Basic Matrix Math

$$\begin{array}{cc} & \begin{array}{ccc}\text{Column 1} & \text{Column 2} & \text{Column 3}\end{array} \\ \begin{array}{c}\text{Row 1}\\ \text{Row 2}\\ \text{Row 3}\\ \text{Row 4}\end{array} & \begin{pmatrix} 3 & 2 & 7 \\ 4 & 5 & 3 \\ -1 & 6 & 1 \\ 0 & 3 & 9\end{pmatrix}\end{array} \quad a_{23}$$

rows columns

4 rows $\times$ 3 columns $= 4 \times 3$ matrix.

General Matrix Element a_{ij}

row column

e.g. a_{23} = Row 2, column 3

(= 3 above)

Usual Convention → use upper case characters for an entire matrix →

$$\widetilde{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Identity Matrix:

$$\mathbb{I} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

Matrix Algebra:

① Addition and Subtraction

→ can only add / subtract matrices of exactly the same shape!

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} + \begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 0 \\ 8 & 0 & 12 \end{pmatrix}$$

→ add / subtract element by element.

② Multiplication

(i) by a scalar

$$k\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$$

$$= \begin{pmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{pmatrix}$$

→ multiply <u>each</u> element by the scalar.

(ii) multiplying two matrices.

$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 0 & 2 & 4 \\ 3 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 0 & 2 & 5 \\ 2 & -2 & 0 & 1 & 1 \\ 3 & -3 & 0 & 5 & 2 \end{pmatrix}$$

Procedure : multiply <u>Rows</u> of first matrix by columns of

second matrix.

$$\begin{pmatrix} 2 & 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (2)(1) + (1)(2) + (3)(3)$$

$$= 13$$

→ start with first row, multiply by each column

→ move to next row ... continue.

$\boxed{\text{REQUIREMENT!}}$ # of columns of first matrix

= # of rows of second matrix!!

$$\tilde{A} \cdot \tilde{B} = \tilde{C}$$

$(m \times n) \cdot (n \times p) \rightarrow (m \times p)$

$(4 \times 3) \cdot (3 \times 5) \Rightarrow \boxed{4 \times 5}$

Topic:

$\tilde{A} \cdot \tilde{B}$ vs. $\tilde{B} \cdot \tilde{A}$

$(m \times n)\ (n \times p)$ $\quad$ $(n \times p)\ (m \times n)$

$m \times p$ $\quad$ $p = m$

$\boxed{m \times m}$ SQUARE MATRIX

So, this only works if $\tilde{A} \rightarrow m \times n$

$\tilde{B} \rightarrow n \times m$

$\therefore \quad \tilde{A} \cdot \tilde{B} \rightarrow m \times m$ result

$\tilde{B} \cdot \tilde{A} \rightarrow n \times n$ result.

different !!

$\therefore \quad \tilde{A} \cdot \tilde{B} \neq \tilde{B} \cdot \tilde{A}$

→ They might not even be the same size!!

But, what if $\tilde{A} \to n \times n$
$\tilde{B} \to n \times n$

Then $\tilde{A} \cdot \tilde{B}$ and $\tilde{B} \cdot \tilde{A}$ are both $n \times n$.

But, <u>still</u> $\tilde{A} \cdot \tilde{B} \neq \tilde{B} \cdot \tilde{A}$ in general!!

E.g.

$$\begin{array}{cc} \tilde{A} & \tilde{B} \\ \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} & \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} 8 & 5 \\ 20 & 13 \end{pmatrix}$$

$$\neq$$

$$\begin{array}{cc} \tilde{B} & \tilde{A} \\ \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} 13 & 20 \\ \text{[illegible]} & \text{[illegible]} \end{pmatrix}$$

$(3 \quad 8)$

Questions: Are there situations when $\tilde{A} \cdot \tilde{B} = \tilde{B} \cdot \tilde{A}$?

What are the properties / characteristics of $\tilde{A}$ and $\tilde{B}$ that would make this the case?

We will come back to this later, because it turns out that the answers are *really* important in physics!!!