KTH ROYAL INSTITUTE
OF TECHNOLOGY

Soft X-ray camera

Tomographic imaging in EXTRAP T2R

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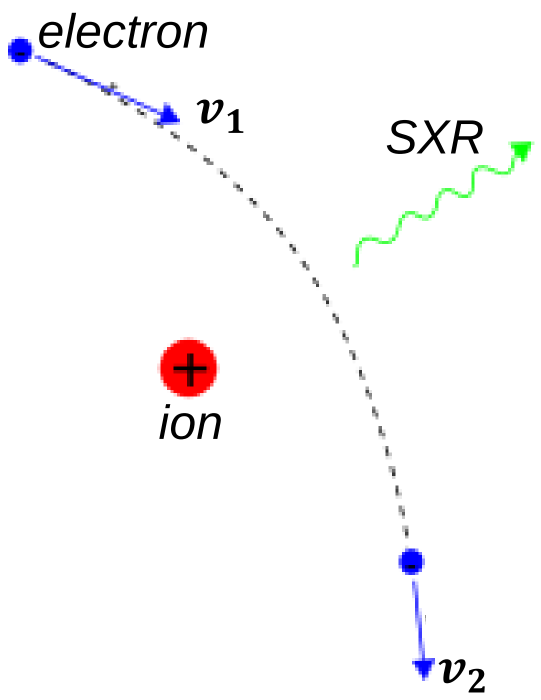
Outline

- Soft X-rays (SXR)
- SXR diagnostic
- Tomographic imaging
  - The inversion algorithm (Cormack)
  - SXR emissivity maps in EXTRAP T2R

SXR sources

- Bremsstrahlung (free-free)
  - Main SXR source
  - Acceleration of electrons in Coulomb collisions
  - Depends on density *n*, the electron temperature T_e, and the charge *Z* :

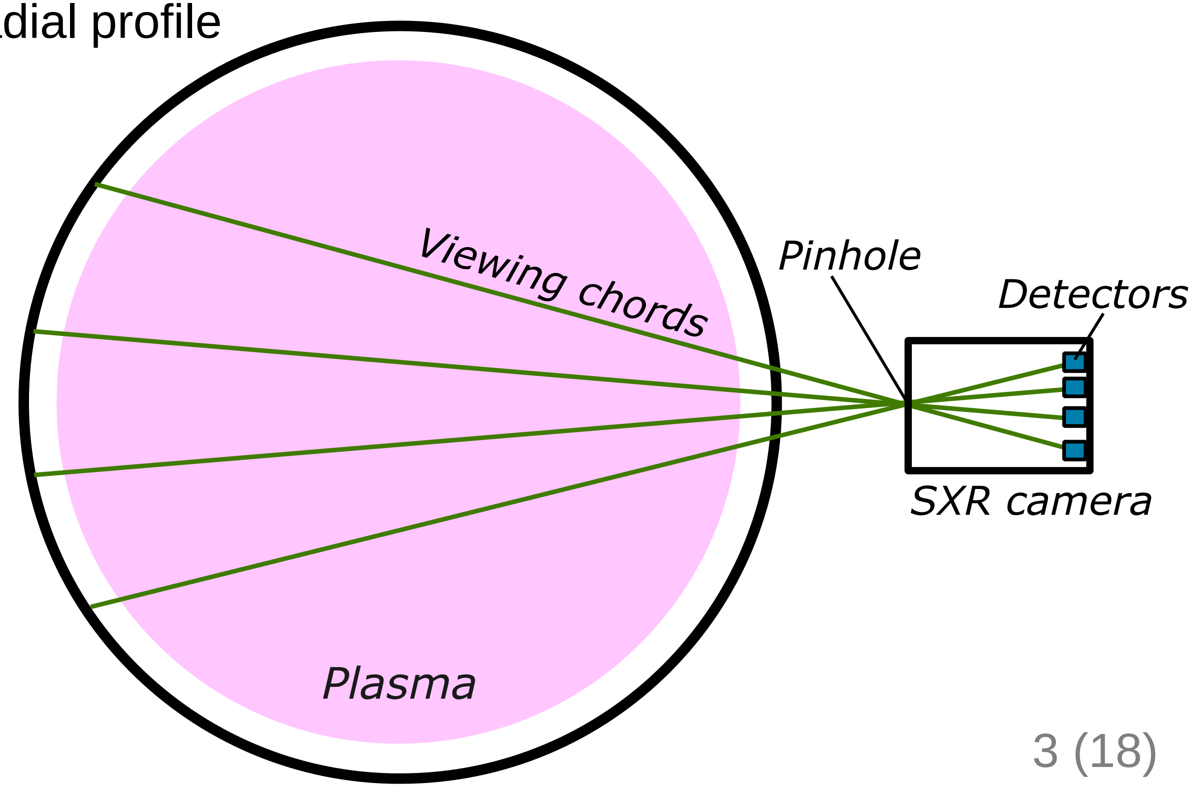
$$\varepsilon_B = \left(\frac{\sqrt{2}}{3\pi^{\frac{5}{2}}}\right)\left(\frac{e^6}{\epsilon_0 c^3 h m_e^{\frac{3}{2}}}\right) Z^2 n_i n_e \sqrt{T_e} \quad [\mathrm{W/m^3}].$$



- Other SXR sources:
  - Recombination radiation (free-bound)
  - Line radiation (bound-bound)

SXR diagnostic

- Detector head
  - Pinhole collimator
  - High-pass filter (Beryllium)
    - ✓ Study the core plasma
  - Silicon photodiode detectors
    - ✓ Low-pass filter (finite diode width)
    - ✓ Many detectors → radial profile

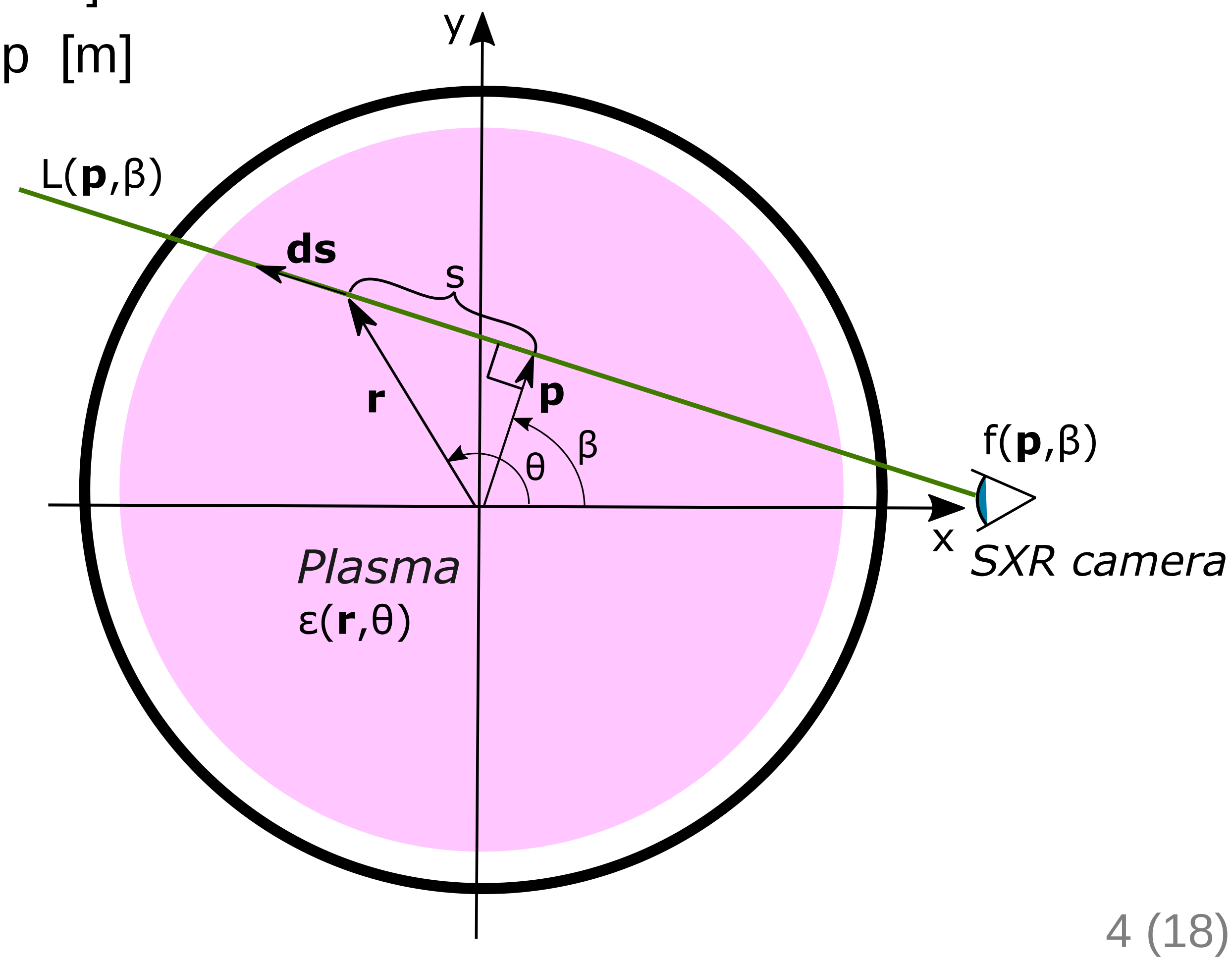


SXR diagnostic

- Measured brightness along a single chord:

$$f(p,\beta) = \int_{L(p,\beta)} \varepsilon(r,\theta)ds \quad \text{[W/m]}$$

  - Emissivity: $\varepsilon(r,\theta)$ [W/m^2]
  - Impact parameter: p [m]

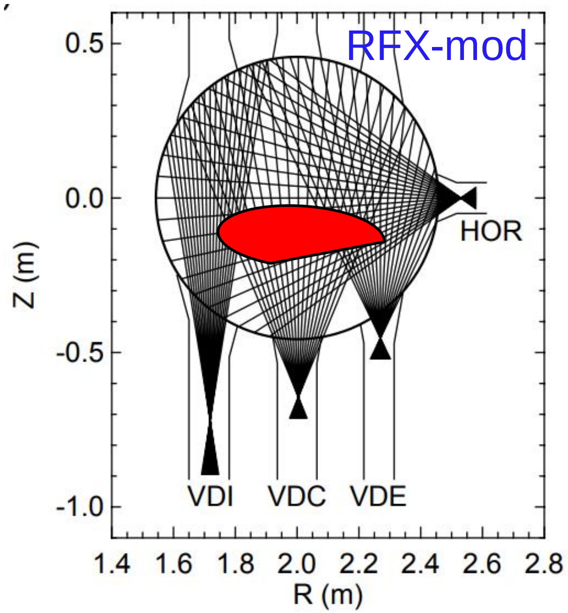


SXR cameras for tomography

Tomographic reconstruction

- Multiple cameras
  - e.g. RFX-mod, ASDEX-U

SXR image at each time instant



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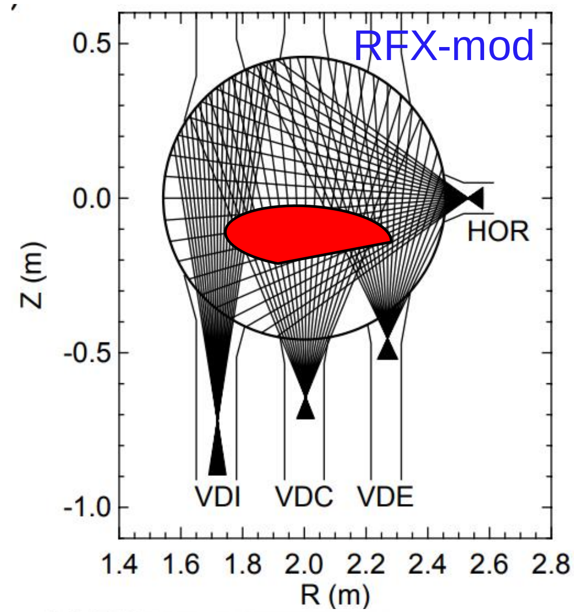
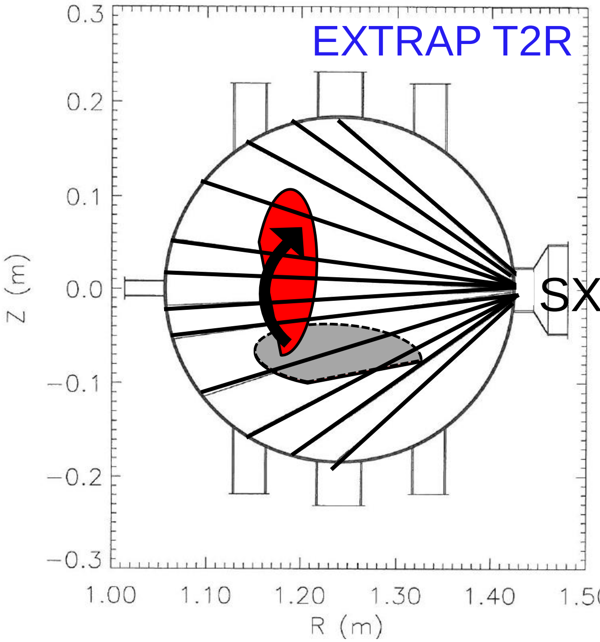


- Simulate additional cameras
  - e.g. MST, EXTRAP T2R
    - ✓ Plasma poloidal rotation

One time instant not enough
→Use two or more measurements



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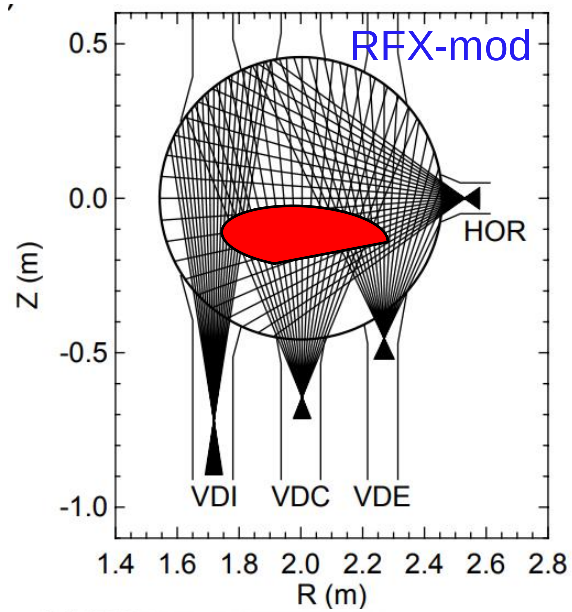

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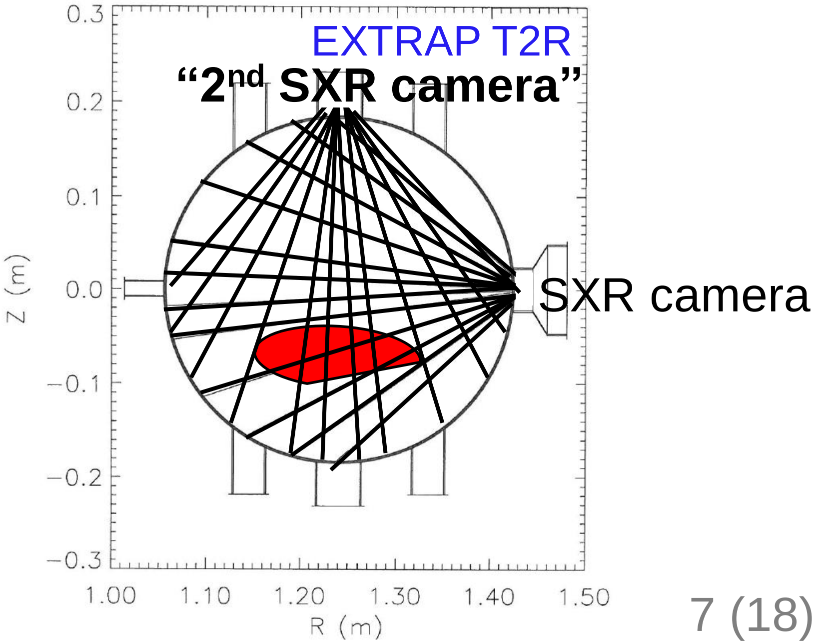


Tomographic imaging of SXR emissivity

Cormack's method:

- **Measured** brightness along a single chord:

$$\boxed{f(p,\beta)} = \int_{L(p,\beta)} \varepsilon(r,\theta) ds$$

- **Model**: expand in Fourier series:

$$\varepsilon(r,\theta) = \sum_{m=0}^{\infty} [\varepsilon_m^c(r)\cos(m\theta) + \varepsilon_m^s(r)\sin(m\theta)],$$

$$\boxed{f(p,\beta)} = \sum_{m=0}^{\infty} [f_m^c(p)\cos(m\beta) + f_m^s(p)\sin(m\beta)].$$

  - $\varepsilon_m^{c,s}(r)$ are then further expanded
    $\varepsilon_m^{c,s}(r) = \sum_{l=0}^{\infty} a_{ml}^{c,s} \varepsilon_{ml}(r)$.

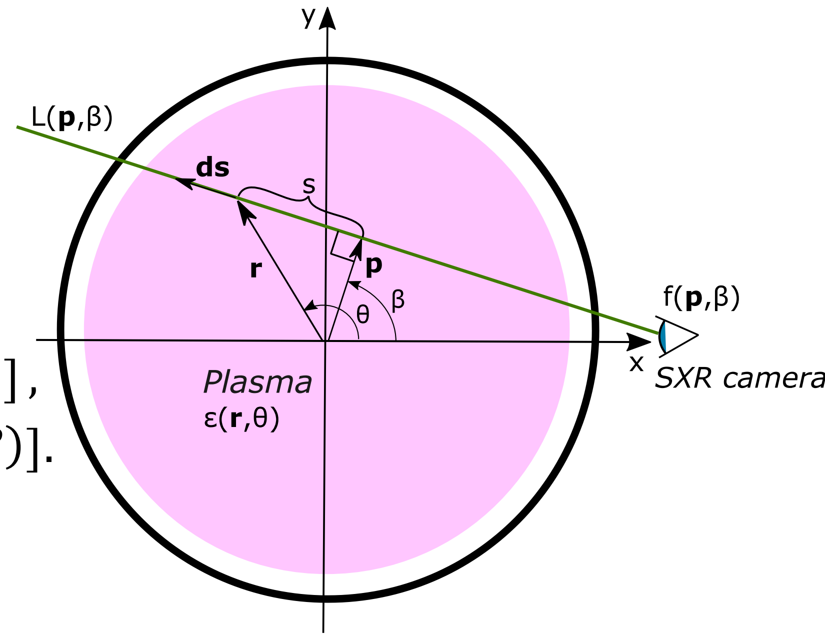
    $\rightarrow f_m^{c,s}(p) = \sum_{l=0}^{\infty} a_{ml}^{c,s} f_{ml}(p)$, and $f_{ml}(p) = 2\int_p^1 \frac{\varepsilon_{ml}(r) T_m\left(\frac{p}{r}\right) r dr}{\sqrt{r^2-p^2}}$,

    where $T_m(x) = \cos(m\cos^{-1}x)$ [Chebyshev polynomial]

  - Chose Bessel functions of the first kind: $\varepsilon_{ml}(r) = J_m(x_{ml} r)$, where $J_m(x_{ml}) = 0$.

    $\rightarrow f_{ml}(p) = -2J_m'(x_{ml}) \int_0^{\cos^{-1} p} \cos(m\theta)\sin[x_{ml}(\cos\theta - p)]\, d\theta$

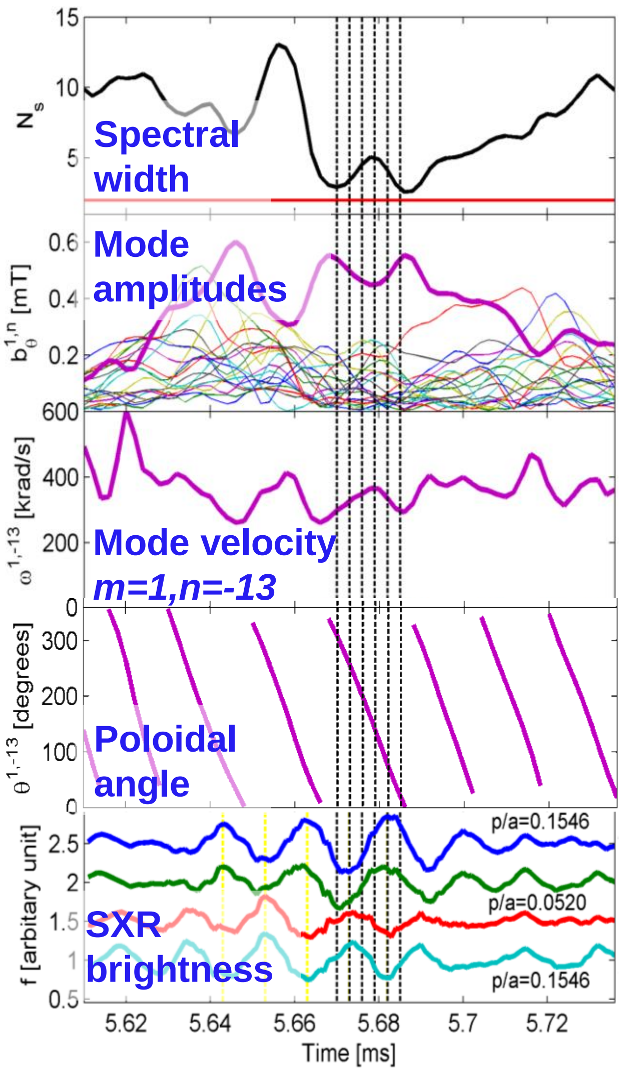
- **Minimizing the squared error (model-measured) in all chords gives $a_{ml}^{c,s}$**



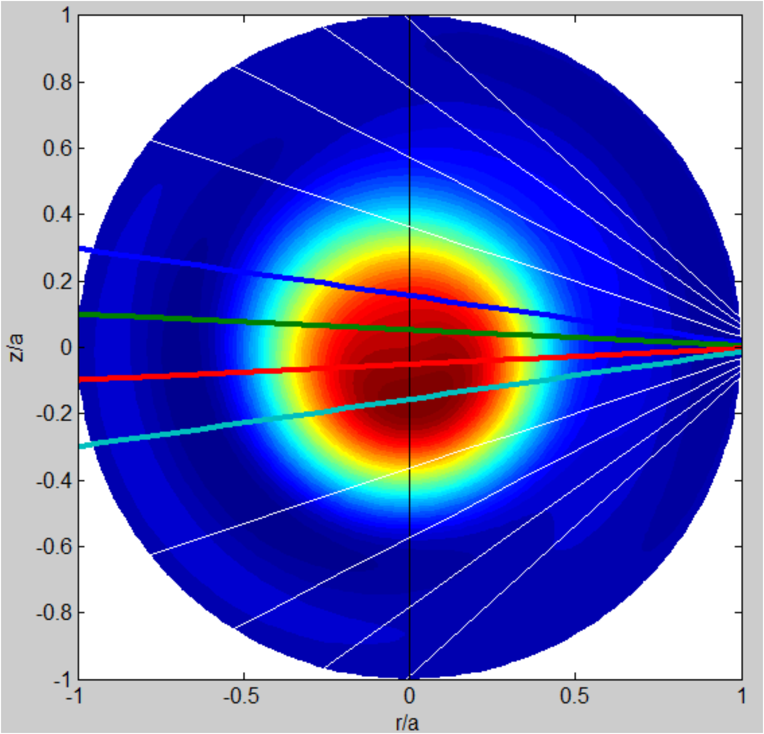
[Cormack A.M., J. Appl. Phys. (1963)] and *[Wang L.*, Rev. Sci instr *(1991)]*

SXR imaging EXTRAP T2R using mode rotation

- Simulate SXR cameras using mode rotation (SXR signals at three time instants)
- Single dominant mode (m=1,n=-13)



- m=1 modes via magnetic pick-up coils
- SXR signal in central chords
  - Up-down chord signals: ~180° phase shifted (rotating m=1 mode)
- SXR phase via the Hilbert transform.



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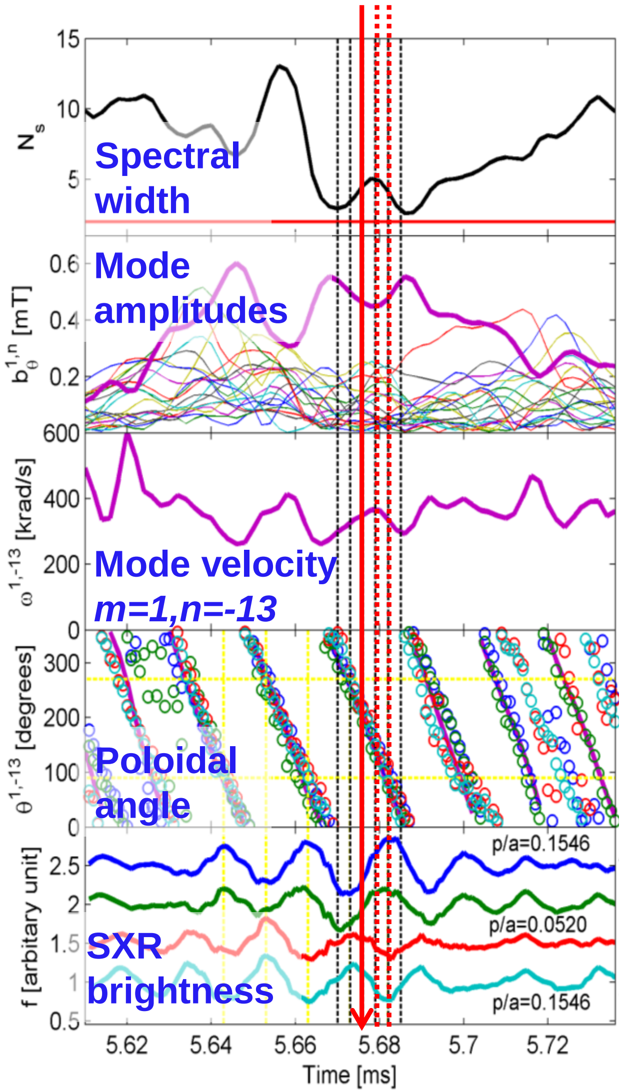
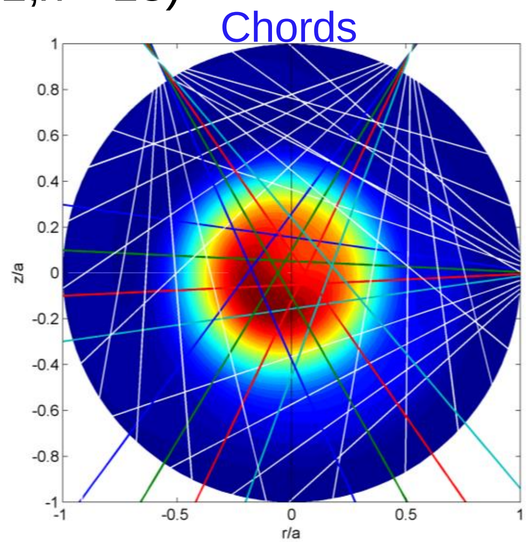
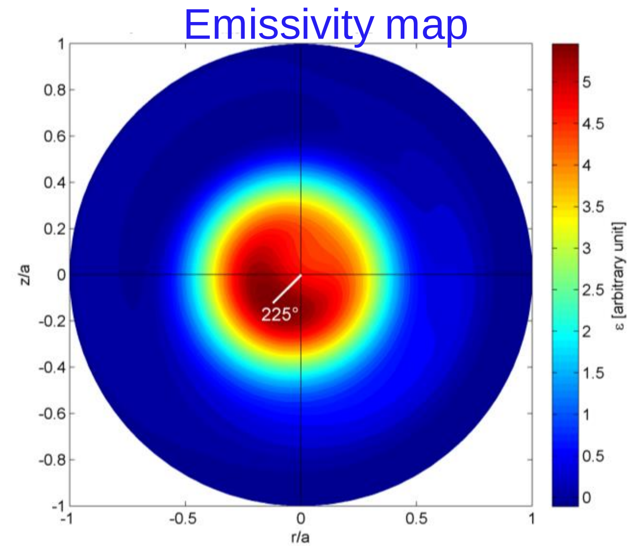
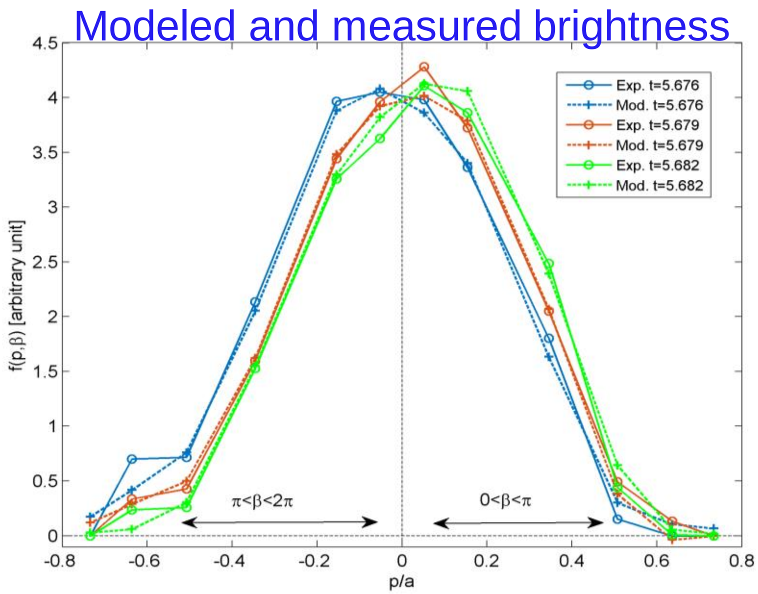


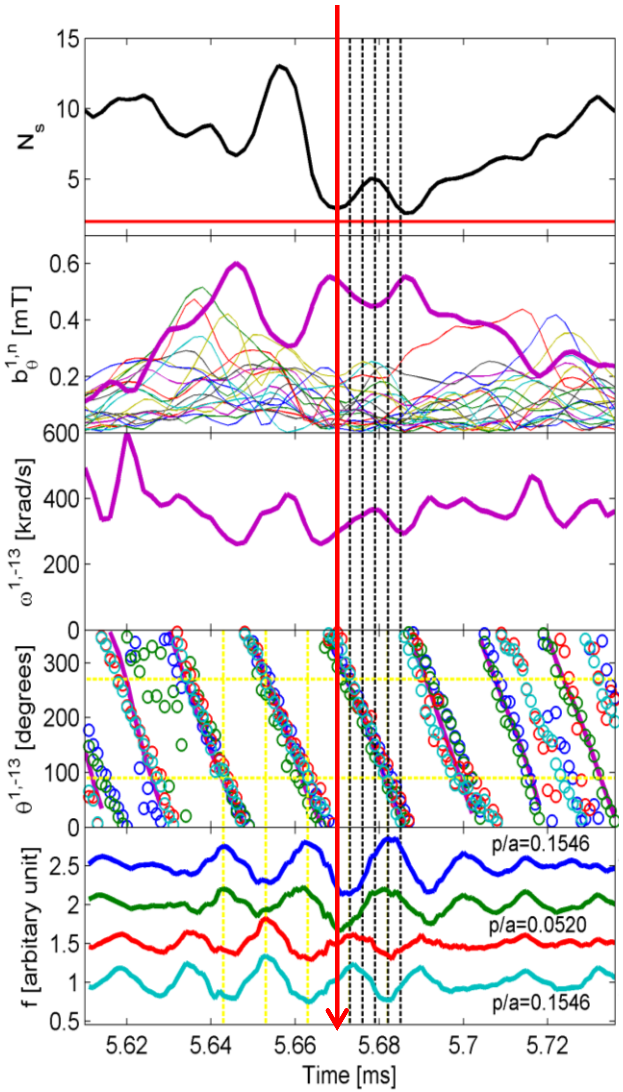


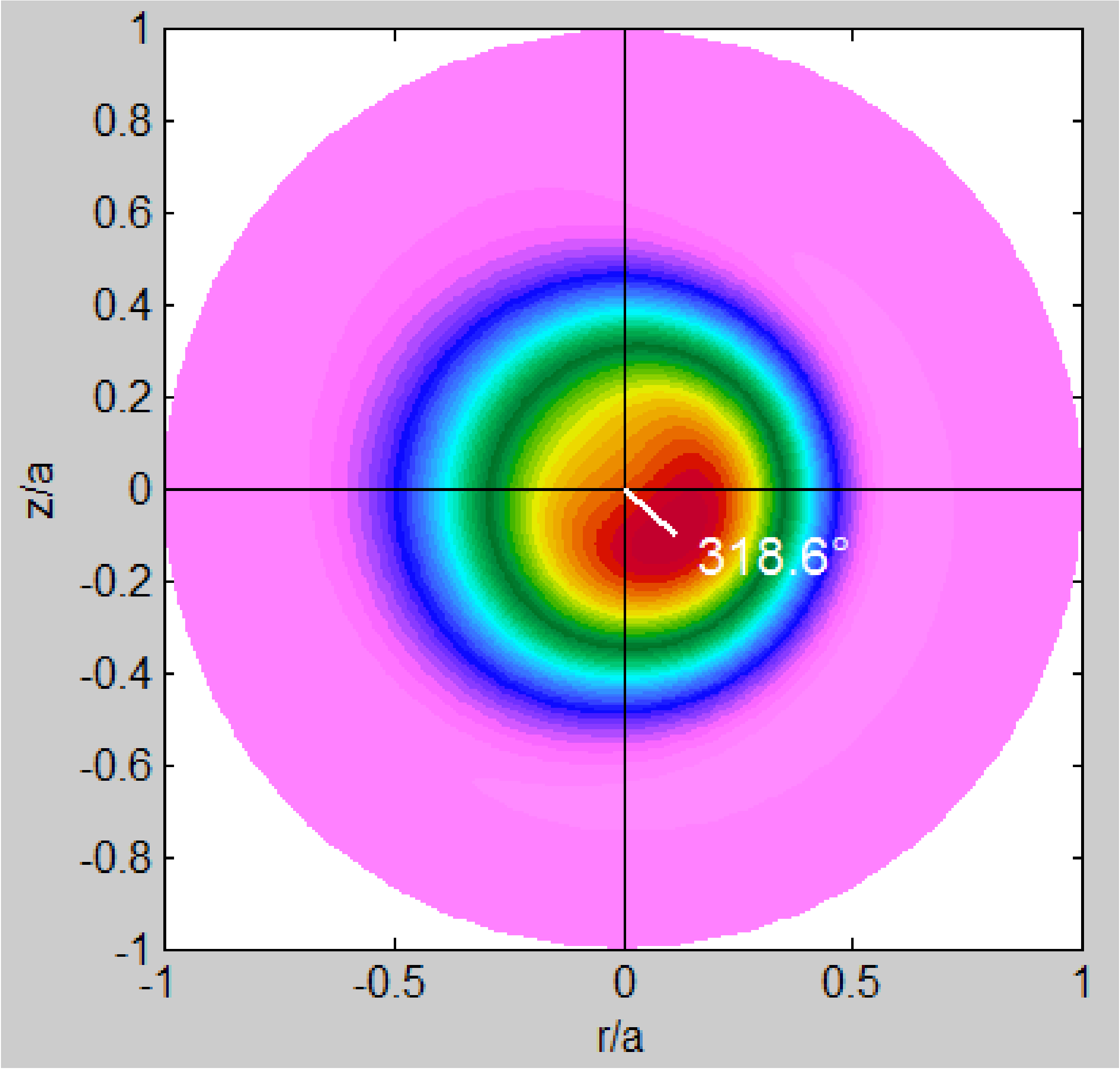




SXR imaging EXTRAP T2R

- Reconstruction of six instants (almost one period in the SXR and magnetic signal)





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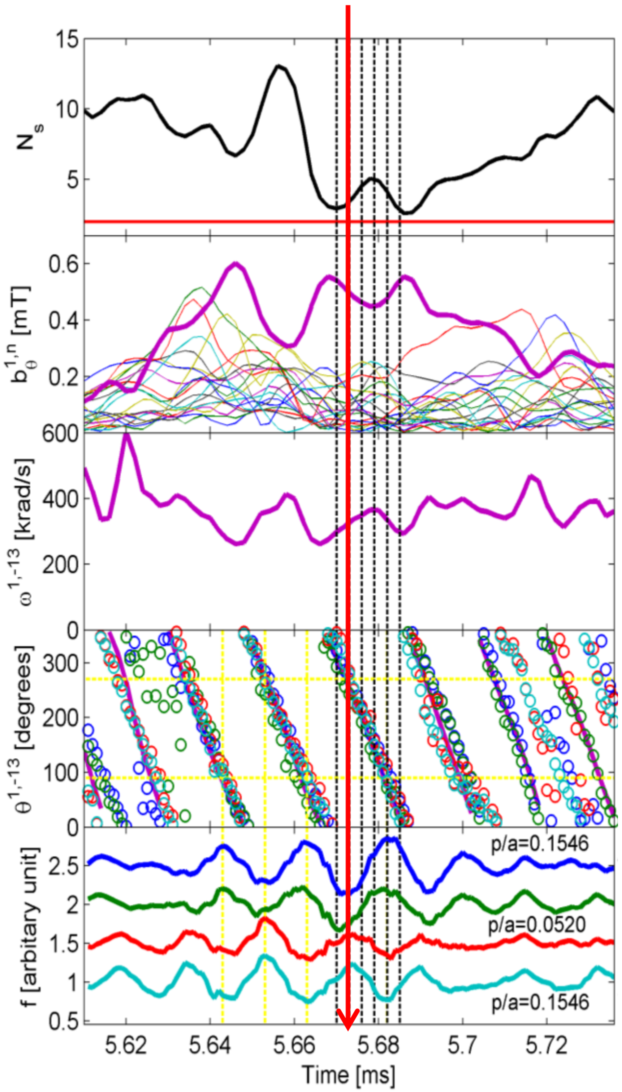

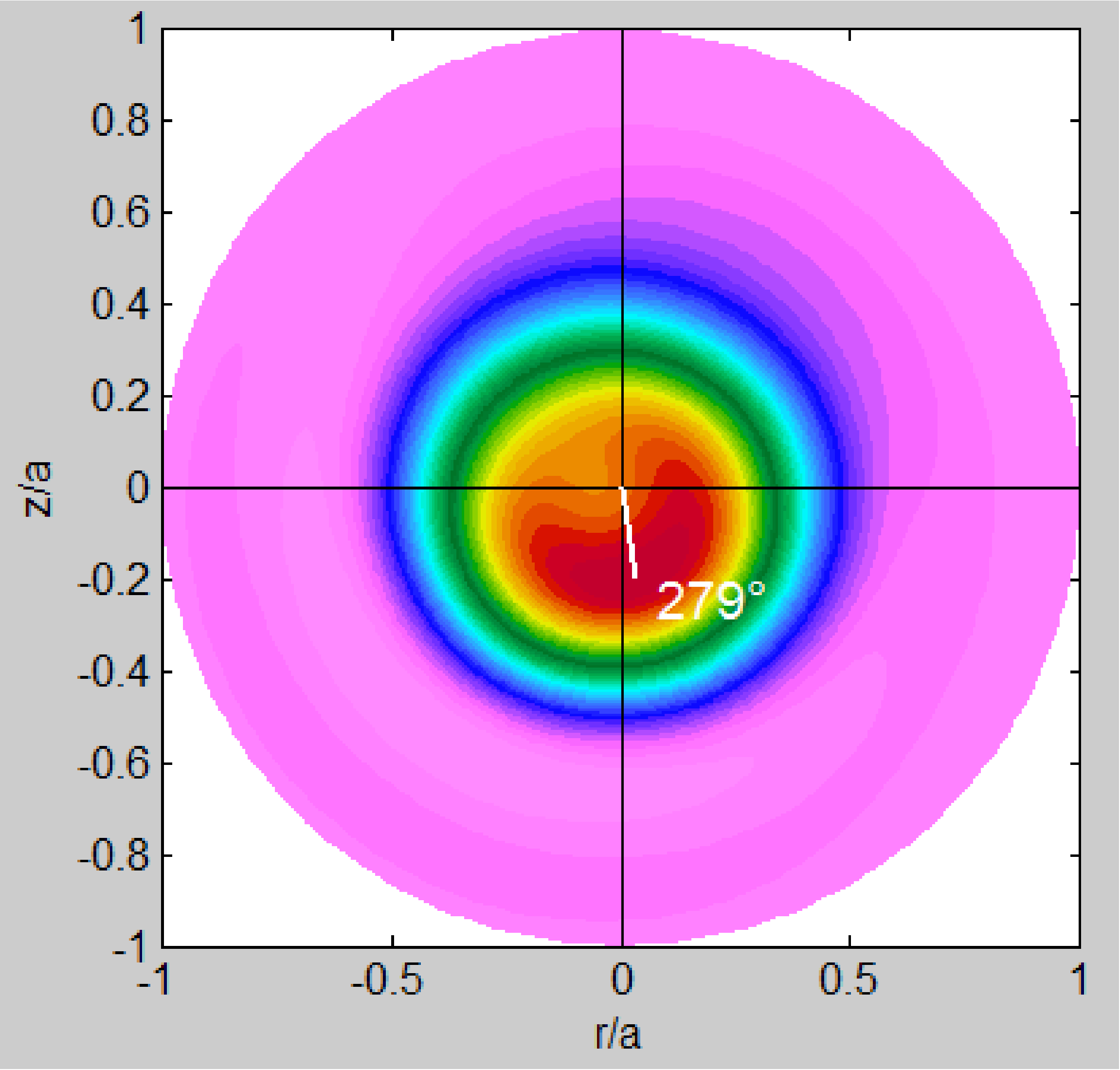

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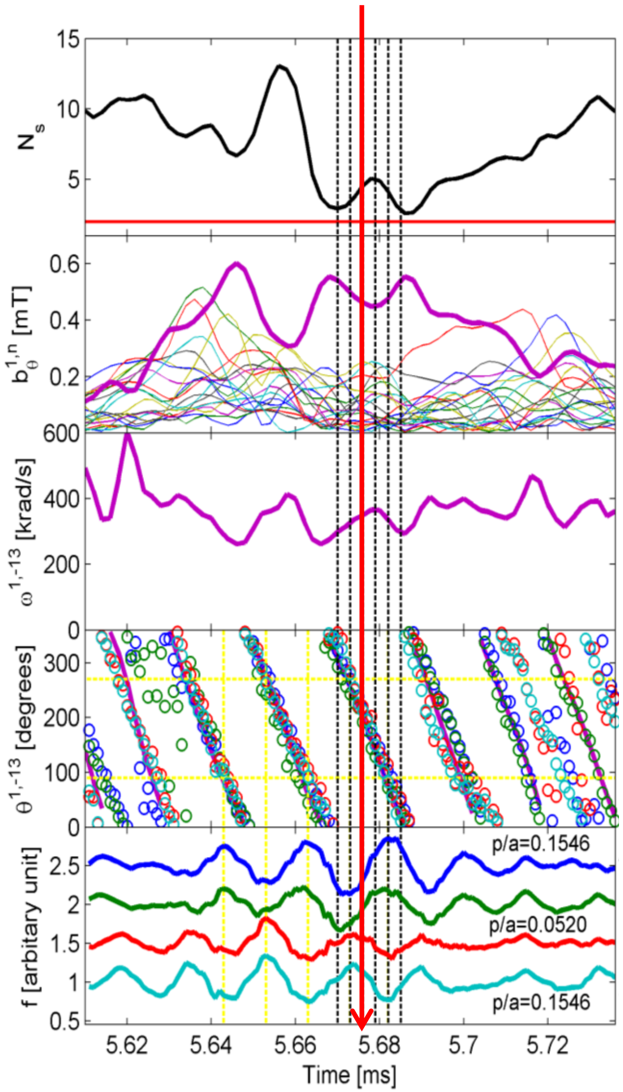

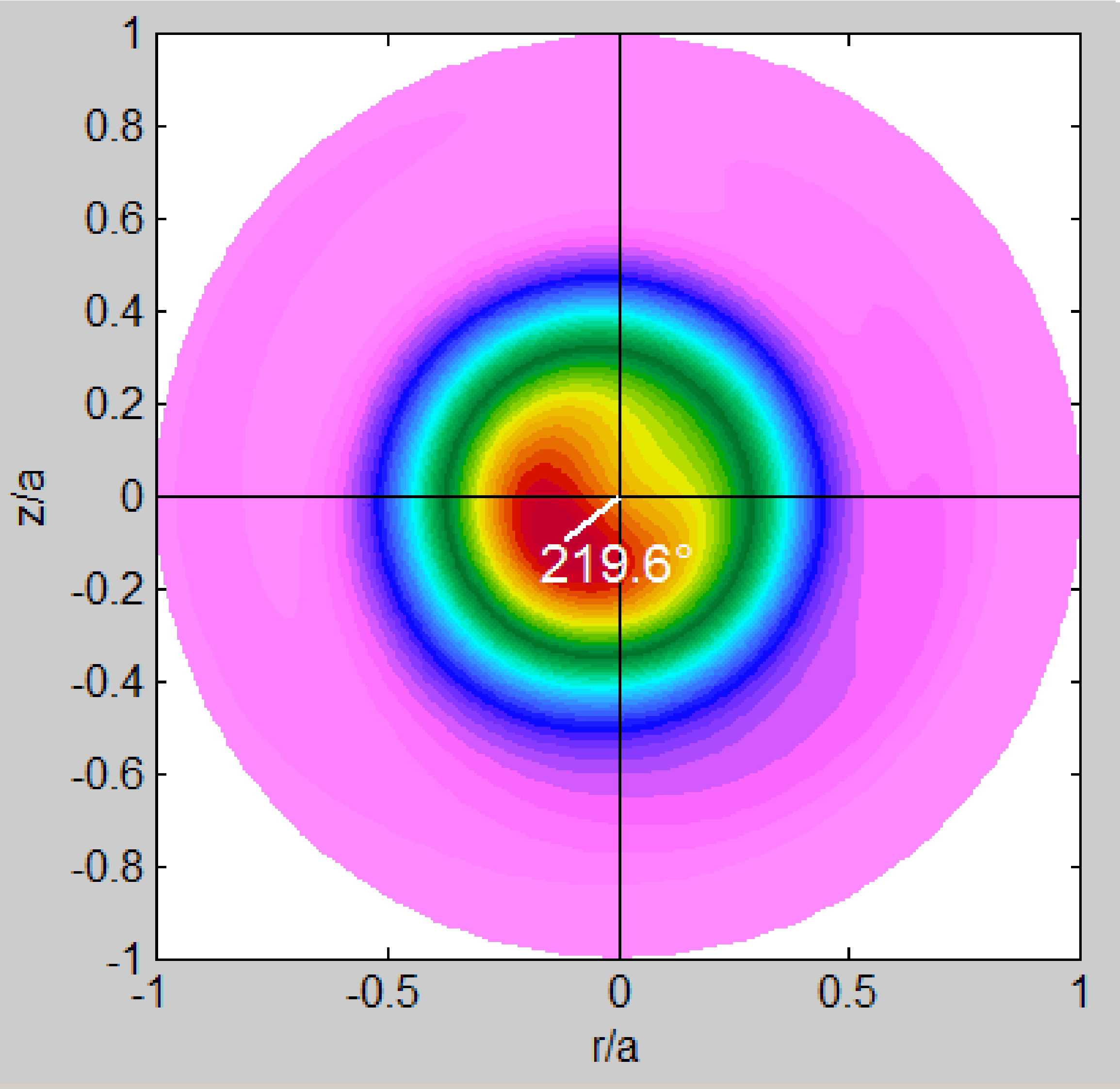

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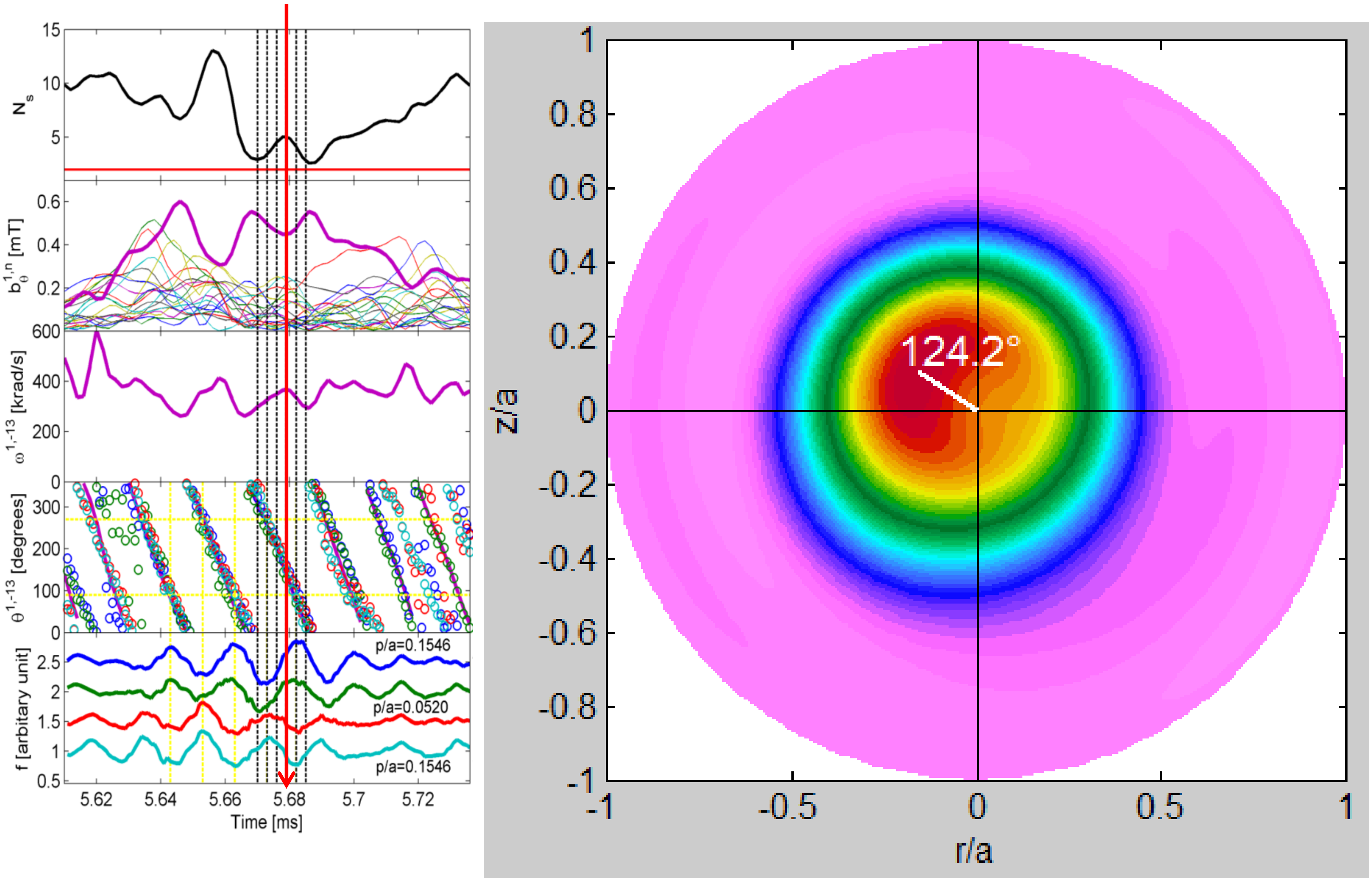

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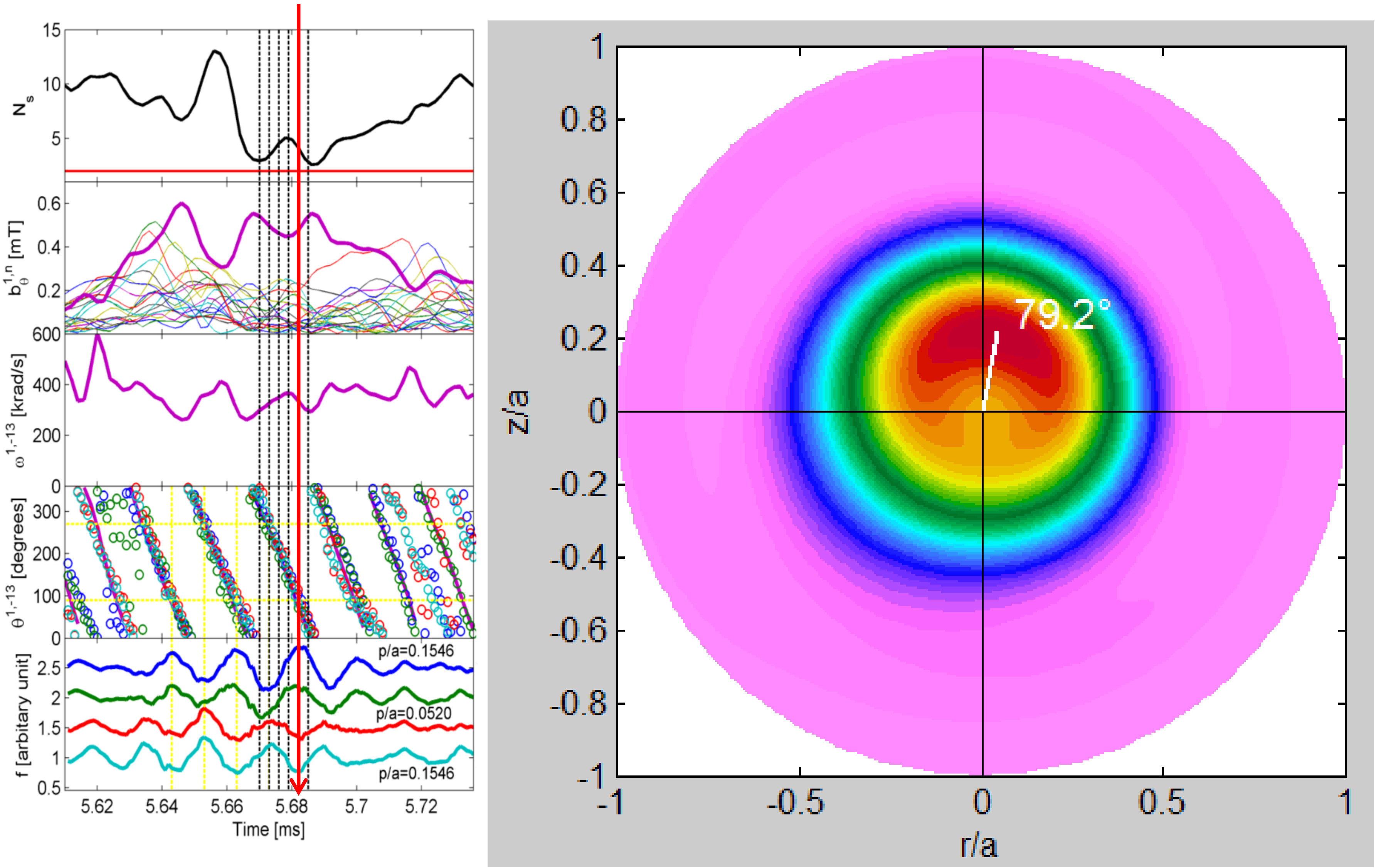

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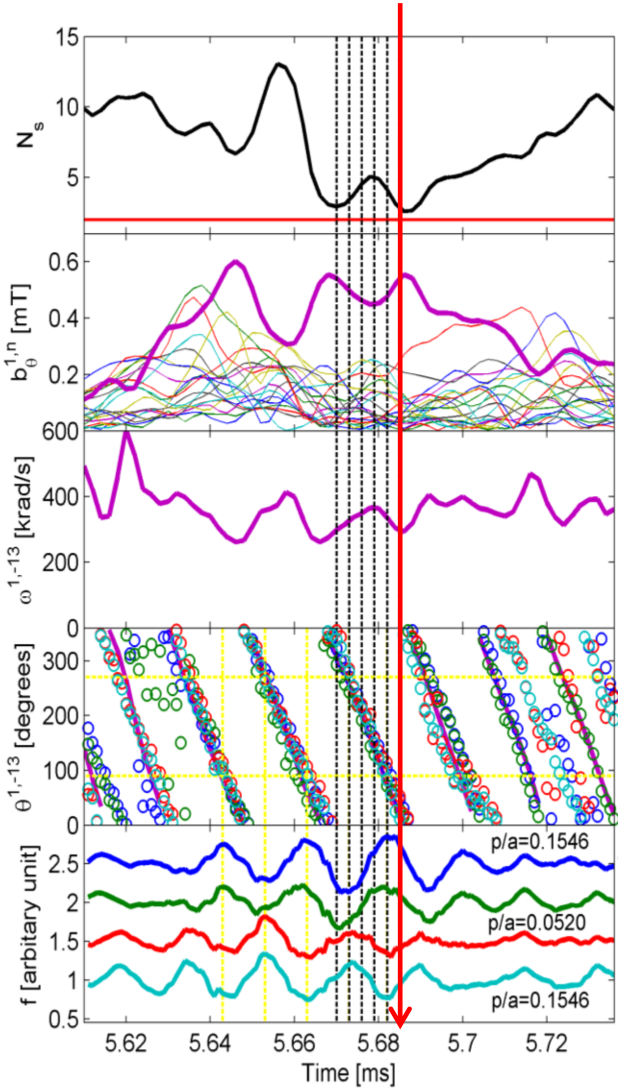
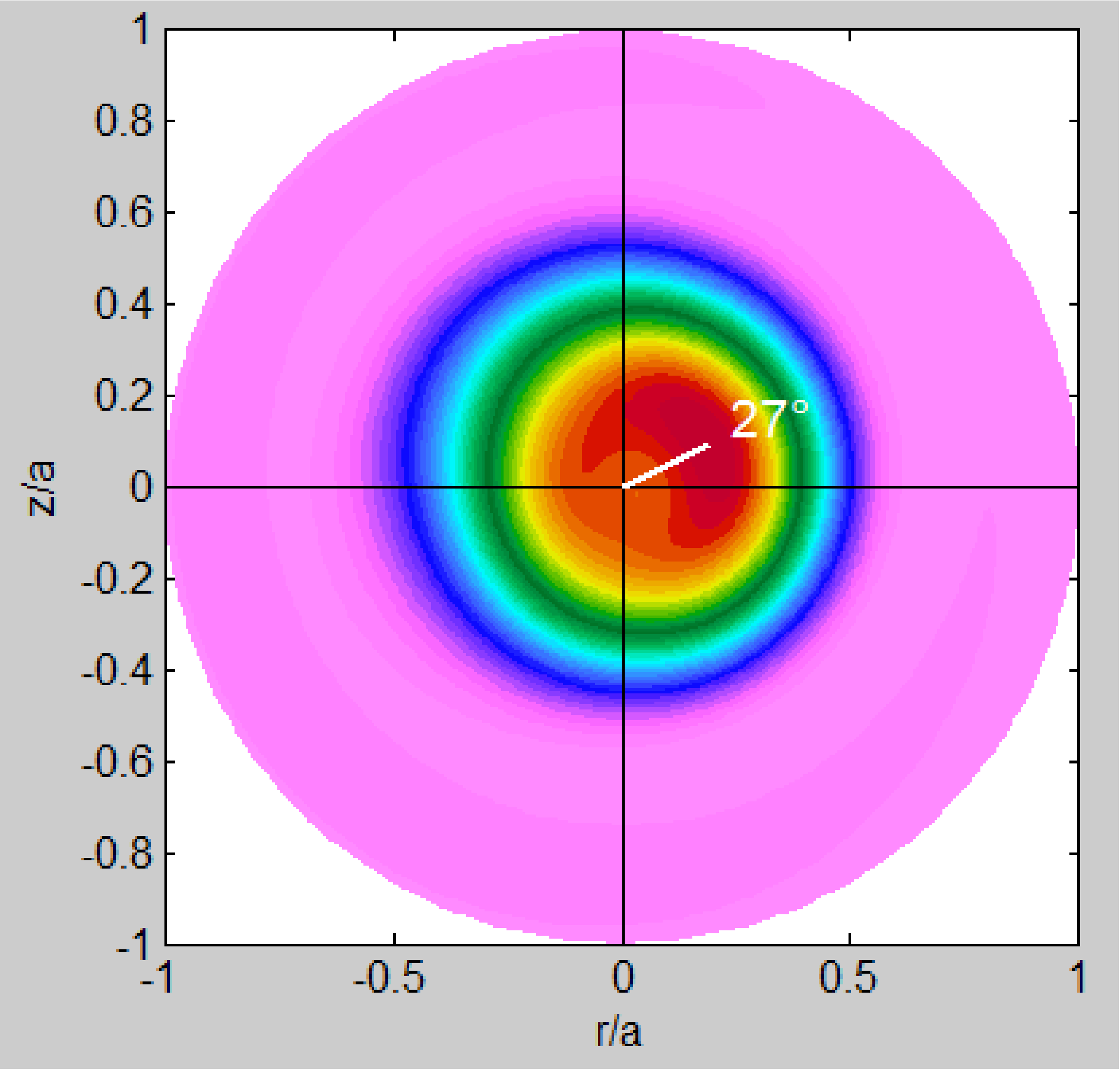
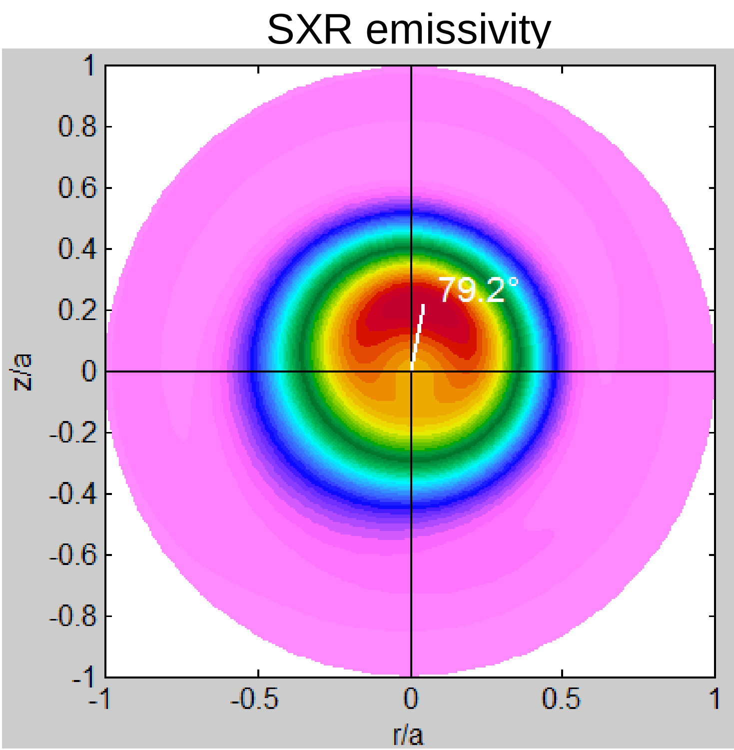
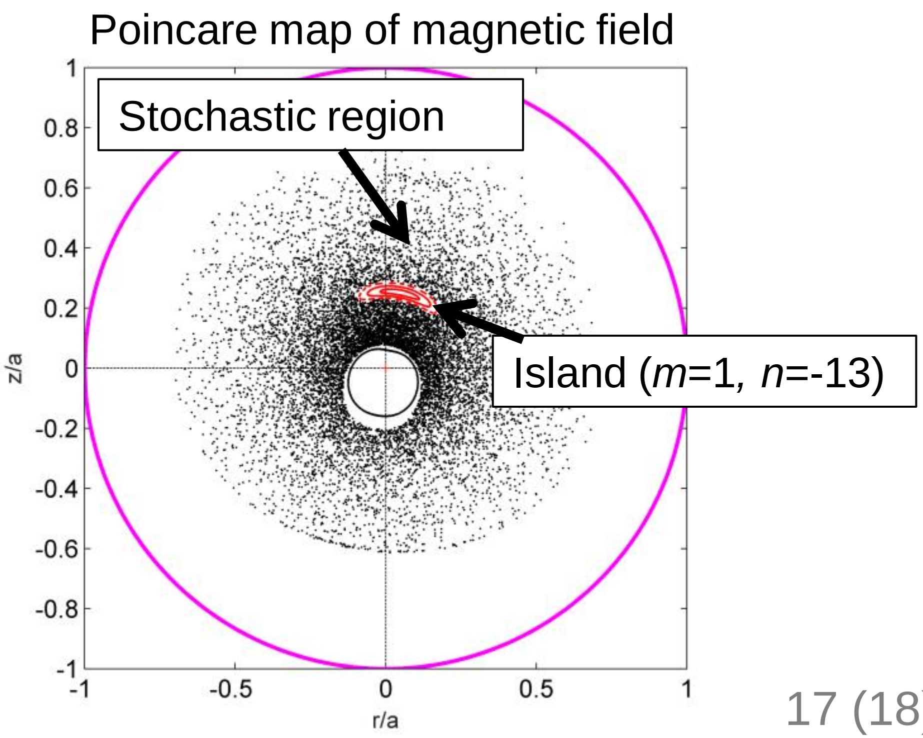




SXR map and Poincare map of magnetic field

- Field line tracing: $\frac{dr}{b_r} = \frac{rd\theta}{B_\phi} = \frac{Rd\phi}{B_\theta}$,
  - Magnetic field: $\bar{B} = B_\theta\hat{\theta} + B_\phi\hat{\phi} + \overline{\delta b}$,
  - perturbation: $\overline{\delta b} = b_r\hat{r} + b_\theta\hat{\theta} + b_\phi\hat{\phi}$, where $b_r = \sum_n b_{r,n}(r)cos(m\theta + n\phi + \alpha_n)$
  - B_θ and B_ϕ from equilibrium reconstruction
  - $b_{r,n}(r)$: profile from Newcomb's equation and amplitude via edge measurements





Conclusion

- In quasi-single helicity:
  - Reconstructed SXR show a *m*=1 structure
  - SXR and magnetic structure has the same
    - ✓ Position (angle and radius)
    - ✓ Rotational velocity