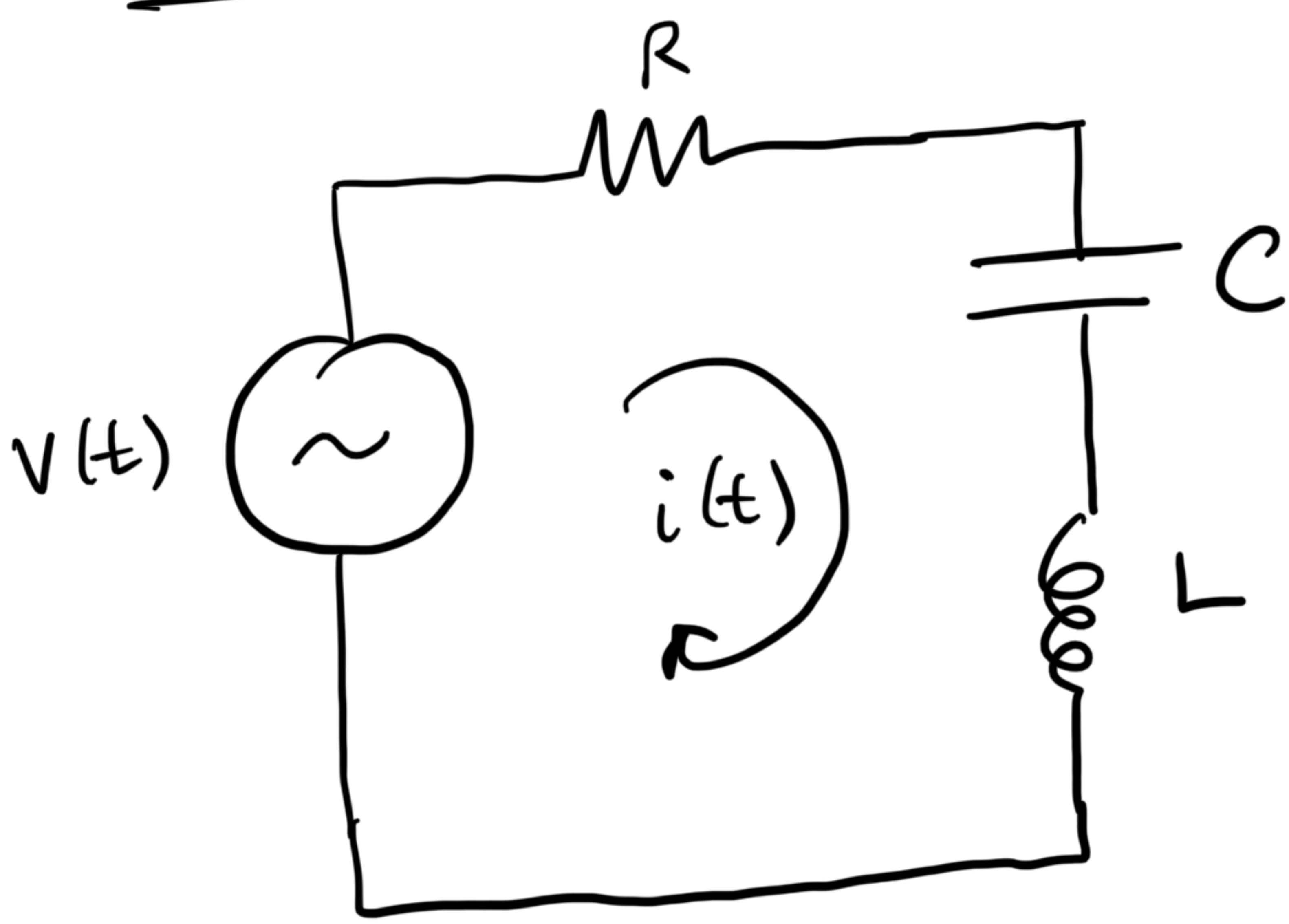
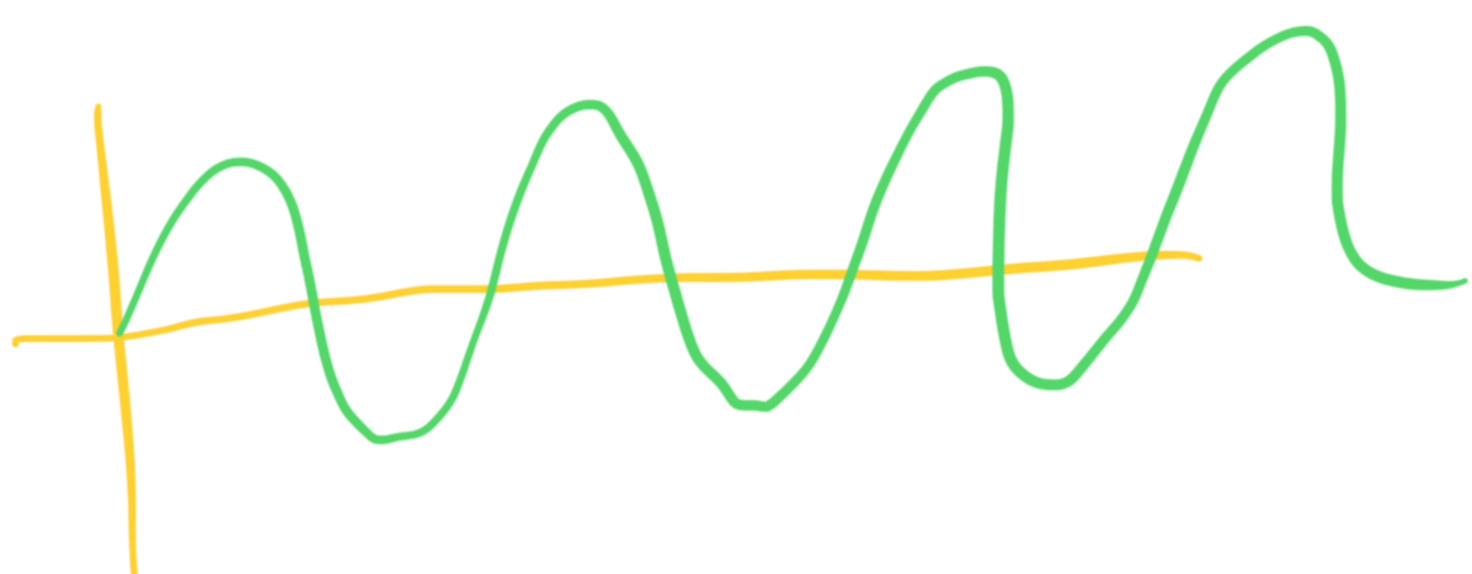
Applications of Complex Numbers.

AC Circuit Analysis.



Suppose $V(t) = V_0 \sin(\omega t)$



What is $i(t)$?

Define the complex impedance, Z

$$Z_R = R \qquad j = \sqrt{-1} \ !!$$

$$Z_L = j\omega L$$

$$Z_C = \frac{1}{j\omega C}$$

The $Z_{circuit} = R + j\omega L + \frac{1}{j\omega C}$

(series!)

$$V(t) = i(t)\, Z_{circuit}$$

$$\boxed{i(t) = \frac{V(t)}{Z_{circuit}}}$$

Let $V_0 = 10 \text{ V}$

$R = 100\ \Omega$

$L = 2 \text{ mH} = 0.002 \text{ H}$

$C = 10\ \mu F = 1 \times 10^{-5} \text{ F}$

$w = 1 \text{ kHz} = 1000 \text{ Hz}$

If $v(t) = V_0 \sin wt$

Then $v(t) = \text{Im}\left(V_0 e^{iwt}\right)$

$I(t) = \text{Im}\left(I_0 e^{i(wt+\phi)}\right)$

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

Interesting Cases:

① $\text{Im}(Z) = 0$

$\therefore$

$$Z = R + j\omega L + \frac{1}{j\omega C} \times \frac{j}{j}$$

$$= R + j\omega L - \frac{j}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega^2 LC = 0 \qquad \omega = \frac{1}{\sqrt{LC}}$$

$L = .002$, $C = 1\times 10^{-5}$

$$\omega = \frac{1}{\sqrt{.2(1\times 10^{-5})}} = 7071.07 \text{ Hz}$$

at $\omega = 7071.07$ Hz

$$Z = R = 100\ \Omega$$

$$i(t) = \frac{v(t)}{100} = \frac{10 \sin \omega t}{100}$$

$$i(t) = 0.1 \sin \omega t$$

or, $100\, i(t) = 10 \sin(\omega t)$

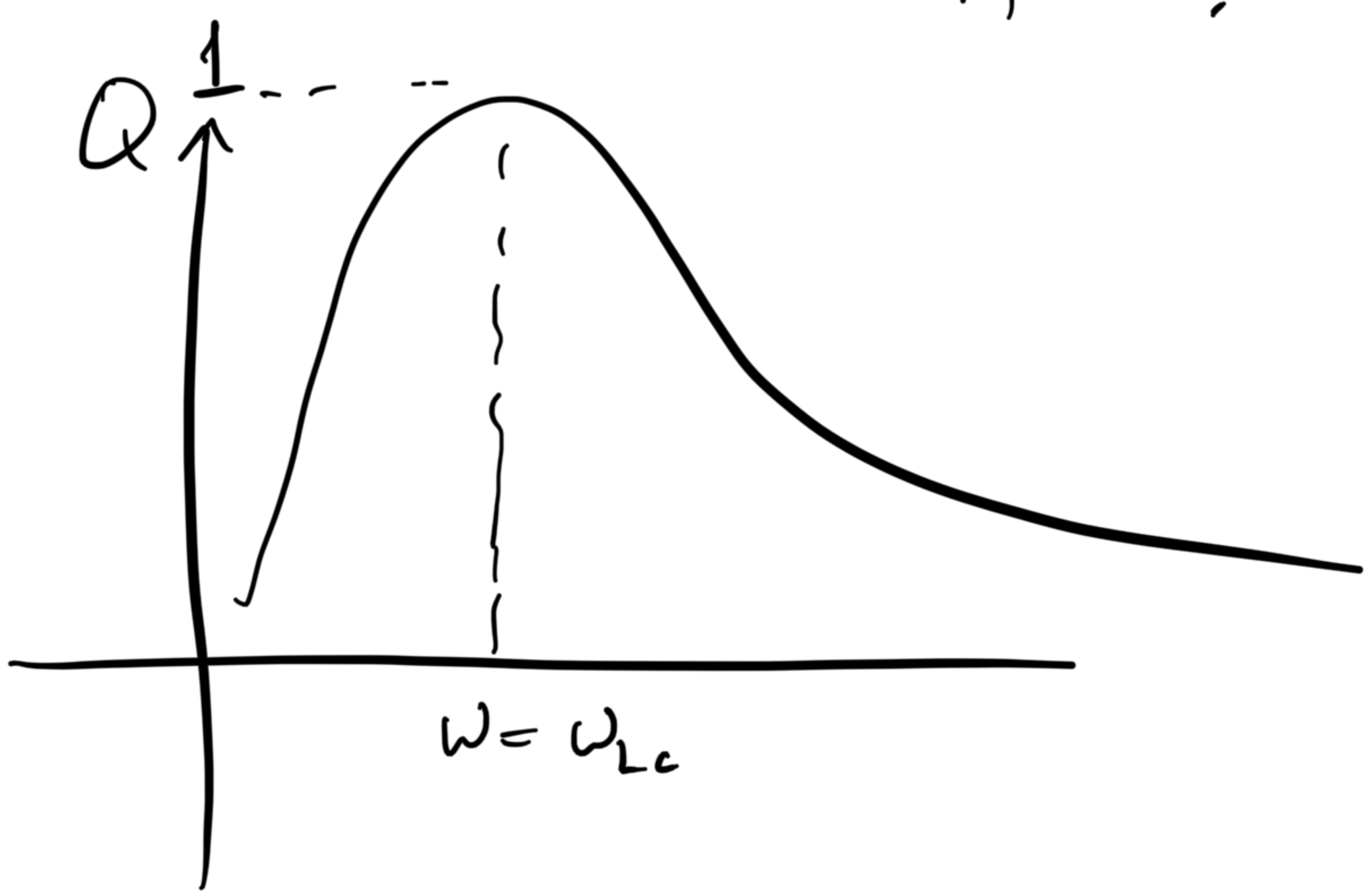
$(|i(t)| = |v(t)|)$

interesting for visualization!!

Define $$Q = \frac{|R I(t)|}{|V(t)|}$$

$Q = 1$ at $\omega = \omega_{LC}$

$Q < 1$ at all other values of ω!



Answering Machine:

press 1, ...

Xmas lights:

Mail S&O Gifts:

- → Walmart Gift Cards
- → S&O gifts → Hobonichi (Olivia)
  → Pouch (Sarah)
- → Pringles
- → Take to Post Office