

Homework 8

ASE 366L

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CJL3282

$$1) \quad \mathbf{r} = 1.678\hat{i} - 0.189\hat{j} + 1.130\hat{k} \text{ DU}$$

$$\mathbf{v} = -0.406\hat{i} - 0.100\hat{j} + 0.570\hat{k} \text{ DU/TU}$$

$$\mathbf{p} = 0.1\hat{r} - 0.1\hat{s} + 0.2\hat{w} \text{ DU}$$

$$\dot{\mathbf{p}} = -0.05\hat{r} + 0.04\hat{s} - 0.1\hat{w} \text{ DU/TU}$$

$$\mathbf{v}_c = \mathbf{v}_o + \mathbf{v}_{rel} + \mathbf{\Omega} \times \mathbf{p}_{rel}$$

$$\dot{\mathbf{p}}_{rel} = \mathbf{v}_{rel} = \mathbf{v}_c - \mathbf{v}_T = \mathbf{p} - \mathbf{\Omega} \times \mathbf{p}_{rel}$$

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = 0.819\hat{i} - 0.00995\hat{j} + 0.565\hat{k}$$

$$\hat{\mathbf{w}} = \frac{\mathbf{r} \times \mathbf{v}}{|\mathbf{r} \times \mathbf{v}|} = -0.0003\hat{i} - 0.985\hat{j} - 0.173\hat{k}$$

$$\hat{\mathbf{s}} = \hat{\mathbf{w}} \times \hat{\mathbf{r}} = -0.574\hat{i} - 0.142\hat{j} + 0.807\hat{k}$$

$$Q_{ijk}^{RSW} = \begin{matrix} & 0.819 & -0.00995 & 0.56503 \\ \begin{matrix} 0.819 \\ -0.00995 \\ 0.56503 \end{matrix} & & & \\ \begin{matrix} -0.574 \\ -0.142 \\ 0.807 \end{matrix} & & & \end{matrix}$$

$$\mathbf{\Omega} = \frac{h}{r_T} \hat{\mathbf{w}} = 0.353 \hat{\mathbf{w}} \frac{\text{radians}}{\text{TU}} = \mathbf{\Omega}$$

$$\dot{\mathbf{p}}_{rel} = \dot{\mathbf{p}} - \mathbf{\Omega} \times \mathbf{p}_{rel} = \dot{\mathbf{p}} - 0.353 \hat{\mathbf{w}} \times \mathbf{p}_{rel}$$

$$\Rightarrow \dot{\mathbf{p}}_{rel} = -0.0854\hat{r} + 0.0047\hat{s} - 0.1\hat{w} \frac{\text{DU}}{\text{TU}}$$

$$1.2) \quad \dot{\mathbf{p}}_{rel,ijk} Q_{ijk}^{RSW} \mathbf{p}_{rel} = -0.0725\hat{i} + 0.106\hat{j} - 0.0272\hat{k} = \dot{\mathbf{p}}_{rel,ijk}$$

$$2) \quad a = 10000 \text{ km} \quad t = 0_s$$

$$\rho = 205 - 10\hat{w} \text{ m} \quad \dot{\rho} = -10\hat{w} \hat{r} - \hat{w} \hat{s}$$

$$n = \sqrt{\frac{\mu}{a^3}} = 6.3135 \times 10^{-4} \text{ rad/s}$$

$$\begin{aligned} x(t) &= (4 - 3 \cos(nt)) x_0 + \frac{\sin(nt)}{n} \dot{x}_0 + \frac{z}{n} (1 - \cos(nt)) \dot{y}_0 \\ y(t) &= 6(\sin(nt) - nt) x_0 + y_0 + \frac{z}{n} (\cos(nt) - 1) \dot{x}_0 \\ &\quad + \frac{1}{n} (4 \sin(nt) - 3nt) \dot{y}_0 \end{aligned}$$

$$z(t) = \cos(nt) z_0 + \frac{\sin(nt)}{n} \dot{z}_0$$

$$\dot{x}(t) = 3n \sin(nt) x_0 + \cos(nt) \dot{x}_0 + 2 \sin(nt) \dot{y}_0$$

$$\dot{y}(t) = 6n (\cos(nt) - 1) x_0 - 2 \sin(nt) \dot{x}_0 + 4 (\cos(nt) - 3) \dot{y}_0$$

$$\dot{z}(t) = -n \sin(nt) z_0 + \cos(nt) \dot{z}_0$$

$$\dot{x}_0 = 10 \cdot n = 6.3135 \times 10^{-3} \quad \dot{y}_0 = \dot{z}_0 = 0$$

$$1) \quad t = P/3 \quad n = \frac{2\pi}{P} \quad nt = \frac{2\pi}{3} \quad \cos(nt) = -0.5$$

$$\sin(nt) = \frac{\sqrt{3}}{2}$$

$$x(P/3) = (4 - 3(-0.5)) x_0 + \frac{\sqrt{3}}{2n} \dot{x}_0 + \frac{z}{n} (1 + 0.5) \dot{y}_0 = 5\sqrt{3} \text{ m}$$

$$y(P/3) = 6 \left(\left(\frac{\sqrt{3}}{2} \right) - nt \right) x_0 + \dot{x}_0 + \frac{z}{n} (-1.5 \dot{x}_0) + \frac{1}{n} \left(\frac{4\sqrt{3}}{2} - 3nt \right) \dot{y}_0$$

$$= -10 \text{ m}$$

$$z(P/3) = -0.5 \dot{z}_0 + \frac{\sqrt{3}}{2n} \dot{z}_0 = 5 \text{ m}$$

$$\dot{x}(P/3) = \frac{3n\sqrt{3}}{2} x_0 + -0.5 \dot{x}_0 + \frac{2\sqrt{3}}{n} \dot{y}_0 = -0.0032 \text{ m/s}$$

$$\dot{y}(P/3) = 6n \cdot -1.5 x_0 - \sqrt{3} \dot{x}_0 + -5 \dot{y}_0 = -10\sqrt{3} \text{ m/s}$$

$$\dot{z}(P/3) = -n \left(\frac{\sqrt{3}}{2} \right) z_0 - 0.5 \dot{z}_0 = 0.00547 \text{ m/s}$$

$$\rho(P/3) = 8.6603 \hat{r} - 10 \hat{s} + 5 \hat{w} \text{ m}$$

$$\dot{\rho}(P/3) = -0.0032 \hat{r} - 0.01095 \hat{s} + 0.00547 \hat{w} \text{ m/s}$$

$$2.2) \quad t = P \quad n = \frac{2\pi}{P} \cdot t = 2\pi$$

$$\cos(nt) = 1 \quad \sin(nt) = 0$$

$$x(P) = x_0 + 0 + 0 = x_0 = 0$$

$$y(P) = \frac{1}{2} - 12\pi x_0 + y_0 + 0 = \frac{6\pi z_0}{n} = 20$$

$$z(P) = z_0 + 0 = z_0 = 10$$

$$\dot{x}(P) = 0 + \dot{x}_0 + 0 = \dot{x}_0$$

$$\dot{y}(P) = 0 - 0 + \dot{y}_0 = 0$$

$$\dot{z}(P) = 0 + \dot{z}_0 = 0$$

$$\therefore \quad p(P) = 20 \hat{s} - 10 \hat{w}_m$$

$$p_{rel}(P) = 6.2135 \times 10^{-3} \hat{R}^2/s$$

$$3) \quad \frac{2\pi}{90} \frac{\text{rad}}{\text{min}} \quad \rho = 5 \text{ m}$$

$$z = 0 \text{ m} \quad \dot{z} = 0 \text{ m/s}$$

$$y_{c0} = y_0 - \frac{z \dot{x}_0}{n} = 0 \quad y_0 = \frac{z \dot{x}_0}{n} \quad \dot{x}_0 = 0$$

$$\dot{y}_c = -b_n x_0 \rightarrow \dot{y}_0 = 0 \quad \dot{y} = -z n x_0 = -10 \text{ m} = \frac{-20}{270} \text{ m/s} = \dot{y}$$

$$4) \quad x_c = -\frac{2\dot{y}_c}{3n} \quad y_{c0} = y_0 - \frac{2x_0}{n} \quad \dot{y}_c = -6n x_0 - 3\dot{y}_0$$

$$y_c(t) = y_{c0} + \dot{y}_c(t - t_0)$$

$$1) \quad \dot{x}_0 \quad \boxed{\dot{x}_0 = 0} \quad \text{when } y_{c0} = y_0$$

$$2) \quad x_0 \quad y_{ct} = -y_0 = y_0 + \dot{y}_c \frac{2\pi}{n} \quad \dot{y}_c = -\frac{y_0 n}{\pi}$$

$$x_c = \frac{2}{3} x_0 = -\frac{2\dot{y}_c}{3n} = \frac{2y_0}{3\pi} = x_c$$

$$\Rightarrow \boxed{x_0 = \frac{4y_0}{9\pi}}$$

$$3) \quad \dot{y}_0 - 6n x_0 - 3\dot{y}_0 = -\frac{y_0 n}{\pi} \Rightarrow \dot{y}_0 = -\frac{y_0 n}{\pi} + 6n x_0$$

$$\Rightarrow \boxed{\dot{y}_0 = \frac{y_0 n}{3\pi} - 2n x_0}$$

$$? \quad 4) \quad n = 8000 \text{ km} \quad y_0 = 20$$

$$x_0 = 2.2829$$

$$\dot{x}_0 = 0$$

$$y_0 = 20$$

$$\dot{y}_0 = -0.0031$$

$$? \quad 5)$$

$$x\left(\frac{t_c}{2}\right) = 5.659$$

$$x(t_c) = 2.829$$

$$y\left(\frac{t_c}{2}\right) = 0$$

$$y(t_c) = -20$$