High Frequency Price Movement Strategy

Adam, Hujia, Samuel, Jorge

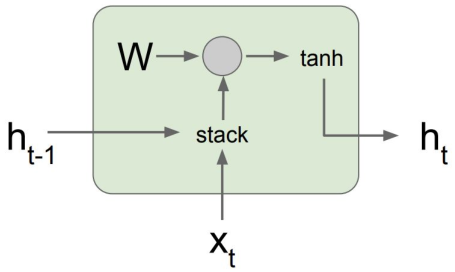
Overview

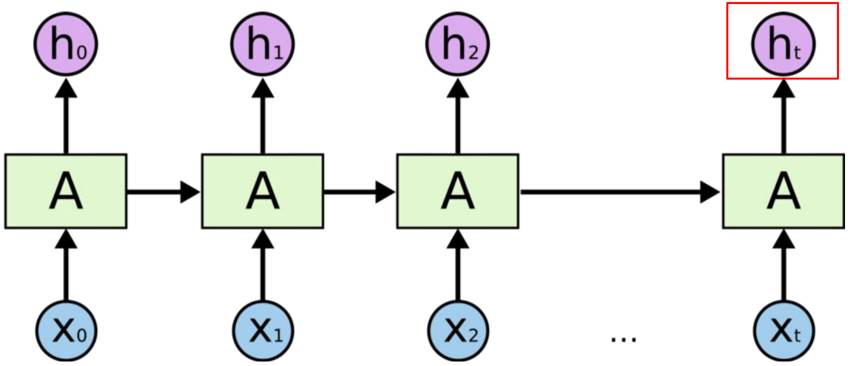
Deep Learning Strategy	● RNN Overview ● Feature and Label Generation ● Model Formation ● Strategy ● Results
Statistical Arbitrage Strategy	● Statistical Arbitrage Overview ● Finding Correlated Pairs ● Stochastic Control ● Parameter Tuning ● Results
Conclusion	● Future Work

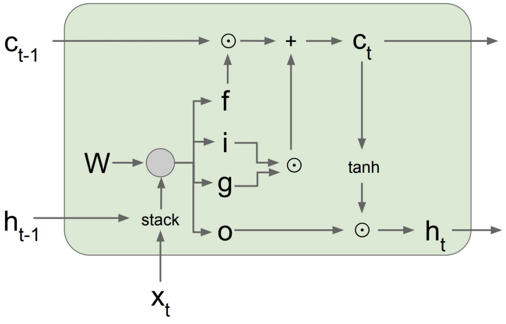
RNN Strategy

Recurrent Neural Networks (RNN)

- Family of Neural Network specialized for sequence data
- ‘Many-to-One’ architecture
- ‘Vanilla’ vs. Long short-term memory (LSTM)







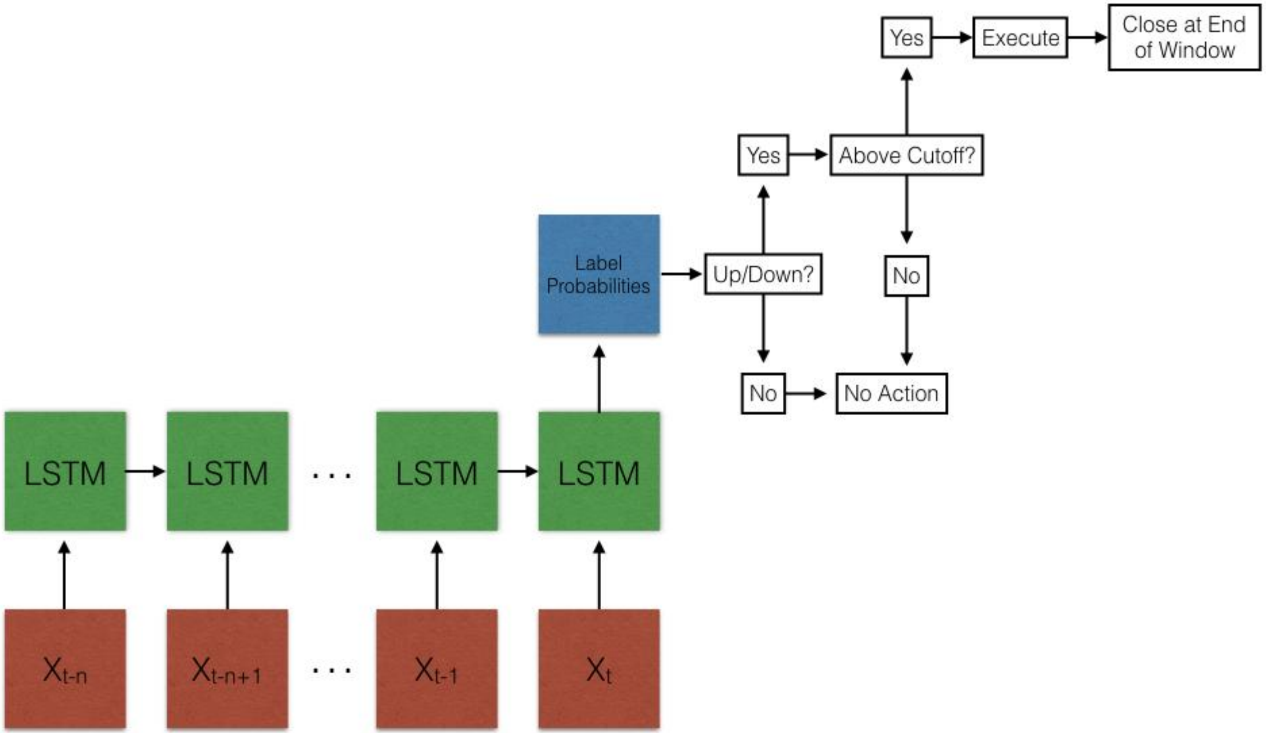
RNN: Feature and Label Generation

Features	Labels (Classification & Regression)
• Bid/Ask Prices and Spread (10 levels) • Volumes (10 levels) • Mean Prices and Volumes • Accumulated Price and Volume Differences • Price and Volume Changes • Order Imbalance Changes • VWAP	• Mid-Price Movement • Volume Weighted Average Price (VWAP) Movement • Settled on classifying VWAP movement over the next time ‘window’

RNN: Model Formation

- Cost Function: **Weighted Cross Entropy**
  - Helps solve challenge of having an imbalanced dataset
- Output: **Softmax Layer**
  - Outputs a predicted probability for each label
- Unit: **LSTM**
  - Long short-term memory (LSTM) units to model longer term dependencies
- Hyperparameters:
  - Number of Units
  - Prediction Window for Label
  - **Trade Probability Cutoff**
  - Cross Entropy Weights
  - Other (e.g. Learning Rate, Dropout)

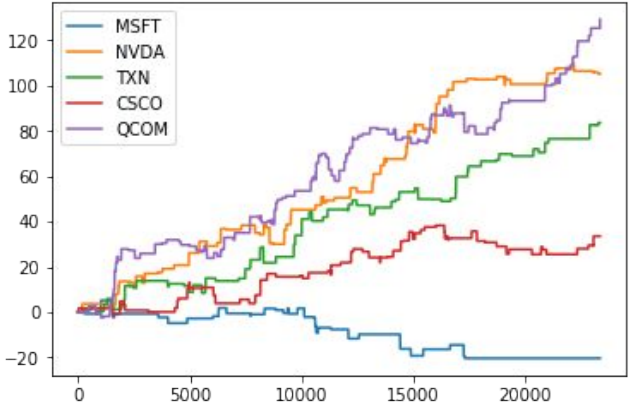
RNN: Strategy



RNN: Results and Next Steps

- **The Good**:
  - Profitable for majority of stocks on the test set
  - Generally steady profits throughout day
  - Results consistent with baseline
- **The Bad**:
  - Traded at the mid-price!
  - Couldn't use Thesys backtester
  - Scalability
- **Next Steps**:
  - Incorporate magnitude of movement
  - Regularization
  - Different ML Models

Single Day Chart



Execution Accuracy (Entire Test Set)

MSFT	NVDA	TXN	CSCO	QCOM
47.6%	53.7%	53.9%	51.5%	54.6%

Statistical Arbitrage Strategy

Baseline model: Pairs trading with Avellaneda-Lee

- Linearly **regress the mid-price** returns of a pair of historically correlated stocks
- **Fit the residuals** to a OU-process (using AR(1) model)

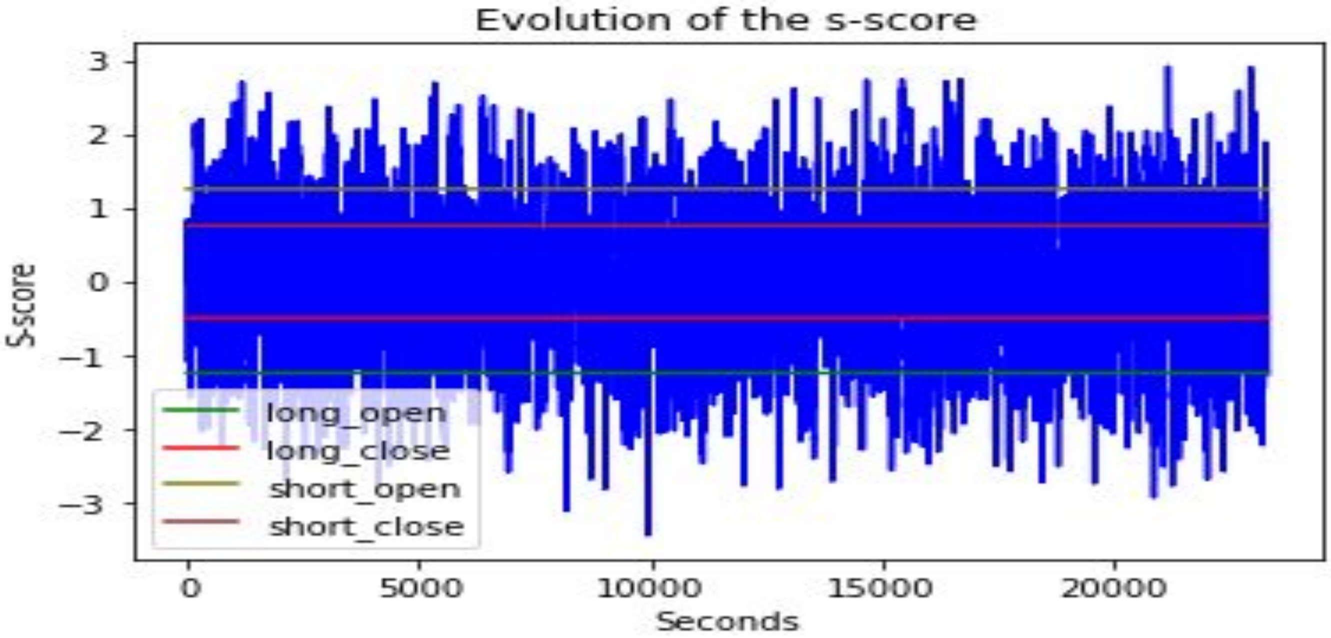
$$dr_t = \kappa(a - r_t)dt + \sigma dW_t$$

- **Mispricing** (and execution) if the last observation is far from the equilibrium

$$|S - Score| = \frac{|r_n - \overline{OU}|}{\sigma(OU)} > Threshold$$

:

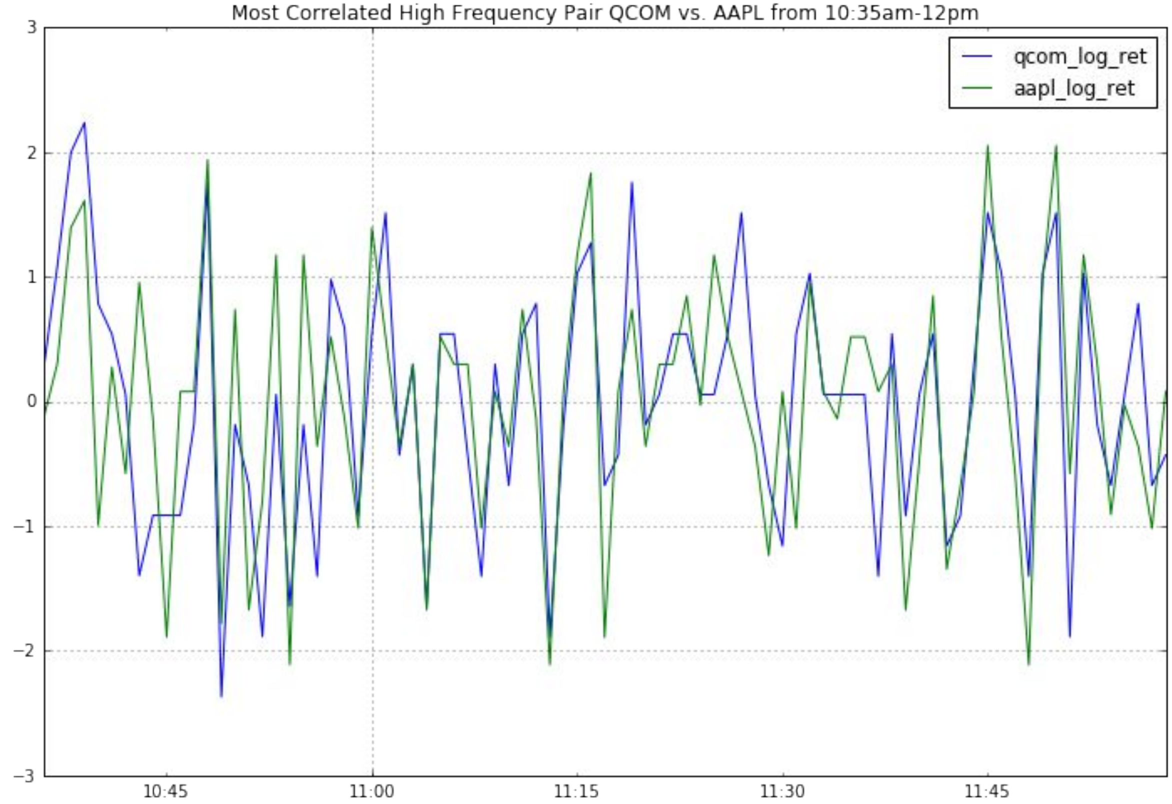
Example of execution process

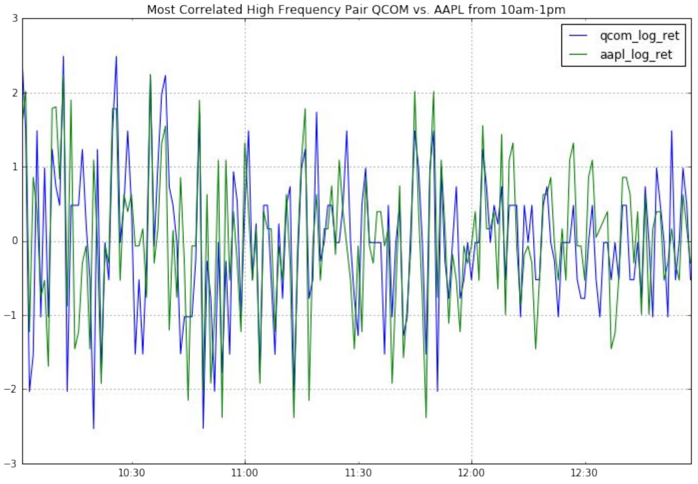


What’s New?

- Identifying **most correlated pairs** to trade
- **Stochastic control** to incorporate dynamically optimal thresholds
- **Hyperparameter tuning** (frequency, training size, leverage, etc.)

Most Correlated High Freq Pair Example





QCOM AAPL
Correlation: 0.701826
60 seconds interval

Date: 20170407

Most Correlated Pair for a 1-day Window.

Time Interval(s)	Most Correlated Pair	Correlation
1	(AAPL, FB)	0.253802
5	(AAPL, MSFT)	0.420605
10	(QCOM, TXN)	0.50241
20	(FB, AAPL)	0.565774
30	(QCOM, TXN)	0.609936
60	(QCOM, AAPL)	0.701826

Table 1. Most Correlated Pair for a 1-day Window for Different Time Intervals.

- Note that the most correlated pair is different for different time intervals.

Stochastic control (Cartea-Jaimungal-Peñalva)

- Motivation: Now **fixed, ad-hoc thresholds,** requiring calibration
- Idea: **choose automatically and dynamically the best** thresholds
- Technique: **stochastic control** (i.e. maximize the expected utility of the strategy)

Criteria for exiting a long/short position

$$H_+(t,r) = \sup_{\tau_+} \mathbb{E}_{t,r}[e^{-\rho(\tau_+ - t)}(r_{\tau_+} - c)] \qquad H_-(t,r) = \sup_{\tau_-} \mathbb{E}_{t,r}[e^{-\rho(\tau_- - t)}(-r_{\tau_-} - c)]$$

Criteria for entering the position

$$G(t,r) = \sup_{\tau} \mathbb{E}_{t,r}[e^{-\rho(\tau_+ - t)}(H_+(\tau_+, r_{\tau_+}) - r_{\tau_+} - c)\mathbb{1}_{\tau_+ \wedge \tau_- = \tau_+} + e^{-\rho(\tau_- - t)}(H_-(\tau_-, r_{\tau_-}) + r_{\tau_-} - c)\mathbb{1}_{\tau_+ \wedge \tau_- = \tau_-}]$$

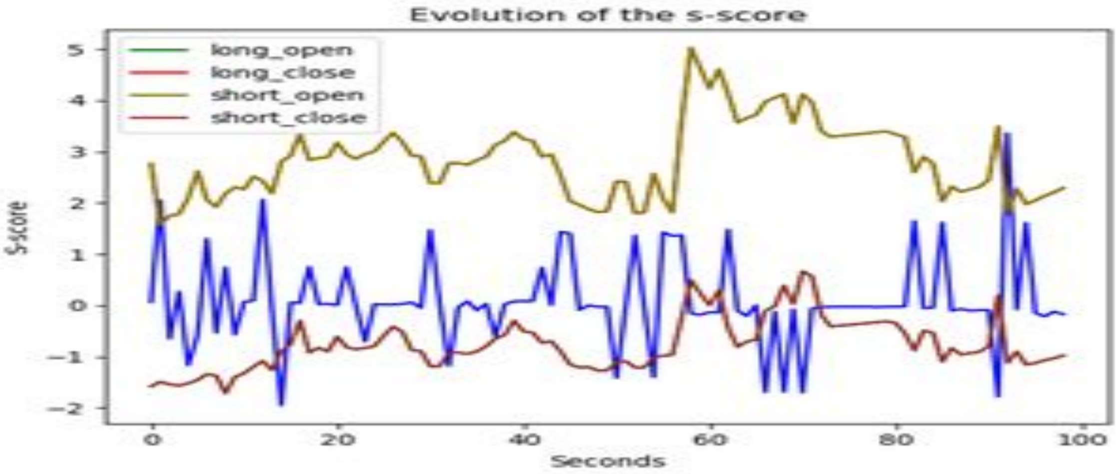
Stochastic control (2)

- **Optimal times are given by thresholds** depending on the OU parameters (so dynamic and automatically computable)
- They are found by **solving Hamilton-Jacobi-Bellman variational inequalities**

$$\max\{(\mathcal{L}-\rho)H_+(r);(r-c)-H_+(r)\}=0=\max\{(\mathcal{L}-\rho)H_-(r);(-r-c)-H_-(r)\}$$
$$\max\{(\mathcal{L}-\rho)G(r);(H_+(r)-r-c)-G(r);(H_-(r)+r-c)-G(r)\}=0$$

$$\mathcal{L}=\kappa(a-r)\partial_r+\frac{1}{2}\sigma^2\partial_{rr}$$

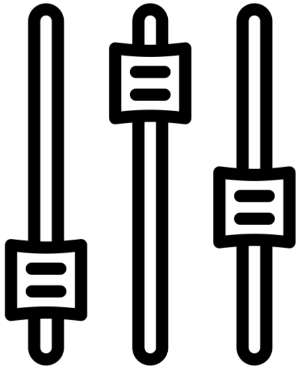
Stochastic control (3)



- **Numerically difficult problem,** computational issues
- **Depends** on the **calibration** method, the **utility function,** and the **numerical methods**

Parameter tuning

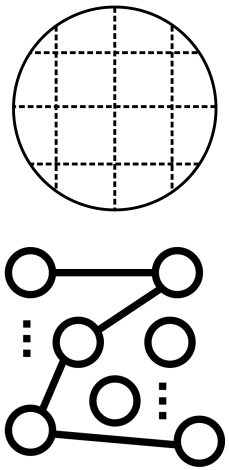
- Different environment than Avellaneda-Lee
- 4 - 5 parameters plus stock picking
  - Thresholds for trading
  - Time length for returns
  - Training size
  - Urgency parameter for stochastic control
  - Pairs to trade



Parameter tuning

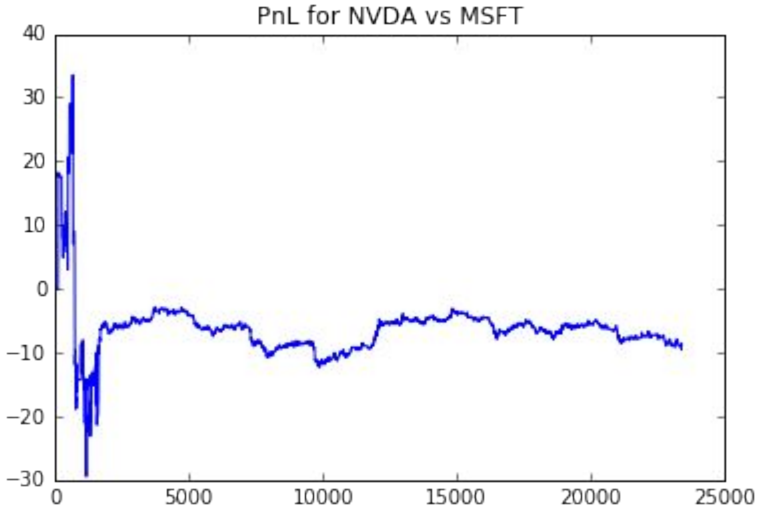
Two approaches to parameter tuning:

- Grid search
  - Systematic exploration
  - Enables for sensitivity analysis
  - Inefficient
- Random search
  - Black-box method
  - Explore larger subspace

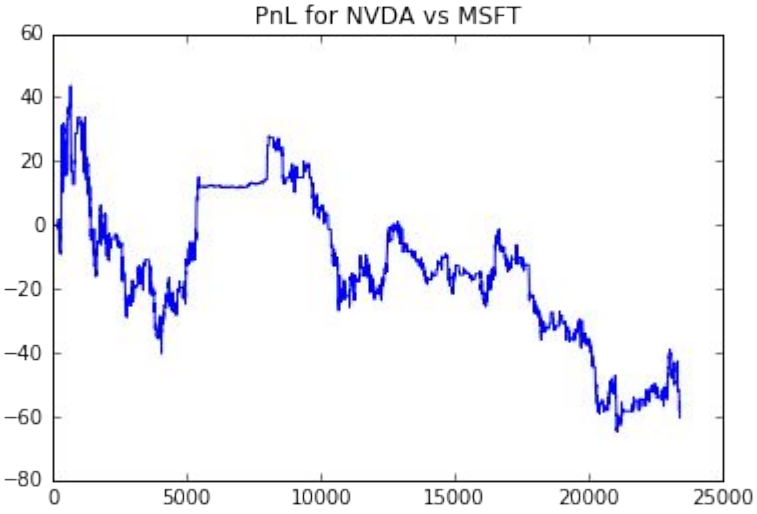


Parameter tuning

- Highly sensitive to changes in parameter values

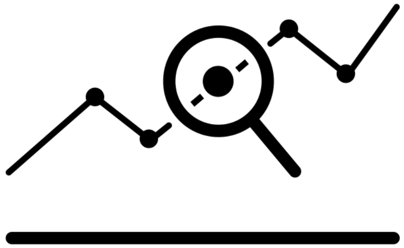


Training size of 100



Training size of 150

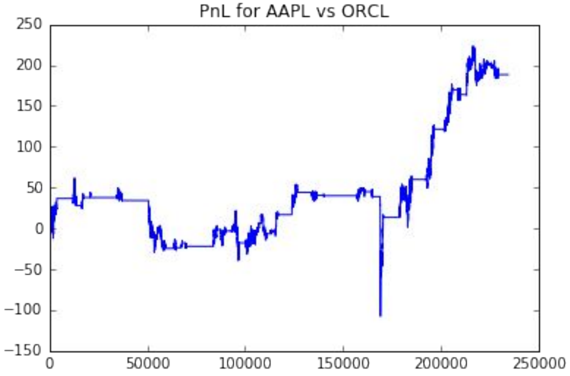
Validation set

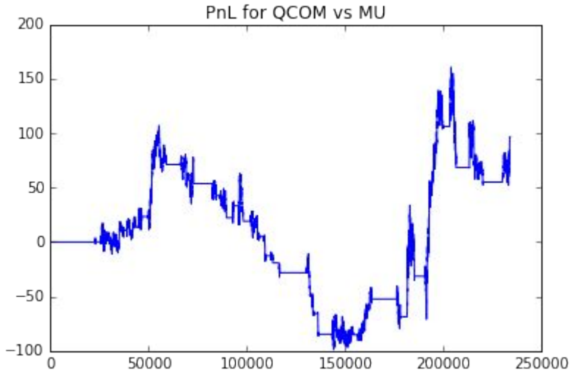


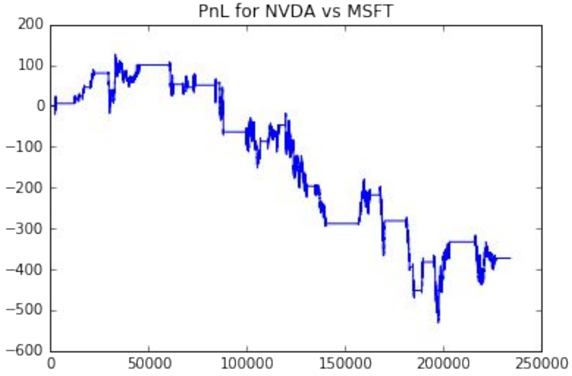
- **Evaluation metric - dollar per trade**
- **Evaluated 4 models**
  a. Fixed thresholds, pairs picked by performance
  b. Fixed thresholds, pairs picked by correlation
  c. Stochastic control thresholds, pairs picked by performance
  d. Stochastic control thresholds, pairs picked by correlation
- **Model a. performed best on validation set**

Test set results

Inconclusive results







Pairs Trading – Unstable Correlation for High Freq Pairs

Time Interval(s)	Most Correlated Pair	Correlation
1	(AAPL, FB)	0.253802
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Table 1. Most Correlated Pair for a 1-day Window.

Time Interval (s)	QCOM AAPL
1	0.185366
5	0.312954
10	0.408662
20	0.491515
30	0.588978
60	0.701826

Table 2. Correlation between QCOM AAPL for a 1-day window from different time intervals.

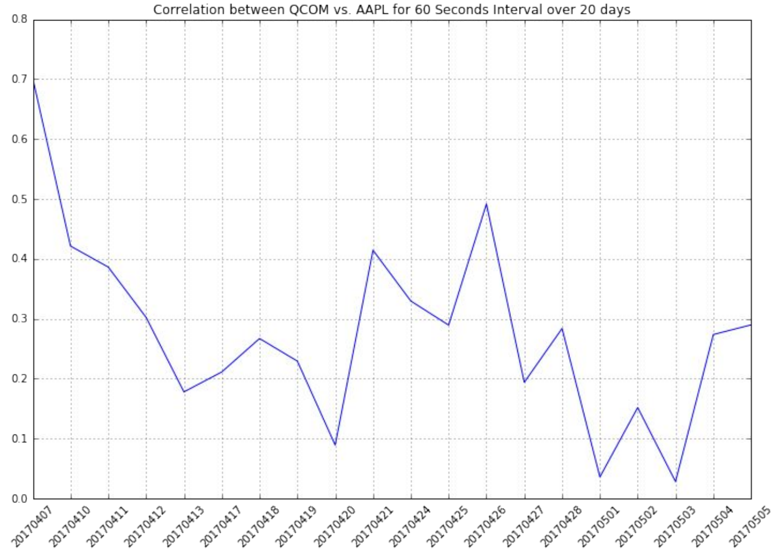
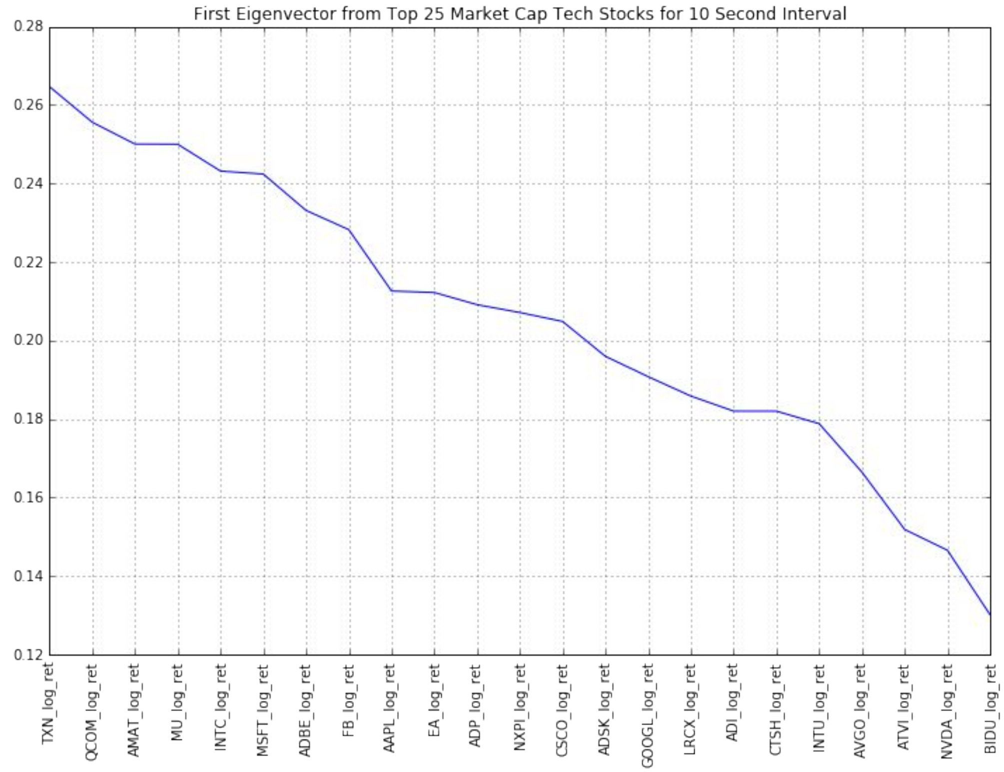


Figure 1. Correlation for same pair for 20-day Window.

Conclusion:

1. Most correlated pairs differ by time intervals.
2. Correlation for same pair changes by time intervals.
3. No pattern in correlation over different days.

Incorporating PCA – Eigenportfolio



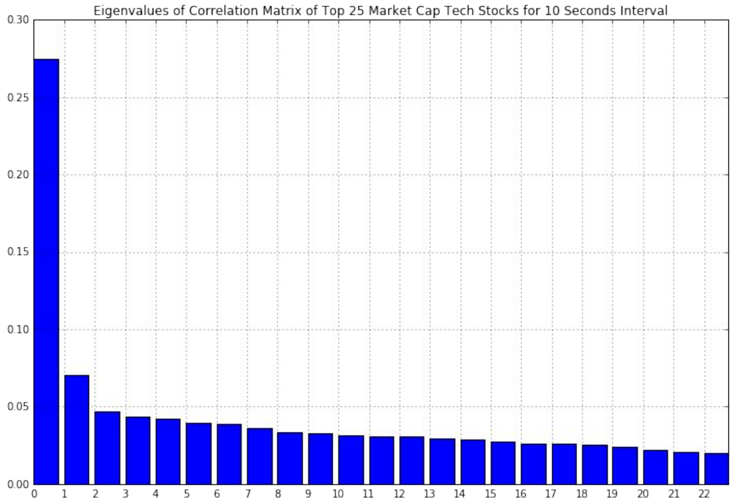


Figure 2 (Left). First eigenvector sorted by coefficient size from top 25 market cap tech stocks for 10 seconds interval. **(Right)** Eigenvalues of this pool of stocks for 10 seconds.

Future Work

- To trade based on factors from PCA eigenportfolio and its eigenvalues:
  - take a variable number of eigenvectors, truncate to explain a given percentage of the total variance of the system

- Implement a more dynamic strategy
  - Using the correlation from yesterday to decide which pairs to trade today.
  - Or observe the market for a couple of hours and then start trading based on earlier correlation

Thank you!

Questions?

References

[1] Avellaneda, M., & Lee, J. H. (2010). *Statistical Arbitrage in the US Equities Market*. Quantitative Finance, 10(7), p.761-782.

[2] Cartea, A., Jaimungal, S., and Peñalva, J. (2015). *Algorithmic and high frequency trading*. Cambridge University Press, chapter 11.

[3] Kercheval, A. and Zhang, Y. Modeling high-frequency limit order book dynamics with support vector machines. University of Florida, 2013