MKT 680 - Marketing Analytics

Project 4: Marketing Models

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1. Business Understanding

Background

Pernalonga, a giant in the supermarket industry, currently operates over 400 stores in Lunitunia and offers more than 10,000 options to customers in over 400 product categories. Promotions are an important part of Pernalonga's business strategy, and a big portion of the promotion sales is brought by its joint-funded promotion programs with supplier partners. Previously, the majority of Pernalonga's promotions are conducted in a chain-wide manner, however, personalized promotions have become a new direction and are in experimentation recently.

Mahou San Miguel, a Spanish brewer, is one of the supplier partners in Pernalonga's regularly promoted beer category. Currently, there are three San Miguel beer products offered in Pernalonga's stores, and promotion methods such as weekly flyers, in-store displays, and email are primary drivers of San Miguel's sales.

Problem Definition

Mahou San Miguel utilizes many promotion channels, from traditional media like TV and Radio to internet-based media like email and online ads. San Miguel wonders which marketing activities are the biggest sales drivers among all these channels, so that it can spend marketing expenditure more wisely in future. In general, we want to have a close look at each channel and analyze the effects of product seasonality and holidays on these vehicles to determine the ones that drive the best results.

2. Data Preparation

TV and Radio Promotions

To properly prepare our data for our most complex problem to date, we decided to break the data preparation process into steps. First, we calculated the alpha for both TV and Radio channels. We then calculated the adstocked GRP for the above mentioned two categories. Lastly, we transformed the adstocked GRP to reach before inserting it into our regression models.

Through our EDA, we found that Pernalonga runs promotions every week. We were able to derive a list of week-indexes from the promo_ad dataset, which gave us about 106 weeks of data in total. The alpha for both TV and Radio ended up being 0.16 and 0.08 respectively (given the 8 and 4-week half-lives respectively). We were also able to confirm that TV and Radio promotions cover all three San Miguel beer products.

We then moved onto the calculation of adstocked GRP of TV and Radio promotions. We stored all our information in a table labeled TV_adstock. A key assumption that we made here is that there are no adstocked GRP left before the initial promotion of this period. After doing small tweaks like replacing all missing values with 0 (for calculation only) and re-adstocking the GRP with the given half-lives, we were able to finally convert all adstocked GRP to reach, which will be useful as a casual in our regression later on.

week	GRP	adstocked GRP	reach
2016-06-05	500	41.498	0.5357
2016-06-12	0	38.054	0.5062
2016-06-19	0	34.896	0.4773
2016-06-26	0	31.999	0.4491
2016-07-03	0	29.344	0.4217
2016-07-10	0	26.908	0.3954
2016-07-17	0	24.675	0.3700
2016-07-24	0	22.627	0.3458
2016-07-31	0	20.749	0.3227
2016-08-07	0	19.027	0.3007

Sample of the TV_adstock table

We effectively repeated the exact same process for Radio promotions, with the final adstock table shown below:

week	GRP	adstocked GRP	reach
2016-06-05	200	31.821	0.49379
2016-06-12	0	26.758	0.43898

2016-06-19	0	22.501	0.38720
2016-06-26	0	18.921	0.33919
2016-07-03	0	15.910	0.29536
2016-07-10	0	13.379	0.25586
2016-07-17	0	11.250	0.22065
2016-07-24	0	9.460	0.18956
2016-07-31	0	7.955	0.16232
2016-08-07	0	6.689	0.13860

Sample of the Radio_adstock table

Other Casuals - Promotion and List Price

We also made sure to look at other influences on our final modeling process. The first and foremost is promotions and list price. To prepare our data for our regression models in this regard, we first took all the relevant information from the transaction table, which included variables such as sales quantity, discount and price. We then merged it back to our week table in-order to aggregate our data by week. Next, we calculated the weekly list price and weekly discount for each product, which resulted in our final weekly_price and weekly_discount variables, which like above will be used as casuals in our regression modeling process.

prod id	tran prod sale amt	tran prod sale qty	tran prod discount amt	prod unit price	week	weekly price	weekly discount
138936951	0.69	1	0.1	0.69	12/27/2015	0.69	0.1
138936952	15.96	4	2	3.99	12/27/2015	3.99	0.5
138936951	1.38	2	0.2	0.69	12/27/2015	0.69	0.1
138936952	11.97	3	1.5	3.99	12/27/2015	3.99	0.5
138936953	29.98	2	4	14.99	12/27/2015	14.99	2

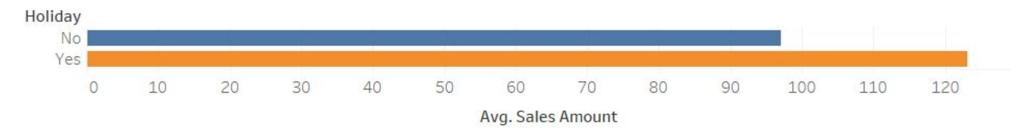
138936952	7.98	2	1	3.99	12/27/2015	3.99	0.5
138936951	5.52	8	0.8	0.69	12/27/2015	0.69	0.1
138936952	3.99	1	0.5	3.99	12/27/2015	3.99	0.5
138936952	3.99	1	0.5	3.99	12/27/2015	3.99	0.5
138936953	14.99	1	2	14.99	12/27/2015	14.99	2

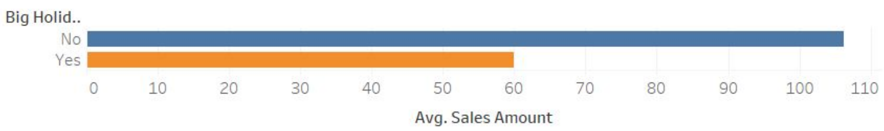
First 10 rows of the Promotion and List Price Variables

Other Casuals - Seasonality and Holiday

Our next step was to look at seasonality and holiday influences within our data. We first added seasonality to our data, followed by holiday indicators, and finally highlighted some of the holidays as more important than others for our analysis (the impact of Christmas vs Ides of March on sales):

The first and simplest step was to merge the seasonality data table, as we will be using the seasonality index as a casual for our model. Before we just blindly merge the holiday data into our model data, we wanted to take a look at the effect of holidays on sales. In fact, different holidays have different effects on sales, which can be seen below (product 13896952 as an example):





From the above bar plots, you can see that we generated a new variable “big_holiday”, which stands for Christmas and New Year. This is because we found that while other holidays, such as July 4th, have a significantly positive effect on beer sales, Christmas and New Year’s Day seem to pose a negative effect. One a assumption is that people choose other beverages such as wine during important holidays. To better distinguish the effects of each holiday, we generated another variable “big_holiday”. And finally, we merged both holiday and big_holiday back into our data.

week	prod_id	season index	holiday	big holiday	weekly price	weekly discount	sales_qty
2015-12-27	138936951	5601	0	1	0.69	0.10	11
2015-12-27	138936952	5601	0	1	3.99	0.50	11
2015-12-27	138936953	5601	0	1	14.99	2.00	3
2016-01-03	138936952	5435	0	0	3.99	0.00	78
2016-01-03	138936951	5435	0	0	0.69	0.00	206
2016-01-03	138936953	5435	0	0	14.99	0.00	7
2016-01-10	138936951	5335	0	0	0.69	0.00	187
2016-01-10	138936953	5335	0	0	14.99	0.00	3
2016-01-10	138936952	5335	0	0	3.99	0.00	83
2016-01-17	138936952	5262	0	0	3.99	0.00	100

First 10 rows of the Holiday & Seasonality Variables

Other Casuals - Flyers, Store Displays, Emails, Web Displays, Paid Search, misc.

The last thing we wanted to look at before diving into modeling is the effect of other miscellaneous items such as web displays, emails, store displays, etc. To do so, we first divided the model data into the three products as they will be modeled separately. We then merged the TV/Radio reach data prepared above with the miscellaneous items like flyer and store display. We created a new variable called base price, which accounts for the earliest price of each time period. Next, we transformed y as we assume that we will be building multiplicative models and logit models. We transformed y to log form for the multiplicative model and we assumed that maximum sales quantity is 10% more than historical maximum when transforming y for the logit models, which ensured that the target will be positive within the logarithm. After all this our data is finally prepared for modeling. A snapshot of product 13896951 is shown as an example below:

Name	week	prod_id	seasonality	holiday	big_holiday	weekly_price	weekly_discount	sales_amount	Email	Flyer	Paid_Search	Web_Display	TV	Radio	base_price	sales_log	max_sales_qty	sales_transformed
Data Type	<date>	<int>	<dbl>	<chr>	<dbl>	<dbl>	<dbl>	<int>	<dbl>	<int>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>		<dbl>
1	42365	1.39E+08	5601	0	1	0.69	0.1	11	0	0	19444	0	0	0	0.69	2.398	358.6	-3.453157
2	42372	1.39E+08	5435	0	0	0.69	0	206	0	0	47733	0	0	0	0.69	5.328	358.6	0.300056
3	42379	1.39E+08	5335	0	0	0.69	0	187	0	0	29342	0	0	0	0.69	5.231	358.6	0.085942
4	42386	1.39E+08	5262	0	0	0.69	0	182	0	0	31367	0	0	0	0.69	5.204	358.6	0.030119
5	42393	1.39E+08	5252	0	0	0.69	0	193	0	0	24650	0	0	0	0.69	5.263	358.6	0.153115
6	42400	1.39E+08	5328	0	0	0.69	0	162	0	0	3288	0	0	0	0.69	5.088	358.6	-0.193575
7	42407	1.39E+08	5516	1	0	0.69	0.1	237	0	0	17507	0	0	0	0.69	5.468	358.6	0.667323
8	42414	1.39E+08	5687	0	0	0.69	0	180	0	0	23529	0	0	0	0.69	5.193	358.6	0.007808
9	42421	1.39E+08	5868	0	0	0.69	0	184	0	0	36556	0	0	0	0.69	5.215	358.6	0.052438
10	42428	1.39E+08	6017	0	0	0.69	0	205	0	0	1660	0	0	0	0.69	5.323	358.6	0.288658

First 10 rows of our Final Modeling Data (Product 13896951))

3. Modelling Approach

Model selection

We decided to build both a multiplicative model and a logit model for each product, so that in the following model diagnosis, we can choose that one that fits our data better.

For the multiplicative model, there is a simple implicit interaction assumed between the causal variables which is true to reality where there are interactions among different types of promotions. The dependent variable in the multiplicative model is unbound. However, sales have limits. So, the multiplicative model may misrepresent the reality.

For logit the model, complex interactions are assumed among the variables which is possible in real promotions. The dependent variable is bound. In our case we assumed the upper limit is 110% of the historical maximum volume. This assumption is based on our data. For product I, the standard deviation of sales is 38, which is about 10% of its historical maximum sales. And for product II and product III, the situation is similar. So, we believe that a 10% markup is a reasonable assumption.

Dealing with unreasonable coefficients

Initially, we tried to build linear regression models with raw data and all available variables, and we discovered that some of the coefficients are negative. For example, below is the summary of logit model for product I:

```
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      2.56e+00   1.21e+00    2.11    0.038 *
weekly_price    -3.70e+00   1.66e+00   -2.23    0.028 *
weekly_discount  2.78e+00   1.90e+00    1.46    0.146
Flyer            4.11e-01   3.87e-01    1.06    0.290
Email            2.68e-06   1.43e-06    1.87    0.064 .
Web_Display      3.39e-07   6.64e-07    0.51    0.611
Paid_Search     -1.04e-06   3.93e-06   -0.26    0.792
TV               2.20e+00   9.60e-01    2.29    0.024 *
Radio           -1.05e+00   6.70e-01   -1.57    0.119
seasonality      3.67e-05   3.78e-05    0.97    0.334
holiday1        -4.22e-02   1.71e-01   -0.25    0.806
big_holiday     -2.55e+00   3.99e-01   -6.37  6.8e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Table: Summary of Raw Logit Model for Product I

Apparently, the coefficients make no sense. According to law of demand, high prices will induce low sales. But all marketing vehicles should have a non-negative effect on sales. So, we decided to dive deep into this problem. We took the following three steps.

First, we took a look at the correlation between causals and the target variable. Let's still use product I as an example:

Price	Discount	Flyer	Email	Web Display	Paid Search	TV	Radio	Seasonality	Holiday	Big Holiday
-0.073	0.238	0.294	-0.079	0.174	0.016	0.319	0.180	0.372	0.216	-0.405

Table: Correlation with sales of product I variables

From the correlation analysis, we found some of the correlations are negative. After consulting with Prof. Lim, we know that the existence of outliers might be causing the negative correlation and coefficients.

So, our second step is to remove the outliers from our data. Here we define outliers as data lies beyond 3 standard deviations of the mean. And we did the correlation analysis again:

Price	Discount	Flyer	Email	Web Display	Paid Search	TV	Radio	Seasonality	Holiday	Big Holiday
-0.109	0.433	0.344	0.104	0.189	0.071	0.306	0.146	0.356	0.203	0.167

Table: Correlation with sales of product I variables after removing outliers

And the correlations have been corrected and become positive (except high price will cause low sales, according to the law of demand).

The last step is to run all models after removing outliers from datasets. In our appendix, you can find the summary of all three final models that we chose for our marketing mix model, which we have ensured all the coefficients make business sense.

Model Diagnosis

As mentioned in the previous step, after appropriately dealing with unreasonable coefficients, we have built both a multiplicative model and a logit model for each product. To determine which one to use and to justify the validity of the model, we have performed a comprehensive diagnosis of our six models (two for each product). We have looked at R-squared, adjusted R-squared, RMSE, MAPE, F value and the Durbin-Watson statistic. Below is a summary of diagnostic metrics for each model:

Model	***R-squared***	***adjusted R-squared***	***RMSE***	***MAPE***	***F stat***	***Durbin-Watson Stat***
Product 1-multiplicative	0.423	0.368	28.999	0.107	7.672	2.180
Product 1-logit	0.427	0.365	28.921	0.109	6.918	2.164
Product 2-multiplic	0.604	0.562	21.080	0.164	14.464	2.002

ative						
Product 2-logit	0.608	0.567	20.950	0.222	14.762	1.882
Product 3-multiplicative	0.790	0.767	6.467	0.498	34.307	2.138
Product 3-logit	0.843	0.826	5.593	0.505	48.931	2.107

Table: Summary of Diagnostic Metrics

R-squared and adjusted R-squared

R-squared measures the proportion of the variation in our dependent variable, which is transformed sales, explained by our independent variables. Adjusted R-squared adjusts the statistic based on the number of independent variables in the model. R-squared and adjusted R-squared are consistent in our results, because we basically used the same number of independent variables in all models.

We can see from the results that the models have an adjusted R-squared value ranging from 0.42 to 0.84, which are not bad. A decent proportion of the independent variable can be well-explained by the predictors. In terms of adjusted R-squared, for both product II and product III, the logit models outperform the multiplicative models, while for product I, the logit model is slightly worse than the multiplicative model. We will take other metrics into consideration, while for now, a decent R-squared value indicates that our model has reasonable predicting power.

RMSE and MAPE

RMSE is the square root of the average squared error, while MAPE calculates the mean absolute percentage error.

Mean squared error	$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n} e_t^2$
Root mean squared error	$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}$
Mean absolute error	$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n} \lvert e_t \rvert$
Mean absolute percentage error	$\text{MAPE} = \frac{100\%}{n}\sum_{t=1}^{n} \left\lvert \frac{e_t}{y_t} \right\rvert$

Table: Formulas of Common Regression Metrics

Both metrics are very useful for validating model performance. In terms of MAPE, the first four models have MAPE less than 23%, which is really good. The models for product three have a larger MAPE and we have dive deep to find the reasons. It turned out that there are still outliers in the observations, whose errors are shared by the other observations when MAPE was computed, resulting in a high MAPE. To avoid losing further information, we decided not to remove the outliers. Instead, we chose to look at RMSE. The RMSE for product III is acceptable, so we have reasons to believe the validity of our models.

As for model selection, we have decided to focus more on RMSE because RMSE punishes large errors and we would like to avoid large errors in our marketing mix model. From the results, we observe that for all three products, the logit models outperform the multiplicative models in terms of RMSE.

F-test

Basically, the F-test for linear regression tests whether any of the independent variables in a multiple linear regression model are significant. We have compared the p-value for the F-tests of all our models to the significance level, and found that all p-values are less than the significance level. So, the conclusion is that all of our models fit the data better than models with no independent variables – our models are valid in terms of F-test.

Durbin-Watson Statistic

The Durbin-Watson statistic is a test statistic used to detect the presence of autocorrelation at lag 1 in the prediction errors from a regression analysis. The Durbin-Watson Statistic ranges from 0 to 4 with 2 being a strong indication of no autocorrelation.

We can observe from the results that the Durbin-Watson Statistic of all our models are close to 2. We have further confirmed the DWs are all in the range of critical values, which indicate the lack of autocorrelation. Also, the Durbin-Watson Statistics of logit models are closer to 2 than those of multiplicative models.

In conclusion, our models are valid in terms of autocorrelation and logit models outperformed multiplicative models in this aspect.

Variance Inflation Factor and T-Statistic

Finally, we used the VIF and T-Statistic to take a look at each independent variable in our models:

	VIF-multiplicative	*VIF-logit*	*Pr(>|t|)-multiplicative*	*Pr(>|t|)-logit*
Price	1.73	1.73	3.54E-02	3.93E-03
Discount	4.00	4.00	1.37E-02	2.61E-02
Flyer	2.53	2.53	1.00E+00	4.11E-01
Email	1.61	1.61	4.50E-01	2.95E-01
Web Display	1.32	1.32	4.82E-01	6.81E-01
Paid Search	1.17	1.17	8.71E-02	2.13E-01
TV	8.54	8.54	5.97E-01	3.39E-01
Radio	6.65	6.65	1.00E+00	1.00E+00
Seasonality	2.38	2.38	9.77E-03	1.16E-02
Holiday	2.32	2.32	1.00E+00	1.00E+00
Big holiday	1.39	1.39	1.54E-01	1.27E-01

Table: VIF and T-Stat for product 138936951

	VIF-multiplicative	*VIF-logit*	*Pr(>|t|)-multiplicative*	*Pr(>|t|)-logit*
Price	1.73	1.73	3.35E-03	2.83E-04
Discount	3.81	3.81	4.59E-03	4.59E-02
Flyer	2.30	2.30	1.00E+00	1.00E+00
Email	1.59	1.59	7.67E-03	9.28E-01
Web Display	1.35	1.35	1.00E+00	1.00E+00
Store Display	1.34	1.34	9.65E-01	3.68E-01
Paid Search	1.13	1.13	5.16E-01	5.83E-01

TV	8.67	8.67	1.00E+00	6.62E-01
Radio	6.67	6.67	9.93E-01	8.69E-01
Seasonality	2.51	2.51	5.99E-06	5.57E-06
Holiday	2.17	2.17	8.41E-01	3.16E-02
Big holiday	1.57	1.57	1.56E-08	1.00E+00

Table: VIF and T-Stat for product 138936952

	VIF-multiplicative	*VIF-logit*	*Pr(>|t|)-multiplicative*	*Pr(>|t|)-logit*
Price	1.7544	1.7544	5.33E-09	1.70E-11
Discount	9.9171	9.9171	1.79E-02	1.02E-02
Flyer	3.9609	3.9609	7.40E-01	7.28E-01
Email	1.6804	1.6804	5.54E-01	8.57E-01
Web Display	1.3763	1.3763	4.71E-01	3.14E-01
Store Display	5.4853	5.4853	1.00E+00	1.00E+00
Paid Search	1.1506	1.1506	5.56E-01	4.79E-01
TV	9.0648	9.0648	1.00E+00	8.14E-01
Radio	7.0417	7.0417	1.00E+00	1.00E+00
Seasonality	2.5083	2.5083	8.15E-11	3.70E-12
Holiday	2.2761	2.2761	8.50E-01	5.70E-01
Big holiday	2.051	2.051	4.21E-01	1.00E+00

Table: VIF and T-Stat for product 138936953

From the above results, we have found that TV and Radio tend to have a larger VIF than 5, which indicates that they have a high level of collinearity with each other. It is reasonable because from a marketing perspective, TV and Radio are large-scale promotional campaigns, thus they are usually implemented in combination with other promotions such as Paid Search, also they usually happen during special events such as holidays. That is why the effects of these variables in our models are similar to others.

We have attempted to remove TV or radio from our regression model, but it resulted in a significantly worse model performance. For example, removing TV from the logit model for product I resulted in a decrease of 10% in R-squared. From a business standpoint, we are interested in analyzing both TV's effects and Radio's effects on sales, thus it is not ideal to remove them either. So, we have decided to keep both TV and radio in our final model.

After running a comprehensive model diagnosis, we have not only tested the validity of our models, but also select the better model form from the multiplicative model and logit model. It turns out that the logit models outperform the multiplicative models in almost all perspectives. This decision is also based on our finding that TV and Radio have implicit interaction with other variables. Logit models are bounded and they tend to perform well within the range, and they are good at depicting the implicit complex interactions among causal variables, just like in our case.

As a result, we have built a logit model for each product, and we will analyze the coefficients and DueTos in the next section.

Model Decomposition

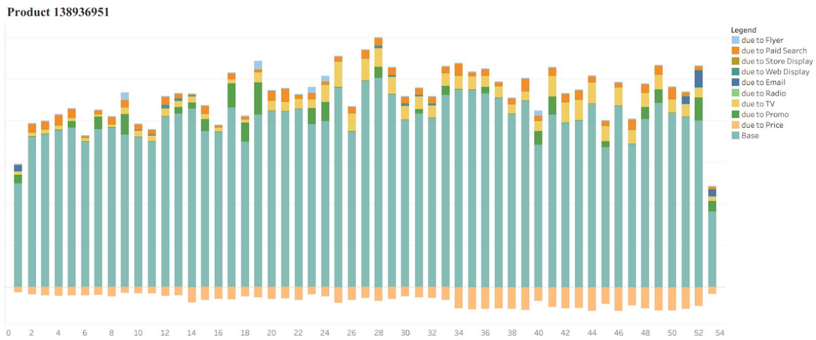
The ultimate goal of marketing mix modelling is to decompose a measure of the marketing goal, in our case sales, into its base and other components due to each marketing vehicle. We have taken the following steps to calculate DueTos:

a. We created the baseline as the base price plus seasonality, holiday and important holiday. Basically, we used the initial price in the two-year period as the base price, with other causal variables replaced with 0, to re-run the regression and get the baseline.

b. We calculated the DueTos of price, promotion, radio, TV, store-display, web-display, email, paid-search and flyer. For example, DueTo-TV is the difference between the quantities estimated using the real ad stocked TV reach and the quantities estimated using TV equal to zero while the other variables remain unchanged. In this way, we are able to isolate the effect of each vehicle.

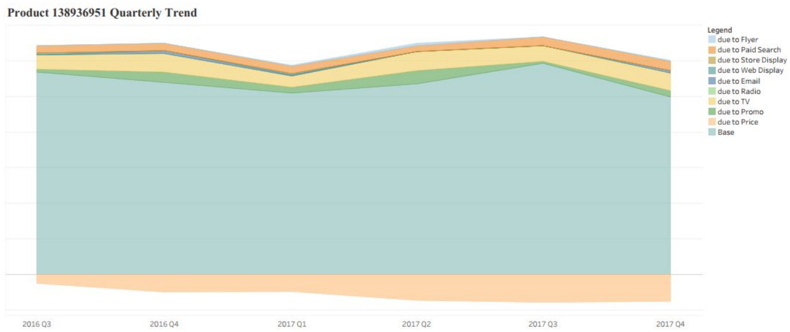
c. At last, we visualized the DueTos weekly and quarterly using Tableau. While weekly DueTos decomposition gives the viewers a lot of details, the purpose of quarterly aggregated visualization is that we want to observe the general trend as well.

Now, we are able to analyze the results of marketing mix model from the visualizations: (The X-axis represents week number, “1” being the week of 2015-12-27 and “53” being the week of 2017-12-31.)



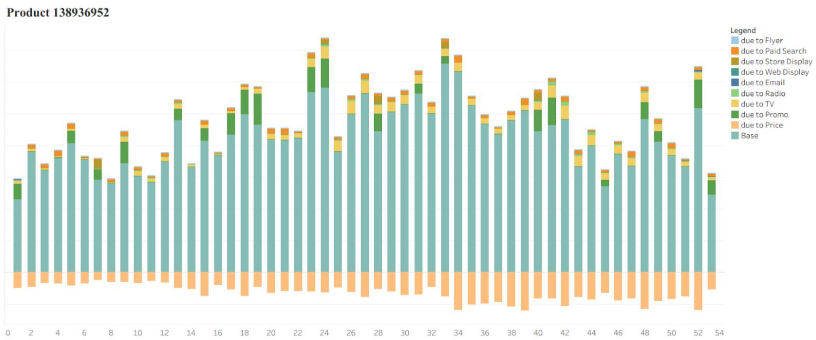
Graph: Weekly DueTos for product 13896951

For product I, the most effective marketing vehicles are TV, discounts and paid search.



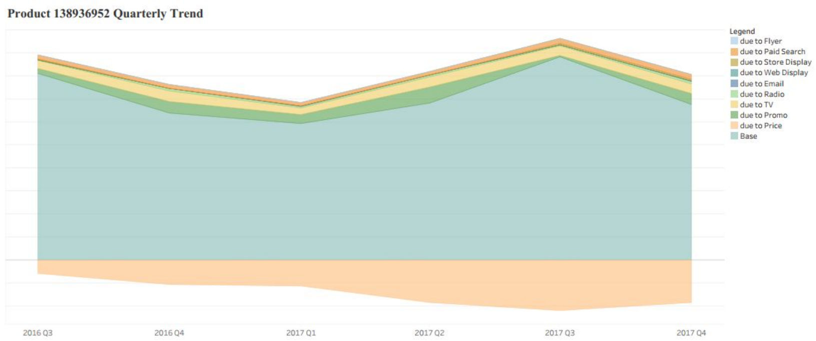
Graph: Quarterly trend of DueTos for product 13896951

For product I, we can see the general trend of DueTos is almost consistent. TV consistently makes up a large proportion of marketing effects.



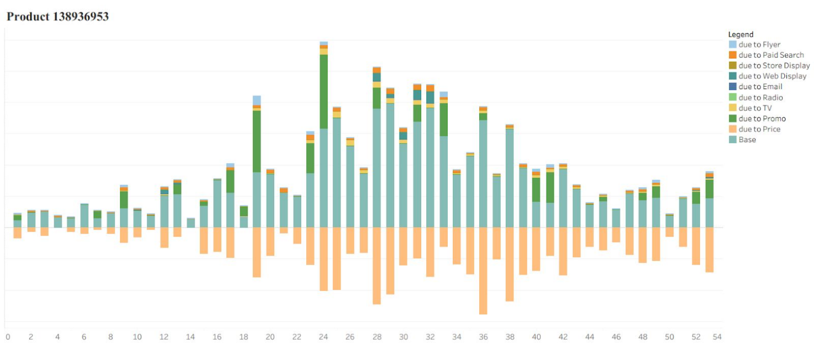
Graph: Weekly DueTos for product 13896952

For product II, the most effective marketing vehicles are TV, discounts and paid search.



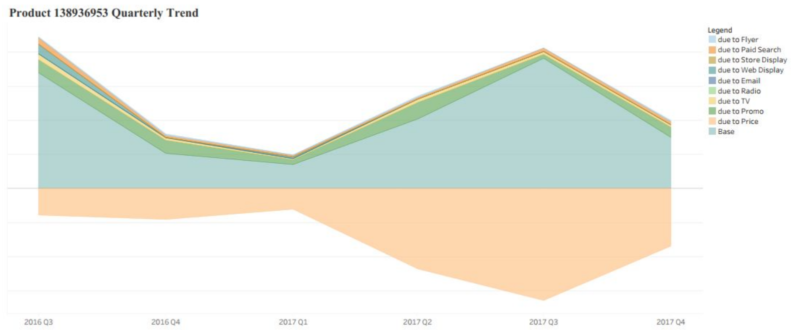
Graph: Quarterly trend of DueTos for product 13896952

For product II, we can see the general trend is changing but almost steady. It seems like that price is posing an increasingly negative effect on sales. We know that from 2016 to 2017 product II has increased its price from \$3.99 to \$4.49. It is possible that the price should be adjusted.



Graph: Weekly DueTos for product 13896953

For product III, the most effective marketing vehicles are TV, discounts, paid search and Email.



Graph: quarterly trend of DueTos for product 13896953

For product III, we can see that Email is an effective marketing vehicle. When there are email campaigns, email is made up of a decent proportion of marketing effects. It may be worth bringing back email campaigns for product III. Also, the price of product III increased to $16.99 around the second quarter and third quarter of 2017, which immediately posed a negative influence. We observed that the price was brought down afterwards, so Pernalonga might have noticed the negative effects too.

Appendix – Final Model Summary

```
Formula: sales_transformed ~ param_X.Intercept. * X.Intercept. + param_weekly_price *
    weekly_price + param_weekly_discount * weekly_discount +
    param_Flyer * Flyer + param_Email * Email + param_Web_Display *
    Web_Display + param_Paid_Search * Paid_Search + param_TV *
    TV + param_Radio * Radio + param_seasonality * seasonality +
    param_holiday * holiday + param_big_holiday * big_holiday

Parameters:
                      Estimate Std. Error t value Pr(>|t|)
param_X.Intercept.     2.25e+00   9.13e-01    2.47   0.0155 *
param_weekly_price    -3.68e+00   1.24e+00   -2.96   0.0039 **
param_weekly_discount  3.21e+00   1.42e+00    2.26   0.0261 *
param_Flyer            2.39e-01   2.90e-01    0.83   0.4115
param_Email            1.22e-06   1.15e-06    1.05   0.2948
param_Web_Display      2.05e-07   4.96e-07    0.41   0.6809
param_Paid_Search      3.77e-06   3.01e-06    1.25   0.2128
param_TV               6.92e-01   7.19e-01    0.96   0.3386
param_Radio            0.00e+00   5.01e-01    0.00   1.0000
param_seasonality      7.29e-05   2.83e-05    2.58   0.0116 *
param_holiday          0.00e+00   1.29e-01    0.00   1.0000
param_big_holiday      7.26e-01   4.71e-01    1.54   0.1266
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Table: Final Logit Model for Product 13896951

```
Formula: sales_transformed ~ param_X.Intercept. * X.Intercept. + param_weekly_price *
    weekly_price + param_weekly_discount * weekly_discount +
    param_Flyer * Flyer + param_Email * Email + param_Web_Display *
    Web_Display + param_Store_Display * Store_Display + param_Paid_Search *
    Paid_Search + param_TV * TV + param_Radio * Radio + param_seasonality *
    seasonality + param_holiday1 * holiday1 + param_big_holiday *
    big_holiday

Parameters:
                      Estimate Std. Error t value Pr(>|t|)
param_X.Intercept.     2.32e+00   1.08e+00    2.15  0.03416 *
param_weekly_price    -9.83e-01   2.60e-01   -3.77  0.00028 ***
param_weekly_discount  6.40e-01   3.16e-01    2.02  0.04592 *
param_Flyer            0.00e+00   1.72e-01    0.00  1.00000
param_Email            1.16e-07   1.28e-06    0.09  0.92770
param_Web_Display      0.00e+00   6.09e-07    0.00  1.00000
param_Store_Display    2.32e-01   2.56e-01    0.90  0.36833
param_Paid_Search      1.95e-06   3.54e-06    0.55  0.58313
param_TV               3.82e-01   8.70e-01    0.44  0.66170
param_Radio            9.98e-02   6.05e-01    0.17  0.86930
param_seasonality      1.68e-04   3.49e-05    4.82  5.6e-06 ***
param_holiday1         3.28e-01   1.50e-01    2.18  0.03155 *
param_big_holiday      0.00e+00   3.53e-01    0.00  1.00000
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Table: Final Logit Model for Product 13896952

```
Formula: sales_transformed ~ param_X.Intercept. * X.Intercept. + param_weekly_price *
    weekly_price + param_weekly_discount * weekly_discount +
    param_Flyer * Flyer + param_Email * Email + param_Web_Display *
    Web_Display + param_Store_Display * Store_Display + param_Paid_Search *
    Paid_Search + param_TV * TV + param_Radio * Radio + param_seasonality *
    seasonality + param_holiday1 * holiday1 + param_big_holiday *
    big_holiday

Parameters:
                      Estimate Std. Error t value Pr(>|t|)
param_X.Intercept.     7.74e+00   1.69e+00    4.57  1.5e-05 ***
param_weekly_price    -8.45e-01   1.10e-01   -7.71  1.7e-11 ***
param_weekly_discount  4.07e-01   1.55e-01    2.63     0.01 *
param_Flyer            1.25e-01   3.60e-01    0.35     0.73
param_Email            3.19e-07   1.77e-06    0.18     0.86
param_Web_Display      8.40e-07   8.30e-07    1.01     0.31
param_Store_Display    0.00e+00   6.38e-01    0.00     1.00
param_Paid_Search      3.50e-06   4.93e-06    0.71     0.48
param_TV               2.83e-01   1.20e+00    0.24     0.81
param_Radio            0.00e+00   8.41e-01    0.00     1.00
param_seasonality      3.81e-04   4.74e-05    8.03  3.7e-12 ***
param_holiday1         1.20e-01   2.10e-01    0.57     0.57
param_big_holiday      0.00e+00   5.43e-01    0.00     1.00
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Table: Final Logit Model for Product 13896953