# [Submission]

1. Write a python code, make a comment, present results for each meaningful block of your code at Jupyter Notebook.

   [New > Python 3]

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# [Taylor Approximation]

1. Define a differentiable function that maps from real number to real number.
2. Define a domain of the function.
3. Plot the function.
4. Select a point within the domain.
5. Mark the selected point on the function.
6. Define the first-order Taylor approximation at the selected point.
7. Plot the Taylor approximation with the same domain of the original function.

In [6]:
```
import numpy as np
import matplotlib.pyplot as plt
```

### 1. Define a differentiable function that maps from real number to real number.

→ The function is $\sin x$

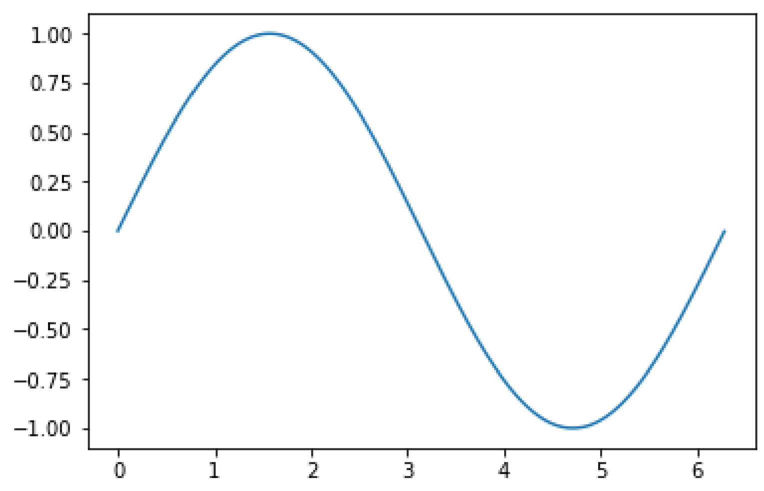
### 2. Define a domain of the function.

→ a domain of function is $[0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}, 2\pi]$

### 3. Plot the function.

In [7]:
```
x=np.arange(0,2*np.pi,0.01)
y=np.sin(x)
plt.plot(x,y)
```

Out[7]: `[<matplotlib.lines.Line2D at 0x2567c99a5c0>]`

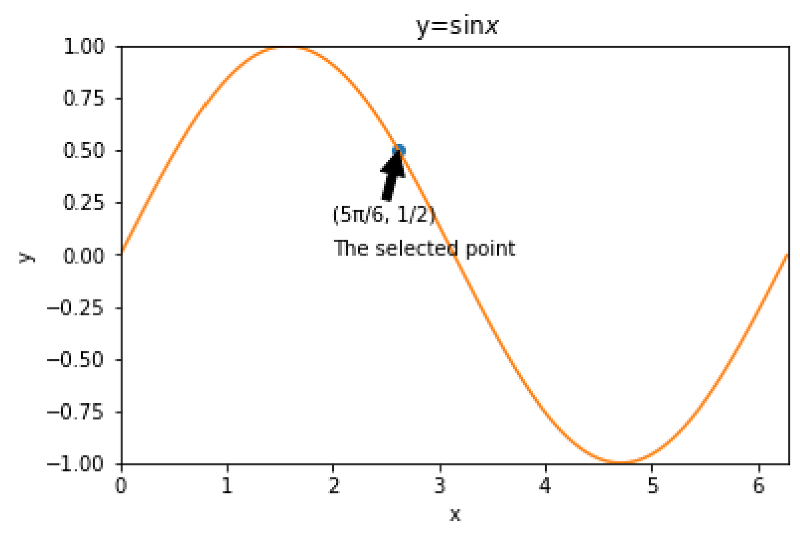


### 4. Select a point within the domain.

→ The selected point is $(\frac{5\pi}{6}, \frac{1}{2})$

### 5. Mark the selected point on the function.

In [12]:
```
fig = plt.figure()

ax = fig.add_subplot(111)
fig.subplots_adjust(top=0.85)
ax.set_title(' y=$\sin x$')

ax.set_xlabel('x')
ax.set_ylabel('y')

ax.text(2, -9/6+3/2, 'The selected point')

ax.plot([5*np.pi/6], [1/2], 'o')
ax.annotate('(5π/6, 1/2)', xy=(5*np.pi/6, 1/2), xytext=(2, -4/3+3/2),
            arrowprops=dict(facecolor='black', shrink=0.005))

ax.axis([0, 2*np.pi, -1, 1])

x=np.arange(0,2*np.pi,0.01)
y=np.sin(x)

plt.plot(x,y)

plt.show()
```




### 6. Define the first-order Taylor approximation at the selected point.

suppose $f: R^n \to R$

the first-order Taylor approximation :

$$\hat{f}(x) = f(x) + \frac{\partial f}{\partial x_1}(z)(x_1 - z_1) + \cdots + \frac{\partial f}{\partial x_n}(z)(x_n - z_n)$$

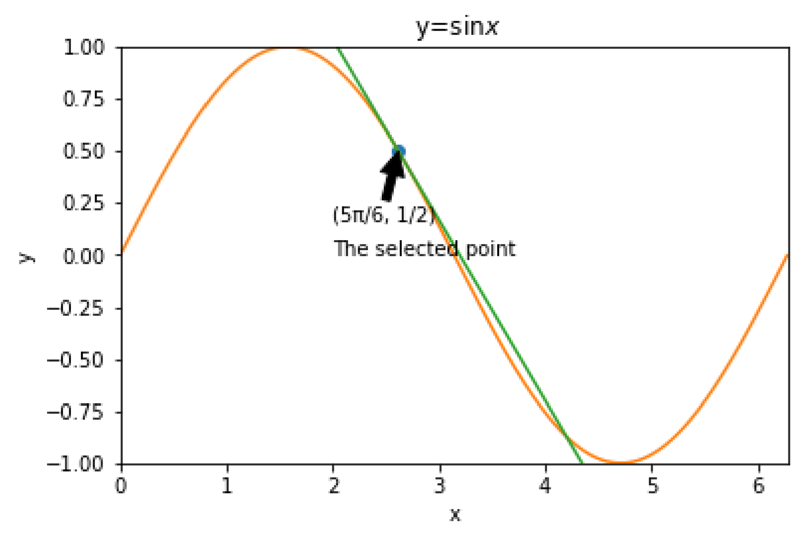
n is 1 because $f(x) = sinx$

therefore $\hat{f}(x) = f(\frac{5\pi}{6}) + \frac{\partial f}{\partial x}(\frac{5\pi}{6})(x - \frac{5\pi}{6})$

$$\therefore \hat{f}(x) = \frac{1}{2} - \frac{\sqrt{3}}{2}(x - \frac{5\pi}{6})$$

### 7. Plot the Taylor approximation with the same domain of the original function.

In [13]:
```
fig = plt.figure()

ax = fig.add_subplot(111)
fig.subplots_adjust(top=0.85)
ax.set_title(' y=$\sin x$')

ax.set_xlabel('x')
ax.set_ylabel('y')

ax.text(2, -9/6+3/2, 'The selected point')

ax.plot([5*np.pi/6], [1/2], 'o')
ax.annotate('(5π/6, 1/2)', xy=(5*np.pi/6, 1/2), xytext=(2, -4/3+3/2),
            arrowprops=dict(facecolor='black', shrink=0.005))

ax.axis([0, 2*np.pi, -1, 1])

x1=np.arange(0,2*np.pi,0.01)
y1=np.sin(x1)

plt.plot(x1,y1)

x2=np.arange(0,2*np.pi,0.01)
y2=1/2-((3**(0.5))/2)*(x2- 5*np.pi/6)

plt.plot(x2,y2)

plt.show()
```




In [ ]:

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