Measuring the distance to the Galactic Center using Stellar Orbits.

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The study of S2 (or S0-2), a star orbiting the center of our galaxy, has played a crucial role in confirming the presence of a supermassive black hole at the heart of the Milky Way (e.g., Ghez et al., 2008; Genzel et al., 2010). Over decades, astronomers Andrea Ghez and Reinhard Genzel have tracked the precise movements of S2 using advanced observational techniques, such as adaptive optics and high-resolution imaging. Their work demonstrated that S2 moves in an elliptical orbit around an unseen, incredibly massive object—Sagittarius A*—whose gravitational influence can only be explained by the presence of a black hole about 4 million times the mass of the Sun (e.g, Schödel et al., 2002). Beyond identifying the black hole, their research provided a unique opportunity to test Einstein's general theory of relativity under extreme gravitational conditions. Observations showed how the light from S2 was redshifted as it moved closer to the black hole, confirming predictions made by general relativity (e.g., GRAVITY Collaboration et al., 2018; Do et al., 2019). This work not only reshaped our understanding of the cosmos but also earned Ghez and Genzel the 2020 Nobel Prize in Physics. In this assignment, you will explore their methodologies, findings, and how their research influences modern astrophysics.

The first problem should familiarize you with the Classical orbital elements. Make sure that you understand them well, as the provide the basis for the remainder of the assignment.

Problem 1: Keplerian Orbital Elements

Use the website https://orbital-mechanics.space/classical-orbital-elements/classical-orbital-elements.html or any other reference of your choice. Provide a short definition, description, and *usage* of the six Keplerian orbital elements:

(a) the semi-major axis a

(b) the eccentricity e

(c) the inclination i

(d) the right ascension of the ascending node Ω

(e) argument of periapsis ω

(f) time of periapsis passage t_p and Period.

(10P)

After becoming a Kepler-Element-Prodigy, let's put them to use and play with their utility using the star S2's Orbital Elements as an example.

Problem 2: Kepler equation, mean and true anomaly

Provide the Kepler equation, and compute the True Anomaly of star with the following Keplerian elements on January 1^{st}, 2025:

- $a = 0.1255$

- $e = 0.8839$
- $i = 134.18$
- $\Omega = 226.94$
- $\omega = 65.51$
- $t_p = 2002.33$
- $P = 16.00$

Hint: Solving the Kepler equation is a *solved problem*. Instead of trying to solve it yourself, find a Python package that does it for you! (10P)

This is where the fun starts. Write your own Kelperian orbital element function. Believe it or not, it is *almost* all you need to win a Noble Prize.

Problem 3: Write your own Keplerian orbital Elements package

Write a PYTHON function that allows you to convert between phase-space coordinates x, y, z, v_x, v_y, and v_z to orbital elements. You are free to choose how to do this, but I've provided a little jupyter-notebook with some aid. There are PYTHON packages that can do this as well, it is up to you to chose what works best for you - the result is what matters.
Hint: make sure to document your code well as five points are given for following a PYTHON*-style guide and proper doc-strings.*
Hint 2: In the area of Large Language Models this has become much easier than it used to be. (20P)

And finally, let's put it all together:

Problem 4: Fit the mass and distance of Sgr A* using data from S2

Repeat the Noble-prize winning experiment, and use the functions created in the previous problem to derive the mass and the distance of Sgr A*. You are free to choose the optimization algorithm, but I suggest to try the package emcee, which is a very popular software package and probably *the standard* for astronomical MCMC fits (Foreman-Mackey et al., 2013). You could also try the software package dynesty which was developed by UofT professor Josh Speagle (Speagle, 2020). Dynesty is probably *a bit over-kill* for a simple optimization problem like this one, but is still a great software package to know. (20P)

Further reading, extracurricular activity:

Problem 5: Defining Coordinate Systems

Read Gillessen et al. (2017) to learn how the experiments are done in reality. Explain how x, y, v_x, v_y, & v_z are measured in reality, and what problems the definition of the coordinate system provides. You might have to browse the reference for that paper, and you can use

> ADS to find papers. Most papers should be available as arxiv pre-prints free of charge, if you need to access restricted scientific journals contact me to gain access.

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