

Homework 8

ASE 366L

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CJL3282

$$1) \quad 1) \quad r = 1.678\hat{i} - 0.189\hat{j} + 1.130\hat{k} \text{ nm}$$

$$v = -0.406\hat{i} - 0.100\hat{j} + 0.570\hat{k} \text{ nm/ps}$$

$$\rho = 0.1\hat{R} - 0.1\hat{s} + 0.2\hat{w} \text{ nm}$$

$$\dot{\rho} = -0.05\hat{R} + 0.04\hat{s} - 0.1\hat{w} \text{ nm/ps}$$

$$v_c = v_0 + v_{rel} + \omega \times \rho_{rel}$$

$$\dot{\rho}_{rel} = v_{rel} = \underbrace{v_c - v_r}_{\rho} - \omega \times \rho_{rel} = \dot{\rho} - \omega \times \rho_{rel}$$

$$\hat{R} = \frac{r}{r} = 0.819\hat{i} - 0.00995\hat{j} + 0.565\hat{k}$$

$$\hat{w} = \frac{r \times v}{|r \times v|} = -0.0003\hat{i} \cancel{- 0.985\hat{j}} - 0.173\hat{k}$$

$$\hat{s} = \hat{w} \times \hat{R} = -0.574\hat{i} - 0.142\hat{j} + 0.807\hat{k}$$

$$Q_{ijk}^{RSW} = \begin{matrix} 0.819 & \cancel{-0.574} & -0.0003 \\ -0.0995 & -0.142 & -0.985 \\ 0.565 & 0.807 & -0.173 \end{matrix}$$

$$\omega = \frac{h}{r^2} \hat{w} = \boxed{0.353 \hat{w} \frac{\text{rad/ps}}{\text{nm}} = \omega}$$

$$\dot{\rho}_{rel} = \dot{\rho} - \omega \times \rho_{rel} = \dot{\rho} - 0.353 \hat{w} \times \rho_{rel}$$

$$\Rightarrow \dot{\rho}_{rel} = -0.0854\hat{R} + 0.0047\hat{s} - 0.1\hat{w} \frac{\text{nm}}{\text{ps}}$$

$$1.2) \quad \dot{\rho}_{rel,ijk} = Q_{ijk}^{RSW} \dot{\rho}_{rel} = \boxed{-0.0725\hat{i} + 0.106\hat{j} - 0.0272\hat{k}} \\ = \dot{\rho}_{rel,ijk}$$

$$2) \quad a = 10,000 \text{ m/s}^2 = 0$$

$$\rho = 20\hat{x} - 10\hat{w} \text{ m per unit } \text{m/s}$$

$$n = \sqrt{\frac{m}{L}} = 6.3135 \times 10^{-4} \text{ rad/s}$$

$$\begin{aligned} x(t) &= (4 - 3 \cos(nt)) x_0 + \frac{\sin(nt)}{n} \dot{x}_0 + \frac{1}{n} (1 - \cos(nt)) y_0 \\ y(t) &= 6 (\sin(nt) - nt) x_0 + y_0 + \frac{n}{n} (\cos(nt) - 1) \dot{x}_0 \\ &\quad + \frac{1}{n} (4 \sin(nt) - 3 nt) \dot{y}_0 \end{aligned}$$

$$z(t) = \cos(nt) z_0 + \frac{\sin(nt)}{n} \dot{z}_0$$

$$\dot{x}(t) = n \sin(nt) x_0 + \cos(nt) \dot{x}_0 + 2 \sin(nt) \dot{y}_0$$

$$\dot{y}(t) = 6n (\cos(nt) - 1) x_0 - 2 \sin(nt) \dot{x}_0 + 4 (\cos(nt) - 3) \dot{y}_0$$

$$\dot{z}(t) = -n \sin(nt) z_0 + \cos(nt) \dot{z}_0$$

$$\dot{x}_0 = 10 \cdot n = 6.3135 \times 10^{-3} \quad \dot{y}_0 = \dot{z}_0 = 0$$

$$1) \quad t = \pi/3 \quad n = \frac{2\pi}{10} = \frac{\pi}{5} \quad nt = \frac{\pi}{3} \quad \cos(\pi/3) = 0.5 \quad \sin(\pi/3) = \frac{\sqrt{3}}{2}$$

$$x(\pi/3) = (4 - 3(-0.5)) x_0 + \frac{\sqrt{3}}{2} n \dot{x}_0 + \frac{1}{n} (1 + 0.5) y_0 = 5\sqrt{3} n$$

$$y(\pi/3) = 6 \left(\left(\frac{\sqrt{3}}{2} \right) - nt \right) x_0 + \dot{x}_0 + \frac{1}{n} (-1.5 \dot{x}_0) + \frac{1}{n} \left(\frac{4\sqrt{3}}{2} - 3nt \right) y_0 = 8.6603 n$$

$$z(\pi/3) = -0.5 \cancel{x_0} + \frac{\sqrt{3}}{2} \cancel{\dot{x}_0} = 5 n$$

$$\dot{x}(\pi/3) = -\frac{3n\sqrt{3}}{2} x_0 + -0.5 \dot{x}_0 + \frac{2\sqrt{3}}{n} \dot{y}_0 = -0.0032 n \text{ m/s}$$

$$\dot{y}(\pi/3) = 6n - 1.5 x_0 - \sqrt{3} \dot{x}_0 + -5 \cancel{\dot{x}_0} = -10n \sqrt{3} \text{ m/s}$$

$$\dot{z}(\pi/3) = -n \left(\frac{\pi}{5} \right) z_0 + -0.5 \cancel{\dot{x}_0} = 0.00547 n \text{ m/s}$$

$$\rho(\pi/3) = 8.6603 R - 10 s + 5 \hat{w} n$$

$$\rho_{rel}(\pi/3) = -0.0032 R - 0.01095 + 0.00547 \hat{w} n$$

$$2.2) \quad t = P \quad n = \frac{2\pi}{P} = \omega t = 2\pi \\ \cos(n\omega t) = 1 \quad \sin(n\omega t) = 0$$

$$x(t) = x_0 + 0 + 0 = x_0 = 0$$

$$y(t) = -12\pi x_0 + y_0 + 0 = -\frac{6\pi x_0}{\pi} = -20$$

$$z(t) = z_0 + 0 = z_0 = 10$$

$$\dot{x}(t) = 0 + \dot{x}_0 + 0 = \dot{x}_0$$

$$\dot{y}(t) = 0 - 0 + \dot{y}_0 = \dot{y}_0$$

$$\dot{z}(t) = 0 + 0 + \dot{z}_0 = \dot{z}_0$$

$$\therefore p(t) = 20\hat{i} - 10\hat{w}$$

$$\rho_{rel}(t) = 6.3135 \times 10^{-3} \text{ kg/m}^3$$

3) $\frac{2\pi}{90} \frac{\text{rad}}{\text{min}}$ $\rho = 5 \hat{12m}$

$z = \omega n \hat{z} = 0 \text{ rad/s}$ ~~5~~

$$x_{c0} = y_0 - \frac{2\dot{x}_0}{n} = 0 \quad y_0 = \frac{2\dot{x}_0}{n} \quad \dot{x}_0 = 0$$

$$y_c = -6n x_0 \rightarrow \dot{y}_0 = 0 \quad \dot{z} = -2n x_0 = -10n = \boxed{-\frac{10}{270} \text{ rad/s} = \dot{z}}$$

$$4) \quad x_c = -\frac{2y_c}{3n} \quad y_{c0} = y_0 - \frac{2x_0}{n} \quad \dot{y}_c = -6nx_0 - 3\dot{y}_0$$

$$y_c(t) = y_{c0} + \dot{y}_c(t-t_0)$$

1) \dot{x}_0 $\dot{x}_0 = 0$ when $y_{c0} = y_0$

2) $x_0 \quad y_{ct} = -y_0 = y_0 + \dot{y}_c \frac{2\pi}{n} \quad \dot{y}_c = -\frac{y_{0n}}{\pi}$

$$x_c = \frac{3}{2}x_0 = -\frac{2\dot{y}_c}{3n} = \frac{2y_0}{3n} = x_0$$

$$\Rightarrow x_0 = \frac{4y_0}{9n}$$

3) $y_0 \quad -6nx_0 - 3\dot{y}_0 = \frac{-y_{0n}}{\pi} \quad \dot{y}_0 = \frac{-x_{0n}}{\pi} + 6nx_0$

$$\Rightarrow \dot{y}_0 = \frac{y_{0n}}{3\pi} - 2nx_0$$

? 4) $\omega = 8000 \text{ rad} \quad y_0 = 20 \text{ m}$

$$x_0 = 2.2829 \quad \dot{x}_0 = 0$$

$$y_0 = 20 \quad \dot{y}_0 = -0.0031$$

? 5) $x\left(\frac{t+t_0}{2}\right) = 5.659 \quad x(t_0) = 2.829$

$$y\left(\frac{t+t_0}{2}\right) = 0 \quad y(t_0) = -20$$