Homework 9

ASE 366L

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CJL3282

1) $$\dot{\rho}_{rel}(t_f) = \Phi_{\dot{\rho}\rho}\, t_f - \Phi_{\dot{\rho}\dot{\rho}}\, t_f \left(\Phi_{\rho\dot{\rho}}\, t_f\right)^{-1} \Phi_{\rho\rho}\, t_f)\, \rho(t_0)$$

$$+ \Phi_{\dot{\rho}\dot{\rho}}(t_f)\left(\Phi_{\rho\dot{\rho}}\, t_f\right)^{-1} \rho(t_f)$$

$$\rho_f = \Phi_{\rho\rho}(t_f)\rho_0 + \Phi_{\rho\dot{\rho}}(t_f)\dot{\rho}_0$$

$$\dot{\rho}_f = \Phi_{\dot{\rho}\rho}(t_f)\rho_0 + \Phi_{\dot{\rho}\dot{\rho}}(t_f)\dot{\rho}_0$$

$$\dot{\rho}_0 = \left(\Phi_{\rho\dot{\rho}}(t_f)\right)^{-1}\left(\rho_f - \Phi_{\rho\rho}(t_f)\rho_0\right)$$

$$\dot{\rho}_f = \Phi_{\dot{\rho}\rho}(t_f)\rho_0 + \Phi_{\dot{\rho}\dot{\rho}}(t_f)\left(\Phi_{\rho\dot{\rho}}(t_f)\right)^{-1}\left(\rho_f - \Phi_{\rho\rho}(t_f)\rho_0\right)$$

$$= \boxed{\left(\Phi_{\dot{\rho}\rho}(t_f) - \Phi_{\dot{\rho}\dot{\rho}}(t_f)\left(\Phi_{\rho\dot{\rho}}(t_f)\right)^{-1}\Phi_{\rho\rho}(t_f)\right)\rho_0 + \Phi_{\dot{\rho}\dot{\rho}}(t_f)\left(\Phi_{\rho\dot{\rho}}(t_f)\right)^{-1}\rho_f = \dot{\rho}_f}$$

2)

$\rho(t_0) = 10\hat{S} + 5\hat{W}$ m $\quad \dot{\rho}_{rel_0} = 0.1\hat{R} - 0.05\hat{W}$ m/s

$a = 7000$ km

$\Delta t = 60$ m $= 3600$ s $\quad \dot{\rho}_{rel}(t_f) = 0 \quad \rho_{rel_f} = 0$

$\Delta V_1 = -(\phi_{\rho\dot{\rho}}(t_f))^{-1}\phi_{\rho\rho}(t_f)\rho(t_0) - \dot{\rho}_{rel}(t_0)$

$\Delta V_2 = -\dot{\rho}_{rel}(t_f) = -((\phi_{\dot{\rho}\rho}(t_f) - \phi_{\dot{\rho}\dot{\rho}}(t_f)(\phi_{\rho\dot{\rho}}(t_f))^{-1}\phi_{\rho\rho}(t_f))\rho_0$

$\Delta V = |V_1| + |\Delta V_2|$

$\Delta V_1 = [-0.0983,\ 0.000333,\ 0.0441]$ m/s

$\Delta V_2 = [0.00172,\ -0.000334,\ 0.0080]$ m/s

$\boxed{\Delta V = 0.1159 \text{ m/s}}$

3) $\rho = -35m$

$$\Delta V_1 = -\dot{\rho}_{want,t_0} - \dot{\rho}_{rel_0} = \Phi_{\rho\dot{\rho}}(t_f)^{-1}(\rho_f - \Phi_{\rho\rho}(t_f)\rho_0) - \dot{\rho}_{rel_0}$$

$$\Delta V_2 = \dot{\rho}_{rel}(t_f) - \dot{\rho}_{want}(t_f) = -(\Phi_{\dot{\rho}\rho}(t_f) - (\Phi_{\rho\dot{\rho}}(t_f)\Phi_{\rho\dot{\rho}}(t_f)^{-1}\Phi_{\rho\rho}(t_f))\rho_0$$

$$- \Phi_{\dot{\rho}\dot{\rho}}(t_f)(\Phi_{\rho\dot{\rho}}(t_f))^{-1}\rho(t_f)$$

$\Delta V_1 = [-0.0578, 0.000434, 0.0441]$
$\delta V_2 = [0.00224, -0.000434, -0.0080]$

5) 1) ~~$t_i = 100$~~ $t_i = 100\cdot(t-1)$

$t_2 = 100s \quad t_3 = 200s \quad \Delta t_{31} = 200 \quad \Delta t_{32} = 100 \quad \Delta t_{21} = 100$

$$V_2 = -\Delta t_{32}\left(\frac{1}{\Delta t_{21}\Delta t_{31}} + \frac{\mu}{12 r_1^3}\right)\vec{r_1} + (\Delta t_{32} - \Delta t_{21})\left(\frac{1}{\Delta t_{21}\Delta t_{32}} + \frac{\mu}{12 r_2^3}\right)\vec{r_2} + \Delta t_{21}\left(\frac{1}{\Delta t_{32}\Delta t_{31}} + \frac{\mu}{12 r_3^3}\right)\vec{r_3}$$

$$\boxed{V(t_2) = [-1.815,\ 2.515,\ -0.045]\ \text{km/s}}$$

Herrick-Gibbs b/c given 3 pos. & time

2) Gibbs $\quad Z_{12} = r_1 \times r_2 \quad Z_{23} = r_2 \times r_3$

$Z_{31} = r_3 \times r_1$

$$\vec{N} = r_1 Z_{23} + r_2 Z_{31} + r_3 Z_{12}$$
$$= [-3.7404\times10^8,\ -8.205\times10^5,\ 1.377\times10^8]\ \text{km}^3$$

$$\vec{D} = \Sigma \vec{Z} = [-54873,\ -123,\ 20187]\ \text{km}^2$$

$$\vec{S} = (r_2 - r_3)\vec{r_1} + (r_3 - r_1)\vec{r_2} + (r_1 - r_2)\vec{r_3}$$
$$= [-222.448,\ 2621.728,\ -588.691]\ \text{km}^2$$

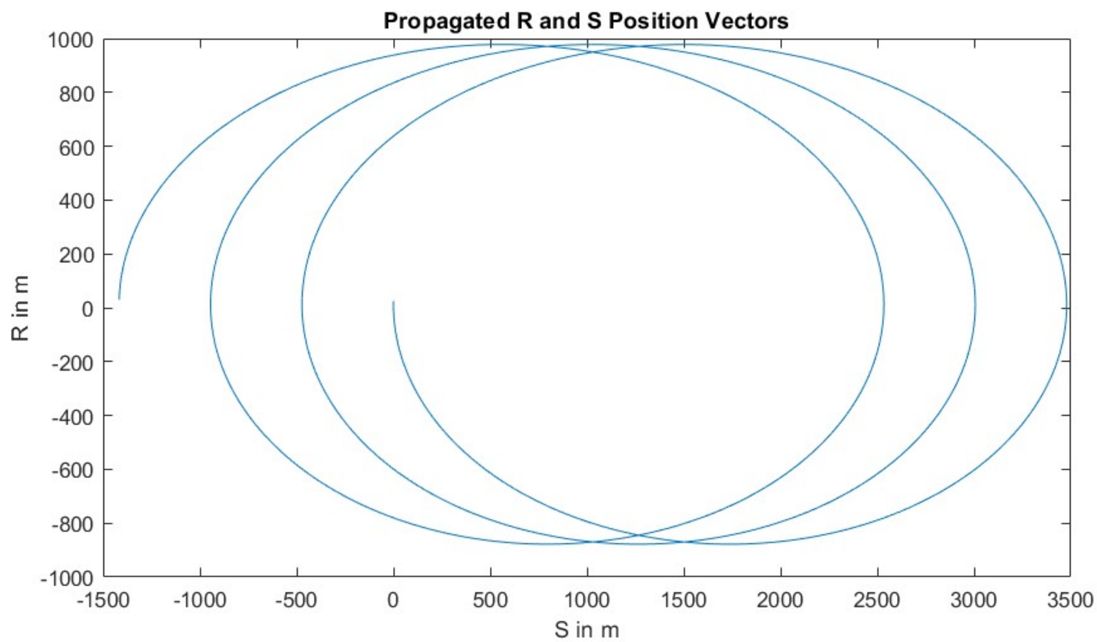
$$B = \vec{D} \times \vec{r_2} = [-6.834\times10^7,\ -3.609^8,\ -1.880\times10^8]\ \text{km}^3$$

$$L_g = \sqrt{\frac{\mu}{ND}} = 1.308\times10^{-4}\ \frac{1}{\text{km s}}$$

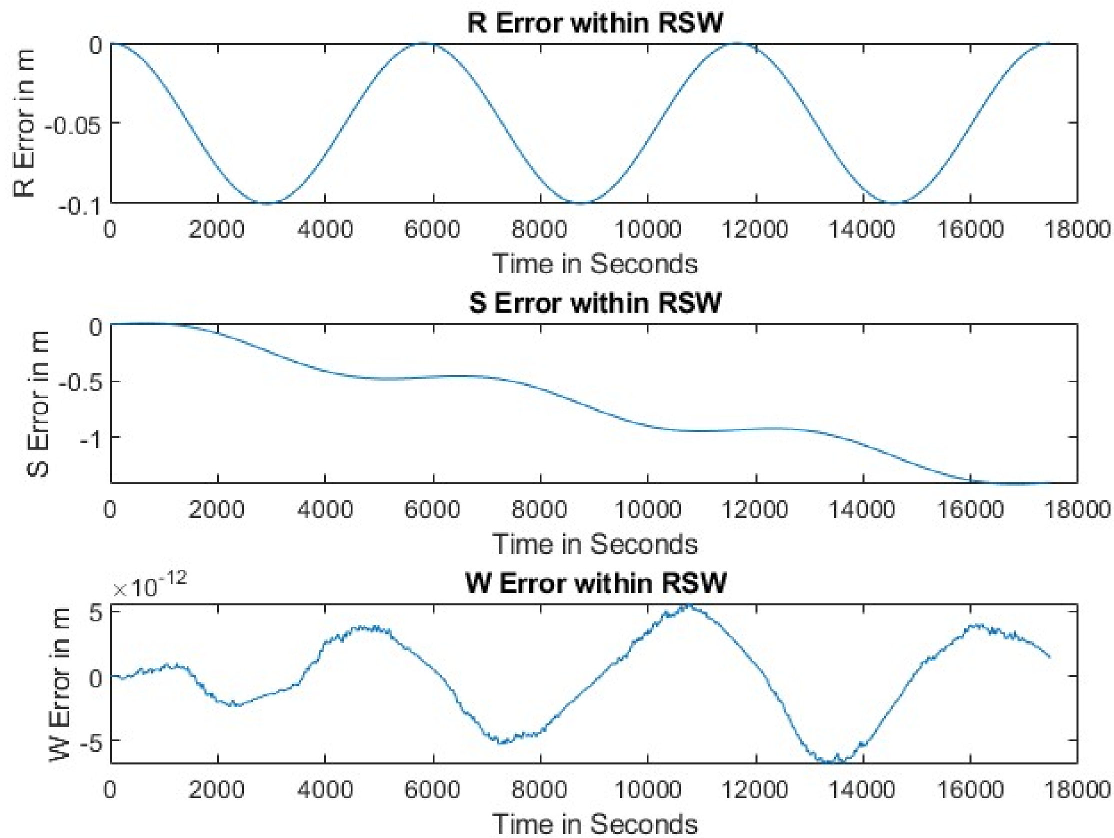
$$\vec{V} = \frac{L_g}{r_2}\vec{B} + L_g\vec{S} = \boxed{[-1.296\ \ -6.345,\ -3.561]\ \text{km/s} = \vec{V_2}}$$

QUESTION 4:

1) r_{ijk} = [7000;0;0] km
   v_{ijk} = [-1.000000000000000e-03; 1.953062306103399; 7.288927756725524] km/s
2) derivative of rho = [25;0;0] m
   derivative of rho = [-1;-0.026950190311671;0] m/s
3)



4)



5) Magnitude of final error = 2.115575989646925 m
6) The computed error values seem way off, but there is certainly considerable error in the CW equations. It grows over time, especially in the S direction meaning the two spacecraft are moving farther apart.