Vectors

You already know that there are many physical quantities that are described by vectors:

$$\vec{F}, \vec{p}, \vec{v}, \vec{a}, \vec{\Delta x}, \dots$$

The mathematics of vectors is actually really interesting !! ... Why ?

Symmetries !!

Noether's theorem → every symmetry in nature is connected to a conservation law.

Example 1: Conservation of Energy

⇔ time invariance

of the laws of physics.

→ the laws of physics, mathematically, don't have to be time-invariant. We just observe that this is the case.

→ A symmetry is a situation that is basically "nicer" than it has to be, mathematically.

Example 2: Conservation of Momentum

⇒ the laws of physics are the same everywhere in space.

→ again, it doesn't have to be this way; but it is.

→ we say that the laws of physics are the same under translations

in 3D space.

Key Point: How the momentum vector transforms under translations in 3D space matters... a LOT!!

Therefore, we need to understand the mathematics of vectors, in order to understand these symmetries!!

It's just so cool!!

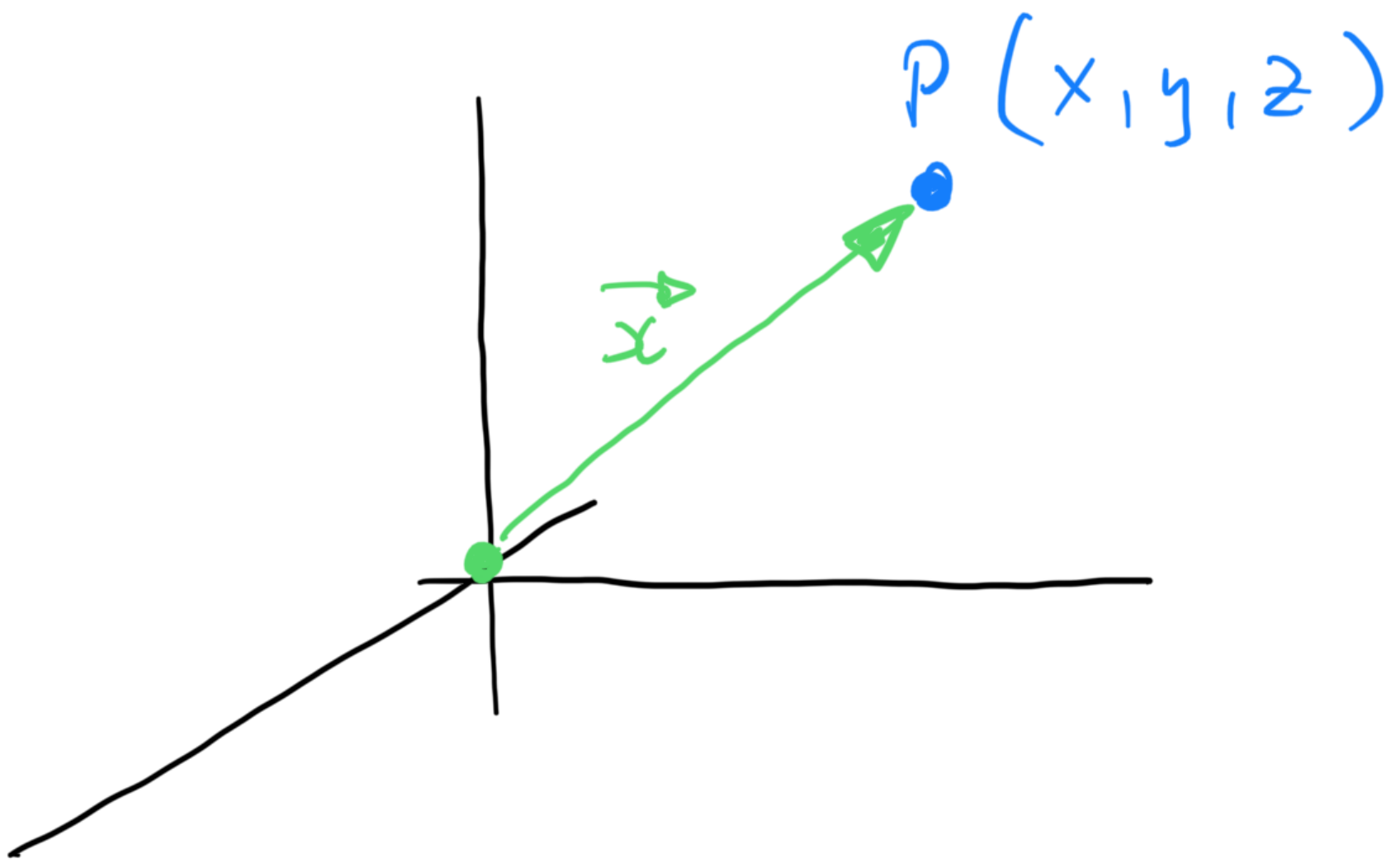
Algebra ← arabic "al jebra" = "the method / way / technique"

As applied to maths, algebra is the method for how to perform the

Operations $+, -, *, \div, (\ ), \dots$

For <u>scalars</u>, the algebra is just the same algebra of real numbers that you've studied for the last 15-20 years.

For <u>vectors</u>, the algebra is totally different !!



We define a vectors as a quantity which points <u>from the origin</u> of some

Coordinate system to a point in the 3D space of that quantity.

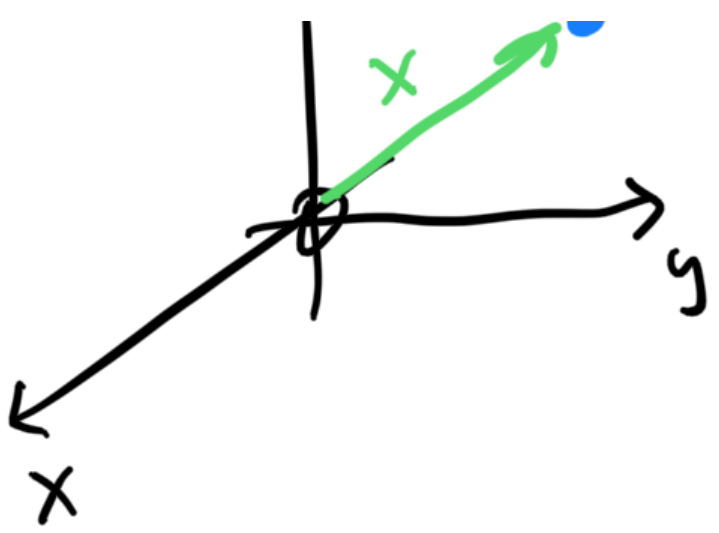
Several Key points:

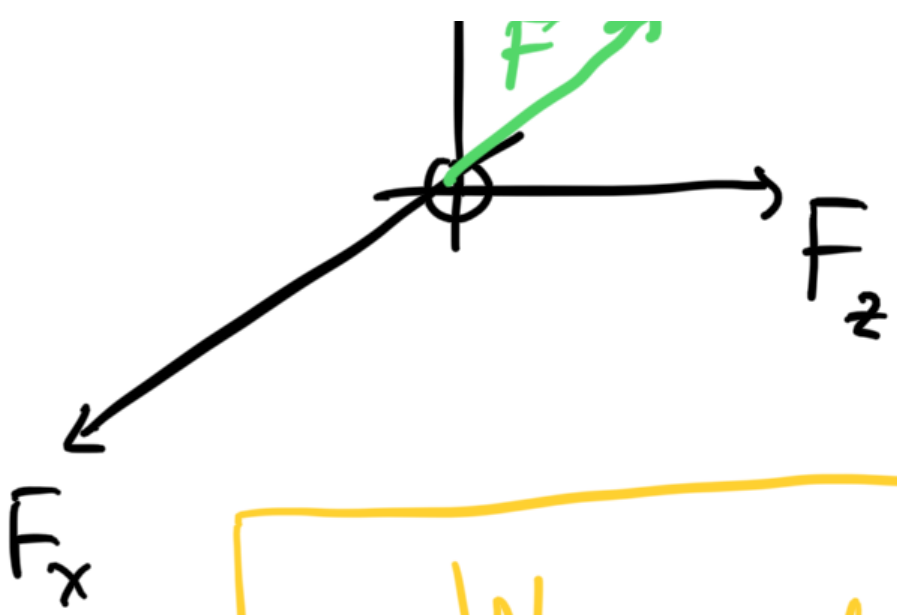
(1) Always from the origin, to the point.

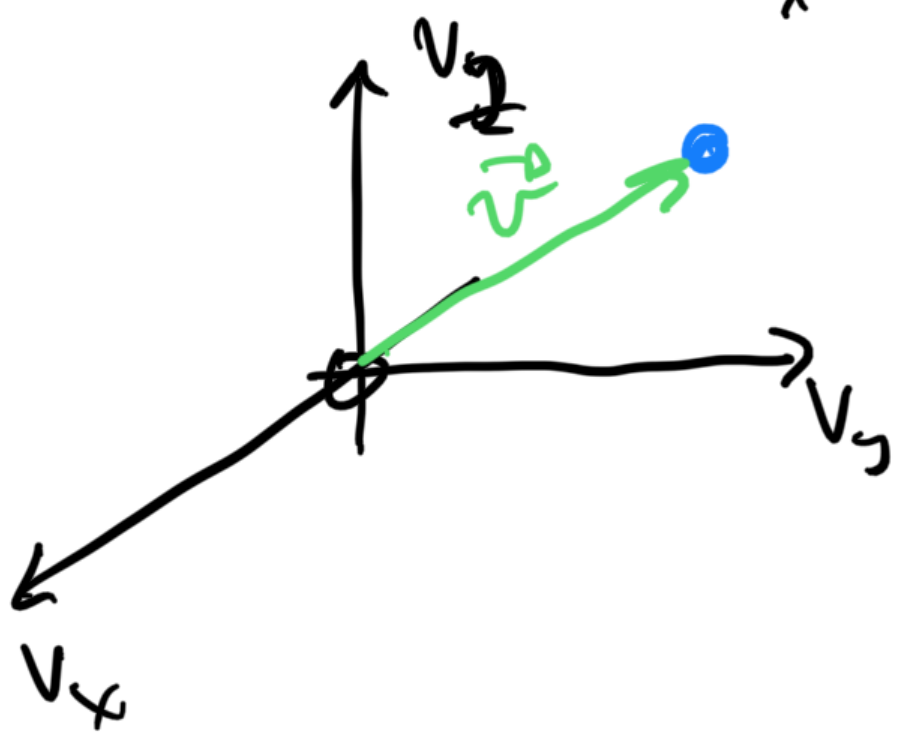
(2) We need to define a coordinate system, with an origin

(Somehow, the answer to the Universe should not depend on this choice!)

(3) For each physical vector quantity, there exists a 3D space of that quantity.







We need to remember which 3D space we are working in!!

Problem: What is $\vec{v} + \vec{F}$?

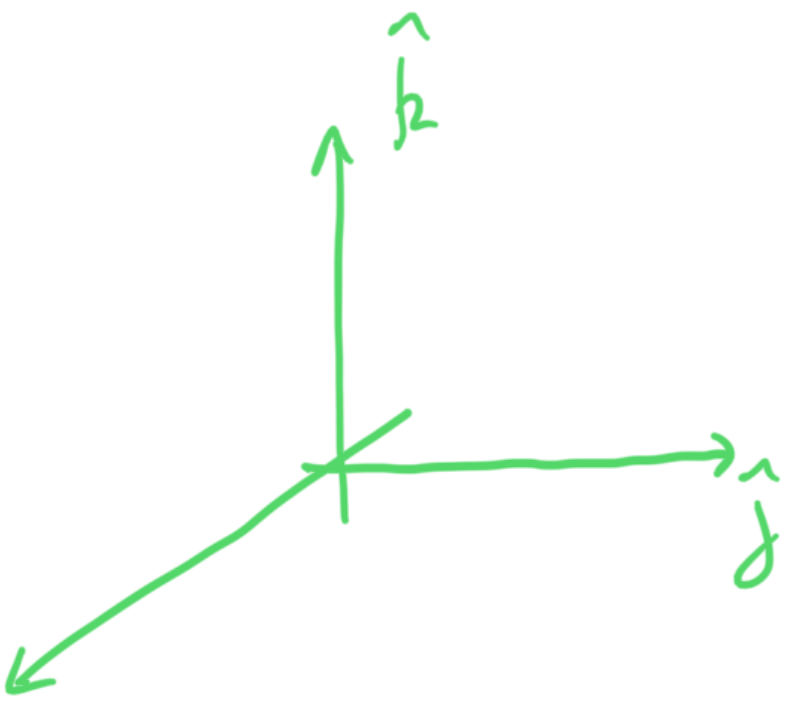
→ makes no sense!!!

Vector Representations:

$$\vec{x} = (1.2, 3.6, -1.8) \text{ (meters)}$$

$$\vec{F} = 3.8\,\hat{i} + 2.9\,\hat{j} - 6.2\,\hat{k} \text{ (Newtons)}$$

$\hat{i}$, $\hat{j}$, $\hat{k}$ → unit vectors.



(1) Length = 1 unit

(2) Unit = whatever space we are working with.

$\hat{i}$

position → $\hat{i}$, $\hat{j}$, $\hat{k}$ are 1 meter long.

velocity → $\hat{i}$, $\hat{j}$, $\hat{k}$ are 1 m/s long!!

In general:

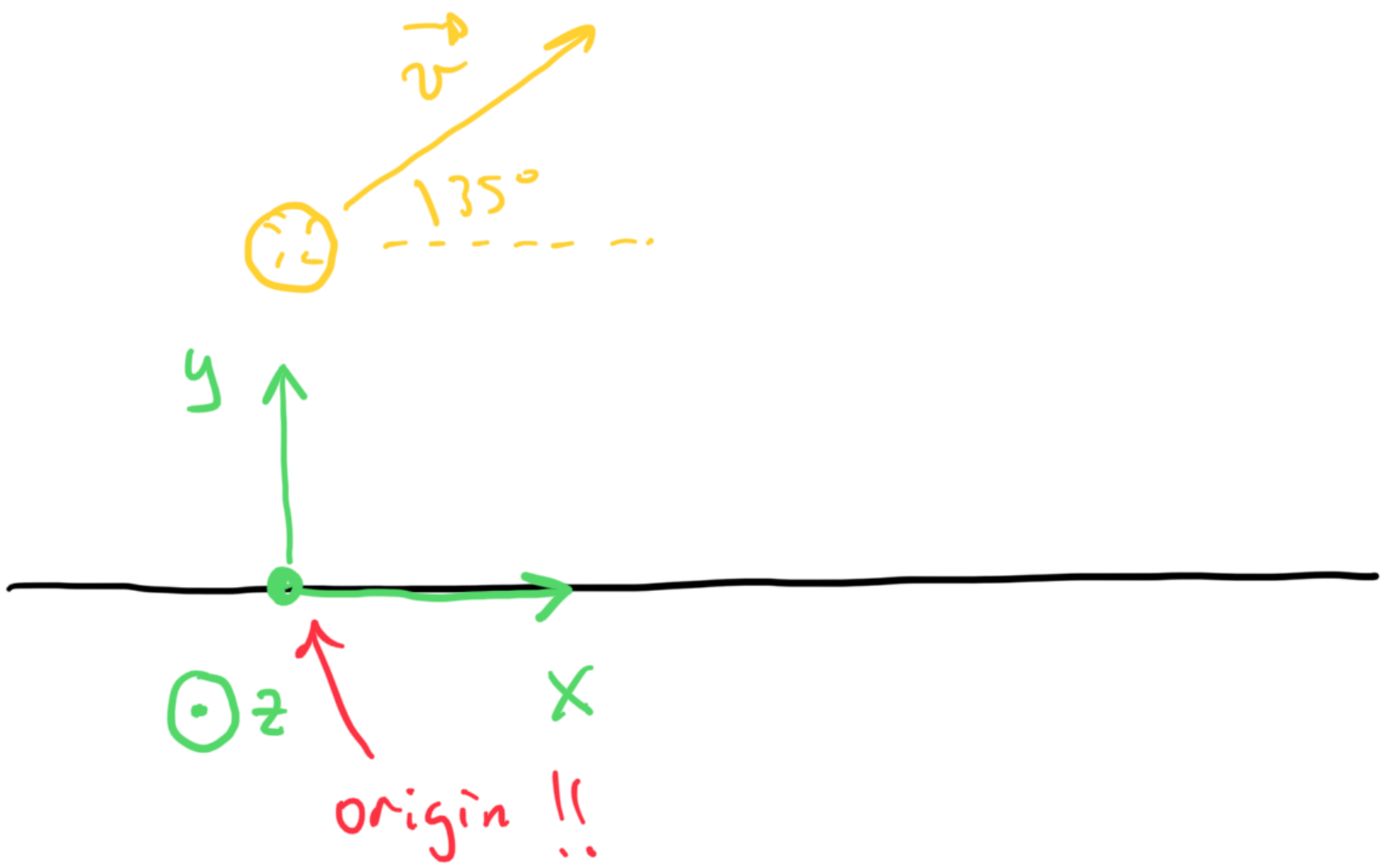
$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Big Hint: In every problem we should:

1. Choose a coordinate system.
2. Express every vector in this form.
3. Do math on these vectors.

Example: A baseball is launched from a height 1.85 m above the ground, with a speed of 40.0 m/s, at an angle of 35° above the horizontal.

Step 1 : Choose a coordinate system.

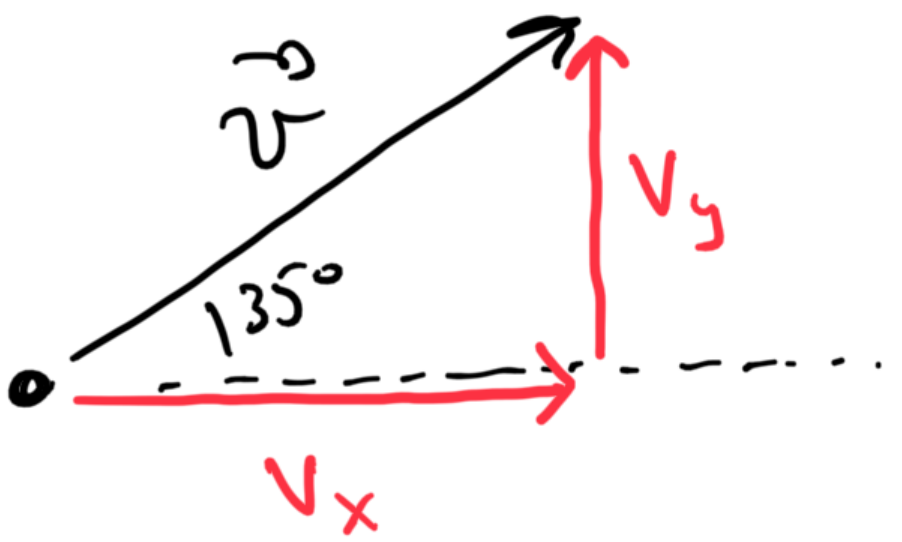


Step 2 : Express the position ($\vec{x}$), and the velocity ($\vec{v}$) in the general form.

$$\vec{x} = 0\,\hat{i} + 1.85\,\hat{j} + 0\,\hat{k}$$

$x = 1.85\ y$

Velocity:



SOHCAHTOA !

$$\sin 35° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{|\vec{v}_y|}{|\vec{v}|}$$

$$\sin 35° = \frac{|\vec{v}_y|}{40.0}$$

$$\therefore |\vec{v}_y| = 40.0 \sin 35°$$

$$= 22.94$$

$$\cos 35° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{|\vec{v}_x|}{|\vec{v}|}$$

$$\cos 35° = \frac{|\vec{v}_x|}{40.0}$$

$$\therefore \quad |\vec{v}_x| = 40.0 \cos 35^\circ$$

$$= 32.77$$

$$\therefore \quad \vec{v} = 32.77\,\hat{i} + 22.94\,\hat{j} + 0\,\hat{k}$$

$$\boxed{\vec{v} = 32.77\,\hat{i} + 22.94\,\hat{j}}$$

Example 2 : Consider the vector

$$\vec{a} = 3.2\,\hat{i} + 1.5\,\hat{j} - 6.8\,\hat{k}$$

(i) What is the size of this vector?

Answer : 3D Pythagorean Theorem.

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$= \sqrt{(3.2)^2 + (1.5)^2 + (-6.8)^2}$$

$$|\vec{a}| = 7.66$$