Problem 1:

程占朋

12132387

① $f(2) = 0$

$f(-2) = 0$ $\quad f(2) = f(-2)$

according to Roll's law $\exists\, c \in [-2, 2]$, $f'(c) = 0$

$f'(x) = 4x^3 - 8x = 4x(x^2 - 2)$ for $f'(x) = 0$

$$\Rightarrow \begin{cases} x = 0 \\ x = \pm\sqrt{2} \end{cases}$$

so c can take value of $0, \sqrt{2}, -\sqrt{2}$.

② $f(0) = 0 = f(2\pi)$

according to Roll's law $\exists\, c \in [0, 2\pi]$ let $f'(c) = 0$

$f'(x) = \cos x + 2\cos 2x = \cos x + 2[2\cos^2 x - 1]$

for $f'(x) = 0$, we get: $\cos x = \frac{-1 \pm \sqrt{33}}{8}$

$$x = \begin{cases} 53.625^\circ = 0.93359 \\ 147.4637^\circ = 2.5738 \\ 306.375^\circ = 5.3472 \\ 212.5343^\circ = 3.709 \end{cases}$$

Problem 2:

Because $P_N(x)$ is the Lagrange polynomial of N-degree $f(x)$

so we have: $P_N(x_i) = f(x_i)$ $(i = 0, 1, 2, \dots N)$

And $E_N(x) = f(x) - P_N(x)$ so $E_N(x_i) = 0$ $(i = 0, 1, \dots N)$

Now let's consider the special function $g(t)$,

$$g(x_i) = f(x_i) - P_N(t) - E_N(x)\frac{(x_i - x_0)\dots(x_i - x_N)}{(x - x_0)\dots(x - x_N)}$$

$$= E_N(x_i) - E_N(x)\cdot 0$$

$$= 0$$

and $g(x) = f(x) - P_N(x) - E_N(x) = E_N(x) - P_N(x) = 0$

so, $g(t)$ has $N+2$ root: $[x_0, x_1, \cdots x_N, x]$

According to the generalized Rolle's theorem, we have. $\exists . \xi$ Let

$$g^{(n+1)}(\xi) = 0$$

and $g^{(n+1)}(\xi) = f^{(n+1)}(\xi) - P_N^{(n+1)}(t) - \frac{E_N(x)\,(n+1)!}{(x-x_0)\cdots(x-x_N)}$

as we know. P_N is N-degree polynomial, so $P_N^{(n+1)}(t) = 0$

so $0 = f^{(n+1)}(\xi) - \frac{E_N(x)\,(N+1)!}{(x-x_0)\cdots(x-x_N)}$

so $E_N(x) = \frac{f^{(n+1)}(\xi)}{(N+1)!}(x-x_0)\cdots(x-x_N)$

Problem 3

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{1}{2a}\frac{b^2-4ac-b^2}{\sqrt{b^2-4ac}+b} = -\frac{2c}{\sqrt{b^2-4ac}+b}$$

$$x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a} = -\frac{1}{2a}\frac{b^2-4ac-b^2}{\sqrt{b^2-4ac}-b} = \frac{2c}{\sqrt{b^2-4ac}-b} = -\frac{2c}{b-\sqrt{b^2-4ac}}$$

Problem 4

shown in hw1-problem4.ipynb.

Problem 5

As problem shown: $f(x) = x^3 - A$, we get,

$f'(x) = 3x^2$.

if we use newton-Raphson method: we have

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^3-A}{3x^2} = \frac{2x^3-A}{3x^2} = \frac{2x - A/x^2}{3}$$

so we have ~~iterater~~ iterative formula

$$p_k = \frac{2p_{k-1} - \frac{A}{p_{k-1}^2}}{3} \quad \text{for } k = 1, 2, \ldots$$

Problem 6:

shown in hw1- problem 6. ipynb.

Problem 7:

① According to ~~above~~:

$$0 = f(P_k) + f'(P_k)(P-P_k) + \frac{1}{2} f''(c_k)(P-P_k)^2$$

if we assume $f'(x) \neq 0 \Rightarrow f'(P_k) \neq 0$

$$\Rightarrow 0 = \frac{f(P_k)}{f'(P_k)} + (P-P_k) + \frac{1}{2}\frac{f''(c_k)}{f'(P_k)}(P-P_k)^2$$

$$\Rightarrow (P-P_k) + \frac{f(P_k)}{f'(P_k)} = -\frac{1}{2}\frac{f''(c_k)}{f'(P_k)}(P-P_k)^2.$$

② for Newton-Raphson method:

$$P_{k+1} = P_k - \frac{f(P_k)}{f'(P_k)}$$

~~So~~ $\cancel{E_{k+1} = (P_{k+1} - P)}$

$$\cancel{= \left(P_k - P - \frac{f(P_k)}{f'(P_k)}\right)}$$

$$\equiv$$

So $E_{k+1} = P - P_{k+1}$

$$= \left(P - P + \frac{f(P_k)}{f'(P_k)}\right)$$

$$= -\frac{\cancel{2}f''(c_k)}{2f'(P_k)}(P-P_k)^2.$$

$$= -\frac{\cancel{2}f''(c_k)}{2f'(P_k)}E_k^2$$

And we assume $f'(P_k) \approx f'(P)$ $f''(c_k) \approx f''(P)$

so $E_{k+1} \approx -\frac{\cancel{2}f''(P)}{2f'(P)}E_k^2.$

Problem 8 shown in hw1 – problem 8. ipynb.