GNR 607 Project

Problem Statement

Compute the Fourier transform of an image and generate a low pass filtered image using Gaussian and Butterworth filters.

Fourier Transform of a Image

In 1-D Fourier transform equation is: $$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

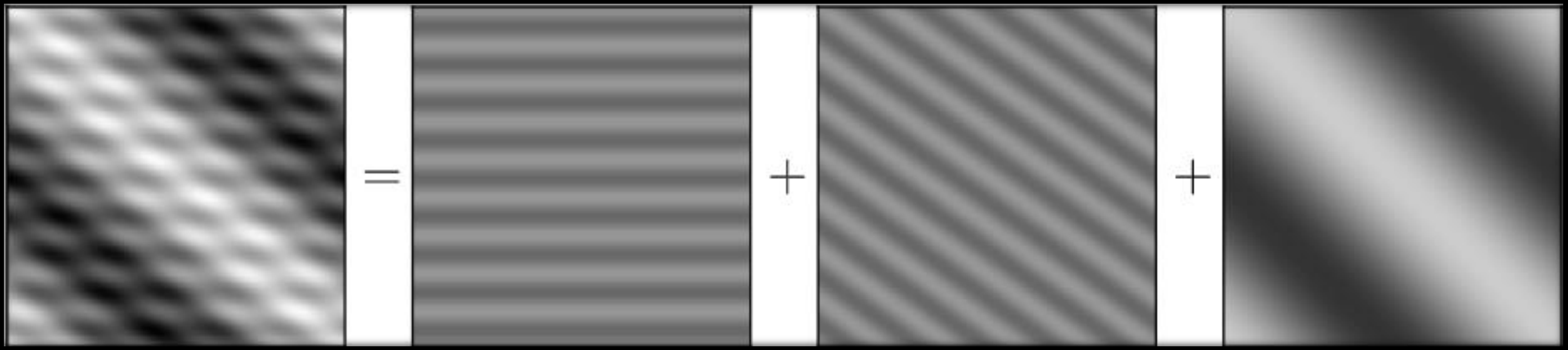
In 2-D Fourier transform equation is: $$F(\omega_1, \omega_2) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f(m,n)e^{-j\omega_1 m}e^{-j\omega_2 n}.$$

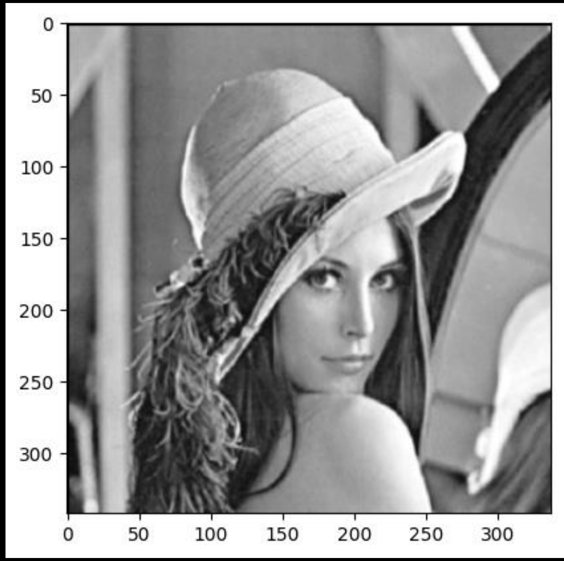
Images can be thought of as composed of different spatial frequencies.

High frequencies represent details and edges, while low frequencies correspond to smoother areas.

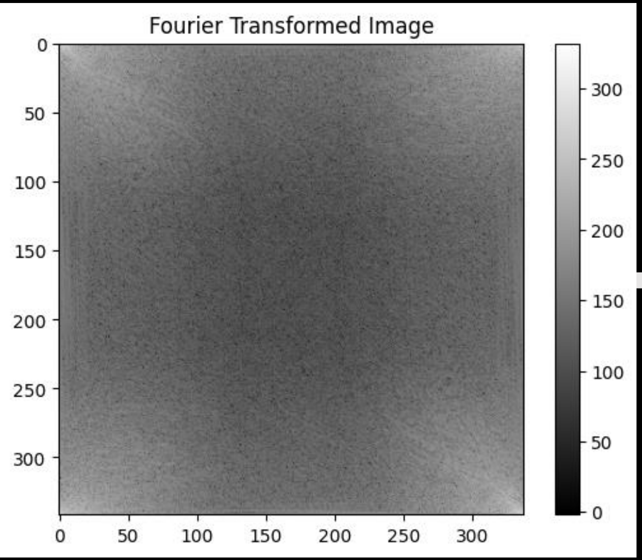
By applying Fourier Transform, you can analyze which frequencies are dominant in an image.

For example

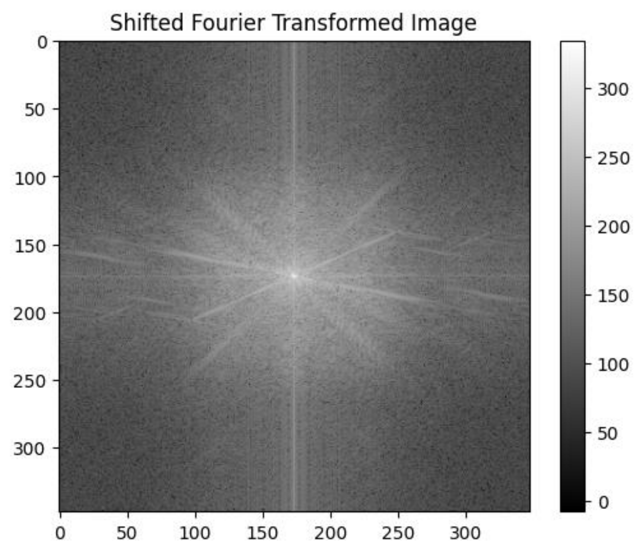




F.T



Shifted the origin to centre of image

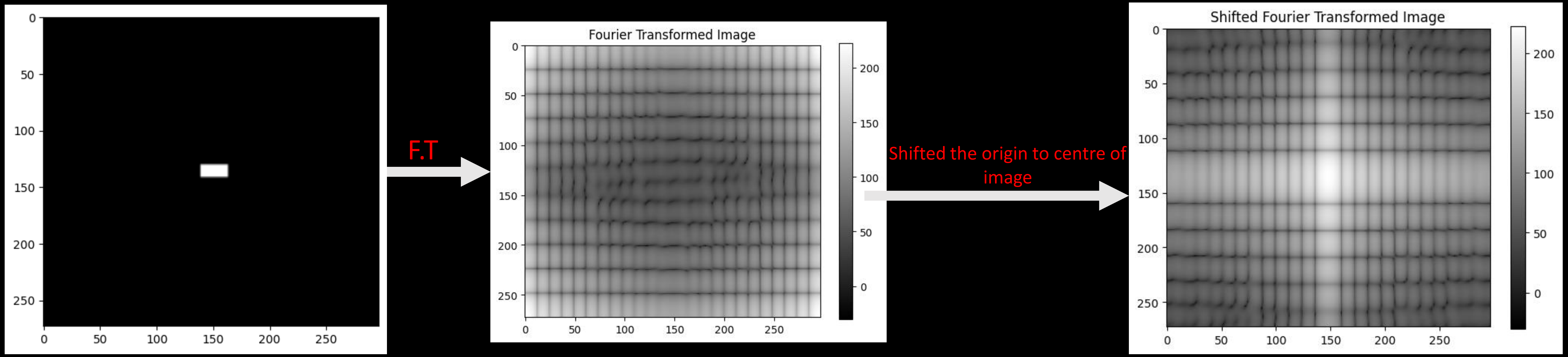


(Logarithmic Expansion applied)

The central part includes the low frequency components. The bright pixels in frequency image (above) indicate strong intensity at the corresponding positions in spatial image (original image).

The low frequency domain corresponds to the smooth areas in the image (such as skin, hat, background, etc.). The high frequency domain, which is shown in the image away from the central part, includes those sharp edges or some structure with the changes of intensity dramatically along one orientation (such as the hairs, boundary of hat, etc.).

But since the intensity for those parts is not high enough compared with the smooth structure, the corresponding regions appear dark on the shifted transformed image. Note that the camera lens focuses on Lenna, so the background is blurred. If the focus region is at background, the vertical lines behind Lenna would be clear, and the sharp edges of the lines will contribute to high frequency magnitude, thus region away from the center on the right image would be bright.

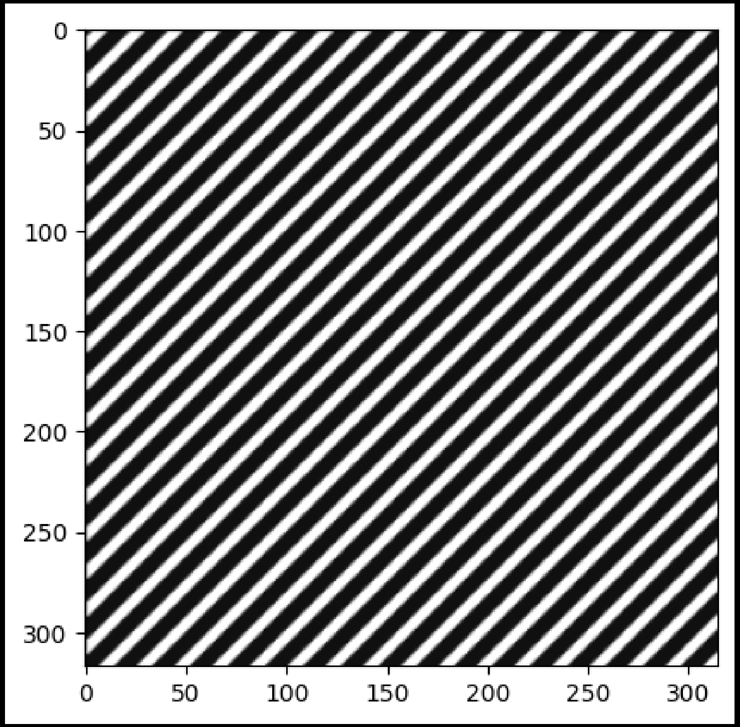


(Logarithmic Expansion applied)

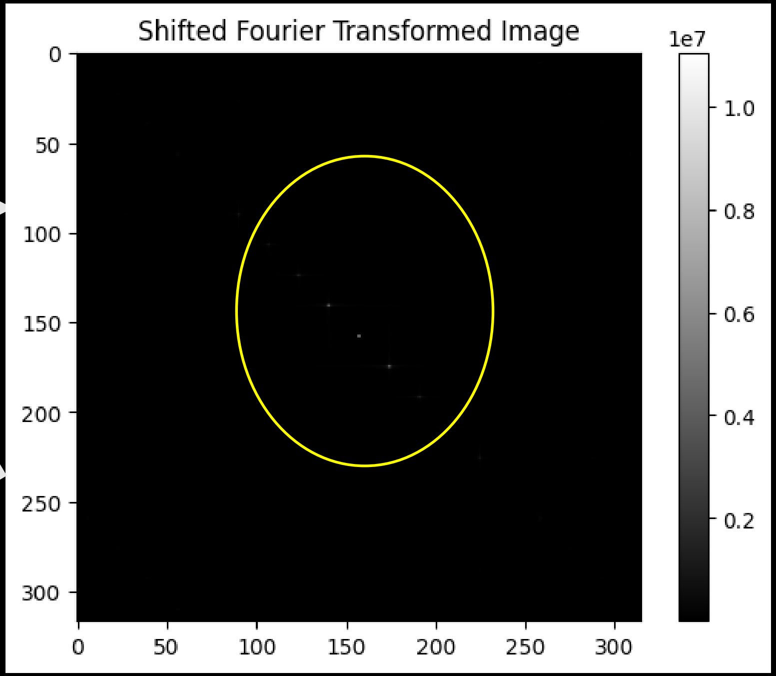
A **black** image has predominantly low-frequency components since there's minimal variation in pixel values. When you introduce a white square (an area with high pixel value contrast) in the middle, it introduces high-frequency components since there's a sudden change in pixel values within that area.

The varying intensity within the square (from black to white) creates higher spatial frequencies, and these translate into higher frequency components in the Fourier Transform, which manifest as the checkered pattern.

This pattern reflects the presence of both low and high-frequency information, with the low frequencies mainly in the surrounding black region and higher frequencies primarily concentrated in the square.

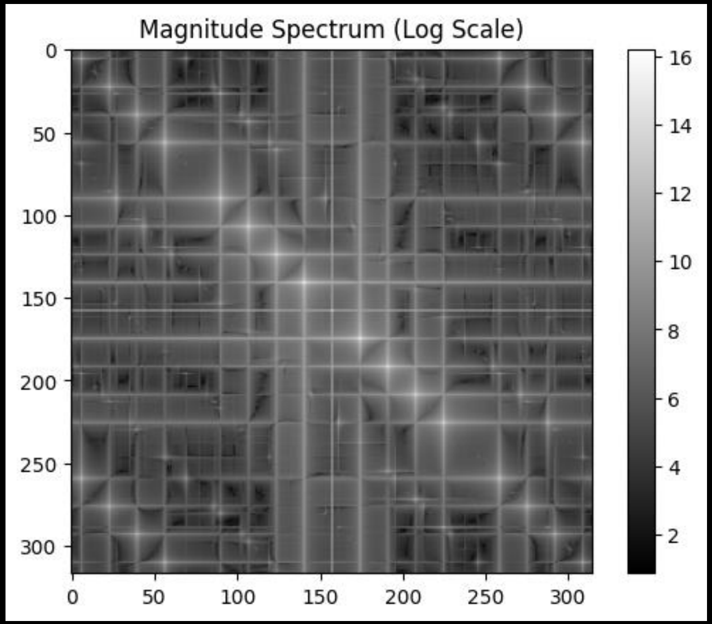


F.T, shift



(**WITHOUT** Logarithmic Expansion)

Logarithmic Expansion



When applying the Fourier Transform to this striped pattern, The main components of the transformed image are the DC-value and the **two points corresponding to the frequency of the stripes**. However, the logarithmic transform of the Fourier Transform, shows that now the image contains many minor frequencies. The main reason is that a diagonal can only be approximated by the square pixels of the image, hence, additional frequencies are needed to compose the image. The logarithmic scaling makes it difficult to tell the influence of single frequencies in the original image

Gaussian Filter

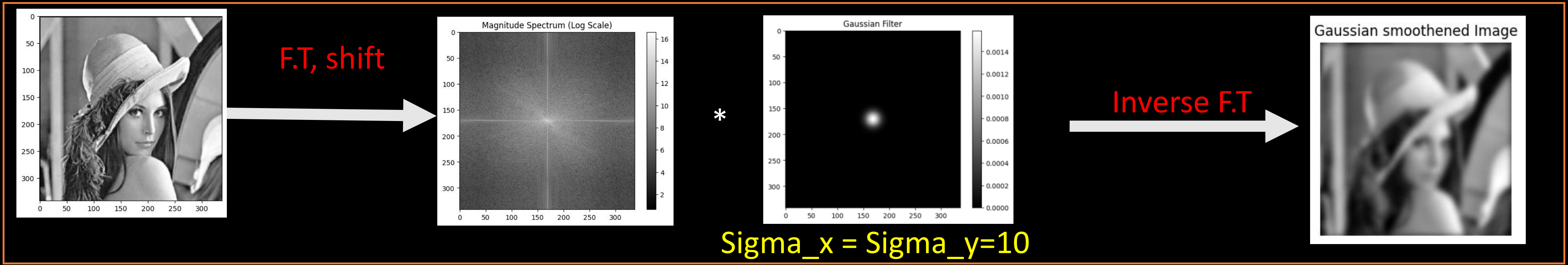
A Gaussian Filter is a low pass filter used for reducing noise (high frequency components) and blurring regions of an image.Since we already have image in the Frequency/Fourier domain, we can directly multiply the image pixel matrix with the filter matrix, Hence the filter matrix is made of appropriate size for each image so that matrix multiplication

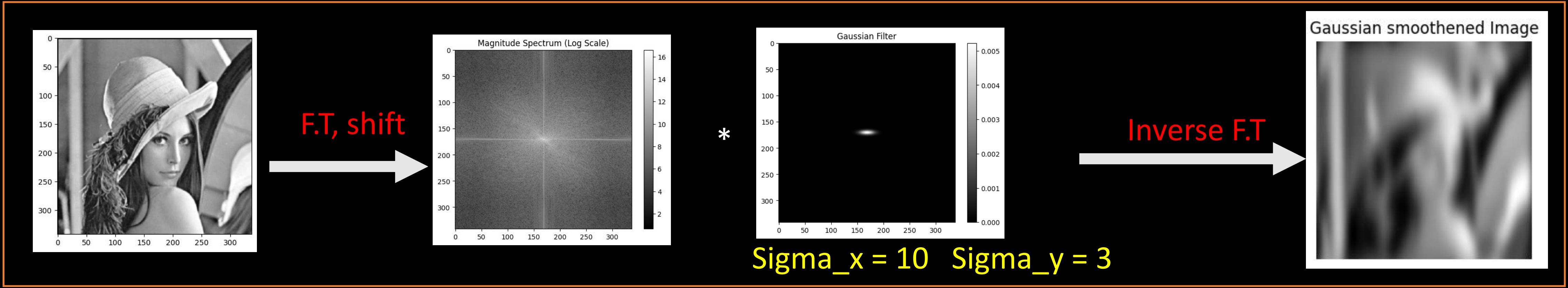
To obtain the Gaussian filter, we can apply the formula $g(x,y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)}$ by choosing appropriate sigma.

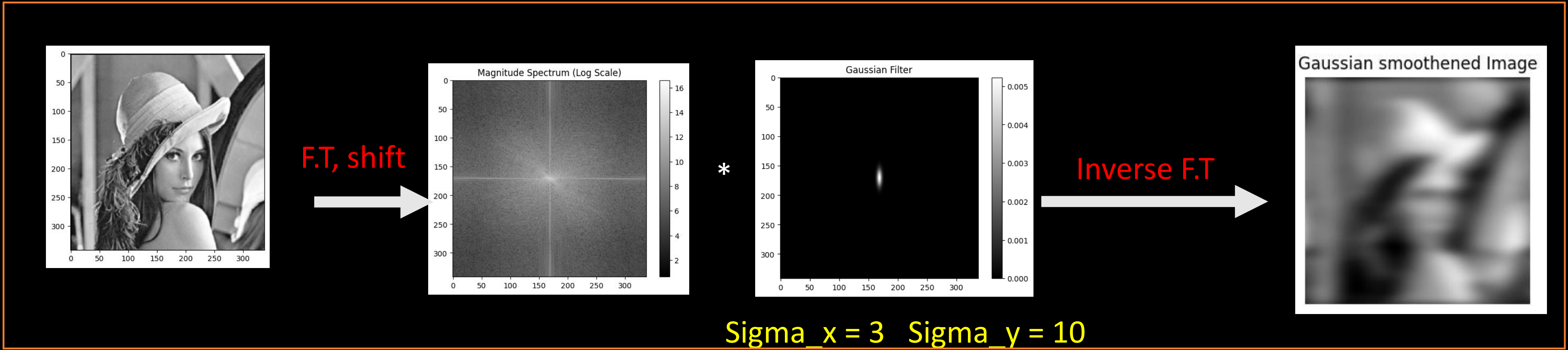
More the sigma, more will the cutoff frequency, more will be the spread of the filter, Basically cutoff frequency will be very high and less blurring will take place.

If sigma is very low, then cutoff frequency will be very low and the image will be highly blurred.

We have made the algorithm so as to inculcate different sigma's for different dimensions, so effectively the formula used for making the filter is $\frac{1}{2\pi\sigma_x\sigma_y} \cdot \exp\left(-\frac{\left(x-\frac{l-1}{2}\right)^2}{2\sigma_x^2} - \frac{\left(y-\frac{b-1}{2}\right)^2}{2\sigma_y^2}\right)$. Some demonstrations are given on next slide.


F.T, shift
Magnitude Spectrum (Log Scale)
*
Gaussian Filter
Inverse F.T
Gaussian smoothened Image
Sigma_x = Sigma_y=10


F.T, shift
Magnitude Spectrum (Log Scale)
*
Gaussian Filter
Inverse F.T
Gaussian smoothened Image
Sigma_x = 10 Sigma_y = 3

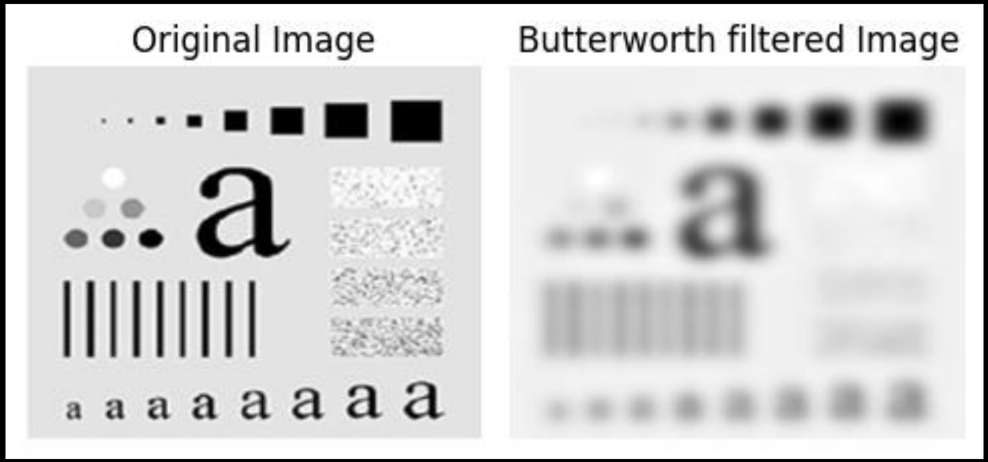

F.T, shift
Magnitude Spectrum (Log Scale)
*
Gaussian Filter
Inverse F.T
Gaussian smoothened Image
Sigma_x = 3 Sigma_y = 10

Butterworth Filter

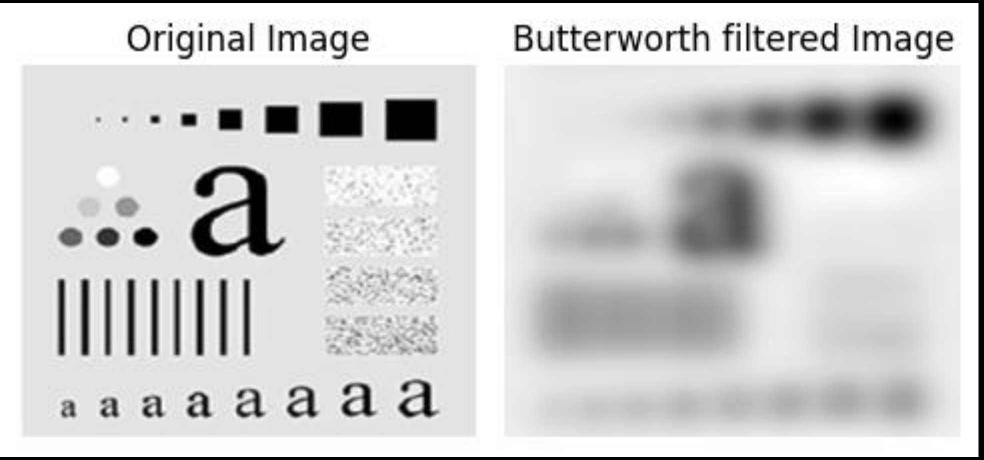
Another version of smoothing/ sharpening filters are the Butterworth filter. A Butterworth filter of order n and cutoff frequency D_0 is defined as*: $$H(u,v) = \frac{H_0}{1+\left(\frac{D(u,v)}{D_0}\right)^{2n}}$$

A Butterworth low pass filter keeps frequencies inside radius D_0 and discard value outside it introduces a gradual transition from 1 to 0 to reduce ringing artifacts. The transfer function of a Butterworth low pass filter (BLPF) of order "n", and with cutoff frequency at distance D_0 from the origin.

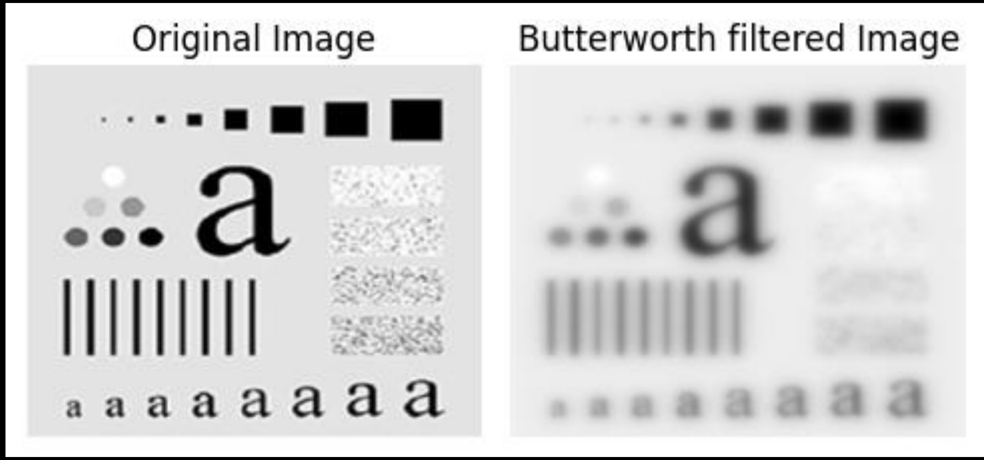
*Assume Ho =1



Order= 2 Cutoff Frequency= 10



Order=2 Cutoff Frequency=5



Order= 1 Cutoff Frequency= 10