

Lecture 6: Training Neural Networks, Part I

Разбор задачи с самостоятельной

Для заданной функции записать вычислительный граф, рассчитать прямое и обратное распространение по графу для заданного значения входов. При обратном распространении считать начальное значение градиента равным 1. При необходимости результаты, и промежуточные, и окончательные, округлять до второго знака.

Вариант 2.

$$f(\mathbf{x}, \mathbf{y}) = \frac{x_0}{e^{-(x_0 y_0 + x_1 y_1)}}$$

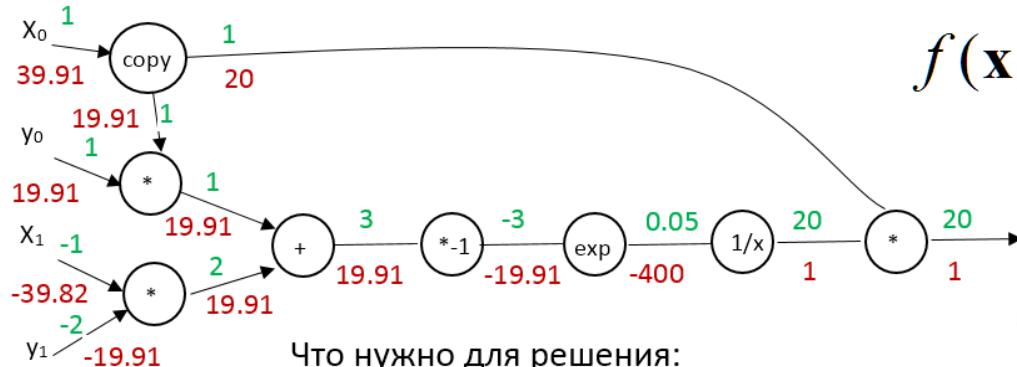
Рассчитать прямое распространение и градиенты

$$\partial f / \partial x_0, \quad \partial f / \partial x_1, \quad \partial f / \partial y_0, \quad \partial f / \partial y_1$$

Начальные значения: $x_0 = 1, \quad x_1 = -1, \quad y_0 = 1, \quad y_1 = -2$

Разбор задачи с самостоятельной

Решение:



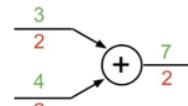
Что нужно для решения:

$$\begin{array}{lll} f(x) = e^x & \rightarrow & \frac{df}{dx} = e^x \\ f_a(x) = ax & \rightarrow & \frac{df}{dx} = a \end{array} \quad \left| \quad \begin{array}{lll} f(x) = \frac{1}{x} & \rightarrow & \frac{df}{dx} = -1/x^2 \\ f_c(x) = c + x & \rightarrow & \frac{df}{dx} = 1 \end{array} \right.$$

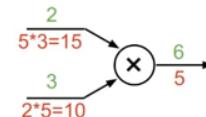
Задача:

$$f(\mathbf{x}, \mathbf{y}) = \frac{x_0}{e^{-(x_0y_0 + x_1y_1)}}$$

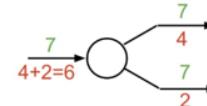
add gate: gradient distributor



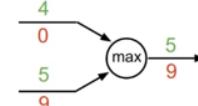
mul gate: "swap multiplier"



copy gate: gradient adder

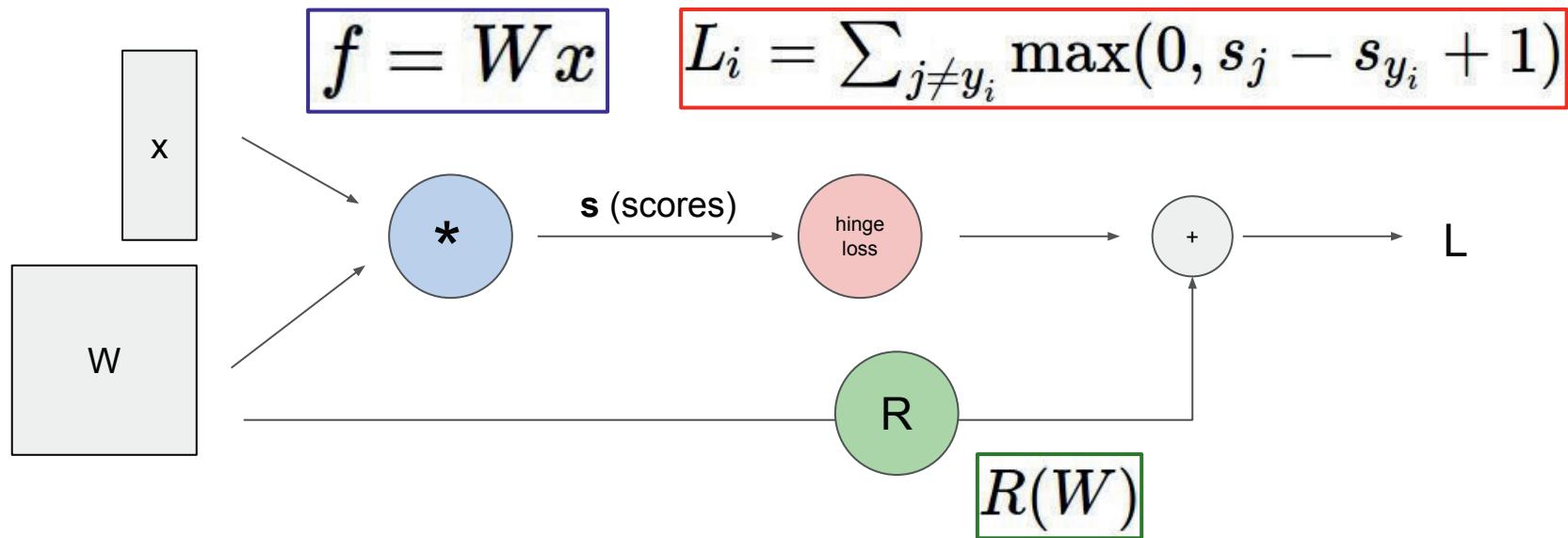


max gate: gradient router



Where we are now...

Computational graphs



Where we are now...

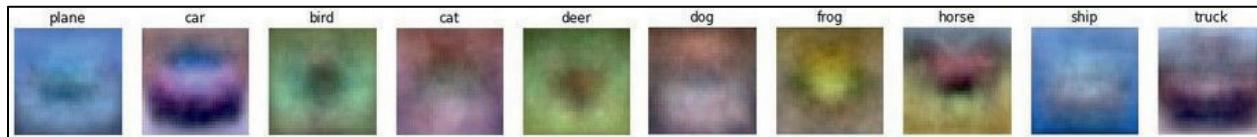
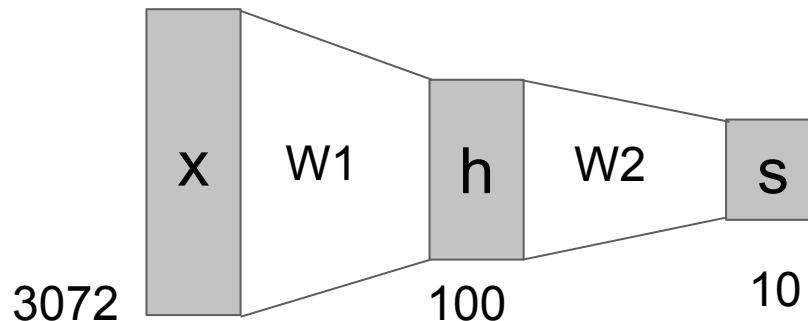
Neural Networks

Linear score function:

$$f = Wx$$

2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$



Where we are now...

Convolutional Neural Networks

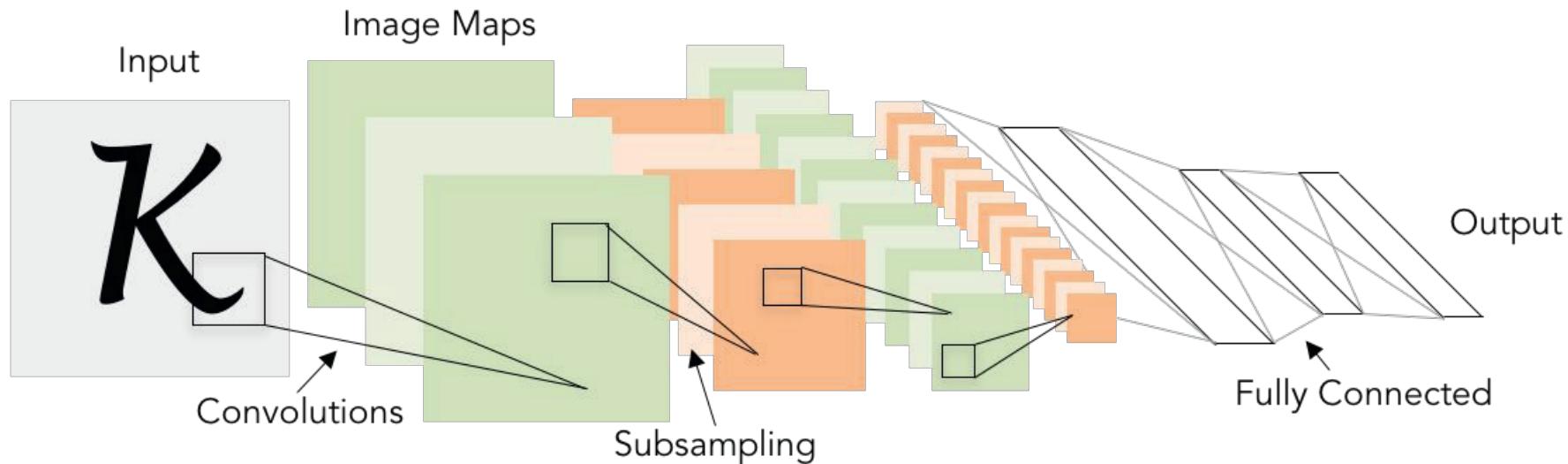
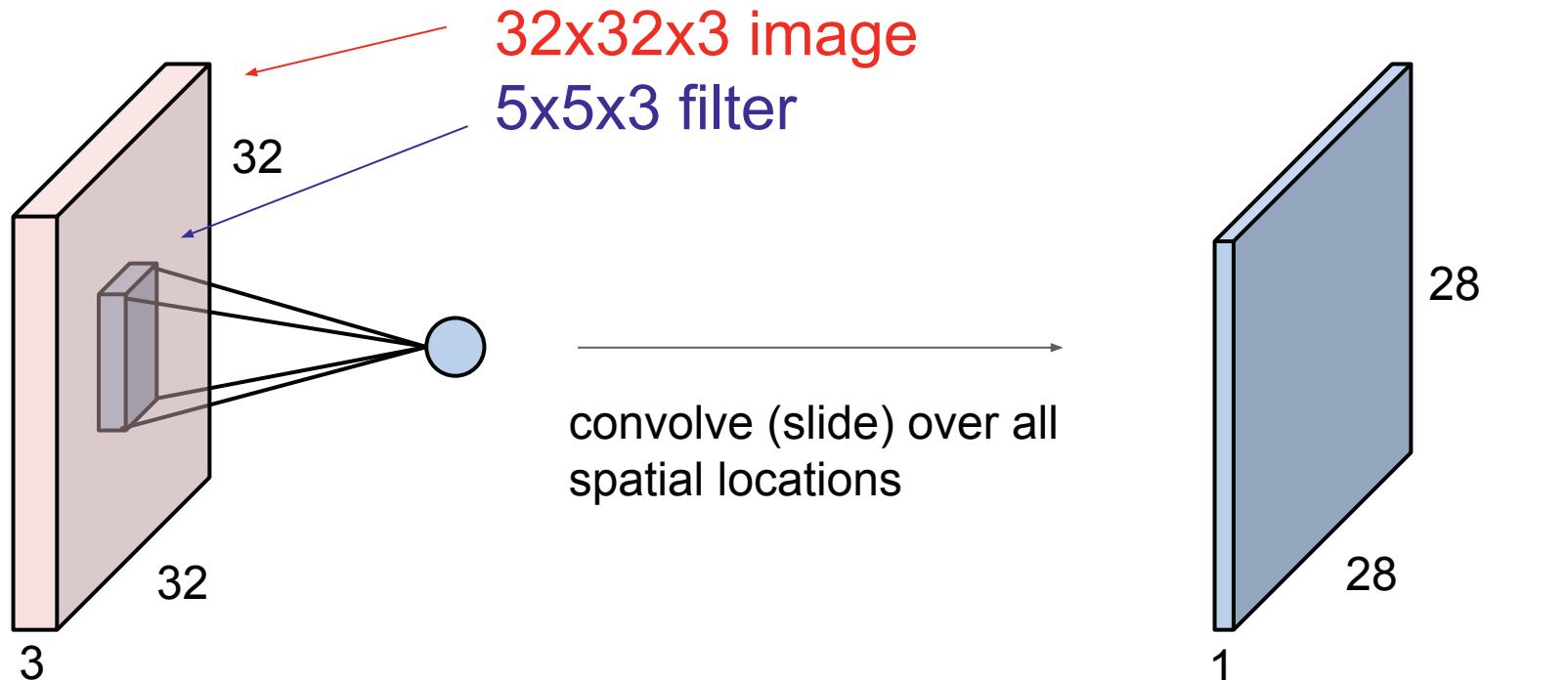


Illustration of LeCun et al. 1998 from CS231n 2017 Lecture 1

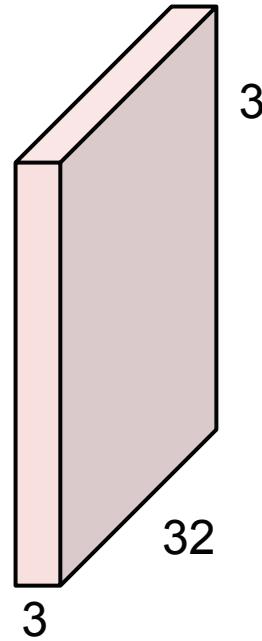
Where we are now...

Convolutional Layer



Where we are now...

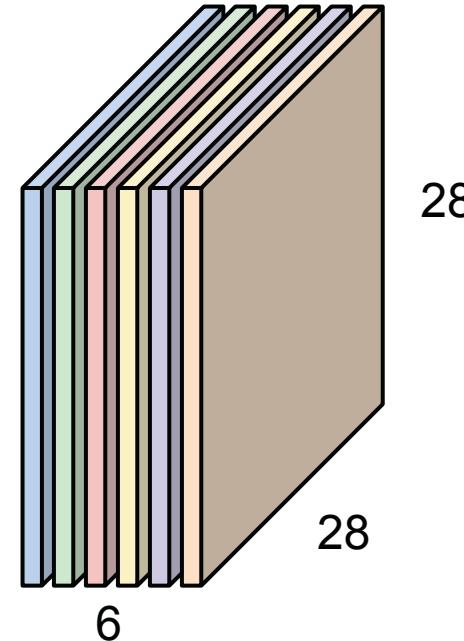
Convolutional Layer



Convolution Layer

For example, if we had 6 5×5 filters, we'll get 6 separate activation maps:

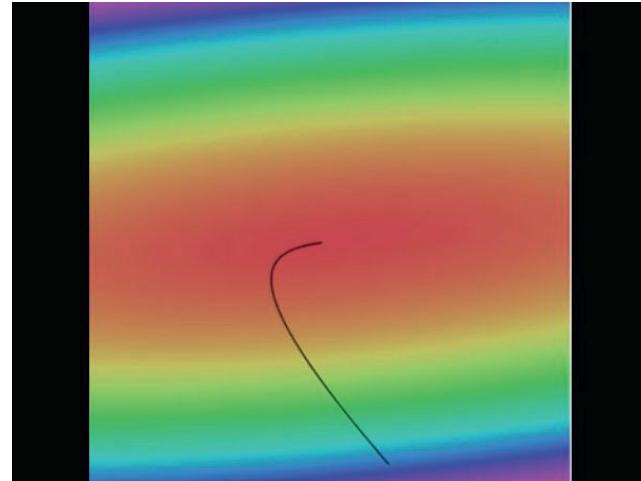
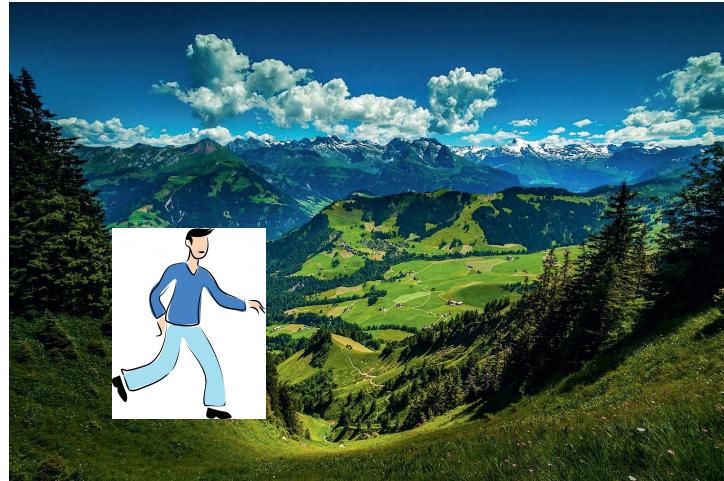
activation maps



We stack these up to get a “new image” of size $28 \times 28 \times 6$!

Where we are now...

Learning network parameters through optimization



```
# Vanilla Gradient Descent

while True:
    weights_grad = evaluate_gradient(loss_fun, data, weights)
    weights += - step_size * weights_grad # perform parameter update
```

Landscape image is [CC0 1.0](#) public domain
Walking man image is [CC0 1.0](#) public domain

Where we are now...

Mini-batch SGD

Loop:

1. **Sample** a batch of data
2. **Forward** prop it through the graph
(network), get loss
3. **Backprop** to calculate the gradients
4. **Update** the parameters using the gradient

Where we are now...

Hardware + Software



PyTorch



TensorFlow

Next: Training Neural Networks

Overview

1. One time setup

activation functions, preprocessing, weight initialization, regularization, gradient checking

2. Training dynamics

transfer learning, babysitting the learning process, parameter updates, hyperparameter optimization

3. Evaluation

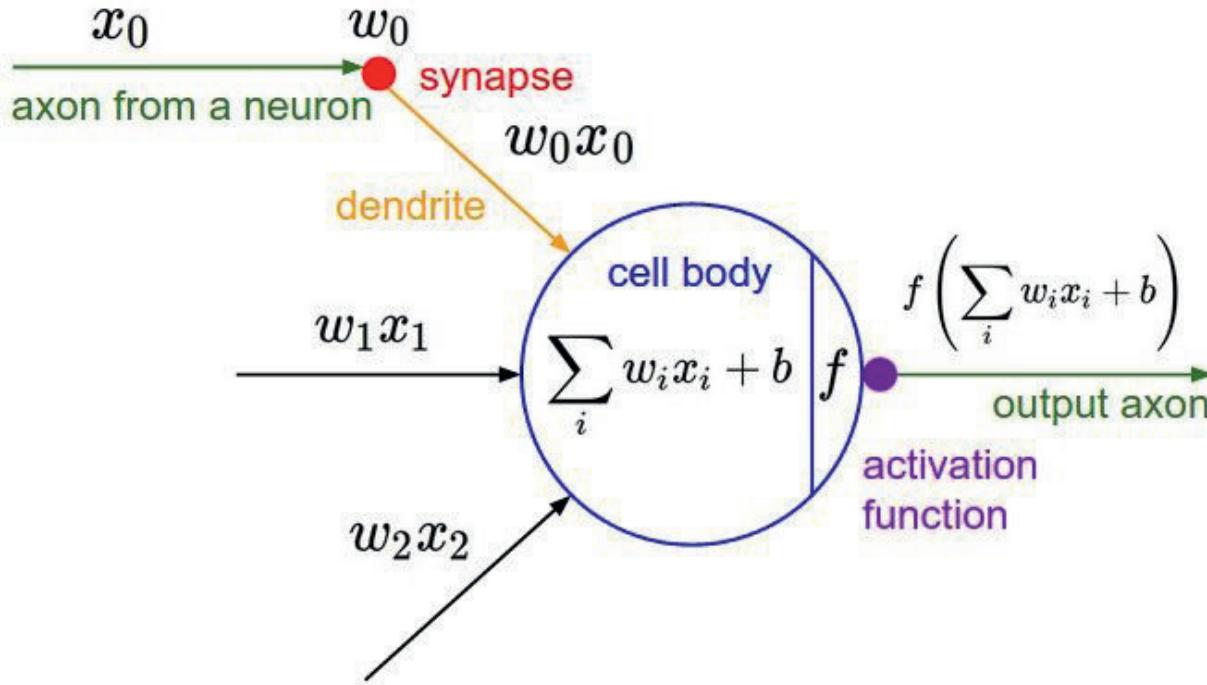
model ensembles, test-time augmentation

Part 1

- Activation Functions
- Data Preprocessing
- Weight Initialization
- Batch Normalization
- Transfer learning

Activation Functions

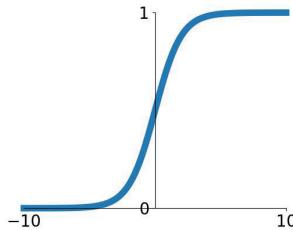
Activation Functions



Activation Functions

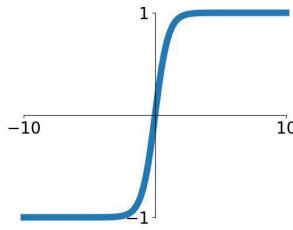
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



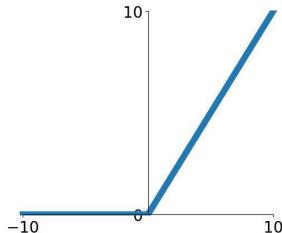
tanh

$$\tanh(x)$$



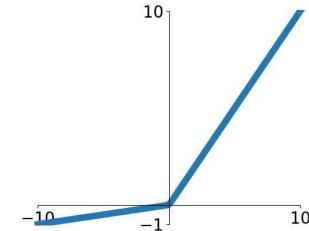
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

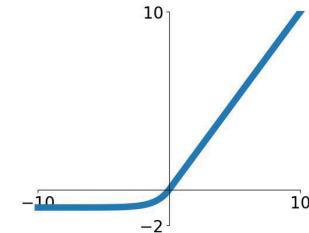


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

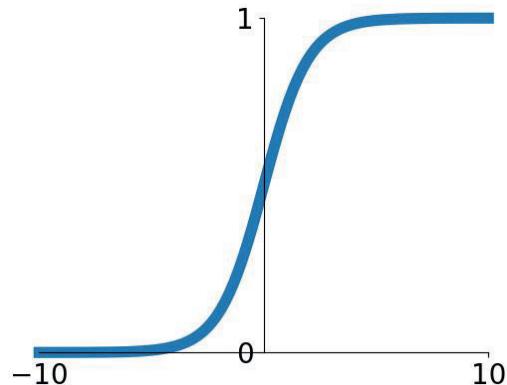
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$

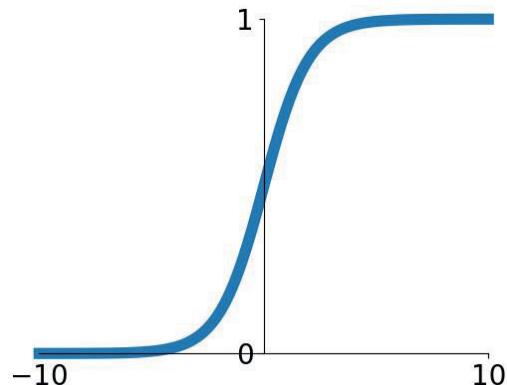
- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron



Sigmoid

Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$

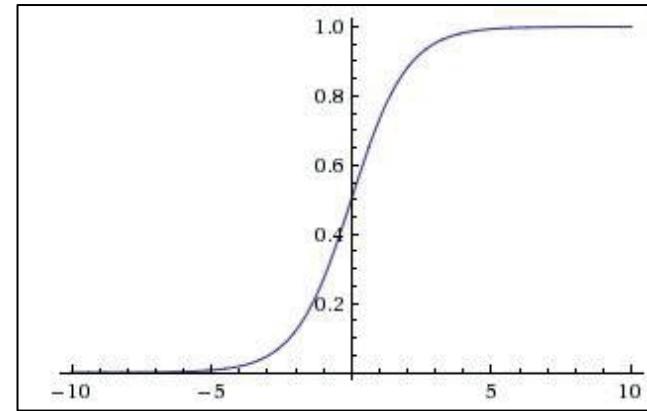
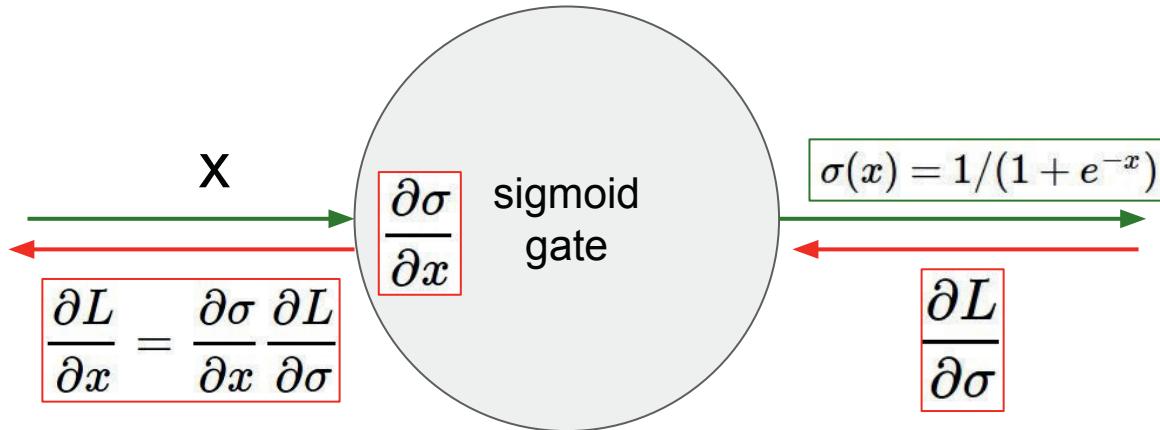


Sigmoid

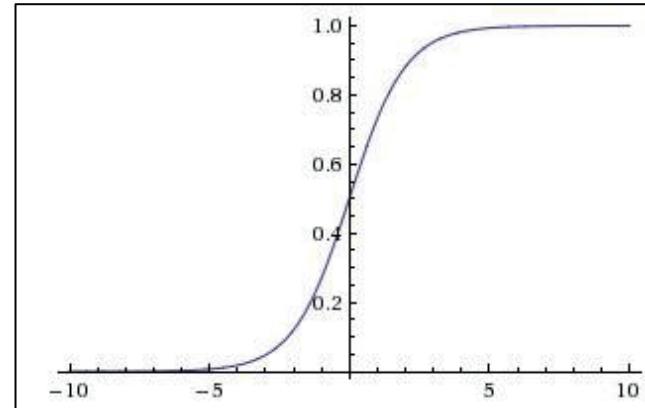
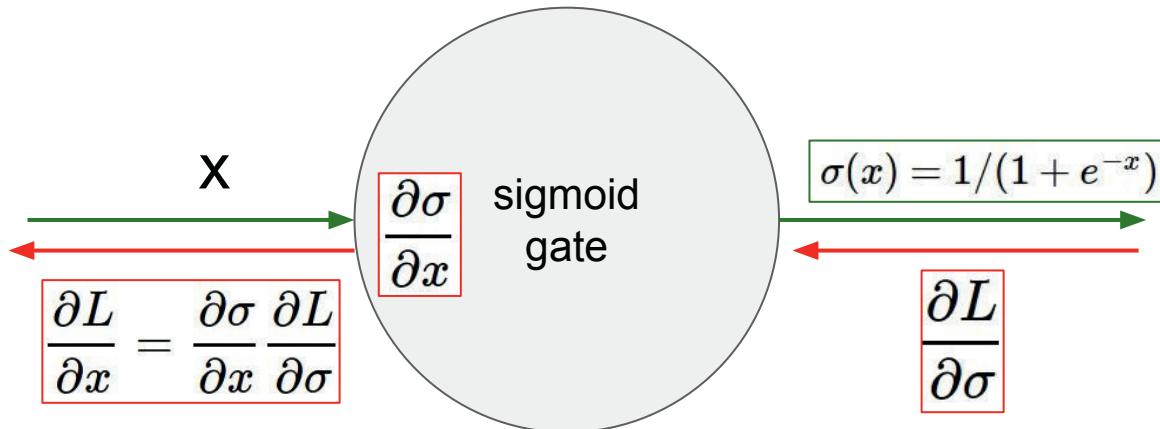
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3 problems:

1. Saturated neurons “kill” the gradients

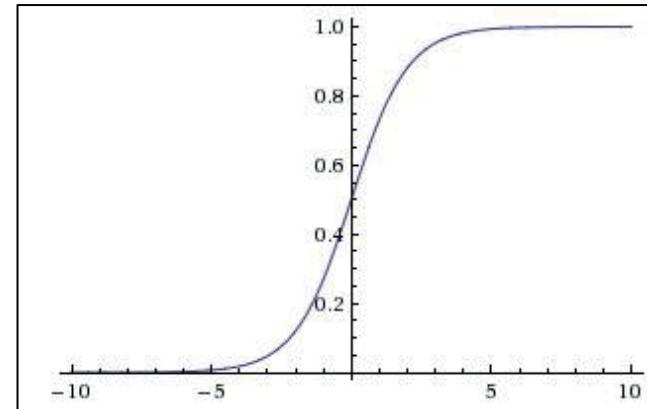
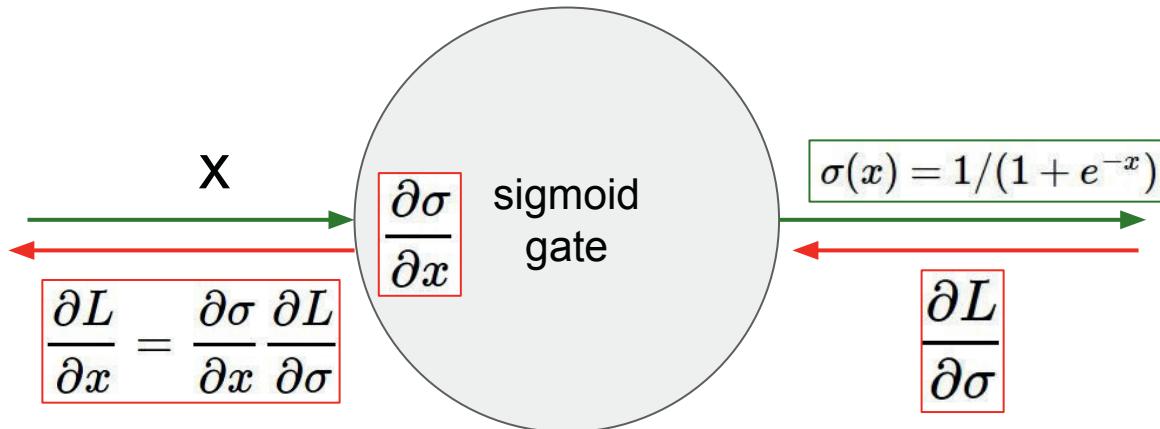


$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1 - \sigma(x))$$



What happens when $x = -10$?

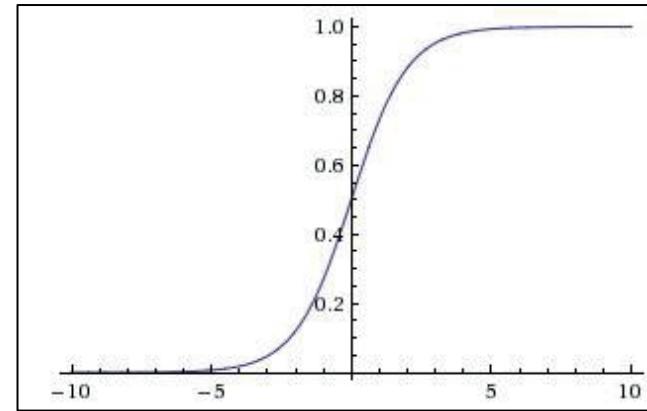
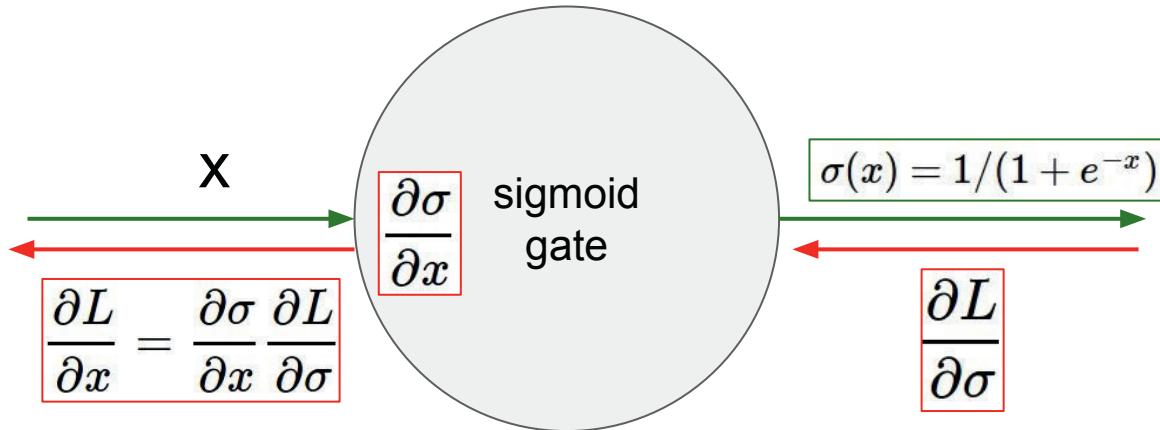
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What happens when $x = -10$?

What happens when $x = 0$?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1 - \sigma(x))$$

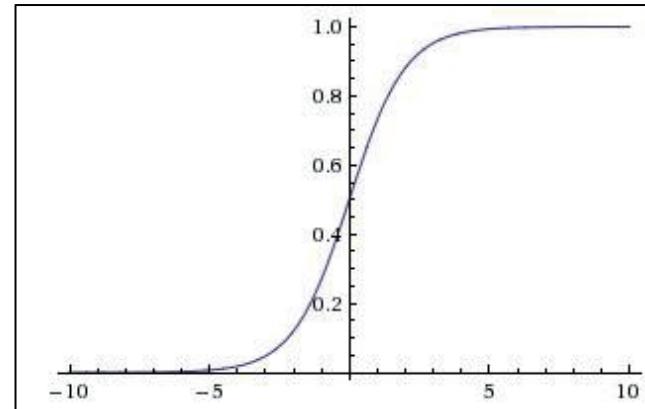
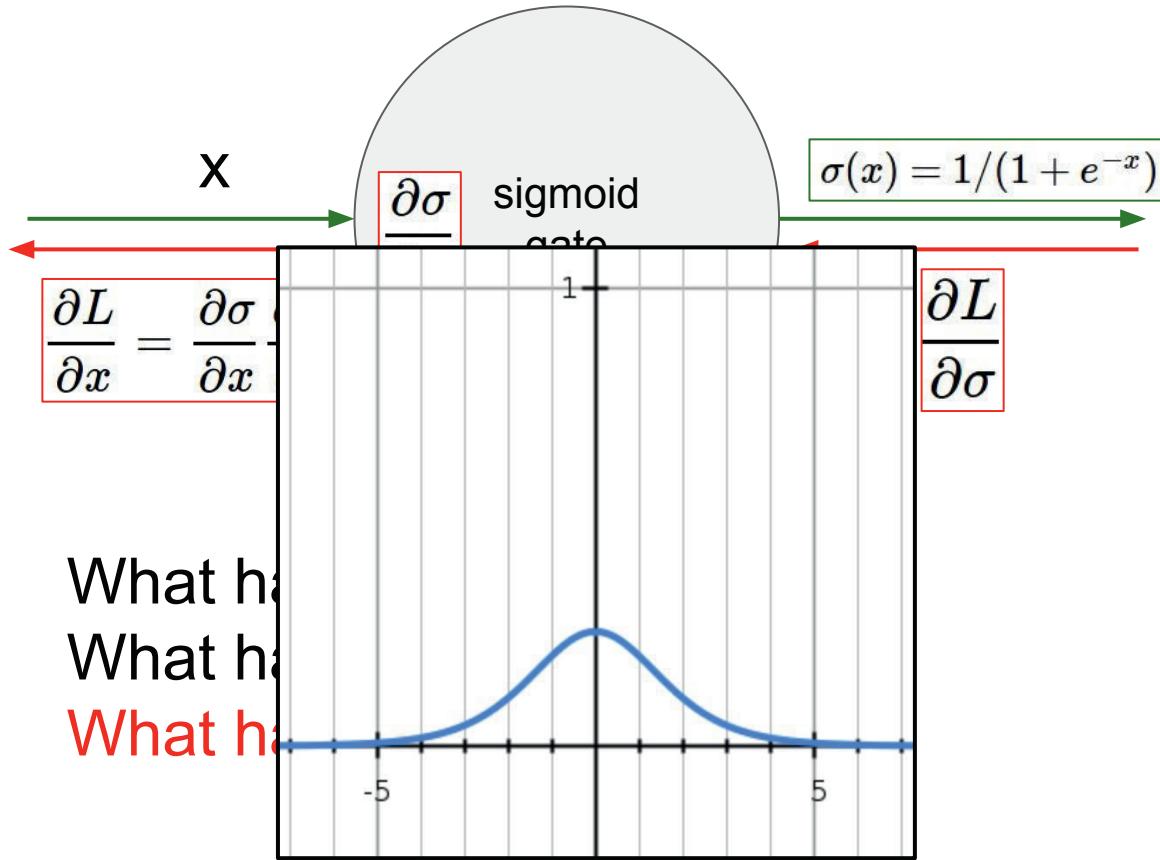


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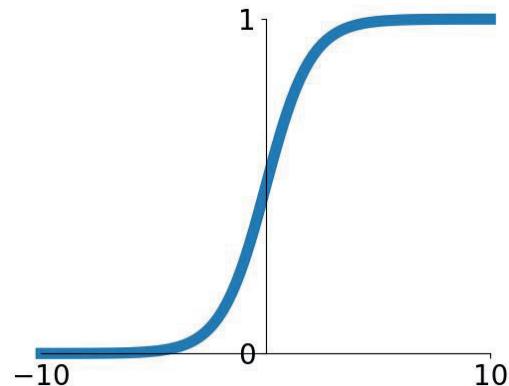
What happens when $x = 10$?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$



$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1 - \sigma(x))$$

Activation Functions



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

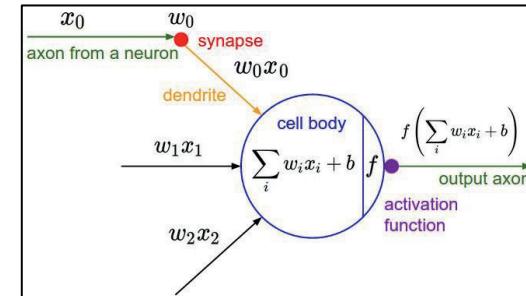
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- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

Consider what happens when the input to a neuron is always positive...

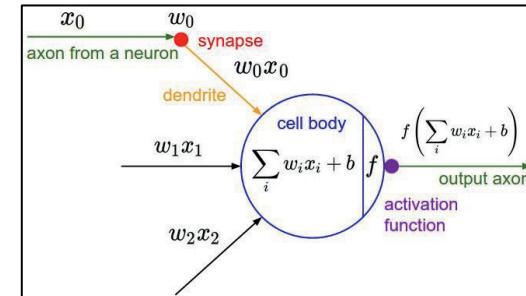
$$f \left(\sum_i w_i x_i + b \right)$$



What can we say about the gradients on w ?

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$

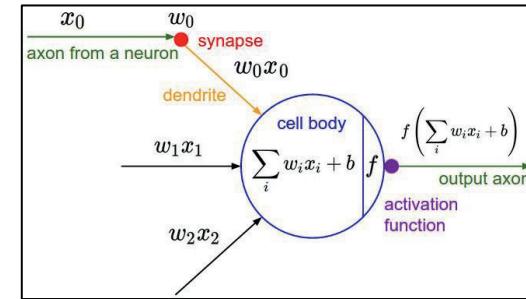


What can we say about the gradients on w ?

$$\frac{\partial L}{\partial w} = \sigma(\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))x \times \text{upstream_gradient}$$

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$



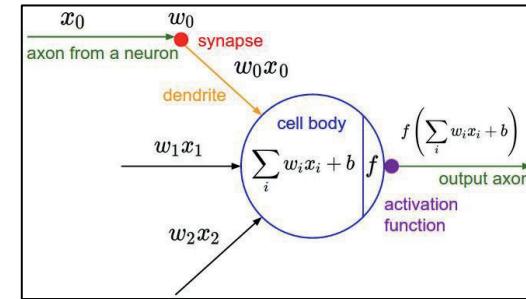
What can we say about the gradients on w ?

We know that local gradient of sigmoid is always positive

$$\frac{\partial L}{\partial w} = \boxed{\sigma(\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))x \times \text{upstream_gradient}}$$

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$



What can we say about the gradients on w ?

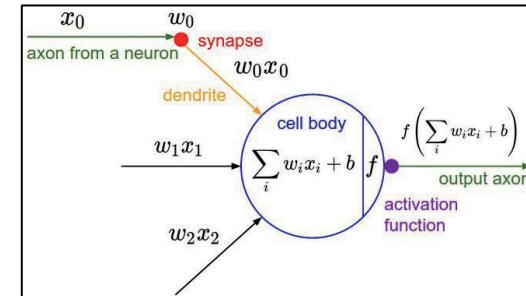
We know that local gradient of sigmoid is always positive

We are assuming x is always positive

$$\frac{\partial L}{\partial w} = \boxed{\sigma(\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))x} \times \text{upstream_gradient}$$

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$



What can we say about the gradients on w ?

We know that local gradient of sigmoid is always positive

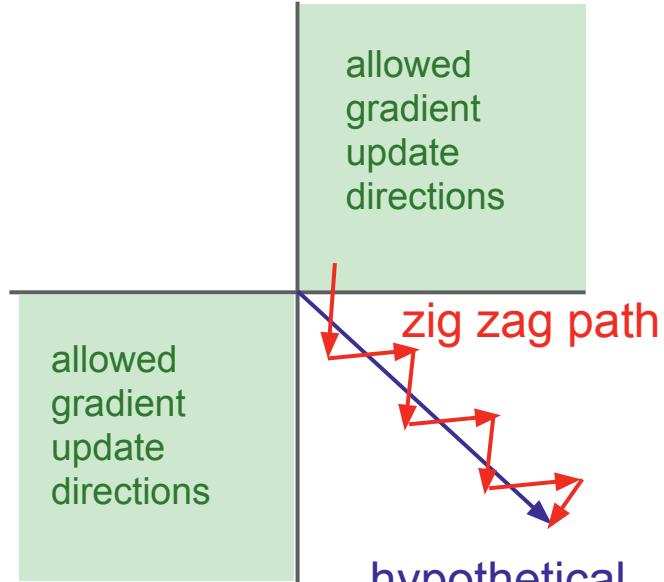
We are assuming x is always positive

So!! Sign of gradient for all w_i is the same as the sign of upstream scalar gradient!

$$\frac{\partial L}{\partial w} = \sigma(\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))x \times \text{upstream_gradient}$$

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$



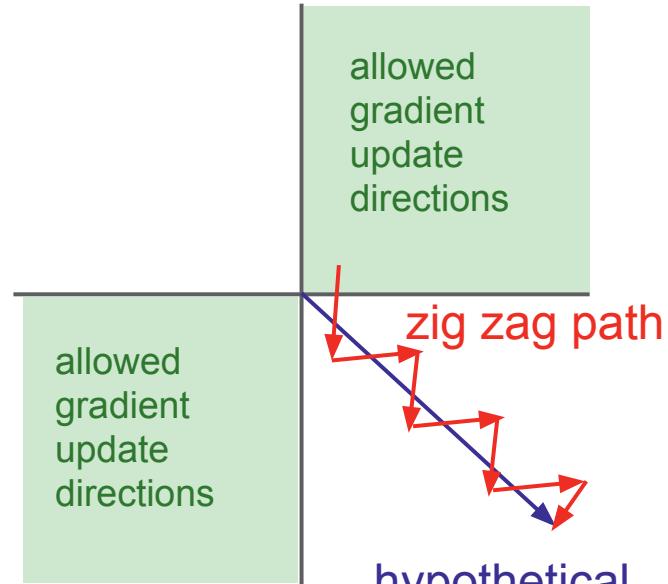
What can we say about the gradients on w ?

Always all positive or all negative :(

hypothetical
optimal w
vector

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$

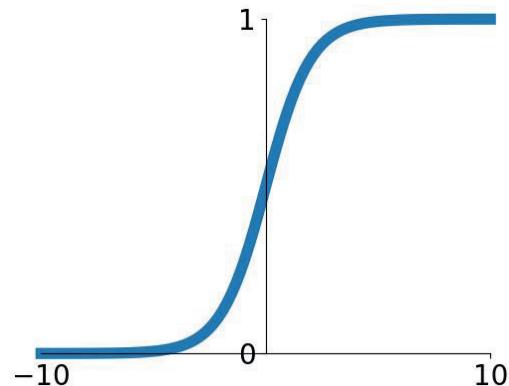


What can we say about the gradients on w ?

Always all positive or all negative :(

(For a single element! Minibatches help)

Activation Functions



Sigmoid

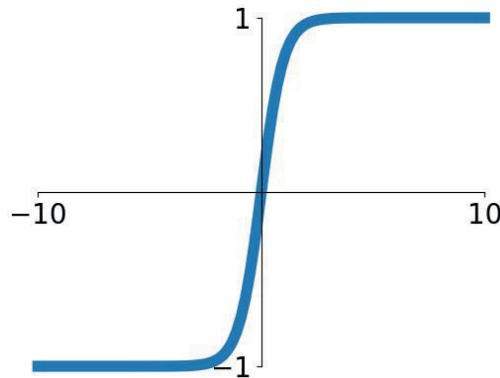
$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0, 1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered
3. $\exp()$ is a bit compute expensive

Activation Functions

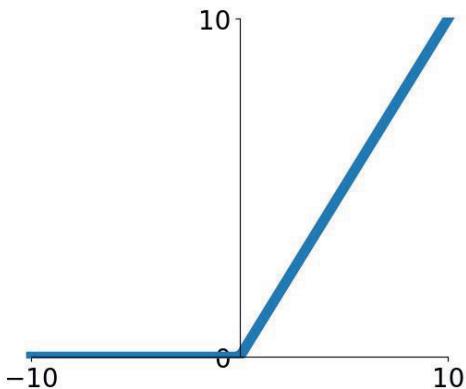


tanh(x)

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

Activation Functions

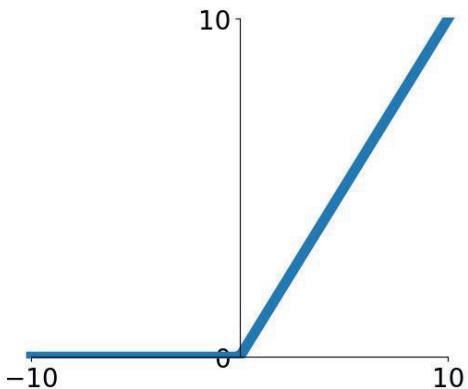


- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

ReLU
(Rectified Linear Unit)

[Krizhevsky et al., 2012]

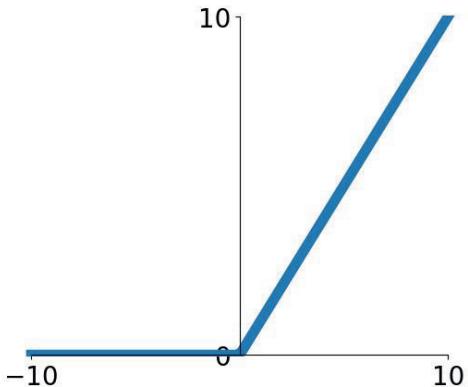
Activation Functions



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- Not zero-centered output

Activation Functions

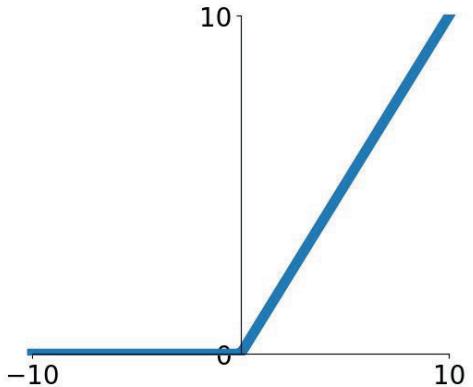
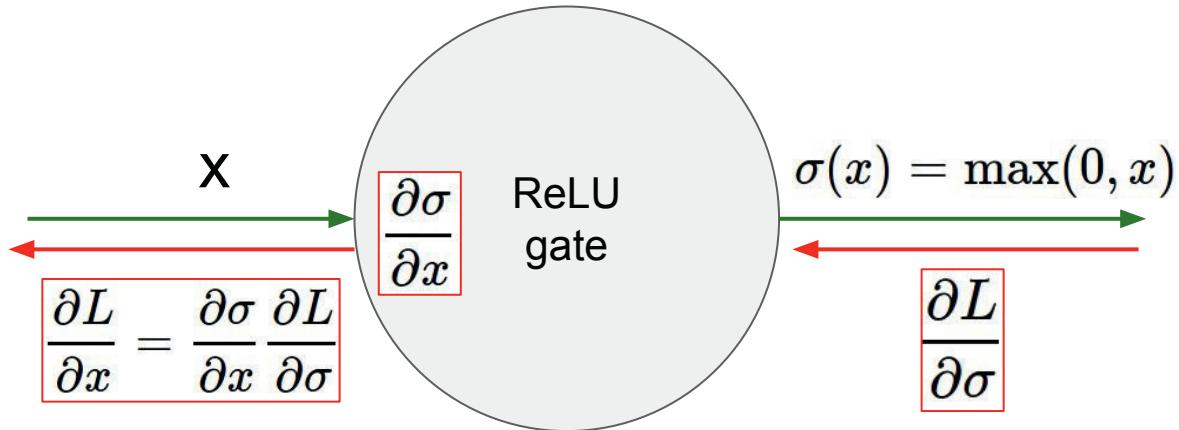


ReLU
(Rectified Linear Unit)

- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

- Not zero-centered output
- An annoyance:

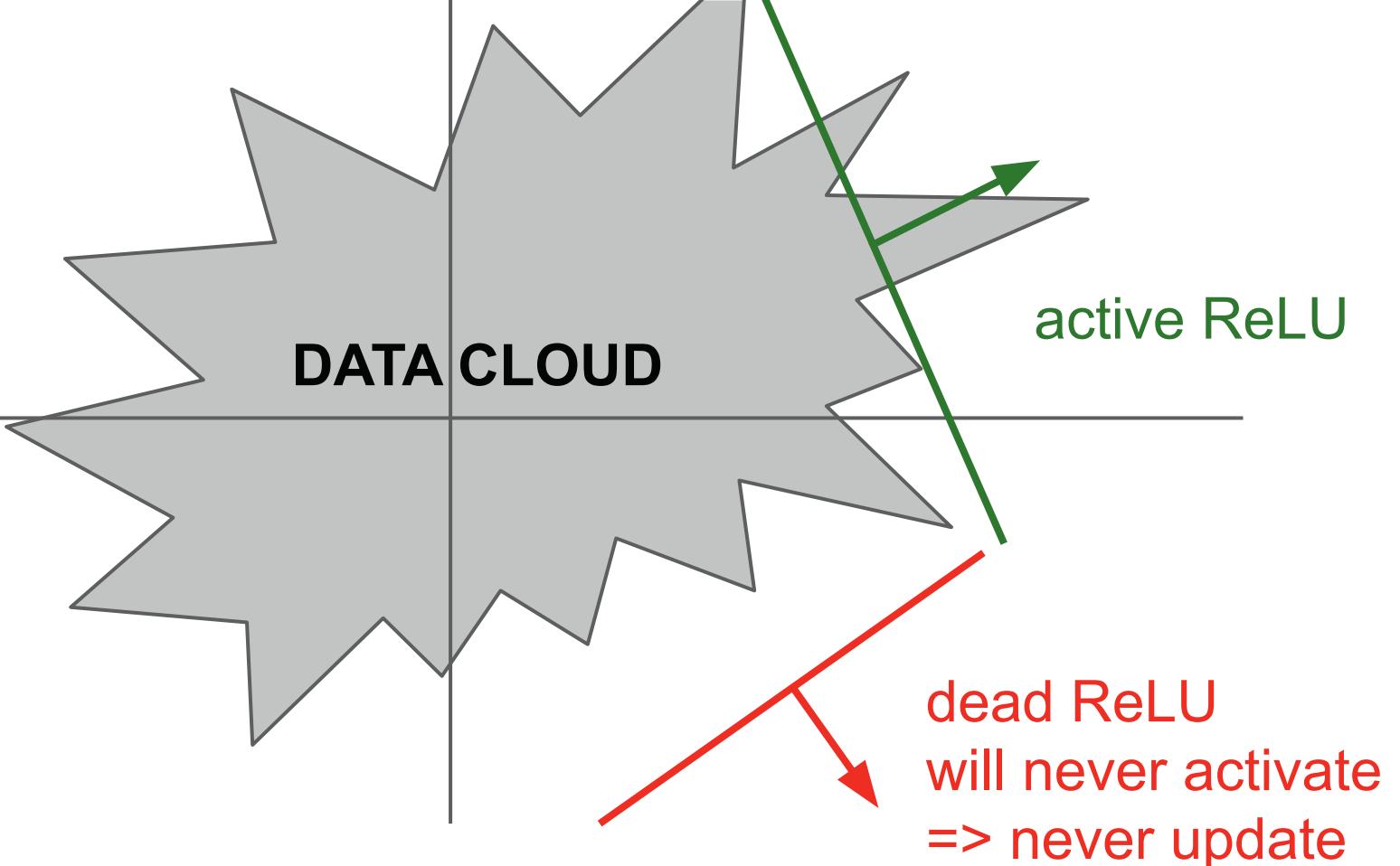
hint: what is the gradient when $x < 0$?

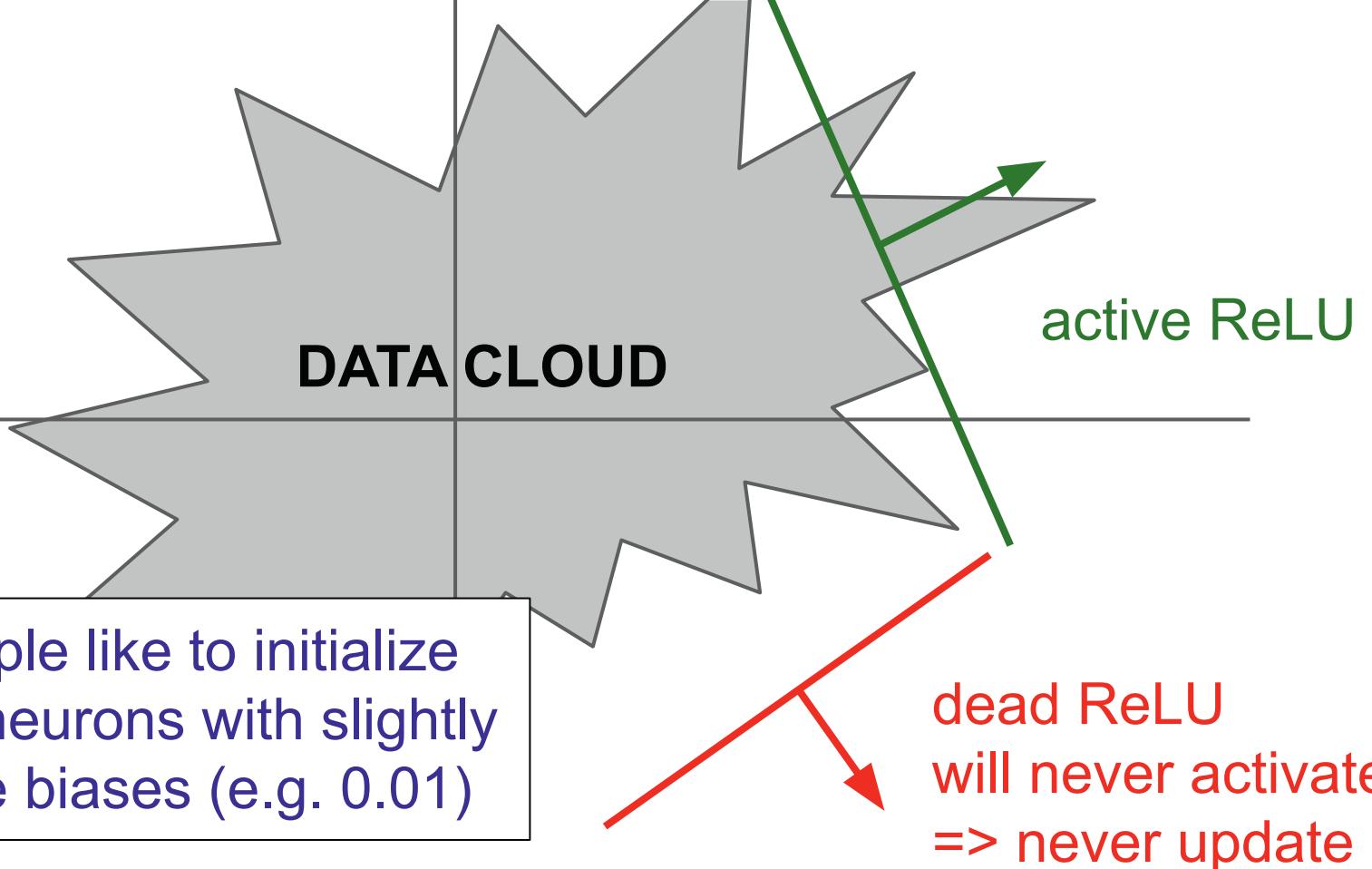


What happens when $x = -10$?

What happens when $x = 0$?

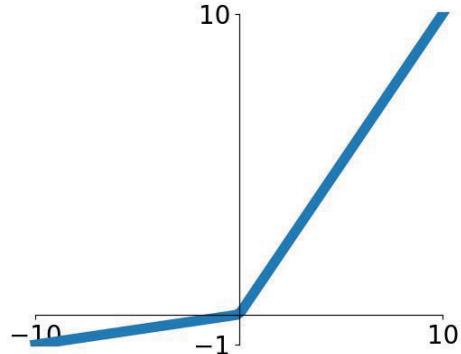
What happens when $x = 10$?





Activation Functions

[Mass et al., 2013]
[He et al., 2015]



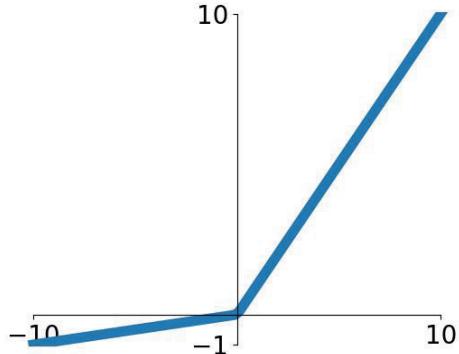
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Activation Functions

[Mass et al., 2013]
[He et al., 2015]



Leaky ReLU

$$f(x) = \max(0.01x, x)$$

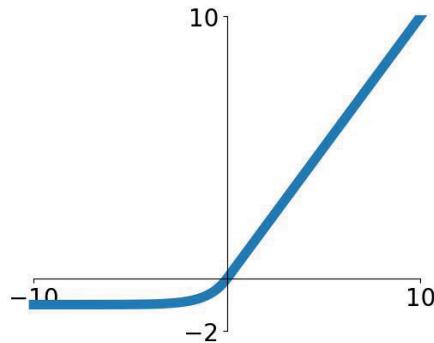
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into α
(parameter)

Exponential Linear Units (ELU)



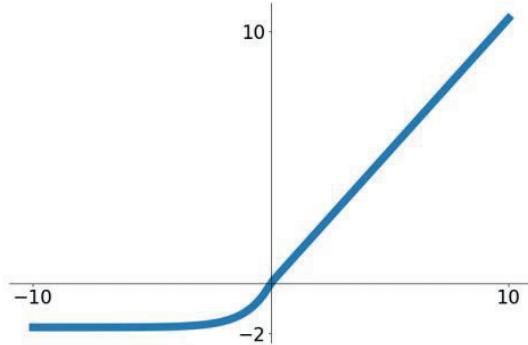
- All benefits of ReLU
- Closer to zero mean outputs
- Negative saturation regime compared with Leaky ReLU adds some robustness to noise

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$

(Alpha default = 1)

- Computation requires $\exp()$

Scaled Exponential Linear Units (SELU)



$$f(x) = \begin{cases} \lambda x & \text{if } x > 0 \\ \lambda\alpha(e^x - 1) & \text{otherwise} \end{cases}$$

$$\alpha = 1.6733, \lambda = 1.0507$$

- Scaled version of ELU that works better for deep networks
- “Self-normalizing” property;
- Can train deep SELU networks without BatchNorm
 - (will discuss more later)

Maxout “Neuron”

[Goodfellow et al., 2013]

- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

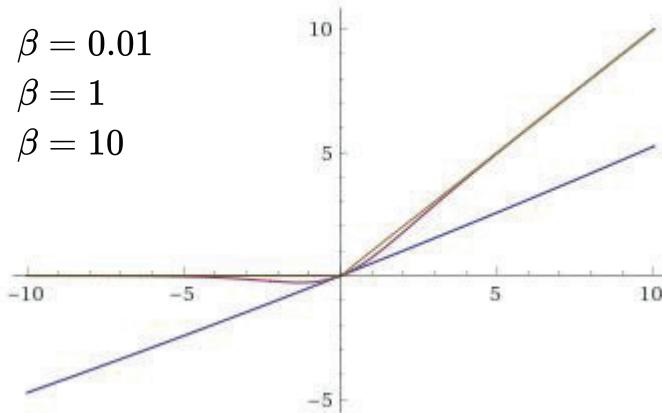
Problem: doubles the number of parameters/neuron :(

Activation Functions

[Ramachandran et al. 2018]

Swish

$$\begin{aligned}\beta &= 0.01 \\ \beta &= 1 \\ \beta &= 10\end{aligned}$$



$$f(x) = x\sigma(\beta x)$$

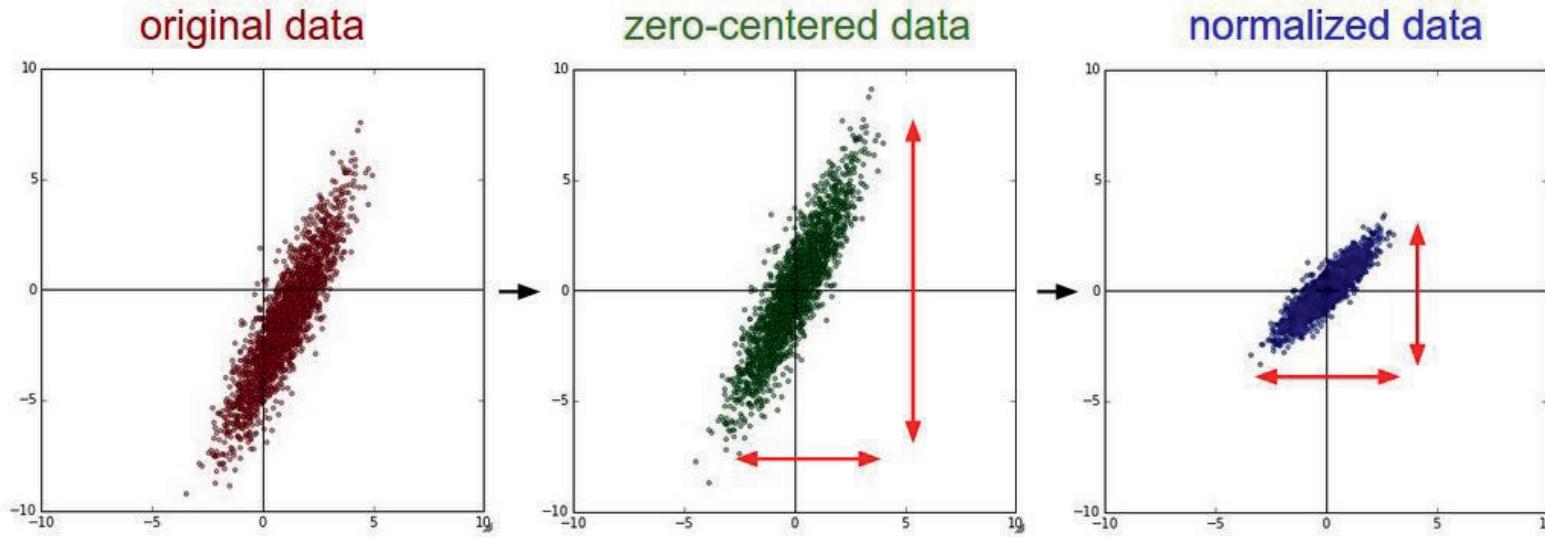
- They trained a neural network to generate and test out different non-linearities.
- Swish outperformed all other options for CIFAR-10 accuracy

TLDR: In practice:

- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / Maxout / ELU / SELU
 - To squeeze out some marginal gains
- Try PReLU with smaller learning rate
- Don't use sigmoid or tanh

Data Preprocessing

Data Preprocessing



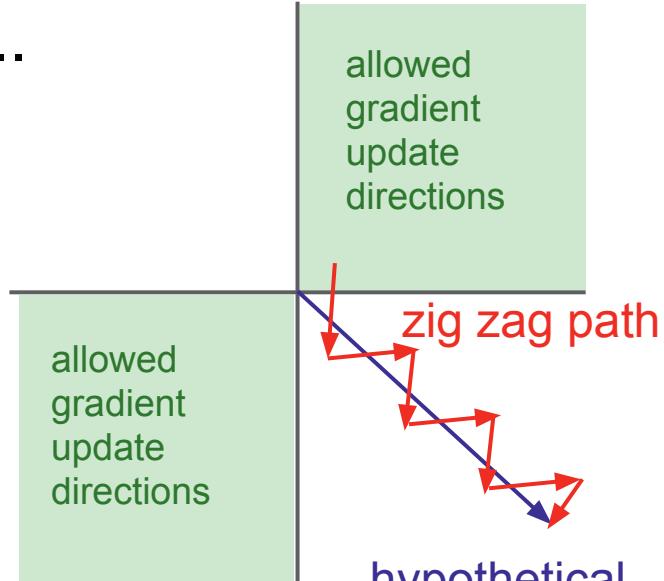
```
X -= np.mean(X, axis = 0)
```

```
X /= np.std(X, axis = 0)
```

(Assume X [NxD] is data matrix,
each example in a row)

Remember: Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$

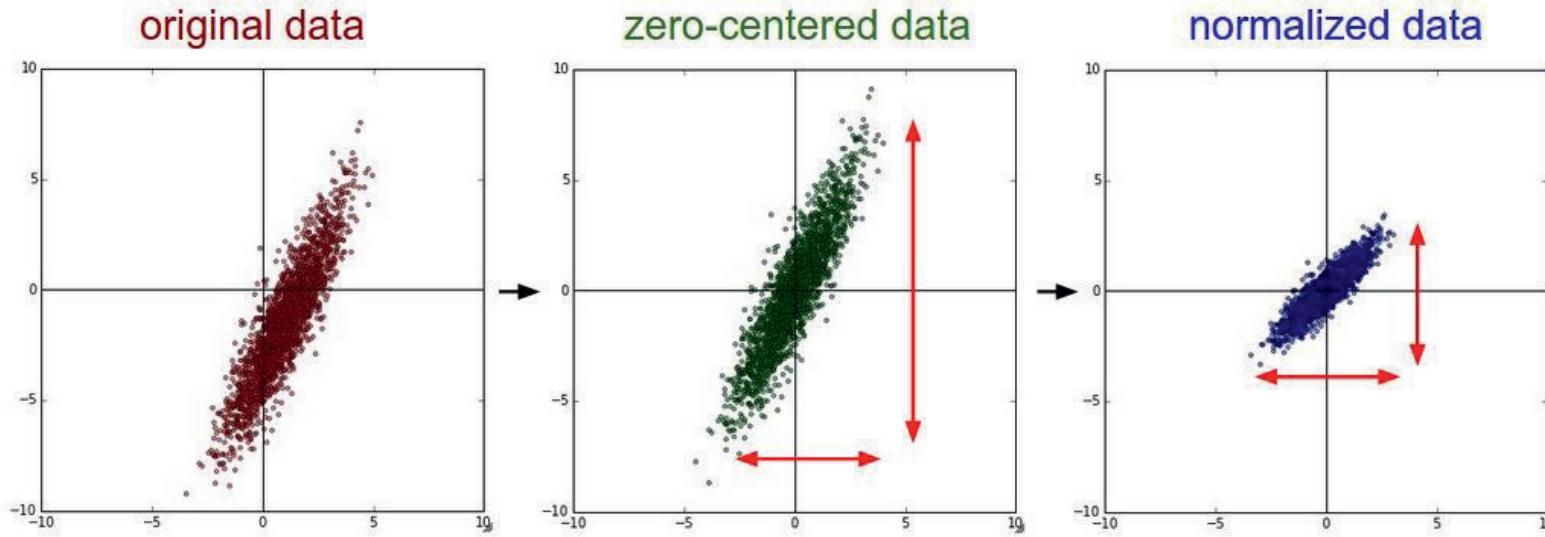


What can we say about the gradients on w ?

Always all positive or all negative :(

(this is also why you want zero-mean data!)

Data Preprocessing



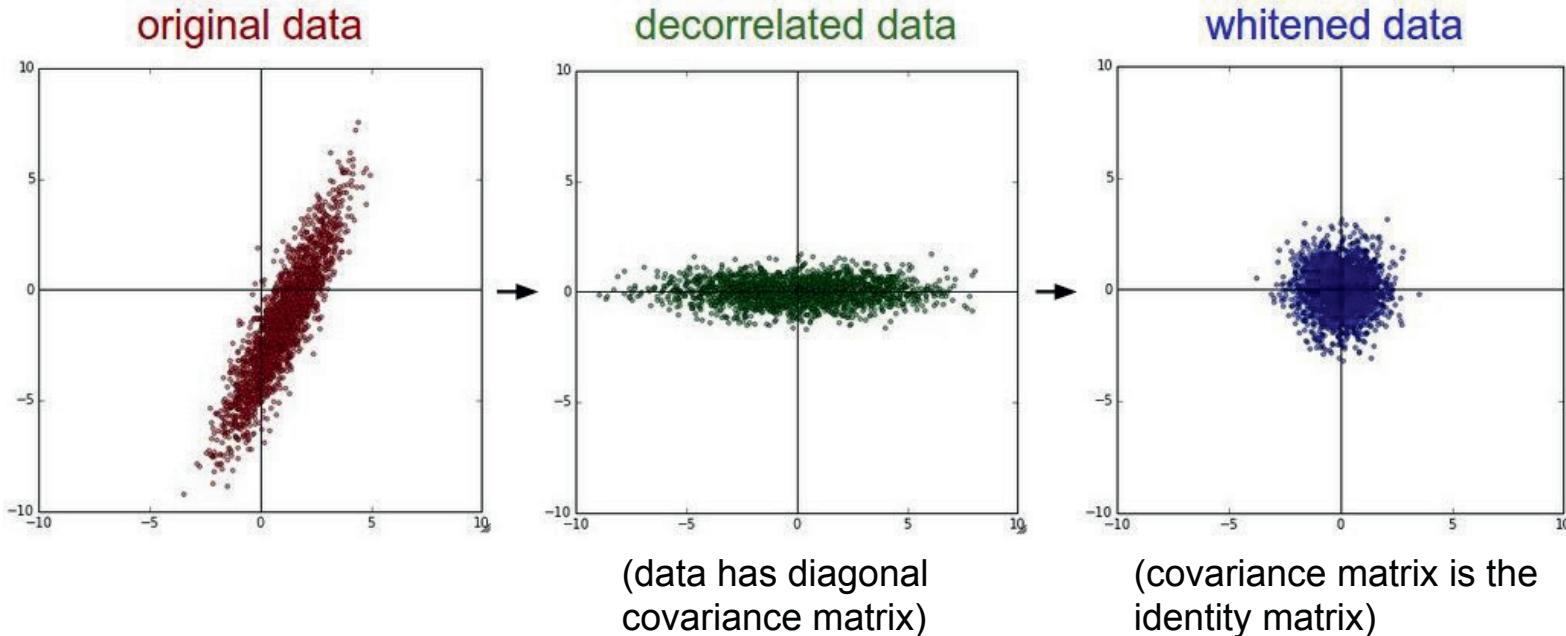
```
X -= np.mean(X, axis = 0)
```

```
X /= np.std(X, axis = 0)
```

(Assume X [$N \times D$] is data matrix, each example in a row)

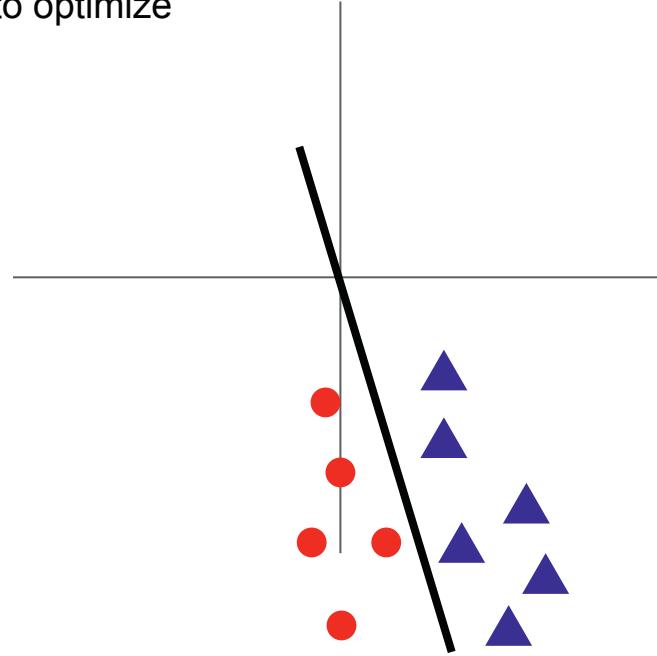
Data Preprocessing

In practice, you may also see **PCA** and **Whitening** of the data

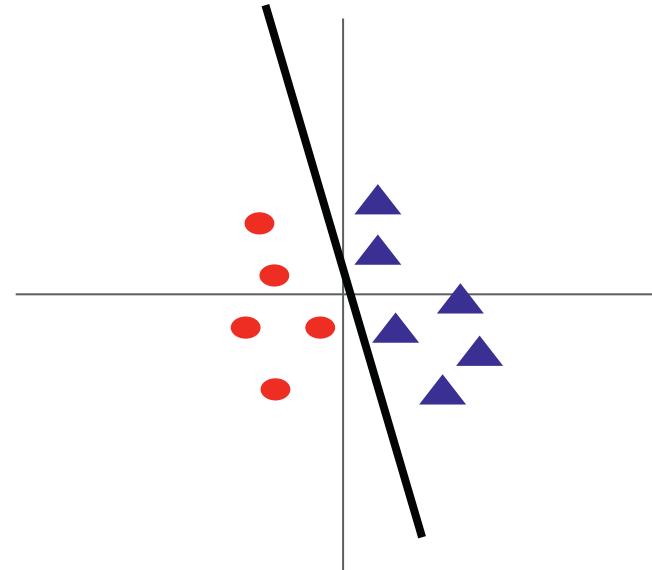


Data Preprocessing

Before normalization: classification loss very sensitive to changes in weight matrix; hard to optimize



After normalization: less sensitive to small changes in weights; easier to optimize



TLDR: In practice for Images: center only

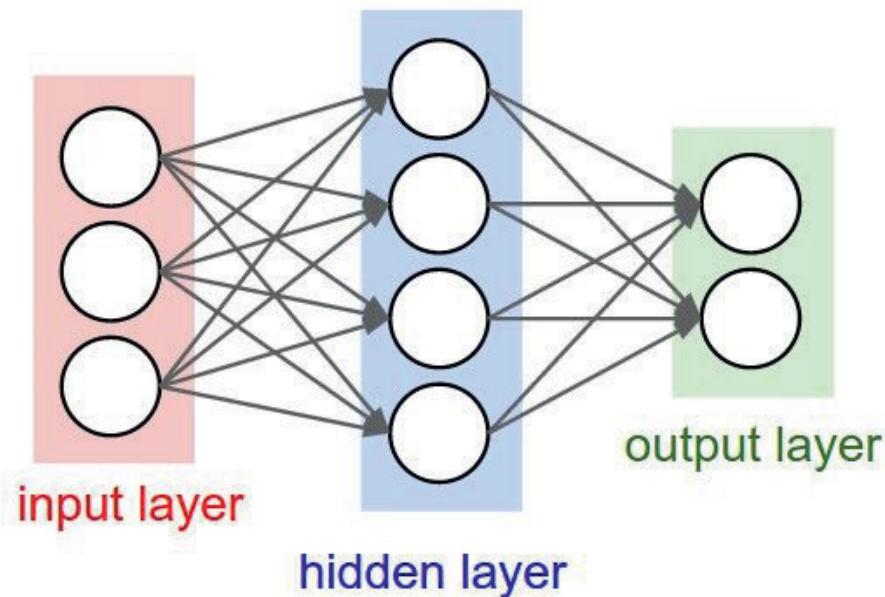
e.g. consider CIFAR-10 example with [32,32,3] images

- Subtract the mean image (e.g. AlexNet)
(mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet)
(mean along each channel = 3 numbers)
- Subtract per-channel mean and
Divide by per-channel std (e.g. ResNet)
(mean along each channel = 3 numbers)

Not common
to do PCA or
whitening

Weight Initialization

- Q: what happens when $W=\text{constant init}$ is used?



- First idea: **Small random numbers**
(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

- First idea: **Small random numbers**
(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

Works ~okay for small networks, but problems with deeper networks.

Weight Initialization: Activation statistics

```
dims = [4096] * 7      Forward pass for a 6-layer
hs = []                  net with hidden size 4096
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.01 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

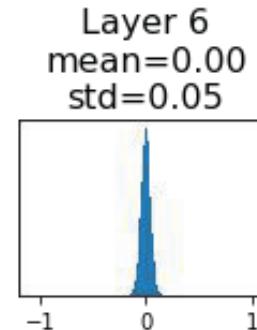
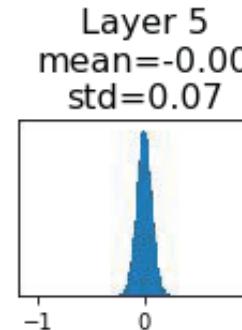
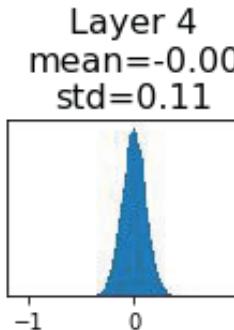
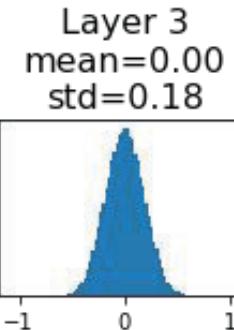
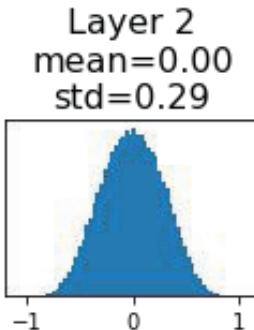
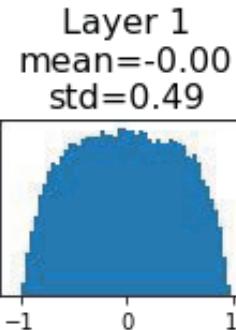
What will happen to the activations for the last layer?

Weight Initialization: Activation statistics

```
dims = [4096] * 7      Forward pass for a 6-layer  
hs = []                  net with hidden size 4096  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = 0.01 * np.random.randn(Din, Dout)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

All activations tend to zero for deeper network layers

Q: What do the gradients dL/dW look like?



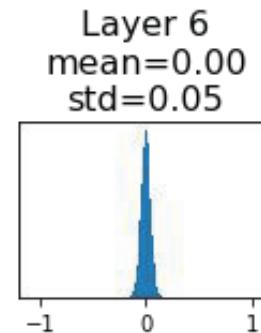
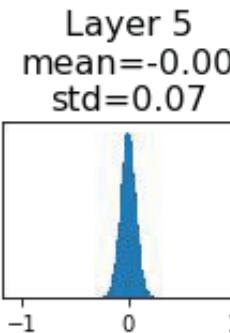
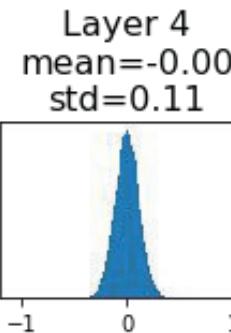
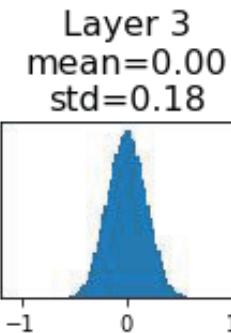
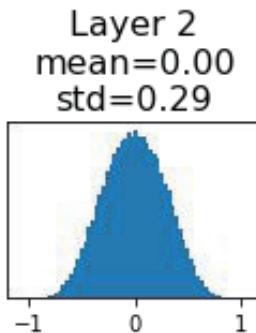
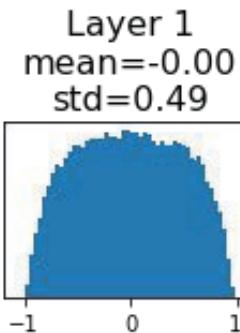
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```

All activations tend to zero for deeper network layers

Q: What do the gradients dL/dW look like?

A: All zero, no learning =(



Weight Initialization: Activation statistics

```
dims = [4096] * 7    Increase std of initial  
hs = []                weights from 0.01 to 0.05  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = 0.05 * np.random.randn(Din, Dout)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

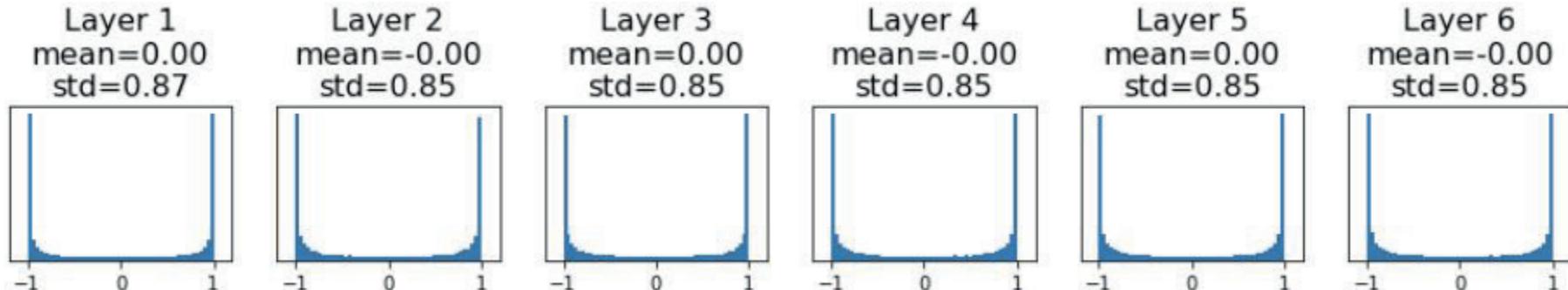
What will happen to the activations for the last layer?

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```

All activations saturate

Q: What do the gradients look like?



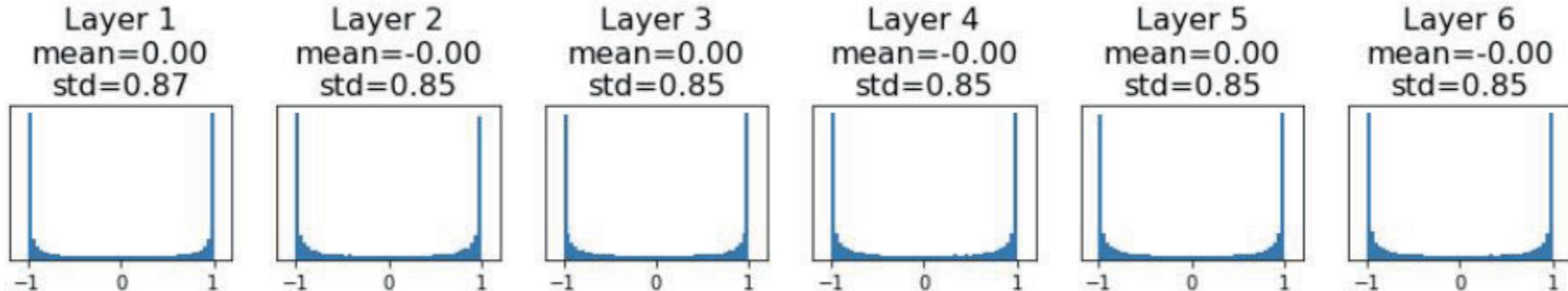
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```

All activations saturate

Q: What do the gradients look like?

A: Local gradients all zero,
no learning =(

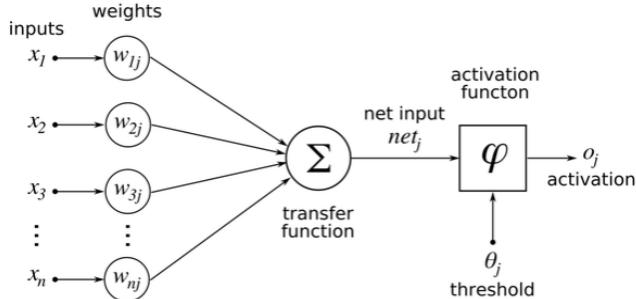


Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7           "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Обоснование инициализации Ксавьера



$$s = \sum_i^n w_i x_i$$

$$\text{Var}(s) = \text{Var}\left(\sum_i^n w_i x_i\right)$$

$$= \sum_i^n \text{Var}(w_i x_i)$$

$$= \sum_i^n [E(w_i)]^2 \text{Var}(x_i) + [E(x_i)]^2 \text{Var}(w_i) + \text{Var}(x_i) \text{Var}(w_i)$$

$$= \sum_i^n \text{Var}(x_i) \text{Var}(w_i)$$

$$= (n \text{Var}(w)) \text{Var}(x)$$

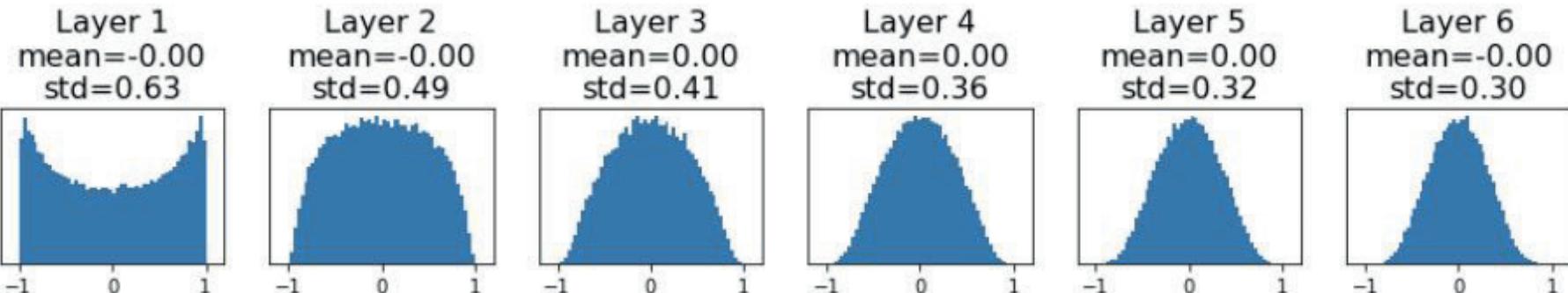
$$\text{Var}(aX) = a^2 \text{Var}(X)$$

`w = np.random.randn(n) / sqrt(n)`

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    x = np.tanh(x.dot(W))  
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```

“Just right”: Activations are nicely scaled for all layers!



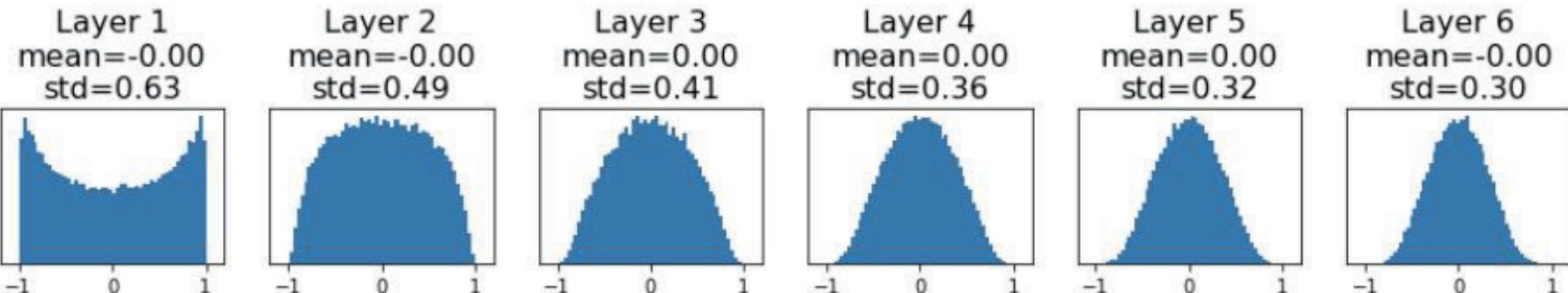
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For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$



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For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$

Derivation:

$$y = Wx$$

$$h = f(y)$$

$$\text{Var}(y_i) = D_{in} * \text{Var}(x_i w_i)$$

[Assume x, w are iid]

Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

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$$\begin{aligned}\text{Var}(y_i) &= \text{Din} * \text{Var}(x_i w_i) && [\text{Assume } x, w \text{ are iid}] \\ &= \text{Din} * (\mathbb{E}[x_i^2]\mathbb{E}[w_i^2] - \mathbb{E}[x_i]^2 \mathbb{E}[w_i]^2) && [\text{Assume } x, w \text{ independant}]\end{aligned}$$

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Derivation:

$$y = Wx$$
$$h = f(y)$$

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If $\text{Var}(w_i) = 1/\text{Din}$ then $\text{Var}(y_i) = \text{Var}(x_i)$

Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Weight Initialization: What about ReLU?

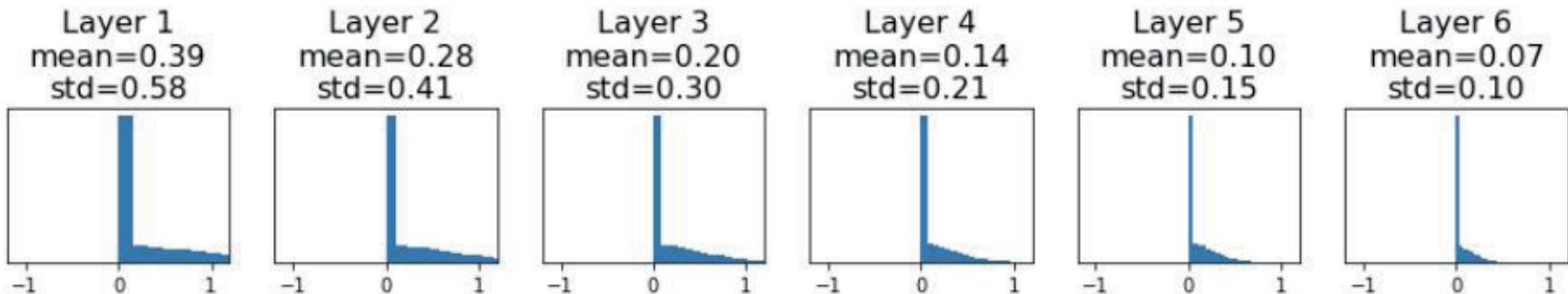
```
dims = [4096] * 7      Change from tanh to ReLU
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

Weight Initialization: What about ReLU?

```
dims = [4096] * 7      Change from tanh to ReLU
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

Xavier assumes zero centered activation function

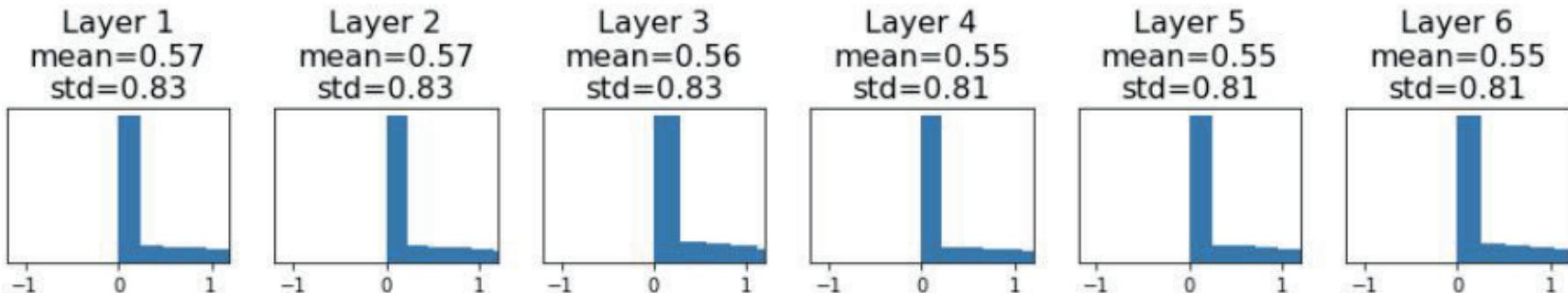
Activations collapse to zero again, no learning =(



Weight Initialization: Kaiming / MSRA Initialization

```
dims = [4096] * 7  ReLU correction: std = sqrt(2 / Din)
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) * np.sqrt(2/Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

“Just right”: Activations are nicely scaled for all layers!



He et al, “Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification”, ICCV 2015

Proper initialization is an active area of research...

Understanding the difficulty of training deep feedforward neural networks

by Glorot and Bengio, 2010

Exact solutions to the nonlinear dynamics of learning in deep linear neural networks by Saxe et al, 2013

Random walk initialization for training very deep feedforward networks by Sussillo and Abbott, 2014

Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification by He et al., 2015

Data-dependent Initializations of Convolutional Neural Networks by Krähenbühl et al., 2015

All you need is a good init, Mishkin and Matas, 2015

Fixup Initialization: Residual Learning Without Normalization, Zhang et al, 2019

The Lottery Ticket Hypothesis: Finding Sparse, Trainable Neural Networks, Frankle and Carbin, 2019

Batch Normalization

Batch Normalization

[Ioffe and Szegedy, 2015]

“you want zero-mean unit-variance activations? just make them so.”

consider a batch of activations at some layer. To make each dimension zero-mean unit-variance, apply:

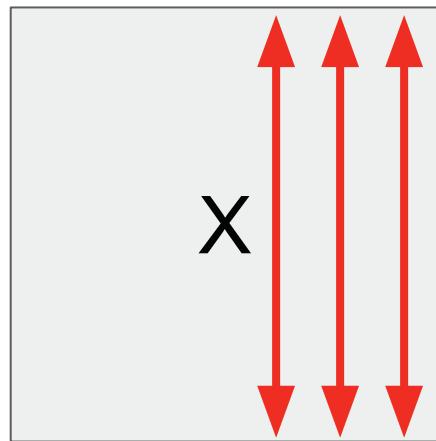
$$\hat{x}^{(k)} = \frac{x^{(k)} - \text{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

this is a vanilla
differentiable function...

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: $x : N \times D$



$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j}$$

Per-channel mean,
shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$

Per-channel var,
shape is D

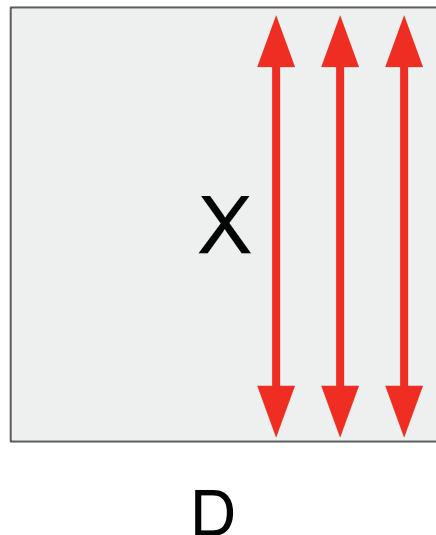
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x,
Shape is $N \times D$

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: $x : N \times D$



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$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$

Per-channel var,
shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x,
Shape is $N \times D$

Problem: What if zero-mean, unit variance is too hard of a constraint?

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: $x : N \times D$

Learnable scale and shift parameters:

$\gamma, \beta : D$

Learning $\gamma = \sigma$,
 $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j}$$

Per-channel mean,
shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$

Per-channel var,
shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x,
Shape is $N \times D$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output,
Shape is $N \times D$

Batch Normalization: Test-Time

Estimates depend on minibatch;
can't do this at test-time!

Input: $x : N \times D$

Learnable scale and shift parameters:

$\gamma, \beta : D$

Learning $\gamma = \sigma$,
 $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j}$$

Per-channel mean,
shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$

Per-channel var,
shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x,
Shape is $N \times D$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output,
Shape is $N \times D$

Batch Normalization: Test-Time

Input: $x : N \times D$

$$\mu_j = \text{(Running) average of values seen during training}$$

Per-channel mean,
shape is D

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

$$\sigma_j^2 = \text{(Running) average of values seen during training}$$

Per-channel var,
shape is D

During testing batchnorm becomes a linear operator!
Can be fused with the previous fully-connected or conv layer

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

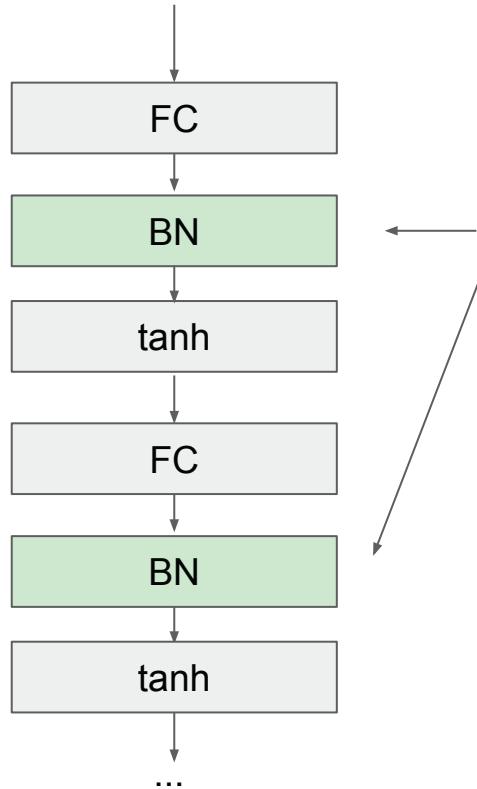
Normalized x,
Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output,
Shape is N x D

Batch Normalization

[Ioffe and Szegedy, 2015]

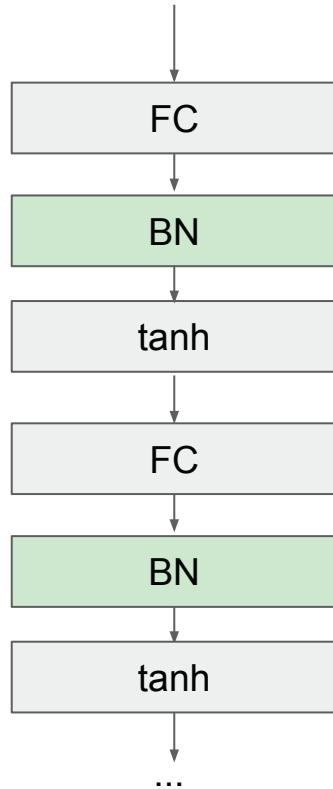


Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\hat{x}^{(k)} = \frac{x^{(k)} - \text{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]



- Makes deep networks **much** easier to train!
- Improves gradient flow
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!
- Behaves differently during training and testing: this is a very common source of bugs!

Batch Normalization for ConvNets

Batch Normalization for
fully-connected networks

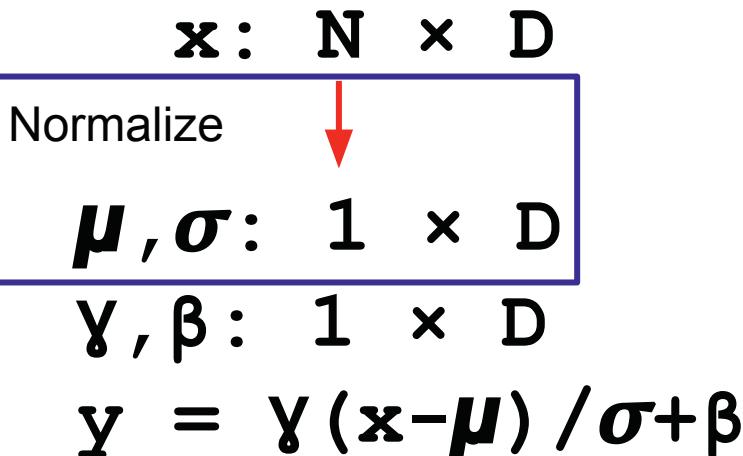
$$\begin{aligned}\mathbf{x} &: N \times D \\ \text{Normalize} &\quad \downarrow \\ \boldsymbol{\mu}, \sigma &: 1 \times D \\ \gamma, \beta &: 1 \times D \\ \mathbf{y} &= \gamma(\mathbf{x} - \boldsymbol{\mu}) / \sigma + \beta\end{aligned}$$

Batch Normalization for
convolutional networks
(Spatial Batchnorm, BatchNorm2D)

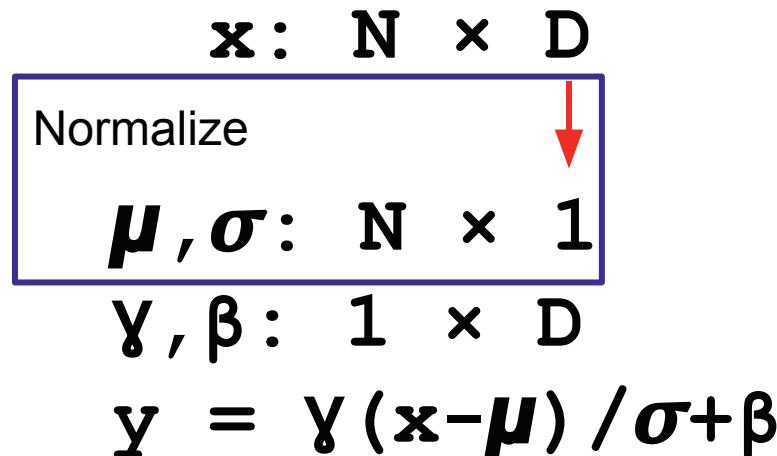
$$\begin{aligned}\mathbf{x} &: N \times C \times H \times W \\ \text{Normalize} &\quad \downarrow \quad \downarrow \quad \downarrow \\ \boldsymbol{\mu}, \sigma &: 1 \times C \times 1 \times 1 \\ \gamma, \beta &: 1 \times C \times 1 \times 1 \\ \mathbf{y} &= \gamma(\mathbf{x} - \boldsymbol{\mu}) / \sigma + \beta\end{aligned}$$

Layer Normalization

Batch Normalization for
fully-connected networks



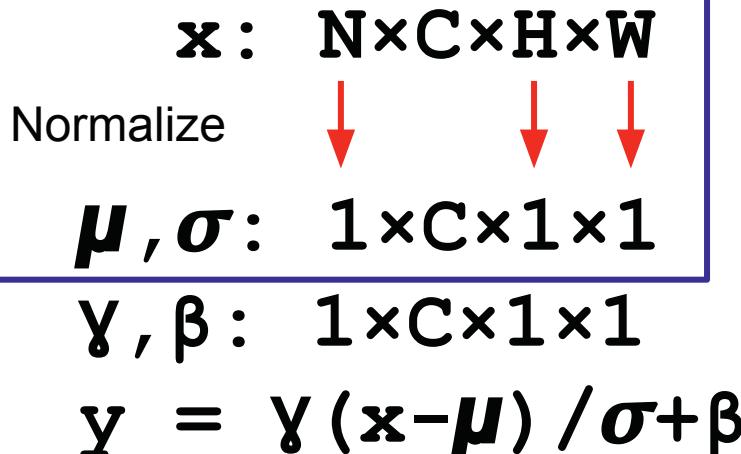
Layer Normalization for
fully-connected networks
Same behavior at train and test!
Can be used in recurrent networks



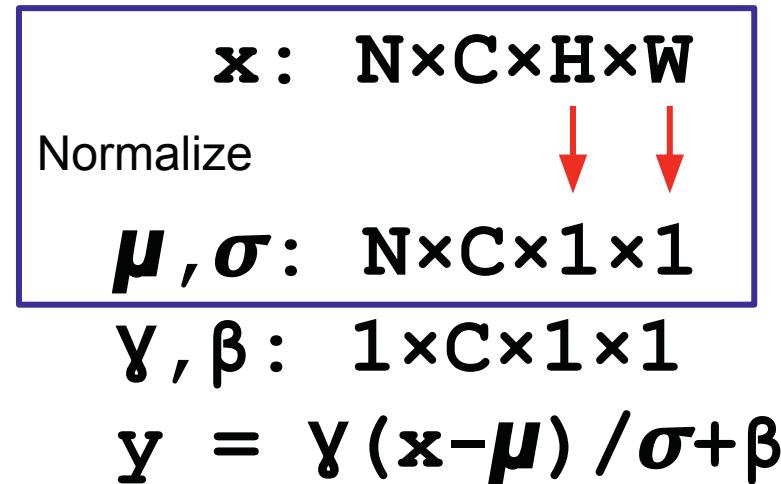
Ba, Kiros, and Hinton, “Layer Normalization”, arXiv 2016

Instance Normalization

Batch Normalization for convolutional networks

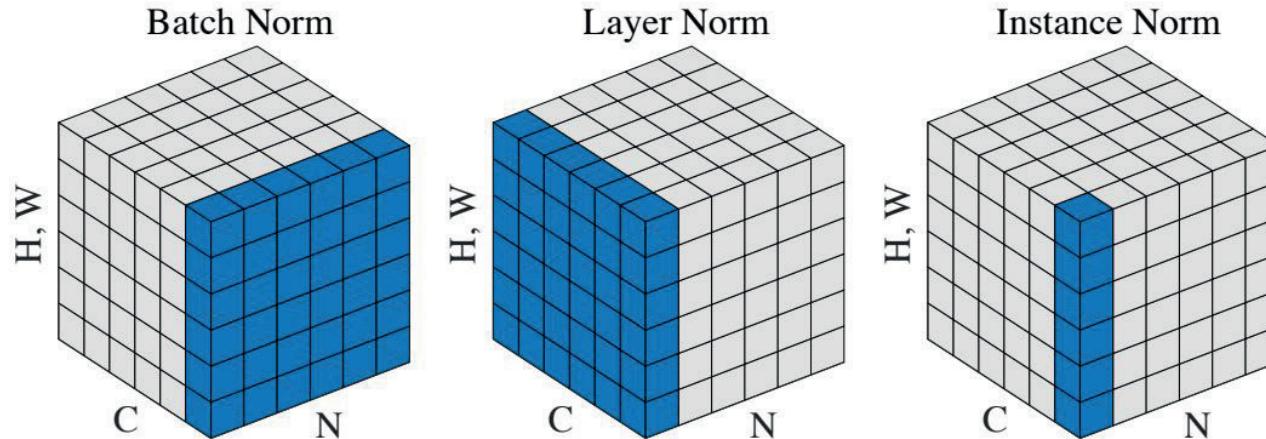


Instance Normalization for convolutional networks
Same behavior at train / test!



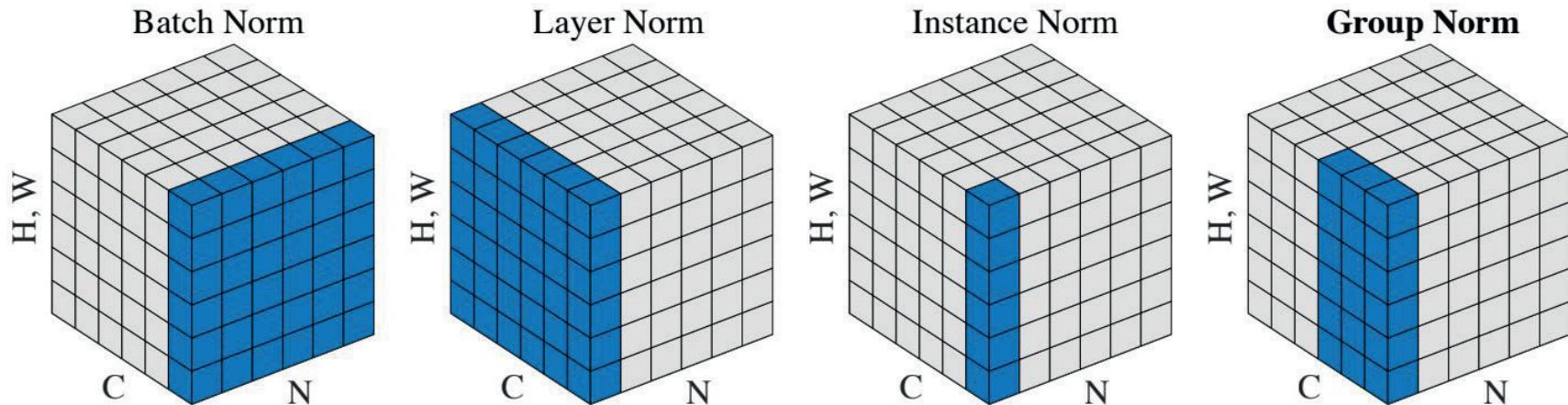
Ulyanov et al, Improved Texture Networks: Maximizing Quality and Diversity in Feed-forward Stylization and Texture Synthesis, CVPR 2017

Comparison of Normalization Layers



Wu and He, "Group Normalization", ECCV 2018

Group Normalization



Wu and He, "Group Normalization", ECCV 2018

Transfer learning

“You need a lot of data if you want to
train/use CNNs”

“You need a lot of ~~data~~ if you want to
train/use CNNs”

BUSTED

Transfer Learning with CNNs

Donahue et al, "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014
Razavian et al, "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014

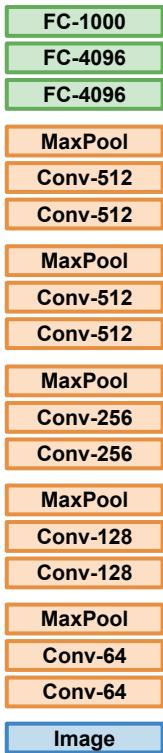
1. Train on Imagenet



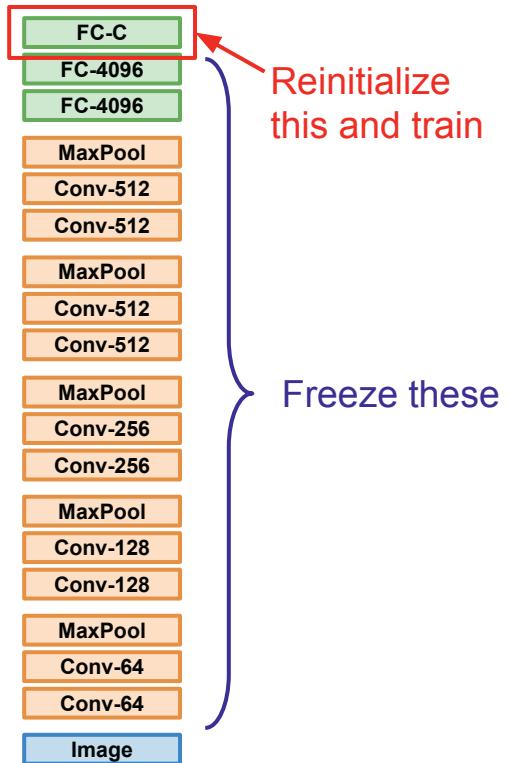
Transfer Learning with CNNs

Donahue et al, "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014
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1. Train on Imagenet

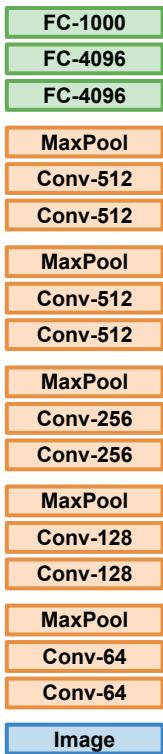


2. Small Dataset (C classes)

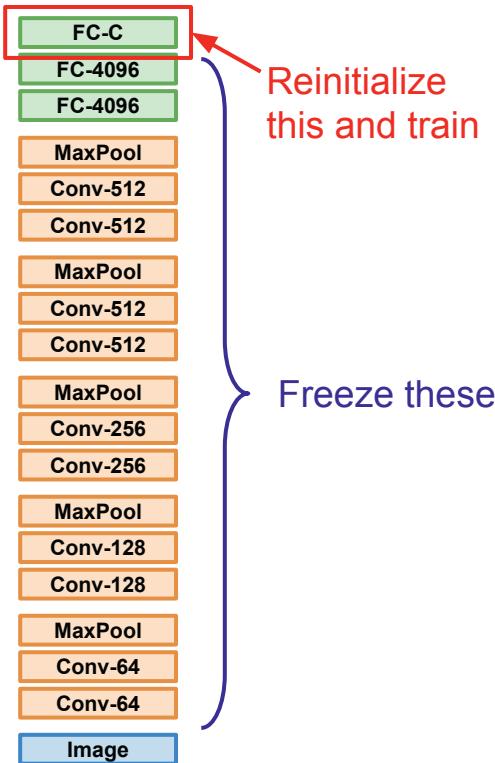


Transfer Learning with CNNs

1. Train on Imagenet

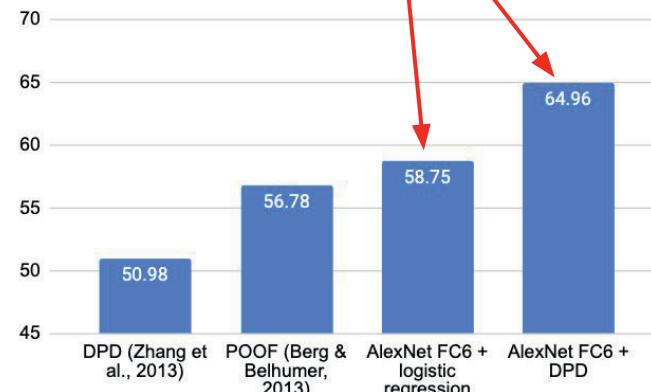


2. Small Dataset (C classes)



Donahue et al, "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014
Razavian et al, "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014

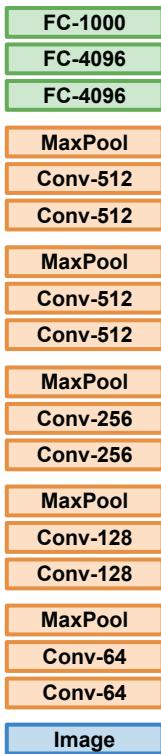
Finetuned from AlexNet



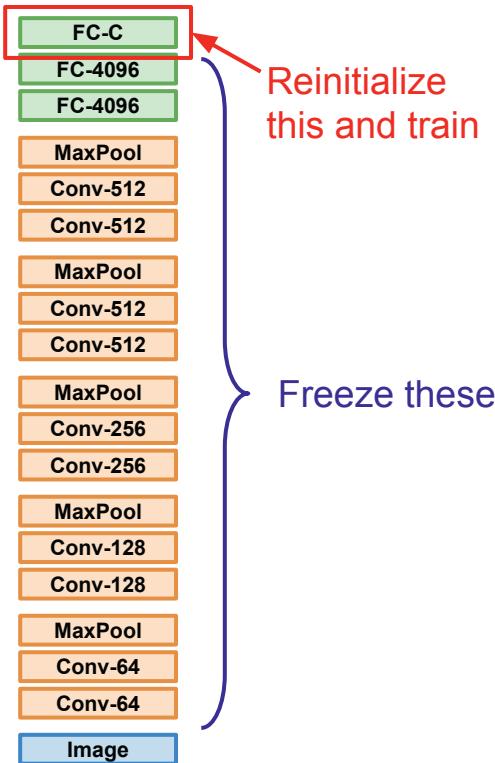
Donahue et al, "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014

Transfer Learning with CNNs

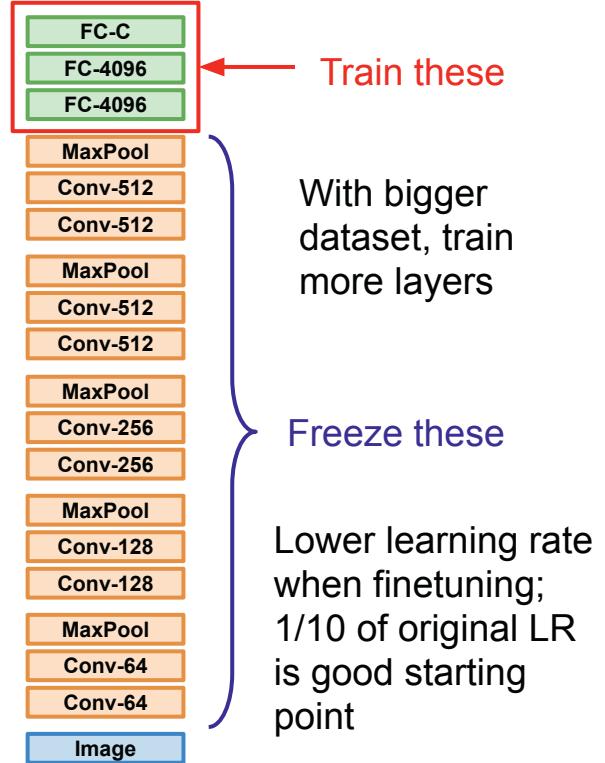
1. Train on Imagenet



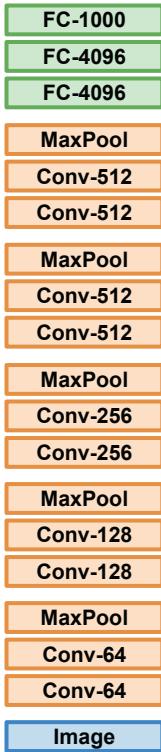
2. Small Dataset (C classes)



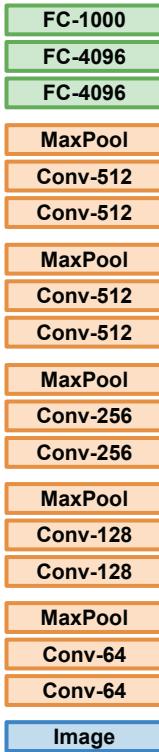
3. Bigger dataset



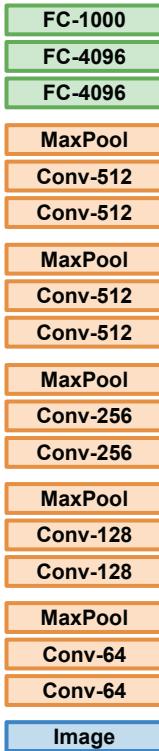
Donahue et al, "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014
Razavian et al, "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014



	very similar dataset	very different dataset
very little data	?	?
quite a lot of data	?	?



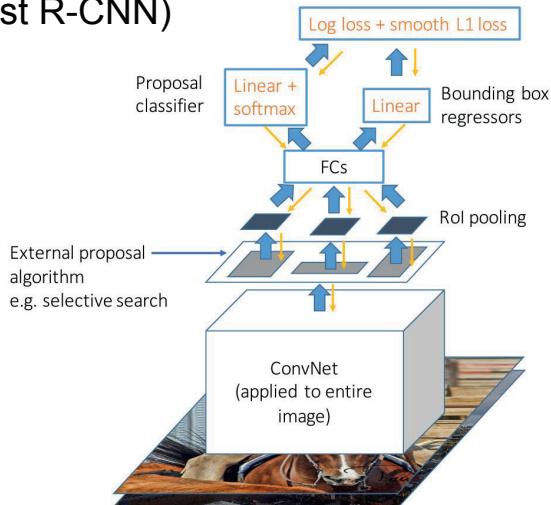
	very similar dataset	very different dataset
very little data	Use Linear Classifier on top layer	?
quite a lot of data	Finetune a few layers	?



	very similar dataset	very different dataset
very little data	Use Linear Classifier on top layer	You're in trouble... Try linear classifier from different stages
quite a lot of data	Finetune a few layers	Finetune a larger number of layers

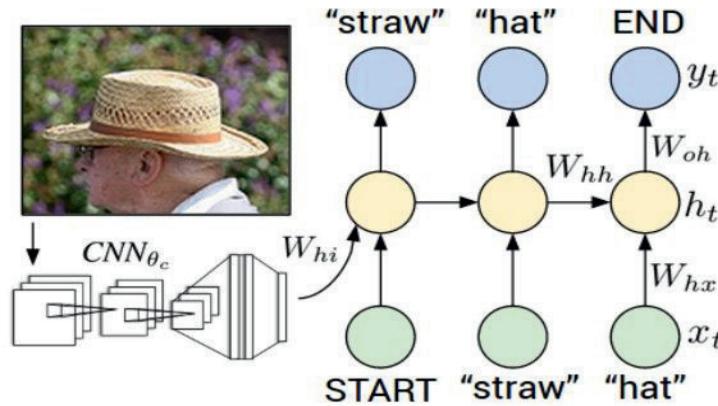
Transfer learning with CNNs is pervasive... (it's the norm, not an exception)

Object Detection (Fast R-CNN)



Girshick, "Fast R-CNN", ICCV 2015
Figure copyright Ross Girshick, 2015. Reproduced with permission.

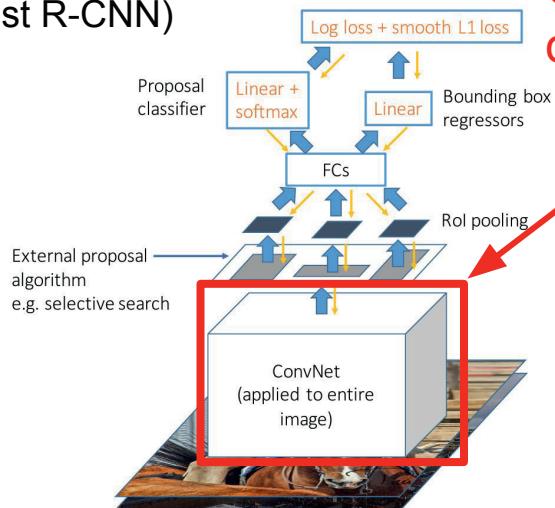
Image Captioning: CNN + RNN



Karpathy and Fei-Fei, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015
Figure copyright IEEE, 2015. Reproduced for educational purposes.

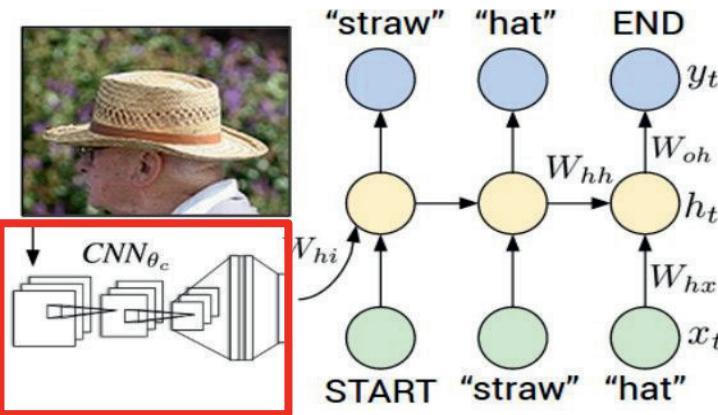
Transfer learning with CNNs is pervasive... (it's the norm, not an exception)

Object Detection
(Fast R-CNN)



CNN pretrained
on ImageNet

Image Captioning: CNN + RNN

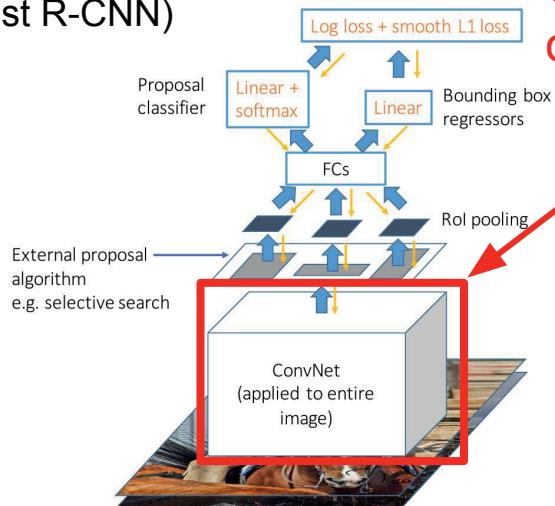


Girshick, "Fast R-CNN", ICCV 2015
Figure copyright Ross Girshick, 2015. Reproduced with permission.

Karpathy and Fei-Fei, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015
Figure copyright IEEE, 2015. Reproduced for educational purposes.

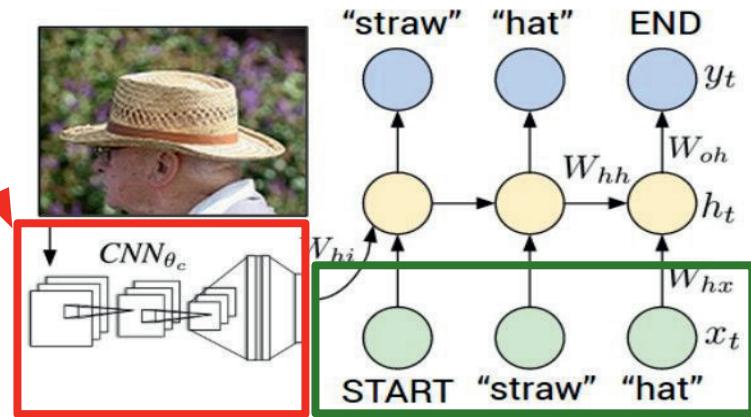
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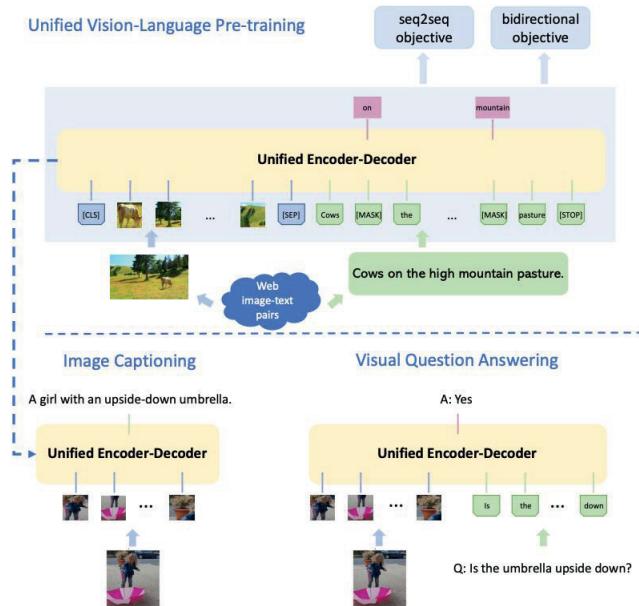


Word vectors pretrained
with word2vec

Girshick, "Fast R-CNN", ICCV 2015
Figure copyright Ross Girshick, 2015. Reproduced with permission.

Karpathy and Fei-Fei, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015
Figure copyright IEEE, 2015. Reproduced for educational purposes.

Transfer learning with CNNs is pervasive... (it's the norm, not an exception)



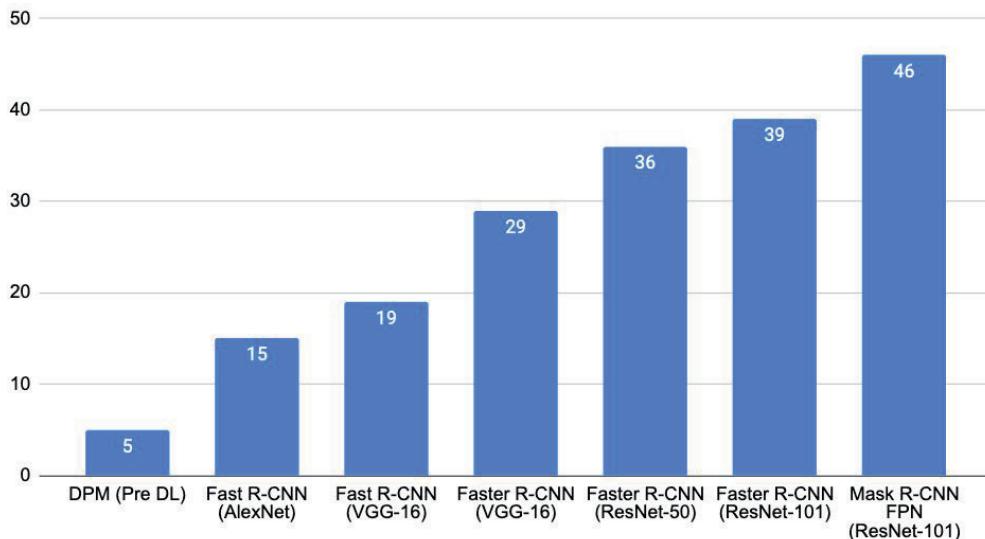
Zhou et al., "Unified Vision-Language Pre-Training for Image Captioning and VQA" CVPR 2020
Figure copyright Luwei Zhou, 2020. Reproduced with permission.

1. Train CNN on **ImageNet**
2. Fine-Tune (1) for object detection on **Visual Genome**
3. Train **BERT** language model on lots of text
4. Combine(2) and (3), train for joint image / language modeling
5. Fine-tune (4) for image captioning, visual question answering, etc.

Krishna et al., "Visual genome: Connecting language and vision using crowdsourced dense image annotations" IJCV 2017
Devlin et al. "BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding" ArXiv 2018

Transfer learning with CNNs - Architecture matters

Object detection on MSCOCO

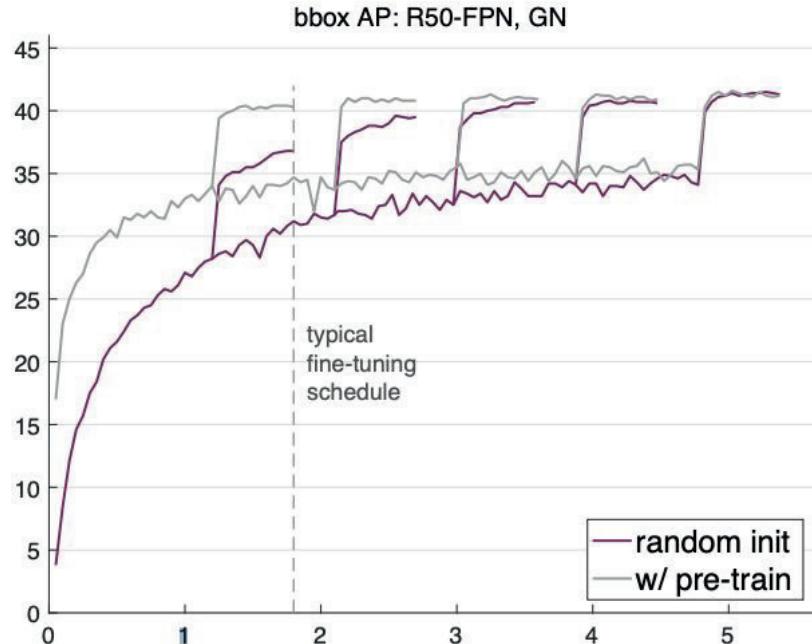


We will discuss different architectures in detail in two lectures

Girshick, "The Generalized R-CNN Framework for Object Detection", ICCV 2017 Tutorial on Instance-Level Visual Recognition

Transfer learning with CNNs is pervasive...

But recent results show it might not always be necessary!



Training from scratch can work just as well as training from a pretrained ImageNet model for object detection

But it takes 2-3x as long to train.

They also find that collecting more data is better than finetuning on a related task

He et al, "Rethinking ImageNet Pre-training", ICCV 2019
Figure copyright Kaiming He, 2019. Reproduced with permission.

Takeaway for your projects and beyond:

Transfer learning be like



Source: AI & Deep Learning Memes For Back-propagated Poets

Takeaway for your projects and beyond:

Have some dataset of interest but it has < ~1M images?

1. Find a very large dataset that has similar data, train a big ConvNet there
2. Transfer learn to your dataset

Deep learning frameworks provide a “Model Zoo” of pretrained models so you don’t need to train your own

TensorFlow: <https://github.com/tensorflow/models>

PyTorch: <https://github.com/pytorch/vision>

Summary

TLDRs

We looked in detail at:

- Activation Functions (use ReLU)
- Data Preprocessing (images: subtract mean)
- Weight Initialization (use Xavier/He init)
- Batch Normalization (use this!)
- Transfer learning (use this if you can!)

Next time:

Training Neural Networks, Part 2

- Parameter update schemes
- Learning rate schedules
- Gradient checking
- Regularization (Dropout etc.)
- Babysitting learning
- Evaluation (Ensembles etc.)
- Hyperparameter Optimization
- Transfer learning / fine-tuning