Homework 4

ASE 366L

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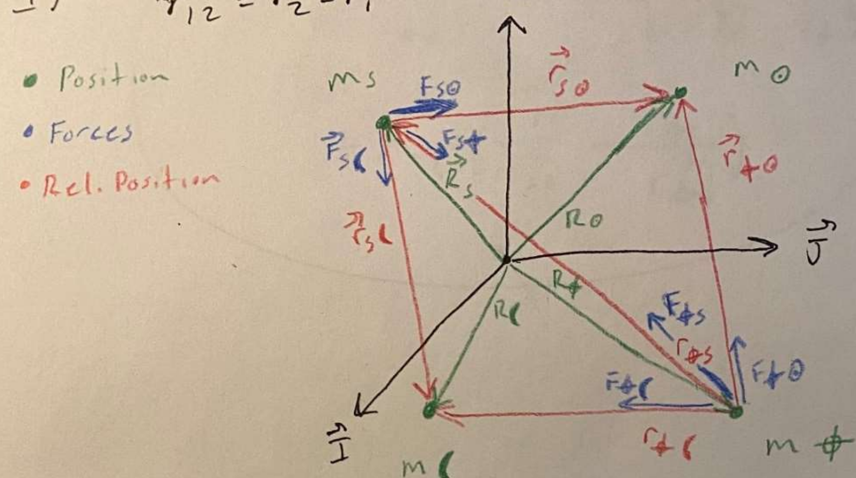
Cjl3282

1) $r_{12} = r_2 - r_1$

a)



b) $a_{\oplus s}$ $\qquad a_{\oplus s} = \frac{d^2}{dt^2}\vec{r}_{\oplus s} = \vec{A}_s - \vec{A}_\oplus$

$$\vec{A}_s = \frac{1}{m_s}\Sigma F = \frac{1}{m_s}\left(\vec{F}_{s\oplus} + \vec{F}_{s\leftmoon} + \vec{F}_{s\odot}\right)$$

$$\vec{A}_\oplus = \frac{1}{m_\oplus}\left(\vec{F}_{\oplus s} + \vec{F}_{\oplus\leftmoon} + \vec{F}_{\oplus\odot}\right)$$

$$\vec{F}_{s\oplus} = \frac{-G M_s M_\oplus}{r_{\oplus s}^2}\hat{r}_{\oplus s} \qquad \vec{F}_{s\odot} = \frac{G M_s M_\odot}{r_{s\odot}^2}\hat{r}_{s\odot}$$

$$\vec{F}_{s\leftmoon} = \frac{G m_s m_{\leftmoon}}{r_{s\leftmoon}^2}\hat{r}_{s\leftmoon} \qquad \vec{F}_{\oplus s} = \frac{G M_s M_\oplus}{r_{\oplus s}^2}\hat{r}_{\oplus s} \qquad \vec{F}_{\oplus\odot} = \frac{G M_\oplus M_\odot}{\vec{r}_{\oplus\odot}^{\,2}}\hat{r}_{\oplus\odot}$$

$$\vec{F}_{\oplus\leftmoon} = \frac{G M_\oplus M_{\leftmoon}}{\vec{r}_{\oplus\leftmoon}^{\,2}}\hat{r}_{\oplus\leftmoon}$$

M_s & $M_\oplus$ cancel.

$$a_{\oplus s} = -\frac{G M_\oplus}{\vec{r}_{\oplus s}^{\,2}}\hat{r}_{\oplus s} + \frac{G M_\odot}{\vec{r}_{s\odot}^{\,2}}\hat{r}_{s\odot} + \frac{G M_{\leftmoon}}{r_{sc}^2}\hat{r}_{sc}$$

$$- \frac{G m_s}{r_{\oplus s}^2}\hat{r}_{\oplus s} - \frac{G M_\odot}{\vec{r}_{\oplus\odot}^{\,2}}\hat{r}_{\oplus\odot} - \frac{G M_{\leftmoon}}{\vec{r}_{\oplus c}^{\,2}}\hat{r}_{\oplus c}$$

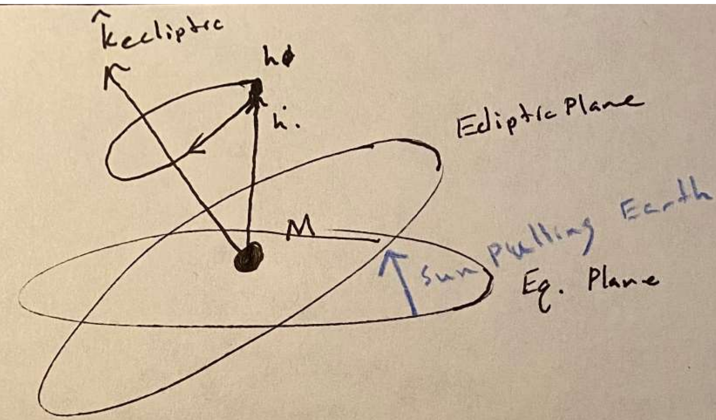
$$a_{\oplus s} = \frac{-G\left(M_\oplus + M_s\right)}{\vec{r}_{\oplus s}^{\,2}}\hat{r}_{\oplus s} + G M_\odot\left(\frac{\hat{r}_{s\odot}}{\vec{r}_{s\odot}^{\,2}} - \frac{\hat{r}_{\oplus\odot}}{\vec{r}_{\oplus\odot}^{\,2}}\right) + G M_{\leftmoon}\left(\frac{\hat{r}_{sc}}{\vec{r}_{sc}^{\,2}} - \frac{\hat{r}_{\oplus c}}{r_{\oplus c}^2}\right)$$

1)   2)

$$a_{\oplus s} = -\frac{\mu_\oplus}{r_{\oplus s}^3}\vec{r} + \mu_\odot\left(\frac{\vec{r}_{s\odot}}{r_{s\odot}^3} - \frac{\vec{r}_{\oplus\odot}}{r_{\oplus\odot}^3}\right)$$

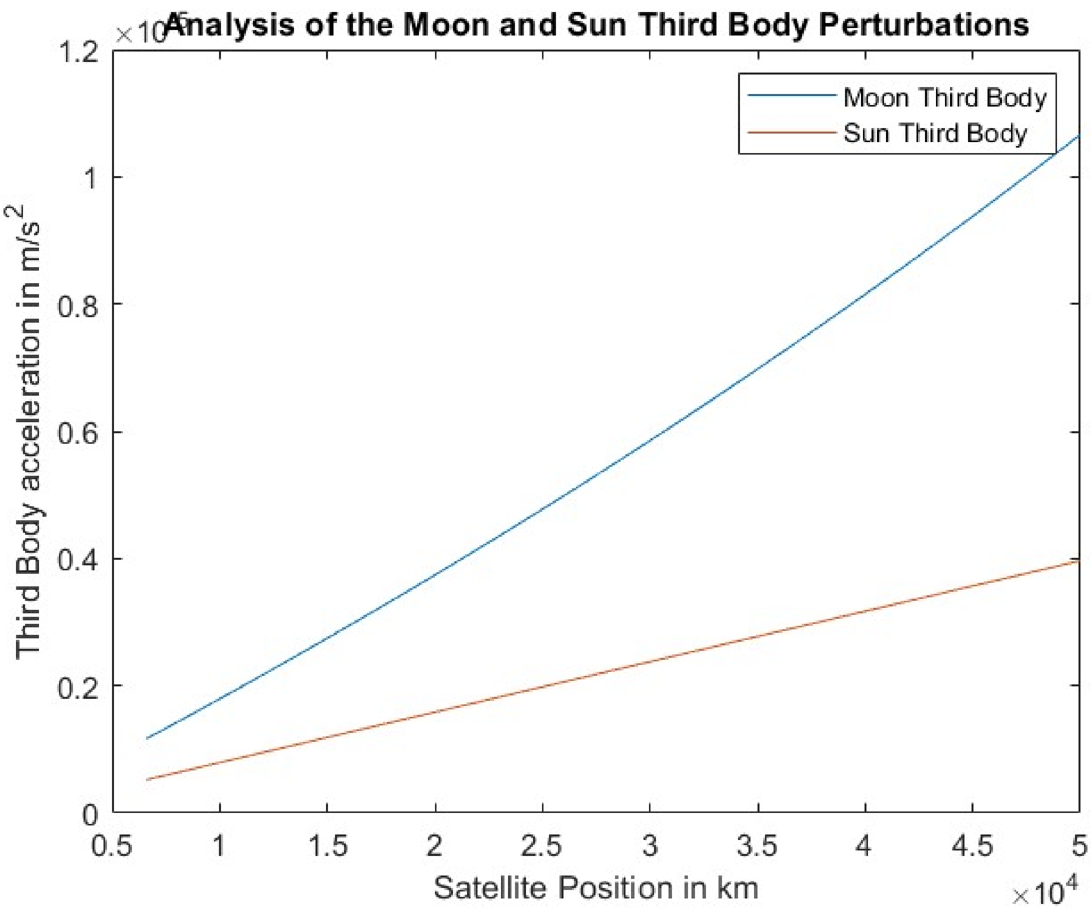
$$+ \mu_{\text{c}}\left(\frac{\vec{r}_{s\text{c}}}{r_{s\text{c}}^3} - \frac{\vec{r}_{\oplus\text{c}}}{r_{\oplus\text{c}}^3}\right)$$

2)

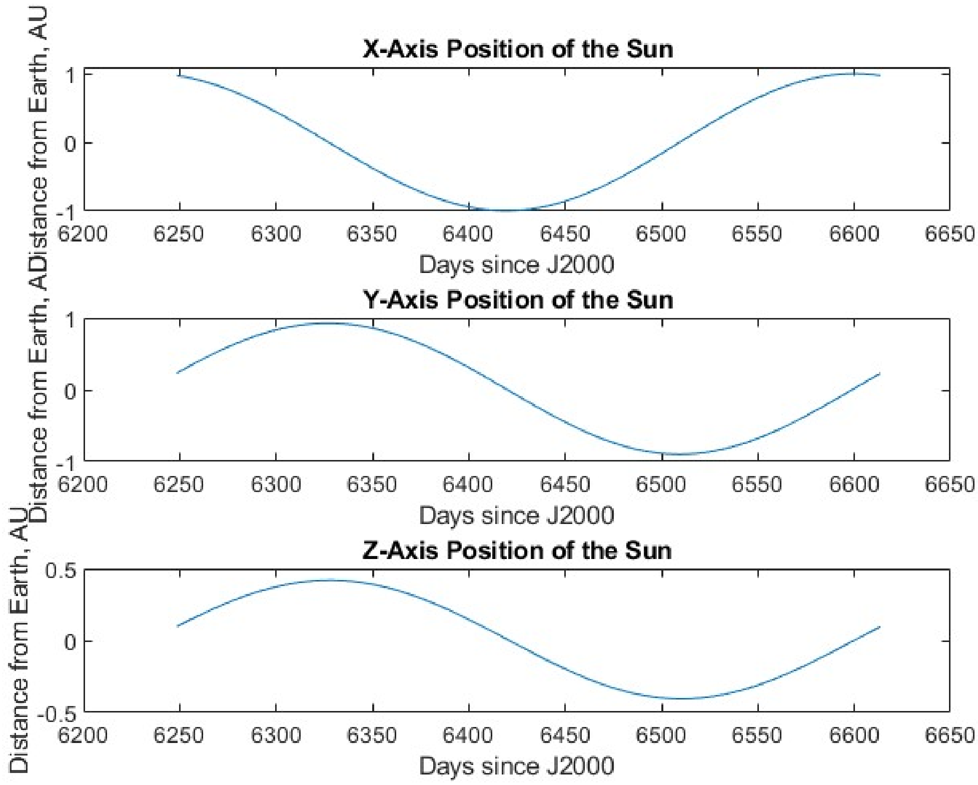


The Sun lies along the ecliptic plane while the Earth has a mass bulge along the equatorial plane due to rotation. The gravitational force of the Sun pulls that bulge toward the ecliptic plane. See Blue Arrow. This force results in a torque on the Earth, and because it's angular momentum and velocity do not change (would be catastrophic), its direction/orientation must. This torque comes directly out of the diagram by the right hand rule resulting in a clockwise precession about the pictured $\hat{k}$.

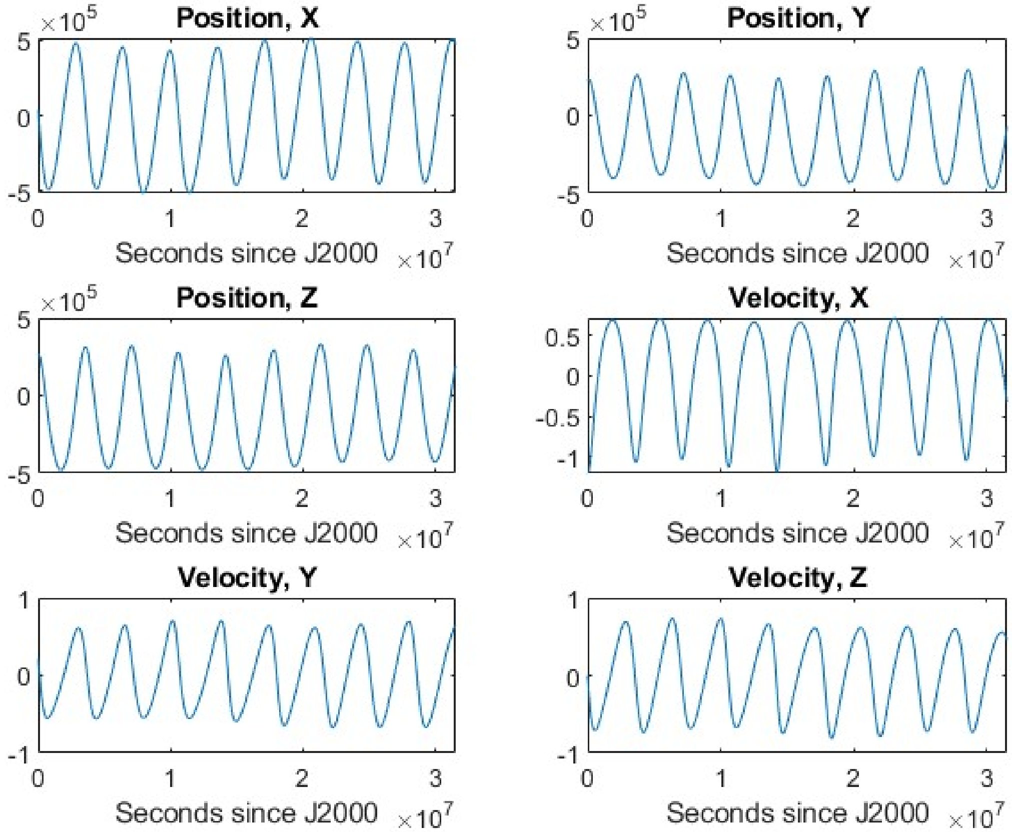
3) The figure below shows the magnitudes of the third body perturbations caused by the sun and moon on a satellite. With the given satellite, the moon has much stronger perturbations and these perturbations increase with increased distance from the Earth.



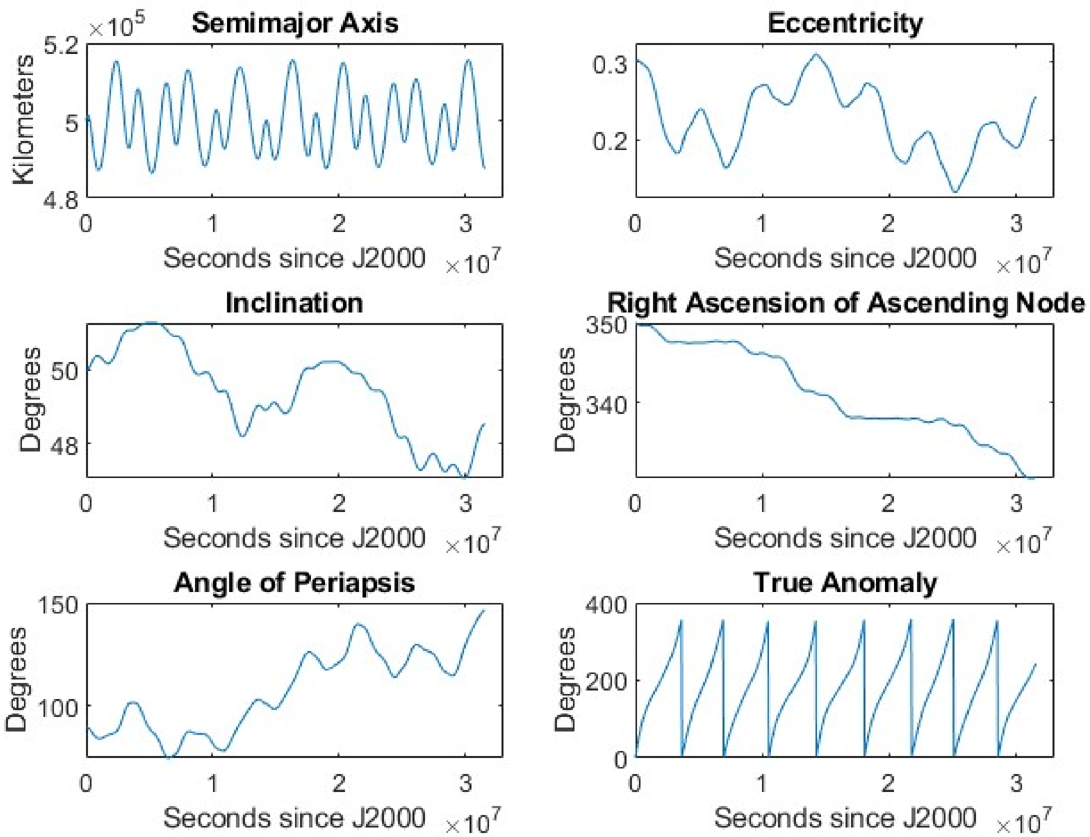
4) rsunvecGCRF = [0.7578, -0.5796, -0.2513] AU



5) First set of plots below is the propagated position and velocity components over time. Second set of plots are the propagated orbital elements over time. Most of the results were as expected. The general regularity of the position and velocity components fit as expected in a regular orbit. The generally regularity of the semimajor axis and eccentricity also fits a regular orbit with regular perturbations. I'm not sure why the inclination and angle of periapsis tend to generally decrease and increase, respectively. That could be a result of different strengths of forces between the Sun and Earth.



Position, X
Position, Y
Position, Z
Velocity, X
Velocity, Y
Velocity, Z
Seconds since J2000
×10^5
×10^7


Semimajor Axis
×10^5
5.2
5
4.8
Kilometers
0
1
2
3
Seconds since J2000
×10^7
Eccentricity
0.3
0.2
0
1
2
3
Seconds since J2000
×10^7
Inclination
50
48
Degrees
0
1
2
3
Seconds since J2000
×10^7
Right Ascension of Ascending Node
350
340
Degrees
0
1
2
3
Seconds since J2000
×10^7
Angle of Periapsis
150
100
Degrees
0
1
2
3
Seconds since J2000
×10^7
True Anomaly
400
200
0
Degrees
0
1
2
3
Seconds since J2000
×10^7