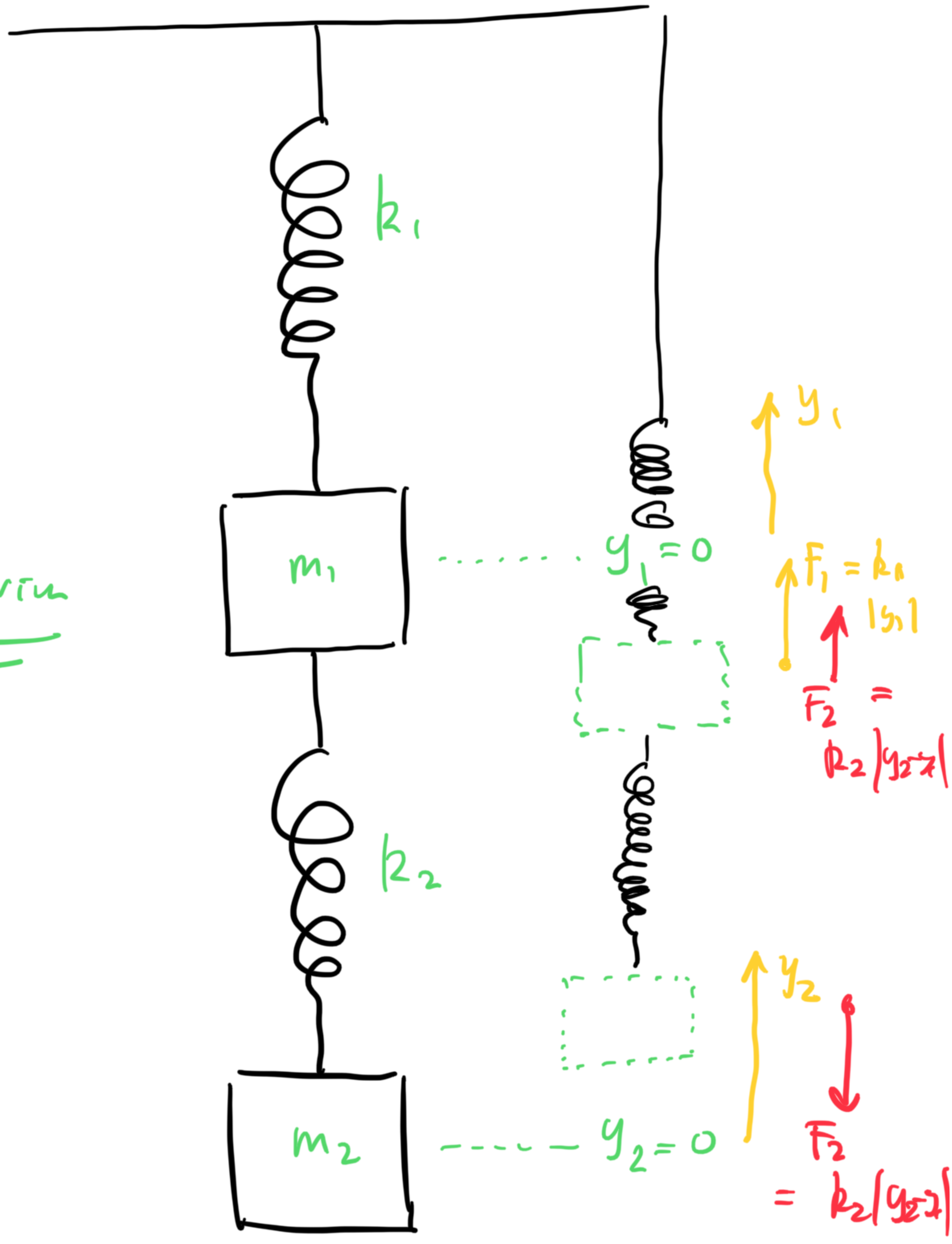
In equilibrium



$$m_1 \ddot{y}_1 = -k_1 y_1 + k_2 (y_2 - y_1)$$

$$m_2 \ddot{y}_2 = -k_2 (y_2 - y_1)$$

$$\left(\ddot{\;} \right) \quad \left(-k_1 - k_2 \quad k_2 \right) \left(x \right)$$

$$\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} = \begin{pmatrix} \frac{\quad}{m_1} & \frac{\quad}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} \end{pmatrix} \begin{pmatrix} \quad \\ y \end{pmatrix}$$

<u>Suppose</u> :

$$m_1 = m_2 = 1\,kg$$

$$k_1 = 3\,\frac{N}{m} \quad , \quad k_2 = 2\,\frac{N}{m}$$

$$y_1(0) = 1 \quad , \quad y_2(0) = 2$$

$$\dot{y}_1(0) = -2\sqrt{6} \quad , \quad \dot{y}_2(0) = \sqrt{6}$$

$$\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$$

$$\det(A-\lambda I) = \det\begin{pmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{pmatrix}$$

$$= (5+\lambda)(2+\lambda) - 4$$

$$= 10 + 7\lambda + \lambda^2 - 4$$

$$0 = \lambda^2 + 7\lambda + 6$$

$$\lambda = -6, -1$$

$$\Lambda_1 = \begin{pmatrix} -6 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(A - \lambda_1 I) v_1 = 0$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} v_1 = 0$$

$$v_{1x} + 2v_{1y} = 0 \qquad v_{1y} = -\frac{v_{1x}}{2}$$

$$v_{1x}^2 + v_{1y}^2 = 1$$

$$v_{1x}^2 + \frac{v_{1x}^2}{4} = 1$$

$$v_{1x}^2 = \frac{4}{5}$$

$$\boxed{\begin{aligned} v_{1x} &= \frac{2}{\sqrt{5}} \\ v_{1y} &= \frac{1}{\sqrt{5}} \end{aligned}}$$

$$(A - \lambda_2 I) v_2 = 0$$

$$\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} v_{2x} \\ v_{2y} \end{pmatrix} = 0$$

$$-4 v_{2x} + 2 v_{2y} = 0$$

$$2 v_{2x} - v_{2y} = 0$$

$$v_{2y} = 2 v_{2x}$$

$$v_{2x}^2 + 4 v_{2x}^2 = 1$$

$$\boxed{\begin{aligned} v_{2x} &= \frac{1}{\sqrt{5}} \\ v_{2y} &= \frac{2}{\sqrt{5}} \end{aligned}}$$

$$S = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

column

$$\begin{bmatrix} \sqrt{5} & \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$\begin{pmatrix} \dot{r} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} -6 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix}$$

$$r = C_1 \sin\sqrt{6}\,t + C_2 \cos\sqrt{6}\,t$$

$$s = K_1 \sin t + K_2 \cos t$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = S \begin{pmatrix} r \\ s \end{pmatrix}$$

$$S = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$

$$\det S = \frac{1}{\sqrt{5}} (5) = \sqrt{5}$$

$$S^{-1} = \frac{1}{\sqrt{\;}} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} r \\ s \end{pmatrix} = S^{-1} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\begin{pmatrix} r \\ s \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\therefore \quad r(0) = \frac{1}{\sqrt{5}} \left[2y_{10} - y_{20} \right] = \frac{1}{\sqrt{5}} \left(2(1) - 2 \right) = 0$$

$$s(0) = \frac{1}{\sqrt{5}} \left[y_{10} + 2y_{20} \right] = \frac{1}{\sqrt{5}} \left(1 + 2(2) \right) = \sqrt{5}$$

$$r_0 = C_2 = 0 \quad \therefore \quad C_2 = 0$$

$$s_0 = K_2 = \sqrt{5} \quad \therefore \quad K_2 = \sqrt{5}$$

$$\therefore \quad \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} C_1 \sin \sqrt{6} t \\ K_1 \sin t + \sqrt{5} \cos t \end{pmatrix}$$

$$\begin{pmatrix} \dot r \\ \dot s \end{pmatrix} = \begin{pmatrix} \sqrt{6}\, c_1 \cos\sqrt{6}\, t \\ K_1 \cos t - \sqrt{5} \sin t \end{pmatrix}$$

$$\begin{pmatrix} \dot r \\ \dot s \end{pmatrix} = S^{-1} \begin{pmatrix} \dot y_1 \\ \dot y_2 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \dot y_1 \\ \dot y_2 \end{pmatrix}$$

$$\begin{pmatrix} \dot r(0) \\ \dot s(0) \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} -2\sqrt{6} \\ \sqrt{6} \end{pmatrix}$$

$$\begin{pmatrix} \dot r(0) \\ \dot s(0) \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} -5\sqrt{6} \\ 0 \end{pmatrix}$$

$$\therefore \quad \frac{-5\sqrt{6}}{\sqrt{5}} = \sqrt{6}\, c_1$$

$$\boxed{c_1 = -\sqrt{5}}$$

$$0 = K_1 \quad \therefore K_1 = 0$$

$$\therefore \begin{pmatrix} r \\ \end{pmatrix} = \begin{pmatrix} -\sqrt{5} \sin\sqrt{6}\, t \\ \end{pmatrix}$$

$$\begin{pmatrix} s \end{pmatrix} \quad \begin{pmatrix} \sqrt{5}\cos t \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = S \begin{pmatrix} r \\ s \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -\sqrt{5}\sin\sqrt{6}t \\ \sqrt{5}\cos t \end{pmatrix}$$

$$= \frac{1}{\sqrt{5}} \begin{pmatrix} -2\sqrt{5}\sin\sqrt{6}t + \sqrt{5}\cos t \\ \sqrt{5}\sin\sqrt{6}t + 2\sqrt{5}\cos t \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2\sin\sqrt{6}t + \cos t \\ \sin\sqrt{6}t + 2\cos t \end{pmatrix}$$

$$\boxed{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \sin\sqrt{6}t \\ \cos t \end{pmatrix}}$$