$$T(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$T - \lambda \mathbb{I} = \begin{pmatrix} \cos\theta - \lambda & -\sin\theta \\ \sin\theta & \cos\theta - \lambda \end{pmatrix}$$

$$\det(T - \lambda \mathbb{I}) = (\cos\theta - \lambda)^2 + \sin^2\theta$$

$$= \cos^2\theta - 2\lambda\cos\theta + \lambda^2 + \sin^2\theta$$

$$= \lambda^2 - 2\lambda\cos\theta + 1 = 0$$

$$\lambda = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$$b^2 - 4ac = 4\cos^2\theta - 4 = 4(\overbrace{\cos^2\theta - 1}^{-\sin^2\theta})$$

$$= -4\sin^2\theta$$

$$\lambda = \frac{2\cos\theta \pm 2i\sin\theta}{2}$$

$$\boxed{\lambda = \cos\theta \pm i\sin\theta}$$

or:

$$\boxed{\lambda = e^{\pm i\theta}}$$

Eigenvectors:

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = e^{\pm i\theta} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\cos\theta\, v_x - \sin\theta\, v_y = e^{\pm i\theta} v_x$$

$$\sin\theta\, v_x + \cos\theta\, v_y = e^{\pm i\theta} v_y$$

$$\left(\cos\theta - e^{\pm i\theta}\right) v_x - \sin\theta\, v_y = 0$$

$$\sin\theta\, v_x + \left(\cos\theta - e^{\pm i\theta}\right) v_y = 0$$

$$\ldots e^{\pm i\theta} = \cos\theta - \left(\cos\theta \pm i\sin\theta\right)$$

$\cos\theta - e$

$$= \mp i\sin\theta$$

$$\mp i \cancel{\sin\theta}\, v_x - \cancel{\sin\theta}\, v_y = 0$$

$$\cancel{\sin\theta}\, v_x + (\mp i \cancel{\sin\theta})\, v_y = 0$$

$$\boxed{v_y = \mp i v_x}$$

<u>So, we can choose</u>:

$$\vec{v}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \end{pmatrix}$$

$$\vec{v}_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix}$$
