

LAB REPORT ON PLASTIC HINGES FORMED ON BEAMS AND PORTAL FRAMES

Ciyang WANG

3rd year student MEng Engineering and Architectural Design

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INTRODUCTION

This experiment aims at observing and exploring the formation of plastic hinges in both beams and portal frames. Three support conditions of beams and three load ratios on portal frames are studied. Before tests, theoretical yielding loads and failure loads are calculated with the given dimensions and yield strength of specimens. The results are compared with wizard and reasons of difference are accounted for. The formation of plastic hinges in theory is also calculated and predicted. During tests, loads are exerted on the specimens incrementally and loads and displacements are recorded. Plastic hinges occur consecutively, and the sequence should be observed. Force-displacement curves can be plotted after gaining the experimental data, and further analysis of the results can be carried out. First, the experimental and theoretical values are compared, and possible reasons for differences in them are discussed. Second, the changes of slope and turning points on the curves are associated with the formation of plastic hinges, and various phases of the specimens under loading are identified and analyzed. By carrying out the tests, the plastic behaviors of beams and portal frames are better studies and more deeply understood.

1. CALCULATION

1.1 BEAMS

In this experiment, point load P is imposed on the midspan of beams, which are made of steel with yielding strength $f_{yk} = 355\text{MPa}$. The beams have spans $L = 750\text{mm}$ and cross sections of $7.9\text{mm} \times 7.9\text{mm}$. The support conditions are simple support, propped cantilever, and fixed end.

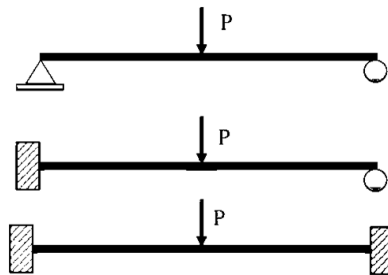


Figure 1. Beams with three support conditions.

1.1.1 Compute the value of the yielding load (P_y) for each restraint condition.

1.1.1.a Simply supported beam

The bending moment diagram of a simply supported beam with central point load is shown in Figure 2.

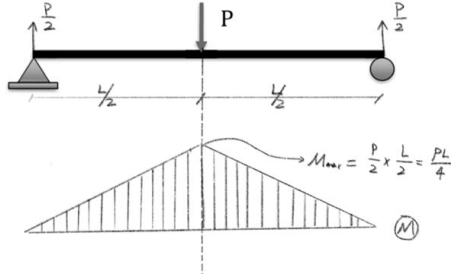


Figure 2. Bending moment diagram of a simply supported beam.

At midspan of the beam where the bending moment M is at its maximum, M can be calculated as:

$$M = \frac{PL}{4}$$

As the value of load P increases to yielding load P_y , the bending moment M at the midspan of the beam also reaches yielding moment M_y . The relationship can be expressed as :

$$M_y = \frac{P_y L}{4} \text{ or}$$

$$P_y = \frac{4M_y}{L} \text{ (eq. 1)}$$

The second moment of inertia I for the beam used in the experiment can be calculated with the equation:

$$I = \frac{bh^3}{12}$$

Where b is the breadth of the beam and h is the height of the beam, and the values are given by $b = h = 7.9\text{mm}$. The stress σ in the fibres of the beam can be expressed as:

$$\sigma = \frac{M}{I} \times y$$

Where y represents the distance between the fibre and the neutral axis of the beam. The extreme fibre in the beam first reaches yielding strength σ_y , and the distance y between the extreme fibre and neutral axis of this beam is $\frac{h}{2}$. Thus the yielding strength σ_y can be expressed as:

$$\sigma_y = \frac{M_y}{I} \times \frac{h}{2}$$

Reorganizing the equation, the value of yielding moment M_y can be expressed as:

$$M_y = \frac{2\sigma_y}{h} \times I = \frac{2\sigma_y}{h} \times \frac{bh^3}{12} = \sigma_y \times \frac{bh^2}{6}$$

Bringing the expression of M_y to eq.1, the value of yielding load P_y can be calculated as:

$$P_y = \frac{4M_y}{L} = \sigma_y \times \frac{2bh^2}{3L}$$

$$= f_{yk} \times \frac{2bh^2}{3L}$$

$$= 355\text{MPa} \times \frac{2 \times 7.9\text{mm} \times (7.9\text{mm})^2}{3 \times 750\text{mm}}$$

$$= 155.58\text{N}$$

1.1.1.b Propped cantilever

The bending moment diagram of a simply supported beam with central point load is shown in Figure 3. At the fixed end of the beam, the bending moment M reaches maximum. M can be expressed as:

$$M = \frac{3PL}{16}$$

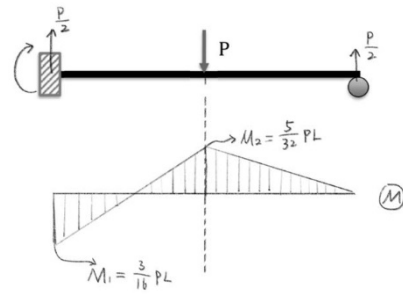


Figure 3. Bending moment diagram of a propped cantilever.

As the value of load P increases to yielding load P_y , the bending moment M at the midspan of the beam also reaches yielding moment M_y . The relationship can be expressed as :

$$M_y = \frac{3P_y L}{16} \text{ or}$$

$$P_y = \frac{16M_y}{3L}$$

Similarly to the calculations for the simply supported beam, the value of yielding moment M_y can be expressed as:

$$M_y = \sigma_y \times \frac{bh^2}{6}$$

Thus the value of yielding load P_y can be calculated as:

$$P_y = \frac{16M_y}{3L} = \sigma_y \times \frac{8bh^2}{9L} = f_{yk} \times \frac{8bh^2}{9L}$$

$$= 355\text{MPa} \times \frac{8 \times 7.9\text{mm} \times (7.9\text{mm})^2}{9 \times 750\text{mm}}$$

$$= 207.44\text{N}$$

1.1.1.c Fixed end beam

The bending moment diagram of a simply supported beam with central point load is shown in Figure 4.

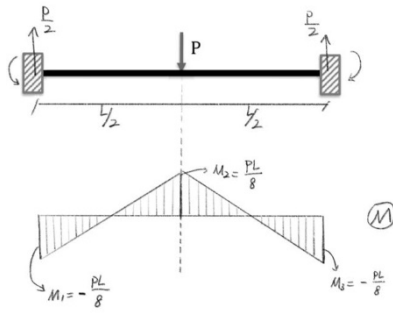


Figure 4. Bending moment diagram of fixed end beam

At fixed ends and the midspan of the beam, the bending moment M reaches maximum. M can be expressed as:

$$M = \frac{PL}{8}$$

As the value of load P increases to yielding load P_y , the bending moment M at the midspan of the beam also reaches yielding moment M_y . The relationship can be expressed as :

$$M_y = \frac{P_y L}{8} \text{ or } P_y = \frac{8M_y}{L}$$

Similarly to the calculations for the simply supported beam, the value of yielding moment M_y can be expressed as:

$$M_y = \sigma_y \times \frac{bh^2}{6}$$

Thus the value of yielding load P_y can be calculated as:

$$\begin{aligned} P_y &= \frac{8M_y}{L} = \sigma_y \times \frac{4bh^2}{3l} \\ &= f_{yk} \times \frac{4bh^2}{3l} \\ &= 355MPa \times \frac{4 \times 7.9mm \times (7.9mm)^2}{3 \times 750mm} \\ &= 311.16N \end{aligned}$$

As a result, the yielding load for the three restraint conditions are:

Simply supported beam	155.58N
Propped cantilever	207.44N
Fixed end beam	311.16N

1.1.2 Identify number and position of the plastic hinge/ hinges

	r	n	Position of plastic hinges
Simply supported beam	0	1	C
Propped cantilever	1	2	A,C
Fixed end beam	2	3	A,B,C

where r represents redundancy, n represents the number of plastic hinges. For the fixed end beam, the redundancy is originally calculated as 3 (with 6 unknown values, 1 member and 3 equilibriums). However, the symmetry will reduce the level of redundancy by 1. This gives the actual redundancy which is 2.

1.1.3 Compute the value of Z_p and obtain the corresponding value of M_p

The total plastic moment M_p of a beam can be expressed as:

$$M_p = \frac{1}{2} A(y_1 + y_2) \sigma_y \text{ (eq. 2)}$$

where A is the area of the cross section, y_1 and y_2 are the distance between the equal area axis and centroids of the two parts of the cross section separated by the equal area axis, and σ_y is the yield stress of the frame material. It is given that the cross section of the beam has a dimension of $7.9mm \times 7.9mm$, and that the yield strength $f_{yk} = 355MPa$. Since the cross section of beam is symmetrical, $y_1 + y_2$ equals the half height of the beam. As a result,

$$\begin{aligned} M_p &= \frac{1}{2} A(y_1 + y_2) \sigma_y \\ &= \frac{1}{2} \times (7.9mm)^2 \times \frac{7.9mm}{2} \times 355MPa \\ &= 43.76Nm \end{aligned}$$

Elastic section modulus Z_p is defined by:

$$M_p = \sigma_y \times Z_p$$

Therefore, the value of Z_p can be calculated as:

$$\begin{aligned} Z_p &= \frac{M_p}{\sigma_y} = \frac{43.76Nm}{355MPa} \\ &= 123.26 \text{ mm}^3 \end{aligned}$$

1.1.4 On the basis of 1.1.2 estimate the failure load P_p for each restraint condition

It is assumed that when plastic collapse occurs, the internal deformation occurs at the plastic hinges, and other parts of the beam remain rigid. According to the principle of virtual work and energy balance,

$$W_e = W_i$$

$$\sum_i P_i \delta_i = \sum_j M_j \varphi_j$$

where W_e and W_i are the virtual work done by the external forces and internal forces respectively, P_i represents applied loads which are in equilibrium with the resultant moments M_j , and δ_i and φ_j are corresponding displacements and rotations.

1.1.4.a Simply supported beam

The displacement and rotation must be compatible with the geometry, which is illustrated in Figure 5.

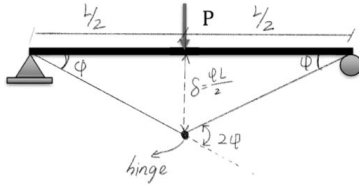


Figure 5. The mechanism formed by the simply supported beam.

The displacement and rotation can be expressed as:

$$\delta_i = \frac{l}{2} \times \tan \varphi \approx \frac{l}{2} \times \varphi$$

$$\varphi_j = 2\varphi$$

As a result,

$$\sum_i P_i \delta_i = \sum_j M_j \varphi_j$$

$$P_p \times \frac{l}{2} \times \varphi = M_p \times 2\varphi \quad (eq. 3)$$

$$P_p = M_p \times \frac{4}{l} = 43.76 \text{ Nm} \times \frac{4}{750 \text{ mm}}$$

$$= 233.39 \text{ N}$$

1.1.4.b Propped cantilever

Similar to the calculation process of the simply supported beam, mechanism approach and principle of work balance is applied. The geometry of this case is illustrated in Figure 6.

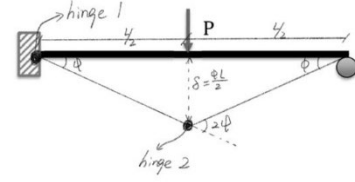


Figure 6. Mechanism formed by the propped cantilever.

The value of P_p can be calculated accordingly:

$$\sum_i P_i \delta_i = \sum_j M_j \varphi_j$$

$$P_p \times \frac{l}{2} \times \varphi = M_p \times \varphi + M_p \times 2\varphi \quad (eq. 4)$$

$$P_p = M_p \times \frac{6}{l} = 43.76 \text{ Nm} \times \frac{6}{750 \text{ mm}}$$

$$= 350.08 \text{ N}$$

1.1.4.c Fixed end beam

The geometry of this case is illustrated in Figure 7. Three hinges are formed in this mechanism.

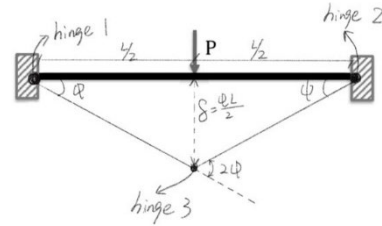


Figure 7. Mechanism formed by the fixed end beam.

The value of P_p can be calculated accordingly:

$$\sum_i P_i \delta_i = \sum_j M_j \varphi_j$$

$$P_p \times \frac{l}{2} \times \varphi = M_p \times \varphi + M_p \times 2\varphi + M_p \times \varphi \quad (eq. 5)$$

$$P_p = M_p \times \frac{8}{l} = 43.76 \text{ Nm} \times \frac{8}{750 \text{ mm}}$$

$$= 466.77 \text{ N}$$

In summary, during this experiment the failure loads P_p of beams with three support conditions under point loads are:

Simply supported beam	233.39N
Propped cantilever	350.08N
Fixed end beam	466.77N

1.2 PORTAL FRAMES

In this experiment, both horizontal forces (H) and vertical forces (V) are imposed on a portal frame of height $h = 0.3m$ and width $l = 0.2m$. The cross section of the frame has a dimension of $3.2mm \times 12.7mm$. The yield stress of the material has a value of $f_{yk} = 355MPa$. In this experiment, three different ratios of H and V are imposed on the frame in order to observe the behaviors of the frame under different load combinations.

1.2.1 Compute the plastic moment M_p of the given section

The plastic moment M_p of the given section can be calculated using the following equation:

$$M_p = \frac{1}{2} A (y_1 + y_2) \sigma_y$$

where A is the area of the cross section, y_1 and y_2 are the distance between the equal area axis (E.A.A) and centroids of the two parts of the cross section separated by E.A.A, and σ_y is the yield stress of the frame material. The dimensions of the cross section are $b = 12.7mm$ and $h = 3.2mm$ (shown in Figure 8), and the yield stress $\sigma_y = 355MPa$.

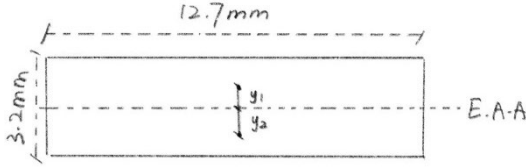


Figure 8. The cross section of the frame.

As a result, the value of plastic moment M_p can be obtained.

$$\begin{aligned} M_p &= \frac{1}{2} A (y_1 + y_2) \sigma_y \\ &= \frac{1}{2} \times (12.7mm \times 3.2mm) \times \frac{3.2mm}{2} \times 355MPa \\ &= 11.54Nm \end{aligned}$$

1.2.2 Using the mechanism approach and the principle of work balance, calculate the collapse load factors (λ_{BEAM} , λ_{SWAY} , $\lambda_{COMBINED}$) considering that the portal frame can deform into three possible mechanisms, namely “beam”, “sway” and “combined”.

Mechanism approach and principle of work balance is applied to obtain the values of collapse load factors. The virtual work done by the internal and external forces must be equal:

$$W_e = W_i$$

$$\sum_i P_i \delta_i = \sum_j M_j \varphi_j$$

where P_i represents applied loads which are in equilibrium with the resultant moments M_j , and δ_i and φ_j are corresponding displacements and rotations.

For the beam mechanism, only the vertical load V is taken into consideration, and the geometry of this case is illustrated in Figure 9. Applying the virtual work principle and energy balance,

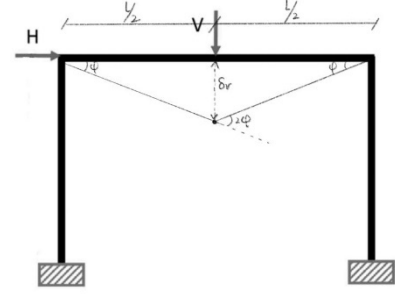


Figure 9. Geometry of the beam mechanism.

$$\sum_i P_i \delta_i = \sum_j M_j \varphi_j$$

$$\lambda_{BEAM} \times V \times \delta_v = M_p \varphi + M_p \varphi + M_p 2\varphi$$

$$\lambda_{BEAM} = \frac{4M_p \varphi}{V \times \delta_v} = \frac{4M_p \varphi}{V \times \frac{l}{2} \times \varphi} = \frac{4M_p}{V \times \frac{0.3}{2}} = \frac{80M_p}{3V}$$

For the sway mechanism, only the horizontal load is taken into consideration. The geometry is shown in Figure 10. and four plastic hinges will be formed.

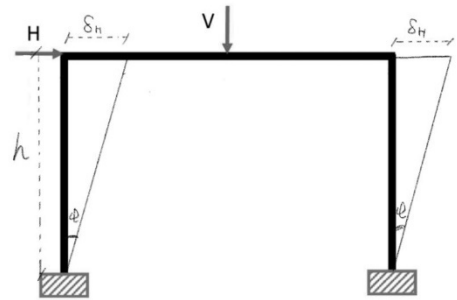


Figure 10. Geometry of sway mechanism.

$$\sum_i P_i \delta_i = \sum_j M_j \varphi_j$$

$$\lambda_{SWAY} \times H \times \delta_H = M_p \times 4\varphi$$

$$\lambda_{SWAY} = \frac{4M_p \varphi}{H \times \delta_H} = \frac{4M_p \varphi}{H \times h \times \varphi} = \frac{4M_p}{H \times 0.2} = \frac{20M_p}{H}$$

For the combined mechanism, both horizontal and vertical loads are considered, and the geometry is shown in Figure 11.

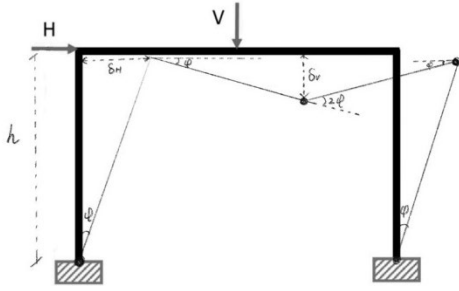


Figure 11. Geometry of the combined mechanism.

$$\sum_i P_i \delta_i = \sum_j M_j \phi_j$$

$$\lambda_{COMBINED} \times (H \times \delta_H + V \times \delta_v) = M_p \times 6\phi$$

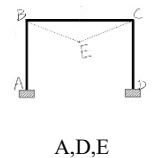
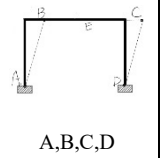
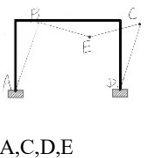
$$\lambda_{COMBINED} = \frac{6M_p \phi}{H \times \delta_H + V \times \delta_v}$$

$$= \frac{6M_p \phi}{H \times h \times \phi + V \times \frac{l}{2} \times \phi}$$

$$= \frac{6M_p}{H \times 0.2 + V \times \frac{0.3}{2}} = \frac{120M_p}{3V + 4H}$$

1.2.3 Determine the number and position of the hinges for each mechanism to occur. Compute the collapse load factors (λ) for each given load ratio and determine which mechanism will develop.

The number and positions of the plastic hinges for each mechanism is listed in the table below.

	beam	sway	combined
r	2	3	3
n	r+1=3	r+1=4	r+1=4
position	 A,D,E	 A,B,C,D	 A,C,D,E

where r represents redundancy and n represents the number of plastic hinges formed. For the case of beam mechanism, the level of redundancy r is originally calculated as 3 (6 unknowns, 1 member and 3 equilibriums). However, the symmetry will reduce the level of redundancy by 1, and thus $r=3-1=2$.

Collapse load factor λ for each given ratio is calculated in the table below.

	λ		
H:V	1:2	2:1	1:3
beam	λ_{BEAM} $= \frac{80M_p}{3V}$ $= 26.67 \frac{M_p}{V}$	λ_{BEAM} $= \frac{80M_p}{3V}$ $= 26.67 \frac{M_p}{V}$	λ_{BEAM} $= \frac{80M_p}{3V}$ $= 26.67 \frac{M_p}{V}$
sway	λ_{SWAY} $= \frac{20M_p}{0.5V}$ $= 40 \frac{M_p}{V}$	λ_{SWAY} $= \frac{20M_p}{2V}$ $= 10 \frac{M_p}{V}$	λ_{SWAY} $= \frac{20M_p}{0.33V}$ $= 60 \frac{M_p}{V}$
combined	$\lambda_{COMBINED}$ $= \frac{120M_p}{3V + 4 \times 0.5V}$ $= 24 \frac{M_p}{V}$	$\lambda_{COMBINED}$ $= \frac{120M_p}{3V + 4 \times 2V}$ $= 10.9 \frac{M_p}{V}$	$\lambda_{COMBINED}$ $= \frac{120M_p}{3V + 4 \times 0.33V}$ $= 27.7 \frac{M_p}{V}$
minimum	$\lambda_{COMBINED}$ $= 24 \frac{M_p}{V}$	λ_{SWAY} $= 10 \frac{M_p}{V}$	λ_{BEAM} $= 26.67 \frac{M_p}{V}$

In the case of load ratio H:V=1:2, the combined mechanism delivers the smallest collapse factor, and thus the combined mechanism will develop. Similarly, when H:V=2:1 the sway mechanism will develop and when H:V=1:3 the beam mechanism will develop.

1.2.4 Compute the ultimate loads (λH and/or λV) for each given load ratios.

According to the collapse load factors calculated above and the value of plastic moment M_p calculated in 1.2.1, the ultimate loads of the beams can be calculated as:

	λV	λH
1:2 (combined)	$= 24 \frac{M_p}{V} \times V$ $= 24M_p$ $= 276.96N$	$= 24 \frac{M_p}{V} \times H$ $= 12M_p$ $= 138.48N$
2:1 (sway)	$= 10 \frac{M_p}{V} \times V$ $= 10M_p$ $= 115.4N$	$= 10 \frac{M_p}{V} \times H$ $= 20M_p$ $= 230.8N$
1:3 (beam)	$= 26.67 \frac{M_p}{V} \times V$ $= 26.67M_p$ $= 307.77N$	$= 26.67 \frac{M_p}{V} \times H$ $= 8.89M_p$ $= 102.58N$

1.2.5 Use the provided excel sheet to check your calculations.

The comparison between calculated results and results provided by the wizard during experiments is shown in the table below.

load	H(N)		V(N)	
	calculation	wizard	calculation	wizard
combined	138.48	138.50	276.96	277.00
sway	230.8	214.93	115.4	107.46
beam	102.58	95.52	307.77	286.57

It can be observed that there exists deviation of data for especially sway and beam mechanism. It is possibly caused by the difference of beam used during the experiments, which will cause difference of cross section areas and thus the value of plastic moment M_p .

2. EXPERIMENT

2.1 EQUIPMENT

The setup for the beam tests is shown in Figure 12. By changing the holders, the supported conditions of the beams can be switched between being fixed and being simply supported. Central point loads can be manually adjusted, and the values can be read from the reader. Before starting the tests, the wizard in the software can provide the values of failure loads by inputting cross section dimensions and support conditions. The loads can then be increased to test the failure loads of in different support conditions and to show the formation of plastic hinges. The basic setup for the portal frames is similar, but both horizontal and vertical loads are exerted on the frames (shown in Figure 13). By manually adjusting load ratios, the formation of different mechanism can be observed and explored.

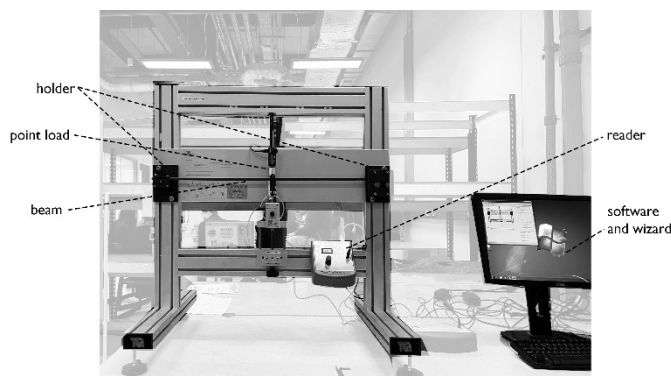


Figure 12. Setup used for tests on beams.

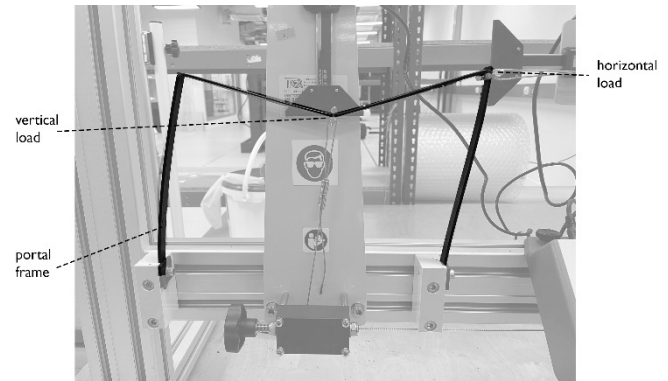


Figure 13. Setup for tests on portal frames.

2.2 PROCEDURE

Beam tests:

- Measure the cross section of the beam and adjust the support condition by changing the holders. Fill the information in the wizard. Read and record the ultimate load and displacement from the wizard.
- Rotate the knob and at the same time read the values of point loads on the reader. Adjust point loads with reasonable steps of increase. Record loads and displacements after each step.
- The ultimate load is reached when the forces on the reader can no longer be increased. Unload the beam with the same step and record.
- Repeat with two other support conditions.

Portal frame tests:

- Measure the cross section of the portal frame. With a similar setup, attach both horizontal and vertical point loads to the frame.
- Read the ultimate loads, displacements, and order of formed plastic hinges and corresponding loads from the wizard.
- Rotate the knobs and adjust both vertical and horizontal loads simultaneously with a certain load ratio and reasonable steps of increase.
- The ultimate load is reached when the forces on the reader can no longer be increased. The value stabilizes at this point.
- Repeat the tests with two other load ratios. Observe the mechanism and plastic hinges formed.
- Compare and discuss the calculated and experimental results.

2.3 RESULT

2.3.1 BEAMS

The data provided by the wizard and experimental results are listed in Table 1. The deformed shapes of beams after tests are shown in Figure 14. From top to bottom are respectively the original beam, fixed end beam, propped cantilever and simply supported beam.

	SIMPLY SUPPORTED BEAM		PROPPED CANTILEVER		FIX END BEAM	
	P(N)	δ (mm)	P(N)	δ (mm)	P(N)	δ (mm)
WIZARD	233.3	30.5	311.2	17.8	466.7	15.3
			350.1	22.9		
EXPERIMENT	0	0.0	0	0.0	0	0.0
	25	2.6	50	2.7	25	1.3
	50	5.6	100	6.3	50	2.5
	75	8.7	150	8.7	75	4.2
	100	12.4	200	12.2	100	5.9
	125	16.3	250	15.3	125	7.4
	150	19.1	230	25.9	150	8.1
	175	21.1	230	28.5	175	9.1
	180	25.3	230	32.3	200	10.1
	180	31.7	230	35.1	225	11.1
	180	32.7	150	30.1	250	12.0
	180	34.8	100	27.4	275	13.1
	100	25.7	50	23.9	300	14.4
	50	18.8	0	21.0	300	17.8
	0	12.7			303	21.5
					324	26.8
					342	33.9
					355	39.5
					363	45.8
					363	51.0
					363	56.0
					363	59.8
					363	60.7
					250	57.5
					200	55.7
					150	53.7
					50	49.4
					0	47.5

Table 1. Experimental results of loads and displacements of beams under different support conditions. P represents loads on beams, and δ represents the deformation of beam under loads.

2.3.2 PORTAL FRAMES

The data provided by the wizard and experimental results are listed in the tables below. It should be noted that the dimensions of frames used during the tests are slightly different from the given values. The deformed portal frame under loads H:V=1:2 is shown in Figure 15.

PORTAL FRAME H:V=1:2

WIZARD

	H (N)	V(N)	δH (mm)	δV (mm)
1	119.91	239.83	8.90	8.20
2	121.95	243.90	9.30	8.50
3	124.30	248.59	10.70	11.60
4	138.50	277.00	39.80	27.90

EXPERIMENT

	H (N)	V(N)	δH (mm)	δV (mm)
	0.00	0.00	0.00	0.00
	12.50	8.90	0.08	0.67
	20.00	40.00	0.48	1.29
	30.50	59.30	1.40	2.40
	40.00	80.00	2.06	3.12
	49.70	109.70	3.40	4.17
	54.90	119.70	3.65	4.56
	59.80	129.70	4.05	4.87
	65.10	140.50	4.47	5.19
	69.70	149.70	4.94	5.54
	75.10	140.50	5.37	6.02
	80.10	149.70	5.66	6.50
	84.70	159.90	6.01	6.80
	89.90	170.00	7.17	7.73
	95.00	179.50	8.34	10.08
	100.00	188.50	10.47	11.28
	104.70	198.50	14.47	15.87
	110.00	205.00	18.96	21.21
	115.00	213.00	18.96	26.06
	120.00	213.00	20.87	28.81
	122.00	231.00	24.15	36.03
	120.00	234.00	28.24	42.07

SECTION

PLASTIC HINGES

PORTAL FRAME H:V=1:3

WIZARD

	H (N)	V (N)	δH (mm)	δV (mm)
1	76.42	229.26	6.30	8.70
2	81.10	243.31	6.70	11.20
3	85.97	257.91	11.10	21.20
4	95.52	286.57	28.3	50.5

SECTION

12.6 mm

3.1 mm

PLASTIC HINGES

EXPERIMENT

H (N)	V (N)	δH (mm)	δV (mm)
10.70	29.40	0.00	0.90
20.30	59.70	0.00	2.43
28.60	90.20	0.15	3.89
29.70	118.10	1.18	5.13
50.40	147.80	2.29	6.77
54.90	157.70	2.72	7.62
60.40	175.00	3.45	9.58
65.00	185.00	3.88	11.38
68.00	203.00	5.05	15.89
75.00	215.00	5.87	19.79
80.00	215.00	7.40	23.67
86.00	215.00	8.67	26.96
94.00	215.00	8.98	28.72
97.00	215.00	9.07	31.19
104.00	221.00	9.35	33.47
110.00	230.00	10.89	39.26
115.00	233.00	12.24	41.16
122.00	232.00	15.44	43.93
118.00	234.00	19.38	49.04

PORTAL FRAME H:V=2:1

WIZARD

	H (N)	V (N)	δH (mm)	δV (mm)
1	160.32	80.15	13.20	3.10
2	184.42	92.22	16.70	3.90
3	190.24	95.12	18.80	4.00
4	214.93	107.46	46.30	9.30

SECTION

12.6 mm

3.1 mm

PLASTIC HINGES

EXPERIMENT

H (N)	V (N)	δH (mm)	δV (mm)
0.00	0.00	0.00	0.00
20.60	10.20	0.00	0.36
40.20	20.00	0.63	0.56
60.30	29.90	2.37	0.85
79.70	40.20	3.80	1.29
99.80	49.80	5.28	1.83
110.20	55.00	6.68	2.28
120.30	60.60	8.53	2.66
123.00	67.10	9.81	3.40
135.00	70.30	11.54	3.91
144.20	75.00	13.05	4.27
141.70	80.40	16.28	5.44
151.10	83.70	18.74	6.29
163.10	90.40	22.39	7.64
159.00	94.50	24.44	8.41
166.10	100.00	27.43	9.59
163.00	105.30	28.81	10.27
168.10	110.00	30.05	10.98
171.00	110.60	32.67	12.65
172.90	113.80	33.39	13.29

Table 2-4. Experimental results of loads and displacements of beams under different support conditions. H and V represents loads on beams, and δ represents the deformation of beam under loads.

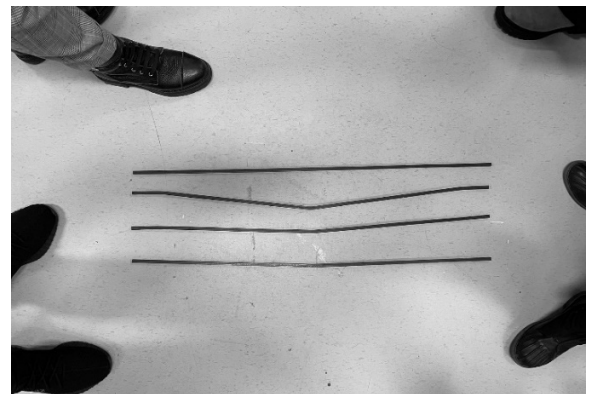


Figure 14. Deformed beams after tests.

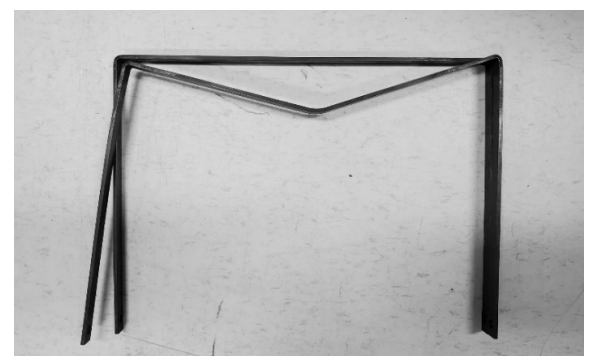


Figure 15. Deformed shape of a portal frame after test

3. ANALYSIS

3.1 BEAMS

3.1.1 Simply support beam

The theoretical load capacity has been calculated in 1.1.4.a. During the experiment, loads stabilize at around $P = 180N$ (as seen in Figure 16). Therefore,

$$P_{theoretical} = 233.39N$$

$$P_{experimental} = 180N$$

$$\Delta = \frac{P_{experimental} - P_{theoretical}}{P_{theoretical}} = -22.9\%$$

The experimental value of ultimate load is 22.9% lower than the theoretical value.

The theoretical displacement at the ultimate load is provided in the wizard data, which gives the value of δ immediately when the ultimate load is reached. This value should be compared with the first value of displacement δ when the load reaches the ultimate load 180N, after which the curve reaches a plateau. It can be obtained from Table 1 that:

$$\delta_{theoretical} = 30.5mm$$

$$\delta_{experimental} = 25.3mm$$

$$\Delta = \frac{\delta_{experimental} - \delta_{theoretical}}{\delta_{theoretical}} = 17\%$$

The experimental value of ultimate displacement is 17% higher than the theoretical value.

The actual yield strength σ_y of the beam can be calculated with experimental ultimate loads. Reorganizing eq. 3,

$$P_p \times \frac{l}{2} \times \varphi = M_p \times 2\varphi \text{ (eq. 3)}$$

$$M_p = \frac{l \times P_p}{4}$$

Reorganizing eq. 2 and bringing the expression for M_p ,

$$M_p = \frac{1}{2} A(y_1 + y_2) \sigma_y \text{ (eq. 2)}$$

$$\begin{aligned} \sigma_y &= \frac{2M_p}{A(y_1 + y_2)} = \frac{l \times P_p}{2A(y_1 + y_2)} \\ &= \frac{750mm \times 180N}{2 \times 7.9mm \times 7.9mm \times \frac{7.9mm}{2}} \\ &= 273.8MPa \end{aligned}$$

Therefore, the actual yield strength σ_y is 22.9% lower than the assumed value which is $\sigma_y = 355MPa$.

Only one plastic hinge is formed during the test, and the phenomenon of full plasticity has occurred, because an obvious turning point in the beam can be observed (seen in Figure 14) and the shape is not restored after the unloading phase.

The force-displacement curve during both loading and unloading phases is plotted in Figure 16, where the phases during the test are also identified. As the step of increase of load is not small enough, precise yielding phase is difficult to identify with low accuracy of the curve. The phases can only be spotted qualitatively.

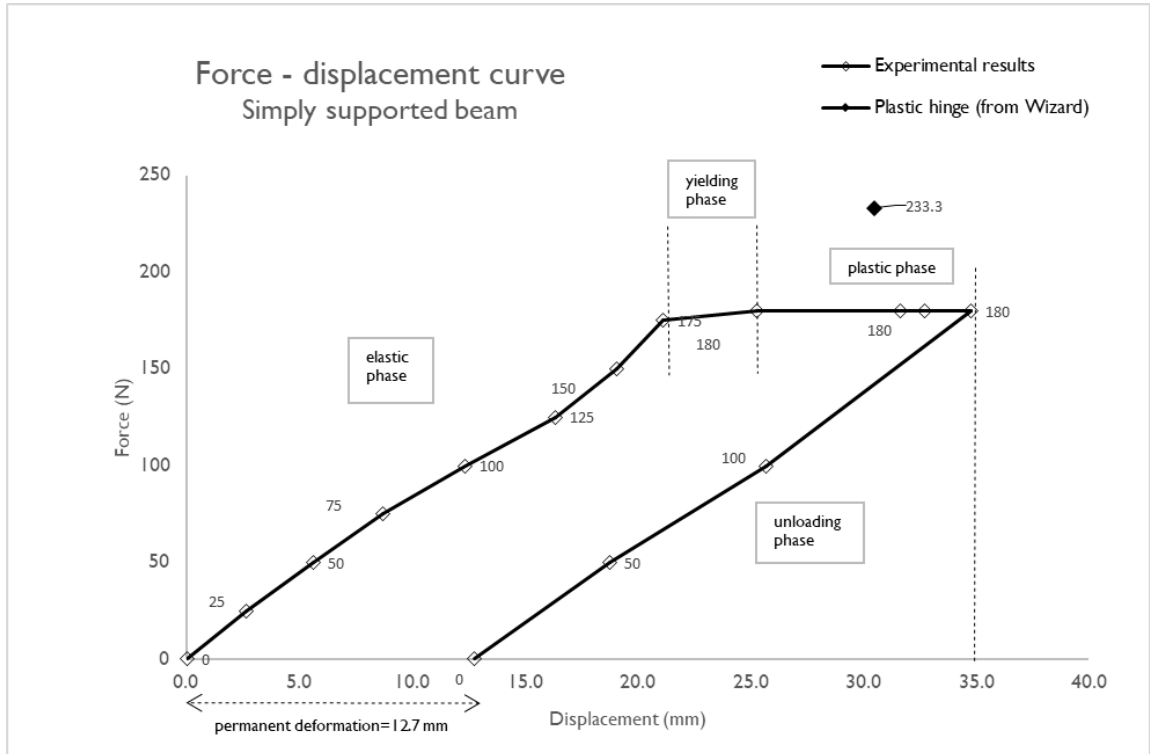


Figure 16. Force-displacement curve of simply supported beam

3.1.2 Propped cantilever

During the experiment, loads stabilize at around $P = 230N$ (as seen in Table 1), which is obtained as the ultimate load of the beam. Different from the simply supported beam, two plastic hinges instead of one should be formed during this test. The two plastic hinges will form consecutively, and two sets of values of loads and displacements can be obtained from the Wizard. The second set corresponds to the situation where both hinges are formed and full plasticity has been reached, and thus should be chosen as the ultimate load and corresponding displacement. Therefore,

$$P_{theoretical} = 350.08N$$

$$P_{experimental} = 230N$$

$$\Delta = \frac{P_{experimental} - P_{theoretical}}{P_{theoretical}} = -34.3\%$$

The experimental value of ultimate load is 34.3% lower than the theoretical value.

Similar to the calculation for simply supported beam, the value of theoretical displacement when the ultimate load is reached can be obtained from Table 1. However, before the ultimate load stabilizes at 230N, it once reaches 250N, which forms a peak on the curve (as seen in Figure 17). This can be seen as the upper yield point (in Figure 18), and the displacement should be obtained when load reaches 230N.

$$\delta_{theoretical} = 22.9mm$$

$$\delta_{experimental} = 25.9mm$$

$$\Delta = \frac{\delta_{experimental} - \delta_{theoretical}}{\delta_{theoretical}} = 13.1\%$$

The experimental value of ultimate displacement is 13.1% higher than the theoretical value.

The actual yield strength σ_y of the beam can be calculated with experimental ultimate loads.

$$P_p \times \frac{l}{2} \times \varphi = M_p \times 3\varphi \text{ (eq. 4)}$$

$$M_p = \frac{l \times P_p}{6}$$

Reorganizing eq. 2 and bringing the expression for M_p ,

$$M_p = \frac{1}{2} A(y_1 + y_2) \sigma_y \text{ (eq. 2)}$$

$$\begin{aligned} \sigma_y &= \frac{2M_p}{A(y_1 + y_2)} = \frac{l \times P_p}{3A(y_1 + y_2)} \\ &= \frac{750mm \times 230N}{3 \times 7.9mm \times 7.9mm \times \frac{7.9mm}{2}} \\ &= 233.2MPa \end{aligned}$$

Therefore, the actual yield strength σ_y is 34.3% lower than the assumed value which is $\sigma_y = 355MPa$.

It can also be observed from Figure 14 that the plastic hinge at the midspan of the beam can be clearly observed, but the one at the fixed end is not very evident. This is in accordance with the fact that the bending moment at the midspan is maximum, and at the fixed end is slightly lower. Therefore, with the increase of load the plastic hinge at the midspan would occur first. However, it cannot be judged for sure if both plastic hinges can be observed or if full plasticity is reached. Clearer photographs should have been taken and closer observations should have been done.

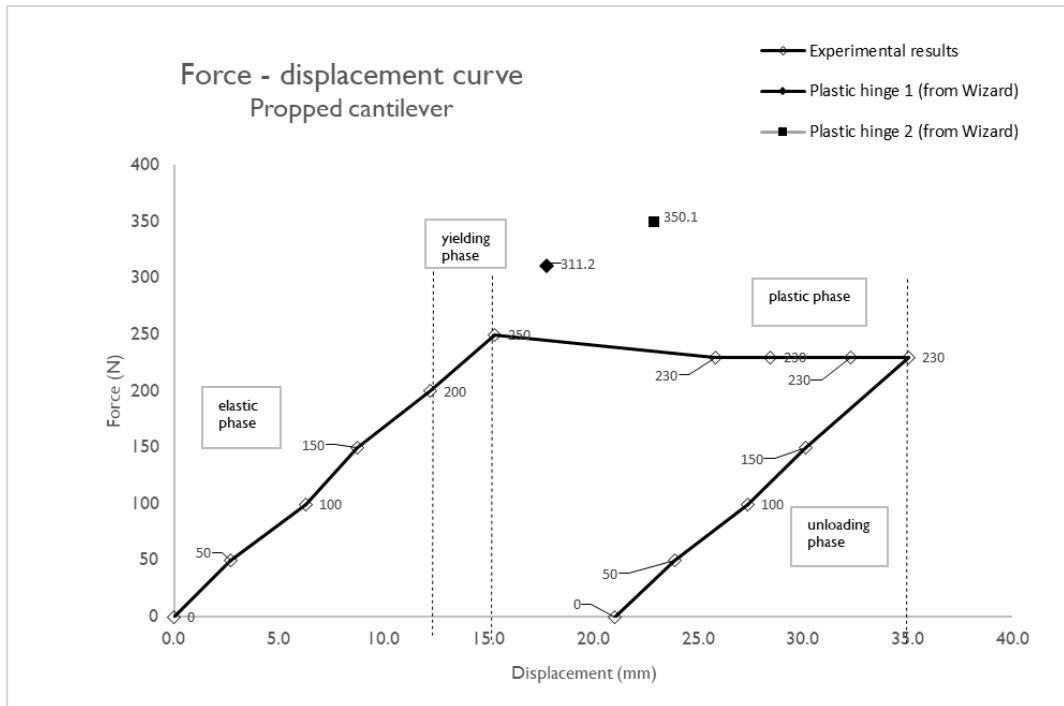


Figure 18. Force-displacement curve of propped cantilever

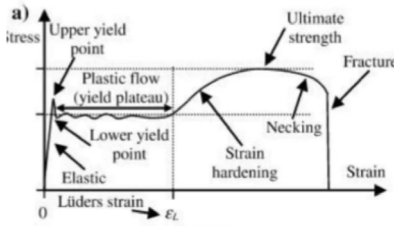


Figure 17. Qualitative stress strain curve for steel

The force-displacement curve during both loading and unloading phases is plotted in Figure 18, where the phases during the test are also identified. As the step of increase of load is not small enough, precise yielding phase is difficult to identify with low accuracy of the curve. The phases can only be spotted qualitatively.

3.1.3 Fixed end beam

During the test, load stabilizes at around $P = 363N$ (as seen in Table 1), which should be obtained as the ultimate load of the beam. Different from the two other tests, three plastic hinges will be formed at the same time. The value of ultimate load and displacement can be obtained from the Wizard.

$$P_{theoretical} = 466.77N$$

$$P_{experimental} = 363N$$

$$\Delta = \frac{P_{experimental} - P_{theoretical}}{P_{theoretical}} = -22.2\%$$

The experimental value of ultimate load is 22.2% lower than the theoretical value.

Similar to previous calculations, the value of theoretical displacement when the ultimate load is reached can be

obtained from Table 1. The experimental displacement at the ultimate load corresponds to the value of δ when the load first reaches 363N. Therefore,

$$\delta_{theoretical} = 15.3mm$$

$$\delta_{experimental} = 45.8mm$$

$$\Delta = \frac{\delta_{experimental} - \delta_{theoretical}}{\delta_{theoretical}} = 199.3\%$$

The experimental value of ultimate displacement is almost two times higher than the theoretical value, which means that there exist large errors during the test. They include the inappropriate operations during the test, high level of impurity of the material, the ends of the frame not being tight fixed, etc.

The actual yield strength σ_y of the beam can be calculated with experimental ultimate loads. Reorganizing eq. 5,

$$P_p \times \frac{l}{2} \times \varphi = M_p \times 4\varphi \text{ (eq. 5)}$$

$$M_p = \frac{l \times P_p}{8}$$

Reorganizing eq. 2 and bringing the expression for M_p ,

$$M_p = \frac{1}{2} A(y_1 + y_2) \sigma_y \text{ (eq. 2)}$$

$$\begin{aligned} \sigma_y &= \frac{2M_p}{A(y_1 + y_2)} = \frac{l \times P_p}{4A(y_1 + y_2)} \\ &= \frac{750mm \times 363N}{4 \times 7.9mm \times 7.9mm \times \frac{7.9mm}{2}} \\ &= 276.1MPa \end{aligned}$$

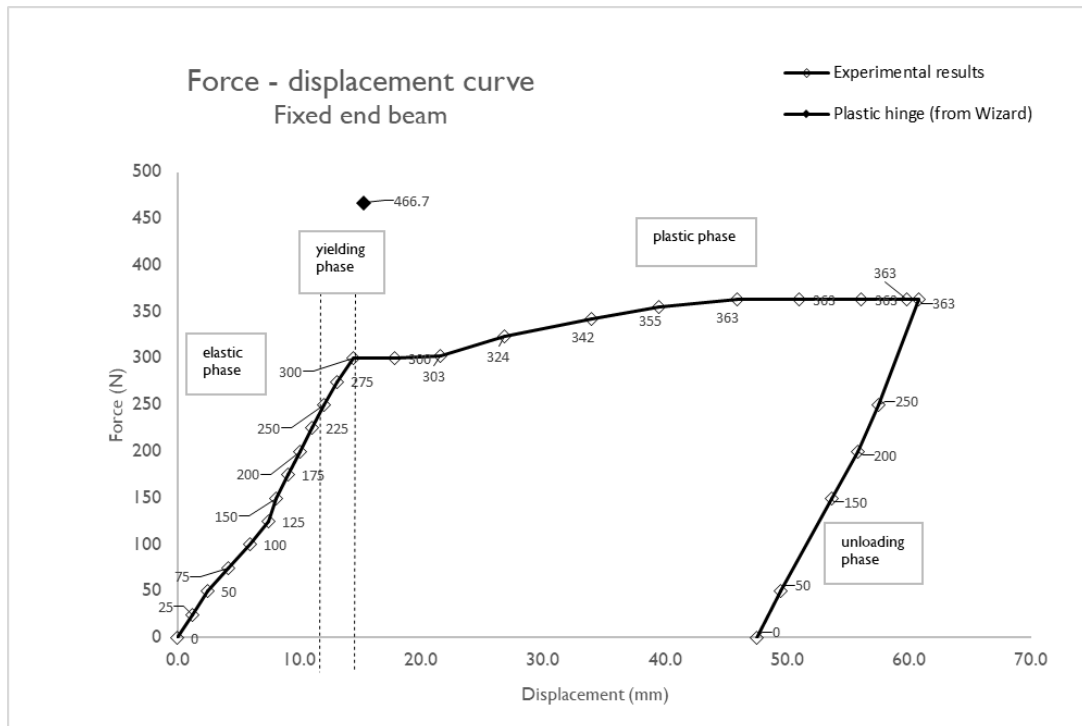


Figure 19. Force-displacement curve of fixed end beam

Therefore, the actual yield strength σ_y is 22.2% lower than the assumed value which is $\sigma_y = 355\text{MPa}$.

It can be seen from Figure 14 that plastic hinges at the midspan and both ends of the beam can be relatively clearly observed. The sequence of formation, however, has not been captured during the tests, and in future tests better pictures and recordings should be taken. In theory all three plastic hinges should be formed at the same time, in accordance with the fact that the bending moment at midspan and both ends of the beam is the same (see in section 1.1.1.c).

The force-displacement curve during both loading and unloading phases is plotted in Figure 19, where the phases during the test are also identified. Similarly, it is only a qualitative identification of the phases.

3.2 PORTAL FRAMES

3.2.1 Beam mechanism ($H:V=1:3$)

The theoretical load capacity is given in the Wizard by the value of load when the last plastic hinge is formed. According to results in section 2.3.2,

$$H_{theoretical} = 95.52\text{N}$$

$$H_{experimental} = 122\text{N}$$

$$V_{theoretical} = 286.57\text{N}$$

$$V_{experimental} = 234\text{N}$$

$$\Delta_H = \frac{H_{theoretical} - H_{experimental}}{H_{theoretical}} = 27.7\%$$

$$\Delta_v = \frac{V_{theoretical} - V_{experimental}}{V_{theoretical}} = -18.3\%$$

The experimental and theoretical values differ for many reasons. Firstly, during the tests the ratios between horizontal and vertical forces are difficult to maintain, which can lead to very different results. Secondly, the readings tend to fluctuate dramatically when ultimate load is about to be achieved, and thus the recorded values can be inaccurate. Thirdly, there might exist impurities within the materials, which will also cause large uncertainties.

The values of displacement at ultimate loads can also be obtained from the Table 2-4.

$$\delta H_{theoretical} = 28.3\text{mm}$$

$$\delta H_{experimental} = 19.38\text{mm}$$

$$\delta V_{theoretical} = 50.5\text{mm}$$

$$\delta V_{experimental} = 49.04\text{mm}$$

$$\Delta_{\delta H} = \frac{\delta H_{theoretical} - \delta H_{experimental}}{\delta H_{theoretical}} = -31.5\%$$

$$\Delta_{\delta v} = \frac{\delta V_{theoretical} - \delta V_{experimental}}{\delta V_{theoretical}} = -2.9\%$$

The sequence of formation of plastic hinges has not been captured and recorded. However, the formation of the first plastic hinge in the middle of the frame has been observed, which complies with the results provided by the wizard. All three plastic hinges are observed and full plasticity has been reached. This can be justified by the obvious 'M' shape of the structure after the test.

The force-displacement curve during both loading and unloading phases is plotted in Figure 20. During the experiment the steps of increase of the loads are not small enough, and as a result only very rough shape of curve is plotted and it is difficult to spot the exact locations of turning points and changes of slope. As a result, the identification of formation of plastic hinges is only qualitative, which corresponds to the observed formation of plastic hinges on the beam. Exact values of loads at which plastic hinges occur can not be obtained for this test.

3.2.2 Sway mechanism ($H:V=2:1$)

The theoretical load capacity is given in the Wizard by the value of load when the last plastic hinge is formed. According to results in section 2.3.2,

$$H_{theoretical} = 214.93\text{N}$$

$$H_{experimental} = 172.9\text{N}$$

$$V_{theoretical} = 107.46\text{N}$$

$$V_{experimental} = 113.8\text{N}$$

$$\Delta_H = \frac{H_{theoretical} - H_{experimental}}{H_{theoretical}} = -19.5\%$$

$$\Delta_v = \frac{V_{theoretical} - V_{experimental}}{V_{theoretical}} = 5.9\%$$

The reasons for the difference between experimental and theoretical values are similar to reasons for the beam mechanism.

The values of displacement at ultimate loads can also be obtained from the Table 2-4.

$$\delta H_{theoretical} = 46.3\text{mm}$$

$$\delta H_{experimental} = 33.39\text{mm}$$

$$\delta V_{theoretical} = 9.3\text{mm}$$

$$\delta V_{experimental} = 13.29\text{mm}$$

$$\Delta_{\delta H} = \frac{\delta H_{theoretical} - \delta H_{experimental}}{\delta H_{theoretical}} = -27.9\%$$

$$\Delta_{\delta v} = \frac{\delta V_{theoretical} - \delta V_{experimental}}{\delta V_{theoretical}} = 43.9\%$$

Plastic hinges can be clearly observed at the fixed ends and top right corner of the frame, as shown in Figure 23. This complied with the sequence of formation of hinges provided by the wizard. Full plasticity is not reached, because the fourth plastic hinge on the top left corner has not occurred. The reasons include impurities of materials, wrong operations during tests, etc.

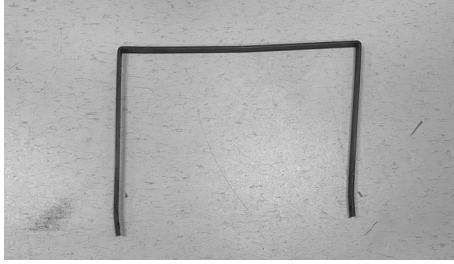


Figure 23. Deformed portal frame under H:V=2: 1

The force-displacement curve during both loading and unloading phases is plotted in Figure 21. Similar to Figure 20, the analysis is only qualitative and no exact values can be obtained.

3.2. Combined mechanism (H:V=1:2)

The theoretical load capacity is given in the Wizard by the value of load when the last plastic hinge is formed. According to results in section 2.3.2,

$$H_{theoretical} = 138.5N$$

$$H_{experimental} = 122N$$

$$V_{theoretical} = 277N$$

$$V_{theoretical} = 234N$$

$$\Delta_H = \frac{H_{theoretical} - H_{experimental}}{H_{theoretical}} = -11.9\%$$

$$\Delta_v = \frac{V_{theoretical} - V_{experimental}}{V_{theoretical}} = -15.5\%$$

The reasons for the difference between experimental and theoretical values are similar to reasons for the beam mechanism.

The values of displacement at ultimate loads can also be obtained from the Table 2-4.

$$\delta H_{theoretical} = 39.8mm$$

$$\delta H_{experimental} = 28.24mm$$

$$\delta V_{theoretical} = 27.9mm$$

$$\delta V_{experimental} = 42.07mm$$

$$\Delta_{\delta H} = \frac{\delta H_{theoretical} - \delta H_{experimental}}{\delta H_{theoretical}} = -29.6\%$$

$$\Delta_{\delta v} = \frac{\delta V_{theoretical} - \delta V_{experimental}}{\delta V_{theoretical}} = 50.5\%$$

It can be observed from the results of all three

mechanisms that the experimental horizontal displacements are all larger than theoretical ones, and experimental vertical displacement are all smaller than theoretical ones. This implies that potentially systematic problems exist in the apparatus.

All four plastic hinges at both fixed ends, middle point and top right corner can be observed. Therefore, full plasticity is achieved. The sequence of formation of the hinges is not captured and recorded during the test.

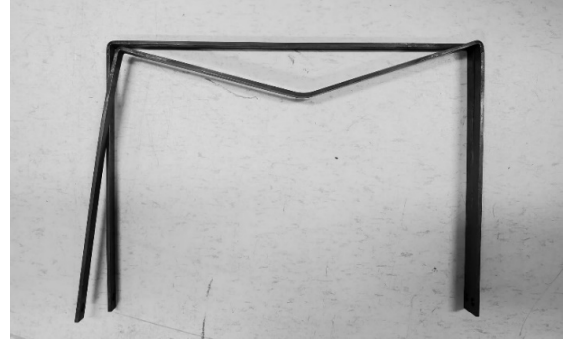


Figure 24. Deformed portal frame under H:V=1:2

The force-displacement curve during both loading and unloading phases is plotted in Figure 22. Similarly, the analysis is only qualitative and no exact values can be obtained.

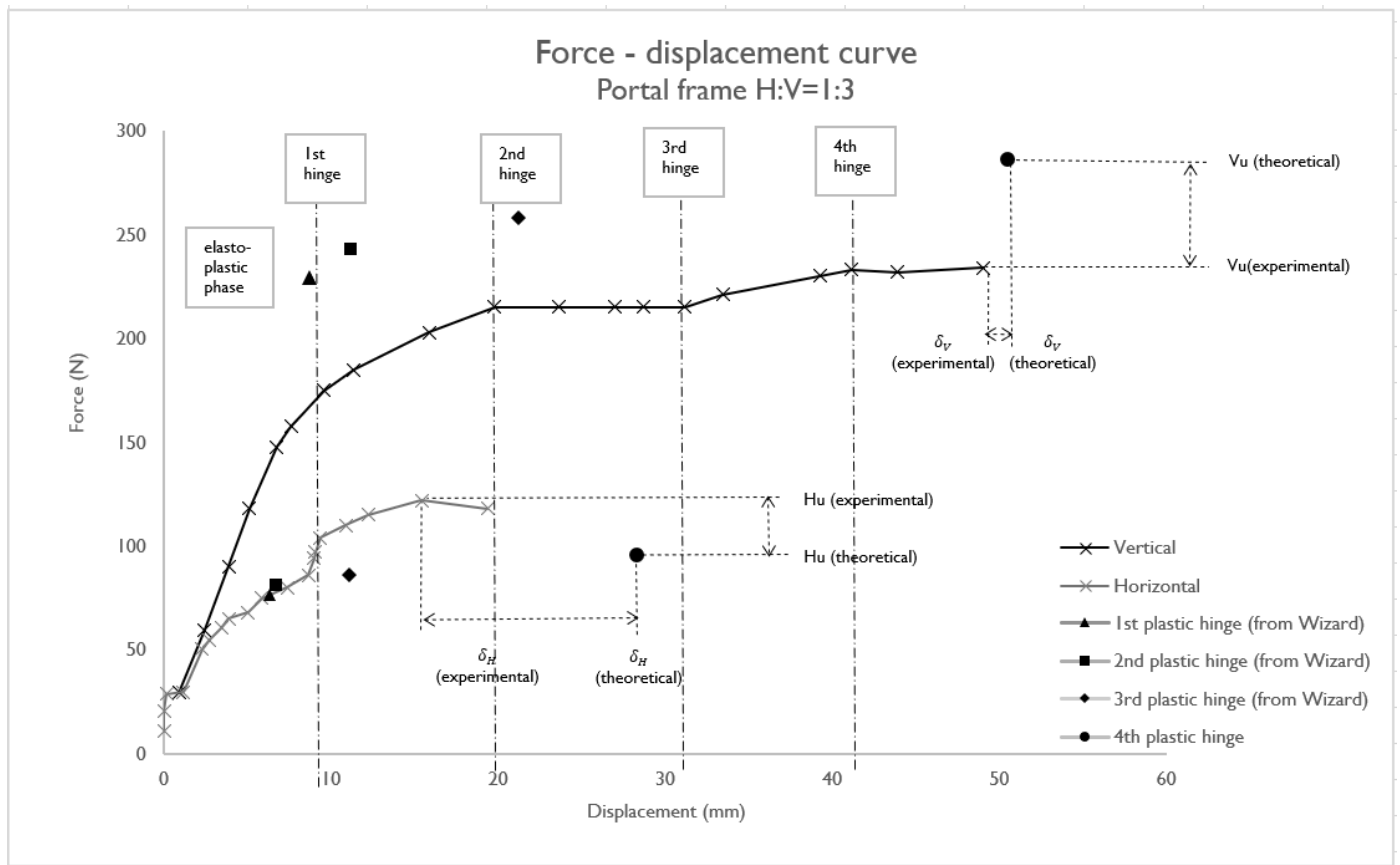


Figure 20. Force-displacement curve of portal frame H:V=1:3 (beam mechanism)

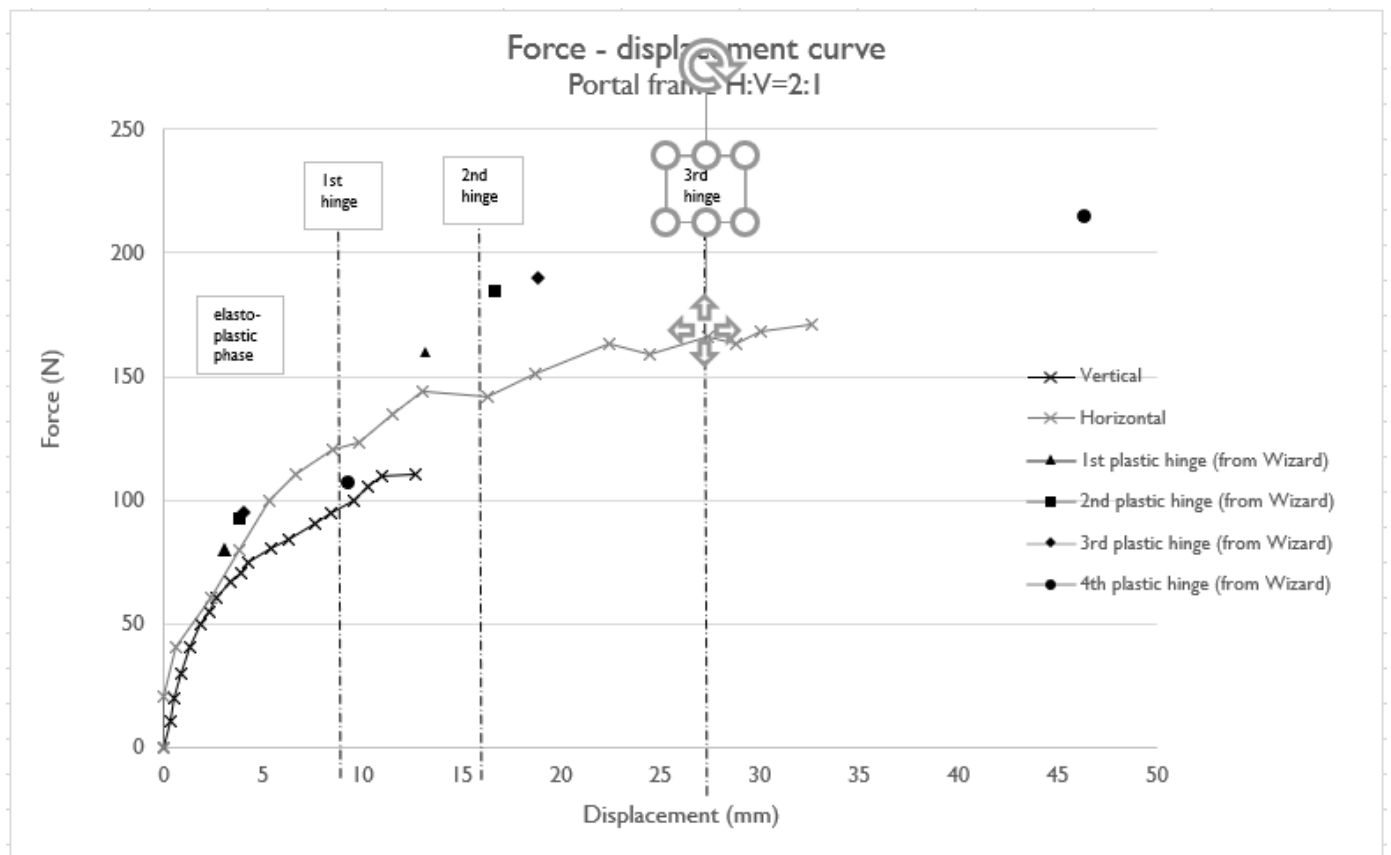


Figure 21. Force-displacement curve of portal frame H:V=2:1 (sway mechanism)

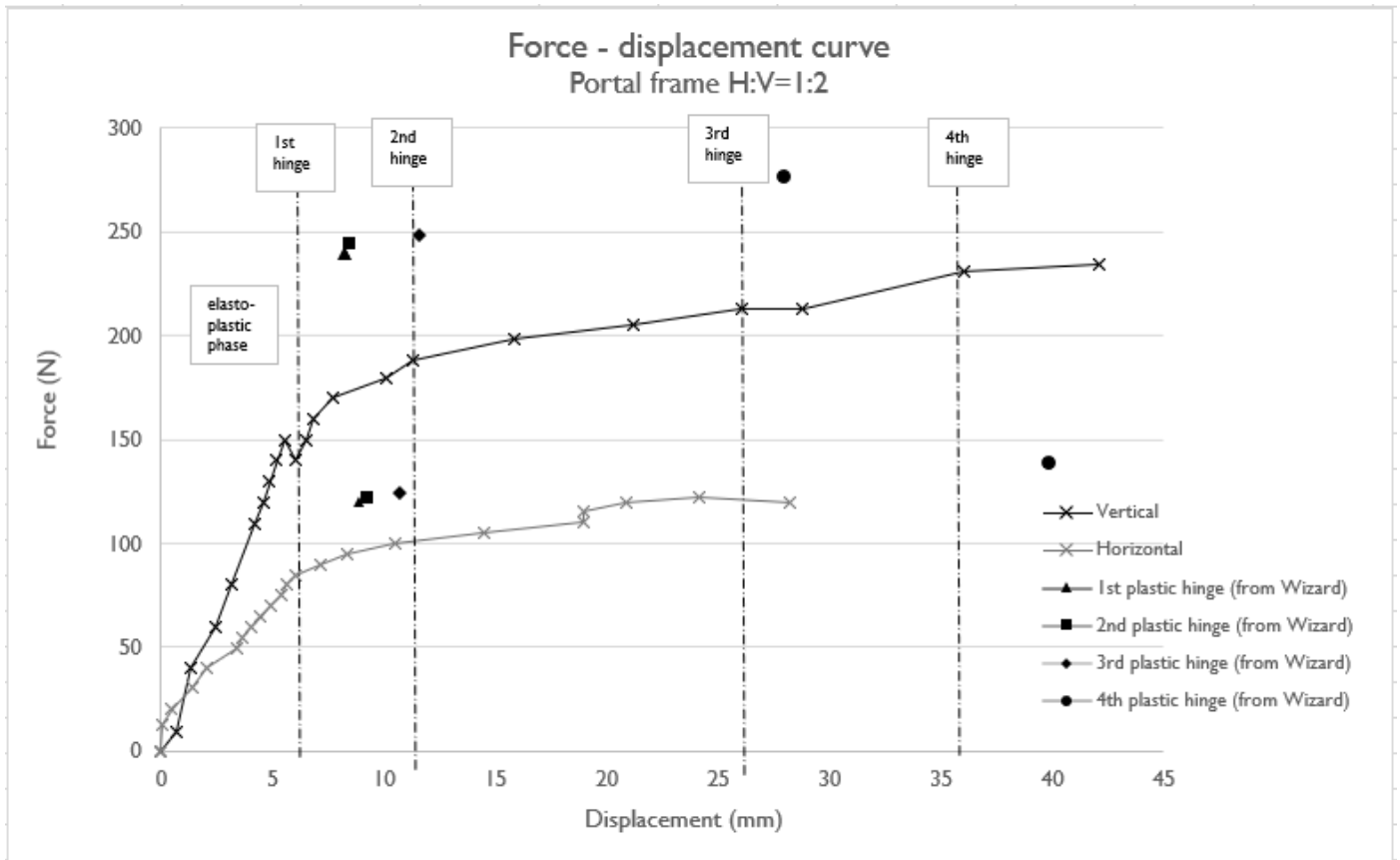


Figure 22. Force-displacement curve of portal frame H:V=1: 2 (combined mechanism)

4. SCANNED DOCUMENTS

CIVIL ENVIRONMENTAL AND GEOMATIC ENGINEERING

$r=0$
 $r=1 \Rightarrow f=1$
 $P_c = \frac{qL^2}{8} = 233.5$

SIMPLY SUPPORTED BEAM
 From the Wizard (i.e., Theoretical results)
 Take the values of Load and Displacement corresponding to the formation of the plastic hinge.

P (N)	δ (mm)
233.5	80.5

Note:

From Experiment (i.e., Experimental results)
 Record the observed values of the Loads and Displacements

P (N)	δ (mm)
0	0
25	2.63
50	5.62
75	8.69
100	12.35
125	15.32
150	19.89
175	21.13
200	25.18
225	31.66
250	32.74
275	34.78
300	25.47
325	16.71
350	12.74
375	
400	
425	
450	
475	
500	
525	
550	
575	
600	
625	
650	
675	
700	
725	
750	
775	
800	
825	
850	
875	
900	
925	
950	
975	
1000	

Table 1

University College London, Gower Street, London WC1E 6BT
 Tel: +44 (0)20 7679 7226
 d.dayala@ucl.ac.uk

CIVIL ENVIRONMENTAL AND GEOMATIC ENGINEERING

Increment: 25 N

FIXED ENDS
 From the Wizard (i.e., Theoretical results)
 Take values of the Loads and Displacements corresponding to the formation of the plastic hinge.

P (N)	δ (mm)
266.79	15.3

Note: hinges are formed at the same time.

From Experiment (i.e., Experimental results)
 Record the observed values of the Loads and Displacements

P (N)	δ (mm)
0	0
25	1.28
50	2.46
75	4.17
100	5.93
125	7.49
150	9.15
175	9.55
200	10.10
225	11.47
250	12.46
275	13.99
300	14.97
325	17.89
350	21.72
375	24.82
400	27.72
425	29.29
450	31.83
475	30.97
500	34.17
525	39.15
550	46.73
575	57.61
600	67.87
625	73.65
650	99.24
675	47.92
700	
725	
750	
775	
800	
825	
850	
875	
900	
925	
950	
975	
1000	

Table 2

University College London, Gower Street, London WC1E 6BT
 Tel: +44 (0)20 7679 7226
 d.dayala@ucl.ac.uk

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QUESTIONS

- How do loads at the ultimate load capacity relate between the theoretical and the experimental results?
- How much do they differ?
- How do displacements at the ultimate load capacity relate between the theoretical and the experimental results?
- Based on the value of the ultimate load capacity obtained for the beam during the experiment, compute the value of the actual yielding strength for the beam and compare it with the initial assumed value.
- Did you manage to observe the formation of the two hinges (i.e., formation of first plastic hinge, formation of first and second plastic hinge or none of the previous)? Clearly explain which stage you could observe and why.
- How would you explain the reason for the sequence of plastic hinge formation observed? Support your answer with sketches if needed.
- Draw the force-displacement curve according to the data you've gathered during the experiment. Plot the curve for both loading and unloading phases and determine the permanent plastic deformation.
- Where possible, identify the experimental forces and displacements values which fall into the following:
 - Elastic phase
 - Yielding phase
 - Plastic phase
 - Unloading

or 1.

University College London, Gower Street, London WC1E 6BT
 Tel: +44 (0)20 7679 7226
 d.dayala@ucl.ac.uk

CIVIL ENVIRONMENTAL AND GEOMATIC ENGINEERING

$r=2$
 $r=3 \Rightarrow f=1$
 $P_c = \frac{qL^2}{8} = 466.8$

hinges: A, B, C

QUESTIONS

- How do loads at the ultimate load capacity relate between the theoretical and the experimental results?
- How much do they differ?
- How do displacements at the ultimate load capacity relate between the theoretical and the experimental results?
- Based on the value of the ultimate load capacity obtained for the beam during the experiment, compute the value of the actual yielding strength for the beam and compare it with the initial assumed value.
- Did you manage to observe the sequence of formation of the three plastic hinges? Clearly explain which stage you could observe and why.
- How would you explain the reason for the sequence of plastic hinge formation observed? Support your answer with sketches if needed.
- Draw the force-displacement curve according to the data you've gathered during the experiment. Plot the curve for both loading and unloading phases and determine the permanent plastic deformation.
- Where possible, identify the experimental forces and displacements values which fall into the following:
 - Elastic phase
 - Yielding phase
 - Plastic phase
 - Unloading

Please scan and submit this handout with your report.

hinges are formed at the same time together

University College London, Gower Street, London WC1E 6BT
 Tel: +44 (0)20 7679 7226
 d.dayala@ucl.ac.uk

