STATS 415 HW 4

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Problem 1

(a) $P(Y=1) = \frac{e^{\beta_0+\beta_1 x_1+\beta_2 x_2}}{1+e^{\beta_0+\beta_1 x_1+\beta_2 x_2}}$
Since, $\beta_0 = -4, \beta_1 = 0.05, x_1 = 5, \beta_2 = 1, x_2 = 3.5$
$P(Y=1) = \frac{e^{-4+0.05*5+1*3.5}}{1+e^{-4+0.05*5+1*3.5}} = \frac{e^{-0.25}}{1+e^{-0.25}} = 0.438$
The probability is around 0.438.

(b) $Odds = \frac{P(Y=1)}{1-P(Y=1)} = \frac{0.438}{1-0.438} = 0.779$
The odds are around 0.779.

(c) $\log(\frac{P(Y=1)}{1-P(Y=1)}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$
Since, $P(Y=1) = 0.5$, $\log 1 = 0 = -4 + 0.05 * x_1 + 3.5$
Thus, $x_1 = 10$, the student has to study 10 hours a week.

Problem 2

(a) iii. is the correct one. Lasso regression shrinks the coefficients toward 0(could be 0), making the model more stable but less flexible compared to the OLS regression.
As λ it increases, the variance will decrease, while bias will increase, before reaching the least MSE, bias increasing is less than the variance decreasing, which improves the prediction accuracy.

(b) iii. is the correct one. Ridge regression shrinks the coefficients toward 0(couldn't be 0), making the model more stable but less flexible compared to the OLS regression.
As λ it increases, the variance will decrease, while bias will increase, before reaching the least MSE, bias increasing is less than the variance decreasing, which improves the prediction accuracy.

(c) ii. is the correct one. Non-linear methods tend to be more complex and have more coefficients than OLS, so the model is more flexible but less stable.
As the complexity of the model increases, if the variance increasing is less than the bias decreasing, the prediction accuracy improves.

Problem 3

(a) iv. is the correct one. RSS_{train} would continually decrease since the constraint s is increasing, and more coefficients can be chosen to make the RSS_{train} less.

(b) ii. is the correct one. RSS_{test} would initially decrease just like RSS_{train} since the model can fit the data better, but after reaching the OLS, RSS_{test} would increase since the model over-fits the data, and performs worse on test data.

(c) iii. is the correct one. As the model fits the train data better, the variance will continually increase, making the model less stable.

(d) iv. is the correct one. As the model fits the train data better, the bias will continually decrease, making the model more flexible.

(e) v. is the correct one. The irreducible error never changes as s changes.