CLEAR PROCESSING OF MOTION BLURRED IMAGES BASED ON INVERSE FILTERING AND WIENER FILTERING TECHNIQUES

THE FINAL YEAR PROJECT

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Content

1. **Research on project background**

1.1 Research Purpose and Significance

In the information society, images, as a medium for information dissemination, have been widely used in daily life due to their ability to convey information more vividly and easily understood and memorized. Therefore, ensuring high-quality images is the key to effective information dissemination and a prerequisite for the continuous development of image processing technology. The general methods for obtaining images include photography, video capture, scanning, etc. However, during the process of obtaining images, due to camera lens shake, shutter speed error, optical system defects, weather conditions, large focal length error, atmospheric turbulence, etc., the obtained images may appear blurry and degraded, resulting in the loss of some information in the image and increasing the difficulty of obtaining information. Therefore, it is of great practical significance to perform clear processing on degraded images in order to make the information of the images as complete as possible.

In order to obtain high-quality images, people add stabilizing devices to the system or device that obtains the images, in order to increase the clarity of the images. But this not only increases the cost of acquiring image systems and equipment, but also cannot completely remove or avoid image blurring. In fact, the issue of motion blur is widespread and closely related to people's lives. For example, in daily life, when people use their phones or cameras to obtain images, the phone or camera will shake when pressing the shutter, or when pressing the shutter, the object being photographed will have relative displacement with it, resulting in blurring of the image; In natural landscapes, raindrops fall, and due to their high speed of movement, the originally water droplet shaped raindrops appear as strips in human vision; In the field of aerospace, motion blur can also occur in images taken by aircraft or rockets during high-speed flight.

It is precisely the universality of motion blur problems that has led to the focus of research on solutions to motion blur. Since 1960, people have been conducting research on motion blurred image restoration technology and applying it to the exploration of space (Jin, 2023). So far, there has been a large amount of theoretical proof that blurred images have specific performance in specific transformation intervals, and can be mathematically simulated to create a degradation model to restore blurred images to clarity. Among them, there is a key function called PSF, whose full name is Point Spread Function, which is a parameter that can describe image blur. Simply put, when an image is presented, it is described as a series of points. If the imaging system is not perfect, the points in the image will spread out, causing blurring of the image. When the PSF is determined, people can establish mathematical models based on image blur information, apply deconvolution to solve for unique variables, and restore blurred images. This method is also known as non blind deconvolution; When the PSF is unknown, it is blind deblurring and difficult.

1.2 Research background and development status of the subject

1.2.1 Research background and development status of image deblurring technology

At present, image deblurring technology has attracted much attention. Its core idea is to use the fuzzy information known in natural images, establish mathematical models, and use physical methods to restore damaged images. In this field, especially in the study of motion blur, it has become a focus of research due to its complex causes and the difficulty of obtaining prior knowledge. Domestic and foreign research has made some progress in motion image deblurring technology. According to different classification standards, existing blurred images can be divided into the following three types:

(1) Image blur range

According to the different blurred areas in the image, it can be divided into local blur and global blur. Local blur refers to the blurring of certain regions or individual objects in an image, with the blur kernel changing with the position of the blur (Wang, 2022). Global blur refers to the blur caused by global relative motion when the object being photographed maintains the same motion trend as the camera, and the blur kernel of this image is unique.

(2) Information source

According to the different information required for deblurring, it can be divided into single frame method and multi frame method. Single frame method is a common image restoration method that mainly relies on a single image and does not require additional information (Li, 2015). Multi frame image deblurring not only relies on the information of a single image, but also requires other information, such as continuous multi frame image information or a noisy and non blurry image, mainly using the correlation between information for deblurring (Huang et al, 2015).

(3) Image blur kernel

According to whether PSF is known, it can be divided into non blind deblurring and blind deblurring. Non blind deblurring is generally determined by PSF, and a mathematical model can be established based on blurred image information. The use of deconvolution can solve for unique variables and restore clear images. Blind deblurring is generally applicable to PSF unknowns, where constraints can only be determined based on a small amount of image information, and corresponding mathematical models can be established. After solving for PSF, non blind methods can be used to restore blurred images. Compared to other methods, blind deblurring is a serious pathological problem. As it does not rely on the transfer function characteristics in degraded systems, conducting fuzzy kernel estimation has more research and application value in practical situations (Lu, 2011).

1.2.2 Research background and development status of non blind deblurring methods

Non blind deblurring, also known as non blind deconvolution, is generally divided into two categories: spatially invariant PSF methods and spatially variable PSF methods. There are four classic space invariant PSF methods.

Inverse filtering, also known as reverse filtering, mainly involves converting the image to be processed from the spatial domain to the frequency domain, performing reverse filtering, and then transferring to the spatial domain to obtain the restored image information (Li, 2020). As early as the 1960s and 1970s, inverse filtering was widely used in image restoration. American scientist Nathan used inverse filtering to restore images captured by space equipment at that time. The inverse filtering method is simple in design, but it is only suitable for restoring blurry images without noise interference. It cannot achieve satisfactory results in restoring blurry images with noise interference.

Wiener filtering is the optimal linear filtering method proposed by N. Wiener in 1942 based on the minimum mean square error criterion. Wiener filtering is a commonly used filtering technique in signal and image processing, named after American mathematician Norbert Wiener. The main goal of Wiener filtering is to recover the original signal from signals containing noise for further analysis or processing. This method is based on statistical principles and achieves signal recovery by minimizing the mean square error between the original signal and the estimated signal. Wiener filtering has extensive applications in many fields, such as communication, image processing, speech processing, and so on. However, this algorithm has some drawbacks. The improved results do not fully conform to the characteristics of human vision, but instead treat the signal and noise as stationary processes, giving the same weight to all errors, which cannot achieve good image processing. The obtained image intuitively does not meet the needs of the human eye. In fact, people's sensitivity to each region is different, such as the sensitivity of the human eye in darkness being higher than in light (Mao, 2018).

R-L restoration algorithm. In the 1970s, Richardson and Lucy proposed an image restoration method based on maximum likelihood (ML) Bayesian theory, namely the R-L restoration algorithm (Lucy, 1974). The core idea of the R-L restoration algorithm is to iteratively convolution the observed image with a pre-defined blur function, and then compare it with the actual observed image, continuously updating the estimated original image until it converges to the optimal restoration result. This algorithm can effectively improve the quality of images under certain conditions, especially when the image is heavily affected by blur or noise interference, and has good results.

Kalman filtering. Kalman filtering is an optimal estimation method that can process measurement data containing noise and provide optimal estimation of system state. The proposal and development of this algorithm have wide applications in control theory and signal processing fields. Rudolf E. Kálmán is a Hungarian American mathematician whose research mainly focuses on control theory and system identification. In 1960, Kálmán published a paper detailing this filtering algorithm (Kalman, 1960). This paper proposes a recursive estimation method for system state, which continuously updates the state estimation by combining the system model and actual observation data, gradually approaching the true state.

The PSF non blind deblurring method with spatial variation breaks away from the traditional PSF spatial invariance criterion, believing that the imaging system is influenced by multiple factors, and therefore the PSF of the system will change with spatial variation. Based on this spatial variability criterion, first estimate the PSF, and then use a non blind deblurring algorithm for image restoration. Although this method improves the quality of image restoration, it also increases the difficulty and computational complexity of mathematical modeling.

As early as 1965, Lohmann and Paris first discussed and analyzed the issue of spatial differences in images (Lohmann & Paris, 1965). In 1978, Trussell and Hunt (1978) proposed dividing images into PSF invariant sub blocks, transforming the problem of spatial variation into a problem of spatial invariance, and then using the Wiener method for image restoration. Subsequently, Hertz et al. (1993) used advanced processing platforms to restore images of spatially varying PSFs. In 1996, Boden and Redding et al. (1996) analyzed PSF block interpolation and used the R-L algorithm for image restoration. In 1997, Costello and Mikhael (1997) used the isohalo image segmentation method for image restoration. In order to solve the problem of block effects caused by image segmentation algorithms, Guo and Lee et al. (1997) used the EM (Expectation Maximization) method in 1997 to solve the PSF of each sub block of the image, and used the least squares method for restoration. Finally, the sub blocks were pieced together to obtain a clear image. In 2002, Kim and Tsai et al. proposed a combination of alternating minimization method and Mumford Shah regularization method to determine PSF for image restoration (Kim et al, 2002). In 2007, Bar and Sochen et al. further investigated and suppressed the ringing effect on this basis.

Considering the shortcomings of image segmentation algorithms, Berger et al. proposed an adaptive method that introduces regularization terms to determine PSF, and uses convex set projection for restoration, thereby improving the restoration effect (Ozkan et al, 1994). In 2005, Welk and Theis et al. (2005) used the non convex Perona Malik regularization method for direct restoration, which showed good robustness. In 2006, Klappde et al. (2006) analyzed in detail the spatial variation PSF differences caused by rotation in blurred images under defocus and aberration conditions. In recent years, an increasing number of scholars have proposed various spatial variation PSF non blind deblurring methods based on previous research, which have improved restoration quality and robustness, and expanded their applicability.

2. Noise in the image

After the degradation of the role of the image, the existence of the noise is because the image in the acquisition, transmission, due to imaging equipment and transmission equipment and other performance by the weather and other different reasons for the formation of the role, such as in the use of equipment to pick up the image, by the sunlight and the impact of the climate, so that the acquisition of the image of the existence of the different intensity of the noise component, and when the image is transmitted, the transmission of the information by the channel of the interference will also be made to the original image subject to the interference of the effective information in the original image suffered interference, for example, in the transmission of the image of the wireless network with the intensity of the light will be the transmission of the image of the contamination caused by the transmission.

2.1 Characterization of noise

The recovery process of motion blurred images involves an important and complex problem in the field of image processing. In this process, one is usually confronted with the presence of noise which may cause unwanted interference in image restoration. Therefore, understanding the nature of noise and dealing with them effectively is a part of image processing that cannot be ignored.

First of all, noise is regarded in image restoration as a component that should not be present in an image, or a component whose presence is not desired during image processing. The presence of noise may originate from a variety of factors, including sensor errors in image acquisition equipment, interference in signal transmission, and data loss in image storage. When recovering motion blurred images, these noises may be introduced, thus affecting the accuracy and quality of the recovery.

In the theoretical analysis, noise is considered as a random error that cannot be measured in advance and can only be dealt with in the form of probabilistic statistics. This means that the generation of noise is of a random nature and we cannot precisely predict its exact value. Therefore, in order to deal with noise effectively, we need to employ probability statistics to better understand and characterize it.

Probability Density Function (PDF) and Cumulative Distribution Function (CDF) are two commonly used concepts when dealing with noise. The Probability Density Function describes the probability distribution of the noise over a range of values, while the CDF gives the cumulative probability of the noise up to a certain value point. The use of these two functions helps us to analyze and model the noise in more depth.

In order to minimize the effect of noise on image restoration, several methods are used in the field of image processing. One of the common methods is the application of filters such as inverse filtering and Wiener filtering. These filters smooth the image and reduce the effect of localized noise. However, the selection of filters needs to be adjusted according to the specific type of noise and image characteristics in order to obtain the best restoration effect.

In addition, deep learning techniques have shown strong potential in the field of image restoration. By using deep neural networks, especially Convolutional Neural Network (CNN), complex features in an image can be learned and noise can be effectively removed. The strength of deep learning methods lies in their ability to model complex nonlinear relationships, making them better suited to different types of noise and image restoration tasks.

When dealing with color noise, in addition to filters and deep learning methods, the technique of color space conversion can also be considered. Converting an image from RGB color space to other color spaces such as YUV or Lab helps to better separate and process color noise, which improves the restoration.

In summary, for the recovery processing of motion blurred images, strategies such as understanding the nature of noise, adopting probabilistic statistical methods, selecting appropriate filters or deep learning models, and considering color space conversion are all critical steps. The combined application of these methods can effectively improve the quality of image restoration, reduce the interference of noise, and make the processed image more consistent with the characteristics of the actual scene.

2.2 Classification of noise

Images are usually generated or transmitted with the addition of many noises, which are formed for different reasons and are not identical in nature. The generation of noise is usually considered to be random, so its generation process can be described by adopting a stochastic process, i.e., it is considered to be a random variable characterized by a probability density function (PDF). Based on this characterization noise can be classified into the following broad categories:

2.2.1 Gaussian noise

Gaussian noise is a random noise that conforms to a Gaussian distribution, also known as normally distributed noise. Mathematically, the distribution of Gaussian noise satisfies the Gaussian distribution function, i.e. the bell curve. Its mathematical expression is determined by two parameters, the mean (μ) and the standard deviation (σ), while the probability density of the noise value is highest near the mean, and the probability gradually decreases as the distance from the mean increases. Gaussian noise is characterized as symmetric, continuous and unbounded, so it can take values in any real number range. The PDF expression for Gaussian noise is:

$$\mathrm{P(x)} = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\bar{x})^2}{2\sigma^2}} \quad (2.1)$$

In equation (2.1), σ is the standard deviation of x, $\bar{x}$ is the expected value of x.

The following are the relevant codes:

```
% 读取图像
image = imread('input_image.jpg');

% 添加高斯噪声
noisy_image = imnoise(image, 'gaussian', 0, 0.01);

% 使用高斯滤波去除噪声
denoised_image = imgaussfilt(noisy_image, 2);

% 显示图像
imshowpair(noisy_image, denoised_image, 'montage');
title('Noisy Image vs. Denoised Image');
```

Imread ('input_image. jpg '): Read the input image.

IMNOISE (image,'Gaussian ', 0,0.01): Add Gaussian noise to the image, where 0.01 is the variance of the Gaussian noise.

Imgaussfilter (noisy_image, 2): Smoothes noisy images using a Gaussian filter, where 2 is the standard deviation of the filter.

Imshowpair (noisy_image, denoised_image,'montage '): Display the original noisy image and the denoised image in a concatenated form.

Gaussian noise has a wide range of applications. In the field of communication, Gaussian noise is often used to model random interference in communication channels. In wireless communication, wired communication and optical communication systems, Gaussian noise modeling can be used to study the performance indexes such as bit error rate and signal-to-noise ratio in signal transmission. In finance, Gaussian noise is widely used to model random fluctuations in asset prices. The Brownian motion model is an example, in which asset prices are considered to wander randomly and their fluctuations obey a Gaussian distribution. In image processing, Gaussian noise is commonly used to model random noise introduced by devices such as image sensors and cameras. Dealing with Gaussian noise helps to improve image quality, for example by applying a Gaussian filter to smooth the image.

2.2.2 Salt and pepper noise

Salt and pepper noise is the sudden appearance of black and white clutter in an image that makes parts of the image unusually bright or unusually dark. This kind of noise is usually caused by image sensor failure, interference in signal transmission, and damage to storage media, so it is widely studied and dealt with in the field of image processing. The PDF expression for salt and pepper noise is:

$$P(x) = \begin{cases} P_a, x = a \\ P_b, x = b \\ 0, \text{else} \end{cases} \tag{2.2}$$

From equation (2.1), it can be seen that, when $a > b$, The grayscale level a will appear in the image as dark spots, while the grayscale level b will appear in the image as bright spots, when P_a or P_b equal to 0, at this point, the pulse is called a monopole pulse. When neither is zero, or when $P_a \approx P_b$, at this point, the pulse noise presents a random distribution state, scattered throughout the entire image like salt and pepper particles, hence it is called salt and pepper noise.

The following are the relevant codes:

```
% 读取图像
image = imread('input_image.jpg');

% 添加椒盐噪声
noisy_image = imnoise(image, 'salt & pepper', 0.02);

% 使用中值滤波去除噪声
denoised_image = medfilt2(noisy_image, [3, 3]);

% 显示图像
imshowpair(noisy_image, denoised_image, 'montage');
title('Noisy Image vs. Denoised Image');
```

Noise (image,'salt&pepper ', 0.02): Add salt and pepper noise to the image, where 0.02 is the density of salt and pepper noise.

Medfilt2 (noisy_image, [3, 3]): Use a 3x3 median filter to perform median filtering on noisy images.

Imshowpair (noisy_image, denoised_image,'montage '): Display the original noisy image and the denoised image in a concatenated form.

Salt and pepper noise is a common problem in digital image processing. In the fields of digital photography, medical images, satellite images, etc., images may be affected by salt and pepper noise due to equipment failure or signal transmission instability. In order to improve the image quality and visualization, the removal of salt and pepper noise becomes crucial. In computer vision applications, salt and pepper noise may negatively affect tasks such as target detection and object recognition. The handling of salt and pepper noise is a critical issue for applications such as autonomous driving, surveillance systems, etc. that require extraction of critical information from images. In medical imaging, salt and pepper noise may appear in various medical images such as X-ray images and MRI images. For accurate medical diagnosis, removal of the noise to maintain the clarity and readability of the image is essential.

2.2.3 Color noise

Color noise is an undesired color shift or stray color in an image. This noise usually appears as grainy stray colors in the image and can be caused by sensor damage, electromagnetic interference in signal transmission, and loss of information due to compression algorithms. Color noise differs from single-channel luminance noise in that it affects the overall color balance of an image, giving it an unnatural or distorted appearance.

The following are the relevant codes:

```
% 读取图像
image = imread('input_image.jpg');

% 添加色彩噪声
noise = randn(size(image)) * 25;
noisy_image = imadd(image, noise);

% 转换为HSV空间，对饱和度通道进行中值滤波
hsv_image = rgb2hsv(noisy_image);
hsv_image(:, :, 2) = medfilt2(hsv_image(:, :, 2), [5, 5]);
denoised_image = hsv2rgb(hsv_image);

% 显示图像
imshowpair(noisy_image, denoised_image, 'montage');
title('Noisy Image vs. Denoised Image');
```

Randn (size (image)) * 25: Generate Gaussian distribution noise of the same size as the input image.

Imagdd (image, noise): Add the generated noise to the original image.

Rgb2hsv (noisy_image): Convert RGB images with noise into HSV color space.

Medfilt2 (hsv_image (:,:, 2), [5,5]): Perform a 5x5 median filter on the saturation channel of the HSV image.

Hsv2RGB (hsv_image): Convert the processed HSV image back to the RGB color space.

Imshowpair (noisy_image, denoised_image,'montage '): Display the original noisy image and the denoised image in a concatenated form.

In digital photography, color noise can be caused by high sensitivity shooting, image acquisition in low light conditions, and other factors. Removal of color noise is essential to maintain color fidelity and detail in photographs. In medical imaging, such as X-rays and CT scans, image quality is critical for accurate diagnosis. The presence of color noise may interfere with the doctor's interpretation of the image, so dealing with color noise in medical image processing is necessary. In the field of video transmission and broadcasting, color noise may be caused due to interference in signal transmission, compression algorithms etc. Dealing with color noise helps to improve the quality of television and radio programs.

Reducing or removing image noise is an important task in image processing. This can be accomplished by using filters, reducing ISO sensitivity, enhancing hardware quality, and using denoising algorithms. The goal of denoising is to reduce or eliminate the effects of noise while preserving as much image detail as possible.

3. **Introduction to the principle of inverse filtering and related mathematical models**

3.1 Introduction to inverse filtering

Inverse filtering is a technique for image processing and restoration that aims to recover the original image as much as possible by reversing the filtering effects introduced by the system. The method is commonly used to deal with image degradation due to motion blur, blurring or other factors to improve image quality. The principle of inverse filtering involves frequency domain analysis of the image and modeling of the system to infer and counteract the filtering effects of the system. The core principle of inverse filtering is closely related to frequency domain analysis. In the frequency domain, the behavior of images and systems can be represented by the Fourier transform. The Fourier transform of an image represents the components of the image at different frequencies, while the transfer function of the system describes the response of the system to different frequency components. The goal of inverse filtering is to counteract the effect of filtering by the inverse of the transfer function of the system, thus restoring the frequency domain components of the original image. Inverse filtering is mainly used in image restoration, medical image processing, remote sensing image recovery and other fields. In medical images, inverse filtering can help improve the clarity of X-ray or MRI images. In remote sensing images, inverse filtering helps to recover image blurring caused by factors such as atmospheric disturbances (Tiwari et al, 2013).

Overall, inverse filtering, as a classical image restoration technique, provides an effective means in the field of image processing by analyzing and modeling the frequency domain of the image, as well as using regularization techniques to balance the signal and noise. In practical applications, it is necessary to select appropriate inverse filtering methods according to specific problems and image characteristics, and pay attention to controlling the introduction of noise in order to obtain more accurate and clear image restoration (Likhterov & Kopeika, 2004).

3.2 Mathematical modeling of inverse filtering

The effectiveness of inverse filtering usually relies on the accuracy of the degeneracy model. By understanding the degradation model, we can better evaluate the performance of the inverse filtering algorithm in practical applications. Comparing the difference between the image recovered by inverse filtering and the blurred image generated by the degenerate model can help us understand the robustness and applicability of the algorithm. Below is the expression for image degradation:

$$g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)$$

f(x,y) : Input image

h(x,y) : Degenerate function

η(x,y) : Noise item

g(x,y) : Degraded images

converted to the frequency domain by the convolution theorem:

$$G(u,v) = H(u,v) * F(u,v) + N(u,v)$$

The inverse filtering operation is the removal of the degenerate function:

$$\hat{F} = \frac{G(u,v)}{H(u,v)} = F(u,v) \quad + \frac{N(u,v)}{H(u,v)}$$

According to the above equation, there are two key problems with inverse filtering:

（1） The estimation of the degenerate function H(x,y) is indeed accurate, the clearer the recovery.

（2） When the degenerate function H(x,y) tends to 0, the value of the latter term becomes too large, and the final equation is more affected by the noise term N(x,y) (Altaie, 2021).

3.3 Estimating Degenerate Models for Motion Blur

When the image is in uniform linear motion in the x-direction with a rate of $x_0(t) = at/T$,and at the same time is in uniform linear motion in the y-direction with a rate of $y_0(t) = bt/T$,

$$H(u,v) = \frac{T}{\pi(ua+vb)} sin[\pi(ua+vb)] e^{-j\pi(ua+vb)}$$

$\pi(ua+vb)$ is the frequency component of the blurring direction, which controls the degree of blurring.

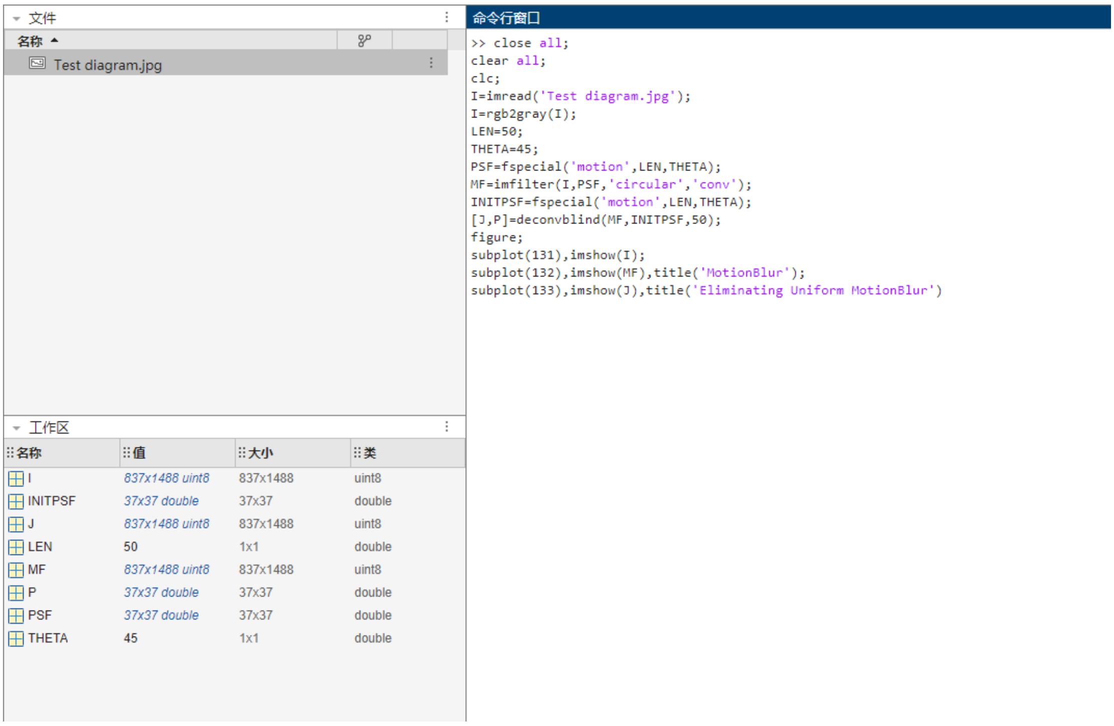
$sin[\pi(ua+vb)]$ is the frequency used to modulate the direction of the blur, ensuring that the blur is dynamic.

$e^{-j\pi(ua+vb)}$ denotes the phase factor, which is related to the phase of the frequency response.

3.4 Practical Applications of Inverse Filtering in MATLAB

Next we will proceed with the practical application of the inverse filtering technique in a MATLAB program, we first take the following diagram as a generalized TEST DIAGRAM below.

We set the motion displacement to 50 pixels, set the motion angle to 45°, and then build a 2D simulated linear motion filter PSF, which is used to generate degraded images. Then eliminate the uniform motion blur. The code is shown below:



The specific code is shown in the appendix.

The final result was obtained as shown in the figure below:

MotionBlur

Eliminating Uniform MotionBlur

3.5 Shortcomings of the inverse filtering technique

The inverse filtering technique, as a classical method for image restoration, has some significant defects and limitations, although it has achieved remarkable results in some cases. The existence of these defects mainly stems from the sensitivity of the inverse filtering method to noise, the inaccuracy of the system model, and the numerical stability in the frequency domain processing. The following are some of the major drawbacks of inverse filtering techniques:

(1) Sensitivity to noise

Inverse filtering techniques are very sensitive to noise during image restoration. Since inverse filtering is performed by the inverse operation of the system transfer function, it is easy to amplify the noise in the image together. In practice, the image may be disturbed by various forms of noise, such as additive noise, quantization noise, etc., which requires additional means in inverse filtering to control and suppress the introduction of noise.

(2) Uncertain system modeling

The successful establishment of inverse filtering relies on the accurate modeling of the system transfer function. However, in practice, the transfer function of the system is usually unknown or can only be approximated. If there is an error in the system model, inverse filtering will produce inaccurate results. This uncertainty may arise from factors such as the non-ideal nature of the image acquisition equipment, optical system, sensors, and changes in environmental conditions.

(3) Ringing effect

The operation of inverse filtering in the frequency domain may lead to the ringing effect, i.e., the presence of too many high-frequency components in the image. This is due to the fact that the goal of inverse filtering is to recover information at all frequencies, including high frequencies, but this can also lead to an unnatural ringing effect at the edges of the image, which reduces the visual quality of the image.

(4) Numerical instability

Numerical computations during inverse filtering may introduce numerical instability, especially in frequency domain computations. Since image restoration usually involves division operations, it may lead to numerical amplification as the value of the transfer function approaches zero. This can cause noise in the image to be amplified, leading to unstable results.

(5) Dependence on precise a priori information

The effectiveness of inverse filtering relies heavily on the accuracy of the a priori information. The a priori information includes the statistical properties of the image, the noise model, the system transfer function, and so on. If this information is inaccurate or difficult to obtain, the results of inverse filtering may be unsatisfactory. This makes the application of inverse filtering in real scenarios more complicated.

(6) Require high computational cost

Some inverse filtering methods may involve high computational cost operations such as frequency domain transforms, inverse transforms, etc., especially when dealing with large size images. This makes inverse filtering in practice may require a large amount of computational resources, limiting its application in real-time processing or resource-constrained environments.

(7) Strong dependence on image content

Inverse filtering methods tend to have a strong dependence on image content. It requires the image to have certain structural and statistical properties, otherwise the effect of inverse filtering may be greatly reduced. In real-world scenarios, the content of images may be varied, which limits the generalization of inverse filtering methods.

3.6 Methods to overcome the defects of inverse filtering

In order to overcome these shortcomings of inverse filtering, it is usually necessary to use some improved techniques and methods. For example, incorporating regularization techniques to suppress noise, employing adaptive filters to adapt to the characteristics of different frequencies, and introducing a priori information to better describe the system model. The development of deep learning technology also brings new ideas to the field of image restoration, and some limitations of inverse filtering can be alleviated to a certain extent by learning the complex mapping relations of images through deep neural networks.

4. **Introduction to the principles of Wiener filtering and related mathematical models**

4.1 An introduction to Wiener filtering:

In signal processing, Wiener filtering is a commonly used noise reduction method to extract the actual signal from noisy observations, and has important applications in both speech and image signals. Wiener filtering is a linear minimum mean square error (LMMSE) estimation, linear means that this form of estimation is linear, and the minimum variance is the optimization criterion for constructing the filter later, that is to say, the difference $y - \hat{y}$ between the actual signal and the estimation should have the minimum variance. And Wiener filtering is to construct a filter such that the output that can be obtained after the observed signal passes through the filter is the minimum mean square error estimate of the actual signal. Finally, the Wiener filtering described in this paper is based on the discrete time domain, partly because in practice it is generally necessary to deal with the discrete case, and partly because it is more complicated to analyze the case of the continuous domain in an abstract way, so it will not be discussed here (Simmons et al, 2007).

Wiener filtering is a technique that provides an optimal estimation of a smooth process based on the minimum mean square error criterion. By minimizing the mean square error between the filter's output and the desired output, the Wiener filter establishes an excellent filtering system. One of its primary applications is extracting signals in the presence of smooth noise interference. Filtering, a crucial method in signal processing, involves removing noise and interference from continuous (or discrete) input data to extract useful information. The corresponding device for this purpose is called a filter.

Filters are categorized into linear and nonlinear types based on whether their output is a linear function of the input. The Wiener filter falls into the category of linear filters. Linear filters, with their output's linearity concerning the input, play a vital role in various fields such as communication, image processing, and control systems.

In summary, Wiener filtering technology holds a significant position in signal processing. Its superior estimation performance and wide-ranging applications in different domains make it a prominent area of interest in both research and practical applications..

4.2 Mathematical modeling of Wiener filtering:

The most important feature of Wiener filtering is that it is filtered using the minimum mean square error approach, which has the expression for the minimum mean square error e^2:

$$e^2 = minE\{[f(x,y) - \hat{f}(x,y)]^2\}$$

can be obtained by further transformations:

$$F（u,v） = [\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}]G(u,v)$$

Where $H\ (u,v)$ denotes the degenerate function. $H^*\ (u,\ v)$ is the complex conjugate of $H\ (u,\ v)$, thus $|H(u,v)|^2 = H^*(u,v)\,H(u,v)$.

$S_\eta(u,v) = |N(u,v)|^2$ denotes the power spectrum of the noise;

$S_f(u,v) = |F(u,v)|^2$ denotes the power spectrum of the undegraded image.

The algorithm for Wiener filtering the original image is to first estimate the mean and variance values within the M*N neighborhood of each pixel point. There are two expressions calculated as follows:

(1) Estimation of average values: $\mu = \frac{1}{MN}\sum_{n_1,n_2\in\eta} a(n_1,n_2)$ μ is the estimated average. M and N are the dimensions of the neighborhood. $a(n_1,n_2)$ is the pixel value of the original image at position (n_1,n_2).

(2) Estimation of variance: $\sigma^2 = \frac{1}{MN}\sum_{n_1,n_2\in\eta} a^2(n_1,n_2) - \mu^2$, $\sigma^2 a$ is the estimated variance. μ is the average of the above calculations. M and N are the dimensions of the neighborhood. $a(n_1,n_2)$ is the pixel value of the original image at position (n_1,n_2).

Calculate the output grayscale value using the following expression. Where $a(n_1,n_2)$ is the gray value before adjustment and $b(n_1,n_2)$ is the gray value after adjustment.

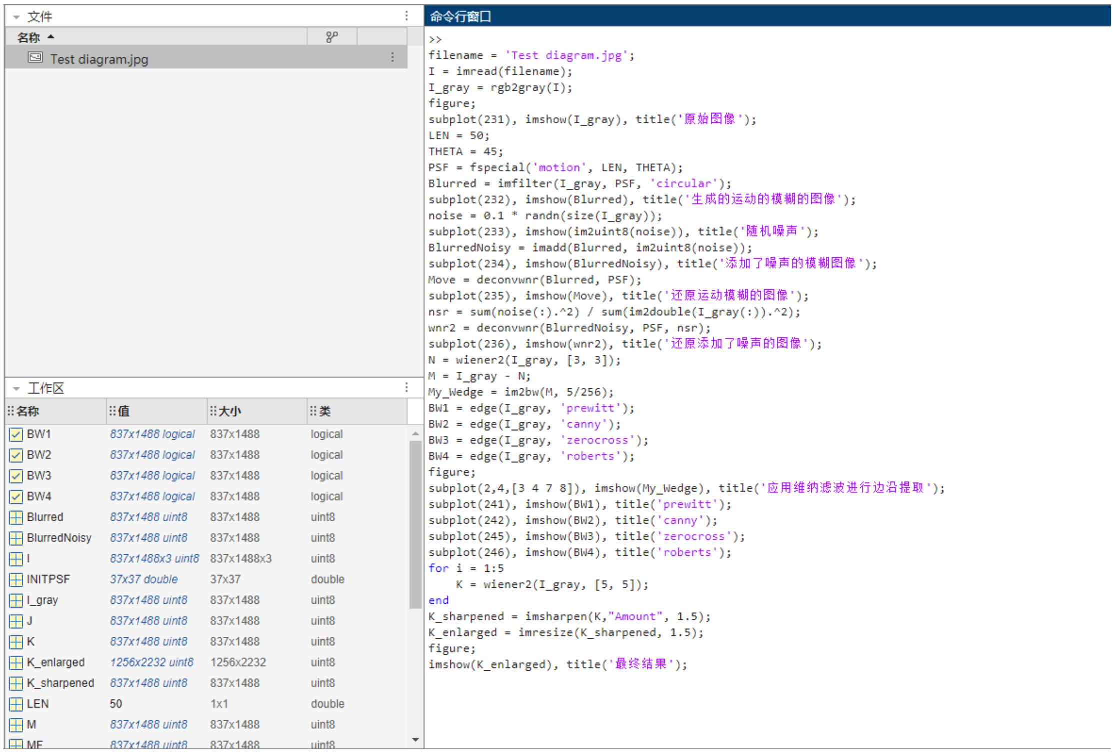
$$b(n_1,n_2) = \mu + \frac{\sigma^2 - v^2}{\sigma^2}(a(n_1,n_2) - \mu)$$

v^2 is an adjustment parameter that balances the effects of mean and variance.

Wiener filtering operates on the principle that the observed signal y (t) includes both the desired signal x (t) and statistically independent white noise ω (t). Its objective is to employ Wiener filtering to reconstruct the desired signal x (t) from the observed signal y (t). Wiener filtering stands out as a fundamental method among various techniques for extracting signal waveforms from noise. It is applicable when the goal is to isolate the useful signal from the entirety of the signal (waveform), not merely a few of its covariates. The Wiener filter takes a random signal with noise as input. The disparity between the desired and actual outputs constitutes the error, with the mean square of this error representing the mean square error. Consequently, a smaller mean square error indicates superior noise filtering. The key to minimizing the mean square error lies in determining the impulse response. By satisfying the Wiener-Hoff equation, optimal optimization of the Wiener filter can be achieved (Nagu & Shanker, 2014).

4.3 Practical application of Wiener filtering in MATLAB

We still use the test diagram in the inverse filtering application. after reading the image file and converting it to grayscale, we still set the motion displacement to 50 pixels and the motion angle to 45° in the generation of the motion blur image. Then random noise is generated and added to the blurred image and the noise power spectral ratio is calculated to generate the "Reduced Noise Added Image". Finally, Wiener filtering is applied for edge extraction and image enhancement. In order to ensure that the images in the paper are easy to view, the resulting images are zoomed and enhanced for clarity. The code is shown below:

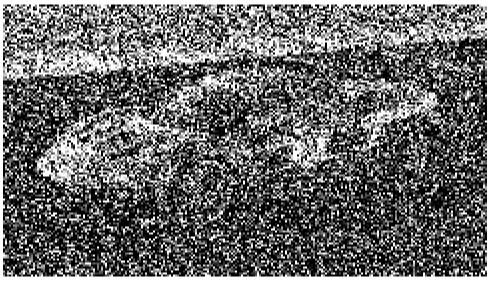


The generated image is shown below:

随机噪声

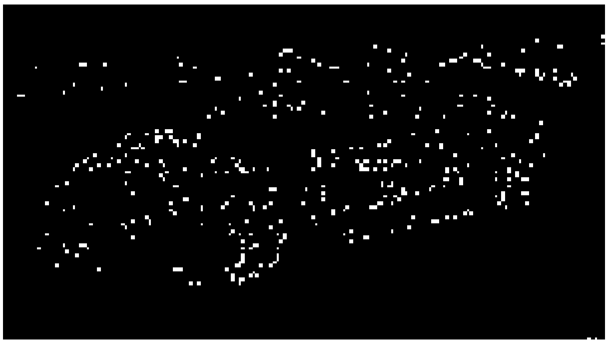
添加了噪声的模糊图像

还原运动模糊的图像

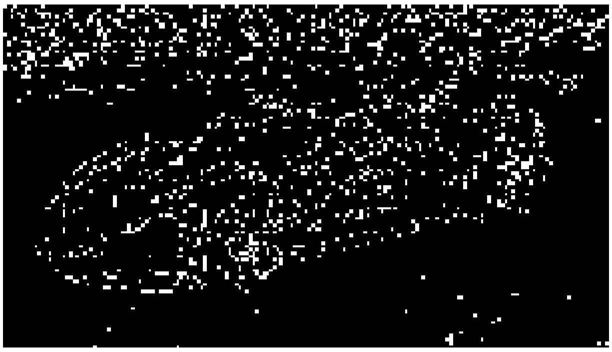


还原添加了噪声的图像

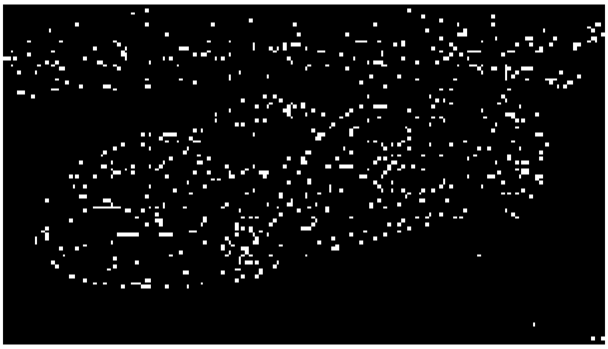
prewitt



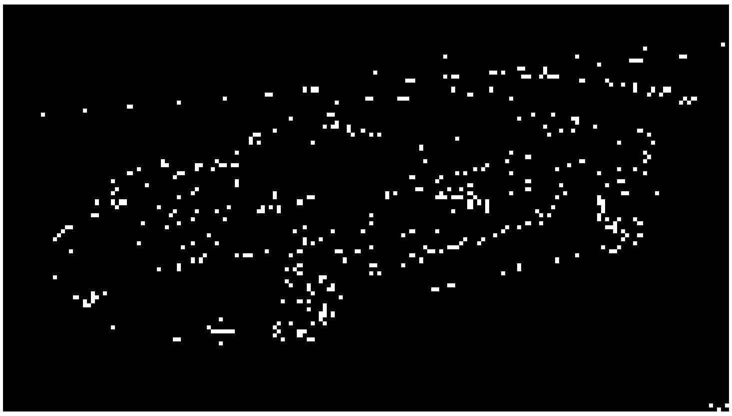
canny



zerocross



roberts



应用维纳滤波进行边沿提取

4.4 Advantages and disadvantages of Wiener filtering

The Wiener filtering algorithm has many advantages in the field of signal processing and image processing, making it a commonly used filtering technique. The following are the main advantages of Wiener filtering:

(1) Minimum mean square error criterion:

The Wiener filtering algorithm is based on the minimum mean square error criterion, which is able to minimize the mean square error between the signal and the noise by optimizing the design of the filter. This means that the characteristics of the original signal are preserved as much as possible during the filtering process, making the filtered signal closer to the real signal.

(2) Frequency domain processing:

Wiener filtering models and processes the signal and noise in the frequency domain, taking full advantage of the nature of the Fourier transform. This frequency-domain processing makes Wiener filtering very effective in dealing with periodic signals and periodic noise, and can be more flexible to deal with the effects of different frequency components.

(3) Applicable to linear systems:

The Wiener filtering algorithm assumes that the system is linear, which holds true in many practical applications. For linear systems, Wiener filtering can be accurately modeled and processed to achieve better filtering results.

(4) Widely used in image restoration:

In the field of image processing, Wiener filtering is widely used in image restoration, which can effectively remove the noise in the image and improve the clarity and quality of the image. This makes Wiener filter has important application value in medical imaging, satellite images and other fields.

(5) Solid theoretical foundation:

The Wiener filtering algorithm has a solid mathematical theoretical foundation, which is based on the principles of statistics and signal processing, and is therefore able to clearly explain the filtering process and analyze its performance in theory.

Although Wiener filtering excels in many aspects, it has some drawbacks that limit its application in certain situations.

(1) High requirement of power spectral density:

The Wiener filtering algorithm requires high accuracy in estimating the signal and noise power spectral densities, and these parameters are often difficult to obtain accurately in practical applications. The introduction of errors may lead to a decrease in the effectiveness of Wiener filtering.

(2) Assumption of linear system:

Wiener filtering assumes that the system is linear, while in some practical situations, the system may be nonlinear. For nonlinear systems, the applicability of Wiener filtering is limited.

(3) Dependence on a priori knowledge of the statistical properties of the signal and noise:

Wiener filtering requires some a priori knowledge of the statistical properties of the signal and noise at the design stage, which may be difficult to obtain in some practical applications.

(4) Limitations on noise modeling:

Wiener filtering assumes that the noise is smooth Gaussian white noise, which does not always hold in some practical scenarios. For non-Gaussian noise or non-smooth noise, Wiener filtering may not be as effective as expected.

(5) Cannot handle non-smooth signals:

The Wiener filtering algorithm is relatively weak for non-smooth signals because its frequency-domain processing based on the Fourier transform may fail in non-smooth situations.

In summary, the Wiener filtering algorithm performs well under certain conditions, but when faced with complex, nonlinear, and nonsmooth real-world problems, it needs to be carefully selected or combined with other filtering techniques to improve performance (Khetkeeree & Liangrocapart, 2019).

5. Summary and outlook

Image restoration is an important problem in the field of computer vision, where the goal is to recover the original clear image from degraded and blurred images. Inverse filtering and Wiener filtering are two commonly used methods for image restoration, based on the inverse filtering operation and the minimum mean square error criterion, respectively. In the summary, we summarize the principle, mathematical model, advantages, disadvantages of inverse filtering and Wiener filtering, and provide an outlook on future directions.

5.1Mathematical Model and Problems of Inverse Filtering

Inverse filtering is a method based on the convolution theorem whose goal is to recover a clear image from a blurred image by removing the degenerate function. The mathematical model of inverse filtering involves an input image f(x,y), a degenerate function h(x,y), a noise term η(x,y) and a degenerate image g(x,y). Through the convolution theorem, these relations can be transformed into the frequency domain, where the inverse filtering operation is to remove the degenerate function. However, there are two key problems with inverse filtering: the first is the accurate estimation of the degeneracy function, and the second is the increased numerical instability and susceptibility to noise terms when the degeneracy function tends to zero.

Estimating the degradation model of motion blur is a common application of inverse filtering. When the image has uniform linear motion in both the x- and y-directions, the degradation model of motion blur can be estimated by a correlation code. This model involves parameters such as rate, frequency components, phase factor, etc., which control the degree and direction of image blur.

5.2 Shortcomings of the inverse filtering technique

The inverse filtering technique, although excellent in some cases, has some significant drawbacks. First, inverse filtering is very sensitive to noise and tends to amplify the noise in the image, requiring additional means to control and suppress the introduction of noise. Second, there is a high dependence on accurate modeling of the system model, while in practice the transfer function of the system is usually unknown or can only be approximated. Frequency-domain manipulations may lead to ringing effects, i.e., the presence of too many high-frequency components in the image, which degrades the visual quality. Instability in numerical computation and dependence on precise a priori information are also among the drawbacks of inverse filtering.

5.3 Principle and mathematical model of Wiener filtering

Wiener filtering is a noise reduction method based on the minimum mean square error criterion, which is widely used in signal processing and image processing. The core idea is to construct a filter, so that the observed signal through the filter can get the minimum mean square error estimate of the actual signal. The mathematical model of Wiener filtering is based on the frequency domain representation and involves parameters such as the original signal, the observed signal, the system transfer function, and the noise power spectrum. The Wiener filtering algorithm first estimates the local mean and variance, and then computes the output of the filter from these estimates and eventually adjusts the gray value of the output.

5.4 Advantages and disadvantages of Wiener filtering

Wiener filtering has the following advantages: based on the minimum mean square error criterion, frequency domain processing, applicable to linear systems, widely used in image restoration, and has a solid mathematical theoretical foundation. However, it also has the disadvantages of high requirements on power spectral density, assumptions on linear systems, reliance on a priori knowledge of statistical properties, limitations on noise modeling, and inability to deal with non-stationary signals.

5.5 Future Outlook

In the future, the field of image restoration can be expanded and improved in the following directions:

5.5.1 Combination of deep learning and image restoration

Utilizing deep learning techniques, especially structures such as Convolutional Neural Networks (CNN), to learn more complex image mapping relationships and improve the performance of image restoration.

5.5.2 Modeling of nonlinear systems

Research on recovery methods that are more suitable for nonlinear systems that exist in reality, in order to improve the applicability of the algorithms.

5.5.3 Further research on adaptive filters

When dealing with images with different frequency characteristics, further improve the performance of adaptive filters to improve the quality of recovered images.

5.5.4 Data-driven approaches

Utilizing larger data sets, data-driven approaches will improve the learning ability of system models and noise models, and reduce the reliance on a priori information.

5.5.5 Real-time and computational efficiency

Optimize algorithms to improve real-time and computational efficiency for practical application needs, making image restoration techniques easier to deploy in real-world scenarios.

In summary, inverse filtering and Wiener filtering in the field of image restoration are two classical methods, each with its own advantages and disadvantages. The future research direction should pay more attention to the innovation of deep learning, nonlinear system modeling, adaptive filters, etc., in order to promote the image restoration technology to achieve greater breakthroughs in practical applications.

Related Code

```
% Application of Inverse Filtering in MATLAB
close all;
clear all;
clc;
I=imread('Test diagram.jpg');
I=rgb2gray(I);
LEN=50;
THETA=45;
PSF=fspecial('motion',LEN,THETA);
MF=imfilter(I,PSF,'circular','conv');
INITPSF=fspecial('motion',LEN,THETA);
[J,P]=deconvblind(MF,INITPSF,50);
figure;
subplot(131),imshow(I);
subplot(132),imshow(MF),title('MotionBlur ');
subplot(133),imshow(J),title(' Eliminating Uniform MotionBlur')
```

```
% Application of Wiener filtering in MATLAB
filename = 'Test diagram.jpg';
I = imread(filename);
I_gray = rgb2gray(I);
figure;
subplot(231), imshow(I_gray), title('原始图像');
LEN = 50;
THETA = 45;
PSF = fspecial('motion', LEN, THETA);
Blurred = imfilter(I_gray, PSF, 'circular');
subplot(232), imshow(Blurred), title('生成的运动的模糊的图像');
```

```
noise = 0.1 * randn(size(I_gray));
subplot(233),  imshow(im2uint8(noise)),  title('随机噪声');
BlurredNoisy = imadd(Blurred, im2uint8(noise));
subplot(234),  imshow(BlurredNoisy),  title('添加了噪声的模糊图像');
Move = deconvwnr(Blurred, PSF);
subplot(235),  imshow(Move),  title('还原运动模糊的图像');
nsr = sum(noise(:).^2) / sum(im2double(I_gray(:)).^2);
wnr2 = deconvwnr(BlurredNoisy, PSF, nsr);
subplot(236),  imshow(wnr2),  title('还原添加了噪声的图像');
N = wiener2(I_gray, [3, 3]);
M = I_gray - N;
My_Wedge = im2bw(M, 5/256);
BW1 = edge(I_gray, 'prewitt');
BW2 = edge(I_gray, 'canny');
BW3 = edge(I_gray, 'zerocross');
BW4 = edge(I_gray, 'roberts');
figure;
subplot(2,4,[3  4 7 8]), imshow(My_Wedge),  title('应用维纳滤波进行边沿提取');
subplot(241), imshow(BW1), title('prewitt');
subplot(242), imshow(BW2), title('canny');
subplot(245), imshow(BW3), title('zerocross');
subplot(246), imshow(BW4), title('roberts');
for i = 1:5
    K = wiener2(I_gray, [5, 5]);
end
K_sharpened = imsharpen(K,"Amount", 1.5);
K_enlarged = imresize(K_sharpened, 1.5);
figure;
imshow(K_enlarged),  title('最终结果');
```

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