Homework 8

ASE 366L

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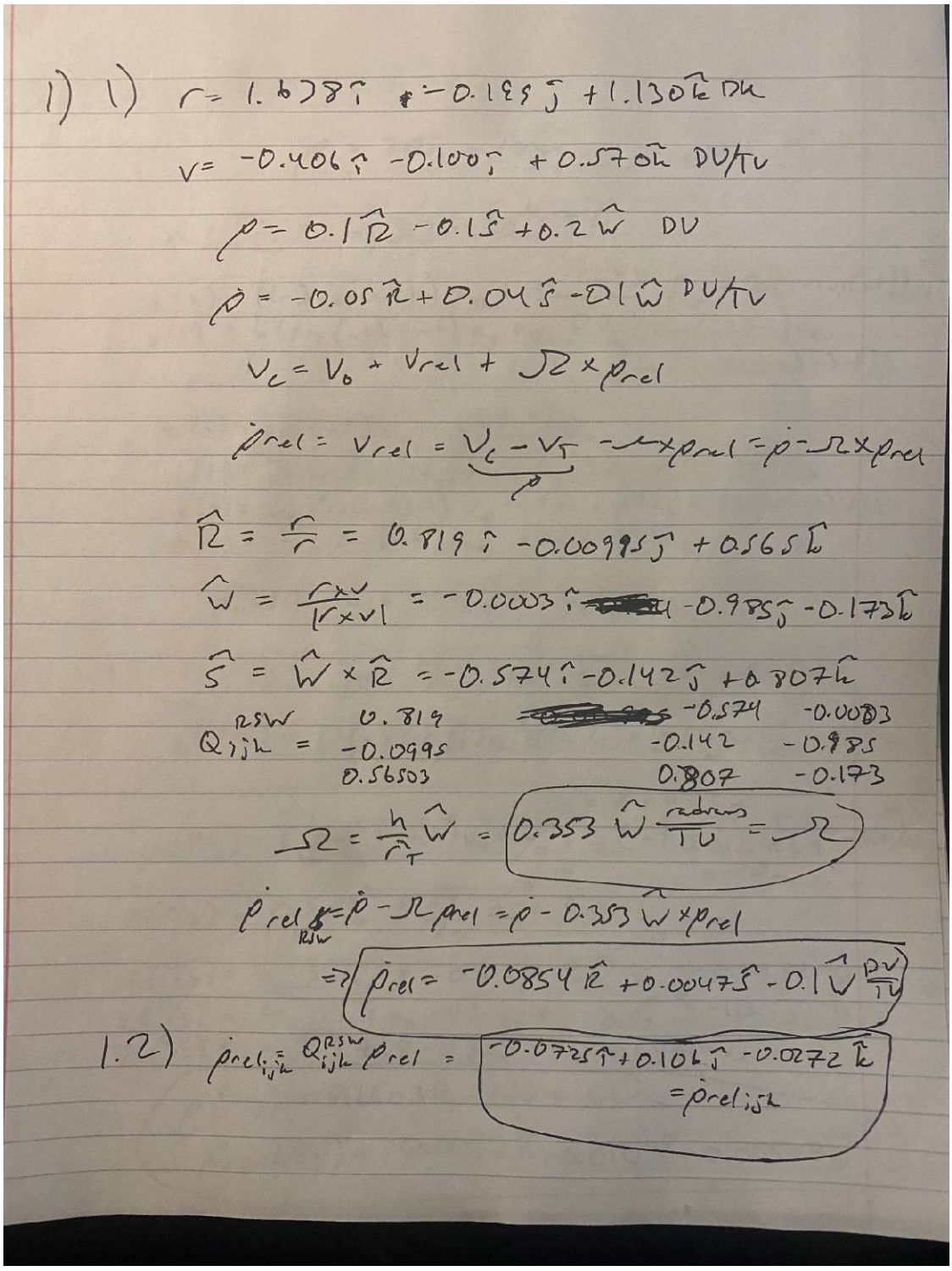

2) $a = 10{,}000$ km $\quad t = 0$ s

$\rho = 20\hat{I} - 10\hat{K}$ m $\quad \dot{\rho}_{rel} = 10n\hat{J}$ m/s

$n = \sqrt{\frac{\mu}{a^3}} = 6.3135 \times 10^{-4}$ rad/s

$x(t) = (4 - 3\cos(nt))x_0 + \frac{\sin(nt)}{n}\dot{x}_0 + \frac{2}{n}(1 - \cos(nt))\dot{y}_0$

$y(t) = 6(\sin(nt) - nt)x_0 + y_0 + \frac{2}{n}(\cos(nt) - 1)\dot{x}_0 + \frac{1}{n}(4\sin(nt) - 3nt)\dot{y}_0$

$z(t) = \cos(nt)z_0 + \frac{\sin(nt)}{n}\dot{z}_0$

$\dot{x}(t) = 3n\sin(nt)x_0 + \cos(nt)\dot{x}_0 + 2\sin(nt)\dot{y}_0$

$\dot{y}(t) = 6n(\cos(nt) - 1)x_0 - 2\sin(nt)\dot{x}_0 + 4(\cos(nt) - 3)\dot{y}_0$

$\dot{z}(t) = -n\sin(nt)z_0 + \cos(nt)\dot{z}_0$

$\dot{x}_0 = 10n = 6.3135 \times 10^{-3} \quad \dot{y}_0 = z_0 = 0$

1) $t = P/3 \quad n = \frac{2\pi}{10} \quad nt = \frac{2\pi}{3} \quad \cos(nt) = -0.5 \quad \sin(nt) = \frac{\sqrt{3}}{2}$

$x(P/3) = (4 - 3(-0.5))x_0 + \frac{\sqrt{3}/2}{n}\dot{x}_0 + \frac{2}{n}(1 + 0.5)\dot{y}_0 = 5\sqrt{3}$ m $= 8.6603$ m

$y(P/3) = 6\left(\left(\frac{\sqrt{3}}{2}\right) - nt\right)x_0 + y_0 + \frac{2}{n}(-1.5\dot{x}_0) + \frac{1}{n}\left(\frac{4\sqrt{3}}{2} - 3nt\right)\dot{y}_0$ $= -10$ m

$z(P/3) = -0.5z_0 + \frac{\sqrt{3}}{2n}\dot{z}_0 = 5$ m

$\dot{x}(P/3) = \frac{3n\sqrt{3}}{2}x_0 + -0.5\dot{x}_0 + \frac{2\sqrt{3}}{n}\dot{y}_0 = -0.0032$ m/s

$\dot{y}(P/3) = 6n \cdot -1.5x_0 - \sqrt{3}\dot{x}_0 + -5\dot{y}_0 = -10n\sqrt{3}$ m/s

$\dot{z}(P/3) = -n\left(\frac{\sqrt{3}}{2}\right)z_0 + -0.5\dot{z}_0 = 0.00547$ m/s

$\rho(P/3) = 8.6603\hat{R} - 10\hat{S} + 5\hat{W}$ m

$\dot{\rho}_{rel}(P/3) = -0.0032\hat{R} - 0.01095\hat{S} + 0.00547\hat{W}$ m/s

7.2) $t = P \quad n = \frac{2\pi}{P} \Rightarrow nt = 2\pi$

$\cos(nt) = 1 \quad \sin(nt) = 0$

$x(P) = x_0 + 0 + 0 = x_0 = 0$

$y(P) = -12x_0 + y_0 + 0 = \frac{6\dot{x}_0}{n} = 20$

$z(P) = z_0 + 0 = z_0 = 10$

$\dot{x}(P) = 0 + \dot{x}_0 + 0 = \dot{x}_0$

$\dot{y}(P) = 0 - 0 + \dot{y}_0 = 0$

$\dot{z}(P) = 0 + \dot{z}_0 = 0$

$\therefore \boxed{\rho(P) = 20\hat{s} - 10\hat{w}\ m}$

$\dot{\rho}_{rel}(P) = 6.7135 \times 10^{-3}\hat{R}\ m/s$

3) $\frac{2\pi}{90}\ \frac{rad}{min}$ $\quad \vec{r} = 5\,\hat{i}\,m$

$z = 0\ m \quad \dot{z} = 0\ m/s$ ✱

$y_{co} = y_0 - \frac{2\dot{x}_0}{n} = 0 \quad y_0 = \frac{2\dot{x}_0}{n} \quad \dot{x}_0 = 0$

$\dot{y}_c = -6n x_0 - 3\dot{y}_0 = 0 \quad \dot{y} = -2n x_0 = -10n = \frac{-\pi}{270}\ m/s = \dot{y}$

4) $x_c = \frac{-2\dot{y}_c}{3n}$ $\quad y_{c0} = y_0 - \frac{2\dot{x}_0}{n}$ $\quad \dot{y}_c = -6n x_0 - 3\dot{y}_0$

$$y_c(t) = y_{c0} + \dot{y}_c(t - t_0)$$

1) $\dot{x}_0$ $\quad \boxed{\dot{x}_0 = 0}$ when $y_{c0} = y_0$

2) x_0 $y_{c\frac{T}{2}} = -y_0 = y_0 + \dot{y}_c \frac{2\pi}{n}$ $\quad \dot{y}_c = \frac{-y_0 n}{\pi}$

$$x_c = \frac{3}{2} x_0 = \frac{-2\dot{y}_c}{3n} = \frac{2y_0}{3\pi} = x_c$$

$$\Rightarrow \boxed{x_0 = \frac{4y_0}{9\pi}}$$

3) $\dot{y}_0$ $\quad -6nx_0 - 3\dot{y}_0 = \frac{-y_0 n}{\pi}$ $\quad -3\dot{y}_0 = \frac{-y_0 n}{\pi} + 6nx_0$

$$\Rightarrow \boxed{\dot{y}_0 = \frac{y_0 n}{3\pi} - 2nx_0}$$

? 4) $a = 8000\,km$ $\quad y_0 = 20$

$x_0 = 2.2829$ $\quad \dot{x}_0 = 0$

$y_0 = 20$ $\quad \dot{y}_0 = -0.0031$

? 5)

$x\left(\frac{t_f}{2}\right) = 5.659$ $\quad x(t_f) = 2.829$

$y\left(\frac{t_f}{2}\right) = 0$ $\quad y(t_f) = -20$