Homework 10

ASE 366L

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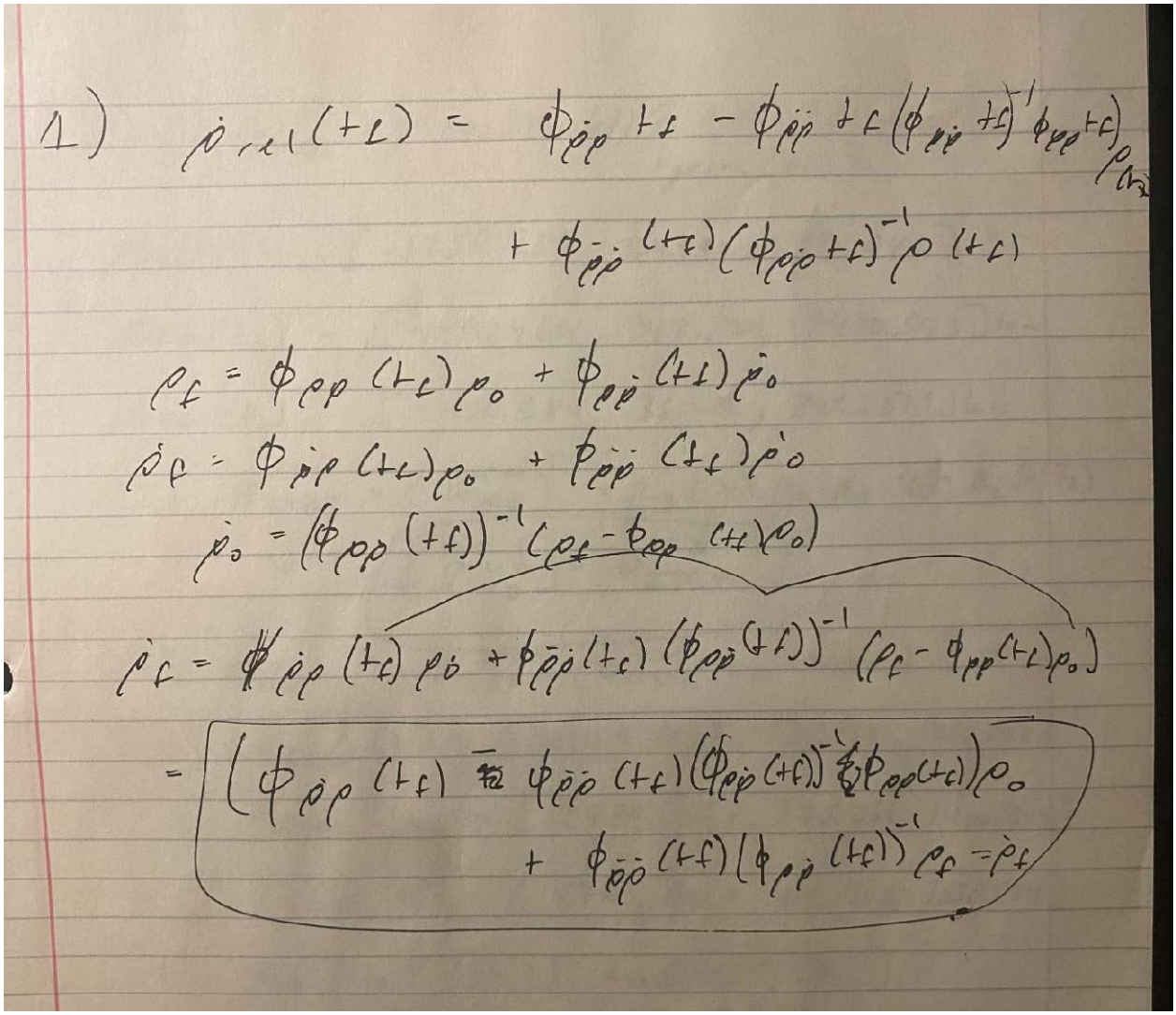

1) 1) $\rho_{SEZ} = -\rho\cos(\beta)\cos(el)\,\hat{S} + \rho\sin(A)\cos(el)\,\hat{E} + \rho\sin(el)\,\hat{Z}$

$\rho^{\circ}_{SEZ}(t_1) = [-1250.208,\ 334.992,\ 8494.049]$ km

$\rho_{SEZ}(t_2) = [-1400.464,\ 342.924,\ 8480.543]$ km

$\rho_{SEZ}(t_3) = [-1550.584,\ 350.861,\ 8455.519]$ km

2) $Q^{SEZ}_{ITRF} = \left(Q^{ITRF}_{SEZ}\right)^T = R_3(-\lambda)\,R_2(\phi_{gd} - \frac{\pi}{2}) = R_2(-\pi/2)$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = Q^{SEZ}_{ITRF}$$

3) $r_{GSEZ} = Q^{ITRF}_{GCRF}\left(Q^{SEZ}_{ITRF}\rho_{SEZ} + R_{ITRF}\right)$

$r_{GCRF}(t_1) = [14794.048,\ 334.992,\ 1250.208]$

$r_{GCRF}(t_2) = [14779.757,\ 375.261,\ 1400.464]$

$r_{GCRF}(t_3) = [14763.8427,\ 415.466,\ 1550.584]$

4) $\alpha_{12} = \cos^{-1}\left(\frac{r_1 \cdot r_2}{r_1 r_2}\right) = 0.6027$ $\quad \alpha_{23} = \cos^{-1}\left(\frac{r_2 \cdot r_3}{r_2 r_3}\right) = 0.6027$

Both $< 1^{\circ}$ therefor Herrick-Gibbs

2) $\hat{\rho}_* = [\cos\delta_* \cos\alpha_* \quad \cos\delta_* \sin\delta_* \quad \sin\delta_*]^T$

$\hat{\rho}_1 = [0.698 \quad -0.680 \quad 8465.519]$

$\hat{\rho}_2 = [0.698 \quad -0.680 \quad 8465.519]$

$\Theta = \pi - \cos^{-1}(\hat{r}_2 \cdot \hat{\rho})$

$\Theta_1 = 134.229° \quad \Theta_2 = 137.027°$

$range_1 = 37362.282 \quad range_2 = 37170.332$

$\hat{r}_1 = [0.7705 \quad -0.6365 \quad -0.0350]$

$\hat{r}_2 = [0.7975 \quad -0.6020 \quad -0.0391]$

$\hat{h} = \frac{\hat{r}_1 \times \hat{r}_2}{|\hat{r}_1 \times \hat{r}_2|} = \frac{0.003817 \quad 0.00221 \quad 0.0438}{0.04398}$

$\Omega = \tan^{-1}\left(\frac{\hat{h}_i}{-\hat{h}_j}\right) = 120.006°$

$i = \cos^{-1}(\hat{h}_k) = 5.731°$

$\hat{n} = \begin{bmatrix} \cos\Omega \\ \sin\Omega \\ 0 \end{bmatrix} \quad u_1 = \cos^{-1}(\hat{n} \cdot \hat{r}_1) \quad u_2 = \cos^{-1}(\hat{n} \cdot \hat{r}_2)$

$\hat{n} = \begin{bmatrix} -0.5001 \\ 0.76598 \\ 0 \end{bmatrix} \Rightarrow u_1 = 200.5301$

$u_2 = 203.0512$

$a = 42000.0$ km $\quad u_1 = 200.5301°$

$e = 0 \quad u_2 = 203.0512°$

$i = 5.731°$

$\Omega = 120.0059$

3) $r_1 = 2\hat{i} + \hat{j}$ DU $r_2 = -3\hat{i} + 2\hat{j}$ DU

$\mu = 1 \text{ DU}^3/\text{TU}^2$

1) a_{min} $c = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\Delta v)} = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \frac{r_1 \cdot r_2}{r_1 r_2}}$

$= 5.0990$ DU

$a_{min} = \frac{r_1 + r_2 + c}{4} = 2.7352$ DU $= a_{min}$

2) $\cos(\Delta v) = \frac{r_1 \cdot r_2}{r_1 r_2} = -0.49614$

$\sin(\Delta v) = +_m \sqrt{1 - \cos^2(\Delta v)} = 0.86824$

$p_{min} = \frac{r_1 r_2}{c}(1 - \cos(\Delta v)) = 2.3656$ DU

$f = 1 - \frac{r_2}{p_{min}}(1 - \cos(\Delta v)) = -1.2804$

$g = \frac{r_1 r_2 \sin \Delta v}{\sqrt{\mu p_{min}}} = 4.5512$ TU

$v_1 = \frac{1}{g}(r_2 - f r_1) = [-0.0965 \;\; 0.7208 \;\; 0]$ DU/TU

3) $\sin \Delta v = +_m \sqrt{1 - \cos^2(\Delta v)} = -0.86824$

$g = -4.5512$ TU

$v_1 = [0.0965 \;\; -0.7208 \;\; 0]$ DU/TU

4) $t_{ab} = \frac{1}{3}\sqrt{\frac{2}{m}}\left(s^{3/2} - (s-c)^{3/2}\right)$ $s = \frac{r_1 + r_2 + c}{2}$

$t_{ab_{min}} = 5.9247$ TU

5) $r_1' = 2a - r_1$ $r_2' = 2a - r_2$

$R_1 = \begin{bmatrix} r_{1i} \\ r_{1j} \end{bmatrix}$ $R_2 = \begin{bmatrix} r_{2i} \\ r_{2j} \end{bmatrix}$ $\gamma = \frac{r_1'^2 - r_2'^2 + c^2}{2c}$

$= 3.3765$ DU

$\delta = \sqrt{r_1'^2 - \gamma^2} = 1.6632$ DU $p = R_1 + \frac{1}{c}(R_2 - R_1)$

$= \begin{bmatrix} -1.3101 \\ 1.6622 \\ 0 \end{bmatrix}$ DU

$$F' = p \pm \begin{bmatrix} \frac{\delta}{2}(r_{2,j} - r_{1,j}) \\ \frac{\delta}{2}(r_{1,i} - r_{2,i}) \end{bmatrix} DU$$

$$e = \frac{FF'}{2a} \Rightarrow |F' - F| \;\&\; F = 0$$

$$e = |F'| \frac{1}{6}$$

$$e_1 = 0.5729$$
$$e_2 = 0.2729$$