Physics 340 Final Exam

Problem 1

1) a) $Z_1 = X_1 + iY_1$

$Z_2 = X_2 + iY_2$

$Z_1 Z_2 = Re(Z) \neq 0$

Show that there exits an r that $Z_1 = r Z_2^*$

$Z_1 Z_2 = (X_1 + iY_1)(X_2 + iY_2)$

$= X_1 X_2 + iX_1 Y_2 + iX_2 Y_1 + i^2 Y_1 Y_2$

$Z_1 Z_2 = (X_1 + X_2 - Y_1 Y_2) + i(X_1 Y_2 + X_2 Y_1)$

b) $Z_1 = r Z_2^*$ * complex conjugate

$Z_2^* = X_2 - iY_2$

plug in

$X_1 + iY_1 = r(X_2 - iY_2)$

$X_1 + iY_1 = rX_2 - riY_2$

Real

$rX_2 = X_1$

$\boxed{r = \frac{X_1}{X_2}}$

Imaginary

$riY_2 = -iY_1$

$\boxed{r = \frac{-Y_1}{Y_2}}$

But from the equation,

$Z_1 Z_2 = \ldots$

Since this is real & non-zero,

$X_1 Y_2 + X_2 Y_1 = 0$

$X_2 Y_1 = -X_1 Y_2$

$\frac{X_1}{X_2} = \frac{-Y_1}{Y_2}$

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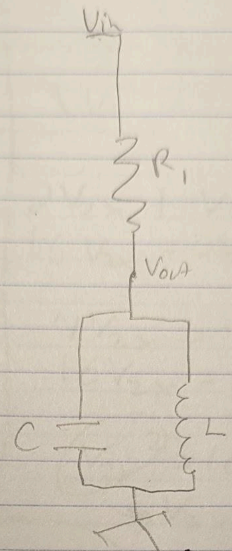
problem 2

2) $V_{in} = 1V$
$R_1 = 1\times10^{3}\Omega$
$C = 1\times10^{-6}F$
$L = 1\times10^{-6}$ Hertz
$V_{out} = ?$

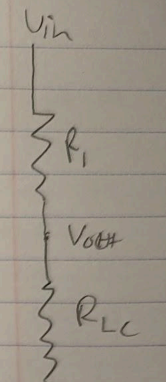
$$\frac{1}{Z_{eff}} = \frac{1}{Z_L} + \frac{1}{Z_c}$$

$$Z_{eff} = \frac{Z_L Z_c}{Z_L + Z_c}$$

Where $Z_L = j\omega L$
$Z_c = \frac{1}{j\omega c}$
$Z_R = R$



is the sames as



$R_{eff} = R_1 + R_{LC}$

$$V_{out} = V_{in}\left(Z_{eff}\right)\left(\frac{1}{Z_R + Z_{eff}}\right)$$

plugged into my code and graphed

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problem #3

3) $\dot{x} = 2x + y$
$\dot{y} = 16x + 2y$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 16 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$\lambda_1, \lambda_2 = ?$

$$\begin{vmatrix} 2-\lambda & 1 \\ 16 & 2-\lambda \end{vmatrix} = 0$$

$(2-\lambda)(2-\lambda) - 16 = 0$
$4 - 4\lambda + \lambda^2 - 16 = 0$
$\lambda^2 - 4\lambda - 12 = 0$
$\lambda = \frac{4 \pm \sqrt{16+48}}{2}$
$= \frac{4 \pm 8}{2} = 2 \pm 4 = 6, -2$

$\boxed{\lambda_1 = 6, \quad \lambda_2 = -2}$

$$\begin{pmatrix} 2 & 1 \\ 16 & 2 \end{pmatrix} \begin{pmatrix} V_{1x} \\ V_{1y} \end{pmatrix} = 6 \begin{pmatrix} V_{1x} \\ V_{1y} \end{pmatrix}$$

$2V_{1x} + V_{1y} = 6V_{1x}$
$16V_{1x} + 2V_{1y} = 6V_{1y}$

$V_{1y} = 4V_{1x} \quad V_{1y}^2 = 16V_{1x}^2$
$4V_{1x} = V_{1y}$

$V_{1x}^2 + V_{1y}^2 = 1$
$V_{1x}^2 + 16V_{1x}^2 = 1$
$17V_{1x}^2 = 1$
$\boxed{V_{1x} = \frac{1}{\sqrt{17}} \quad V_{1y} = \frac{4}{\sqrt{17}}}$

$$\begin{pmatrix} 2 & 1 \\ 16 & 2 \end{pmatrix} \begin{pmatrix} V_{2x} \\ V_{2y} \end{pmatrix} = -2 \begin{pmatrix} V_{2x} \\ V_{2y} \end{pmatrix}$$

$2V_{2x} + V_{2y} = -2V_{2x}$
$16V_{2x} + 2V_{2y} = -2V_{2y}$

$4V_{2x} = -V_{2y}$
$16V_{2x} = -4V_{2y}$

$V_{2y}^2 = 16V_{2x}^2$
$V_{2x}^2 + V_{2y}^2 = 1$
$17V_{2x}^2 = 1$
$\boxed{V_{2x} = \frac{1}{\sqrt{17}}}$

$\frac{16}{\sqrt{17}} = -4V_{2y}$
$\boxed{V_{2y} = \frac{-4}{\sqrt{17}}}$

$$\boxed{\begin{aligned} V_1 &= \frac{1}{\sqrt{17}} \begin{pmatrix} 1 \\ 4 \end{pmatrix} \\ V_2 &= \frac{1}{\sqrt{17}} \begin{pmatrix} 1 \\ -4 \end{pmatrix} \end{aligned}}$$

$$S = \begin{pmatrix} V_{1x} & V_{2x} \\ V_{1y} & V_{2y} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{17}} & \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} & \frac{-4}{\sqrt{17}} \end{pmatrix}$$

$$S^{-1} = \begin{pmatrix} \frac{-4}{\sqrt{17}} & \frac{-1}{\sqrt{17}} \\ \frac{-4}{\sqrt{17}} & \frac{1}{\sqrt{17}} \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} 6 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = S \Lambda S^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \dot{r} \\ \dot{s} \end{pmatrix} = \Lambda \begin{pmatrix} r \\ s \end{pmatrix}$$

$$\begin{pmatrix} \dot{r} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix}$$

$$\dot{r} = 6r$$

$$\boxed{\begin{aligned} r &= c_1 e^{6t} \\ s &= c_2 e^{-2t} \end{aligned}}$$

$$S^{-1} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{17}} & \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} & \frac{-4}{\sqrt{17}} \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix}$$

$$x = \frac{1}{\sqrt{17}} r + \frac{1}{\sqrt{17}} s$$

$$y = \frac{4}{\sqrt{17}} r - \frac{4}{\sqrt{17}} s$$

eneral

$$\boxed{\begin{aligned} x &= \frac{1}{\sqrt{17}} c_1 e^{6t} + \frac{1}{\sqrt{17}} c_2 e^{-2t} \\ y &= \frac{4}{\sqrt{17}} c_1 e^{6t} \quad \frac{-4}{\sqrt{17}} c_2 e^{-2t} \end{aligned}}$$

$$x(0) = 1$$
$$y(0) = 0$$

$$x(0) = 1 = \frac{1}{\sqrt{17}} c_1 + \frac{1}{\sqrt{17}} c_2$$

$$y(0) = 0 = \frac{4}{\sqrt{17}} c_1 \; \frac{-4}{\sqrt{17}} c_2$$

$$\frac{4}{\sqrt{17}} c_1 = \frac{4}{\sqrt{17}} c_2$$

$$c_1 = c_2$$

$$1 = \frac{1}{\sqrt{17}} 2c_1$$

$$c_1 = \frac{\sqrt{17}}{2} = c_2$$

$$x(t) = \frac{1}{\cancel{\sqrt{17}}} \cdot \frac{\cancel{\sqrt{17}}}{2} e^{6t} + \frac{1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{2} e^{-2t}$$

$$y(t) = \frac{\cancel{4}^{2}}{\cancel{\sqrt{17}}} \cdot \frac{\cancel{\sqrt{17}}}{\cancel{2}} e^{6t} - \frac{\cancel{4}^{2}}{\cancel{\sqrt{17}}} \cdot \frac{\cancel{\sqrt{17}}}{\cancel{2}} e^{-2t}$$

$$x(t) = \frac{1}{2} e^{6t} + \frac{1}{2} e^{-2t}$$

$$y(t) = 2e^{6t} - 2e^{-2t}$$

$$\boxed{\begin{aligned} x(t) &= \tfrac{1}{2}(e^{6t} + e^{-2t}) \\ y(t) &= 2(e^{6t} - e^{-2t}) \end{aligned}}$$

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problem 4

4)

a) $a_0 = \frac{1}{2T}\int_{-T}^{T} f(x)\,dx$

$$\boxed{a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x|\,dx}$$

$a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\left(\frac{\pi n x}{T}\right)dx$

$$\boxed{a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\cos(nx)\,dx}$$

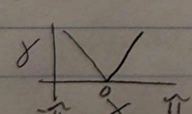
$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\left(\frac{\pi n x}{T}\right)dx$

$$\boxed{b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\sin(nx)\,dx}$$

$f(x) = a_0 + \sum_{n=1}^{n=6}\left(a_n\cos(nx) + b_n\sin(nx)\right)$

$$\boxed{= \frac{1}{2\pi}\int_{-\pi}^{\pi}|x|\,dx + \sum_{n=1}^{n=6}\left(\frac{1}{\pi}\int_{-\pi}^{\pi}\left(|x|\cos(nx)\right)dx\cos(nx) + \frac{1}{\pi}\int_{-\pi}^{\pi}\left(|x|\sin(nx)dx\right)\sin(nx)\right)}$$

b) plot it in matplotlib

c) 

Discontinuity at $x=0$

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problem 5

5) $\iint_S G(x,y,z)\,ds$ $\quad G(x,y,z) = x$

$0 \le x \le 1.4 \quad , \quad 0 \le y \le 4$

$\iint_S G\,n\,ds$

$G = x$

$n = ?$

$F = x^2 + z = 2$

$n = \nabla F = \left(\frac{\partial}{\partial x}(x^2+z), \frac{\partial}{\partial y}(x^2+z), \frac{\partial}{\partial z}(x^2+z)\right)$

$= (2x, 0, 1)$

$|n| = |\nabla F| = \sqrt{4x^2+1}$

$\int_0^4 \int_0^{1.4} x\sqrt{4x^2+1}\,dx\,dy = 8.42$