Assignment 2
Modeling for Computer Graphics CPSC 589/689
Total Marks: 100 + 40 Bonus

2 (10 marks) For the case of a planar B-spline curve, does symmetry of the control polygon with respect to the y-axis imply the same symmetry for the curve?

2 (10 marks) Derive the formula of third order B-spline with uniform, integer knot sequence from deBoor's recursive formula.

3 (10 marks) Show that B-splines of order 3 on {0,0,0,1,1,1} are second degree Berenstein polynomials.

4 (10 marks) Consider the following B-spline surface:

$$S(u,v) = \sum_{i=0}^{100} \sum_{j=0}^{200} P_{i,j} N_{i,4}(u) N_{j,3}(v),$$

with the knot sequences $U = \{u_0, u_1, \ldots u_{104}\}$ and $V = \{v_0, v_1, \ldots v_{203}\}$, and the control points $P_{i,j}$. For fixed values of u and v:

1. How many nonzero terms does $S(u,v)$ have?
2. which terms are these non-zero values? use the δ notation(the index of focus).

5 (10 marks) The first quadratic B-spline basis function defining on positive integer numbers (knot sequence) is given as:

$$N_{0,3}(u) = \begin{cases} \frac{u^2}{2} & 0 \le u < 1 \\ \frac{3}{4} - (u - \frac{3}{2})^2 & 1 \le u < 2 \\ \frac{1}{2}(3-u)^2 & 2 \le u < 3 \\ 0 & \text{otherwise} \end{cases}$$

1. Show that $N_{0,3}(u)$ is really a third order spline function (verify the necessary properties only at $u = 0$).
2. Determine $N_{5,3}(u)$ (try to use $N_{0,3}(u)$). Show your work.

6 (50 marks + 40 bonus) Implement a B-spline curve in 2D with the following specifications.

- User control of the curve's order (k).
- User control of the parameter increment (u).
- Users should be able to add, move, and delete control points through mouse interaction.
- Implement the efficient B-spline algorithm discussed in the lectures.
- Use the standard knot sequence.
- Include an option to demonstrate the algorithm in a geometric fashion.
- (Bonus 1, 20 marks) Extend your program to implement NURBS. This includes creating a good interface for displaying and changing weights.
- (Bonus 2, 20 marks) Extend your program to allow the user control of knot spacing and multiple knots. This will mean showing the basis curves in parameter space as well as the knot positions. In addition, an option to control sharp control points is required. For this option, it is enough to use a multiple knot for the indicated control points.
- (Bonus 3, 20 marks) Extend your program to implement surfaces of revolution based on your B-spline curves.

Although you are free to implement all bonuses, you will only receive bonus marks for two of them (the better bonus implementation will be marked).