# CryptoHack

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## 1 Introduction

First things first, By Symmetry, We can interchange the $c_1$ and $(2^p + 3^q)$.

In the expansion of $(x + y)^n$, We can ignore all terms except first and last. Because all the terms except first and last contain the product of $x$ and $y$, and Since $N = p * q$, their contribution to modulo $N$ is 0. Combining all properties, we get following equations.

$$((2p)^{e_1} + (3q)^{e_1}) \equiv c_1 mod N \tag{1}$$

$$((5p)^{e_2} + (7q)^{e_2}) \equiv c_2 mod N \tag{2}$$

By compatibility with exponentiation, And using above property again (Ignoring middle terms), we get,

$$((2p)^{e_1 e_2} + (3q)^{e_1 e_2}) \equiv c_1^{e_2} mod N \tag{3}$$

$$((5p)^{e_1 e_2} + (7q)^{e_1 e_2}) \equiv c_2^{e_1} mod N \tag{4}$$

Let's define few variables for convenience. $a = 2^{e_1 e_2}$, $b = 3^{e_1 e_2}$, $x = c1^{e_2}$, $c = 5^{e_1 e_2}$, $d = 7^{e_1 e_2}$, $y = c_2^{e_1}$. Note that,

$$x = (ap^{e_1 e_2} + bq^{e_1 e_2}) mod N$$

$$y = (cp^{e_1 e_2} + dq^{e_1 e_2}) mod N$$

$$p = gcd(dx - by, N)$$

$$q = gcd(ay - cx, M)$$