Problem 1

$z_1 = x_1 + i y_1 \quad z_2 = x_2 + i y_2 \quad z_1 z_2 = \mathbb{R} \neq 0$

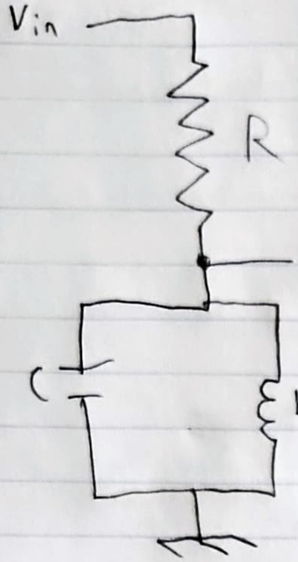
Show that r exist such that $z_1 = r z_2^*$ and is a real numbers

$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1) \quad \therefore \quad x_1 y_2 + x_2 y_1 = 0$

$$r = \frac{x_1 + i y_1}{x_2 - i y_2} = \frac{x_1 x_2 - y_1 y_2}{x_2^2 + y_2^2} + i \underbrace{\frac{x_1 y_2 + x_2 y_1}{x_2^2 + y_2^2}}_{\text{zero!}} =$$

$$\boxed{r = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2}}$$

Problem 2



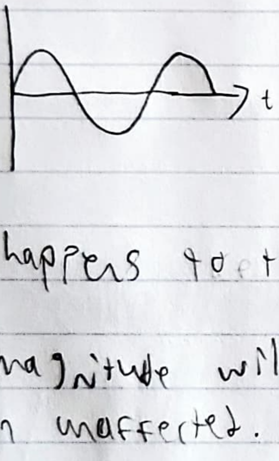
input voltage is a sin source w/ magnitude 1V and frequency ω

a.) Plot magnitude and phase of V_{out} as a function

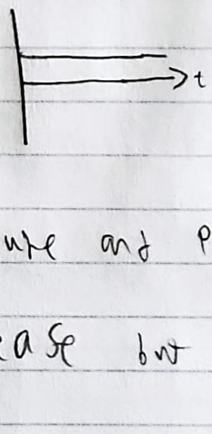
$V_{out} = V_{in} \cdot (1/z)$ $\quad z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{K^2\Omega^2 + (10\mu H - \mu F)^2}$

$V_{in} = V\sin(\omega t)$ $\quad V_{out} = V\sin(\omega t) / \sqrt{(K\Omega)^2 + (10\mu H - \mu F)^2}$

Magnitude



Phase



b.) what happens to the magnitude and phase at $\omega = \frac{1}{\sqrt{LC}}$?

the magnitude will decrease but the phase will remain unaffected.

Problem 3

$$\frac{dx}{dt} = 2x + y \qquad x(0) = 1$$
$$\frac{dy}{dt} = 16x + 2y \qquad y(0) = 0$$

a.) Write system of equations in matrix form

$$\begin{bmatrix}\dot x\\ \dot y\end{bmatrix} = \begin{bmatrix}2 & 1\\ 16 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

b.) find eigenvalues and eigenvectors

$$\begin{vmatrix}2-\lambda & 1\\ 16 & 2-\lambda\end{vmatrix} = 0 = 4 - 4\lambda + \lambda^2 - 16 : \quad \lambda^2 - 4\lambda - 12 \qquad \boxed{\lambda = 6, -2}$$

$$\begin{bmatrix}-4 & 1\\ 16 & -4\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \qquad c_1 = -4c_2 \qquad \boxed{\begin{matrix}A\begin{pmatrix}4\\ -1\end{pmatrix}\\ A\begin{pmatrix}4\\ 1\end{pmatrix}\end{matrix}}$$

$$\begin{bmatrix}4 & 1\\ 16 & 4\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \qquad c_1 = 4c_2$$

c.) write the general solution to the system of equations

$$\tilde{A} = S\Lambda S^{-1} \qquad S = \begin{bmatrix}4 & 4\\ -1 & 1\end{bmatrix} \qquad S^{-1} = \frac{1}{8}\begin{bmatrix}1 & -4\\ 1 & 4\end{bmatrix} \qquad \Lambda = \begin{bmatrix}6 & 0\\ 0 & -2\end{bmatrix}$$

$$\begin{bmatrix}\dot x\\ \dot y\end{bmatrix} = S\Lambda S^{-1}\begin{bmatrix}x\\ y\end{bmatrix} \qquad S^{-1}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}u\\ v\end{bmatrix}$$

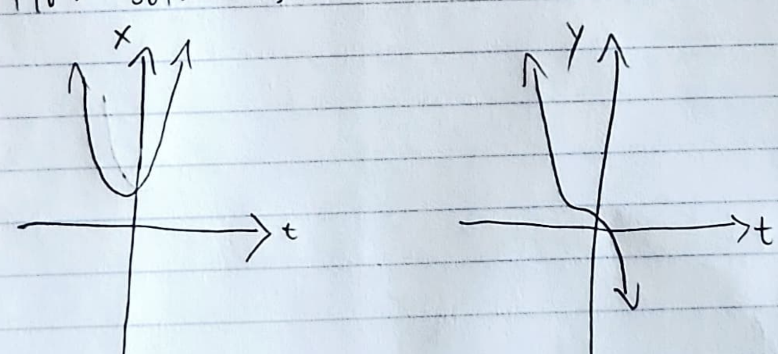
$$\dot u = 6u \qquad u = u_0 e^{6t}$$
$$\dot v = -2v \qquad v = v_0 e^{-2t}$$

$$\boxed{\begin{matrix}x = 4u_0 e^{6t} + 4v_0 e^{-2t}\\ y = v_0 e^{-2t} - u_0 e^{6t}\end{matrix}}$$

d.) Plug in initial conditions and solve

$$x(0) = 1 = 4u_0 + 4v_0 \qquad u_0 + v_0 = \frac{1}{4} \qquad x = \frac{1}{2}e^{6t} + \frac{1}{2}e^{-2t}$$
$$y(0) = 0 = v_0 - u_0 \qquad u_0 = v_0 = \frac{1}{8} \qquad y = \frac{1}{8}e^{-2t} - \frac{1}{8}e^{6t}$$

e.) Plot solutions



Problem 4

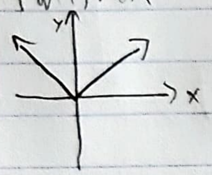
Find the Fourier series of $|x|$ from $-\pi < x < \pi$

a.) compute general form and coefficients (a_0, a_n, b_n) ← plugged function into sympy to solve

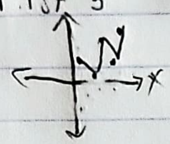
$$\boxed{\begin{array}{l} a_0 = \pi \quad a_n = 0 \quad b_n = \frac{-1}{n} \\ f(x) = \frac{\pi}{2} \sum_{n=1}^{\infty} \left(\frac{-1}{n} \sin\left(\frac{2\pi n x}{T} \right) \right) \end{array}}$$

b.) plot the function and the first 5 non-zero terms of the Fourier series

Function



First 5



c.) there is a discontinuity at $x = 0$ what is the value according to the first 5 of the fourier

the value according to the first 5 should be about 1.

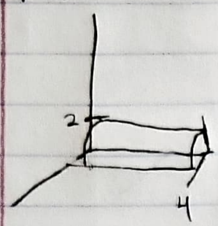
d.) how many non-zero terms do you need for the discontinuity at $x = 0$ to be less than 0.01?

no reasonable number of terms can bring the discontinuity that low.

Problem 5

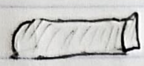
evaluate the surface integral $\iint_S G(x,y,z)\, dS$

$G(x,y,z)=x$; S the portion of the cylinder $z=2-x^2$ in the first octant bound by $x=0$ $y=0$ $y=4$ $z=0$



Step 1:

Step 2



$$\int_0^4 \int_0^{\pi/2} x \, dz\, dy$$

$$= \int_0^4 \frac{\pi}{2} x \, dy$$

$$= \boxed{2\pi x}$$