Small-signal models of power system components for stability analysis

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Contents

1 Introduction

This document contains a detailed guide to explain how to analyze modern converter-based power systems. The analysis is fundamentally based on developing and analyzing both non-linear simulation and linear models. The content will be structured progressively in complexity, starting with simple systems such as RL circuits and moving to more complicated converter and power system models.

2 Main objectives

The key objectives of the document are detailed next:

- Provide a comprehensive guide to learn how to analyze modern power systems.
- Start with simple circuit models, progressively increasing in complexity.
- Show how linearized state-space-based models are able to capture the fundamental dynamics of modern networks
- Explain how to validate linear models
- Explain how to analyze linear models with linear analysis tools, such as eigenvalue analysis, participation factors, etc.
- Showcase the developments in a Matlab/Simulink environment

3 State-space modelling basics

This section will cover the fundamentals of modelling state-space of RL circuits. Before starting, it should be mentioned that the RL circuit that will be considered is already linear, so no linearization process is required. The circuit scheme is detailed next:

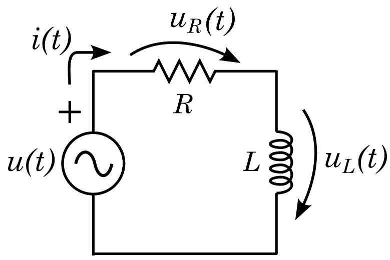

Figure 1: R-L circuit

3.1 RL circuit

3.1.1 Circuit equations

First, we start constructing the differential equation of the circuit. Applying Kirchhoff Voltage Law (KVL) to the circuit, we obtain:

$$u(t) - u_R(t) - u_L(t) = 0 \tag{1}$$

Then, substituting the fundamental behaviour of the RL elements in (1), we obtain:

$$u(t) = R\,i(t) + L\frac{di(t)}{dt} \tag{2}$$

Based on the previous equation, it can be seen that setting the R and L values, the dynamic performance is defined. To illustrate this better, we can isolate the current from (2) obtaining:

$$\frac{di(t)}{dt} = \frac{1}{L}\left(u(t) - R\,i(t)\right) \tag{3}$$

This equation shows indeed that the current evolution (derivative) depends on the applied source voltage $u(t)$ and the circuit values R and L, as mentioned above.

3.1.2 Simulation results

In order to obtain the current $i(t)$ dynamic behaviour, the circuit differential equation (2) can be integrated in a numerical simulation using Matlab Simulink.

Fig. **??** shows that the dynamic response of the current for a step change of 3 V (from 5 to 8 V) is impacted by the selection of different L values ($L = 100$ mH, $L = 50$ mH and $L = 10$ mH), since R is kept constant ($R = 1\ \Omega$) in this case. The dynamic response is what we typically define as a first order system, defined by its time constant, typically called τ. More specifically, the τ time constant is defined as the time that the current magnitude takes to reach the 63.3% of the final value. For RL circuits, this value is defined by the $\frac{L}{R}$ factor.

3.1.3 State-space model derivation

Once we have computed the simulation results, we can proceed to obtain the linear state-space model. The state space model is defined as follows:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) \tag{4}$$

$$y(t) = Cx(t) + Du(t) \tag{5}$$

where A, B, C, D are the state, input, output and feedback matrices. Also, x(t), u(t), y(t) is the state, output, input vector. So, identifying terms with (2), we can obtain:

$$\frac{di(t)}{dt} = \frac{1}{L}v(t) + \frac{R}{L} * i(t) \tag{6}$$

So, basically we have the format of Equation (4) hence, we can write:

$$\left[\frac{di(t)}{dt}\right] = \left[-\frac{R}{L}\right] * i_l + \left[\frac{1}{L}\right] * v(t) \tag{7}$$

where $[\frac{R}{L}]$ is A matrix and, $[\frac{1}{L}]$ is the B matrix. Once we have the values of A, B, C, D matrices as well as state, input, and output variables we can build the linear model. To do so, we can write this code in Matlab:

```
 A_rl=A;
B_rl=B;
C_rl=C;
D_rl=D;
rl_x={'il '};
rl_u={'U'};
rl_y={'i '};
rl= ss(A_rl,B_rl,C_rl,D_rl,'StateName',rl_x,'inputname',rl_u,'outputname',rl_y);
```

Basically, this code will help us to build the linear model in Simulink with the A, B, C, and D matrices that we found, and also the variables that we defined. Once we run the code we are ready to go to the linear model in Simulink. In order to compare both the linear state space model and linear models the same step value has been applied. The model and block can be seen in Fig. 2.
In the end, the comparison between the linear state space model and the linear model can be seen in Figure 3. It is clear that, the both models are matching.

Furthermore, it is crucial to analyze the eigenvalues with different conditions. To do so, Fig. 4 can be used. In the Figure, we can see that, with different L values the response of the signals changes. It is known that, this is a first order response hence the system's transfer function is :

$$G(s) = \frac{1}{\tau s + 1} \tag{8}$$

$$G(s) = \frac{1}{Ls + R} \tag{9}$$

To prove it, different L values have been used and the eigenvalues of the matrix A, which is the state matrix, have been plotted. Moreover, all the eigenvalues have a negative real part only which

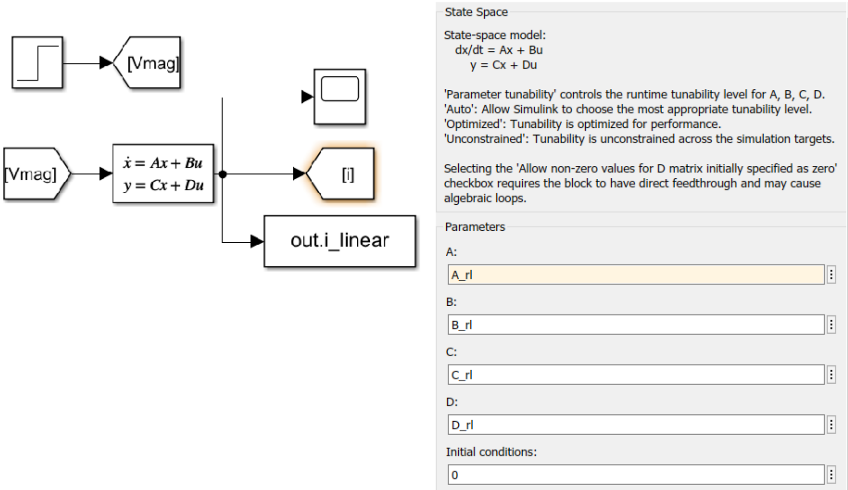

Figure 2: Linear State Space Model in Matlab

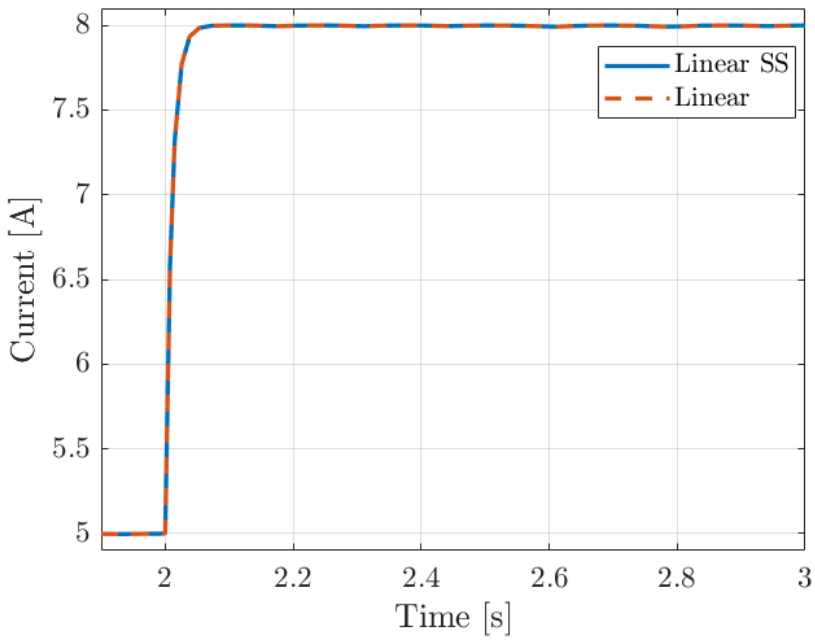

Figure 3: Comparison of Linear state space and linear model

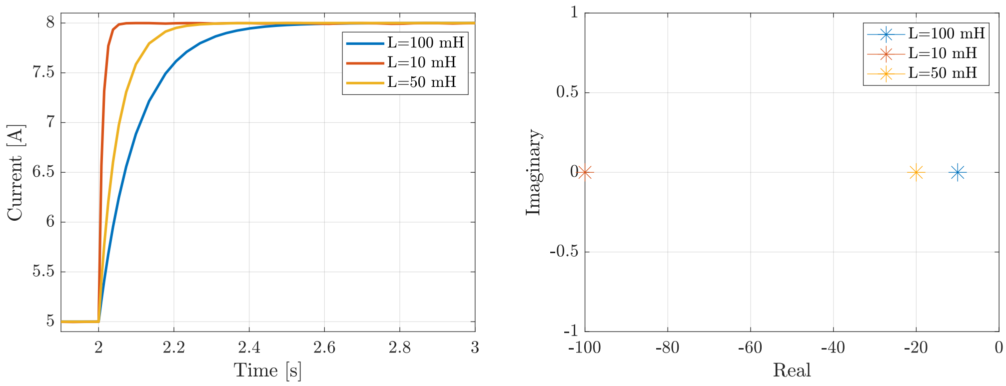

Figure 4: Eigenvalues and the response of the current

means the systems are stable and first order. If we take into account that when L = $100mH$ it can be seen that, the response is slower than the other cases hence its eigenvalue is the closest one to 0. The same observation can be done to other eigenvalues. Furthermore it can be seen that, for L = $50mH$ and L = $10mH$ the response is getting faster hence the eigenvalue is further than 0 respectively.
So, by only looking at the eigenvalues we can see that:

1. According to the number of eigenvalues of the system matrix A, we can say if this is a first-order order system or not. In this case, since we have only one eigenvalue it is a first-order system.

2. The polarity of the real part of the eigenvalues also tells us that, the system is stable since the real part is negative.

3. The magnitude of the real part is also a crucial thing, in this case we can see that the bigger the magnitude the faster the response. In other words the further from the origin the faster the response.

In this part we have covered, by applying KVL gathering the differential equations then taking Laplace of it. Once the equations of the system have been defined we have found the A, B, C, and D matrices and we have built the linear state space model according to those matrices and finally plotted both linear state space and linear model together to check if they are in close agreement. Also, we have covered that, understanding the aspects of the system by observing the eigenvalues. In the next chapter, an RLC circuit will be studied and same methodology will be followed.

3.2 RLC Circuit

In this part, we are going to study an RLC circuit that has a different response than an RL circuit because of the capacitor. In this case, there are two state variables that are the voltage across the capacitor u_c and the current through the inductor i_l.

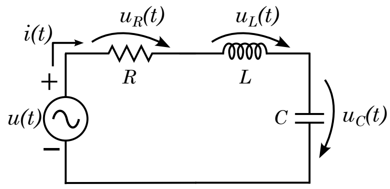

Figure 5: RLC circuit

Then, by applying KVL we can get

$$u(t) = Ri(t) + L\frac{di(t)}{dt} + u_c(t) \tag{10}$$

by isolating the derivative term we get,

$$\frac{di(t)}{dt} = \frac{u(t)}{L} - \frac{1}{L}\frac{Ri(t)}{dt} - \frac{u_c(t)}{L} \tag{11}$$

Also, we can define the voltage across the capacitor as follows

$$u_c(t) = \frac{1}{C}\int i(t)\,dt \tag{12}$$

Next, taking the derivative of the $u_c(t)$ we get,

$$\frac{du_c(t)}{dt} = \frac{i(t)}{C} \tag{13}$$

Until now we have isolated all the state variables of the system and now, we can start building the state-space model of it. Note that as passive elements (R, L, C) are already linear we don't need to linearize the system.

$$\underbrace{\begin{bmatrix} \frac{di(t)}{dt} \\ \frac{du_c(t)}{dt} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i(t) \\ u_c(t) \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u(t) \end{bmatrix}}_{u} \tag{14}$$

Now we have written the A and the B matrices. Moreover, in this case, as an output the voltage across the capacitor will be observed hence the matrices can be written as follows

$$\underbrace{\begin{bmatrix} u_c(t) \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} i(t) \\ u_c(t) \end{bmatrix}}_{\mathrm{x}} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} u(t) \end{bmatrix}}_{u} \tag{15}$$

To show that the state space representation can capture the fundamental dynamics of the system it is compared with the Simulink model. To do so, $L = 100mH$, $R = 5\Omega$ and $C = 5mF$ has been chosen and the signals can be seen in Fig. 7. Clearly, both models are matching and now we can analyze the behavior of the signals by looking at the eigenvalues. To do so, different values for the passive

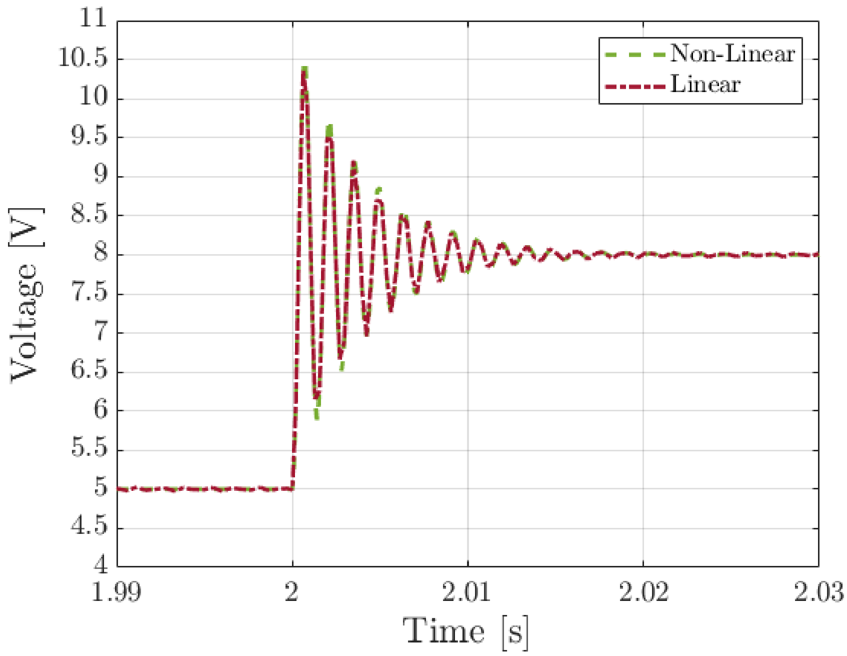

Figure 6: Comparison of Linear and non-linear model

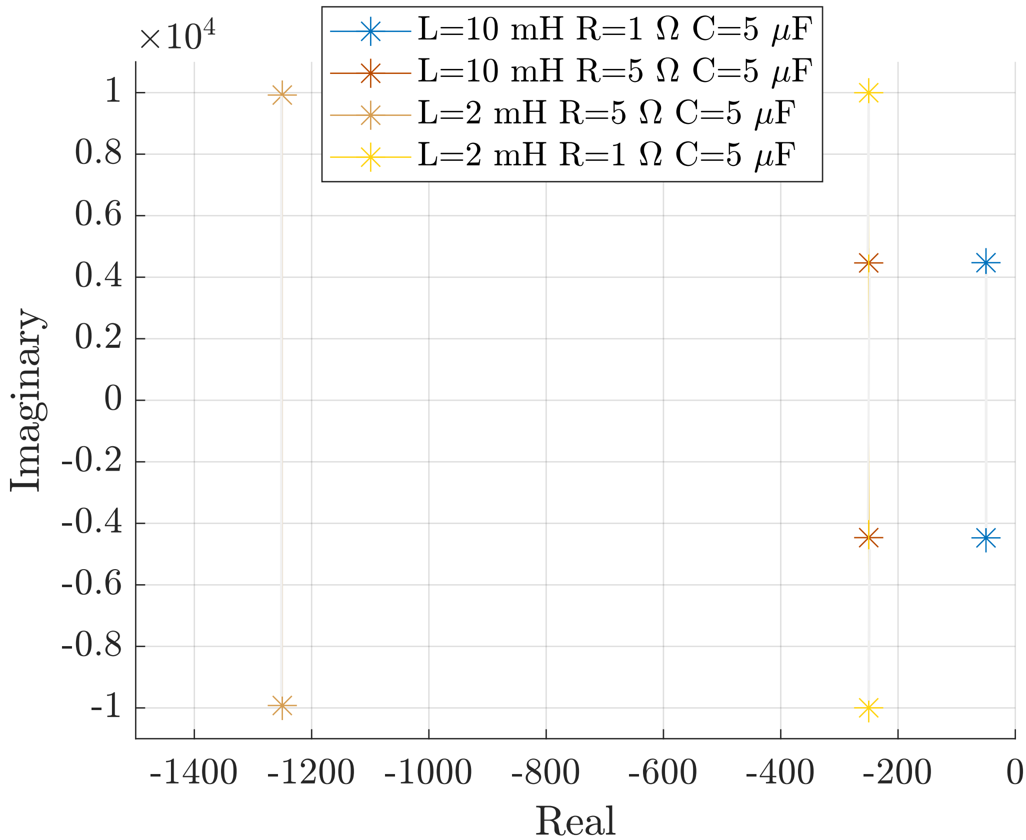

Figure 7: Eigenvalues with different RL values

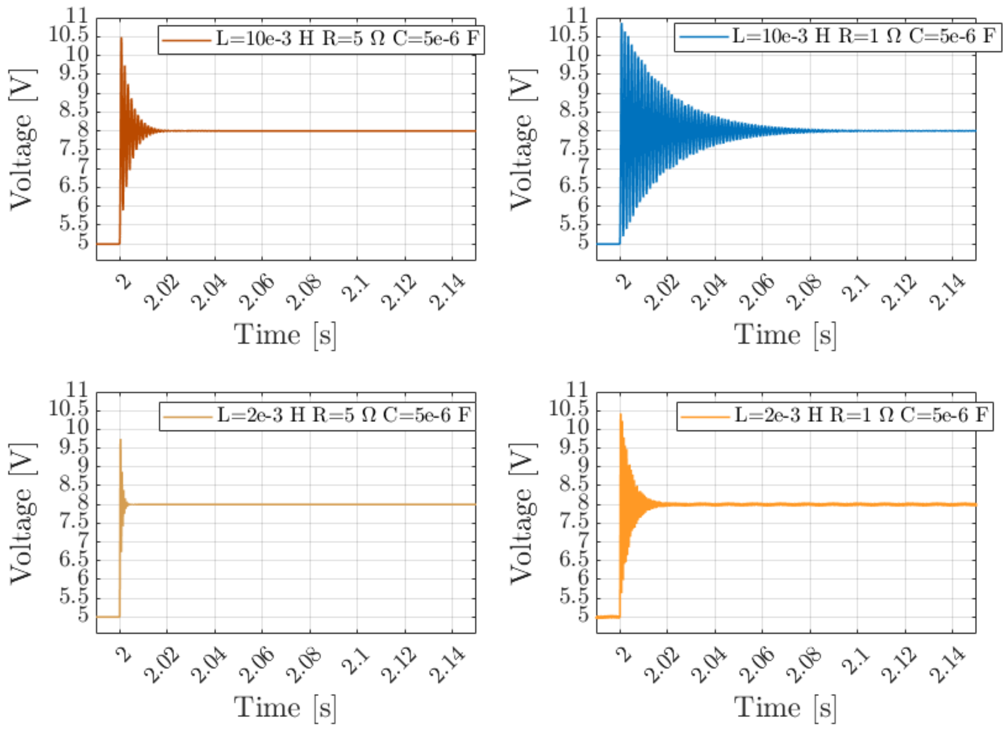

Figure 8: Behaviours of the signals

elements have been used. Now for each case, we have two conjugate poles as the transfer function is second order. Since the objective is to observe the system's behavior by looking at the eigenvalues, it can be observed that all the real parts of the poles are negative hence, we can say that all the cases are stable. Moreover, further from the origin, we have a faster response which can be seen in Fig. 8 (left-up corner). Also, the slowest response once we have real parts of the pole close to origin so, it can be seen in Fig. 8 (right up corner).

Now, it is also important to show the damping ratio and the natural frequencies of the poles. To do so, again we take into account 16.

$$\frac{1}{s^2 + 2\xi Wns + Wn^2} \tag{16}$$

so, by using 17 and 18 we can find the damping ratio and the natural frequency. In this case, they can be defined as:

$$Wn = \sqrt{\frac{1}{LC}} \tag{17}$$

$$\xi = \frac{R\sqrt{C}}{2\sqrt{L}} \tag{18}$$

We have found the natural frequency and damping ratio now we can calculate them with the values that we have used. Another formula also can be used to calculate the damping ratio assuming a+jb:

$$\xi = \frac{-a}{\sqrt{a^2 + b^2}} \tag{19}$$

Table 1: Damping ratio and natural frequency

Real	Imaginary	R (ohm)	L (H)	C (F)	Damping (Eigenvalues)	Damping (RLC values)	Natural Frequency (rad/s)
-50	$\pm$ 4471.86	1	10e-3	5e-6	0.01118	0.01118	4472
-250	$\pm$ 9996.87	1	2e-3	5e-6	0.025	0.025	10000
-1250	$\pm$ 9920	5	2e-3	5e-6	0.125	0.125	10000
-250	$\pm$ 4471.86	5	10e-3	5e-6	0.0559	0.0558	4472

Moreover, to prove our calculations we can observe Fig. 9 which shows all the parameters that we calculated. Also it crucial to observe the eigenvalues and comment about damping ratio and the natural frequency. So, the higher the damping ratio the lower magnitude in the oscillations and vice versa the behavior can be seen in Fig. 8 left down corner and right up corner respectively. Also, we can say that the imaginary parts of the eigenvalues tell about the frequencies. The bigger the imaginary part the bigger the frequency and also vice versa. Moreover, we have seen that the imaginary part is also used to calculate the damping ratio.

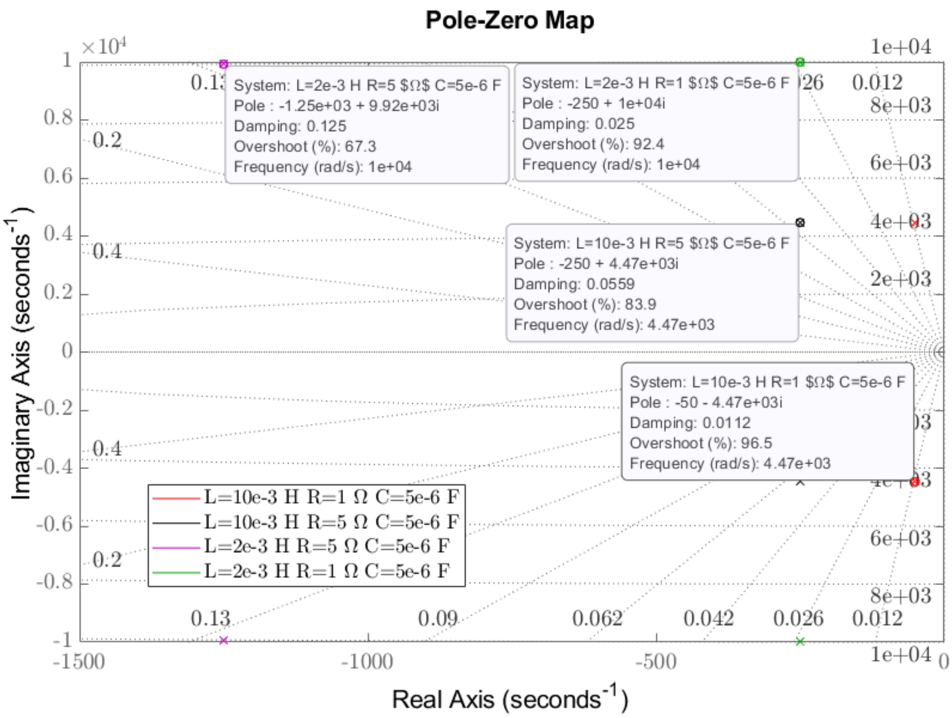

Figure 9: Behaviours of the signals

3.3 RL Circuit with PI contoller

Until now, we have covered how to get the transfer function, commenting about the system by looking at the eigenvalues(response time, damping ratio, frequency, stability,order of the system). In this section, we will try to control the current in an RL circuit with PI controller hence, we will make a small introduction to control and again by building the state-space model of the control we will check the eigenvalues. Noteworthy, since the PI controller is linear still we don't have any nonlinearity.

Fig. 10 represents the block diagram of the RL circuit with the PI controller. To do so, first a random input signal as a current reference is generated, and then, the current through the circuit is fed back to the system.

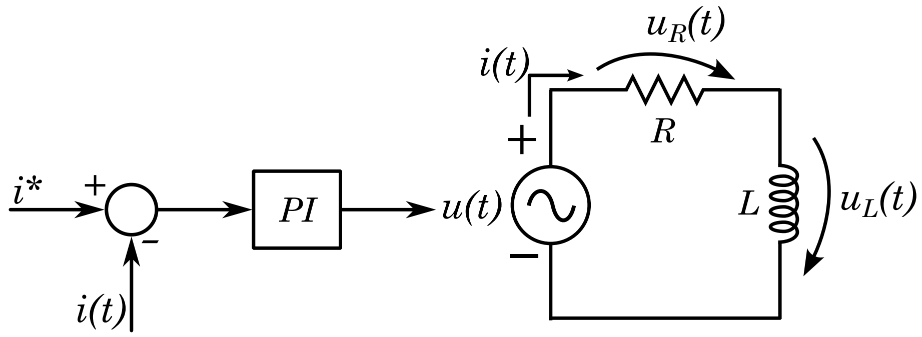

Figure 10: Block diagram in Matlab

Furthermore, previously we have found the transfer function of the RL circuit in equation 9, so we can put PI controller and the transfer function of the circuit together and the block diagram can be seen in Fig. 11.

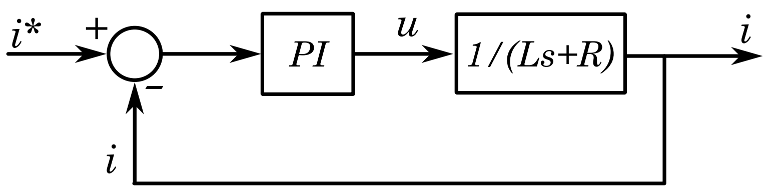

Figure 11: Block diagram of PI controller and RL circuit in Matlab

It is also crucial to explain briefly why we use the PI controller. P as the proportional controller and I as the integral controller can be used separately. However, each controller type has some advantages and drawbacks. The P controller is fast however, it's not accurate on the contrary, the I controller is accurate however it is slow. Hence, by combining these two controllers we eliminate individual drawbacks and achieve a very useful controller. The difference between the controllers can be seen in Fig. **??**.

We can clearly see that P by itself is fast but not accurate, and I is accurate but slow however, the PI controller has no problem in terms of speed and accuracy. After a brief explanation of controllers now we can show that the models in Fig. 10 and Fig. 11 are correct by plotting them together.
It can be seen in Fig. 13, both models are matching however, the controller can be improved. In this

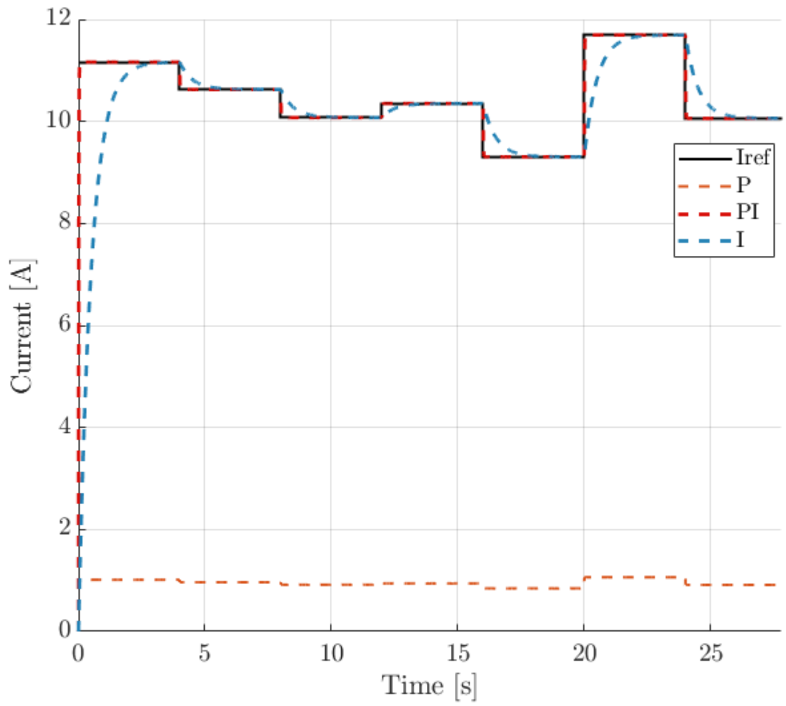

Figure 12: PI, P, I contollers

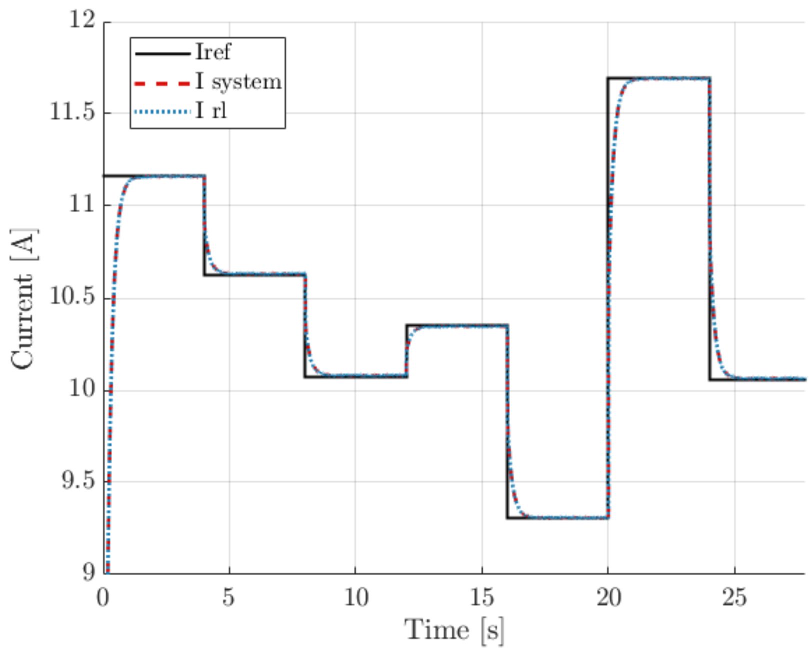

Figure 13: Circuit vs Plant

case, for simplicity, Kp=1 (for proportional) and Ki=10 (for integral) have been chosen. Moreover, these parameters can be better tuned. So, we can choose a time constant and tune Kp and Ki by using:

$$Kp = \frac{L}{\tau} \tag{20}$$

$$Ki = \frac{R}{\tau} \tag{21}$$

In this case, τ has been chosen as 0.01s and the same results with tuned Kp, Ki can be seen in Fig. 14. We can see that with the tuned parameters, the response of the signal is much better. Later, we changed the R and L values from 1 Ω, 10 mH to 5 Ω , 5 mH respectively. Moreover, we want to achieve the same behavior even with these values so Kp, Ki values will change according to R and L values.

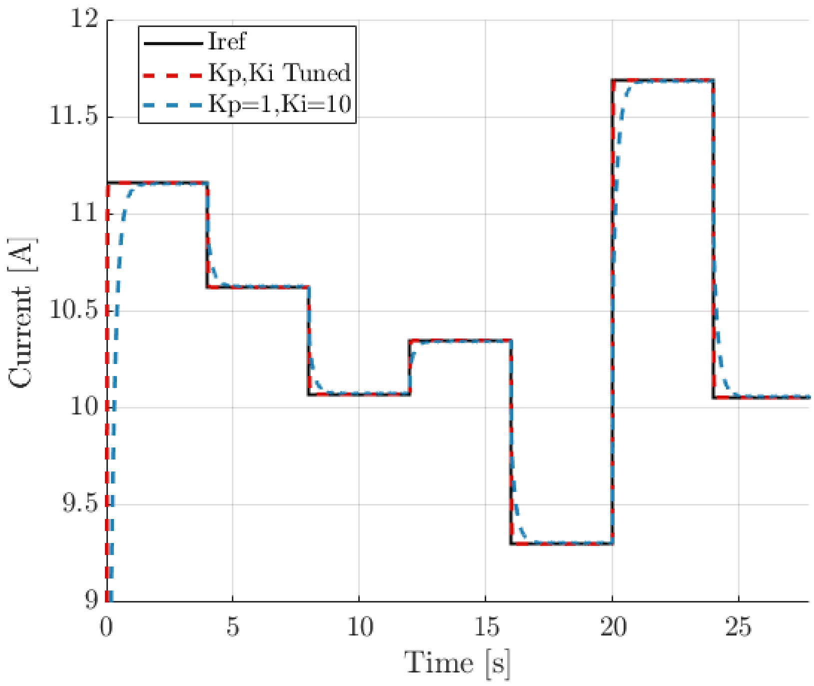

Figure 14: Tuned parameters vs Untuned

Fig. 15, again the comparison between linear state space and nonlinear model is represented with R, L values of 5 Ω, 5 mH respectively. We can see that the controller parameters are properly tuned and still we can achieve the same behavior as we wanted before.

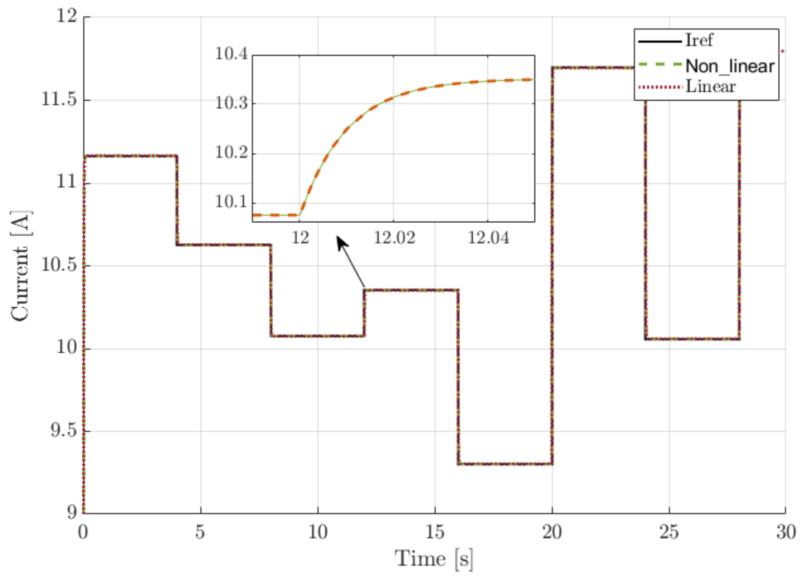

Figure 15: Tuned parameters vs Untuned

4 State space modeling of electrical circuits

4.1 Thevenin-RL line

In this section, an R load that is connected to a Thevenin grid via an RL line will be explained. First, the linear model and then the linear state space model of the system will be studied. Later, by plotting the current through the circuit i_{qd} it will be shown that the linear state-space models can capture the dynamics. Fig. 16 represents the scheme of the model both with individual elements and in aggregated form. Moreover, the Thevenin equivalent has an RL filter connected to an RL line and an active load represented as a resistance. The parameters of the model have been represented in Table 2 note that the values for RL line are chosen randomly and these values are used both in the linear and nonlinear model.

Table 2: Parameters of the system

Parameter	Value	Unit
S_{base}	2.75	MVA
Frequency	50	Hz
V_{base}	690	V
SCR	7	-
X/R	7	-
R_{line}	0.01595	pu/km
L_{line}	0.1721	pu/km
R_{load}	14.28	ohm

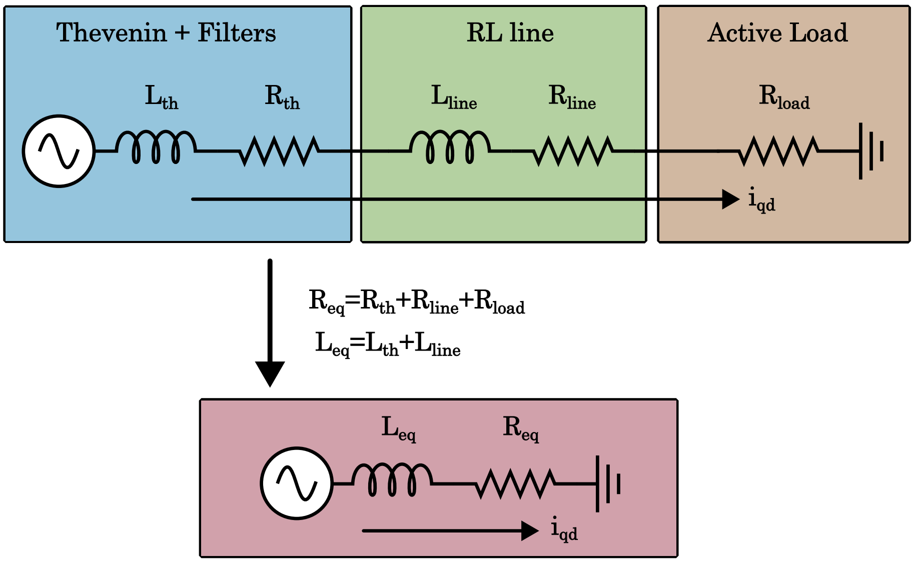

Figure 16: Thevenin RL line with an R load

Once the system parameters have been defined the circuit can be built and then the linear state-space model can be studied. In this case, by working with R_{eq} and L_{eq} the system will be the same as an RL circuit that is explained in Section 3.1. However, in this case, the model has been built in qd frame. Hence the equations are written as follows:

$$u_q^{th}(t) = R_{eq} i_q(t) + L_{eq} \frac{di_q(t)}{dt} + wL_{eq} i_d(t) + u_{0q}(t) \tag{22}$$

$$u_d^{th}(t) = R_{eq} i_d(t) + L_{eq} \frac{di_d(t)}{dt} - wL_{eq} i_q(t) + u_{0d}(t) \tag{23}$$

$$\tag{24}$$

since the load is connected to the ground u_0 is 0, and terms wLi_d and wLi_q occurred due to having an L filter. By substituting the derivatives of i_q and i_d the state space model can be built.

$$\frac{di_q(t)}{dt} = \frac{u_q^{th}(t)}{L_{eq}} - \frac{u_{0q}(t)}{L_{eq}} - \frac{R_{eq} i_q(t)}{L_{eq}} - wi_d(t) \tag{25}$$

$$\frac{di_d(t)}{dt} = \frac{u_d^{th}(t)}{L_{eq}} - \frac{u_{0d}(t)}{L_{eq}} - \frac{R_{eq} i_d(t)}{L_{eq}} - wi_q(t) \tag{26}$$

Now, 25 and 26 can be written in state-space representation.

$$\begin{bmatrix} \dot{i}_q \\ \dot{i}_d \end{bmatrix} = \begin{bmatrix} -\frac{R_{eq}}{L_{eq}} & -w \\ w & -\frac{R_{eq}}{L_{eq}} \end{bmatrix} \begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{eq}} & 0 & -\frac{1}{L_{eq}} & 0 \\ 0 & \frac{1}{L_{eq}} & 0 & -\frac{1}{L_{eq}} \end{bmatrix} \begin{bmatrix} u_q^{th} \\ u_d^{th} \\ u_{0q} \\ u_{0d} \end{bmatrix} \tag{27}$$

$$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_q^{th} \\ u_d^{th} \\ u_{0q} \\ u_{0d} \end{bmatrix} \tag{28}$$

Now, by using 25 and 26 the state space matrices can be written as 27 and 28. In this case, the outputs of the system are i_q and i_d so the C matrix has been identified as a unit matrix. Note that, C and D matrices can be rewritten in order to choose the outputs of the system.

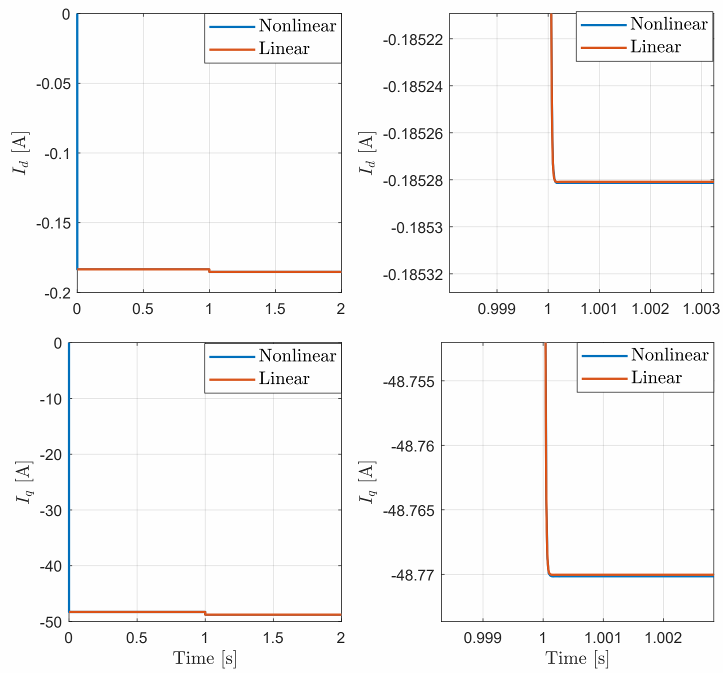

Figure 17: i_q and i_d with 1 % voltage increase in the Thevenin

Fig. 17 the qd components of the current is shown with the magnified graph on the right half plane. In this case, the Thevenin voltage has been increased by 1 % at 1 s and the response of the current has been observed. It is clear that both linear and nonlinear models show a first-order response and both signals are matching.

4.2 Thevenin PI line

In this section, an R load that is connected to a Thevenin equivalent via a PI line will be analyzed. Moreover, the active power and qd components of the load's current and voltage will be observed with 1 % of voltage increase in the Thevenin and in the R load. The scheme of the model can be observed in Fig 18.

PI section line consists of 2 shunt capacitor that are C_{1line} and C_{2line} and a series connected R_{line} and L_{line}. Note that, u_{1q} and u_{1d} are voltages that are across the C_{1line} capacitor. Moreover, the voltages that are denoted as u_{2q} and u_{2d} are the voltages across the C_{2line} capacitor. Table 3 shows the PI section

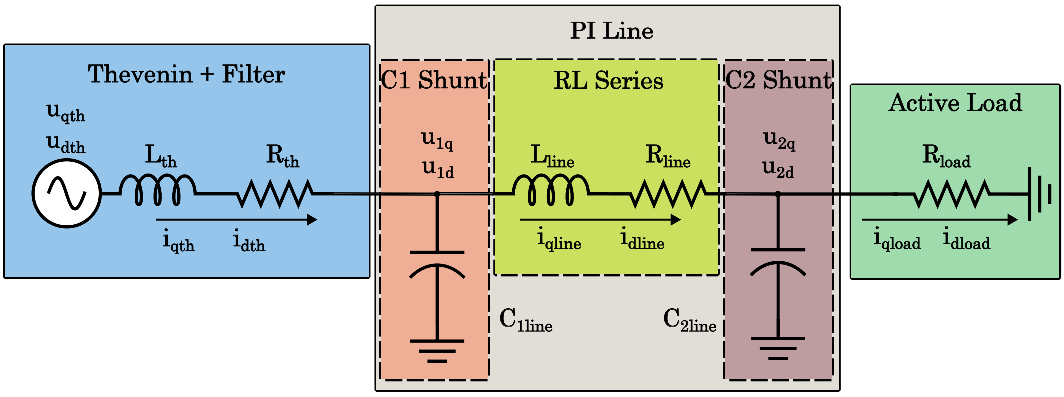

Figure 18: Thevenin PI line connected to an R load

Table 3: Parameters of the PI section line

Parameter	Value	Unit
R_{line}	0.0735	pu/km
L_{line}	0.0054	pu/km
C_{line}	2.2e-9	pu/km

line parameters and the rest of the system has the same values as Table 2. Note that, in this case, the line is 100 km, and again those values have been chosen randomly.

Once the system has been defined, the nonlinear equations can be written for each part in order to build the linear model.

- **Thevenin+ Filter**

  This block can be represented as an RL circuit however, in this case inputs are the voltages across the capacitor C_{1line}.

$$u_q^{th}(t) = R^{th} i_q^{th}(t) + L^{th} \frac{di_q^{th}(t)}{dt} + u_{1q}(t) + wL^{th} i_d^{th}(t) \tag{29}$$

$$u_d^{th}(t) = R^{th} i_d^{th}(t) + L^{th} \frac{di_d^{th}(t)}{dt} + u_{1d}(t) - wL^{th} i_q^{th}(t) \tag{30}$$

Then substituting $\frac{dI_q^{th}(t)}{dt}$ and $\frac{dI_d^{th}(t)}{dt}$ from 29 and 30 respectively 31 and 32 can be achieved.

$$\frac{di_q^{th}(t)}{dt} = \frac{u_q^{th}(t)}{L} - \frac{u_{1q}^{th}(t)}{L} - \frac{R^{th} i_q^{th}(t)}{L} - wi_d^{th}(t) \tag{31}$$

$$\frac{di_d^{th}(t)}{dt} = \frac{u_d^{th}(t)}{L} - \frac{u_{1d}^{th}(t)}{L} - \frac{R^{th} i_d^{th}(t)}{L} + wi_q^{th}(t) \tag{32}$$

Now by using 31 and 32, the Thevenin+Filter block can be represented in state-space as 55 and 56.

$$\begin{bmatrix} \dot{i}_q^{th} \\ \dot{i}_d^{th} \end{bmatrix} = \begin{bmatrix} -\frac{R^{th}}{L^{th}} & -w \\ w & -\frac{R^{th}}{L^{th}} \end{bmatrix} \begin{bmatrix} i_q^{th} \\ i_d^{th} \end{bmatrix} + \begin{bmatrix} \frac{1}{L^{th}} & 0 & -\frac{1}{L^{th}} & 0 \\ 0 & \frac{1}{L^{th}} & 0 & -\frac{1}{L^{th}} \end{bmatrix} \begin{bmatrix} u_q^{th} \\ u_d^{th} \\ u_{1q} \\ u_{1d} \end{bmatrix} \tag{33}$$

$$\begin{bmatrix} i_q^{th} \\ i_d^{th} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_q^{th} \\ i_d^{th} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_q^{th} \\ u_d^{th} \\ u_{1q} \\ u_{1d} \end{bmatrix} \tag{34}$$

In the system, there are 2 state variables which are i_q^{th} and i_d^{th} and 4 inputs that are Thevenin voltage in qd domain u_q^{th} and u_d^{th} and voltage across the C_{1line} capacitor u_{1q} and u_{1d}. Since the Thevenin+Filters block is connected to a shunt capacitor C_{1line}, the output of the system should be the state variables (i_q^{th} and i_d^{th}) because the C shunt block requires a current as an input of its system.

- **C1 shunt**

  Now the equations for the first shunt capacitor C_{1line} of the PI section line can be written as

$$i_q^{th}(t) = C_{1line}\frac{du_{1q}(t)}{dt} + wC_{1line}u_{d1}(t) + i_q^{RL}(t) \tag{35}$$

$$i_d^{th}(t) = C_{1line}\frac{du_{1d}(t)}{dt} - wC_{1line}u_{q1}(t) + i_d^{RL}(t) \tag{36}$$

After that, isolating $\frac{du_{1q}(t)}{dt}$ and $\frac{du_{1d}(t)}{dt}$ from 35 and 36 respectively, 37 and 38 can be written as follow

$$\frac{du_{1q}(t)}{dt} = \frac{i_q^{th}(t)}{C_{1line}} - wu_{d1}(t) - \frac{iq^{RL}(t)}{C_{1line}} \tag{37}$$

$$\frac{du_{1d}(t)}{dt} = \frac{i_d^{th}(t)}{C_{1line}} + wu_{q1}(t) - \frac{i_d^{RL}(t)}{C_{1line}} \tag{38}$$

Now C1 shunt block can be represented in state space by using 37 and 38.

$$\begin{bmatrix} \dot{u_{q1}} \\ \dot{u_{d1}} \end{bmatrix} = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix} \begin{bmatrix} u_{q1} \\ u_{d1} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{1line}} & 0 & -\frac{1}{C_{1line}} & 0 \\ 0 & \frac{1}{C_{1line}} & 0 & -\frac{1}{C_{1line}} \end{bmatrix} \begin{bmatrix} i_q^{th} \\ i_d^{th} \\ i_q^{line} \\ i_d^{line} \end{bmatrix} \tag{39}$$

$$\begin{bmatrix} u_{q1} \\ u_{d1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{q1} \\ u_{d1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_q^{th} \\ i_d^{th} \\ i_q^{line} \\ i_d^{line} \end{bmatrix} \tag{40}$$

In this case, the state variables are u_{q1} and u_{d1}. Moreover, the output of Thevenin+Filters block (i_q^{th} and i_d^{th}) are the first 2 inputs also, the qd components of the current through the series RL line i_q^{line} and i_d^{line} are the remaining inputs. As this C1 shunt block is connected to a series RL, the output of the C1 shunt block should be u_{q1} and u_{d1}.

- **RL series**

This block is basically, the same as the Thevenin+Filters block, however, the state variables and the inputs of the system are different. Again, the equations can be written as

$$u_{q1}(t) = R^{line} i_q^{line}(t) + L^{line} \frac{di_q^{line}(t)}{dt} + u_{2q}(t) + wL^{line} i_d^{line}(t) \tag{41}$$

$$u_{d1}(t) = R^{line} i_d^{line}(t) + L^{line} \frac{di_d^{line}(t)}{dt} + u_{2d}(t) + wL^{line} i_q^{line}(t) \tag{42}$$

Again substituting $\frac{di_q^{line}(t)}{dt}$ and $\frac{di_d^{line}(t)}{dt}$ 43 and 44 can be achieved.

$$\frac{di_q^{line}(t)}{dt} = \frac{u_{q1}(t)}{L} - \frac{u_{d2}^{th}(t)}{L} - \frac{R^{line} i_q^{line}(t)}{L} - wi_d^{line}(t) \tag{43}$$

$$\frac{di_d^{line}(t)}{dt} = \frac{u_{d1}(t)}{L} - \frac{u_{d2}^{th}(t)}{L} - \frac{R^{line} i_d^{line}(t)}{L} - wi_q^{line}(t) \tag{44}$$

Now, 43 and 44 can be represented in state space.

$$\begin{bmatrix} \dot{i}_q^{line} \\ \dot{i}_d^{line} \end{bmatrix} = \begin{bmatrix} -\frac{R^{line}}{L^{line}} & -w \\ w & -\frac{R^{line}}{L^{line}} \end{bmatrix} \begin{bmatrix} i_q^{line} \\ i_d^{line} \end{bmatrix} + \begin{bmatrix} \frac{1}{L^{line}} & 0 & -\frac{1}{L^{line}} & 0 \\ 0 & \frac{1}{L^{line}} & 0 & -\frac{1}{L^{line}} \end{bmatrix} \begin{bmatrix} u_{1q} \\ u_{1d} \\ u_{2q} \\ u_{2d} \end{bmatrix} \tag{45}$$

$$\begin{bmatrix} i_q^{line} \\ i_d^{line} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_q^{line} \\ i_d^{line} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{1q} \\ u_{1d} \\ u_{2q} \\ u_{2d} \end{bmatrix} \tag{46}$$

In this block, i_q^{line} and i_d^{line} are the state variables. Moreover, the system inputs are u_{1q}, u_{1d}, u_{2d} and u_{2d}. Note that, u_{2d} and u_{2d} are the outputs of the C2 shunt block where the RL series block is connected. Also, the outputs of the RL series block are i_q^{line} and i_d^{line} because the C2 shunt block requires a current input.

- **C2 shunt**

This block is the same as the C1 shunt block however the inputs are different. Again, the equations can be written as

$$i_q^{line}(t) = C_{2line} \frac{du_{2q}(t)}{dt} + wC_{2line} u_{d2}(t) + i_q^{line}(t) \tag{47}$$

$$i_d^{line}(t) = C_{2line} \frac{du_{2d}(t)}{dt} - wC_{2line} u_{q2}(t) + i_d^{line}(t) \tag{48}$$

By isolating $\frac{du_{2q}(t)}{dt}$ and $\frac{du_{2d}(t)}{dt}$ 49 and 50 can be achieved.

$$\frac{du_{2q}(t)}{dt} = \frac{i_q^{line}(t)}{C_{2line}} - wu_{d2}(t) - \frac{iq^{load}(t)}{C_{2line}} \tag{49}$$

$$\frac{du_{2d}(t)}{dt} = \frac{i_d^{line}(t)}{C_{2line}} + wu_{q2}(t) - \frac{id^{load}(t)}{C_{2line}} \tag{50}$$

Once 49 and 50 have been found the state space representation can be done.

$$\begin{bmatrix} \dot{u_{q2}} \\ \dot{u_{d2}} \end{bmatrix} = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix} \begin{bmatrix} u_{q2} \\ u_{d2} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{2line}} & 0 & -\frac{1}{C_{2line}} & 0 \\ 0 & \frac{1}{C_{2line}} & 0 & -\frac{1}{C_{2line}} \end{bmatrix} \begin{bmatrix} i_q^{line} \\ i_d^{line} \\ i_q^{load} \\ i_d^{load} \end{bmatrix} \tag{51}$$

$$\begin{bmatrix} u_{q2} \\ u_{d2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{q2} \\ u_{d2} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_q^{line} \\ i_d^{line} \\ i_q^{load} \\ i_d^{load} \end{bmatrix} \tag{52}$$

In this block the state variables are u_{q2} and u_{d2} and the inputs are i_q^{line}, i_d^{line}, i_q^{load} and i_d^{load}. Moreover, i_q^{load} and i_d^{load} are the outputs of the Active Load block. Also, the outputs of the C2 shunt block are u_{q2} and u_{d2} since the Active Load block requires those parameters as input.

- **Active Load**
  This block is the last part of the system and it has only resistance as an active load. The equation of the block can be written as follows

$$u_{q2}(t) = R_{load} i_q^{load}(t) \tag{53}$$

$$u_{d2}(t) = R_{load} i_d^{load}(t) \tag{54}$$

In this case, the state space representation can be found by using 53 and 54.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{q2} \\ u_{d2} \\ 0 \\ 0 \end{bmatrix} \tag{55}$$

$$\begin{bmatrix} i_q^{load} \\ i_d^{load} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{R_{load}} & 0 & 0 & 0 \\ 0 & \frac{1}{R_{load}} & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{q2} \\ u_{d2} \\ 0 \\ 0 \end{bmatrix} \tag{56}$$

In this case, the R load block doesn't have state variables since there are no derivatives or integrals. Moreover, there are 4 inputs but as the load is connected to the ground the voltage is 0 and the only inputs are u_{q2} and u_{d2}. Also, the output is the current i_q^{load} and i_d^{load}.

Figure 19: State Space model of each block of the system

Fig. 19 represents the state space model of each block that is explained and shows how blocks create the overall system connection. Note that, this is one way of representing the overall system, these blocks can be aggregated and built the system which will be explained in the subsequent parts.

These blocks should be modeled (creating the matrices) individually in Matlab after that, there two ways of putting those blocks into Simulink. First, as explained putting each block separately by connecting the related inputs and outputs. The second option is to use the "connect" function in Matlab. The usage of the function can be observed as follows,

```
input={'Uq_TH','Ud_TH'}
output={'Iq_load' 'Id_load'}
 pi_overall=connect(TH_filters,shunt_C1,rl_line ,shunt_C2,rl_load ,input,output)
```

In order to use such a function, the string characters of each input and output should be written correctly. For instance, if a block has an output name of "$block1$" and if this output will be used in another block, exactly the same name has to be written as input. On the other hand, the "connect" function cannot be used. Once the overall system is created by using the "connect" function, A, B, C, and D matrices can be called directly to the Simulink environment. Furthermore, the aggregated matrix can be written as

$$\begin{bmatrix} \dot{i}_q^{th} \\ \dot{i}_d^{th} \\ \dot{u}_{q1} \\ \dot{u}_{d1} \\ \dot{i}_q^{line} \\ \dot{i}_d^{line} \\ \dot{u}_{q2} \\ \dot{u}_{d2} \end{bmatrix} = \begin{bmatrix} -\frac{R^{th}}{L^{th}} & -w & -\frac{1}{L^{th}} & 0 & 0 & 0 & 0 & 0 \\ w & -\frac{R^{th}}{L^{th}} & 0 & -\frac{1}{L^{th}} & 0 & 0 & 0 & 0 \\ \frac{1}{C_{1line}} & 0 & 0 & -w & -\frac{1}{C_{1line}} & 0 & 0 & 0 \\ 0 & \frac{1}{C_{1line}} & w & 0 & 0 & -\frac{1}{C_{1line}} & 0 & 0 \\ 0 & 0 & \frac{1}{L^{line}} & 0 & -\frac{R^{line}}{L^{line}} & -w & -\frac{1}{L^{line}} & 0 \\ 0 & 0 & 0 & \frac{1}{L^{line}} & w & -\frac{R^{line}}{L^{line}} & 0 & -\frac{1}{L^{line}} \\ 0 & 0 & 0 & 0 & \frac{1}{C_{2line}} & 0 & -\frac{1}{C_{2line}R_{load}} & -w \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_{2line}} & w & -\frac{1}{C_{2line}R_{load}} \end{bmatrix} \begin{bmatrix} i_q^{th} \\ i_d^{th} \\ u_{q1} \\ u_{d1} \\ i_q^{line} \\ i_d^{line} \\ u_{q2} \\ u_{d2} \end{bmatrix} + \tag{57}$$

$$\begin{bmatrix} \frac{1}{L^{line}} & 0 \\ 0 & \frac{1}{L^{line}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_q^{th} \\ u_d^{th} \end{bmatrix} \tag{58}$$

$$\begin{bmatrix} i_q^{load} \\ i_d^{load} \\ u_{q2} \\ u_{d2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_{load}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_{load}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_q^{th} \\ i_d^{th} \\ u_{q1} \\ u_{d1} \\ i_q^{line} \\ i_d^{line} \\ u_{q2} \\ u_{d2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_q^{th} \\ u_d^{th} \end{bmatrix} \tag{59}$$

In the aggregated form, there are 8 state variables and 2 inputs and 2 outputs. Until now, a description of the system, building the matrices from the equations, and putting all matrices together to create an overall system has been explained. Moreover, the results of i_q^{load}, i_d^{load}, u_{q2}, u_{d2} and the active power will be shown when increasing the Thevenin voltage by 1 %. Both linear models with blocks and the overall model will be compared with the non-linear one in order to validate the linear models. Note that, depending on the parameters to be observed, the output of the system can be decided by arranging the C matrix.

4.3 Voltage increase in the Thevenin equivalent

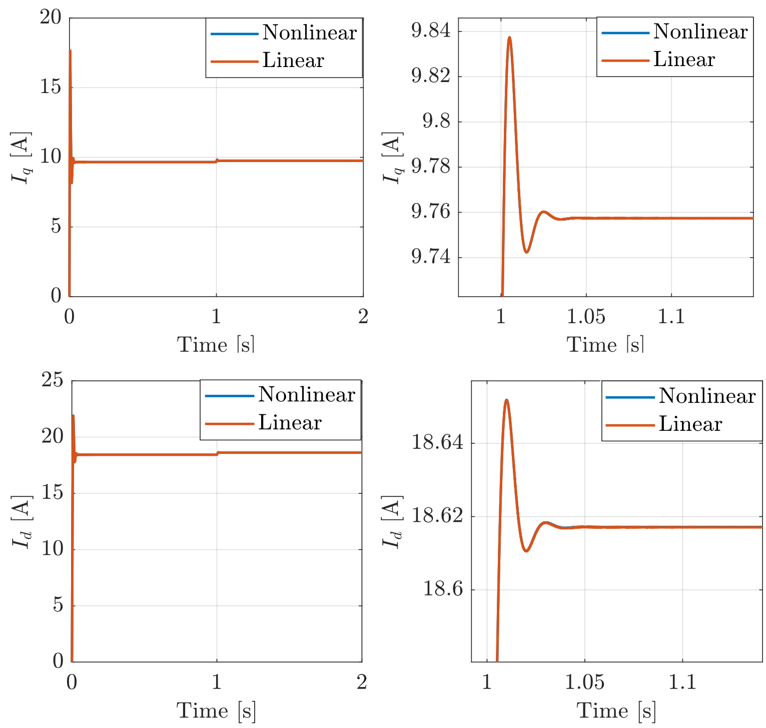

Figure 20: i_q^{load} and i_d^{load} Nonlinear vs Linear (Block by block)

Figures 20 and 21 represent the currents that go to the load (u_{q2}, u_{d2}) and the output voltage of the C2 shunt (u_{q2}, u_{d2}). The 1 % disturbance has been applied at 1 s and both currents and voltages react by increasing their value. In this case, a second-order response can be seen due to capacitance.

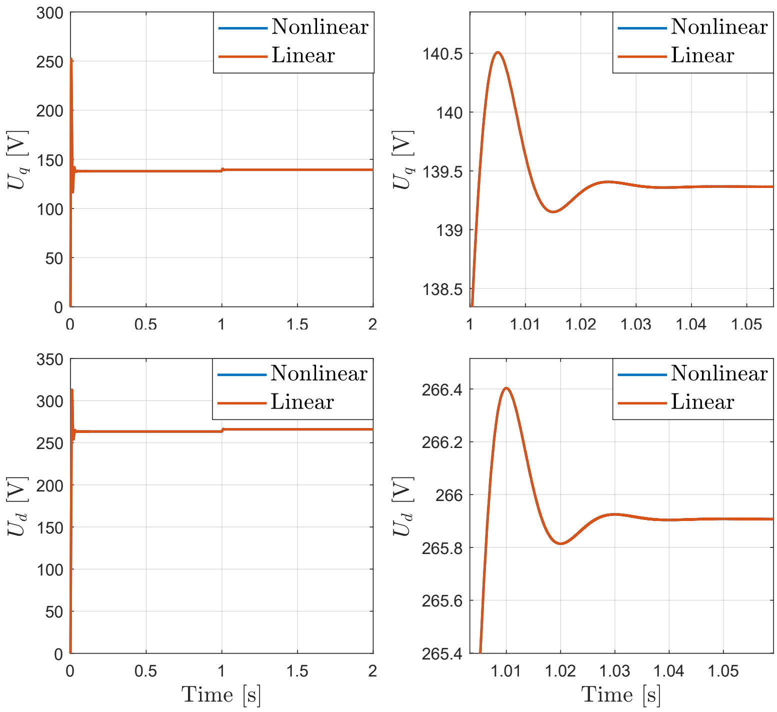

Figure 21: u_{q2}, u_{d2} Nonlinear vs Linear (Block by block)

It is clear that the dynamic response and the steady state of both signals are overlapping indicating that the linear model is valid. In Fig. 22 the response of the active power can be seen. It is clear that,

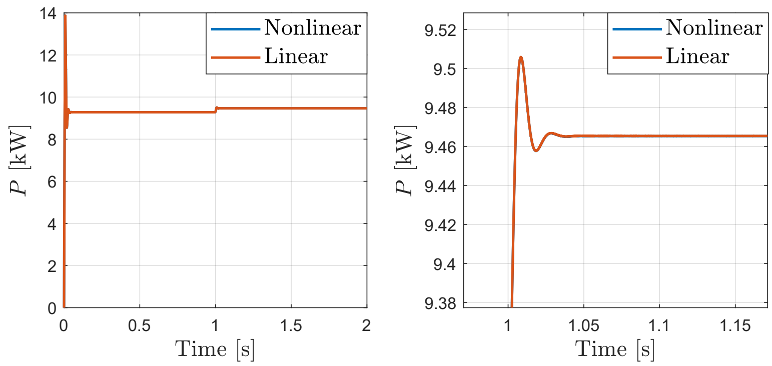

Figure 22: Active power Nonlinear vs Linear (Block by block)

the linear and the nonlinear model are in close agreement. Now, the same results with the aggregated linear model will be shown.

u_{q2}, u_{d2} and u_{q2}, u_{d2} have been shown in Figures 23 and 24 respectively. Again it can be seen, both models show the same response and reach steady-state.
In Fig. 25 the response of the active power can be seen. Again, both signals are overlapping during the transient and in the steady state. Hence, the linear model can be built both block by block or by aggregating such blocks. So, the results of the active load increment will be shown in aggregated

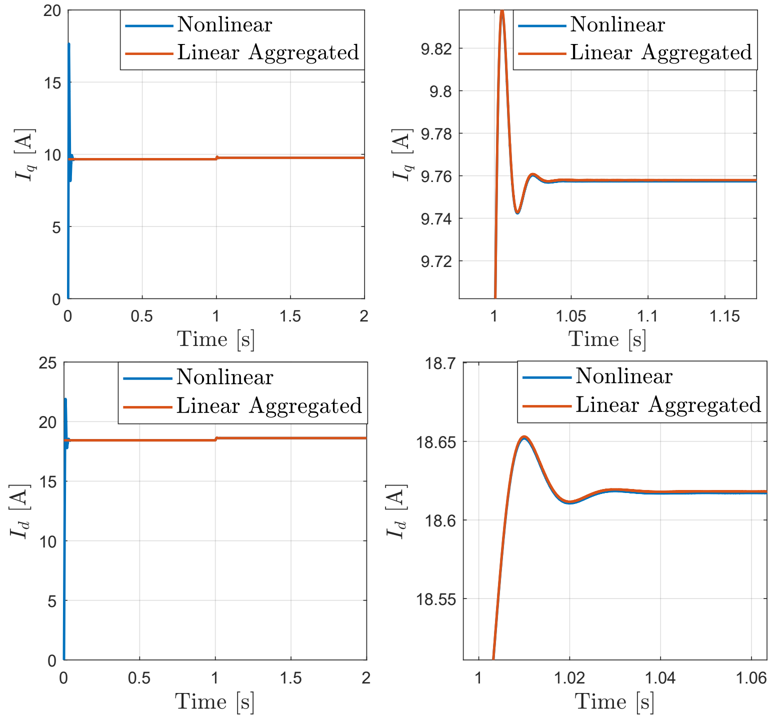

Figure 23: i_q^{load} and i_d^{load} Nonlinear vs Linear (Aggregated)

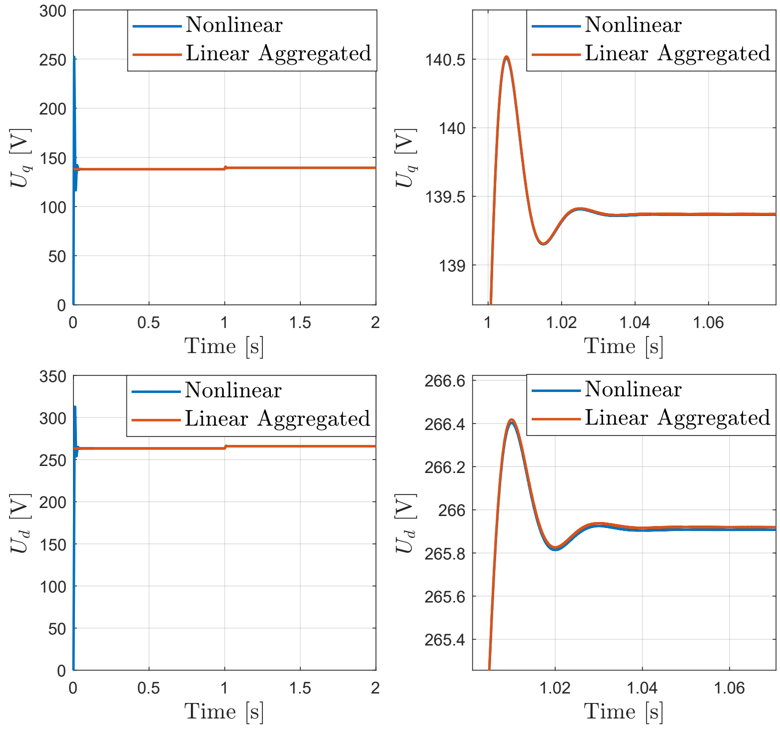

Figure 24: u_{q2}, u_{d2} Nonlinear vs Linear (Aggregated)

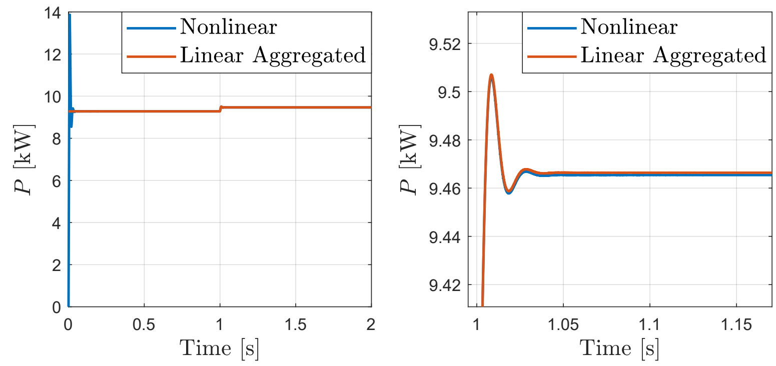

Figure 25: Active power Nonlinear vs Linear (Aggregated)

form.

4.4 Active load increase

In this case, the active load has been incremented 1 % then U_{q2}, U_{d2} which are the outputs of C2 shunt block, I_q^{load} and I_d^{load} that are the output of Active load block and finally the active power have been observed. Until now only the Thevenin voltage has been disturbed so in order to disturb the active load (R load) in the linear model

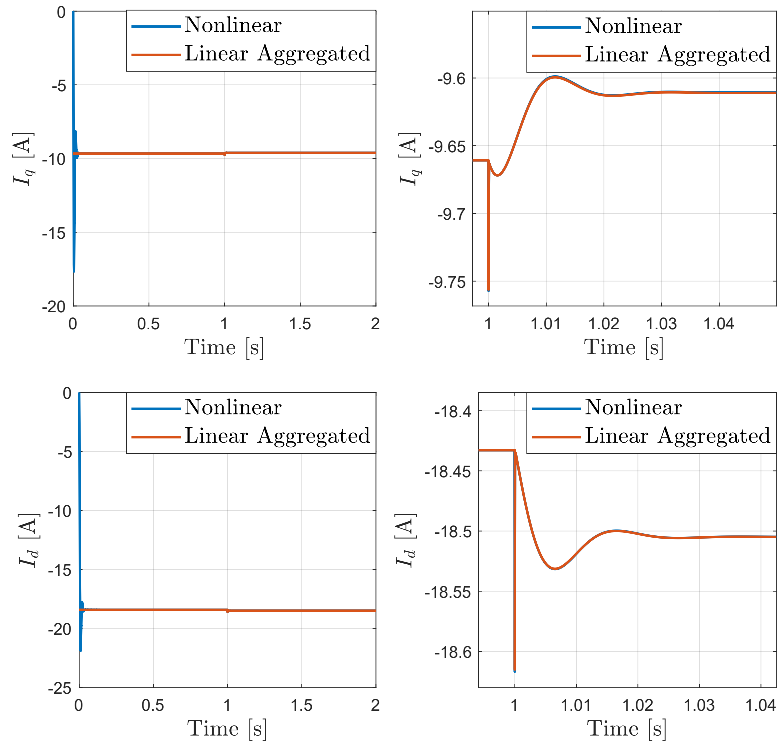

Figure 26: i_q^{load} and i_d^{load} Nonlinear vs Linear active load increase 1 %

Figures 26 and 27 represents the i_q^{load} and i_d^{load} and u_{q2}, u_{d2} respectively. Again it can be seen that the

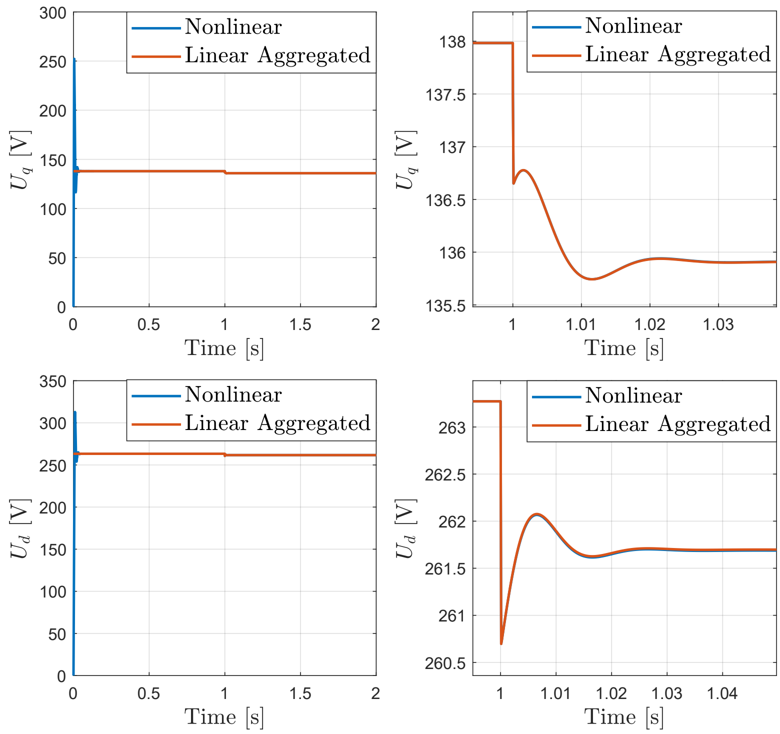

Figure 27: u_{q2}, u_{d2} Nonlinear vs Linear active load increase 1 %

linear and the nonlinear models are matching both in terms of dynamic and steady state.

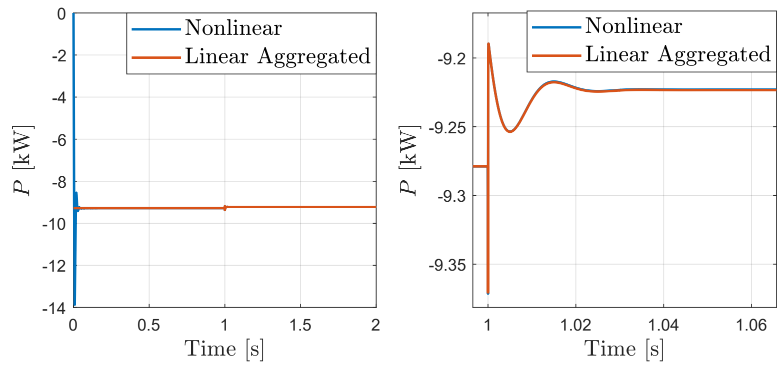

Figure 28: Active power Nonlinear vs Linear active load increase 1 %

Fig. 28 shows the behavior of the active power when active power increases by 1 %. It can be seen that both models are in close agreement.

5 VSC

In this section 2 level Voltage Source Converter (VSC) will be explained in both grid following (GFOL) and grid forming (GFOR) modes. Each section will have 3 parts: 1) Complete model (non-linear) 2) Linearization and state-space 3) Linear analysis. First, a GFOL converter that is connected to the

Thevenin grid via a PI section line will be studied and then, the same scenario will be performed with a GFOR converter. Both scenarios will face various tests and for each case eigenvalues and participation factors will be studied.

5.1 Linearization

Until now, we have not linearized any model yet since we were working on RLC circuits and PI controllers. So, it is crucial to define what are the linear elements.

1. Voltage and Current sources
2. Passive elements such as resistors, inductors, and capacitors
3. Transfer functions such as PI controller and resonant controller
4. Addition and subtraction operations and multiplications by a constant

However, there are several nonlinearities in power systems, for instance, Phase Locked Loop (PLL due to the Park transformation) or power calculations have to be linearized. Normally, the linearization concept is finding an approximation to a function at a given point. Such approximation of a function , in general, can be done by applying the Taylor expansion which can be represented as follows,

$$f(x,y) \approx f(a,b) + \frac{\partial f(x,y)}{\partial x}\Big|_{x,u}(x-a) + \frac{\partial f(x,y)}{\partial u}\Big|_{x,u}(x-b) \tag{60}$$

where $f(a,b)$ is the operating point and $f(x,y)$ is the nonlinear function. Moreover, the calculation of the power 61 is a nonlinear term hence it has to be linearized.

$$P = \frac{3}{2}(V_q I_q + V_d I_d) \tag{61}$$

In order to linearize 61, first we perturb the system and get

$$f(x,u) = \frac{\partial (P+\Delta P)(t)}{\partial t} = \frac{\partial P(t)}{\partial t} + \frac{\partial \Delta P(t)}{\partial t} \tag{62}$$

Then by applying the Taylor we get

$$\frac{\partial (P+\Delta P)(t)}{\partial t} = \underbrace{\frac{3}{2}((Uq_0 + \Delta Uq)(Iq_0 + \Delta Iq) + (Ud_0 + \Delta Ud)(Id_0 + \Delta Id))}_{\text{A}} \tag{63}$$

Let's call such equation A. Moreover, subscript 0 stands for the linearization point, and Δ is the disturbance hence by expanding A we can get

$$\frac{3}{2}(Uq_0 Iq_0 + Uq_0 \Delta Iq + \Delta Uq Iq_0 + \underbrace{\Delta Uq \Delta Iq}_{\text{High Order}} + Ud_0 Id_0 + Ud_0 \Delta Id + \Delta Ud Id_0 + \underbrace{\Delta Ud \Delta Id}_{\text{High Order}}) \tag{64}$$

So, by eliminating the high order terms and taking the first derivate with respect to inputs which in case they are ΔIq, ΔId, ΔUq and ΔUd we can get

$$P_0 + \Delta P = \frac{3}{2}(Uq_0 Iq_0 + Ud_0 Id_0) + \frac{3}{2}(Uq_0 \Delta Iq + Iq_0 \Delta Uq + Ud_0 \Delta Id + Id_0 \Delta Ud) \tag{65}$$

Now 61 is linearized. Such methodology can be applied for performing the network or VSC power by changing the parameters.**It is also important that in the linearized state space model, we work with incremental values. The rest of the document is not represented with 'Δ' but we are incremental.** Moreover in order to validate the model sometimes an active load disturbance that is around 1% can be applied to the system. In this case, the variable resistive load will be a nonlinear equation so, for the active load disturbance case, the equation should be linearized.

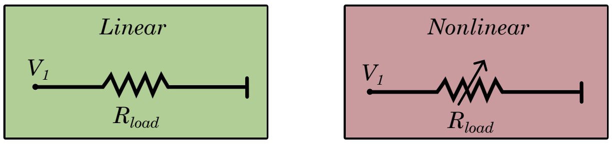

Figure 29: Active load disturbance

Fig. 29 represents the active load disturbance. Hence we can write the equation as follow

$$V_{1q}(t) = i_q(t) * R_{load}(t) \tag{66}$$

$$V_{1d}(t) = i_d(t) * R_{load}(t) \tag{67}$$

The currents i_q and i_d can be observed so by isolating such parameters we achieve

$$i_q(t) = \frac{V_{1q}(t)}{R_{load}(t)} \tag{68}$$

$$i_d(t) = \frac{V_{1d}(t)}{R_{load}(t)} \tag{69}$$

Then by perturbing 68 and 69 and taking the first derivative with respect to inputs that are ΔR_{load}, ΔU_q and ΔU_d we get

$$\Delta i_q = \frac{-V_{q0}}{R_0^2}\Delta R_{load} + \frac{\Delta V_q}{R_0} \tag{70}$$

$$\Delta i_d = \frac{-V_{d0}}{R_0^2}\Delta R_{load} + \frac{\Delta V_d}{R_0} \tag{71}$$

then representing the above equations in state space we get

$$\begin{bmatrix} \Delta i_q \\ \Delta i_d \end{bmatrix} = \begin{bmatrix} \frac{1}{R_0} & 0 & \frac{-V_{q0}}{R_0^2} \\ 0 & \frac{1}{R_0} & \frac{-V_{q0}}{R_0^2} \end{bmatrix} \begin{bmatrix} \Delta U_q \\ \Delta U_d \\ \Delta R_{load0} \end{bmatrix} \tag{72}$$

Furthermore, 72 represents the linearized state space.

5.2 2L-VSC in GFOL mode complete model

There are two operational modes of VSC which are GFOL and GFOR modes. In this section, a VSC in GFOL mode that is connected to a Thevenin grid through a PI line will be studied and analyzed. First,

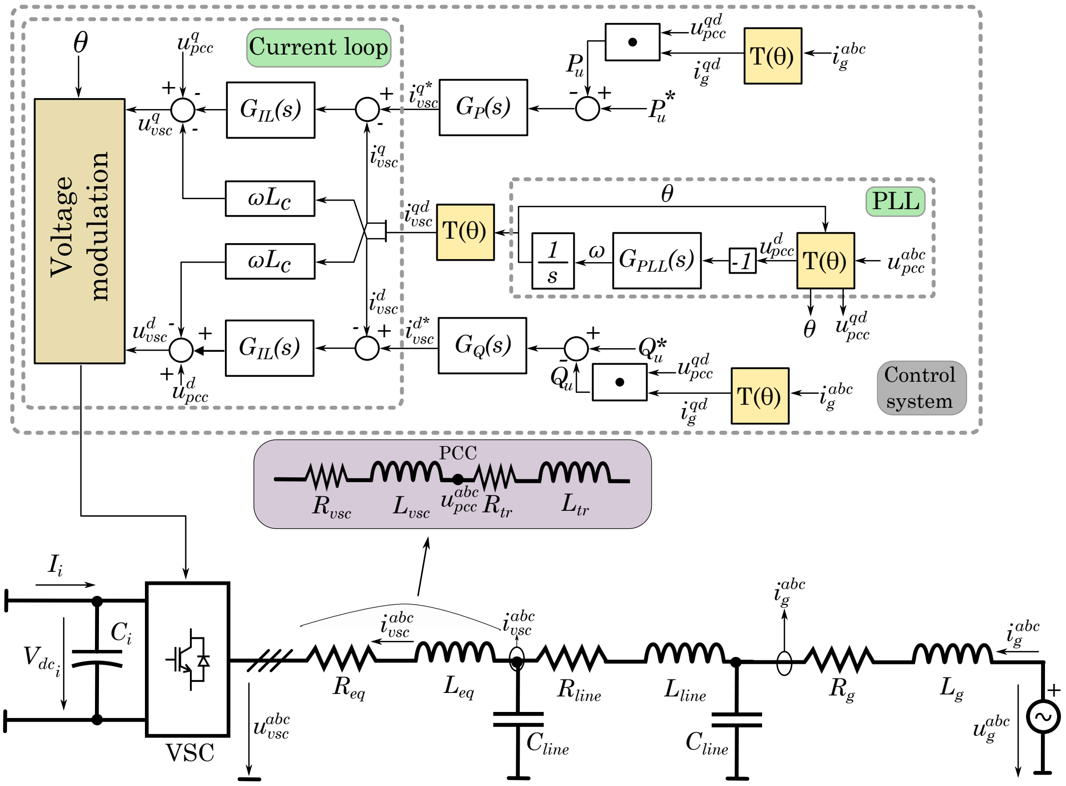

Figure 30: Complete model of a VSC

a GFOL converter with P, Q control with current control will be studied. Then voltage and frequency supports will be added to the system.

Fig. 30 represents the complete model of the VSC operating as GFOL. The converter is connected to a Thevenin grid through a PI line and series RL components that are represented as R_{eq} and L_{eq}. Such components consist of a converter filter R_{vsc} and L_{vsc} and a transformer that is represented as series R_{tr} and L_{tr}. Note that. the Point of Common Coupling (PCC) is between the converter filter and transformer, and this is where the Phase Locked Loop (PLL) connects and measured the voltage u_{pcc}^{abc}. Moreover, the current references i_{vsc}^{q*} and i_{vsc}^{d*} for the current control loop are provided by Active and Reactive power control respectively. By using such references, the current loop provides the voltage for the converter. Also, the blocks that have "G" denotes that is the control block and in this case, such blocks are PI controllers.

5.2.1 Phase Locked Loop (PLL)

GFOL converters use PLL in order to be synchronized with the grid. Moreover, such a component consists of a PI controller, Park transformation, and an integrator. The output of the system provides the angle and such angle is fed again to Park transformation so that it creates a loop. Also, the output of PLL, θ_{PLL}, is used for internal control transformations.

In Fig. 31 the block diagram of the PLL is represented. The input of the system is the voltage that is measured at the PCC (u_{pcc}^{abc}) and the outputs are the angle PLL (θ_{PLL}) and the voltage (u^{qd}). More-

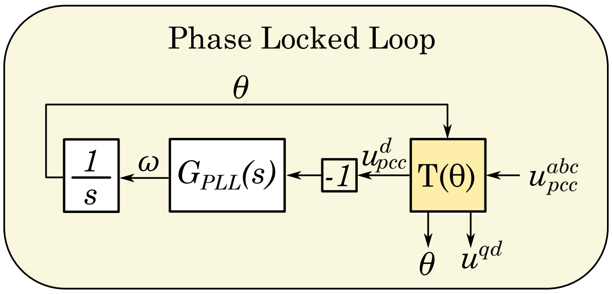

Figure 31: Phase Locked Loop diagram

over, G_{PLL} is the control block of the system and it consists of a proportional and an integral controller (PI). In this case, due to the Park transformation, the system is nonlinear hence it should be linearized.

5.2.2 Active and Reactive Power Control

In this part Active (P) and Reactive (Q) power control blocks will be explained. These blocks provide the reference currents for the current control. The voltage that is obtained by the PLL u^{qd} and the converter current i^{abc}_{vsc} are used to calculate the P and Q generation of the VSC. Moreover, each control blocks consist of PI controllers that are giving the reference currents in qd domain.

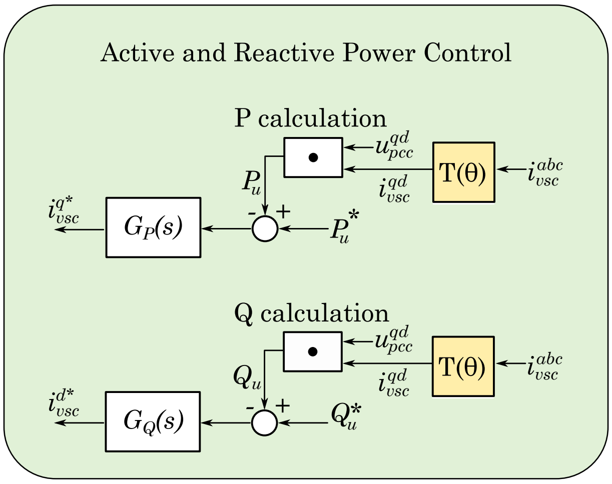

Figure 32: Active and Reactive Power Control Blocks

Fig. 32 represents the block diagram of P and Q control. The P and Q calculation can be represented as follows

$$P = u^q_{pcc} * i^q_{vsc} + u^d_{pcc} * i^d_{vsc} \tag{73}$$

$$Q = u^q_{pcc} * i^d_{vsc} - u^d_{pcc} * i^q_{vsc} \tag{74}$$

As explained in Section 5.1, equations 73 and 74 are nonlinear hence such equations have to be linearized. Also, the current of the converter measured in abc frame has to be converted into qd frame with Park Transformation with the θ_{PLL} so, this block should also be linearized.

5.2.3 Current Control

Once the current references have been received from the P Q control, by using the measured currents of the VSC (i^q_{vsc} i^d_{vsc}) we can control the current and provide the proper voltage modulation to the average VSC model. This block consists of PI controllers both for q and d terms.

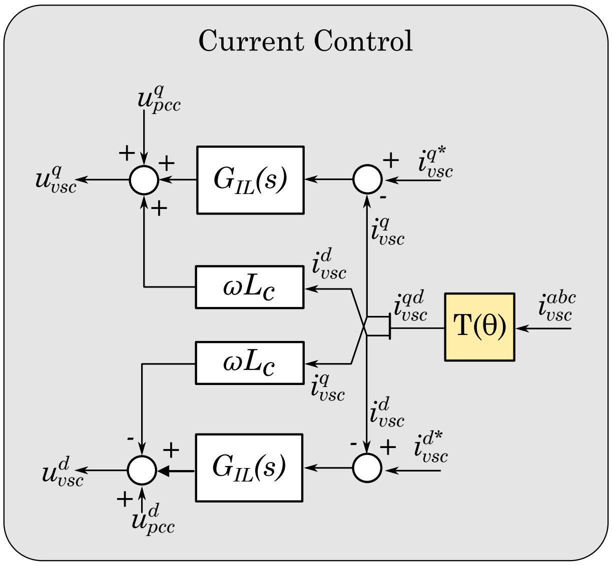

Figure 33: Current Control Diagram

Fig. 33 shows the block diagram of the current control. Note that, the term wL_c which is the decoupling term is fed forward with i^{qd}_{vsc}. Moreover, the u^{qd}_{pcc} is also fed forward after the PI controllers. Finally, the u^{qd}_{vsc} is reached and with the inverse Park Transformation such value is fed to the VSC. Note that, in this block, there is no nonlinearity.

5.2.4 Transformation and rotational matrices

In converter control, for sake of simplicity, the parameters are represented in qd0 reference with Park Transformation and such transformation can be seen as follows

$$\frac{2}{3}\begin{bmatrix} cos(\Theta) & cos(\Theta-\frac{2\pi}{3}) & cos(\Theta+\frac{2\pi}{3}) \\ sin(\Theta) & sin(\Theta-\frac{2\pi}{3}) & sin(\Theta+\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{75}$$

Moerover, 75 represents the Park transformation and this block is only used in the nonlinear model. In the linear state space model, since we work with qd values, this block is not needed. However, in such models, rotational matrices are used to be able to switch between the references. In this case, it is important to split the system into two reference frames. First, qd control reference is denoted as qdc, and the second is the qd grid reference. These two reference frames have an angle between them and such angle is θ_e and can be found by

$$\theta_e = \theta_{PLL} - \theta_{grid} \tag{76}$$

where θ_{PLL} is the output of PLL and θ_{grid} is the grid angle. GFOL converter uses a PLL in order to synchronize with the grid and such a component is connected to the Point of Common Coupling (PCC). PLL measures the voltage at PCC and provides the angle.

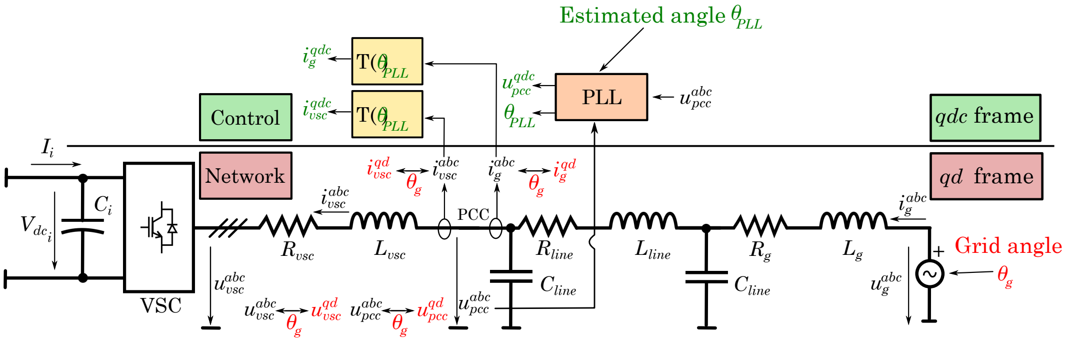

Figure 34: 2L VSC in GFOL mode with qd and qdc frames

Fig. 34 represents the VSC in GFOL operation connected to a Thevenin grid via a PI transmission line. As described before the angle θ_e is the angle between the PLL and the network hence such an angle can be used in the linearized rotational matrices to be able to switch references. For instance, to see the values with respect to the grid perspective we can use

$$\theta_{grid} - \theta_{PLL} \tag{77}$$

or to see from the converter's point of view

$$\theta_{PLL} - \theta_{grid} \tag{78}$$

Furthermore, the rotational matrix can be seen as follows

$$x_{qd}^c = \begin{bmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \sin(\theta_e) & \cos(\theta_e) \end{bmatrix} x_{qd} \tag{79}$$

where x can be current or voltage and superscript c denotes that it is the new value. Such matrix is nonlinear hence it should be linearized to be able to be used in the linearized state space models.

5.3 2L-VSC in GFOL mode linear model

In this section, the linear model of 2L-VSC that is operating in GFOL mode will be explained. There is no need for linearization in the network part of the model (PI line + Thevenin) however, for some parts of the control of VSC requires linearization.

5.3.1 Rotational Matrices

As explained above, network and control references are two references and sometimes we would like to switch between them. To do so, a rotational matrix is used with the angle in between such references. If we expand 79 we can get

$$x_q^c = \cos(\theta_e)x_q - \sin(\theta_e)x_d \tag{80}$$

$$x_d^c = \sin(\theta_e)x_q + \cos(\theta_e)x_d \tag{81}$$

Moreover, 80 and 81 are nonlinear by applying Taylor's theorem we can linearize the equations and represent such equations in state space as follows

$$\begin{bmatrix}0\end{bmatrix} = \begin{bmatrix}0\end{bmatrix}\begin{bmatrix}0\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix}\begin{bmatrix}\Delta x_q\\ \Delta x_d\\ \Delta e\end{bmatrix} \tag{82}$$

$$\begin{bmatrix}\Delta x_q^c\\ \Delta x_d^c\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} + \begin{bmatrix}\cos(\theta e_0) & -\sin(\theta e_0) & -\sin(\theta e_0)x_{q0} - \cos(\theta e_0)x_{d0}\\ \sin(\theta e_0) & \cos(\theta e_0) & \cos(\theta e_0)x_{q0} - \sin(\theta e_0)x_{d0}\end{bmatrix}\begin{bmatrix}\Delta x_q\\ \Delta x_d\\ \Delta e\end{bmatrix} \tag{83}$$

where x_{q0}, x_{d0}, and θe_0 are the values that are taken from the nonlinear model as linearization points. Also, it is crucial to mention that, we are working with incremental values. Similarly, the linearized state space representation of the inverse matrix can be observed as follows,

$$\begin{bmatrix}0\end{bmatrix} = \begin{bmatrix}0\end{bmatrix}\begin{bmatrix}0\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix}\begin{bmatrix}\Delta x_q^c\\ \Delta x_d^c\\ \Delta e\end{bmatrix} \tag{84}$$

$$\begin{bmatrix}\Delta x_q\\ \Delta x_d\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} + \begin{bmatrix}\cos(\theta e_0) & \sin(\theta e_0) & -\sin(\theta e_0)x_{q0}^c + \cos(\theta e_0)x_{d0}^c\\ -\sin(\theta e_0) & \cos(\theta e_0) & -\cos(\theta e_0)x_{q0}^c - \sin(\theta e_0)x_{d0}^c\end{bmatrix}\begin{bmatrix}\Delta x_q^c\\ \Delta x_d^c\\ \Delta e\end{bmatrix} \tag{85}$$

It can be seen that the A and C matrices are zero since there are no state variables (no integration nor derivation).

5.3.2 Active and Reactive Power Control

P and Q control blocks consist of a PI controller and P/Q calculation block that brings nonlinearity. Moreover, the block can be represented with a combination of power calculation and the PI controller. In this case, we have two state variables that are the integrators of the PI controller for both P and Q. The linearized state space representation of both P and Q control can be seen as follows

$$\begin{bmatrix}0\end{bmatrix} = \begin{bmatrix}0\end{bmatrix}\begin{bmatrix}"Ki_P"\end{bmatrix} + \begin{bmatrix}1 & -\frac{3}{2}U_{q0}^c & -\frac{3}{2}U_{d0}^c & -\frac{3}{2}I_{q0}^c & -\frac{3}{2}I_{d0}^c\end{bmatrix}\begin{bmatrix}P_{ref}\\ I_q^c\\ I_d^c\\ U_q^c\\ U_d^c\end{bmatrix} \tag{86}$$

$$\begin{bmatrix}Iq_{ref}\end{bmatrix} = \begin{bmatrix}Ki_P\end{bmatrix}\begin{bmatrix}"Ki_P"\end{bmatrix} + \begin{bmatrix}Kp_P & -\frac{3}{2}U_{q0}^c Kp_P & -\frac{3}{2}U_{d0}^c Kp_P & -\frac{3}{2}I_{q0}^c Kp_P & -\frac{3}{2}I_{d0}^c Kp_P\end{bmatrix}\begin{bmatrix}P_{ref}\\ I_q^c\\ I_d^c\\ U_q^c\\ U_d^c\end{bmatrix} \tag{87}$$

where U^c_{q0}, U^c_{d0}, I^c_{q0}, and I^c_{q0} are the values that are measured in the PCC. Similarly, the same matrices can be written for Q control.

$$\begin{bmatrix}0\end{bmatrix} = \begin{bmatrix}0\end{bmatrix}\begin{bmatrix}"Ki_Q"\end{bmatrix} + \begin{bmatrix}1 & \frac{3}{2}U^c_{d0} & -\frac{3}{2}U^c_{q0} & -\frac{3}{2}I^c_{d0} & \frac{3}{2}I^c_{q0}\end{bmatrix}\begin{bmatrix}Q_{ref}\\ I^c_q\\ I^c_d\\ U^c_q\\ U^c_d\end{bmatrix} \tag{88}$$

$$\begin{bmatrix}Id_{ref}\end{bmatrix} = \begin{bmatrix}Ki_Q\end{bmatrix}\begin{bmatrix}"Ki_Q"\end{bmatrix} + \begin{bmatrix}Kp_Q & \frac{3}{2}U^c_{d0}Kp_Q & \frac{3}{2}U^c_{q0}Kp_Q & -\frac{3}{2}I^c_{d0}Kp_Q & \frac{3}{2}I^c_{q0}Kp_Q\end{bmatrix}\begin{bmatrix}Q_{ref}\\ I^c_q\\ I^c_d\\ U^c_q\\ U^c_d\end{bmatrix} \tag{89}$$

It can be seen that for P and Q control, the outputs of the matrices are Iq_{ref} and Id_{ref} respectively and such outputs will be used in the current control.

5.3.3 VSC

In this case, the VSC is connected to the grid via an RL series filter and a transformer that can be also represented as a series RL. So, in total there are 4 inputs. The first 2 inputs are V^{cc}_q and V^{cc}_d and they are the output of the current control loop and rotated to the network reference because again we are switching references. The other 2 inputs are the voltages after the transformer such as V^{tr}_q and V^{tr}_d. In this block, there is no need for linearization hence the state space equations can be written as follows

$$\begin{bmatrix}\dot{is_q}\\ \dot{is_d}\end{bmatrix} = \begin{bmatrix}-\frac{R_{vsc}+R_{tr}}{L_{vsc}+L_{tr}} & -w\\ w & -\frac{R_{vsc}+R_{tr}}{L_{vsc}+L_{tr}}\end{bmatrix}\begin{bmatrix}"is_q"\\ "is_d"\end{bmatrix} + \begin{bmatrix}\frac{1}{L_{vsc}+L_{tr}} & 0 & \frac{-1}{L_{vsc}+L_{tr}} & 0\\ 0 & \frac{1}{L_{vsc}+L_{tr}} & 0 & \frac{-1}{L_{vsc}+L_{tr}}\end{bmatrix}\begin{bmatrix}V^{cc}_q\\ V^{cc}_d\\ V^{tr}_q\\ V^{tr}_d\end{bmatrix} \tag{90}$$

$$\begin{bmatrix}is_q\\ is_d\\ U^{PLL}_q\\ U^{PLL}_d\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\\ \frac{L_{vsc}R_{tr}-L_{tr}R_{vsc}}{L_{vsc}+L_{tr}} & 0\\ 0 & \frac{L_{vsc}R_{tr}-L_{tr}R_{vsc}}{L_{vsc}+L_{tr}}\end{bmatrix}\begin{bmatrix}"is_q"\\ "is_d"\end{bmatrix} + \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \frac{L_{tr}}{L_{vsc}+L_{tr}} & 0 & \frac{L_{vsc}}{L_{vsc}+L_{tr}} & 0\\ 0 & \frac{L_{tr}}{L_{vsc}+L_{tr}} & 0 & \frac{L_{vsc}}{L_{vsc}+L_{tr}}\end{bmatrix}\begin{bmatrix}V^{cc}_q\\ V^{cc}_d\\ V^{tr}_q\\ V^{tr}_d\end{bmatrix} \tag{91}$$

In this case, the subscripts vsc and tr represent the filter of the converter and the RL of the transformer respectively. In this block, the output is the converter currents is_q and is_d. Also, the U^{PLL}_q and U^{PLL}_d are the outputs of the converter block. Since we modeled the converter filter and the transformer together in order to get the correct voltage for PLL, which is the voltage between the filter and the transformer, the C and D matrices have been adjusted for this case.

5.3.4 PLL and θ_e

In this section, PLL block and θ_e block will be explained. In the complete model, PLL is connected to the PCC which is in between the converter filter and the transformer. In the previous part, the two of the outputs of the VSC block are the input parameters of the PLL however, again since we are changing reference, the parameters should be rotated with the rotation matrix. This is because the parameters are measured in the network and they are going to be used in the converter so a reference change is needed. The state space representation of PLL can be seen as follows,

$$\begin{bmatrix} \dot{Ki_{PLL}} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} "Ki_{PLL}" \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} V_d^c \end{bmatrix} \tag{92}$$

$$\begin{bmatrix} \omega_{PLL} \end{bmatrix} = \begin{bmatrix} -Ki_{PLL} \end{bmatrix} \begin{bmatrix} "Ki_{PLL}" \end{bmatrix} + \begin{bmatrix} -Kp_{PLL} \end{bmatrix} \begin{bmatrix} V_d^c \end{bmatrix} \tag{93}$$

The output of the block is ω_{PLL} and such a parameter hasn't been integrated yet for getting the angle. So, there is only 1 state variable which is the integrator in the PI controller. Moreover, the input of the system V_d^c is the rotated (switched reference) U_d^{PLL} which is one of the outputs of the VSC block. Moreover, we need θe, which is the angle difference of PLL and the grid (see 76), in order to rotate the variables. Such rotation angle can be obtained as

$$\begin{bmatrix} \dot{\theta e} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} "\theta e" \end{bmatrix} + \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \omega_{PLL} \\ \omega_{ref} \end{bmatrix} \tag{94}$$

$$\begin{bmatrix} \theta e \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} "\theta e" \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_{PLL} \\ \omega_{ref} \end{bmatrix} \tag{95}$$

Here we achieve the angle for rotational matrices. The inputs of the system are ω_{PLL} and ω_{ref}. Moreover, ω_{PLL} is the output of the PLL block and ω_{ref} (2*π*50 constant) which is the reference frequency of the grid. Furthermore, there is only one state variable since we have to integrate the differences of the ω to find the θe.

5.3.5 Current Control

In the current control block, there are two state variables because of the integrators of the PI controllers. Moreover, there are 6 inputs Iq_{ref} and Id_{ref} that are the outputs of the PQ control block. Then, I_q^c and I_d^c are the rotated (is_q and is_d) outputs of the VSC block, and finally U_q^c and U_d^c that are the rotated (U_q^{PLL} U_d^{PLL}) outputs of VSC as well. Furthermore, the state space representation can be seen as follows

$$\begin{bmatrix} \dot{Ki_{qcc}} \\ \dot{Ki_{dcc}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} "Ki_{qcc}" \\ "Ki_{dcc}" \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Iq_{ref} \\ Id_{ref} \\ I_q^c \\ I_d^c \\ U_q^c \\ U_d^c \end{bmatrix} \tag{96}$$

$$\begin{bmatrix} V_q^c \\ V_d^c \end{bmatrix} = \begin{bmatrix} Ki_{cc} & 0 \\ 0 & Ki_{cc} \end{bmatrix} \begin{bmatrix} "Ki_{qcc}" \\ "Ki_{dcc}" \end{bmatrix} + \begin{bmatrix} Kp_{cc} & 0 & -Kp_{cc} & wL_{vsc} & 1 & 0 \\ 0 & Kp_{cc} & -wL_{vsc} & -Kp_{cc} & 0 & 1 \end{bmatrix} \begin{bmatrix} Iq_{ref} \\ Id_{ref} \\ I_q^c \\ I_d^c \\ U_q^c \\ U_d^c \end{bmatrix} \tag{97}$$

Moreover, the outputs of the system are V_q^c and V_d^c which are in converter reference. So in order to be fed to the converter that is in the network reference, again by using rotational matrices the values should be rotated.

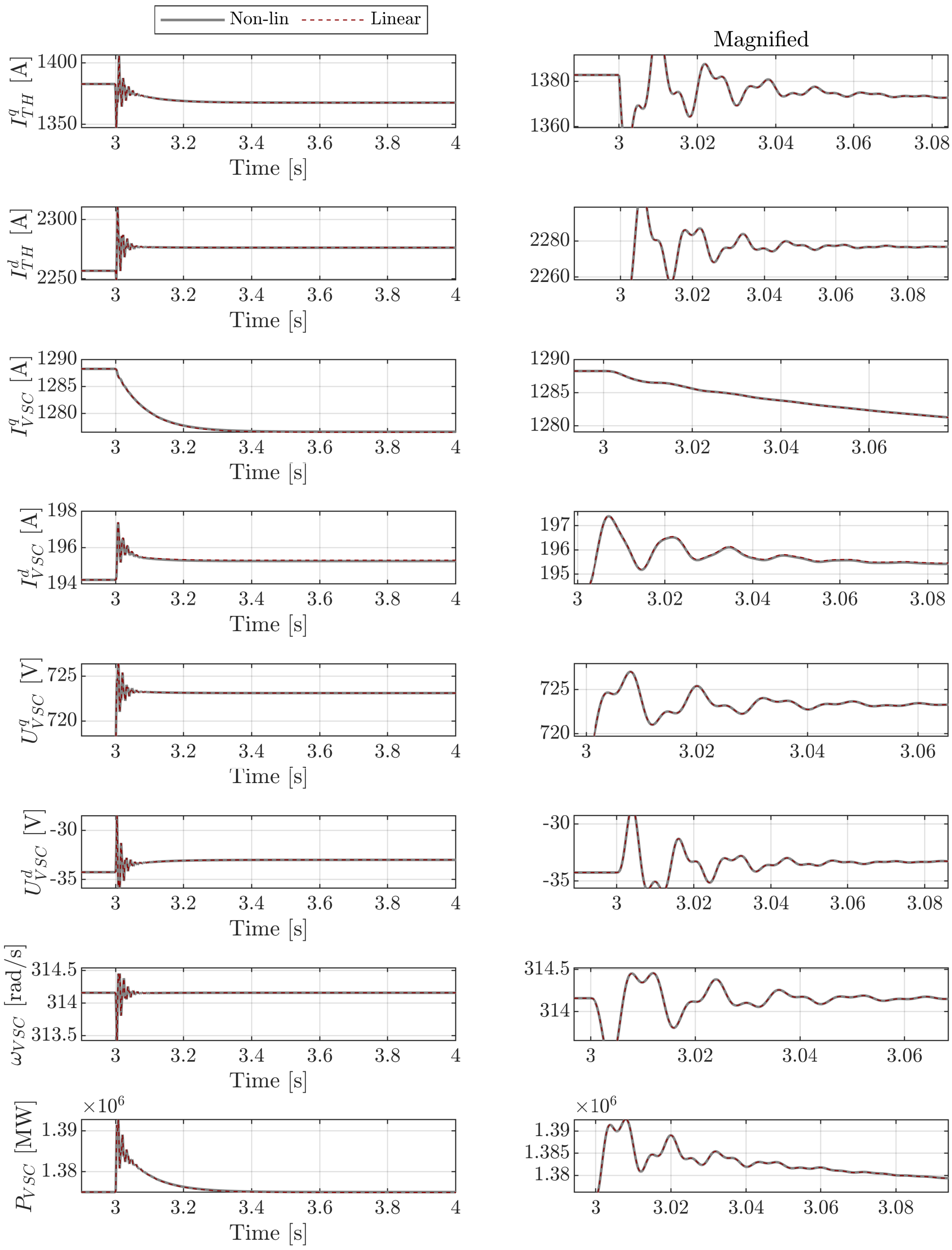

Figure 35: 1 %increase in Thevenin Voltage

Fig. 35 shows the signals of both linear and nonlinear models. Moreover, on the right half plane of the image, the magnified graphs can be observed. It can be seen that both signals (linear and nonlinear) are matching hence the model is validated.

In this case, the reference active power has been increased 1 % and the results can be seen in Fig.. 35

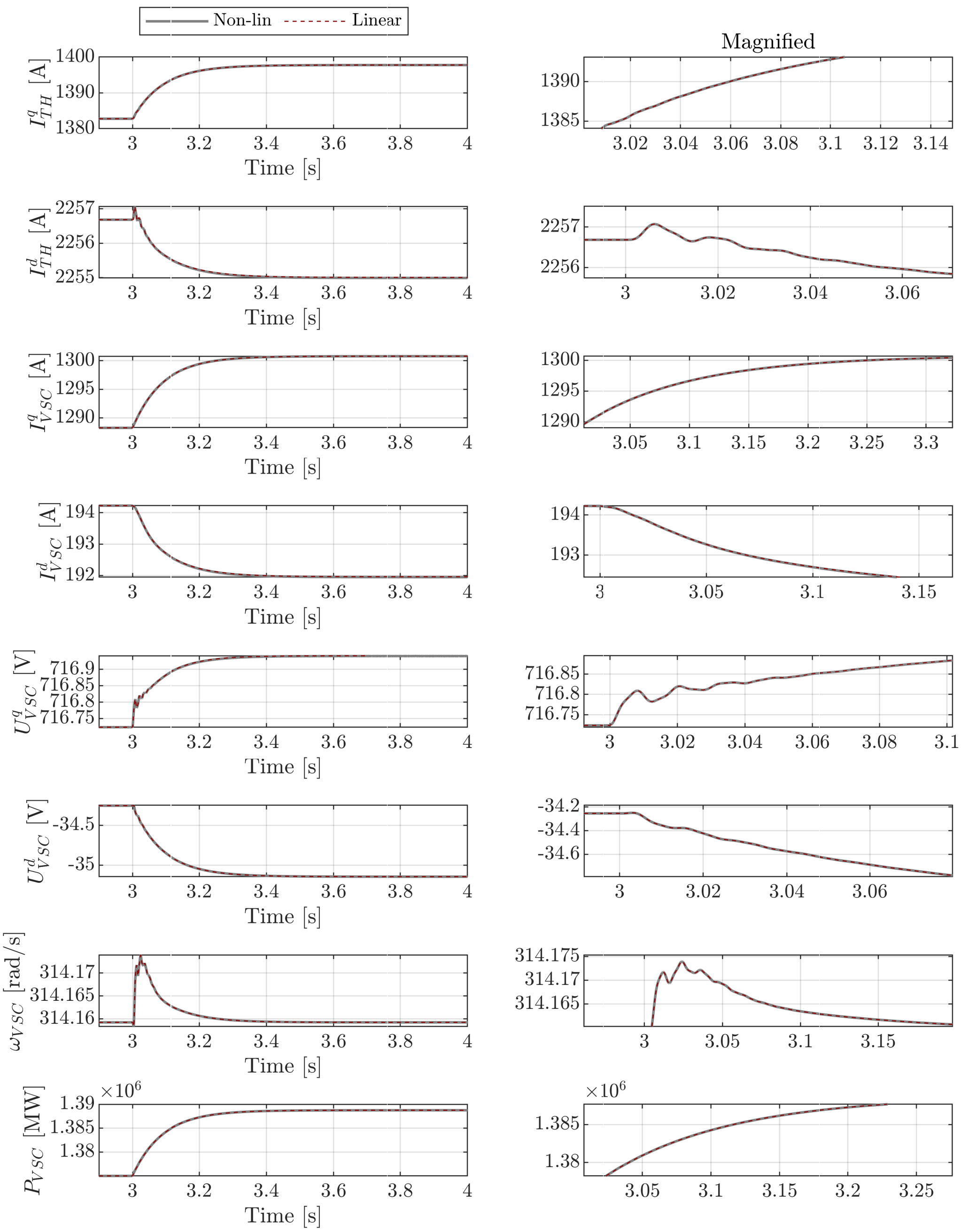

Figure 36: 1 % increase in Active Power reference

5.4 2L-VSC in GFOL mode with Frequency and Voltage Droop complete model

In this section, we will add frequency and voltage support to the GFOL converter while keeping the same system. In this case, the P and Q references will come from frequency and voltage droop respectively and the rest of the system is the same. So, in this section, only the droops will be explained with the complete model, linear model, and linear analysis.

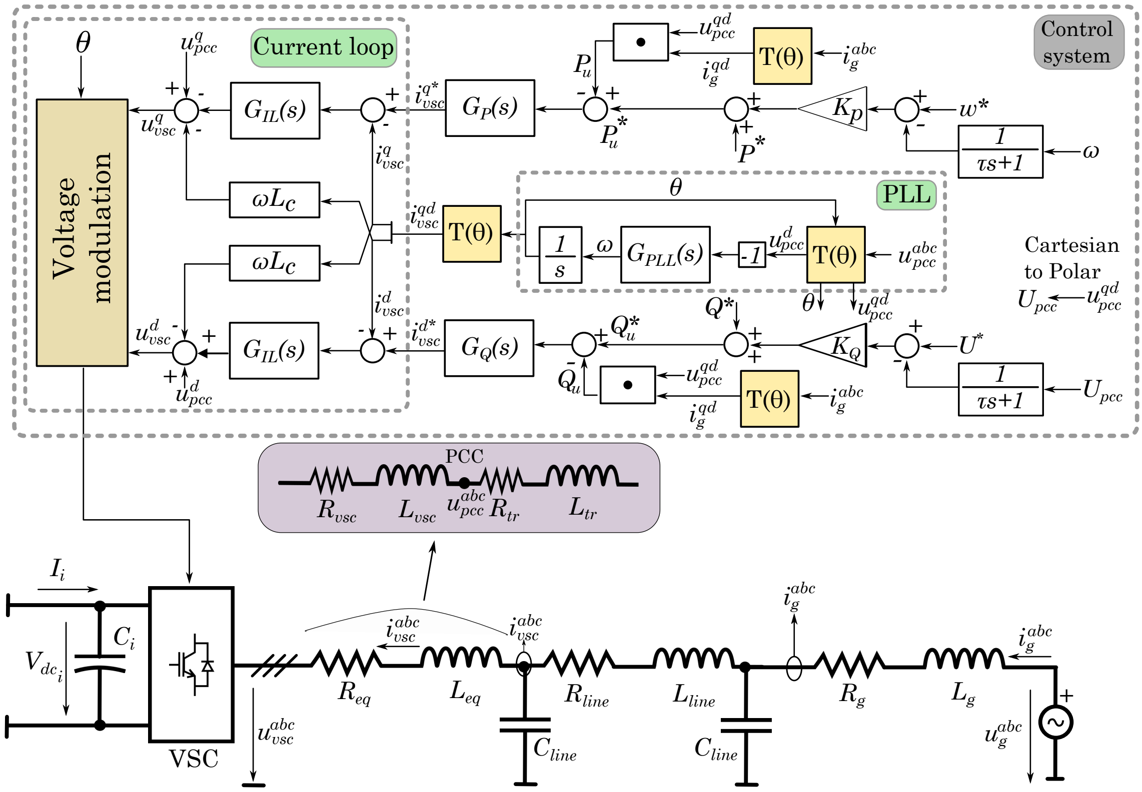

Figure 37: Complete Model of VSC with Droop support

Fig. 37 shows the VSC in GFOL mode with frequency and voltage droop connected to the Thevenin grid with a PI section transmission line. At this time Droop controller that is providing the P and Q reference for the PQ control is added to the system. Moreover, droop consists of a low-pass filter and a droop gain.

5.4.1 f-P Droop

In this case, the reference is the operating frequency (50 Hz) that is shown as w^* and the measured frequency (w) is coming from the PLL. Such frequency is subtracted after the low-pass filter in order to eliminate high-frequency signals. Then multiplied with the droop gain to reach the active power reference.

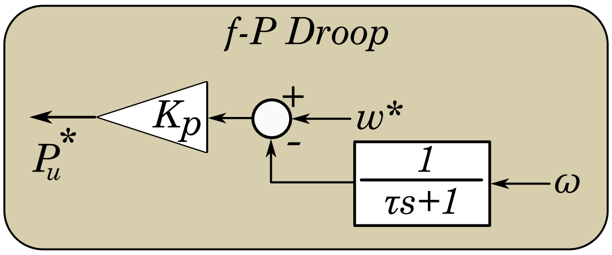

Figure 38: f-P Droop

Fig. 38 represents the f-P droop where τ is the time constant and K_P is the droop gain.

5.4.2 U-Q Droop

In this section, the U-Q droop will be explained. In this case, the reference U^* is the peak voltage and the U_{pcc} is the output of PLL and such output is converted into polar form. Again, the measured voltage in subtracted after the low-pass filter.

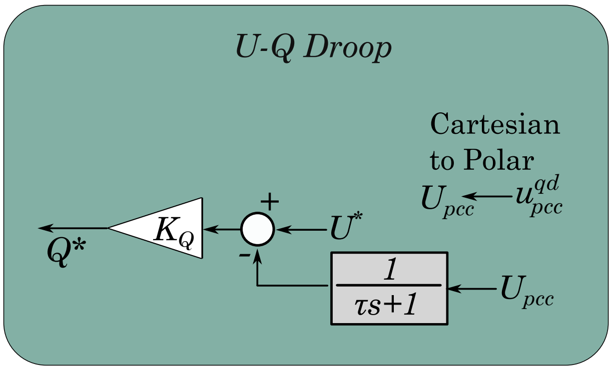

Figure 39: U-Q Droop

5.4.3 2L-VSC in GFOL mode with Frequency and Voltage Droop Linear model

In this section, the linear model of the droops will be explained.

5.4.4 f-P Droop

f-P droop is giving the active power reference for the P control and consists of a low-pass filter and a droop gain. So, the f-P droop can be represented in state space as follows

$$\begin{bmatrix} \dot{\tau_P} \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_P} \end{bmatrix} \begin{bmatrix} "\tau_P" \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w_{ref} \\ w_{PLL} \\ P \end{bmatrix} \tag{98}$$

$$\begin{bmatrix} P_{ref} \end{bmatrix} = \begin{bmatrix} -\frac{K_P}{\tau_P} \end{bmatrix} \begin{bmatrix} "\tau_P" \end{bmatrix} + \begin{bmatrix} K_P & 0 & 1 \end{bmatrix} \begin{bmatrix} w_{ref} \\ w_{PLL} \\ P \end{bmatrix} \tag{99}$$

where τ_P and K_p are the time constant of the filter and the droop coefficient respectively. The system inputs are the w_{ref}, w_{PLL} and P are the reference frequency, the frequecny of the PLL, and the active power. Moreover, the output is the reference power P_{ref}. Also, we only have 1 state variable which is coming from the filter.

5.4.5 U-Q Droop

U-Q droop gives the reactive power reference by taking into account the voltages. Such block consists of a low-pass filter and the droop gain however, there is an additional block which is the calculation of the voltage. In control, we work in qd frame and in this case, we need the magnitude of the voltage which can be calculated as follows

$$U_{mag} = \sqrt{(U_q^c)^2 + (U_d^c)^2} \tag{100}$$

The calculation of the U_{mag} can be seen in 100 and such equation is nonlinear so, it should be linearized.

$$\begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta U_q^c \\ \Delta U_d^c \end{bmatrix} \tag{101}$$

$$\begin{bmatrix} U_{mag} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} \frac{V_{q0}^{vsc}}{\sqrt{(V_{q0}^{vsc})^2 + (V_{d0}^{vsc})^2}} & \frac{V_{d0}^{vsc}}{\sqrt{(V_{q0}^{vsc})^2 + (V_{d0}^{vsc})^2}} \end{bmatrix} \begin{bmatrix} \Delta U_q^c \\ \Delta U_d^c \end{bmatrix} \tag{102}$$

where V_{q0}^{vsc} and V_{q0}^{vsc} are the converter voltage at the PCC and they are linearization points and ΔU_q^c and ΔU_d^c are the incremental inputs. Such inputs were taken from the network and used in the control so, such values are rotated to the converter reference with the rotation matrix. Moreover, the output of the block is U_{mag} and it will be one of the inputs of the U-Q droop block that can be seen as follow

$$\begin{bmatrix} \dot{\tau_Q} \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_Q} \end{bmatrix} \begin{bmatrix} "\tau_Q" \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U_{ref} \\ U_{mag} \\ Q \end{bmatrix} \tag{103}$$

$$\begin{bmatrix} Q_{ref} \end{bmatrix} = \begin{bmatrix} -\frac{K_Q}{\tau_Q} \end{bmatrix} \begin{bmatrix} "\tau_Q" \end{bmatrix} + \begin{bmatrix} K_Q & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{ref} \\ U_{mag} \\ Q \end{bmatrix} \tag{104}$$

where τ_Q and K_Q are the time constant of the low-pass filter and the droop gain respectively. Moreover, U_{ref}, U_{mag}, Q are the inputs and there is only one state variable that is coming from the filter.

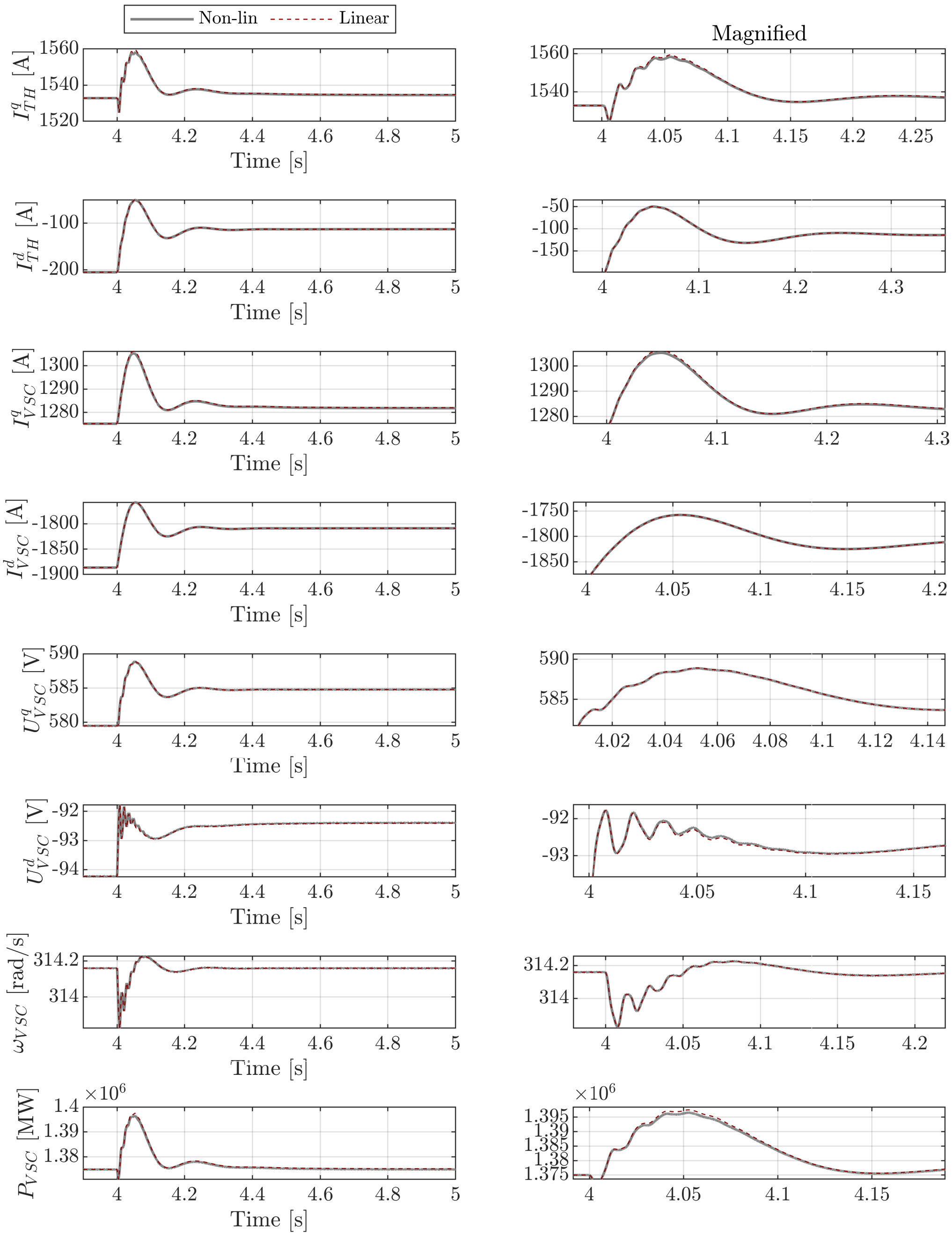

Figure 40: 1 % increase in U_{ref} Droop

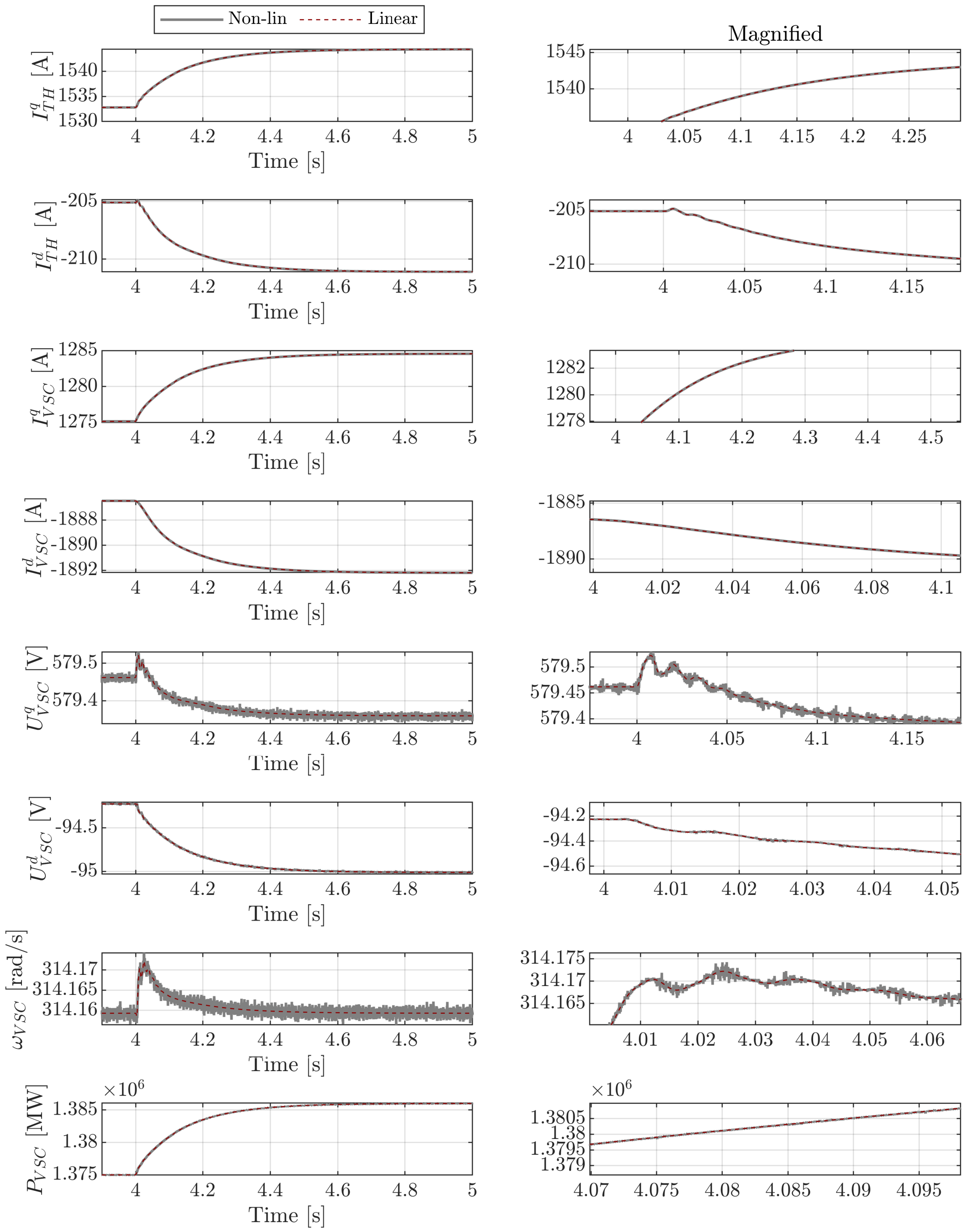

Figure 41: 1 % increase in frequency reference w_{ref}

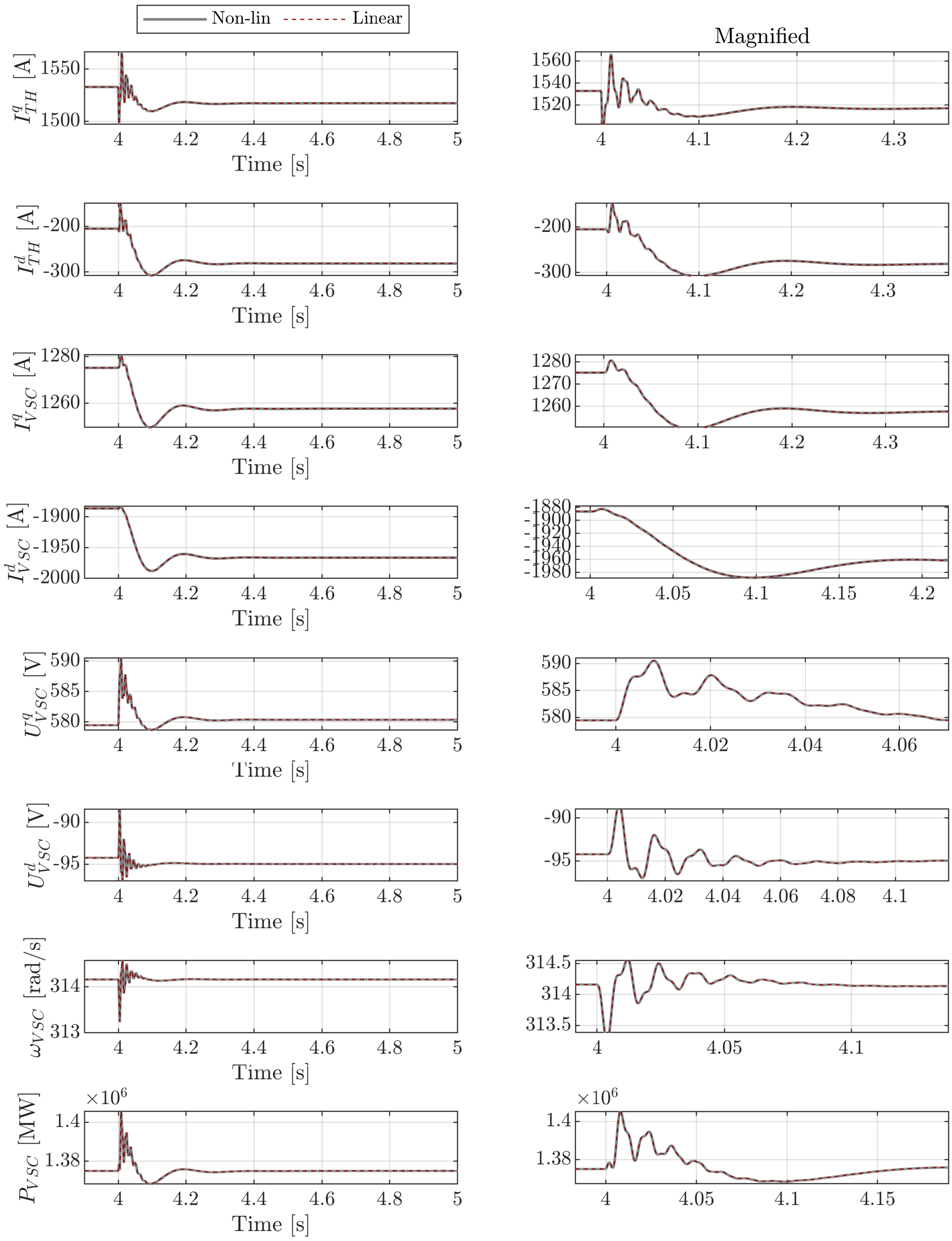

Figure 42: 1 % increase in Thevenin Voltage

It can be seen that the linearized state-space model can capture the essential dynamics in three tests hence the model is validated.

5.4.6 Resistive load disturbance

In this section, we will do an active load disturbance of 1 %. Such disturbance will be added to the Thevenin part of the system. Moreover, to do such disturbance we need some modifications. In this case, 0.64 MW (0.7*Sbase) of the active load was added to the system and such load has been increased by 1 %.

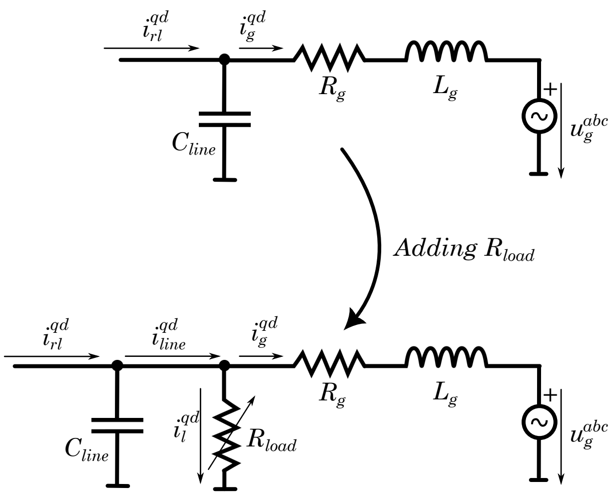

Figure 43: Resistive load disturbance scheme

Fig. 43 represents the circuit when we have a resistive load. In this case, the load is added between the PI section line and the Thevenin grid so, one of the input currents of the shunt capacitor has changed. Moreover, we have to adapt the matrices due to such change. The arranging of the matrices can be done in two ways. Furthermore, we can either arrange the C1 shunt matrices or create another matrix to sum the currents in order to put as an input of C1 shunt block. So without a resistive load disturbance, the C1 shunt equations can be written as

$$i_{rl}^q(t) = C_{line}\frac{du_q(t)}{dt} + wC_{line}u_d(t) + i_g^q(t) \tag{105}$$

$$i_{rl}^d(t) = C_{line}\frac{du_d(t)}{dt} - wC_{line}u_q(t) + i_g^d(t) \tag{106}$$

where, in this case, i_{rl}^{qd} and i_g^{qd} are equal. Moreover, the equations with a resistive load can be written like

$$i_{rl}^q(t) = C_{line}\frac{du_q(t)}{dt} + wC_{line}u_d(t) + i_{line}^q(t) \tag{107}$$

$$i_{rl}^d(t) = C_{line}\frac{du_d(t)}{dt} - wC_{line}u_q(t) + i_{line}^d(t) \tag{108}$$

$$i_{line}^d(t) = i_l^{qd} + i_g^{qd} \tag{109}$$

where i_l^{qd} and i_g^{qd} are the current that goes to load and Thevenin grid respectively. So, we can adjust this case in 2 different ways. The first one is rearranging the C shunt block as follows

$$\begin{bmatrix} \dot{u_q} \\ \dot{u_d} \end{bmatrix} = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix} \begin{bmatrix} "u_q" \\ "u_d" \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{line}} & 0 & -\frac{1}{C_{line}} & 0 & -\frac{1}{C_{line}} & 0 \\ 0 & \frac{1}{C_{line}} & 0 & -\frac{1}{C_{line}} & 0 & -\frac{1}{C_{line}} \end{bmatrix} \begin{bmatrix} i_q^{line} \\ i_d^{line} \\ i_q^g \\ i_d^g \\ i_q^{load} \\ i_d^{load} \end{bmatrix} \tag{110}$$

$$\begin{bmatrix} u_q \\ u_d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} "u_q" \\ "u_d" \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_q^{line} \\ i_d^{line} \\ i_q^g \\ i_d^g \\ i_q^{load} \\ i_d^{load} \end{bmatrix} \tag{111}$$

where inputs i_q^{load} and i_d^{load} are the outputs of the R load block (see 72). Moreover, i_q^g and i_d^g are the outputs of the Thevenin block (see 56). The second way is creating another matrix to sum the currents like 109 and putting the outputs of such matrix as an input of C shunt block.

$$\begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_q^{load} \\ i_d^{load} \\ i_q^g \\ i_d^g \end{bmatrix} \tag{112}$$

$$\begin{bmatrix} i_{sum}^q \\ i_{sum}^d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_q^{load} \\ i_d^{load} \\ i_q^g \\ i_d^g \end{bmatrix} \tag{113}$$

Now, by using the outputs i_{sum}^q and i_{sum}^d as an input of C shunt block we can write

$$\begin{bmatrix} \dot{u_q} \\ \dot{u_d} \end{bmatrix} = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix} \begin{bmatrix} "u_q" \\ "u_d" \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{line}} & 0 & -\frac{1}{C_{line}} & 0 \\ 0 & \frac{1}{C_{line}} & 0 & -\frac{1}{C_{line}} \end{bmatrix} \begin{bmatrix} i_q^{line} \\ i_d^{line} \\ i_{sum}^q \\ i_{sum}^d \end{bmatrix} \tag{114}$$

$$\begin{bmatrix} u_q \\ u_d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} "u_q" \\ "u_d" \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_q^{line} \\ i_d^{line} \\ i_{sum}^q \\ i_{sum}^d \end{bmatrix} \tag{115}$$

Moreover, we can see the validation in Fig. 44.

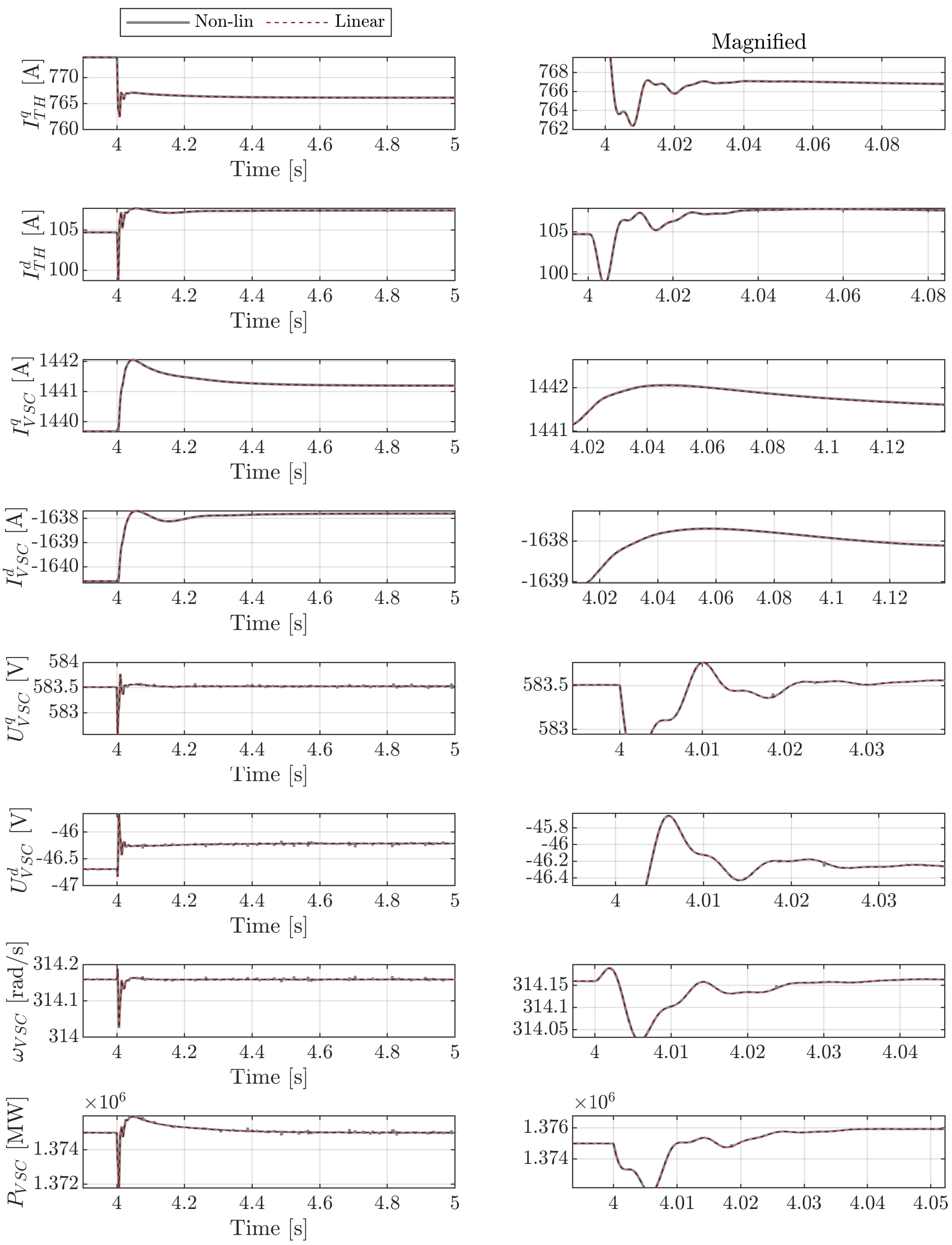

Figure 44: 1 % increase in Resistive Load

5.5 2L-VSC in GFOR mode complete model

In this section, a converter operating in GFOR mode will be explained. Such a converter is connected to a Thevenin grid via a PI section transmission line. In this case, the converter has an LCL filter and voltage controller block. As the GFOR converter imposes its own angle with P-f droop, we will not use PLL.

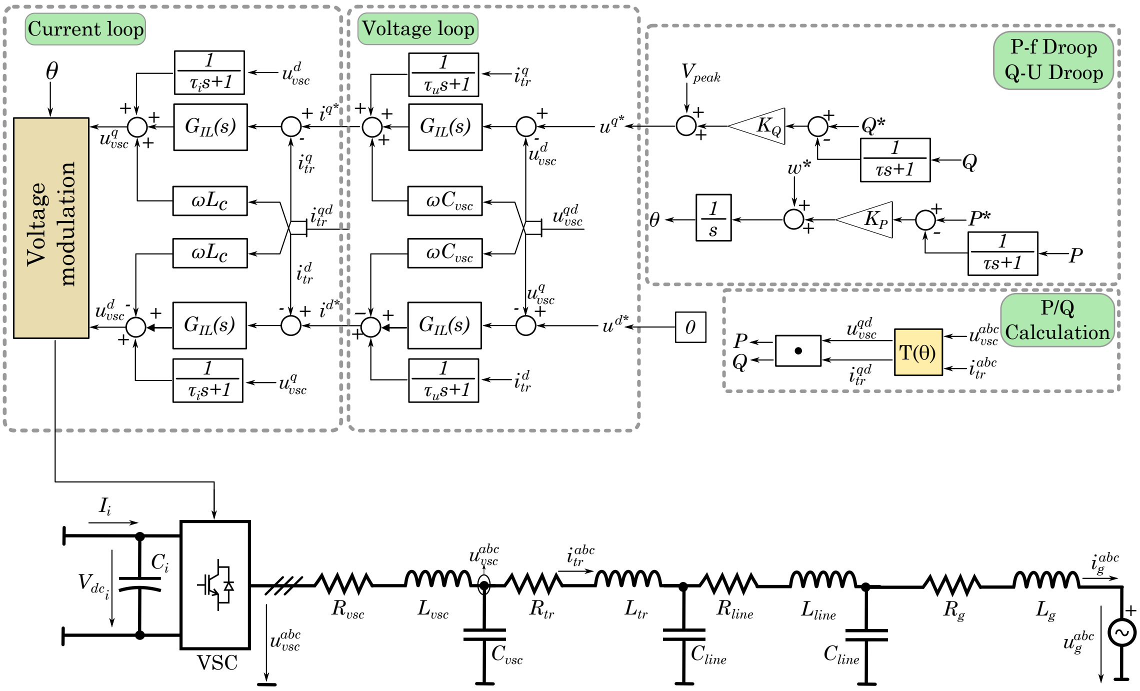

Figure 45: Complete model of the VSC in GFOR mode

Fig. 45 represents the complete model of the system. The VSC has an LCL filter (L_{vsc}, C_{vsc} and L_{tr}) and we have P-f and Q-U droop that give the angle and reference voltages respectively. P/Q calculation is performed with the voltage of the PCC u_{vsc}^{abc} and the current through the transformer i_{tr}^{abc}. Moreover, now we have voltage and current control cascaded and each control is fed forward by a low-pass filter.

5.5.1 P-f and Q-U Droop

In this case, the droop controllers are different in comparison to GFOL droops. Such controllers, in this case, are providing the angle and voltage reference. Inputs of the P-f droop are the reference $(P*)$ and the measured active power (P). Moreover, the P is subtracted after the low-pass filter and then the error is multiplied and with a coefficient. Also, the $w*$ is added and by integrating such value we can achieve the angle.

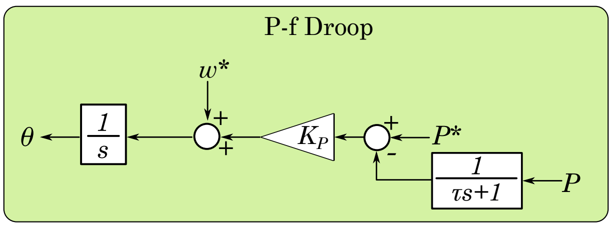

Figure 46: P-f Droop

Fig. 46 represents the P-f droop block. K_P is the coefficient can be calculated as follows

$$K_P = \frac{m_p * w}{Sbase} \tag{116}$$

where m_p is the droop coefficient and the τ in the filter can be found like

$$\tau = \frac{2 * S_{base}}{w * H * K_P} \tag{117}$$

where H is the inertia constant. Moreover, w and S_{base} are the nominal frequency and the base apparent power of the system respectively. Then, we have the Q-U droop that provides the reference voltages for the voltage control. Such droop control again consists of a low-pass filter and a coefficient and at the end, the peak voltage V_{peak} is added.

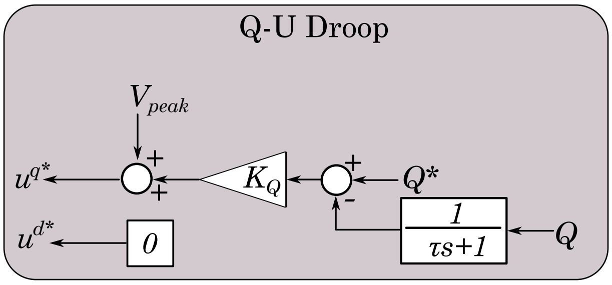

Figure 47: Q-U Droop

Fig. 47 shows the Q-U block diagram. Moreover, the calculation of the coefficient K_Q can be found as

$$K_Q = \frac{m_q * V_{peak}}{S_{base}} \tag{118}$$

where m_q is the droop coefficient and the τ can be calculated as 117. So, the output of the block is the u^{q*} that will go to the voltage control.

5.6 Voltage and Current Control

In this case. the voltage and current control have been used cascaded and each control has a feed-forward parameter with a low-pass filter. The references of the voltage control are the u^{q*} and u^{d*} that are provided by the Q-U droop. As we have a capacitor filter (C_{vsc}), decoupling term wC_{vsc} occurs in the voltage control. Moreover, the measured voltage is u^{qd}_{vsc} and it comes from the PCC. Also, feed-forward terms i^{qd}_{tr} are current that is after the capacitor that goes to the transformer.

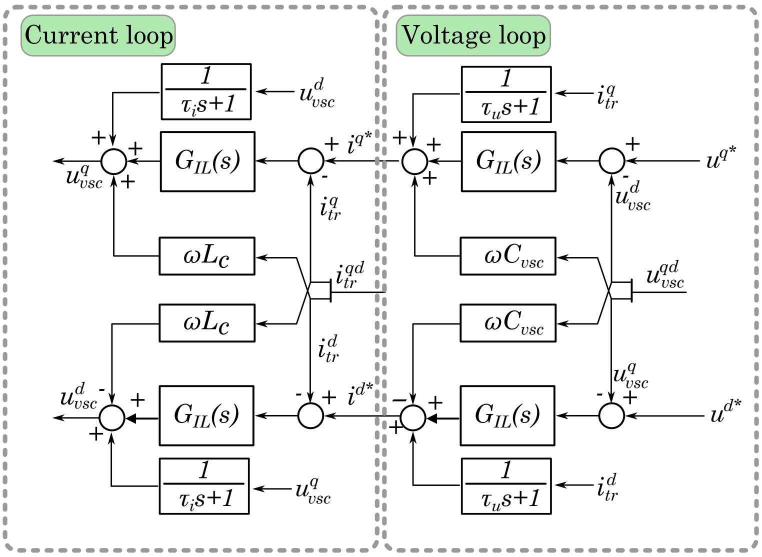

Figure 48: Voltage and Current Control Loops

Fig. 48 represents the cascaded voltage and current control loops. The low-pass filters are used to eliminate high-frequency oscillations. Moreover, such filters have time constants that are τ_u and τ_i. The outputs of the voltage controller are i^{qd*} the current references that go to the current controller.

5.7 2l-VSC in GFOR mode linear model

In this section, the linear model of a VSC operating in GFOR mode that is connected to a Thevenin grid via a PI section line will be discussed. The network part (PI line+Thevenin) and the P/Q calculation are the same as in the previous cases hence, cascaded voltage and current control, droops, and the converter with an LCL filter will be explained.

5.7.1 P-f and Q-U Droop

In this section, the state space model of the droop controllers will be explained. One of the inputs is P/Q measurement and such measurement should be linearized and already explained in the previous parts. We have 1 state variable that is coming from the low-pass filter. So, the state space equations can be written as

$$\begin{bmatrix} \dot{w_P} \end{bmatrix} = \begin{bmatrix} -w_P \end{bmatrix} \begin{bmatrix} "w_P" \end{bmatrix} + \begin{bmatrix} 0 & \frac{3u^q_{vsc0}}{2} & \frac{3u^d_{vsc0}}{2} & \frac{3i^q_{tr0}}{2} & \frac{3i^d_{t0r}}{2} \end{bmatrix} \begin{bmatrix} P_{ref} \\ i^q_{tr} \\ i^d_{tr} \\ u^q_{vsc} \\ u^d_{vsc} \end{bmatrix} \tag{119}$$

$$\begin{bmatrix} w_{droop} \end{bmatrix} = \begin{bmatrix} -K_P * w_P \end{bmatrix} \begin{bmatrix} "w_P" \end{bmatrix} + \begin{bmatrix} K_P & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{ref} \\ i^q_{tr} \\ i^d_{tr} \\ u^q_{vsc} \\ u^d_{vsc} \end{bmatrix} \tag{120}$$

where i_{tr}^q, i_{tr}^d, u_{vsc}^q and u_{vsc}^d are the incremental values for the P calculation. The parameters that have subscript 0 denote the linearization point. Moreover, such block provides w_{droop}, and by using 94 and 95 as an input of w_{droop} we can achieve the angle. Similarly, Q-U droop can be written as follows

$$\begin{bmatrix} \dot{w_Q} \end{bmatrix} = \begin{bmatrix} -w_Q \end{bmatrix} \begin{bmatrix} "w_Q" \end{bmatrix} + \begin{bmatrix} 0 & \frac{3u_{vsc0}^d}{2} & -\frac{3u_{vsc0}^q}{2} & -\frac{3i_{tr0}^d}{2} & \frac{3i_{t0r}^q}{2} \end{bmatrix} \begin{bmatrix} Q_{ref} \\ i_{tr}^q \\ i_{tr}^d \\ u_{vsc}^q \\ u_{vsc}^d \end{bmatrix} \tag{121}$$

$$\begin{bmatrix} U_{ref}^q \end{bmatrix} = \begin{bmatrix} K_Q w_Q \end{bmatrix} \begin{bmatrix} "w_Q" \end{bmatrix} + \begin{bmatrix} K_Q & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Q_{ref} \\ i_{tr}^q \\ i_{tr}^d \\ u_{vsc}^q \\ u_{vsc}^d \end{bmatrix} \tag{122}$$

here again, we have one state variable and the output of the block is U_{ref}^q.

5.7.2 Voltage and Current Control

In this case, the voltage and current control will be explained. As these blocks have a feed-forward filter, for the sake of simplicity, low-pass filters will be represented in another block. The state space equations of a such filter can be written as

$$\begin{bmatrix} \dot{x_{filt}} \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_x} \end{bmatrix} \begin{bmatrix} "x_{filt}" \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} \tag{123}$$

$$\begin{bmatrix} x_{filt}^c \end{bmatrix} = \begin{bmatrix} \frac{1}{\tau_x} \end{bmatrix} \begin{bmatrix} "x_{filt}" \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} \tag{124}$$

where x, the input, u_{vsc}^q, u_{vsc}^d, i_{tr}^q and i_{tr}^d and are rotated with the rotational matrix. Moreover, τ_x is the time constant of the filter. So, the output x_{filt}^c can be, depending the on the input, uq_{filt}^c, ud_{filt}^c, iq_{filt}^c and id_{filt}^c. Also, the state space equations can be written as

$$\begin{bmatrix} \dot{Ki_{qvc}} \\ \dot{Ki_{dvc}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} "Ki_{qvc}" \\ "Ki_{dvc}" \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Uq_{ref} \\ Ud_{ref} \\ iq_{filt}^c \\ id_{filt}^c \\ U_q^c \\ U_d^c \end{bmatrix} \tag{125}$$

$$\begin{bmatrix} Iq_{ref} \\ Id_{ref} \end{bmatrix} = \begin{bmatrix} Ki_{vc} & 0 \\ 0 & Ki_{vc} \end{bmatrix} \begin{bmatrix} "Ki_{qvc}" \\ "Ki_{dvc}" \end{bmatrix} + \begin{bmatrix} Kp_{vc} & 0 & -Kp_{vc} & wC_{vsc} & 1 & 0 \\ 0 & Kp_{vc} & -wC_{vsc} & -Kp_{vc} & 0 & 1 \end{bmatrix} \begin{bmatrix} Uq_{ref} \\ Ud_{ref} \\ iq_{filt}^c \\ id_{filt}^c \\ U_q^c \\ U_d^c \end{bmatrix} \tag{126}$$

where U_q^c and U_d^c are the rotated u_{vsc}^{qd} and the output of the control provides the current references for the current control. Also, we have two state variables because of the integrator in the PI controllers. The inputs of the system Uq_{ref} and Ud_{ref} are provided by the Q-U droop. Moreover, iq_{filt}^c and id_{filt}^c are the output of the low-pass filter which are fed-forward to the control. Similarly, the current control equations can be written as

$$\begin{bmatrix} \dot{Ki_{qcc}} \\ \dot{Ki_{dcc}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} "Ki_{qcc}" \\ "Ki_{dcc}" \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Iq_{ref} \\ Id_{ref} \\ I_q^c \\ I_d^c \\ uq_{filt}^c \\ ud_{filt}^c \end{bmatrix} \tag{127}$$

$$\begin{bmatrix} Uq_{conv} \\ Ud_{conv} \end{bmatrix} = \begin{bmatrix} Ki_{cc} & 0 \\ 0 & Ki_{cc} \end{bmatrix} \begin{bmatrix} "Ki_{qcc}" \\ "Ki_{dcc}" \end{bmatrix} + \begin{bmatrix} Kp_{cc} & 0 & -Kp_{cc} & wL_{vsc} & 1 & 0 \\ 0 & Kp_{cc} & -wL_{vsc} & -Kp_{cc} & 0 & 1 \end{bmatrix} \begin{bmatrix} Iq_{ref} \\ Id_{ref} \\ I_q^c \\ I_d^c \\ uq_{filt}^c \\ ud_{filt}^c \end{bmatrix} \tag{128}$$

where ud_{filt}^c and uq_{filt}^c are the output of the low-pass filter. Moreover, I_d^c and I_q^c are the rotated i_{tr}^{qd}. The outputs of the control are Uq_{conv} and Ud_{conv} that goes to the converter after the rotation.

5.7.3 VSC

In the previous sections, we were working with an L filter only however, now we have an LCL filter. For simplicity, the VSC block is separated into two. First, VSC with a series RL and the parallel capacitor. The VSC block equations can be seen as follows

$$\begin{bmatrix} \dot{is_q} \\ \dot{is_d} \end{bmatrix} = \begin{bmatrix} -\frac{R_{vsc}}{L_{vsc}} & -w \\ w & -\frac{R_{vsc}}{L_{vsc}} \end{bmatrix} \begin{bmatrix} "is_q" \\ "is_d" \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{vsc}} & 0 & \frac{-1}{L_{vsc}} & 0 \\ 0 & \frac{1}{L_{vsc}} & 0 & \frac{-1}{L_{vsc}} \end{bmatrix} \begin{bmatrix} V_q^{cc} \\ V_d^{cc} \\ V_q^{Cvsc} \\ V_d^{Cvsc} \end{bmatrix} \tag{129}$$

$$\begin{bmatrix} is_q \\ is_d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} "is_q" \\ "is_d" \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_q^{cc} \\ V_d^{cc} \\ V_q^{Cvsc} \\ V_d^{Cvsc} \end{bmatrix} \tag{130}$$

where V_q^{Cvsc} and V_d^{Cvsc} are the outputs of the capacitor filter block. Moreover, the outputs are the is_q and is_d and they are two of the inputs of the capacitor filter block. Also, such block can be written as

$$\begin{bmatrix} \dot{V_q^{Cvsc}} \\ \dot{V_d^{Cvsc}} \end{bmatrix} = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix} \begin{bmatrix} "V_q^{Cvsc}" \\ "V_d^{Cvsc}" \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{vsc}} & 0 & \frac{-1}{C_{vsc}} & 0 \\ 0 & \frac{1}{C_{vsc}} & 0 & \frac{-1}{C_{vsc}} \end{bmatrix} \begin{bmatrix} is_q \\ is_d \\ iq_{tr} \\ id_{tr} \end{bmatrix} \tag{131}$$

$$\begin{bmatrix} V_q^{Cvsc} \\ V_d^{Cvsc} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} "V_q^{Cvsc}" \\ "V_d^{Cvsc}" \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} is_q \\ is_d \\ iq_{tr} \\ id_{tr} \end{bmatrix} \tag{132}$$

where iq_{tr} and id_{tr} are the outputs of the transformer block.

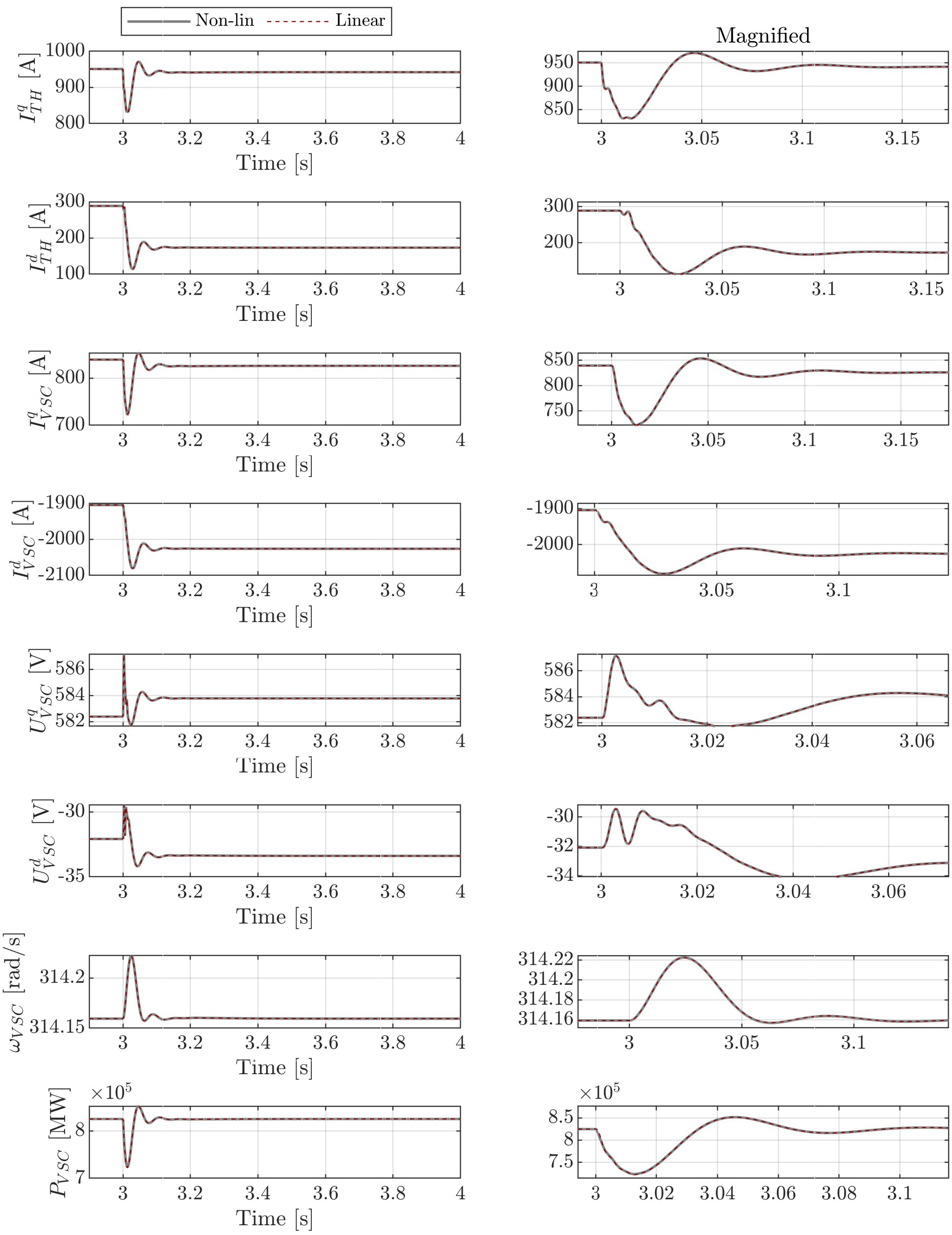

Figure 49: 1 % increase in Thevenin Voltage

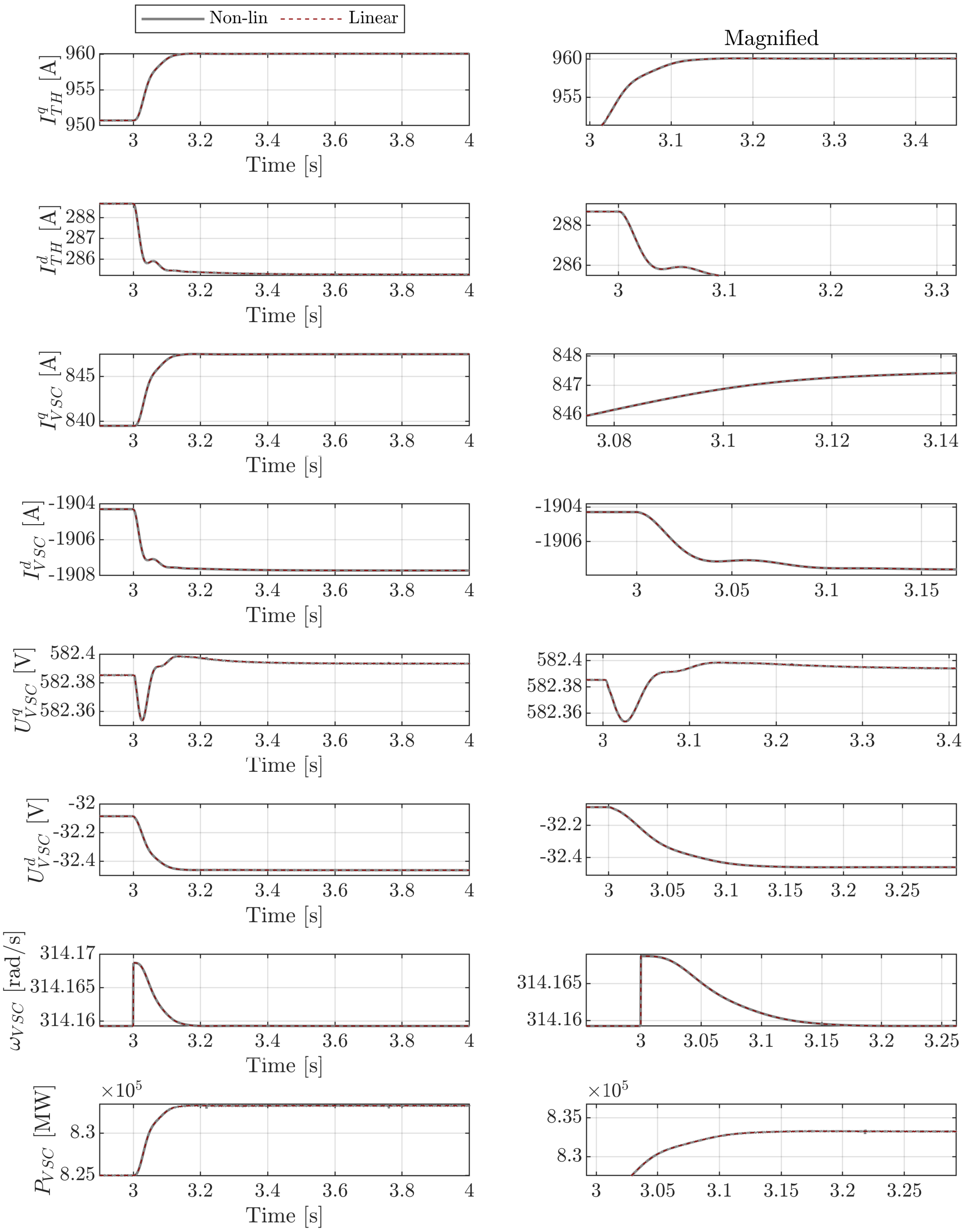

Figure 50: 1 % increase in active power reference

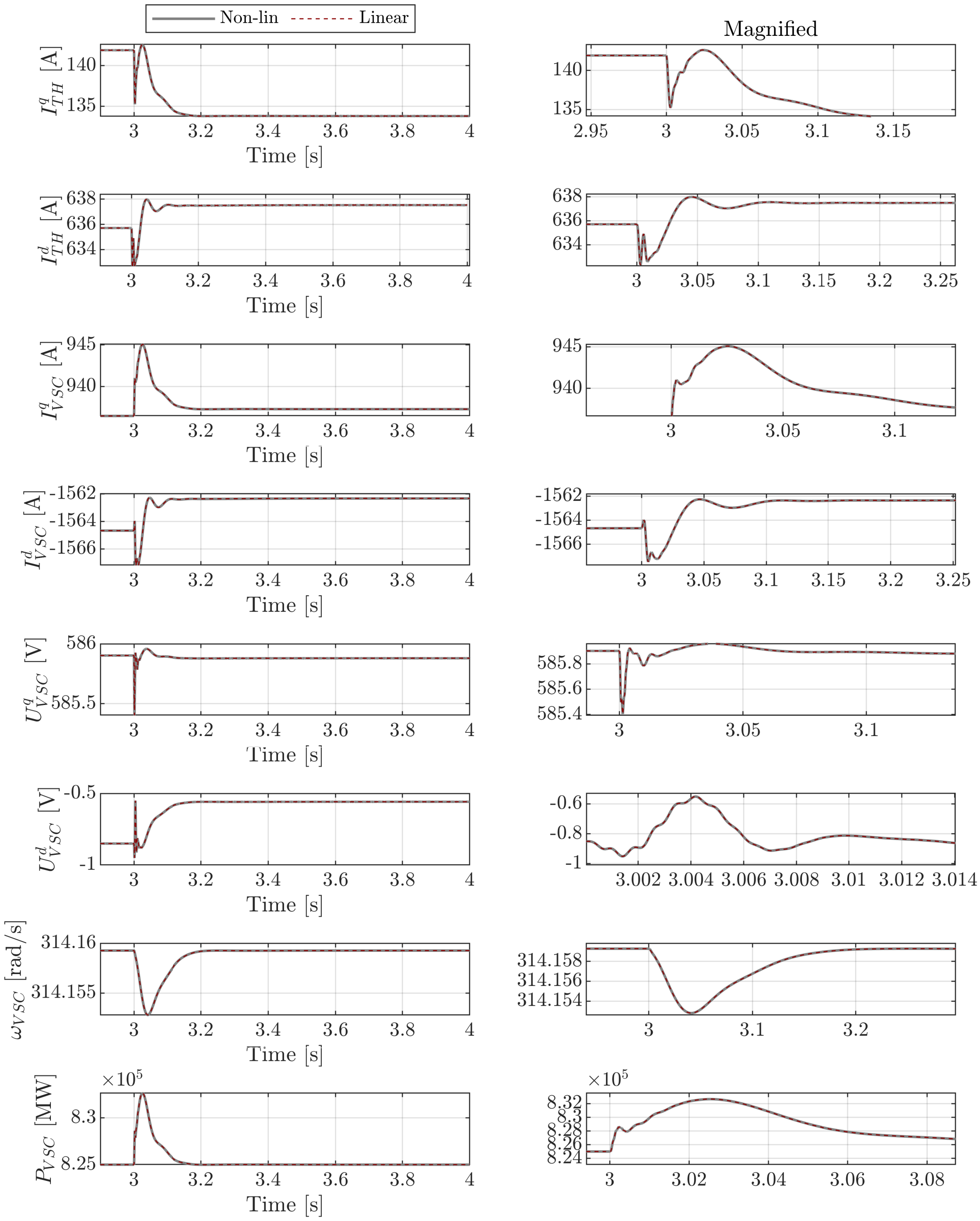

Figure 51: 1 % increase in active load

Figs 49, 50 and 51 represent the validation of the model. On the right half plane of the images, the magnified plots can be observed. Moreover, it can be seen that all dynamics are captured and in steady-state, both signals are the same.

6 Synchronous Generator

In this section, the synchronous generator (SG) model and how to build the linear model of such a system is explained.

The main components of the SG model are depicted in Fig. 52 and described as follows:

- **Electrical machine:** It is the fundamental block of the SG and corresponds to the stator and rotor windings (including field and damper windings).
- **Exciter:** The exciter is responsible for regulating the terminal voltage v_{sg}^{abc} to the desired value. It controls the voltage applied to the field winding v_f^d in order to modify the field current.
- **Governor and Turbine:** The role of the governor is to provide the necessary mechanical power reference to the turbine by measuring the speed of the generator. Then the turbine provides the mechanical power to the SG's shaft. The shaft can be represented with a single-mass or a multi-mass model.

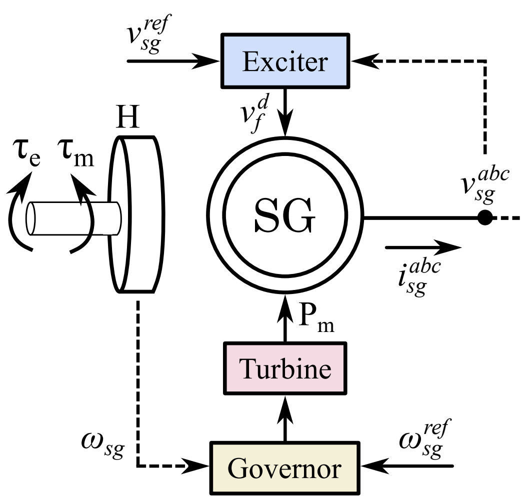

Figure 52: Scheme of a synchronous generator model

In Fig. 53, the modeling of the SG connection to an external grid is represented. A snubber resistance R_{snb} is added before the transformer impedance R_{tr} and L_{tr}. Such resistance is generally selected very big (300 pu) so that no current goes through it, but provides a voltage for the state-space model. In this case, the SG is connected to an infinite bus that has an impedance represented as L_g and R_g also where v_{bx}^{abc} is the voltage of the connection point. Also, parameters i_s^{abc} and i_{sg}^{abc} are the current injected by the SG and current injected by the SG to the grid at bus x, respectively.

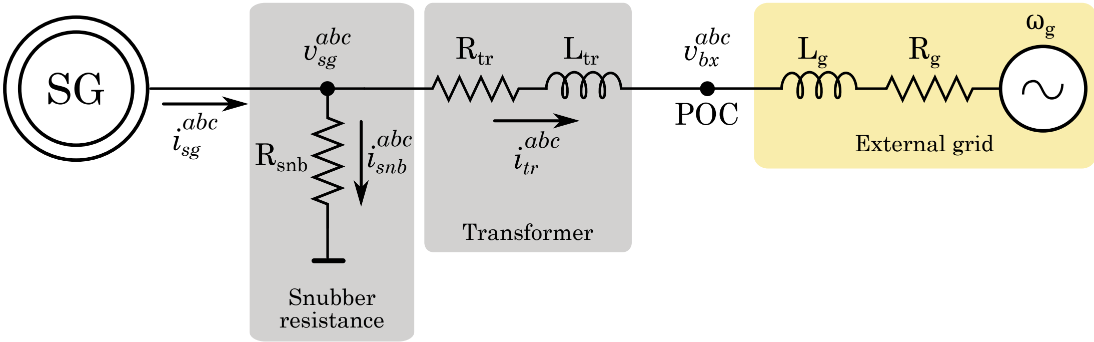

Figure 53: SG connected to infinite bus

6.1 Model description

The equivalent circuit of the GENROU, electrical and mechanical model can be seen in Fig. 54. A round rotor machine has been considered, including the stator winding (q and d axis), the field winding (d axis), and the damper windings (two in the q axis and one in the d axis). Where v_{sg}^{qd} are the qd voltages at the generator terminals. T_m and T_e are the mechanical and electrical torques respectively. The voltage applied by the field winding and current in the field winding are represented as v_f^d and i_f^d respectively. Then currents in the stator windings are shown as i_s^q and i_s^d. The parameters i_{k1}^q, i_{k2}^q and i_k^d are the qd current in the damper windings. Moreover, λ_s^q and λ_s^d are the flux linkages in the stator windings. Also for the electrical components, R_s, R_f, R_{k1}^q,R_{k2}^q and R_k^d are the stator, field winding, and damper winding resistances respectively. Also L_l, L_f, L_{k1}^q,L_{k2}^q and L_k^d are the stator, field winding, and damper winding inductances respectively. Also there are mutual inductances between the stator and rotor windings that are L_m^q and L_m^q. Finally, J is the inertia constant.

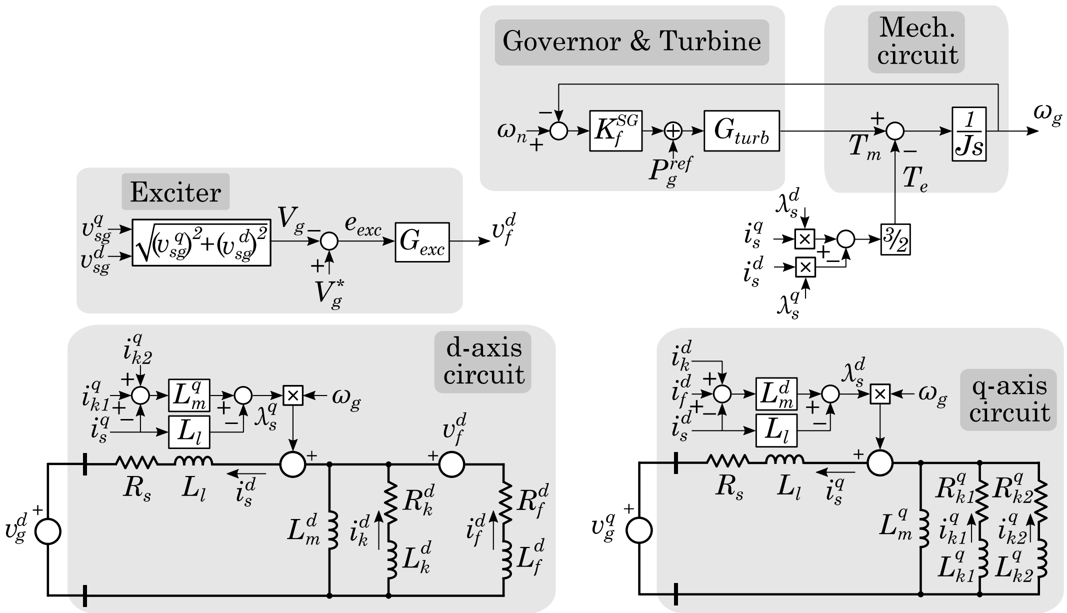

Figure 54: GENROU, electrical and mechanical model

6.2 Excitation System

The Excitation System regulates the terminal voltage v_{sg}^{abc} to the desired value by controlling the field winding voltage v_f^d. By changing the field voltage, the field current i_f^d is modified and, thereby, the rotating magnetic field, which induces alternating voltages in the armature windings. In this way, the terminal voltage of the machine v_{sg}^{abc} is controlled.

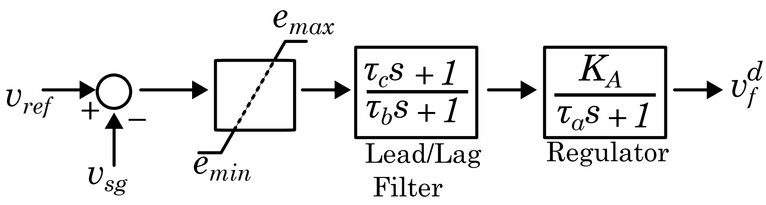

Figure 55: IEEE AC4C excitation system

The excitation system block diagram can be seen in Fig. 55. The inputs are the terminal voltage reference (v_{ref}) and the corresponding measure of terminal voltage magnitude (v_{sg}). The error between the measurement and the reference goes through a voltage limiter (e_{max} and e_{min} are denoted as max. and min. voltage) in order to not exceed the desired voltage level. After the limit, there is a lead-lag filter with time constants of τ_c and τ_b. Such filter then follows up with a regulator that has a regulator gain of K_A and a time constant of τ_a. Finally, the output is the field voltage v_f^d.

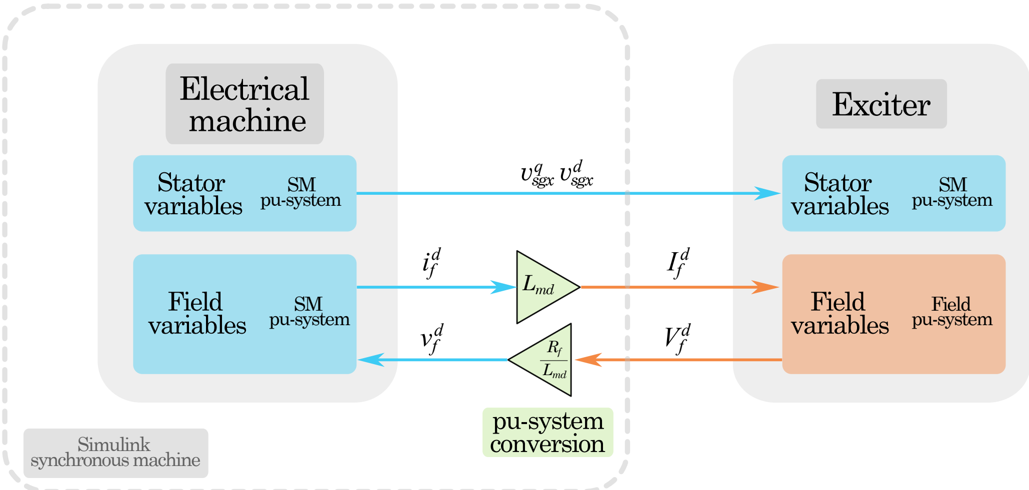

Figure 56: pu system

6.3 Governor and Turbine

The turbine provides the desired mechanical torque to the generator shaft. The governor modulates the power reference of the turbine in order to regulate the output frequency of the terminal voltage. Therefore, these two elements are cascaded. The governor measures the frequency and compares it with the frequency reference and provides the mechanical power reference for the turbine.

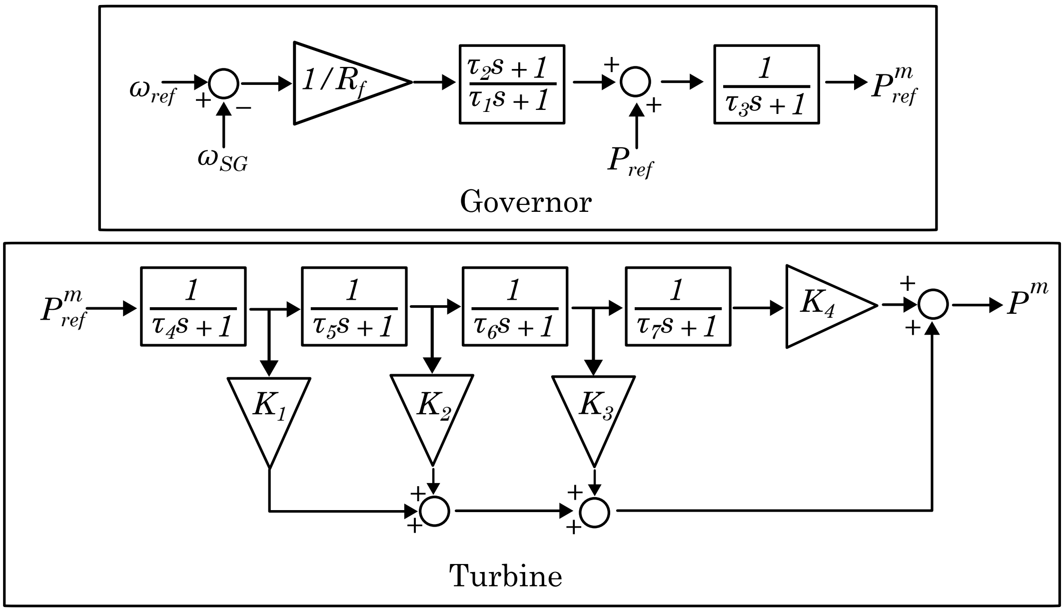

Figure 57: IEEE G1 governor and turbine system

In the Fig. 57, the governor and the turbine model can be observed. It can be seen that the inputs of the governor are the measured and reference frequency that are represented as ω_{ref} and ω_{SG}. Then the error between such variables multiplied by the droop gain $(1/R_f)$ then followed up with a lead-lag filter that has time constants of τ_2 and τ_1. After adding the P_{ref} and considering the servo time constant τ_3 the mechanical power reference P^m_{ref} is obtained. Such reference is the input of the turbine. There are 4 first-order filters cascaded and their time constants of them are represented as τ_4, τ_5, τ_6 and τ_7 which means steam-chest, reheater, reheater/cross-over, cross-over time constants.

6.3.1 A note on Speed Governing

It should be noted that the governor model in Fig. 57 is merely a proportional controller. Thus, it cannot track the frequency reference with zero error. The *Isochronous governor* comprises a proportional-integral controller that can provide this function. However, it cannot be used when there are two or more units connected to the same power system since each generator would have to have precisely the same speed setting. Otherwise, they would fight against each other, each trying to control system frequency to its own setting. That is why the speed-droop characteristic is employed so that the speed of the generator units "merely" drops as the load in the system is increased (and vice-versa). For more details, refer to Chapter-11 "Control of Active Power and Reactive Power" of Kundur.

6.4 Linear model of the SG

In this section the methodology of building the linear model will be explained. As can be seen in Fig. 54 the SG model can be represented as an equivalent circuit and the electrical equations can be written as follows:

$$v^q_{sgx} = -R_s i^q_s + \omega_g \lambda^d_s + \frac{d\lambda^q_s}{dt} \tag{133}$$

$$v_{sgx}^{d} = -R_s i_s^d + \omega_g \lambda_s^q + \frac{d\lambda_s^d}{dt} \tag{134}$$

$$v_{k1}^{q} = R_{k1}^{q} i_{k1}^{q} + \frac{d\lambda_{k1}^{q}}{dt} \tag{135}$$

$$v_{k2}^{q} = R_{k2}^{q} i_{k2}^{q} + \frac{d\lambda_{k2}^{q}}{dt} \tag{136}$$

$$v_{f}^{d} = R_{f}^{d} i_{f}^{d} + \frac{d\lambda_{f}^{d}}{dt} \tag{137}$$

$$v_{k}^{d} = R_{k}^{d} i_{k}^{d} + \frac{d\lambda_{k}^{d}}{dt} \tag{138}$$

where λ are the flux linkages and can be written as;

$$\lambda_s^q = -L_l i_s^d + L_m^q(-i_s^q + i_{k1}^q + i_{k2}^q) \tag{139}$$

$$\lambda_s^d = -L_l i_s^d + L_m^d(-i_s^d + i_f^d + i_k^d) \tag{140}$$

$$\lambda_{k1}^q = L_{k1}^q i_{k1}^q + L_m^q(-i_s^q + i_{k1}^q + i_{k2}^q) \tag{141}$$

$$\lambda_{k2}^q = L_{k2}^q i_{k2}^q + L_m^q(-i_s^q + i_{k1}^q + i_{k2}^q) \tag{142}$$

$$\lambda_f^d = L_f i_f^d + L_m^d(-i_s^d + i_f^d + i_k^d) \tag{143}$$

$$\lambda_k^d = L_k i_k^d + L_m^d(-i_s^d + i_f^d + i_k^d) \tag{144}$$

Now the mechanical equations can be written as follows:

$$T_e = \frac{3}{2}[\underbrace{-L_l i_s^d + L_m^d(-i_s^d + i_{fd}^d + i_k^d)}_{\lambda_s^d}]i_s^q - [\underbrace{-L_l i_s^q + L_m^q(-i_s^q + i_{k1}^q + i_{k2}^q)}_{\lambda_s^q}]i_s^d \tag{145}$$

$$\frac{d\omega_{sg}}{dt} = \frac{T_m - T_e}{2H} \tag{146}$$

Then substituting the flux equations and isolating the $\frac{d}{dt}$ terms we can define the input, state, and output vector.

1. State Vector
   $x(t) = [i_s^q, i_s^d, i_{k1}^q, i_{k2}^q, i_f^d, i_k^d]^T$
2. Input Vector
   $u(t) = [v_{sg}^q, v_{sg}^d, v_{k1}^q, v_{k2}^q, v_f^d, v_k^d]^T$
3. Output Vector
   $y(t) = [i_s^q, i_s^d, \omega_{sg}]^T$

$$A = -L^{-1}A_1 + A_2 \tag{147}$$

$$B = -L^{-1} + B_1 \tag{148}$$

$$C = \left[\begin{array}{cc} \mathrm{I}_{2x2} & 0_{2x5} \\ \hline 0_{2x7} & \\ \hline 0_{1x6} & 1 \end{array}\right] \tag{149}$$

$$D = 0_{2x7} \tag{150}$$

where L^{-1}, A_1, A_2 and B_1 are defined as follow

$$L^{-1} \begin{bmatrix} L_1^{-1} & 0_{6x1} \\ 0_{1x7} & \end{bmatrix} \tag{151}$$

where L_1 is

$$L_1 = \frac{1}{\omega_n} \begin{bmatrix} -(L_l + L_m^q) & 0 & L_m^q & L_m^q & 0 & 0 \\ 0 & -(L_l + L_m^q) & 0 & 0 & L_m^d & L_m^d \\ -L_m^q & 0 & L_{k1}^q + L_m^q & L_m^q & 0 & 0 \\ -L_m^q & 0 & L_m^q & L_{k2}^q + L_m^q & 0 & 0 \\ 0 & -L_m^d & 0 & 0 & L_f^d + L_m^d & \\ 0 & -L_m^d & 0 & 0 & L_m^d & L_k^d + L_m^d \end{bmatrix} \tag{152}$$

$$A_1 = \begin{bmatrix} -R_s & -\omega_0(L_l + L_m^d) & 0 & 0 & \omega_0 L_m^d & \omega_0 L_m^d & -(L_l + L_m^d) i_{s0}^d + L_m^d i_{f0}^d \\ -\omega_0(L_l + L_m^d) & -R_s & -\omega_0 L_m^q & -\omega_0 L_m^q & 0 & 0 & (L_l + L_m^q) i_{s0}^q \\ 0 & 0 & R_{k1}^q & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{k2}^q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_f^d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_f^d & 0 \end{bmatrix} \tag{153}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(L_l + L_m^d) i_{s0}^d + L_m^d i_{f0}^d & -(L_l + L_m^q) i_{s0}^q & L_m^q i_{s0}^d & L_m^q i_{s0}^d & L_m^d i_{s0}^q & L_m^d i_{s0}^q & -\frac{P_{m0}}{w_{g0}^2} \end{bmatrix} \tag{154}$$

$$B1 = \left[\begin{array}{cc} 0_{6x7} & \\ \hline 0_{1x6} & -\frac{1}{2H\omega_0} \end{array}\right] \tag{155}$$

6.5 Linear Model of the Excitation System

In this section, the linear model of the excitation system will be explained. Such a system can be seen in Fig. 58.

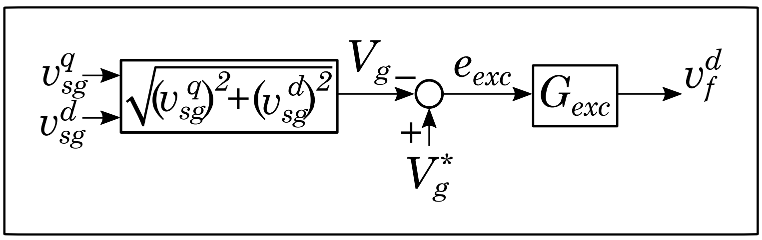

Figure 58: Excitation system

Here the voltage magnitude of the SG is calculated by using the Pythagoras' theorem. Such calculation is nonlinear so it should be linearized (see 102). Also, G_{exc} represents the dynamics of the system so, the system can be divided into two: 1) voltage magnitude and error calculation 2) error as input to G_{exc} to get the v_f^d. The state-space representation of the first part can be seen as follow:

1. State Vector
   $x(t) = [-]$
2. Input Vector
   $u(t) = [V_{ref}, v_{sg}^q, v_{sg}^d]^T$
3. Output Vector
   $y(t) = [e_v]$

$$A = \begin{bmatrix} 0 \end{bmatrix} \tag{156}$$

$$B = \begin{bmatrix} 0_{1x3} \end{bmatrix} \tag{157}$$

$$C = \begin{bmatrix} 0 \end{bmatrix} \tag{158}$$

$$D = \begin{bmatrix} 1 & -\frac{V_{sg}^{q0}}{\sqrt{(V_{sg}^q)^2+(V_{sg}^d)^2}} & -\frac{V_{sg}^{q0}}{\sqrt{(\sqrt{(V_{sg}^q)^2+(V_{sg}^d)^2}}} \end{bmatrix} \tag{159}$$

The excitation dynamics (G_{exc}) can be represented as follows,

$$G_{exc}(s) = \frac{\tau_C s + 1}{\tau_B s + 1} \frac{K_A}{\tau_A s + 1} \frac{R_f^d}{L_m^d} \tag{160}$$

where parameter $\frac{R_f^d}{L_m^d}$ is added in order to transform the excitation voltage from the non-reciprocal per unit system to the reciprocal base. Then the state space representation can be seen as follows,

1. State Vector
   $x(t) = [x_{exc}^1, x_{exc}^2,]^T$
2. Input Vector
   $u(t) = [e_v]$
3. Output Vector
   $y(t) = [v_f^d]$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\tau_A \tau_B} & -\frac{\tau_A+\tau_B}{\tau_A \tau_B} \end{bmatrix} \tag{161}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{162}$$

$$C = \frac{R_f^d}{L_m^d} \begin{bmatrix} \frac{K_A}{\tau_A \tau_B} & \frac{K_A \tau_C}{\tau_A \tau_B} \end{bmatrix} \tag{163}$$

$$D = \begin{bmatrix} 0 \end{bmatrix} \tag{164}$$

6.6 Linear Model of Turbine and Governor

In this section, the linear model of the turbine and governor will be explained. First, the state-space model of the governor and then the turbine will be explained. The governor's model can be seen as follow;

1. State Vector
   $x(t) = [x_{gov1}, x_{gov2}]^T$
2. Input Vector
   $u(t) = [e_w] = [w_{ref} - w_g]$
3. Output Vector
   $y(t) = [Pm_{ref}]$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\tau_1 \tau_3} & -\frac{\tau_1+\tau_3}{\tau_1 \tau_3} \end{bmatrix} \tag{165}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{166}$$

$$C = \frac{1}{R} \begin{bmatrix} \frac{1}{\tau_1 \tau_3} & \frac{\tau_2}{\tau_1 \tau_3} \end{bmatrix} \tag{167}$$

$$D = \begin{bmatrix} 0 \end{bmatrix} \tag{168}$$

where Pm_{ref} is the mechanical power reference which is the input of the turbine. Moreover, the model of the turbine can be seen as follows,

1. State Vector
   $x(t) = [x_{tur1}, x_{tur2}, x_{tur3}, x_{tur4}]^T$
2. Input Vector
   $u(t) = [Pm_{ref}]$
3. Output Vector
   $y(t) = [Pm]$

$$A = \begin{bmatrix} \alpha_{turb1} & \alpha_{turb2} & \alpha_{turb3} & \alpha_{turb4} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{169}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{170}$$

$$C = \begin{bmatrix} 1 & c_{turb2} & c_{turb3} & c_{turb4} \end{bmatrix} \tag{171}$$

$$D = \begin{bmatrix} 0 \end{bmatrix} \tag{172}$$

where

$$\alpha_{turb1} = \sum_{i=4}^{7} \frac{1}{\tau_i} \tag{173}$$

$$\alpha_{turb2} = \sum_{i=4}^{6} \sum_{j=i+1}^{7} \frac{1}{\tau_i \tau_j} \tag{174}$$

$$\alpha_{turb3} = \sum_{i=4}^{5} \sum_{j=i+1}^{6} \sum_{k=j+1}^{7} \frac{1}{\tau_i \tau_j \tau_k} \tag{175}$$

$$\alpha_{turb4} = \prod_{i=4}^{7} \tau_i \tag{176}$$

$$c_{turb2} = \frac{\sum_{i=1}^{2} K_1 \sum_{j=i+4}^{6} \sum_{k=j+1}^{7} \tau_j \tau_k}{K_1 \prod_{i=5}^{7} \tau_i} \tag{177}$$

$$c_{turb3} = \frac{\sum_{i=1}^{3} K_i \sum_{j=i+4}^{7} \tau_j}{K_1 \prod_{i=5}^{7} \tau_i} \tag{178}$$

$$c_{turb4} = \frac{\sum_{i=1}^{4} K_i}{K_1 \prod_{i=5}^{7} \tau_i} \tag{179}$$

7 What we've learned

7.1 Working in the per-unit system

- General recommendation: do not use per-unit unless it is necessary. For instance, Synchronous Generator parameters are in per-unit, and the standard models are usually defined in the per-unit base. So, in this case, it is better to work in per-unit.
- Do not compute power-flow in per-unit. Only the result should be computed in per-unit, if desired.
- If the electrical grid in the non-linear simulation has to be in per-unit always use RMS L-L. Then the qd power computation follows the same equation that in real values. In peak F-N base, it loses the $3/2$.
- Linear model is completely built in the per-unit system:
  - - Grid: system base (RMS L-L)
  - - Elements: in element base. Inputs and outputs should be re-scaled to the system base

7.2 Set the angle reference (slack) in a non-ideal element

First, slack and angle reference are different things, although we generally make them to coincide. If you set the angle reference to an element with a non-constant frequency (this means, you do not put it in a Thevenin) you have to account for the frequency i.e. angle variation in both linear and non-linear.

- - Non-linear: only in the angle qd transformation. Note that the non-linear simulation itself does not have "angles reference"
- - Linear: in the rotation matrices and the linearization point

7.3 Initialization of non-linear simulation

7.4 What to do when Non-linear and linear are not matching?

Welcome to the classic nightmare. The recommended debugging procedure would be something like this:

- Do not spend time trying to magically find the error in the whole system. You'll lose a lot of time. Instead, remove subsystems until you reach a matching model. For instance:
  - Remove all controls and then re-add them one by one
  - Try the individual blocks in a separate simulation with just an infinite bus.
- Be careful with the solvers. Make sure that, you use the same simulation time step for both linear and nonlinear model.
  - - Non-linear: ode45. Powergui in continuous.
  - - Linear: ode1

  These are the solvers we know that work. If you play with other solvers and simulation types, maybe you'll encounter that non-linear and linear models no longer match, or the Simulink beloved "Derivative State Error".
- Make sure that the system is in steady-state before applying the disturbance.
  - Do a vertical zoom (going super zoom) and check if the signal is flat.
  - If the nonlinear model is not in steady-state and even if your state-space model is correct you might have issues for matching the signals.
- Double check the disturbance.
  - 1% of disturbance is recommended. So in the nonlinear model if the step is 100 to 101, in the linear model it should be 0 to 1.
  - Make sure that the disturbance is not leaving the linear regime. Dont make 50% of change!!
- Small mismatches can accumulate.

  - If you wink at the small mismatches in the model, when that model is used in a big system multiple times. The error will be accumulated.

- Double check your state space model.
  - Make sure that the number of state variables that you should have is the same as the number of eigenvalues. If they are not the same something is wrong.
  - Look again at the inputs and outputs of the blocks. Small typos in the naming will not connect the matrices!

- If your nonlinear model is in PSCAD and if you want to compare it with state-space in MATLAB. Be aware:
  - PSCAD's solver is ode1.
  - Putting a resistive disturbance in PSCAD with a breaker has completely different dynamics than using a variable resistor. Don't use a breaker in that case.
  - Be aware of the load model in PSCAD.
  - Bergeron line model loses the approximation above 1500 Hz. So bear in mind you might not capture dynamics faster than 1500 Hz in PSCAD but in MATLAB. So, they wont be validated.