$\boxed{1}$ $z_1 = x_1 + iy_1$ , $z_2 = x_2 + iy_2$ , $z_1 z_2$ is real non-zero

show r, such that $z_1 = r z_2^*$ , find r in terms of $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

$$zz^* = (x+iy)(x-iy) = x^2 + ixy - ixy + y^2 = x^2 + y^2$$

$\Rightarrow$ the product of z & its conjugate is purely real

$$\frac{z_1}{z_2^*} = r \rightarrow \frac{x_1 + iy_1}{x_2 - iy_2} \cdot \frac{x_2 + iy_2}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 + iy_2)}{x_2^2 + y_2^2}$$

conjugate of conjugate

$$\Rightarrow x_1 x_2 + i x_1 y_2 + i x_2 y_1 - y_1 y_2 \rightarrow \frac{(x_1 x_2 - y_1 y_2) + (x_1 y_2 + x_2 y_1)i}{x_2^2 + y_2^2}$$

$$z_1 z_2^* = (x_1 + iy_1)(x_2 - iy_2) = x_1 x_2 - x_1 y_2 i + x_2 y_1 i + y_1 y_2$$
$$\rightarrow (x_1 x_2 + y_1 y_2) + (x_2 y_1 i - x_1 y_2 i)$$

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + (x_2 y_1 i + x_1 y_2 i)$$

$$\Rightarrow \frac{z_1}{z_2^*} = r \rightarrow \frac{(x_1 x_2 - y_1 y_2) + (x_1 y_2 + x_2 y_1)i}{x_2^2 + y_2^2} = \frac{z_1 z_2}{z_2 z_2^*}$$ duh

try
$$z_1 z_2 = \underbrace{(x_1 x_2 - y_1 y_2)}_{\text{Real}} + \underbrace{(x_1 y_2 + x_2 y_1)i}_{0}$$
we know product is <u>real</u> and <u>non-zero</u>

so: $(x_1 x_2 - y_1 y_2)$ is real, $\neq 0$
$(x_1 y_2 + x_2 y_1)i = 0$, need to remove $i$

$$r = \frac{z_1}{z_2^*} = \frac{\overset{\text{real!}}{(x_1 x_2 - y_1 y_2)} + \cancel{(x_1 y_2 + x_2 y_1)i}^{0}}{x_2^2 + y_2^2} \Rightarrow \boxed{\frac{x_1 x_2 - y_1 y_2}{x_2^2 + y_2^2}}$$

$\boxed{4}$ $f(x) = |x|, \quad -\pi \leq x \leq \pi$, goes to $\pi$ or $-\pi$, set $L = \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right) + b_n \sin\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right)\right]$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$f(x) = |x| \rightarrow \begin{cases} x \text{ if } & -\pi < x < 0 \\ x \text{ if } & 0 < x < \pi \end{cases}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\cos(nx)\,dx, \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\sin(nx)\,dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |x|\,dx = \frac{1}{2\cancel{\pi}}\pi^{\cancel{2}} = \frac{\pi}{2} \Big\} \text{ calculator}$$

$$a_n = \frac{2(-1)^n - 2}{\pi n^2}, \quad b_n = 0 \Big\} \text{ calculator}$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{2(-1)^n - 2}{\pi n^2}\cos(nx) + 0\right] \quad ??$$