KTH ROYAL INSTITUTE
OF TECHNOLOGY

# Resonant magnetic perturbation effect on the tearing mode dynamics

Richard Fridström

2017-12-13

# Outline

- **The Reversed-field pinch and tearing modes**
- **Resonant magnetic perturbation and tearing modes**
  - **Papers I-II** (III, V)
- **Viscosity in stochastic magnetic fields**
  - **Papers VI-VII** (IV)

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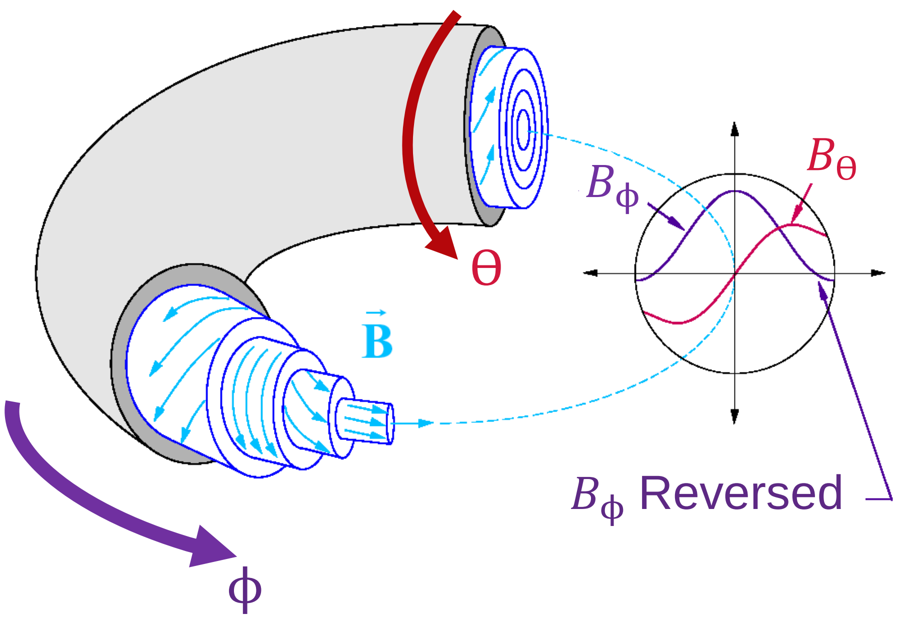
# The reversed field pinch (RFP)

- In a tokamak, $B_\phi > B_\theta$
- RFP similar to a tokamak, but $B_\phi \sim B_\theta$

RFP equilibrium magnetic field




EXTRAP T2R, KTH Stockholm

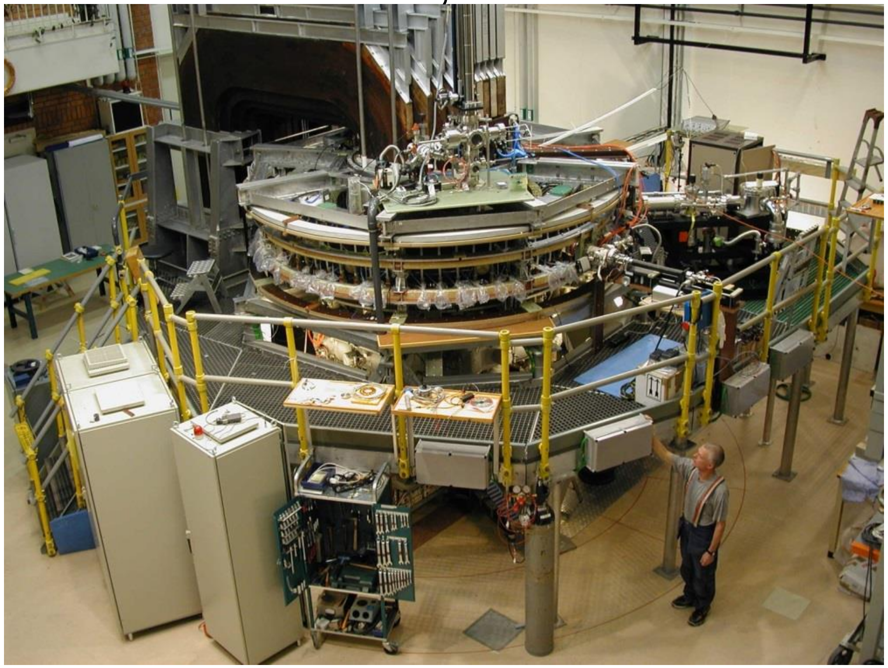


*Figure credit: [MST group, UW-Madison ]*

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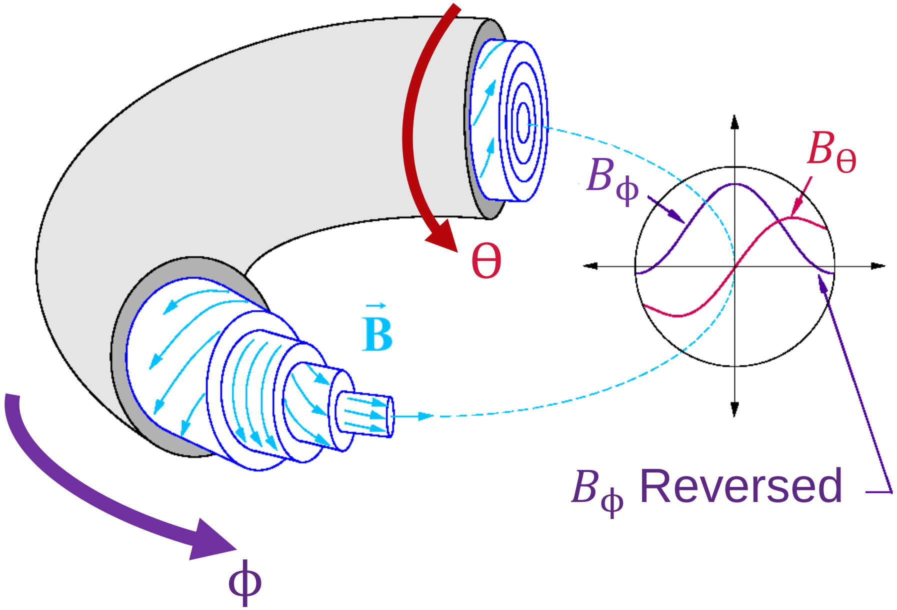
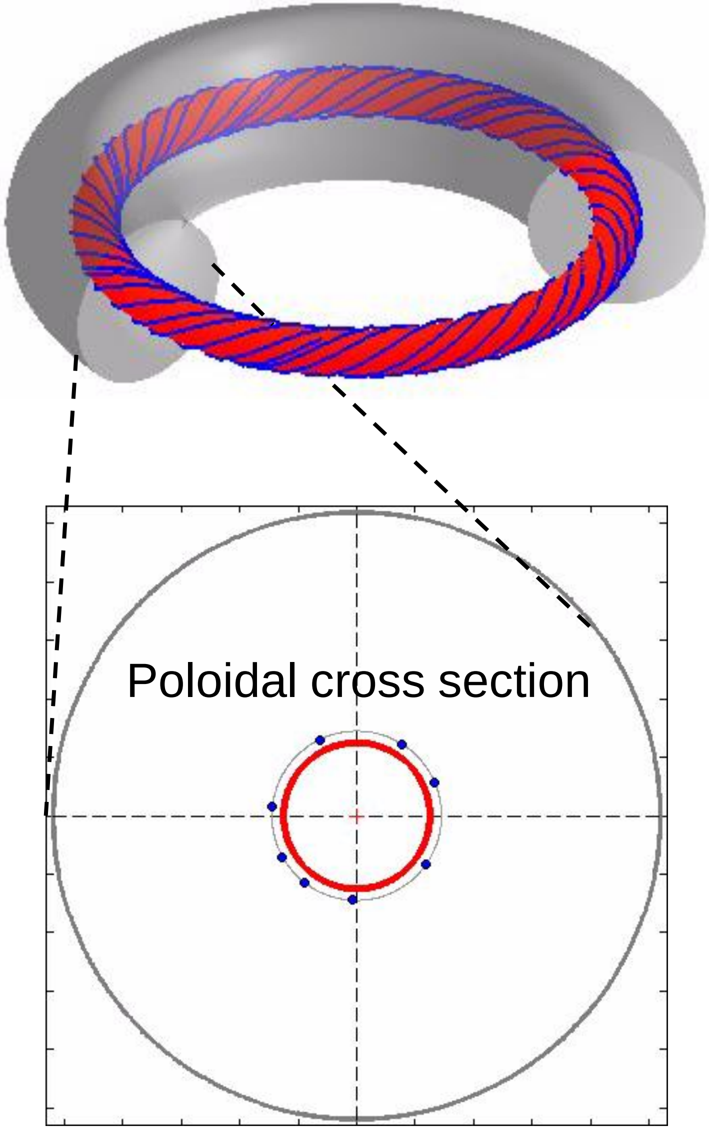


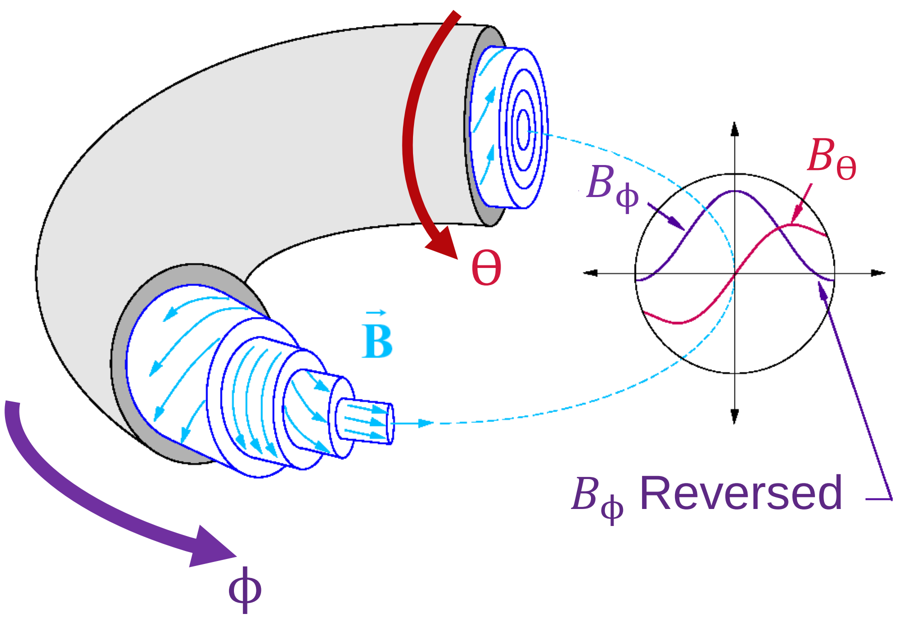

*Figure credit: [MST group, UW-Madison ]*

RFP equilibrium



- Driven unstable by current gradient
- Tearing resonances: $q(r) = \dfrac{rB_\phi}{RB_\theta} = \dfrac{m}{n}$ ← poloidal mode number ← toroidal mode number

RFP equilibrium magnetic field




Safety factor

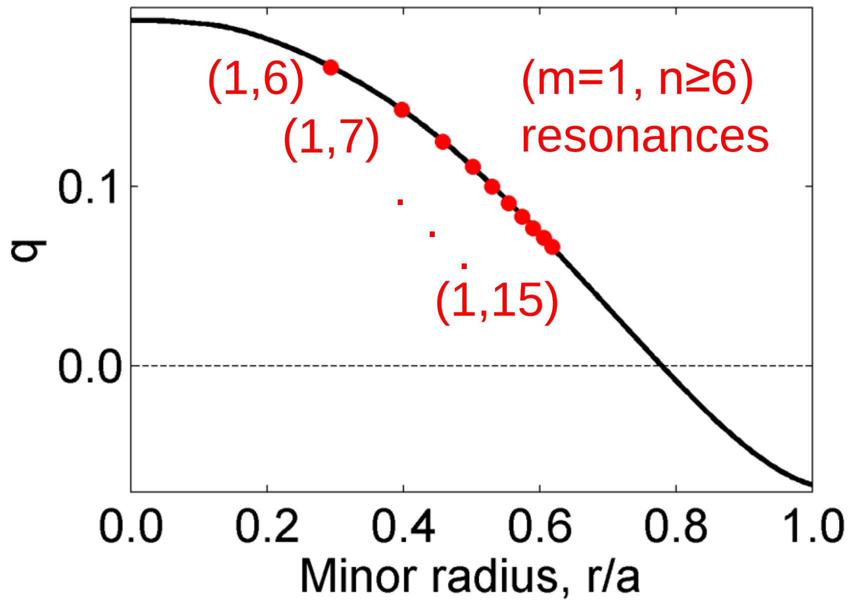



*Figure credit: [MST group, UW-Madison ]*

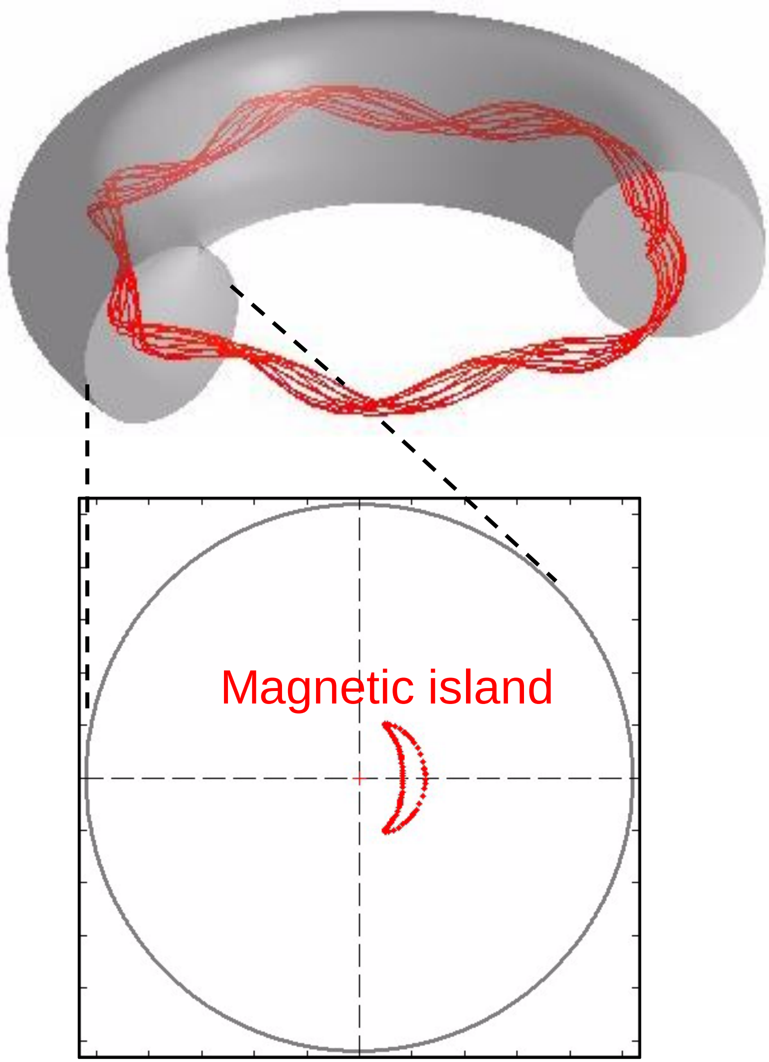
# Magnetic field line - Island

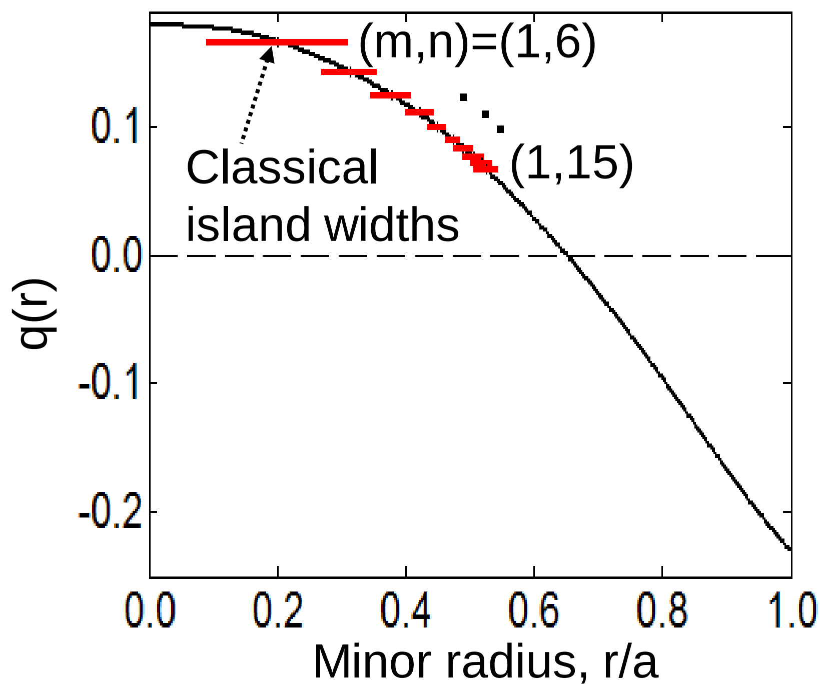
$$\bar{B} = B_\theta \hat{e}_\theta + B_\phi \hat{e}_\phi + \overline{\delta b}$$

Perturbation to equilibrium

$$\overline{\delta b} = b_r \hat{e}_r + b_\theta \hat{e}_\theta + b_\phi \hat{e}_\phi$$

Classical island width: $W_{mn} = 4\sqrt{\frac{r b_{r,mn}}{n B_\theta |q'|}}$



# Magnetic field line - Stochastic

$$\bar{B} = B_\theta \hat{e}_\theta + B_\phi \hat{e}_\phi + \overline{\delta b}$$

Perturbation to equilibrium

$$\overline{\delta b} = b_r \hat{e}_r + b_\theta \hat{e}_\theta + b_\phi \hat{e}_\phi$$

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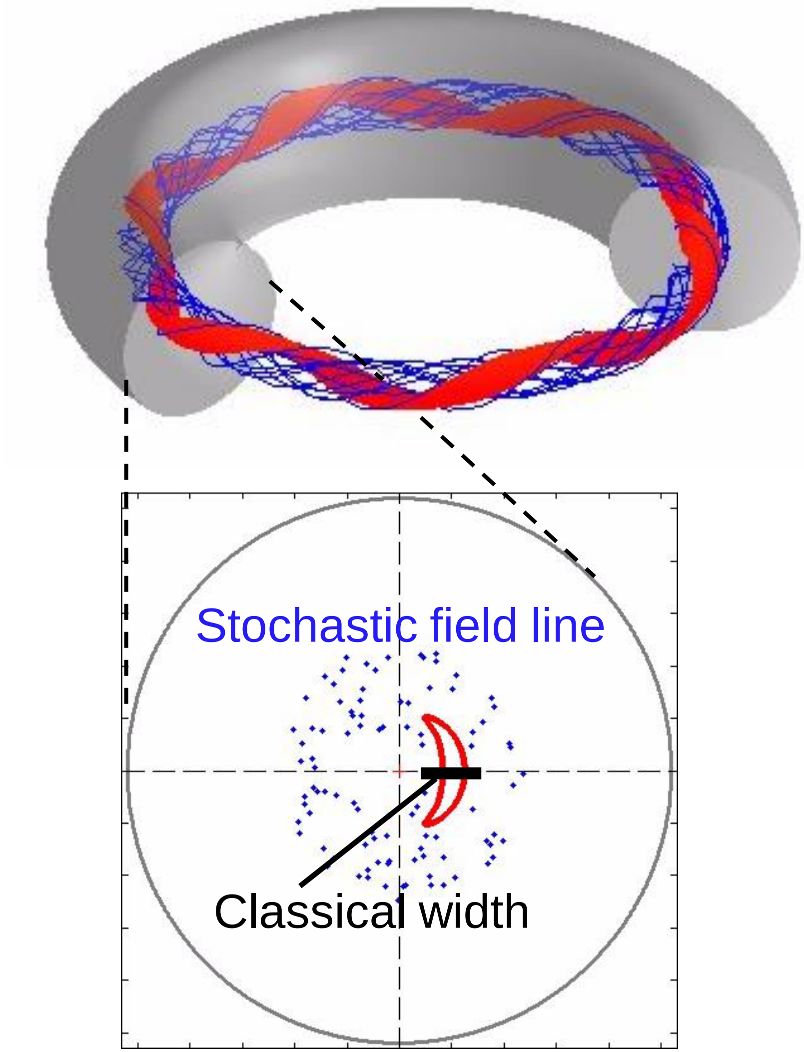
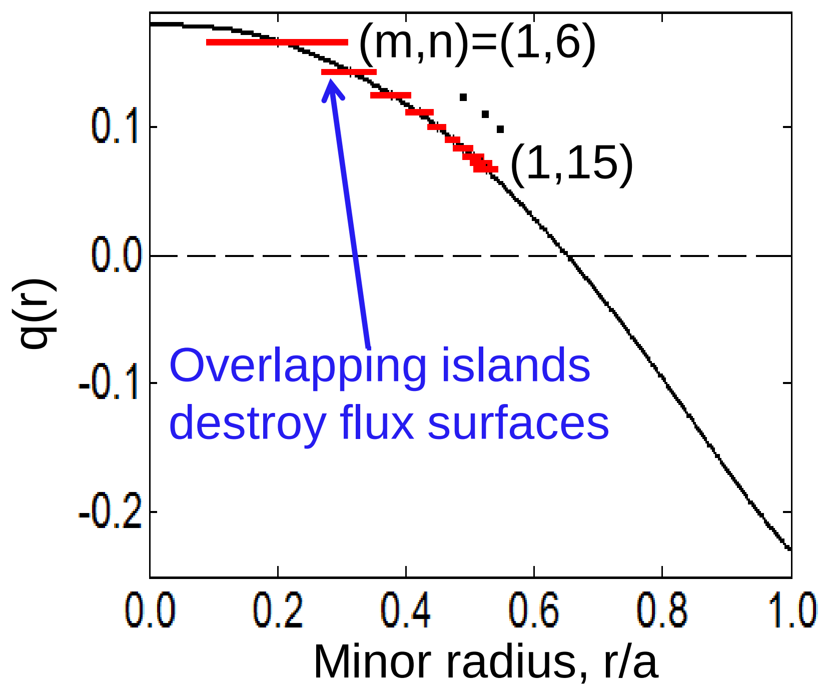
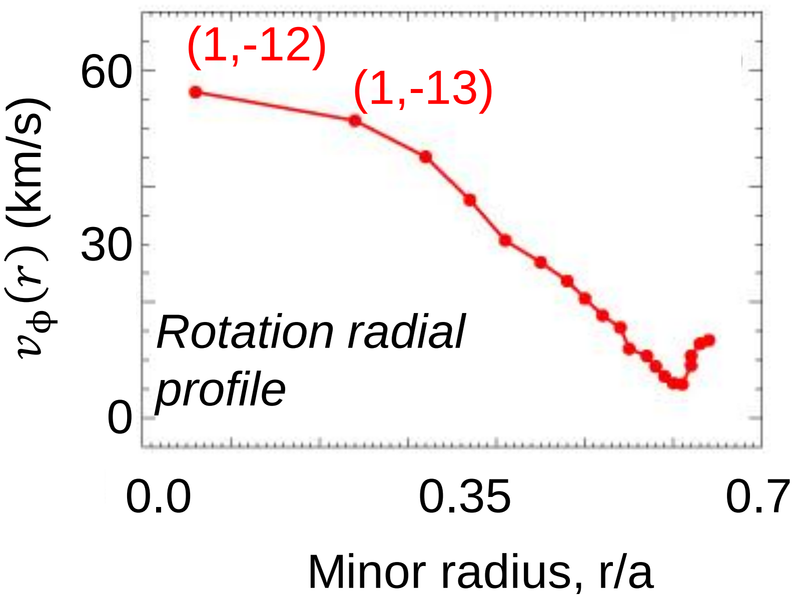


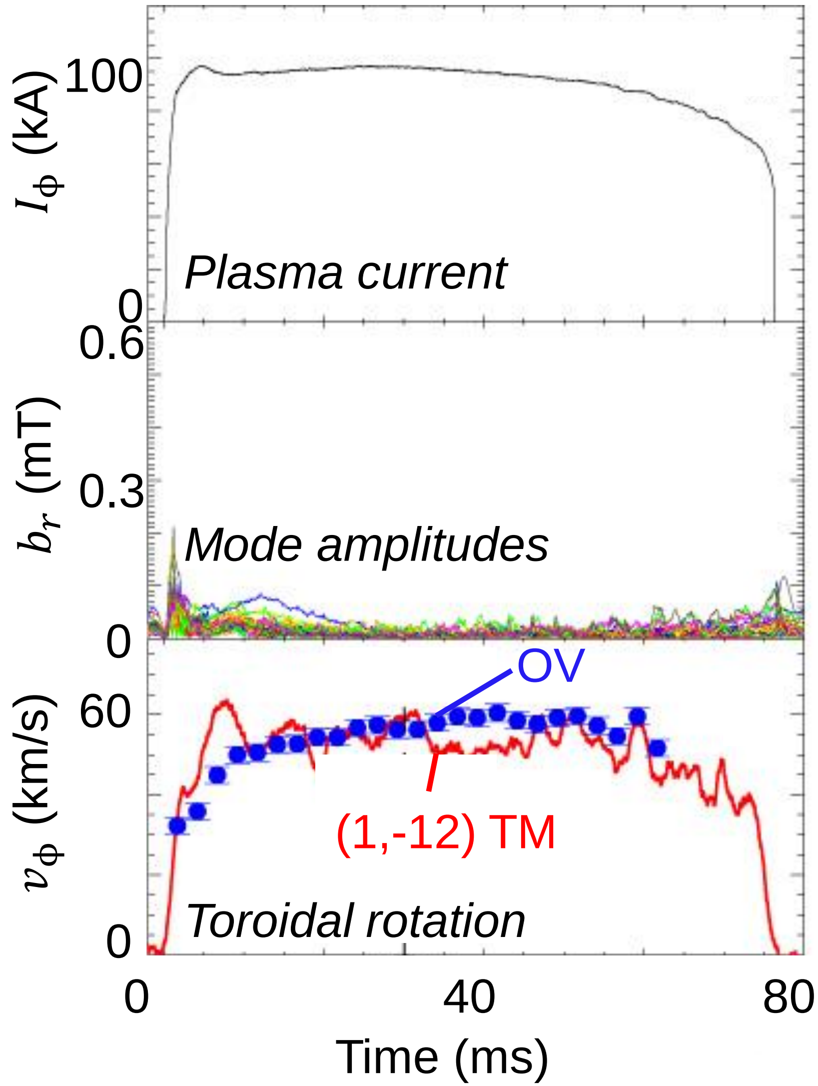
# Tearing mode and plasma rotation

Tearing modes (TMs) in studied RFPs (EXTRAP T2R and MST):

- TMs and plasma almost co-rotate
- TM radial amplitude at wall small due to rotation




EXTRAP T2R discharge [Frassinetti 2012]



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- The RFP and tearing modes
- **Resonant magnetic perturbation and tearing modes**
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# Motivation – Study of RMP effect on TMs

- **Resonant magnetic perturbations (RMPs) useful tool for future machines**
- **Study RMP-Tearing Mode interaction to:**
  - Optimize use of RMP
  - Avoid negative effects of resonant field errors
  - Validate theories
  - Estimate kinematic viscosity
- **Experimental study in RFPs**
  - Naturally occurring TMs
  - Equipped with RMP coils:
    - single-harmonic RMP: Papers I-II (EXTRAP T2R)
    - multi-harmonic RMP: Paper III (Madison symmetric torus)

      Paper IV (EXTRAP T2R

# EXTRAP T2R - OUTLINE

- **Machine description**

  - EXTRAP T2R
  - Feedback system and RMPs

- **Model**

- **TM locking and unlocking to an RMP**
  - RMP effect on TM rotation
  - Comparison of experimental data with models
  - Hysteresis in TM locking-unlocking

- **Summary EXTRAP T2R results**

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# EXTRAP T2R

## THE EXTRAP T2R RFP

R=1.24m

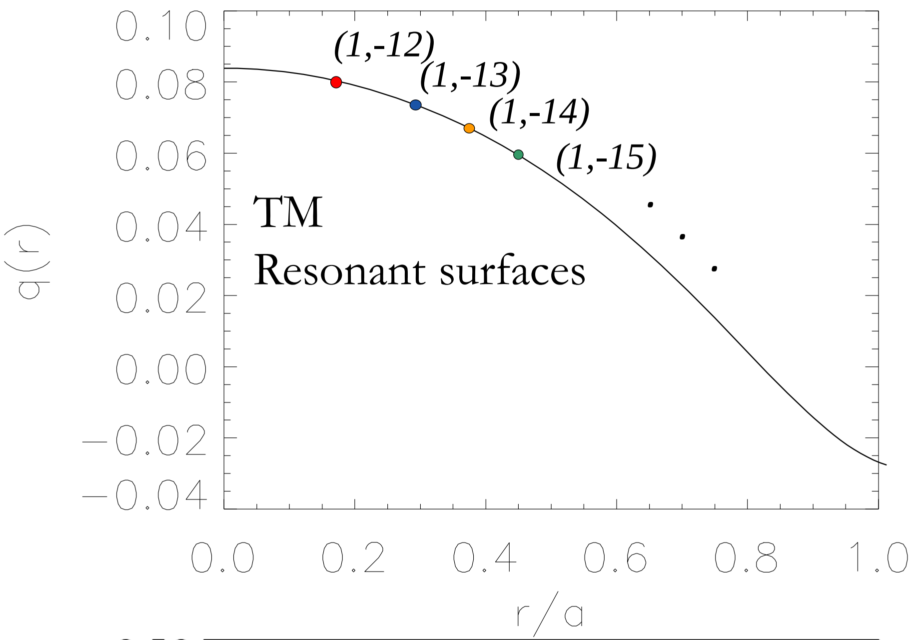
a=0.18m

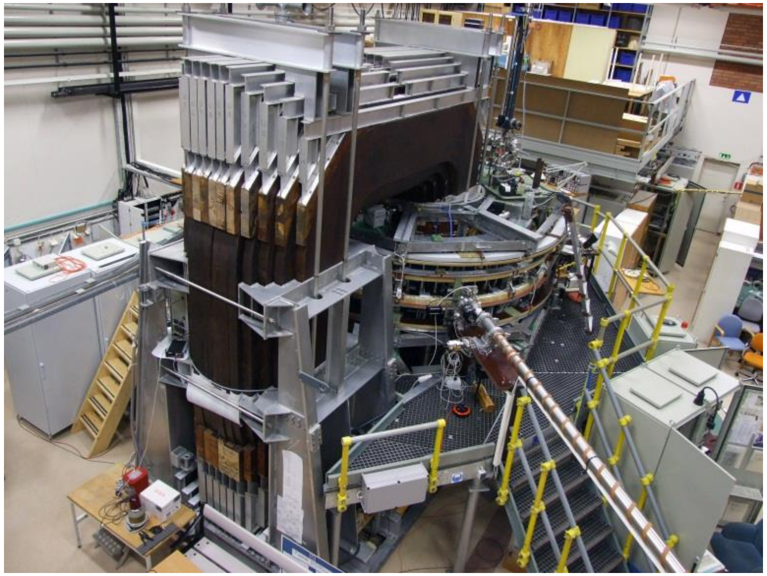
$I_p \approx$ 50-150kA

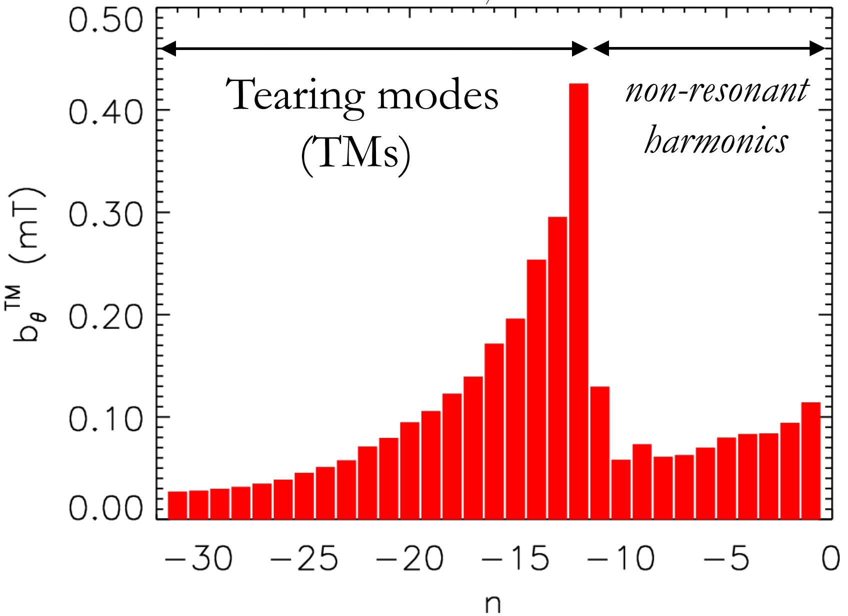
$n_e \approx 10^{19} m^{-3}$

$T_e \approx$ 300-600eV

$t_{pulse} \approx$ 90ms

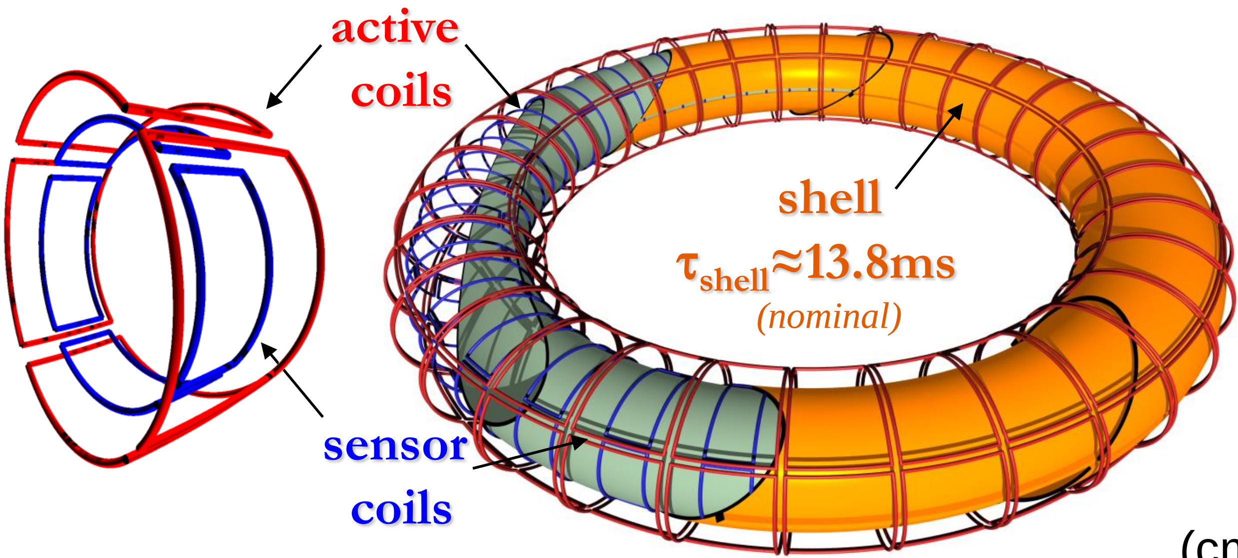


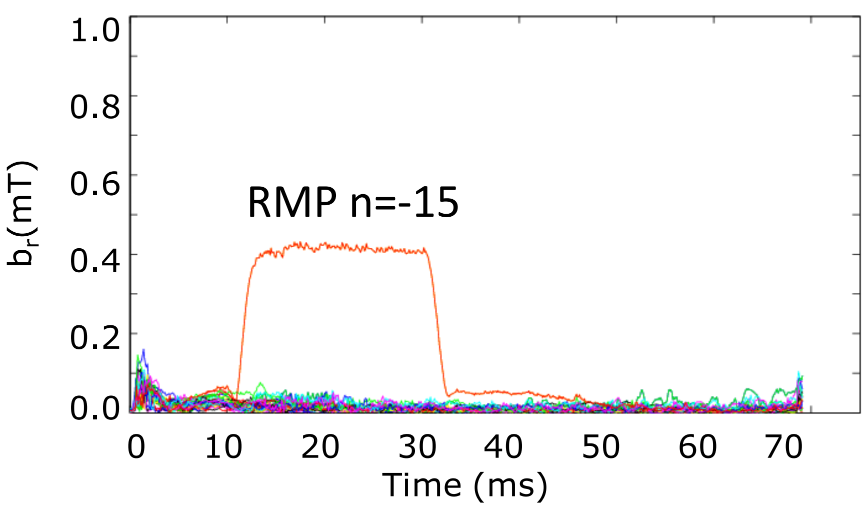



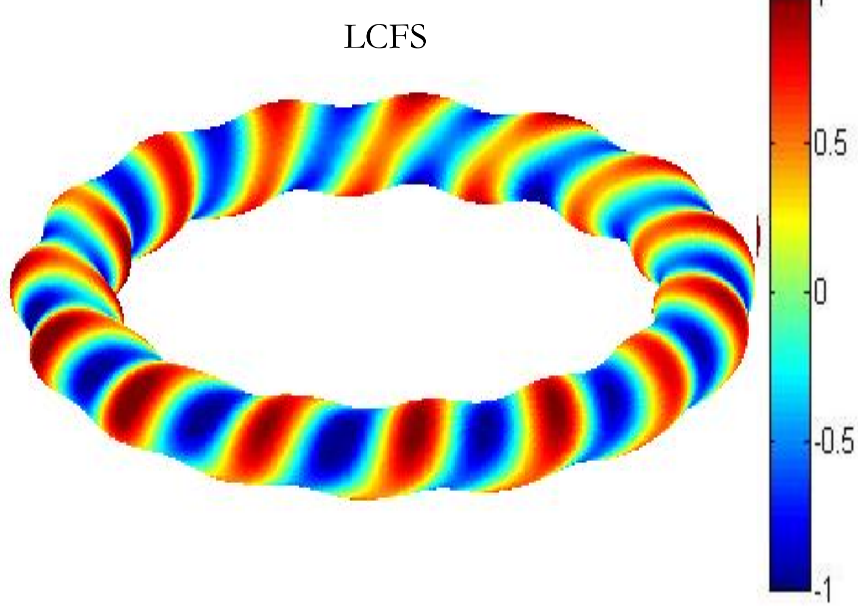


# The magnetic feedback system

- **Sensor coils**
  4 poloidal x 32 toroidal
  located inside the shell
- Digital controller
- **Active coils**
  4 poloidal x 32 toroidal
  located outside the shell




- RMP applied using the RIS algorithm [Olofsson PPCF 2010]

# EXTRAP T2R - OUTLINE

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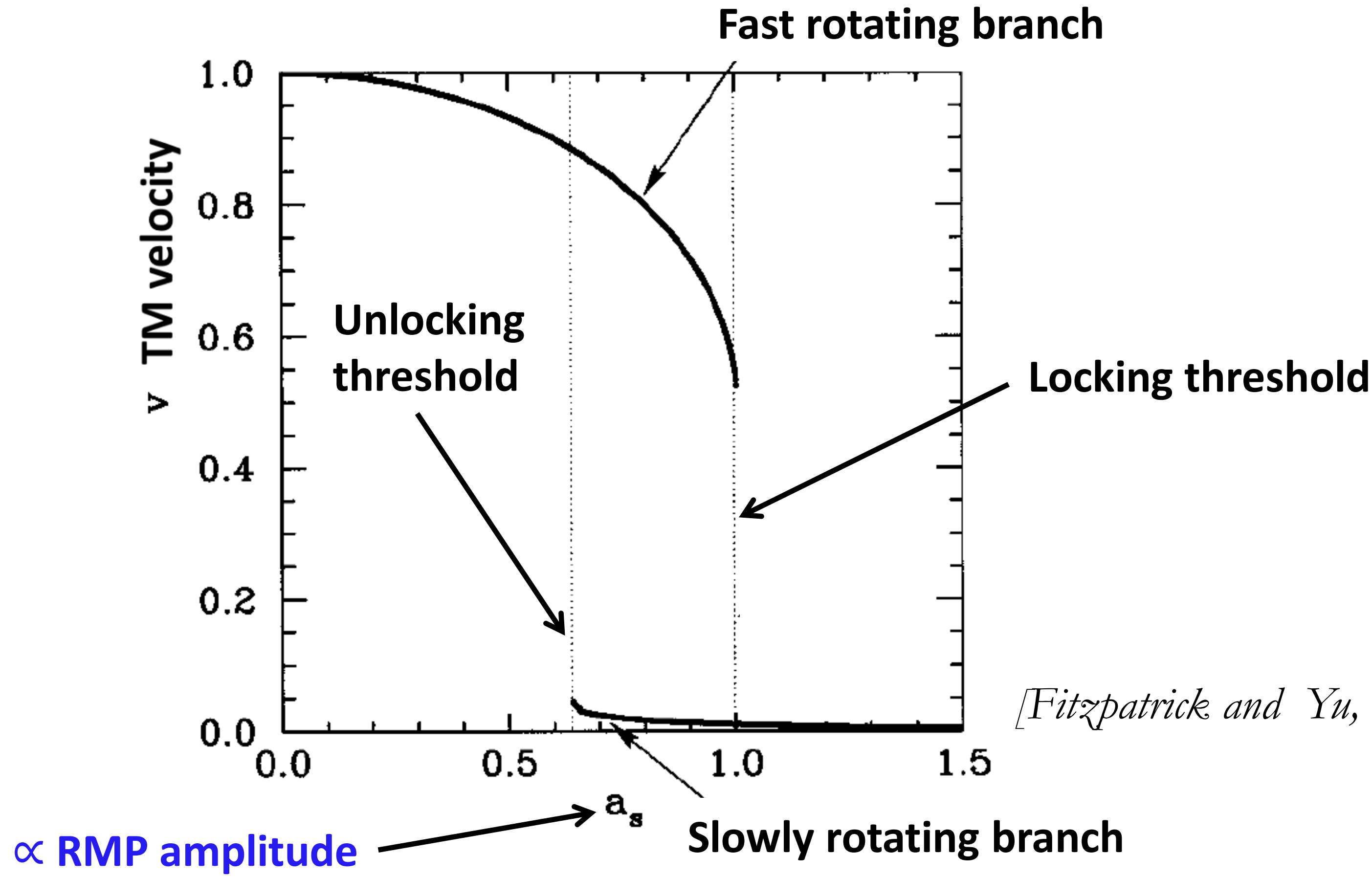
# Earlier results on TM locking-unlocking to RMP

- The TM locking extensively studied in experiments.
- **The unlocking process had been studied in fewer experiments.**
  - COMPASS, JET and EXTRAP T2R
- Theory of locking and unlocking by several authors.




*[Fitzpatrick and Yu, PoP 2002]*

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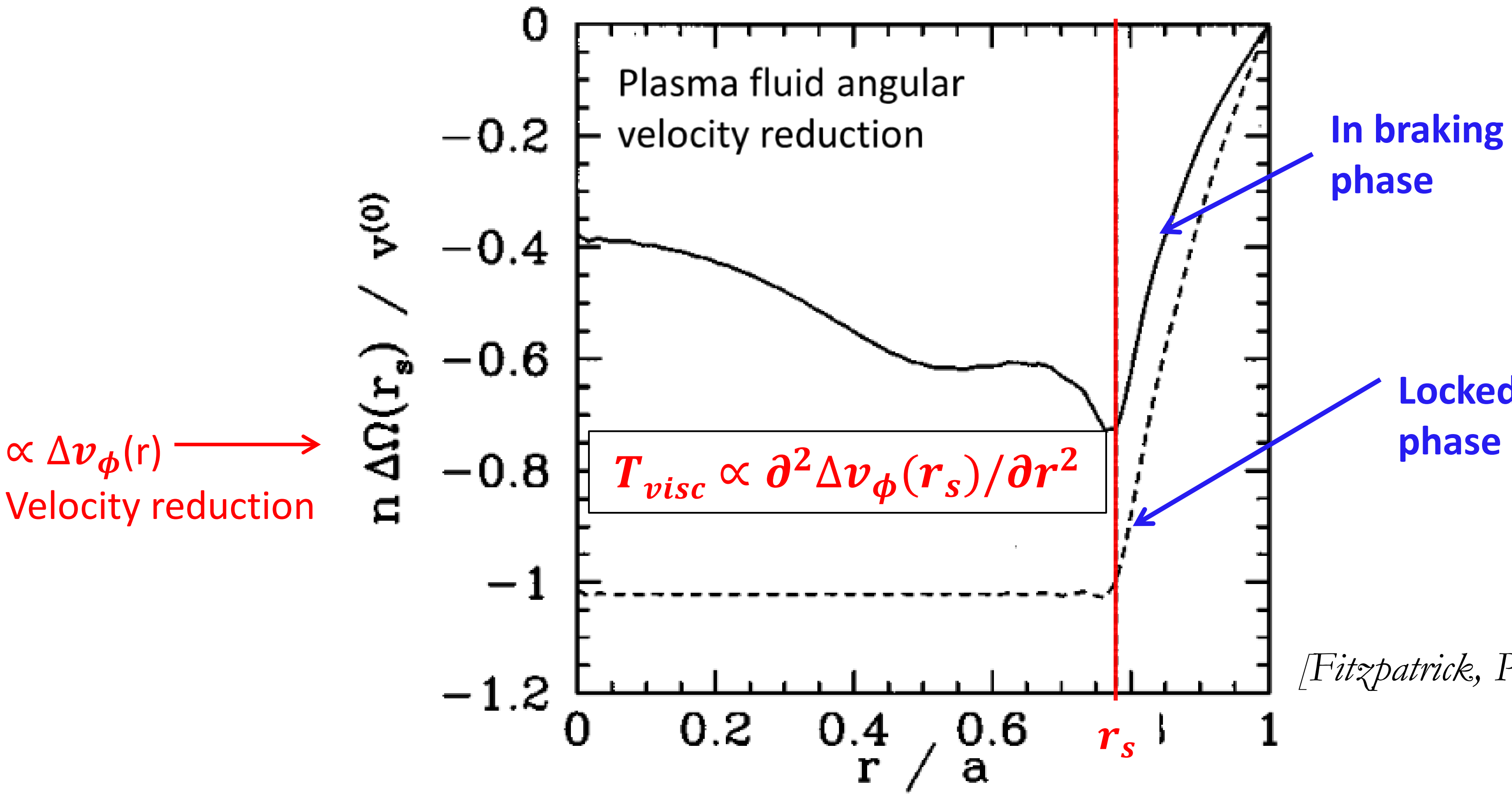


[Fitzpatrick, PoP 2001]

# Modeling the Tearing Mode Dynamics (single TM)

*[R. Fitzpatrick Nucl. Fusion (1993)]*

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**Equation of fluid motion:**

$$\rho \frac{\partial \Delta\Omega}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\nu_{\perp}\frac{\partial \Delta\Omega}{\partial r}\right) + \frac{T_{EM}(t)}{4\pi^2 r R_0^3}\delta(r - r_s)$$

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EM torque: $T_{EM} = T_{wall} + T_{RMP}$

$$T_{RMP} = k|b_{TM}||b_{RMP}|sin(\Delta\alpha(t)) \quad , \; T_{wall} = b_{TM}^2 f(\omega)$$

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**No-slip condition:** $\frac{\mathrm{d}\alpha}{\mathrm{d}t}=n[\Omega(r_s,t_0)+\Delta\Omega(r_s,t)]$

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**Modifield Rutherford equation:**

$$\frac{\mathrm{d}|\Psi_s(t)|}{\mathrm{d}t} = \frac{G}{\Lambda\tau_r}\Bigg(\Big[\underbrace{E_{ss}}_{\substack{\text{Stability}\\ \text{index}}} + \underbrace{D(t)\Big]\sqrt{|\Psi_s|}}_{\substack{\text{Interaction}\\ \text{with wall}}} + \underbrace{\frac{|\Psi_c|}{\sqrt{|\Psi_s|}}E_{sc}\cos\left[\Delta\alpha(t)\right]}_{\text{Interaction with RMP}} - \underbrace{\frac{\Lambda}{G}(\lambda_0)^2\ln\left[\frac{G}{\sqrt{|\Psi_s|}}\right]}_{\substack{\text{Saturation}\\ \text{term}}}\Bigg)$$

where $\Psi_s \propto b_{TM}$ and $\Psi_c \propto b_{RMP}$

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# Modeling the Tearing Mode Dynamics (single TM)

**Equation of fluid motion:**

Free parameter (kinematic viscosity)

$$\underbrace{\rho\frac{\partial\Delta\Omega}{\partial t}}_{\text{Inertia}} = \underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\nu_\perp\frac{\partial\Delta\Omega}{\partial r}\right)}_{\text{Viscous torque}} + \underbrace{\frac{T_{EM}(t)}{4\pi^2 r R_0^3}\delta(r-r_s)}_{\text{Electromagnetic torque}}$$

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# MHD model of TM interaction with RMP

- Model adapted to EXTRAP T2R:

- Equilibrium profiles
- Machine dimensions

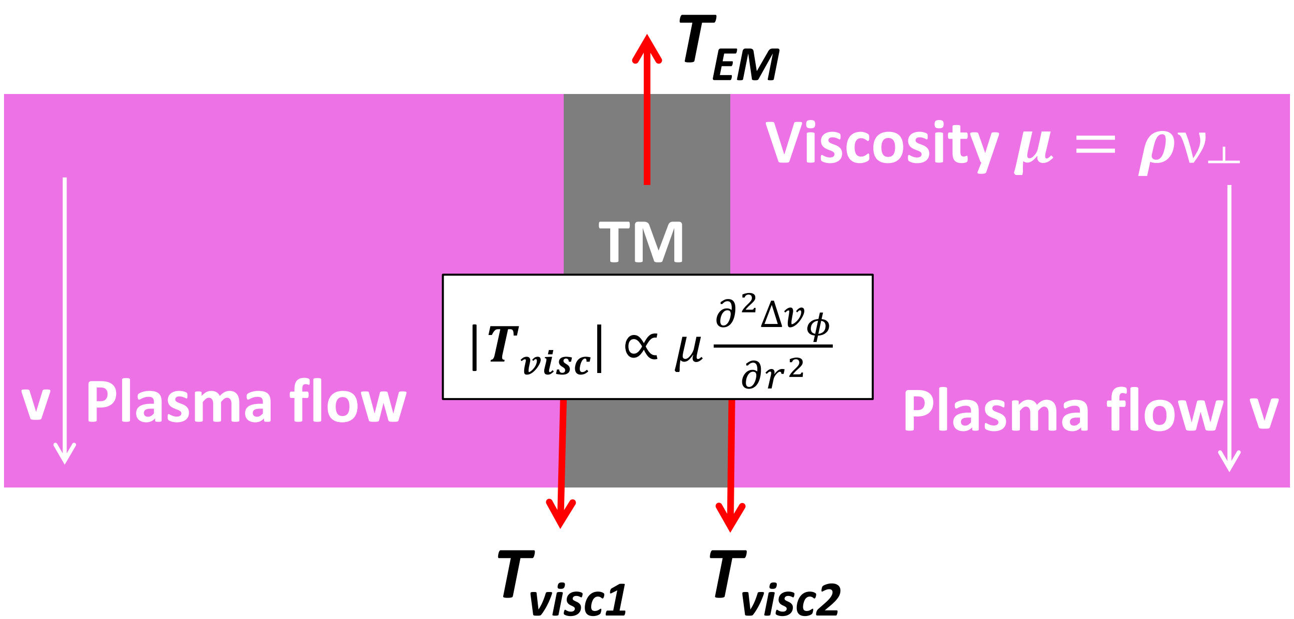
- Newcomb equation
  output: TM eigenfunction

Three coupled equations:

- **Equation of fluid motion (PDE)**
  Torque balance including electromagnetic torques
- **“No-slip condition” (ODE)**
  Co-rotation of TM and fluid at resonant surface.
- **Modified Rutherford equation (ODE)**
  TM amplitude evolution

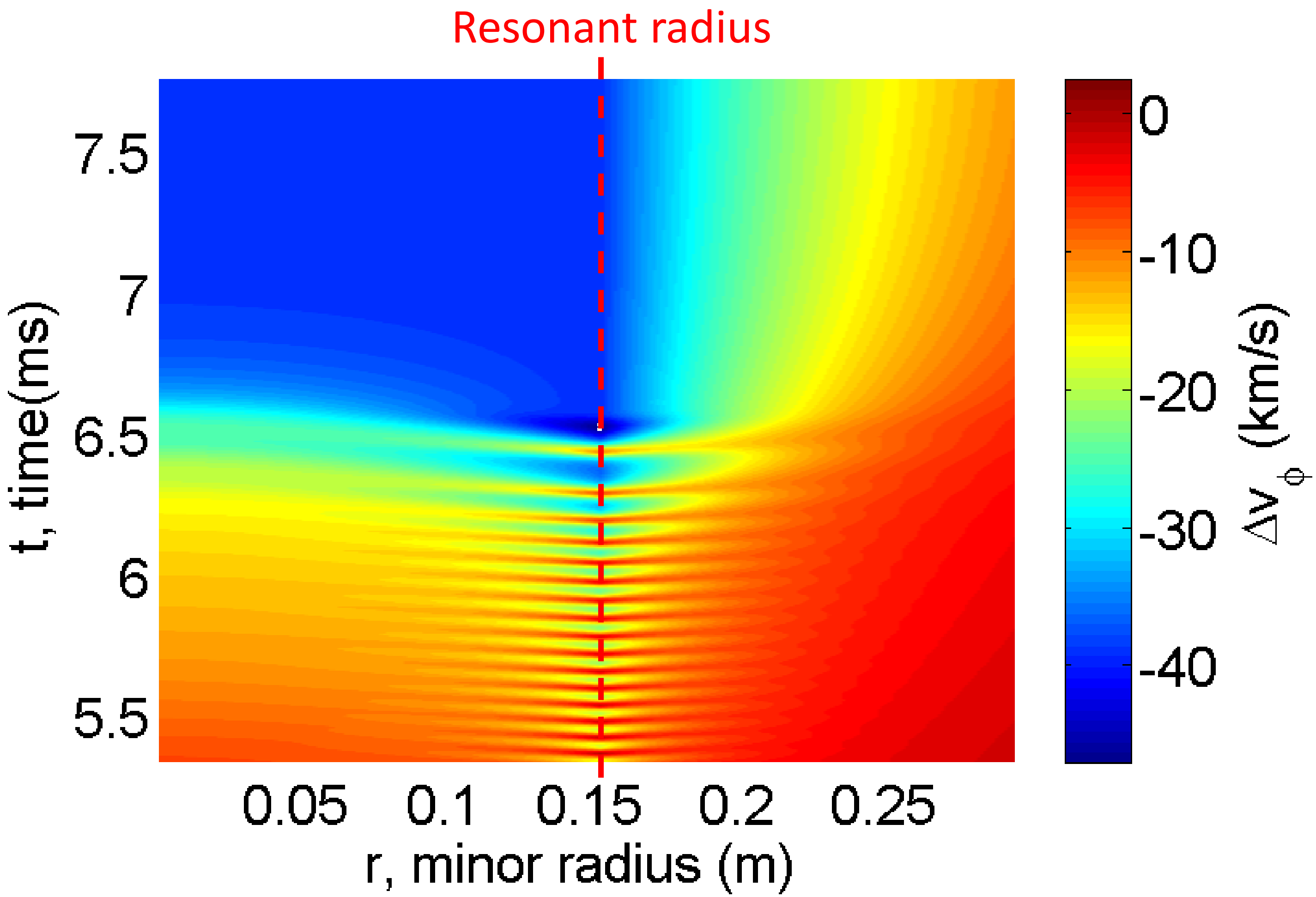
# Electromagnetic (EM) and Viscous torque balance

- $\boldsymbol{T_{EM}}$ → Differential rotation between tearing mode and surrounding plasma
- → Viscous torque from surrounding plasma fluid
  - Tends to equate TM and plasma rotation
    (i.e. restore rotation at the TM resonant surface):
    $\boldsymbol{T_{visc} = T_{visc1} + T_{visc2} \approx -T_{EM}}$



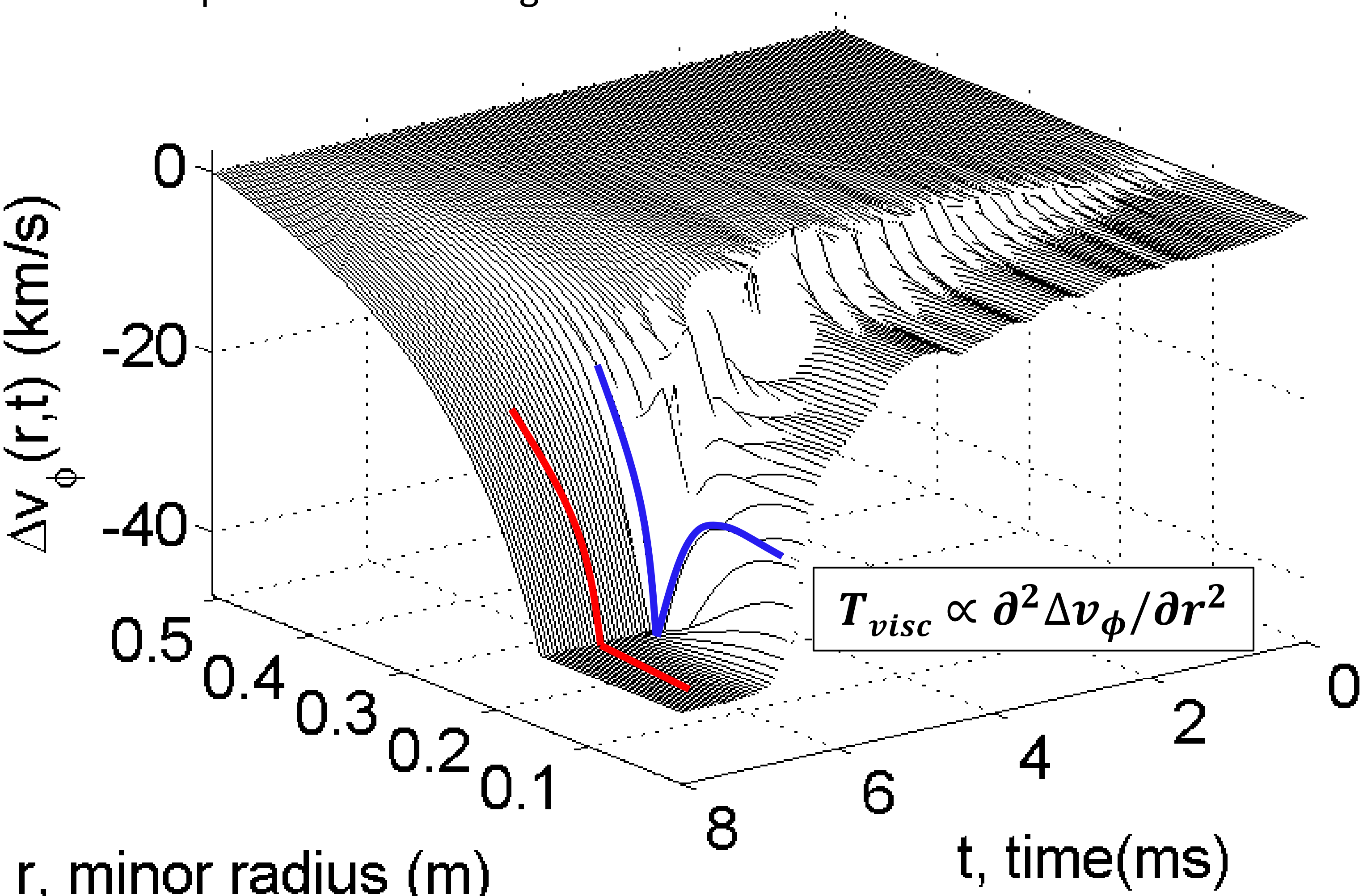
# Modeled plasma flow change

- Modelled evolution of plasma velocity reduction during RMP



# Modeled plasma flow change

- Modelled evolution of plasma velocity reduction during RMP
- Phases of acceleration/deceleration
- Relaxed profile after locking



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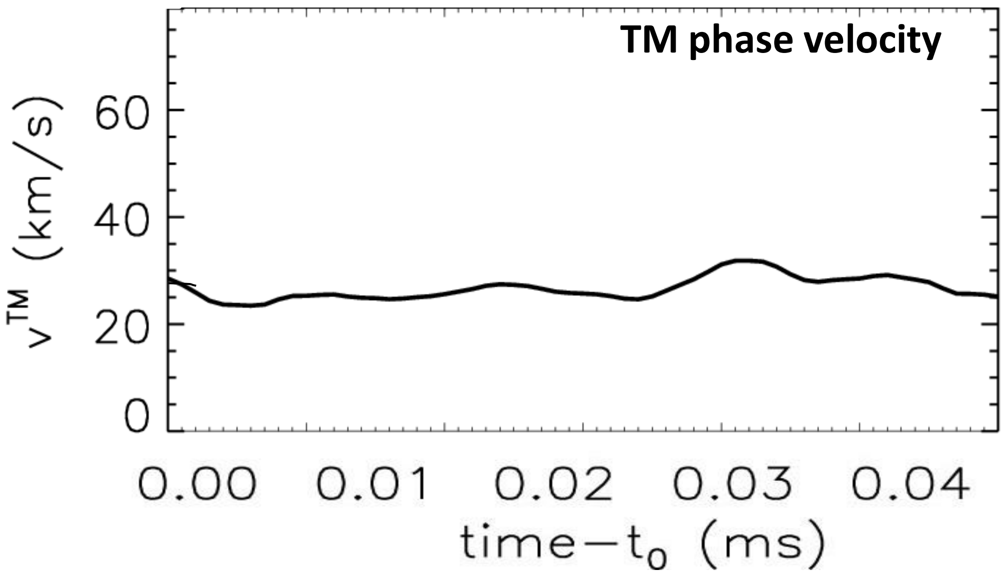
# Tearing mode – RMP interaction

**The RMP cause modulation of the TM velocity.**

- EM Torque : $(\boldsymbol{R} \times \boldsymbol{j} \times \boldsymbol{b_{TM}}) \propto |b_{RMP}||b_{TM}| \sin(\Delta\alpha(t))$




**Applied RMP amplitude:**

$b_{RMP}$ = 0 G ——

*Paper I: [L. Frassinetti, R. Fridström et al PPCF (2014) ]*

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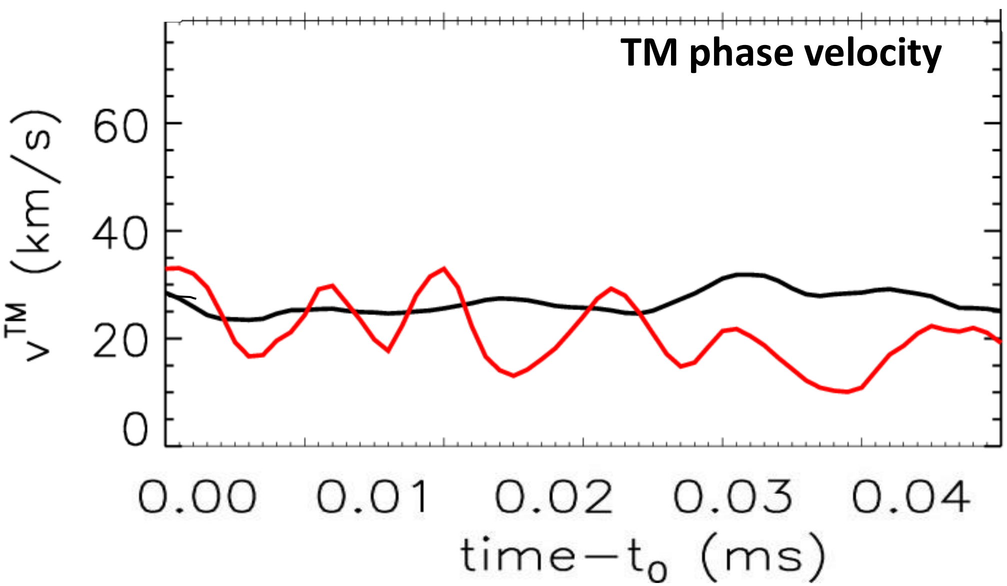


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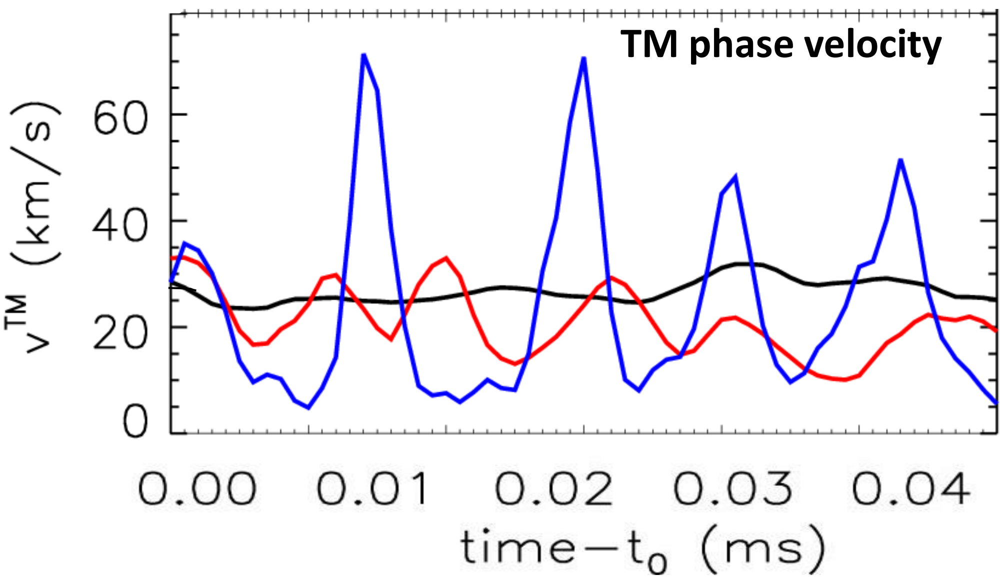


**Applied RMP amplitude:**

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$b_{RMP}$ = 7 G

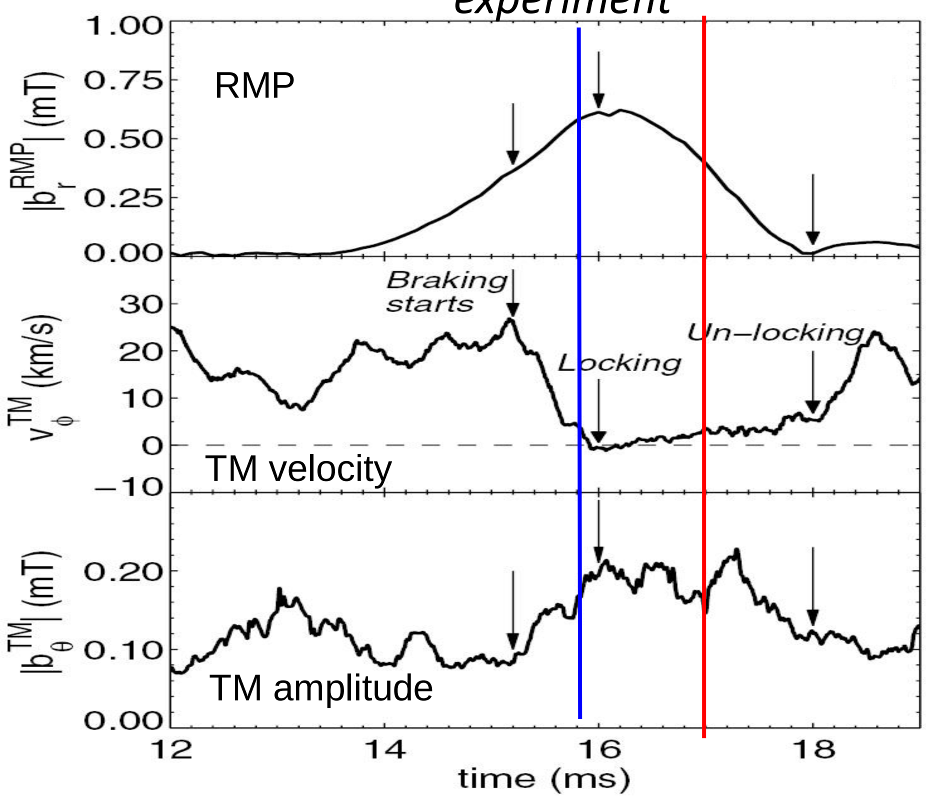
*Paper I: [L. Frassinetti, R. Fridström  et al PPCF (2014) ]*

# Comparison experiment – model: time evolution

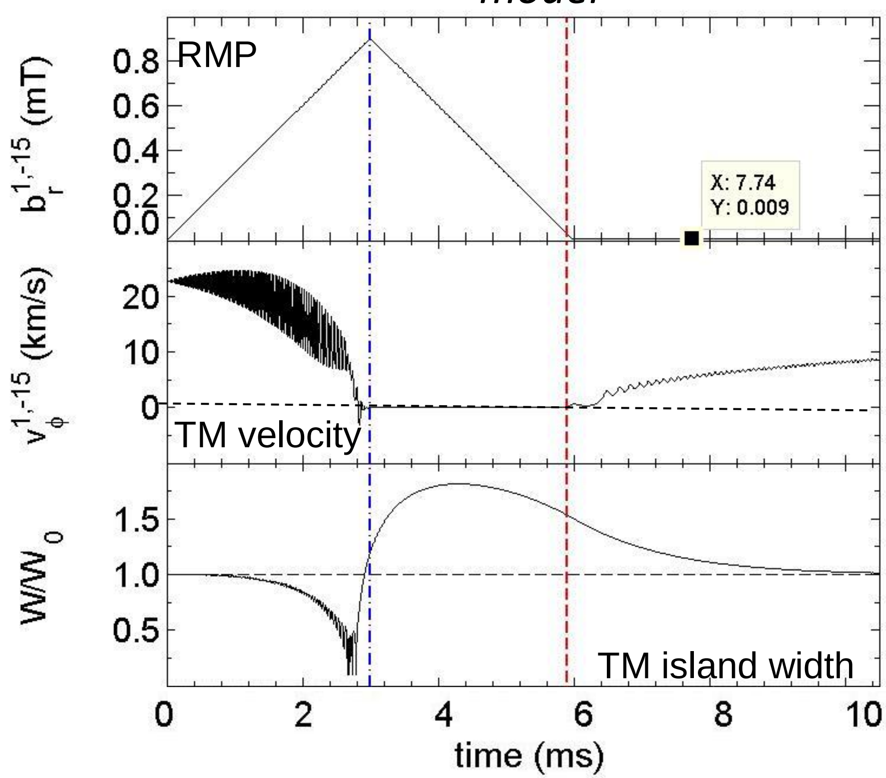
- The model described in [Fitzpatrick et al., PoP (2001)] has been adapted to EXTRAP T2R
- The viscosity is used as a free parameters in order to match the locking threshold
- Reasonable qualitative agreement

$$\begin{cases} \frac{\Lambda_1}{2}\tau_R \frac{d(W/r_s)}{dt} = E_1 - \lambda_s^2 \Lambda_1 \left(\frac{W}{4r_s}\right)\ln\left(\frac{4r_s}{W}\right) + \left(\frac{W_c}{W}\right)^2 \cos(\Delta\alpha(t)) \\ \rho\frac{\partial \Delta\Omega}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial\Delta\Omega}{\partial r}\right) + \frac{T_{EM}}{4\pi^2 R_0^3}\delta(r - r_s) \\ \frac{d\Delta\alpha}{dt} = \Delta\Omega + \omega_0 \end{cases}$$

*experiment*




*model*



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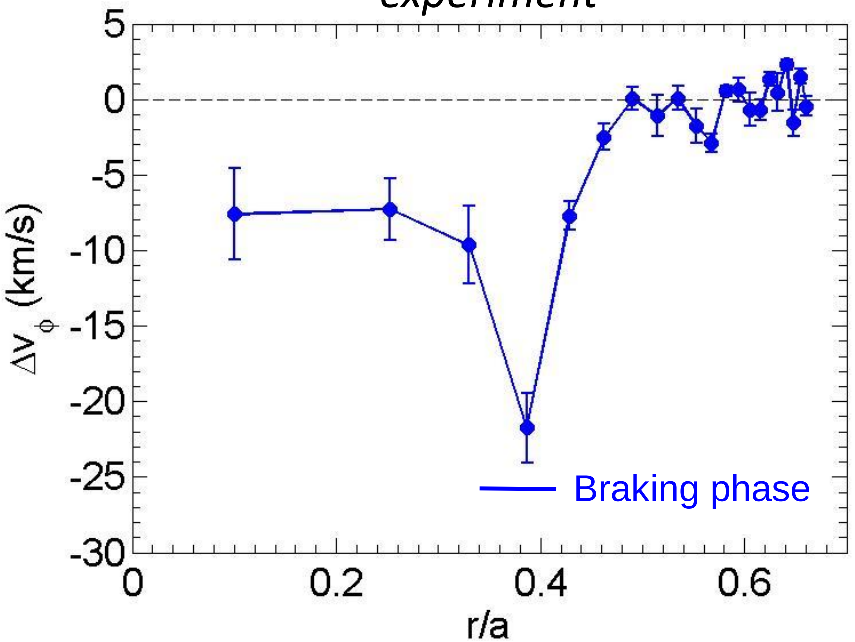


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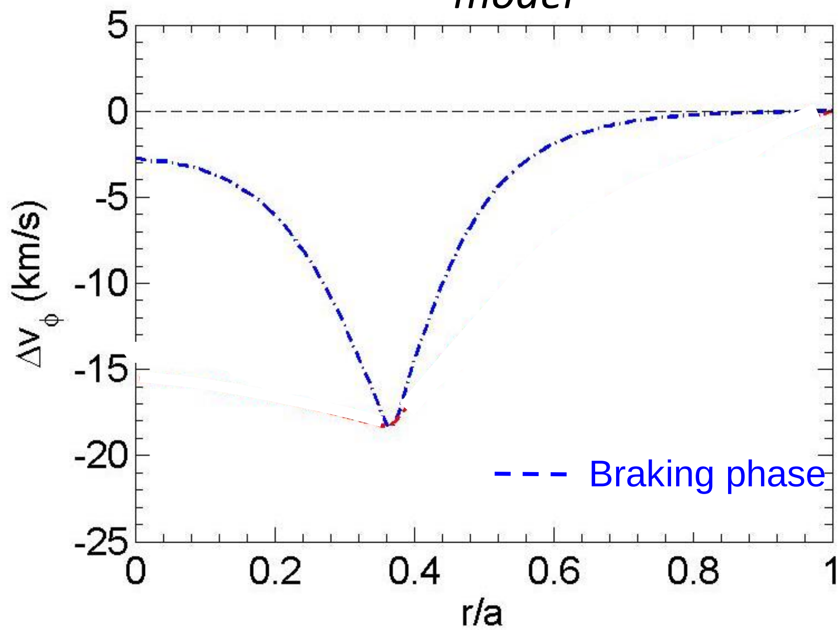

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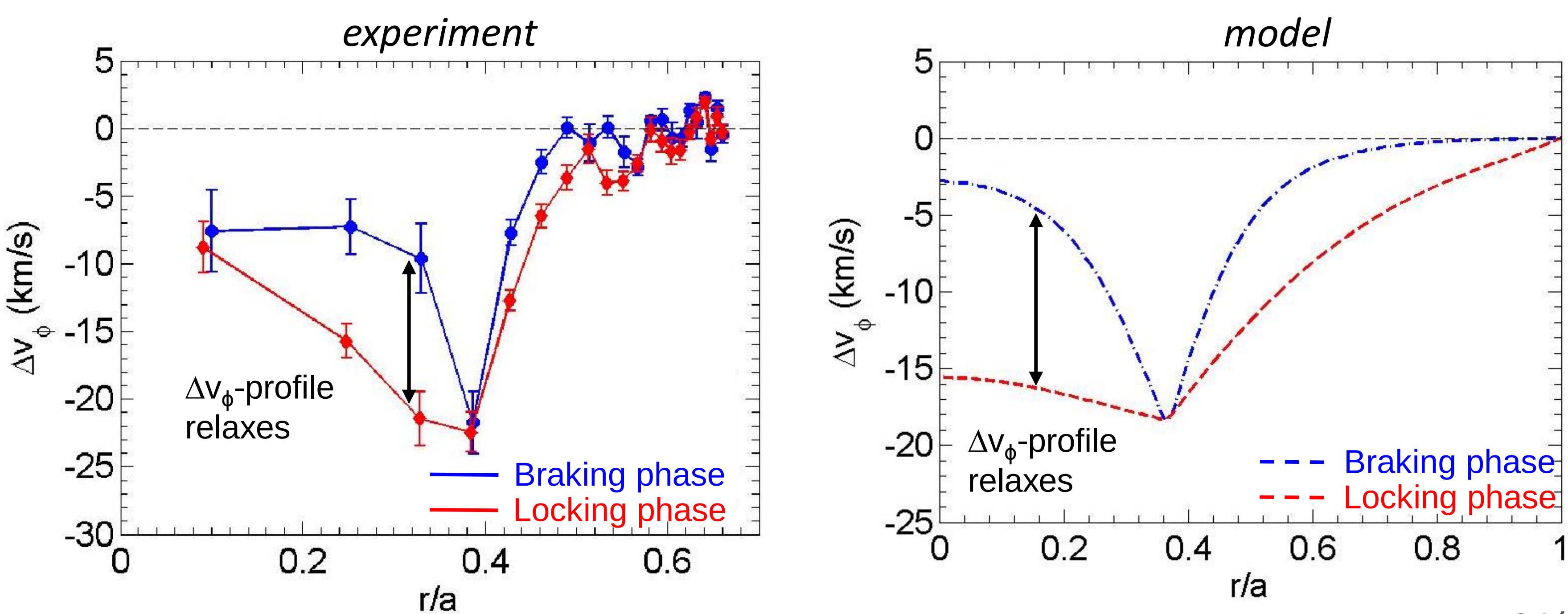
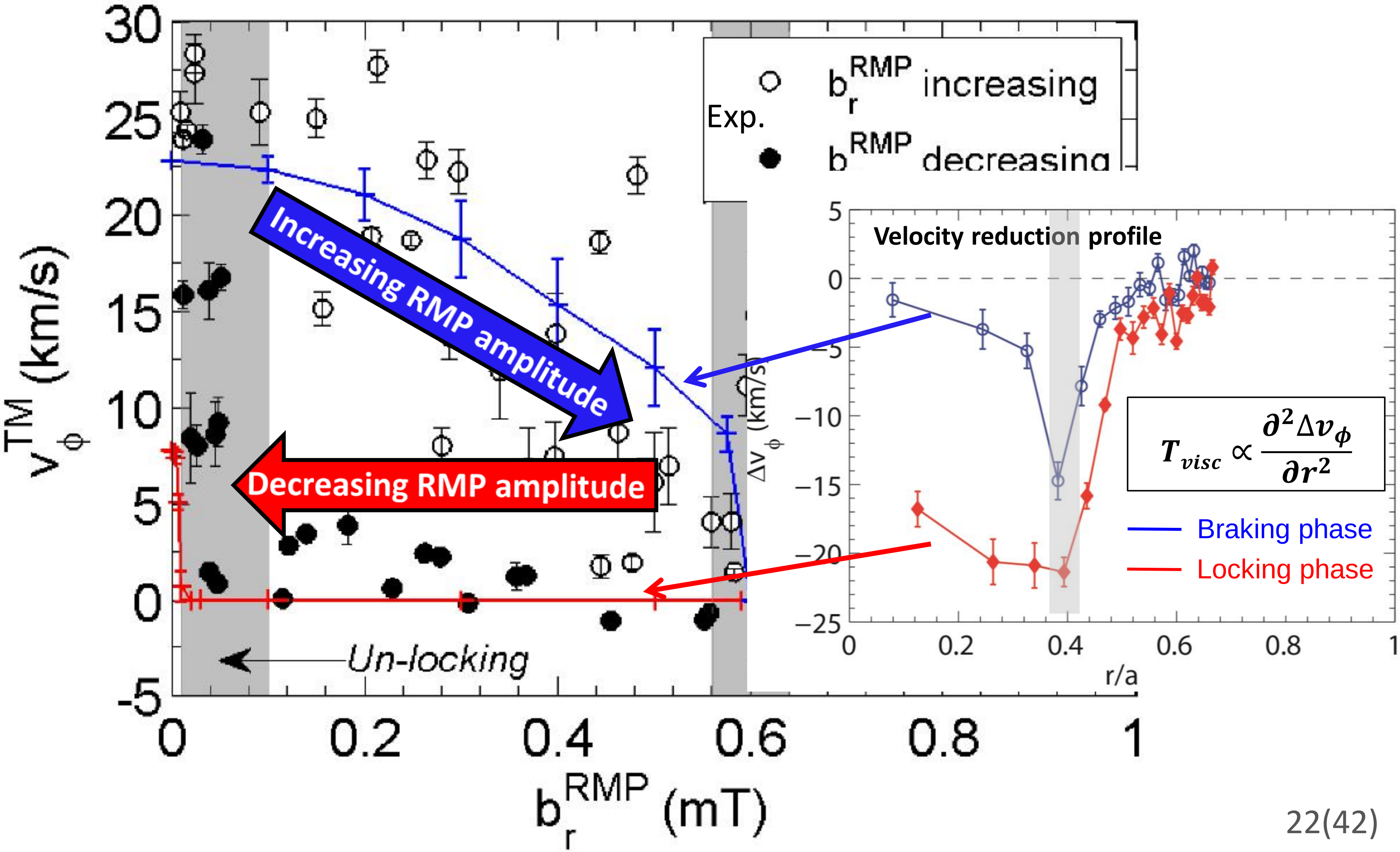
- The viscosity is used as a free parameters in order to match the locking threshold
- Reasonable qualitative agreement

$$\begin{cases} \frac{\Lambda_1}{2}\tau_R \frac{d(W/r_s)}{dt} = E_1 - \lambda_s^2 \Lambda_1 \left(\frac{W}{4r_s}\right) \ln\left(\frac{4r_s}{W}\right) + \left(\frac{W_c}{W}\right)^2 \cos(\Delta\alpha(t)) \\ \rho \frac{\partial \Delta\Omega}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu \frac{\partial \Delta\Omega}{\partial r}\right) + \frac{T_{EM}}{4\pi^2 R_0^3}\delta(r - r_s) \\ \frac{d\Delta\alpha}{dt} = \Delta\Omega + \omega_0 \end{cases}$$



# Comparison experiment – model: hysteresis

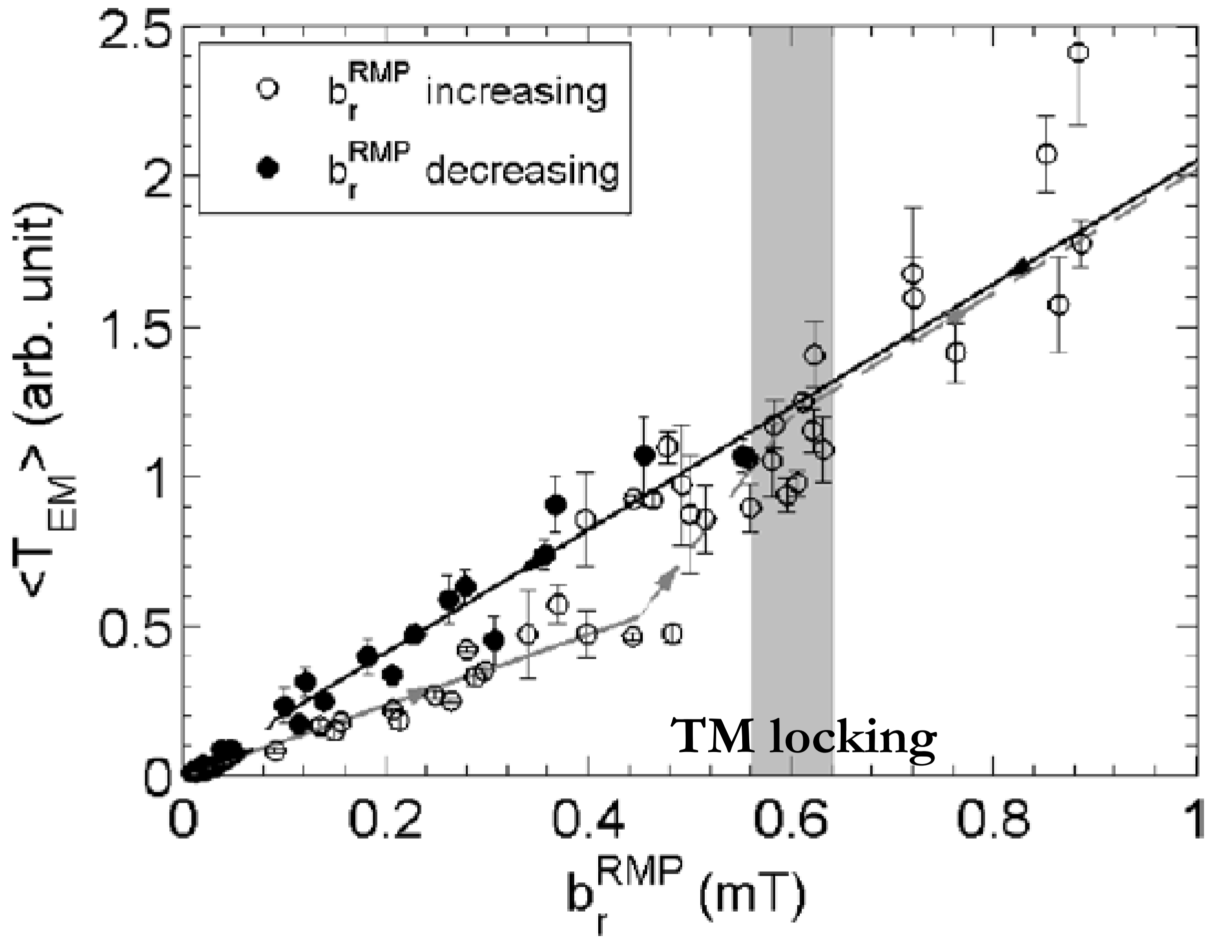
- RMP ramp-up and -down (several shots)



# Increase in TM amplitude deepens the hysteresis

- Increase in TM amplitude after locking → increase $T_{EM}$

$$T_{EM} = \mathrm{k}|b_{RMP}||b_{TM}| \sin(\Delta\alpha(t))$$



# Summary – TM-RMP interaction in EXTRAP T2R

- Studied TM locking-unlocking to RMP
- Hysteresis: $|b^{RMP}|_{\text{unlocking}} << |b^{RMP}|_{\text{locking}}$
- Qualitative agreement with model
- After locking: Drop in $T_{visc}$ and increase in $\max(T_{EM})$ explains the hysteresis
  - Drop in $T_{visc}$ explained by the relaxation of velocity reduction profile.
  - Increase in $\max(T_{EM})$ caused by increased TM amplitude
- Model-required kinematic viscosity 1-2 m$^2$/s
  similar to earlier estimations [Frassinetti et al., NF 2012].

# Outline

- The RFP and tearing modes
- Resonant magnetic perturbation and tearing modes
  - Papers I-II (III, V)
- **Viscosity in stochastic magnetic fields**
  - **Papers VI-VII** (IV)

# Background and motivation

- Stochastic magnetic fields
  - RFP core-region
  - Tokamak: Edge with magnetic perturbation,
    Core during disruption
  - Visco-resistive MHD simulate these plasmas
    - Resistivity well-known
    - Viscosity not well-known

# Background and motivation

Viscosity in stochastic field:

- Theoretical model – sound wave propagation *[J. M. Finn 1992]*
  - Only one-point test (MST RFP plasma) *[A. F. Almagri 1998]*
  - Suggested test varying $\delta b$ in tokamak edge
- Present experiments:
  - Vary $\delta b$ in the RFP core
  - Measure the viscosity via momentum perturbation
    - External torques: RMP and Probe

# Outline

- Introduction and experimental setup
- Viscosity measurements in stochastic magnetic field
- Comparison with Finn model:
  - Momentum diffusion in stochastic magnetic field
- Conclusions

# Madison Symmetric Torus RFP

## Parameters in this study

$R/a \;=\; 1.5/0.5\ \mathrm{m}$

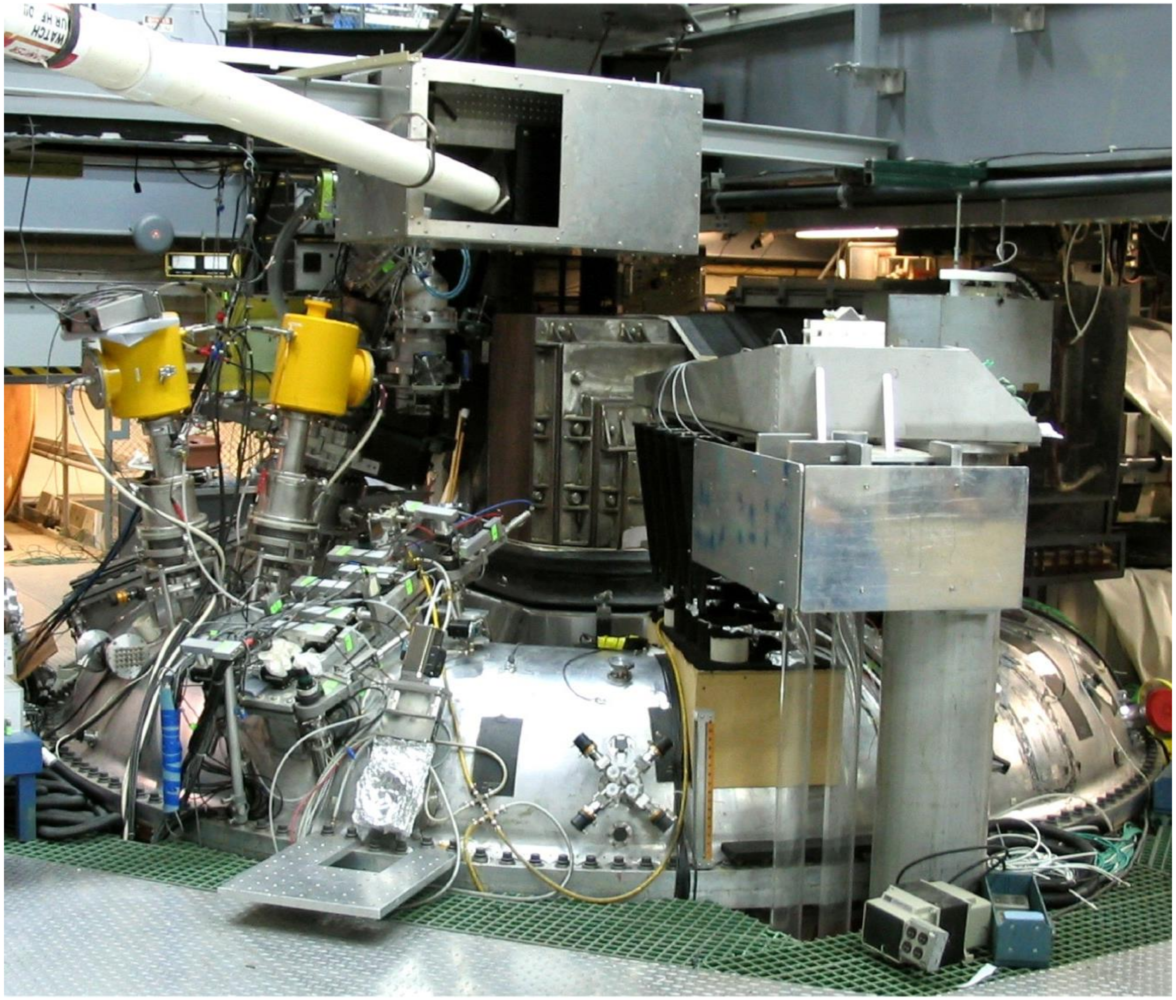
$I_\phi \;\approx\; 50 - 400\ \mathrm{kA}$

$B(0) \;<\; 0.5\ \mathrm{T}$

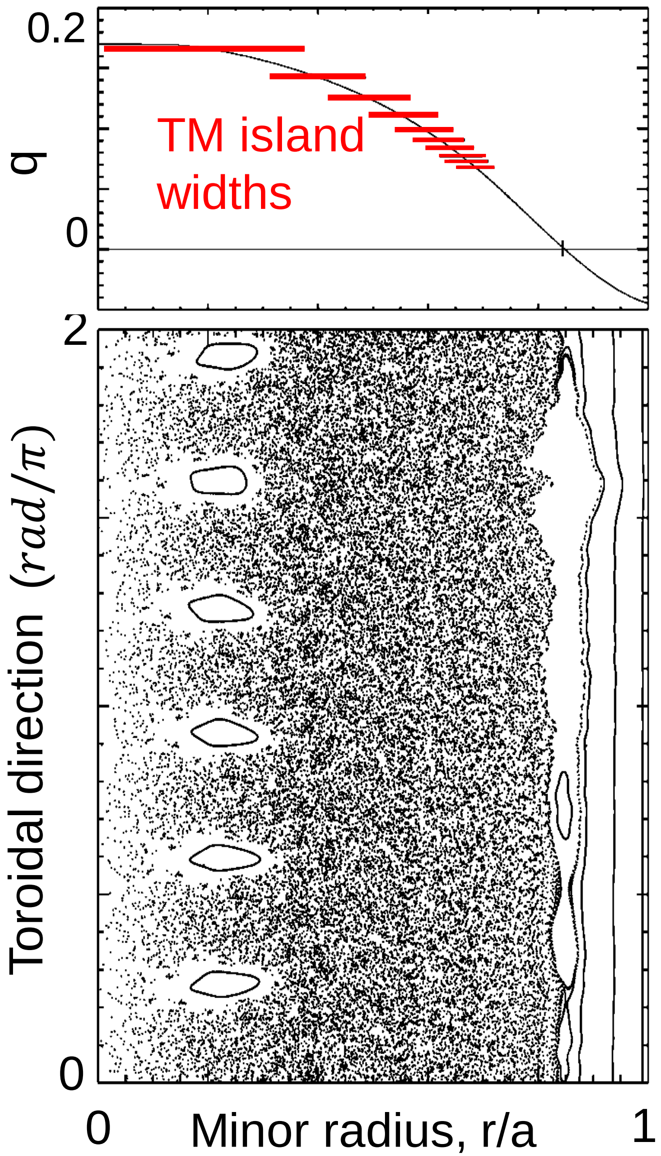
$n_e \approx 0.4 - 1.4 \times 10^{19}\mathrm{m}^{-3}$

$T_e \;\approx\; 50 - 450\ \mathrm{eV}$

$T_i \;\approx\; 50 - 350\ \mathrm{eV}$



# Stochastic magnetic field due to TM islands




*[T. M. Biewer PRL 2003]*

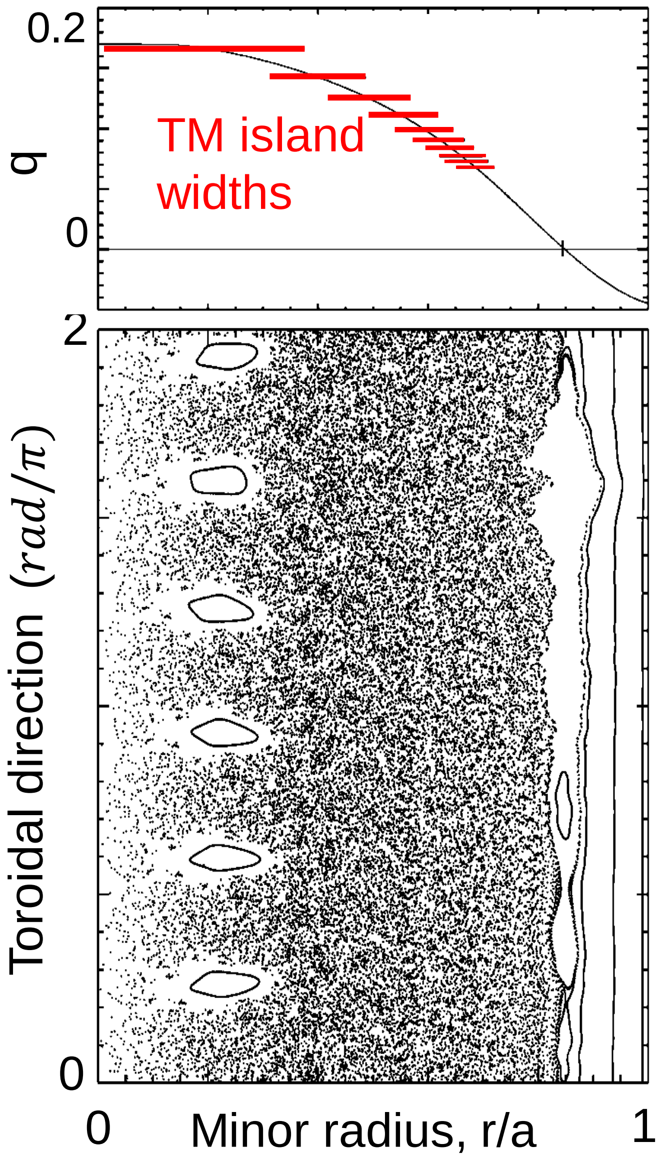
- Island width: $W_{mn} = 4\sqrt{\dfrac{r b_{r,mn}}{n B_\theta |q'|}}$

- Magnetic diffusion coefficient *[M. N. Rosenbluth NF 1966]*:

$$D_m = L_c \sum_{m,n} \left(\frac{b_{r,mn}}{B}\right)^2$$

$L_c$ parallel autocorrelation length

# Stochastic magnetic field due to TM islands

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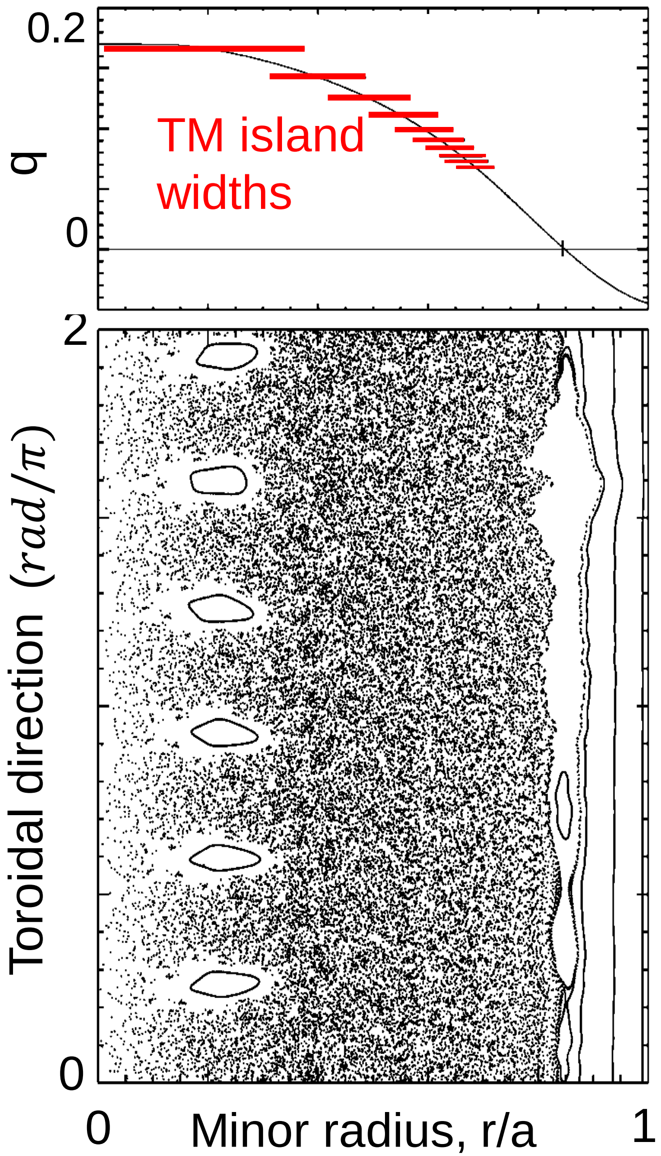
$$D_m = L_c \sum_{m,n} \left(\frac{b_{r,mn}}{B}\right)^2$$

Summed over 3 central TMs in stochastic region $(\boldsymbol{b_r}/\boldsymbol{B})^2$

$L_c$ parallel autocorrelation length

# Finn model – Viscosity in stochastic magnetic field

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Summed over 3 central TMs in stochastic region $(\boldsymbol{b_r}/\boldsymbol{B})^2$

$L_c$ parallel autocorrelation length

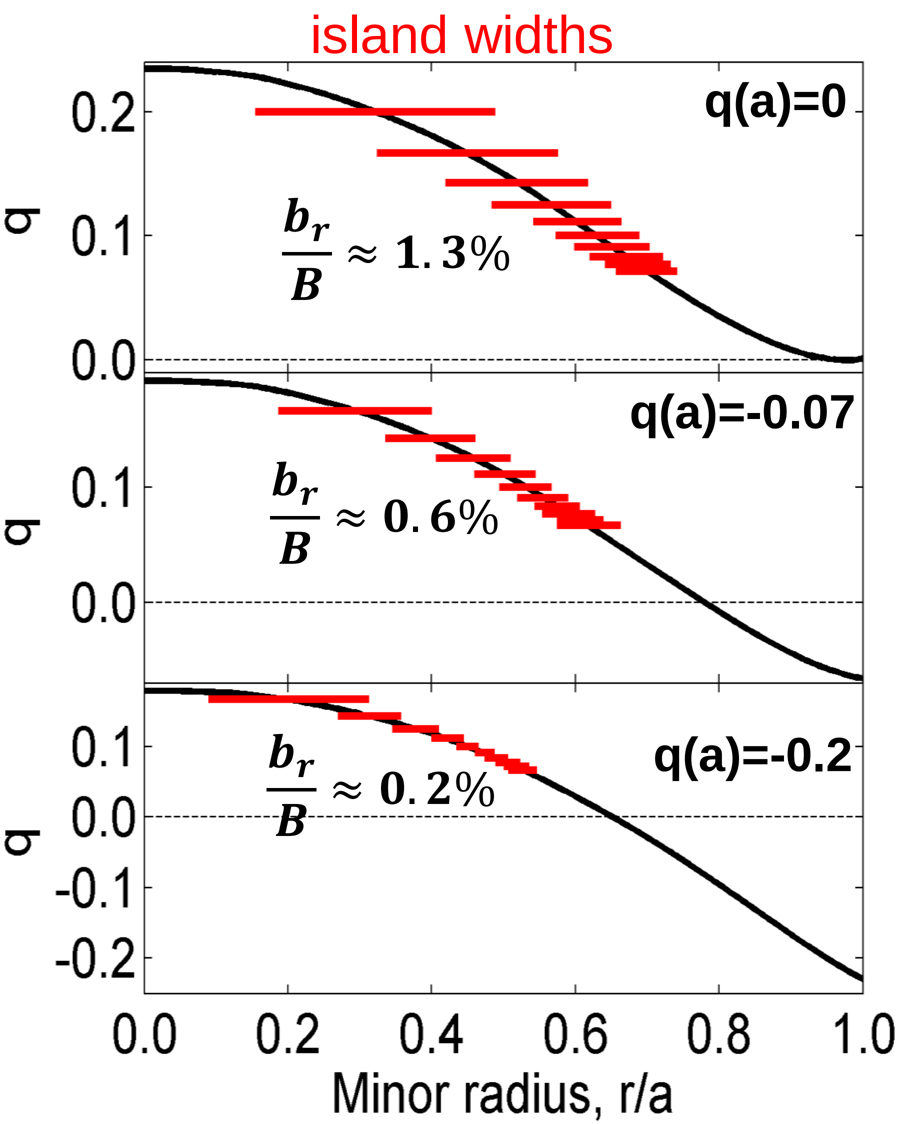
- Kinematic viscosity in Finn's model:

$$\boxed{\nu_\perp = c_s D_m}$$

Sound speed: $c_s \approx \sqrt{\frac{T_e + T_i}{M}}$

*[J. M. Finn, Phys. Fluids B, 1992]*

# Varying magnetic fluctuation and stochasticity

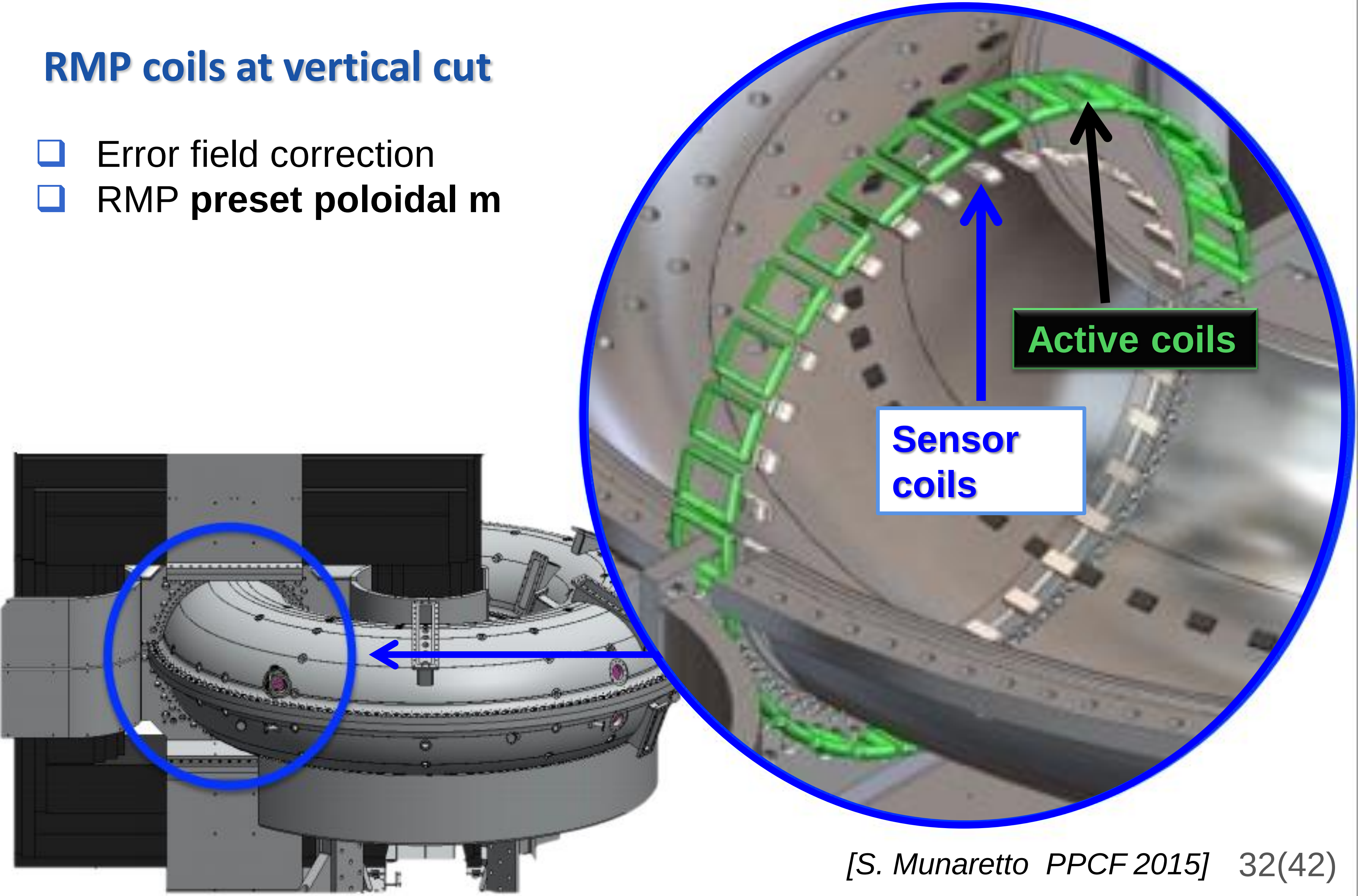



- **~10-fold range in $b_r/B$**
- **Three types of equilibria**
  - **Variation in $b_r/B$ with $I_\phi$**
    - $I_\phi$ range 50 – 400 kA
    - $b_r/B$ range ~ 0.8 – 2.3 %
  - **Inductive $j_\parallel$ profile control**
    - Tokamak-like confinement
    - Reduction to $b_r/B$ ~0.2 %
- Expect: ~100-fold range in $D_m$ and $\nu_\perp$

# Resonant Magnetic Perturbation

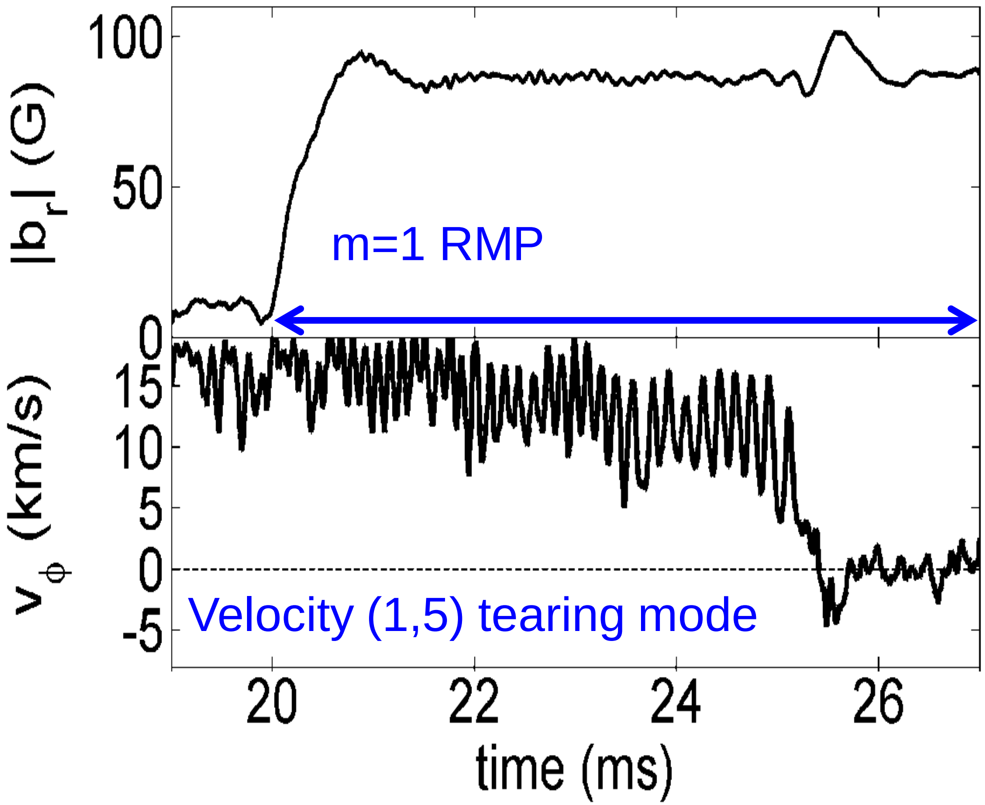
## RMP coils at vertical cut

- Error field correction
- RMP **preset poloidal m**




*[S. Munaretto PPCF 2015]*

# RMP effect on tearing mode rotation in MST




RMP-TM interaction:

- Electromagnetic torque on TMs →
  - Velocity Fluctuations
  - Deceleration
  - Locking to RMP phase

- Similar observations in RFPs and tokamaks:
  - EXTRAP T2R *[L. Frassinetti NF 2010]*
  - AUG *[S. Fietz NF 2014]*

# Model – Plasma flow change via RMP

**Equation of fluid motion:**

$$\boxed{\underbrace{\rho\frac{\partial \Delta v_\phi}{\partial t}}_{\text{Inertia}} = \underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\nu_\perp\frac{\partial \Delta v_\phi}{\partial r}\right)}_{\text{Viscous torque}} + \underbrace{\sum_n T_{EM}{}^{1,n}(t)\delta(r-r_s{}^{1,n})}_{\text{Electromagnetic torque on each resonant surface}}}$$

$$T_{EM} = T_{wall} + T_{RMP}$$

$$T_{wall}{}^{1,n} = |b_{TM}{}^{1,n}|^2 \frac{A\sqrt{v_\phi(r_s,t)}}{Bv_\phi(r_s,t) - C\sqrt{2v_\phi(r_s,t)} + C^2}$$

$$T_{RMP}{}^{1,n} = k|b_{TM}{}^{1,n}||b_{RMP}{}^{1,n}|\sin\left(\alpha_{TM}{}^{1,n} - \alpha_{RMP}{}^{1,n}\right)$$

**No-slip condition:**

***(n equations)***

$$\boxed{\frac{\mathrm{d}\alpha_{TM}{}^{1,n}}{\mathrm{d}t} = \frac{n}{R_0}v_\phi(r_s,t)}$$

*[R. Fitzpatrick Nucl. Fusion (1993)]*

*Adapted to MST: [R. Fridström et al PoP 2016]*

# Model – Plasma flow change via RMP

**Equation of fluid motion:**

**Free parameter: $\nu_\perp$ (kinematic viscosity)**

$$\boxed{\underbrace{\rho\frac{\partial \Delta v_\phi}{\partial t}}_{\text{Inertia}} = \underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\nu_\perp\frac{\partial \Delta v_\phi}{\partial r}\right)}_{\text{Viscous torque}} + \underbrace{\sum_n T_{EM}{}^{1,n}(t)\,\delta(r - r_s{}^{1,n})}_{\text{Electromagnetic torque on each resonant surface}}}$$

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**No-slip condition:**
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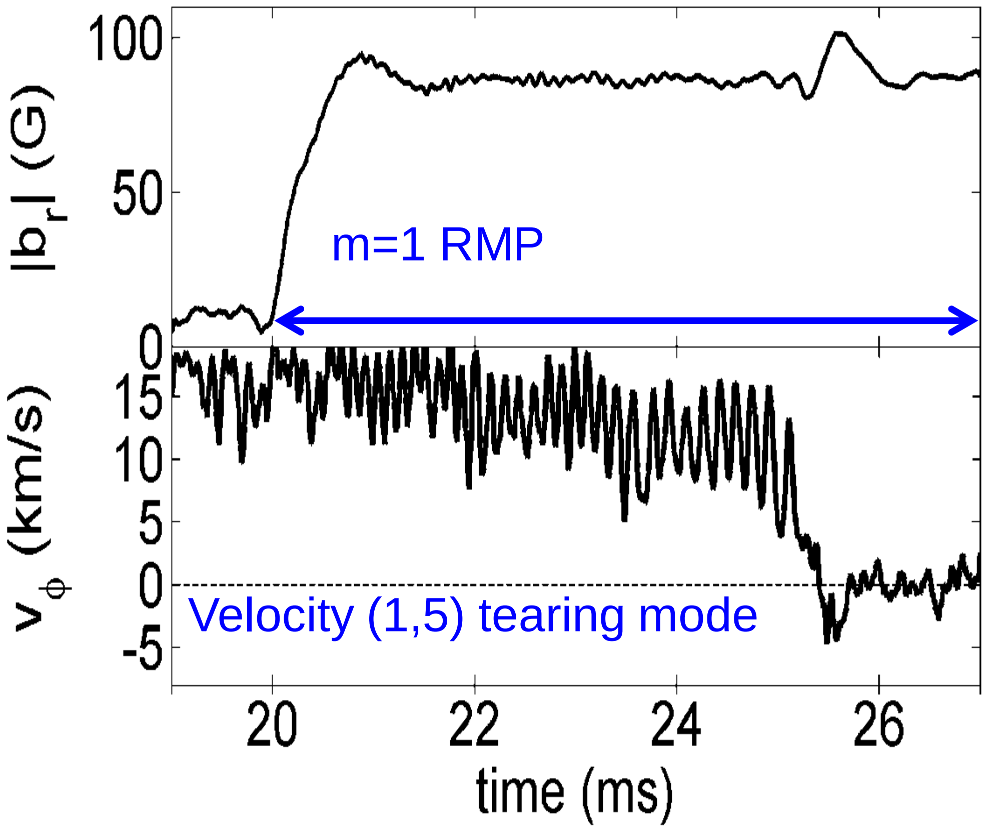
*[R. Fitzpatrick Nucl. Fusion (1993)]*

*Adapted to MST: [R. Fridström et al PoP 2016]*

# Viscosity estimation via an RMP

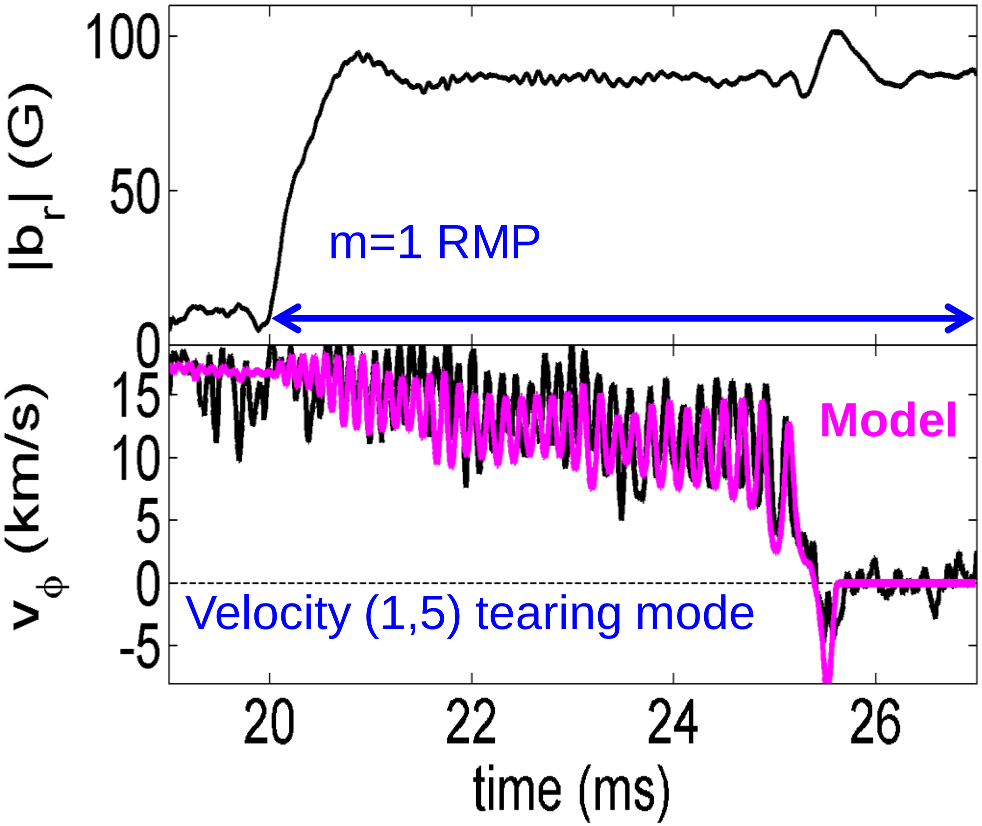



**Plasma with $b_r/B\sim1\%$**

Model best match with $v_\phi$(t) ⇨ $\nu_\perp$

*[R. Fridström et al PoP 2016]*

# Viscosity estimation via an RMP



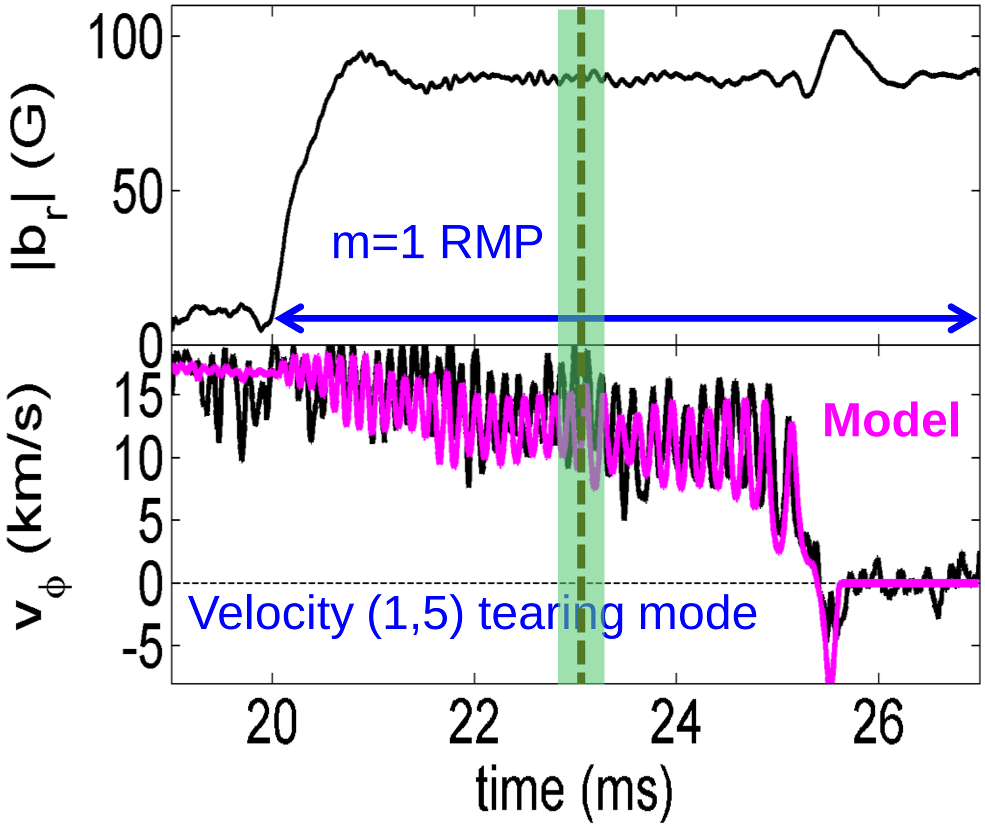

**Plasma with $b_r/B\sim 1\%$**

Model best match with $v_\phi$(t) ⇨ $\nu_\perp$

- **Model includes n=5 to 14**
- **Best match: $\nu_\perp = 14 \pm 6$ m²/s**

*[R. Fridström et al PoP 2016]*

# Viscosity estimation via an RMP




**Plasma with $b_r/B\sim 1\%$**

Model best match with $v_\phi$(t) → $\nu_\perp$

- **Model includes n=5 to 14**
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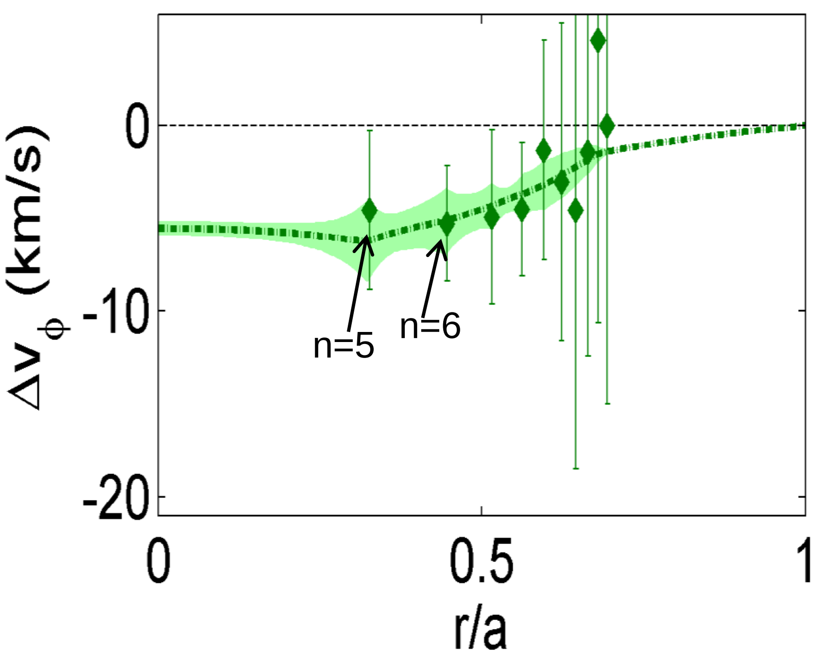
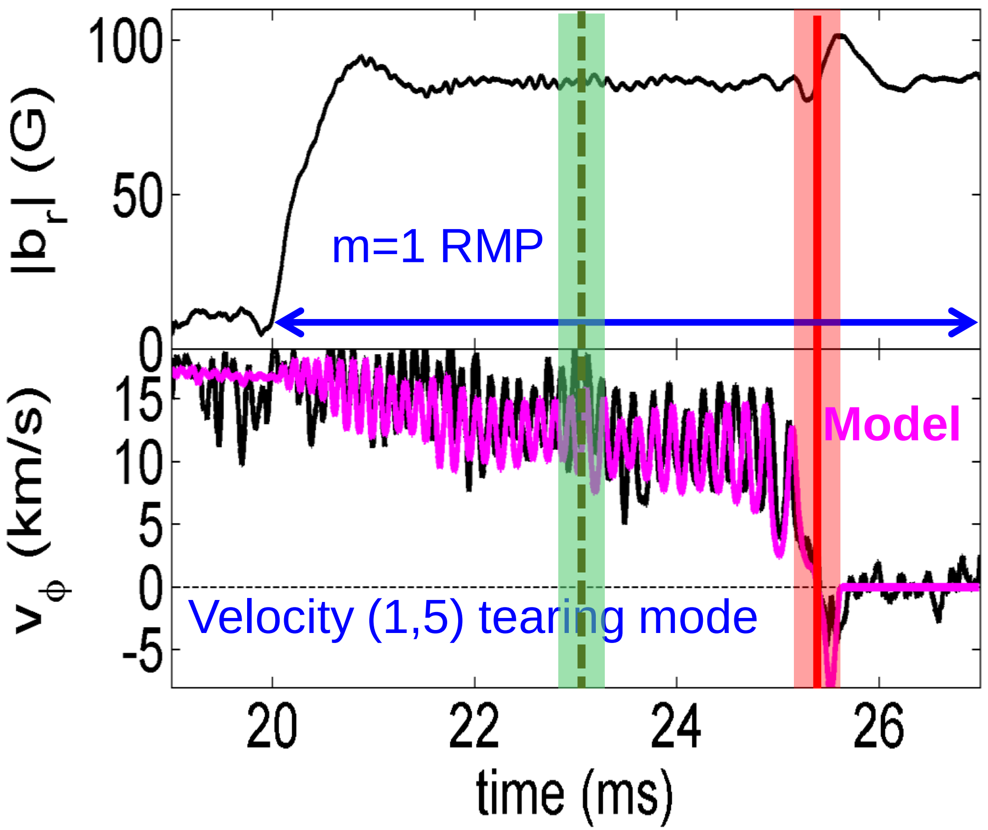
Velocity reduction profile




*[R. Fridström et al PoP 2016]*

# Viscosity estimation via an RMP

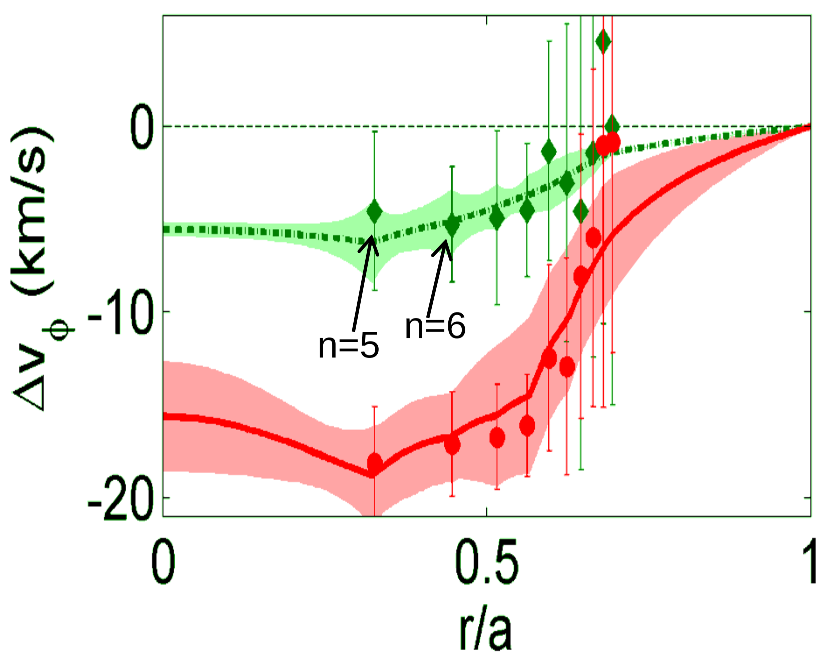



**Plasma with $b_r/B \sim 1\%$**

Model best match with $v_\phi$(t) ⇒ $\nu_\perp$

- **Model includes n=5 to 14**
- **Best match: $\nu_\perp = 14 \pm 6$ m²/s**

Velocity reduction profile




[R. Fridström et al PoP 2016]

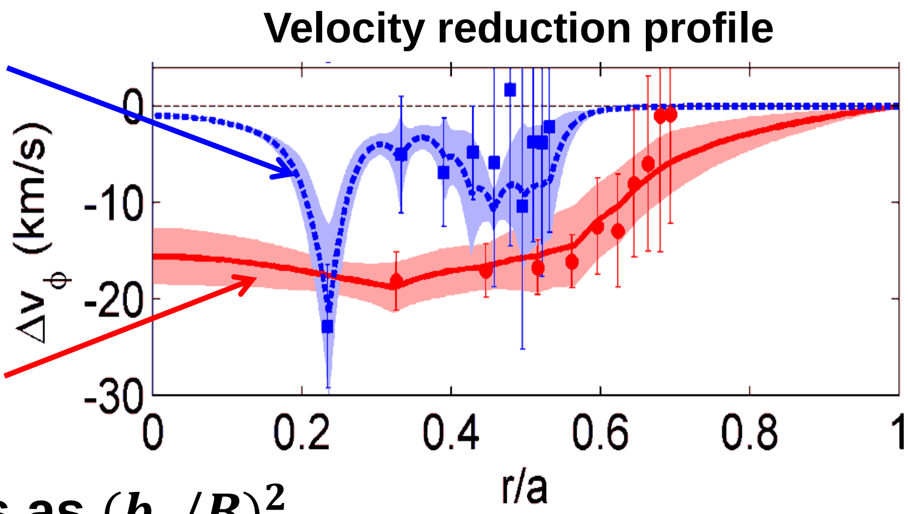
# RMP in two different b/B plasmas

- Same m=1 RMP applied

$b_r/B \sim 0.2\%$ $\nu_\perp \approx 0.6\ \mathrm{m}^2/s$

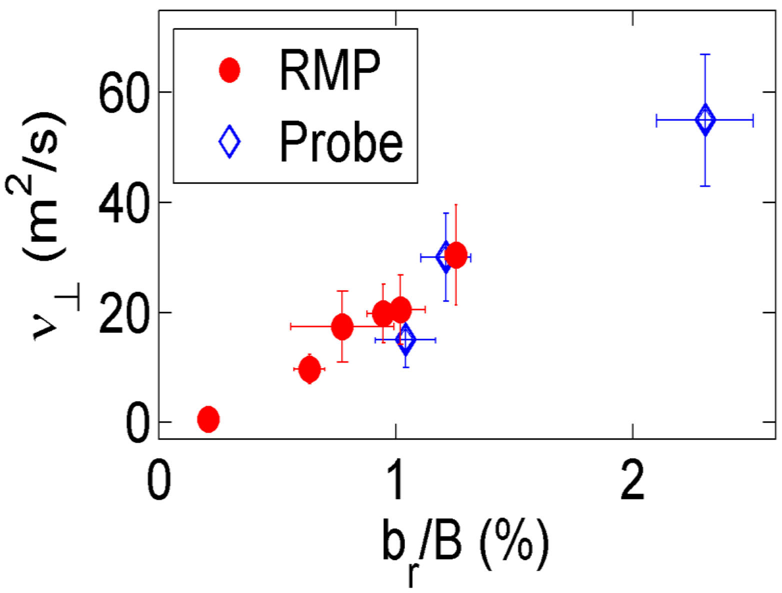
**Different $\nu_\perp$ at locking**

$b_r/B \sim 1\%$ $\nu_\perp \approx 14\ \mathrm{m}^2/s$



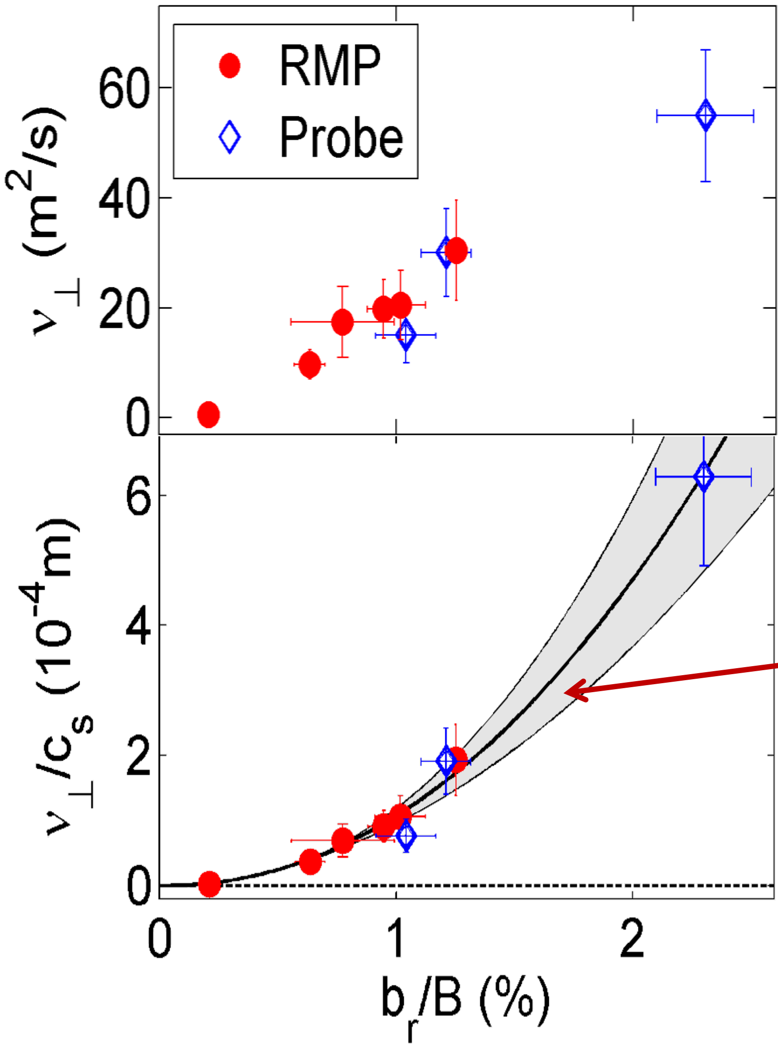

- **Difference in $\nu_\perp$ scales as $(b_r/B)^2$**
  - Factor ~25 span in $\nu_\perp$ and $(b_r/B)^2$

# Similar viscosity estimates from RMP and probe




- Similar result with Probe and RMP
- Magnetic fluctuation dependent
  - $\nu_\perp$ increase with $(b_r/B)$

# Viscosity scales as $c_s(b/B)$^2



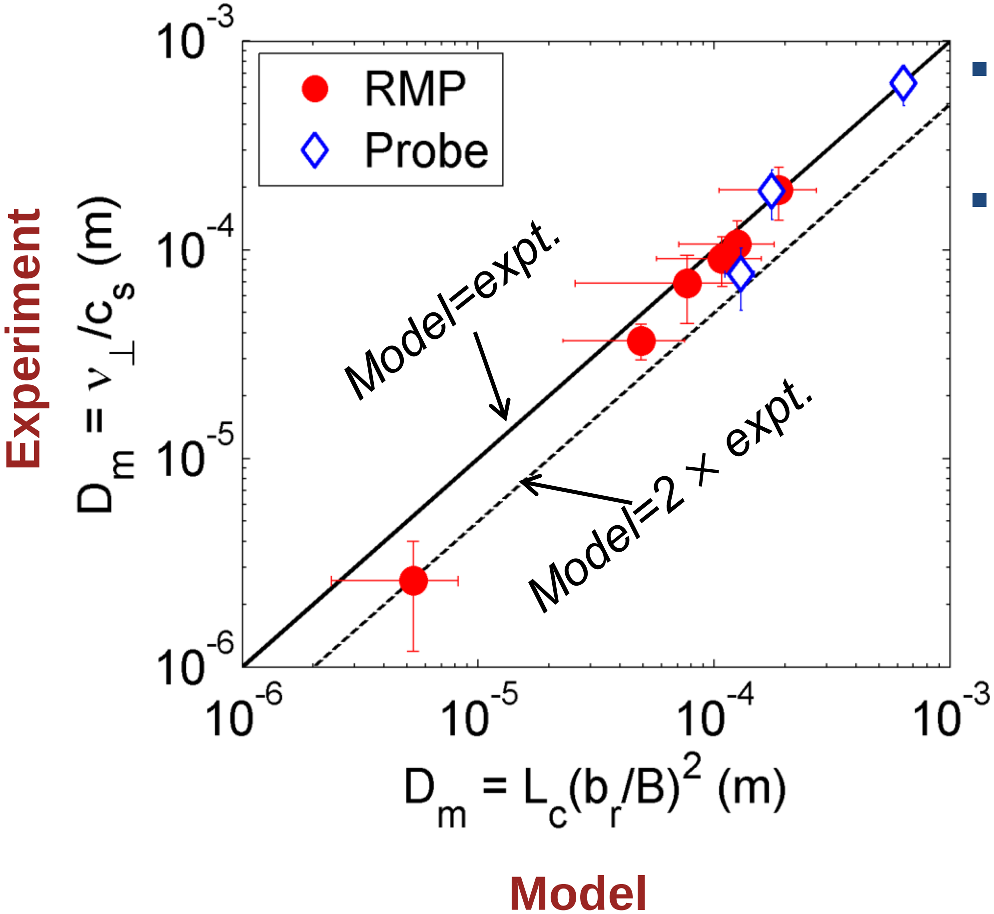

- Similar result with Probe and RMP
- Magnetic fluctuation dependent
  - $\nu_\perp$ increase with $\left(\frac{b_r}{B}\right)$
- Include plasma sound speed $c_s$.

$$\nu_\perp = c_s D_m$$

Experimental: $D_m = \frac{\nu_\perp}{c_s} \propto (b_r/B)^{2.13 \mp 0.19}$

Rosenbluth quasilinear: $D_m \propto (b_r/B)^2$

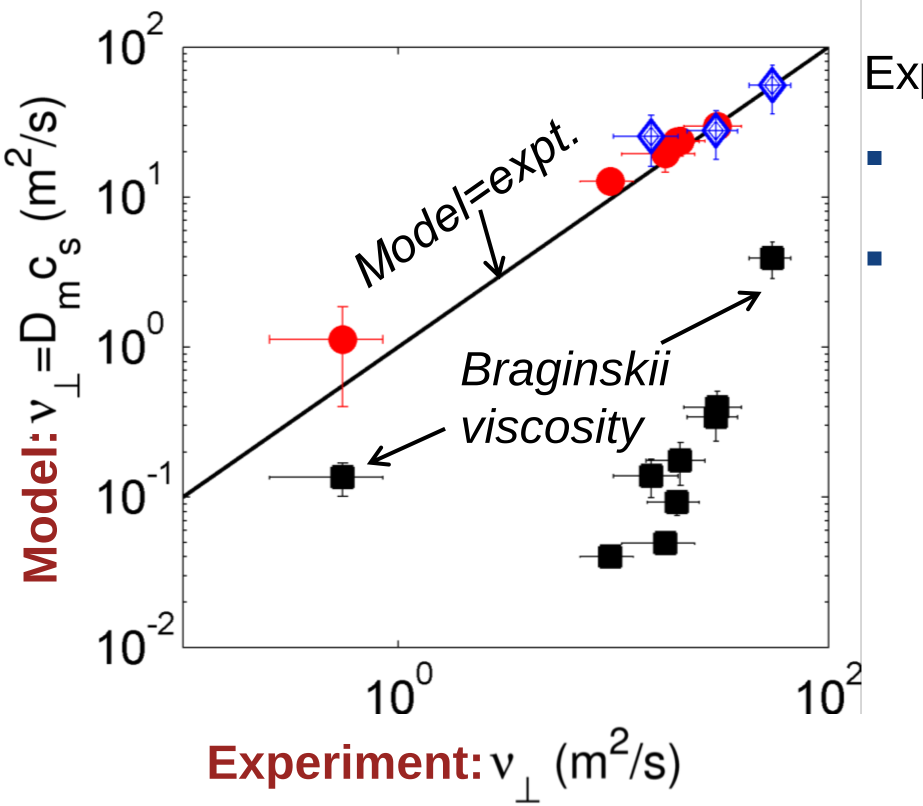
# Viscosity agrees quantitatively with $\nu_\perp = c_s D_m$




- Experiment: $D_m = \frac{\nu_\perp}{c_s}$
- Quasilinear model:

$$D_m = L_c \sum_{m,n} \left( \frac{b_{r,mn}}{B} \right)^2$$

# Viscosity agrees quantitatively with $\nu_\perp = c_s D_m$




Experimental $\nu_\perp$ :

- Agreement with Finn model: $\nu_\perp = c_s D_m$
- ~5 – 200 times larger than Braginskii viscosity
  - The 5 times is at lowest b/B

# Summary and Conclusions

- **Similar $\nu_\perp$ with Probe and RMP**

- **$\nu_\perp$ agrees with Diffusion in Stochastic Field**
  - Finn's model can estimate $\nu_\perp$ in stochastic plasma

# Acknowledgements

- **KTH**: Per Brunsell, Lorenzo Frassinetti, Agung Chris Setiadi, Håkan Ferm, and all colleagues
- **MST, University of Wisconsin-Madison**: Brett Chapman, Stefano Munaretto, Abdulgader Almagri, John Sarff and MST group