$$\frac{dx}{dt} = 2X + y \quad , \quad \frac{dy}{dt} = 16X + 2y \quad , \quad \begin{matrix} X(0) = 1 \\ y(0) = 0 \end{matrix}$$

$$\dot{X} = 2X + y \quad , \quad \dot{y} = 16X + 2y \qquad \vec{V}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{X} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 1 \\ 16 & 2 \end{bmatrix}}_{A} \begin{bmatrix} X \\ y \end{bmatrix} \quad , \quad \det(A - \lambda \mathbb{I}) = 0$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 16 & 2-\lambda \end{bmatrix} \rightarrow [4 - 4\lambda + \lambda^2] - [16] \Rightarrow \lambda^2 - 4\lambda - 12 = 0$$

$$(\lambda - 6)(\lambda + 2)$$

$$\Rightarrow \lambda = -2, 6$$

$$-2: \begin{bmatrix} 4 & 1 \\ 16 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0 \quad , \quad \begin{matrix} 4V_1 + V_2 = 0 \\ 16V_1 + 4V_2 = 0 \end{matrix} \Bigg\} \begin{matrix} 4V_1 = -V_2 \\ V_1 = -1/4\, V_2 \\ V_2 = V_2 \end{matrix}$$

$$\vec{V_1} = \begin{bmatrix} -1/4 \\ 1 \end{bmatrix}$$

$$6: \begin{bmatrix} -4 & 1 \\ 16 & -4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0 \quad , \quad \begin{matrix} -4V_1 + V_2 = 0 \rightarrow V_2 = 4V_1 \\ 16V_1 - 4V_2 = 0 \rightarrow \cancel{16}V_1 = \cancel{4}V_2 \quad V_2 = 1 \\ 4V_1 = V_2 \rightarrow V_1 = 1/4 \end{matrix}$$

$$\vec{V_2} = \begin{bmatrix} 1/4 \\ 1 \end{bmatrix} \qquad \vec{V} = c_1 e^{-2t} \begin{bmatrix} -1/4 \\ 1 \end{bmatrix} + c_2 e^{6t} \begin{bmatrix} 1/4 \\ 1 \end{bmatrix}$$

$$\vec{V}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} -1/4 \\ 1 \end{bmatrix} + c_2 e^{6t} \begin{bmatrix} 1/4 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad 1 = (-1/4)c_1 + (1/4)c_2$$

$$0 = (1)c_1 + (1)c_2 \rightarrow \boxed{c_1 = -c_2} , \boxed{c_2 = -c_1}$$

$$\rightarrow 1 = -\frac{c_1}{4} + \frac{-c_1}{4} \rightarrow -\frac{\cancel{2}}{\cancel{4}}\frac{1}{2} c_1 \rightarrow 1 = -\frac{1}{2} c_1$$

$$\Rightarrow \boxed{c_1 = -2 ,} \quad c_1 = -c_2 \Rightarrow \boxed{c_2 = 2}$$

$$\vec{V} = C_1 e^{-2t}\begin{bmatrix} -1/4 \\ 1 \end{bmatrix} + C_2 e^{6t}\begin{bmatrix} 1/4 \\ 1 \end{bmatrix} \quad , \quad \begin{aligned} C_1 &= -2 \\ C_2 &= 2 \end{aligned}$$

$$\vec{V} = -2e^{-2t}\begin{bmatrix} -1/4 \\ 1 \end{bmatrix} + 2e^{6t}\begin{bmatrix} 1/4 \\ 1 \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} 2e^{-2t}/4 \\ -2e^{-2t} \end{bmatrix} + \begin{bmatrix} 2e^{6t}/4 \\ 2e^{6t} \end{bmatrix} \longrightarrow \longrightarrow \begin{bmatrix} e^{-2t}/2 + e^{6t}/2 \\ -2e^{-2t} + 2e^{6t} \end{bmatrix}$$

$$x = \frac{1}{2}e^{-2t} + \frac{1}{2}e^{6t} \; ; \; y = -2e^{-2t} + 2e^{6t}$$

$\boxed{5}$ $\iint_S G(x,y,z)\,dS$ , $G(x,y,z) = x$

$z = 2 - x^2$

$x=0,\ y=0,\ y=4,\ z=0$

bounds

$z=0,\ z=2-x^2$

$\hookrightarrow 0 = 2 - x^2$

$x^2 = 2 \rightarrow x = \pm\sqrt{2}$

$\iint x\,ds$

$\Rightarrow \iint F \cdot \underbrace{\|F_x \times F_y\|}_{\text{cross product for vectors}}\,dx\,dy \qquad 0 \le y \le 4,\ 0 \le x \le \sqrt{2}$

$$\left.\begin{array}{l} F_x = (1, 0, -2x) \\ F_y = (0, 1, 0) \end{array}\right\} F_x \times F_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -2x \\ 0 & 1 & 0 \end{vmatrix}$$

$= i(0+2x) - j(0) + k(1) \rightarrow \langle 2x, 0, 1 \rangle$

$\|F_x \times F_y\| = \sqrt{4x^2+1} \implies \iint F\sqrt{4x^2+1}\,dx\,dy$

$\Rightarrow \iint \left[ x\sqrt{4x^2+1}\,dx \right] dy \qquad u = 4x^2+1,\ \frac{du}{8x} = dx$

$\Rightarrow \int_0^4 \left[ \frac{1}{8} \int_0^{\sqrt{2}} (u)^{1/2}\,dx \right] dy \rightarrow \qquad u = 4(2)+1 \rightarrow u = 9$

$u = 4(0)+1 \rightarrow u = 1$

$\rightarrow \frac{1}{8}\left( \frac{2}{3}(u)^{3/2} \Big|_1^9 \right. \Rightarrow \left(\frac{1}{12}\, 27\right) - \left(\frac{1}{12}\right) = \frac{26}{12} = \frac{13}{6}$

$\Rightarrow \int_0^4 \frac{13}{6}\,dy \Rightarrow \frac{13}{6} y \Big|_0^4 \Rightarrow \frac{52}{6} \Rightarrow \boxed{\frac{26}{3}}$