Homework 7

ASE 366L

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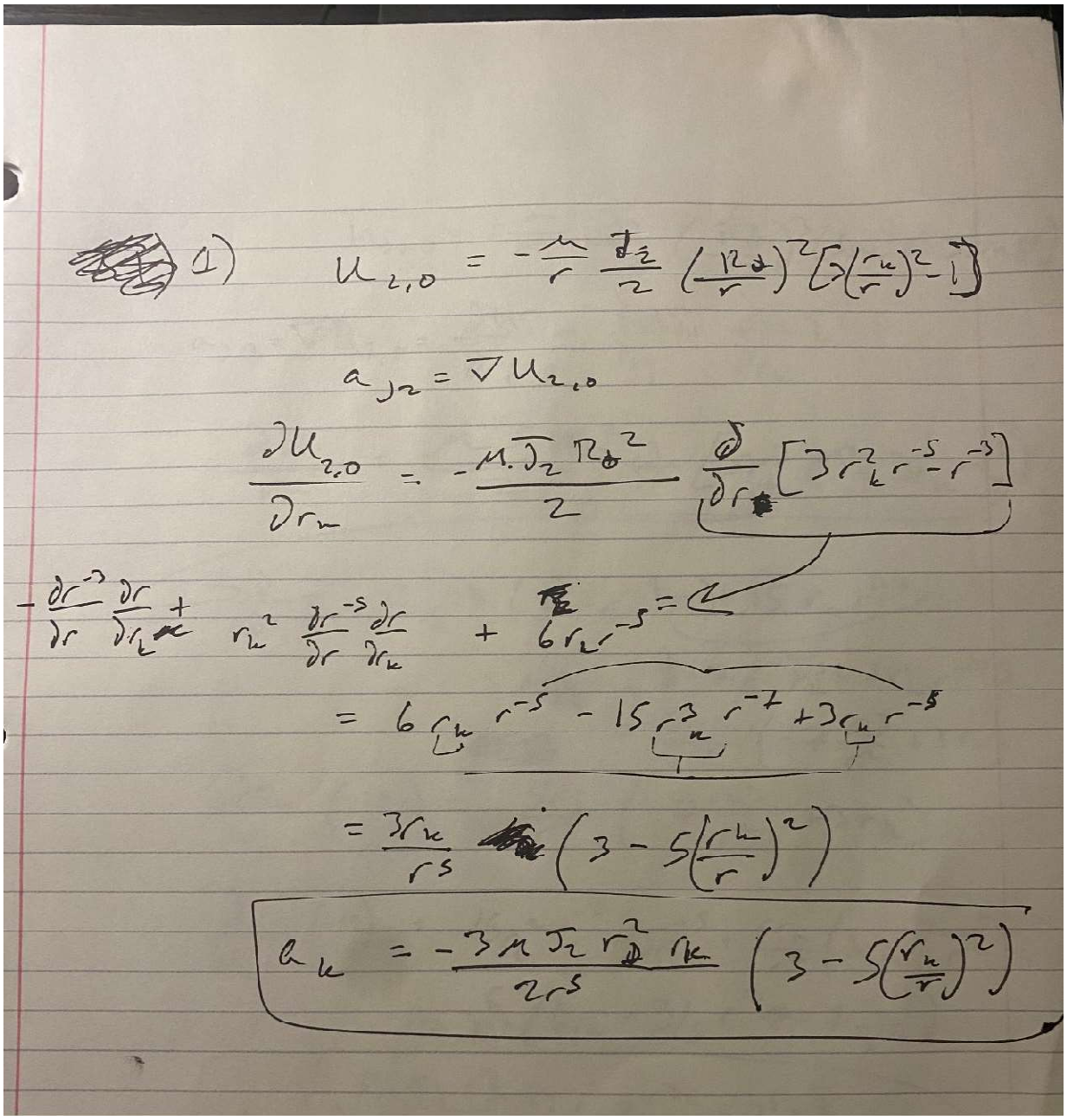
CJL3282



2) $U_{3,0} = -\frac{\mu}{r}\frac{J_3}{2}\left(\frac{R_\oplus}{r}\right)^3(5\sin^3\phi - 3\sin\phi)$

$a_{J_3} = \nabla U_{3,0} = \frac{\partial U}{\partial r_i}\hat{i} + \frac{\partial U}{\partial r_j}\hat{j} + \frac{\partial U}{\partial r_k}\hat{k}$

$\frac{\partial U}{\partial r_i} = \frac{\mu J_3 R_\oplus^2}{2}\frac{\partial}{\partial r}\left[5r_k^3 r^{-7} - 3r_k r^{-5}\right]$

$\frac{\partial}{\partial r_i} = 5r_k^3(-7r^{-9}r_i) + 15 r_k r^{-7} r_i$

$= \frac{5r_i}{r^7}\left(-7r^{-9}r_k^3 + 3r_k r^{-7}\right)$

$= \frac{5r_i}{r^7}\left(-7\frac{r_k^3}{r^2} + 3r_k\right)$

$\frac{\partial}{\partial r_j}(\;) = \frac{5r_j}{r^7}\left(-7\frac{r_k^3}{r^2} + 3r_k\right)$

$\frac{\partial}{\partial r_k}(\;) = 15r_k^2 r^{-7} - 3r^{-5} + \frac{5r_k}{r^7}\left(3r_k - 7\frac{r_k^3}{r^2}\right)$

$= \frac{5}{r^7}\left(6r_k - 3/5\, r^2 - 7\frac{r_k^4}{r^2}\right)$

$a_i = -\frac{5\mu J_3 R_\oplus^3}{2r^7} r_i\left(-\frac{7r_k^3}{r^2} + 3r_k\right)$

$a_j = -\frac{5\mu J_3 R_\oplus^3}{2r^7} r_j\left(-\frac{7r_k^3}{r^2} + 3r_k\right)$

$a_k = -\frac{5\mu J_3 R_\oplus^3}{2r^7}\left(6r_k^2 - \frac{3}{5}r^2 - 7\frac{r_k^4}{r^2}\right)$

$a_{J_3} = a_i\hat{i} + a_j\hat{j} + a_k\hat{k}$

3) $\frac{da}{dt} = \frac{-2a^2 v}{m} \quad F_D = -\frac{a^2 C_D A}{m n} \rho v^3$

$\Rightarrow A = -\frac{m n}{a^2 v^3 \rho C_D} \frac{da}{dt}$

$a = 16000\,km \qquad e = 0.595$

$a_f = \frac{r_p - r_a - 1000}{2} = a - 500$

$da = a_f - a = -500\,km \qquad C_D = 2.0 \quad t = 10\,s$

$r_p = a(1-e) = 6480\,km$

$v_p = \sqrt{m\left(\frac{2}{r_p} - \frac{1}{a}\right)} = 9.8052\ km/s$

$h_p = r_p - R_\oplus = 101.8637\ km$

$\rho_0 = 5.297 \times 10^{-7}\ kg/m^3 \qquad h_0 = 100\,km$

$H = 5.877\,km$

$\rho = \rho_0 e^{-\frac{h_p - h_0}{H}} = 3.358 \times 10^{-7}\ kg/m^3$

$\therefore A = 103.8\,m^2$

4) $r_\oplus \; r_s = 149{,}597{,}870$ km

$a = 10{,}000$ km $\quad e = 0.0 \quad i = 20° \quad \Omega = 0°$

$r_{ijk} = a(\cos(u)\hat{i} + \sin(u)\hat{k})$

$a\sin(\pi - u_{in}) = r_\oplus \quad u_{in} = 2.45$ rad

$u_{out} = 2\pi - u_{in} = 3.833$ rad

$$\frac{3.833 - 2.45}{2\pi} = 0.2201 = 22.01\%$$

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} = 9952 \text{ s}$$

Time in Shadow $= 0.2201 \cdot 9952 =$

2190.49 s $= T_{is}$

5) $r_{\oplus o}$ = same as before

1) $r = -7000\hat{i} - 6400\hat{j} + 600\hat{k}$

$r_{os} = r_s - r_o$

$a = \sin^{-1}\left(\frac{r_\oplus}{r_{os}}\right) = 0.0047$

$b = \sin^{-1}\left(\frac{r_\oplus}{r_s}\right) = 0.736$

$c = \cos^{-1}\left(\frac{r_s \cdot r_o}{r_s r_{os}}\right) = 0.743$

$0.0047 + 0.736 < 0.743$

$\therefore \boxed{\gamma = 1}$

2) $r = -1000\hat{i} + 4685\hat{j} + 4316\hat{k}$

$a = 0.0047$

$b = 1.42$

$c = 1.42$

$x = \frac{c^2 + c^2 - b^2}{2c} = 8.008 \times 10^{-4}$

$y = \sqrt{a^2 - x^2} = 0.00458$

$A = a^2 \cos^{-1}\left(\frac{x}{a}\right) + b^2 \cos^{-1}\left(\frac{c-x}{b}\right) - c_y = 4.143 \times 10^{-5}$

$\gamma = 1 - \frac{A}{\pi a^2} = 1 - \frac{4.143 \times 10^{-5}}{\pi \cdot 0.0047^2} = 0.403 = \gamma$

6) Same Location of Sun

Set: $r = -X\hat{i}$

$$b = \sin^{-1}\left(\frac{r_\phi}{r}\right) \Rightarrow \sin^{-1}\left(\frac{r_\phi}{X}\right)$$

$$a = \sin^{-1}\left(\frac{r_\odot}{|r_{\phi\odot} - r|}\right) \Rightarrow \sin^{-1}\left(\frac{r_\odot}{r_{\phi\odot} + X}\right)$$

$$a = b$$

$$\sin^{-1}\left(\frac{r_\phi}{X}\right) = \sin^{-1}\left(\frac{r_\odot}{r_{\phi\odot} + X}\right)$$

$$\frac{r_\phi}{X} = \frac{r_\odot}{r_{\phi\odot} + X}$$

$$(r_{\phi\odot} + X) r_\phi = r_\odot X$$

$$r_\phi r_{\phi\odot} + X r_\phi = r_\odot X$$

$$r_\phi r_{\phi\odot} = r_\odot X - r_\phi X$$

$$\Rightarrow \boxed{X = \frac{r_\phi r_{\phi\odot}}{r_\odot - r_\phi}}$$