written as:

$$d_x f|_{x_0} \approx \frac{f_{x_0+h} - f_{x_0}}{h}$$

Where h is some small "step". This is found by looking at the Taylor series of $f(x)$ and taking $h \to 0$. A better approximation, when h is finite (rather than infinitessimal) is:

$$d_x f|_{x_0} \approx \frac{f_{x_0+h} - f_{x_0-h}}{2h}$$

Make a python function that has the form `def deriv(f, x0, h)` that takes in a python function, and returns the approximation of the derivative at x0 using stepsize h for each of the methods. Use your functions to take the derivative of the function $f(x) = sin(x)$ for $x_0 = 0.1$ using a variety of values of $h < 1$. Plot the error compared to the analytical derivative (ie abs(d_numerical - d_analytic) / d_analytic) as a function of h for each method on the same loglog plot. What do you notice? What does the slope represent?

[3] Question 2

For each point in the complex plane $c = x + iy$, with $-2 < x < 2$ and $-2 < y < 2$, set $z_0 = 0$ and iterate the equation $z_{i+1} = z_i^2 + c$. Note what happens to the z_i's: some points will remain bounded in absolute value $|z|^2 = \Re(z)^2 + \Im(z)^2$, while others will run off to infinity. Make an image in which your points c that diverge are given one color and those that stay bounded are given another. Make a second image where the points are coloured by a colourscale that indicates the iteration number at which the given point diverged.

[3] Question 3

The SIR model is a simple mathematical model of disease spread in a population. The model divides a fixed population of size N into three groups, which vary as a function of time, t:

- $S(t)$ is those that are susceptible but not yet infected
- $I(t)$ is the number of infected individuals

- $R(t)$ is those individuals that have recovered and are now immune

The model can be described by a set of 3 first order differential equations for each of the variables as

$$\frac{dS}{dt} = -\frac{\beta SI}{N}, \tag{1}$$
$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I, \tag{2}$$
$$\frac{dR}{dt} = \gamma I \tag{3}$$

Using the ODE integrator of your choice (must be callable in Python, we recommend using Scipy as will be covered in lecture on Friday), integrate the equations with $N = 1000$ from $t = 0$ to $t = 200$ for various values of γ and β (at least 3-4 values, justify your choices physically).

Use the initial condition $I(0) = 1, S(0) = 999, R(0) = 0$ (you can also experiment with other initial conditions if you wish). Plot the curves for S, I, R on the same figure with a legend. make separate plots for each choice of the parametersi (hint: use the `subplots()` command).

Bonus (+0.5 mark): Add a 4th parameter D for deaths and justify the addition of a 4th differential equation as well as any RHS terms that must be changed on the initial 3 equations. Integrate the new set of equations for some choice of parameters and comment on the results compared to the SIR model.

[2] Question 4

To practice using the `LaTex`, markup language, we would like you to writeup your results in a .tex file and submit both the `LaTex` code and the associated PDF. For each of the questions, save and insert your figures into a tex file and write a 1 paragraph methods section, which briefly describes what you did as well as a 1 paragraph analysis section which describes what you see in the results.

How to Submit

Submit your assignment by creating a folder called assignment_2 in your repository from Assignment #1 and put the notebook, the `LaTex` file and the PDF output from `LaTex` as well as any other files which are necessary for me to run your code. Commit the files and push to github. No need to email me your repo again since I have the URL from assignment 1.