Deformation Distortion in Disc Records*

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The distortion produced by groove-wall deformation in accordance with the elastic theory of Hertz, combined with the distortion produced by tracing error, is calculated using an extension of the power-series approach of Lewis and Hunt in their treatment of tracing error. The dynamical differential equations for the several components of deformation error representing resonant systems are derived and solved for the distortion components. Numerical results for certain parameters are presented graphically, and the relative importance of various terms is appraised. It is seen that the second harmonic term, for example, arising from tracing error, is reduced by the action of deformation error, away from resonance. For an isotropic stylus impedance there is no cross-coupling between stereo channels in the absence of friction, but a beginning at an analysis of friction suggests a mode of coupling.

INTRODUCTION

WHEN a stylus of a phonograph pickup traces the groove of a disc recording, the groove wall is subjected to a variety of forces. If the disc material is assumed to be ideally rigid, groove-wall deformation produced by such forces may be neglected, and tracing distortion may be calculated from geometric considerations alone. A full analysis, based on this approach, was first given by Lewis and Hunt.[1] Actual disc records are not, however, ideally rigid, and the stylus forces do produce deformations resulting in further distortions of the reproduced waveform. Several analyses, taking the elasticity of the groove wall into account, have been made by Kornei and others,[2,3,5] and, in particular, an anlysis of Miller,[5] was significant for its discussion of resonance involving the elasticity of the groove wall and the equivalent mass of the pickup stylus.

The present work is aimed at extending these analyses, to provide a close and extensive examination of the relations between these stylus forces and their effect upon the waveforms reproduced from the elastic groove wall. The approach of Lewis and Hunt is followed, but both the pressure and mechanical impedance presented by the stylus are taken into account as supplying the forces imposed on the groove wall.

MOTION OF A SPHERICAL STYLUS

In the 45-45 system of making stereo disc recordings, when a signal is recorded in one channel alone, the groove wall on one side is cut in varying depth according to the amplitude of the signal, while the other wall remains smooth.

Let us simulate the stylus tip by means of a sphere, having the same radius as the tip, rolling in the groove. Then, the locus of the center of the sphere will be a plane curve, parallel to the unmodulated groove wall, and lying at a distance $r-\delta_o$ from it, for r being the radius of the sphere, and δ_o being the penetration into the unmodulated wall due to stylus pressure. See Fig. 1. (In this analysis, the deformations of the two groove walls, because of applied forces, will be assumed to be uncorrelated.)

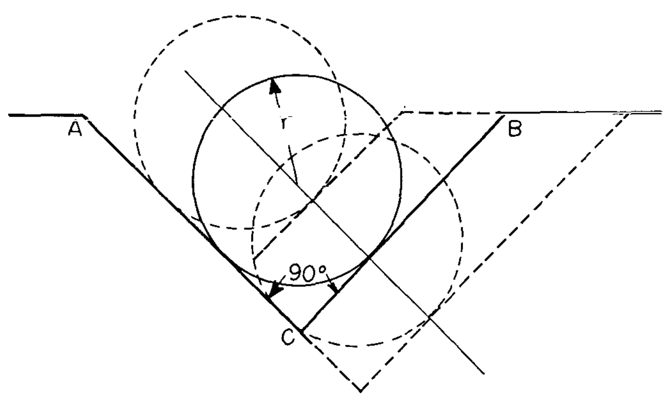

FIG. 1. Motion of a sphere in the groove of a 45-45 stereo record for a signal recorded in only one channel.

On the other hand, as viewed from the modulated wall, the locus of the center of the sphere will appear to differ from the modulation waveform shown in Fig. 2. The locus of the center of the sphere can be given by a relation of the general form

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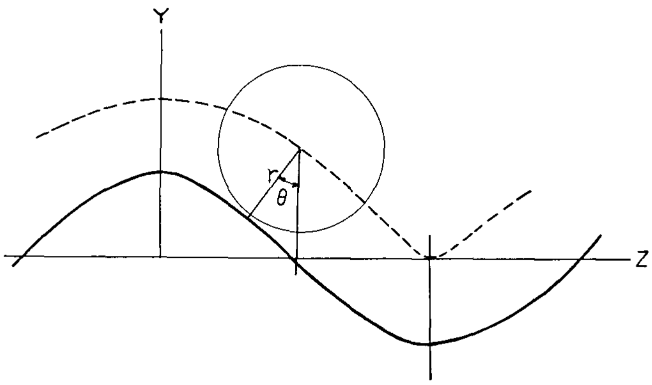

FIG. 2. Locus of the center of a sphere in tracing a sinusoidally modulated groove wall.

$$S(t)=S(q,q',q'',\ldots q^{(n)},\ldots,\delta), \tag{1}$$

where q describes the modulation waveform on the groove wall, and $q',q'',\ldots,q^{(n)},\ldots$ are the derivatives

$$q'=dq/d(Vt),\quad q''=d^2q/d(Vt)^2,$$

etc., δ is the penetration due to the elasticity of the groove wall, and V is the groove speed, so that Vt would be a distance Z along the groove.

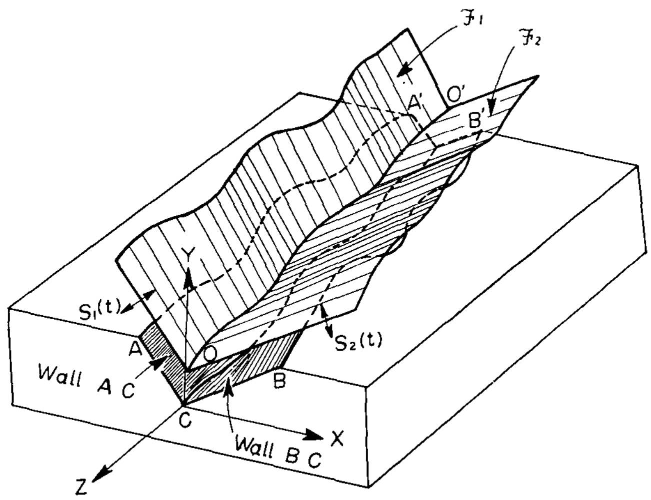

FIG. 3. Locus of the center of a sphere for both groove walls being modulated.

When signals are recorded in both channels, as shown in Fig. 3, the locus of the center of the sphere is described as the intersection of surfaces $\mathcal{J}_1$ and $\mathcal{J}_2$ containing the individual trajectories $S_1(t)$ and $S_2(t)$, corresponding to groove walls AC and BC, given in Eq. (1). The surface $\mathcal{J}_1$ contains the locus S_1 when the sphere rolls on groove wall AC, and the surface $\mathcal{J}_2$ contains the locus as the sphere rolls on BC. The space curve $00'$ denotes the intersection of $\mathcal{J}_1$ with $\mathcal{J}_2$. In this way, it is seen that the mutually perpendicular groove walls act independently of each other.

The motion $S_1(t)$ of the sphere, in response to undulations in wall AC, lies in a plane at right angles to every line in the groove parallel to AC so that $\mathcal{J}_1$ is given by

$$\mathcal{J}_1=y+x-[S_1(t)+r]\sqrt{2}=0, \tag{2}$$

and in a similar manner, the motion in response to BC is given by

$$\mathcal{J}_2=y-x-[S_2(t)+r]\sqrt{2}=0. \tag{3}$$

From these, one obtains the intersection as having the coordinates†

$$x=[S_1(t)-S_2(t)]/\sqrt{2},\qquad y=r\sqrt{2}+[S_1(t)+S_2(t)]/\sqrt{2}. \tag{4}$$

This set of equations shows how the motion of the sphere can be presented in a rectangular system of coordinates in terms of the loci of centers individually obtained for spheres rolling on the two groove walls.

ELASTIC DEFORMATION OF THE GROOVE WALL

When a pickup stylus traces a groove wall, the groove wall is subjected to various forces. These forces include not only the static tracking force, but also inertial forces deriving from the equivalent masses of the pickup stylus and arm, together with the elastic restoring force for the stylus, as well as a force (skating) which develops as a result of an angular difference between the groove direction and the direction to the arm pivot. All of these forces develop in a vertical plane normal to the direction of tracing, or groove direction. The static tracking force is normal to the disc; inertial forces are in the same direction as the stylus motion; the elastic stylus restoring force is in a direction determined by the ratio of vertical and lateral compliances of the stylus, and the skating force also develops laterally in a vertical plane.

The tracking force F_G can be decomposed into two forces, each normal to a groove wall. The force in the direction of the Y-axis in Fig. 4 is, therefore, $F_G/\sqrt{2}$. When the vertical and lateral stylus compliances are equal, the elastic restoring force is also aligned with the Y-axis. Otherwise, the independency of S_1 and S_2 would be perturbed. The inertial forces for both stylus and arm lie in the same direction as the stylus motion, i.e., the Y-axis of Fig. 4. If the arm were of finite length, the independency of S_1 and S_2 would again be perturbed. An infinite length is, therefore,

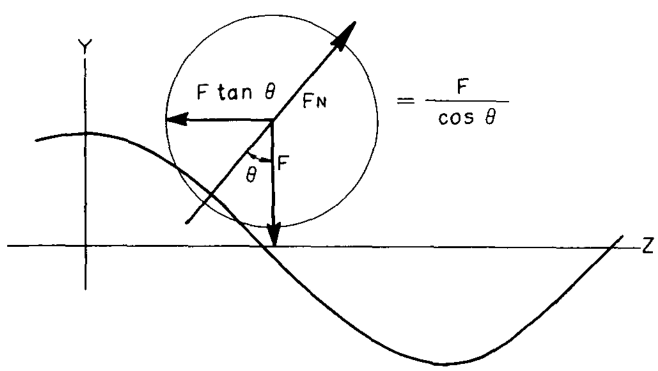

FIG. 4. Resolution of the forces applied by the stylus.

† The positive sense is referred to the outward wall normal for both S_1 and S_2. In another convention, the inward normal is the reference for one of the two walls, so that x would involve the sum of S_1 and S_2, and y would involve the difference.

assumed here. The above three forces are taken into account in the derivation which follows, and they are all assumed to act in the Y direction.

In rolling on the groove wall, as shown in Fig. 4, with a force F applied in the Y direction, the locus of the sphere's center may be perturbed by the elastic deformation of the groove wall. In the playback equipment, the pickup stylus is held fixed at a point on the Z axis by the arm, so that it may not move in that direction while the groove motion is towards $-Z$ with speed V. The Z component of the force resulting from this condition is borne entirely by the arm. This requires a force $F(\tan\theta)$ be applied to the sphere by the arm to prevent it from rolling further down the slope, shown as negative in Fig. 4. Meanwhile, the groove wall is deformed by the force F_N acting along the local wall normal, and the sphere penetrates the wall. According to Hertz, when a rigid-body sphere, of radius r, is pressed on a cylindrically curved surface, of local radius of curvature ρ, with a force F_N, the total deformation δ_N is

$$\delta_N = [(1+r/2\rho)F_N{}^2/(rH^2)]^{1/3}, \tag{5}$$

in which H, which may be called the Hertzian modulus, is given by

$$H = 4E/[3(1-\nu^2)],$$

in which E is Young's modulus, and ν is the Poisson ratio. Since Eq. (5) is not applicable for wavelengths shorter than the radius of the sphere, the applications to be derived must be restricted to regions of validity for (5). The magnitude of F_N will vary with θ, as must δ_N.

In beginning an extension of the rigid-body approach of Lewis and Hunt, so that the output locus deriving from elastic walls may be given, reference is made to Fig. 5.

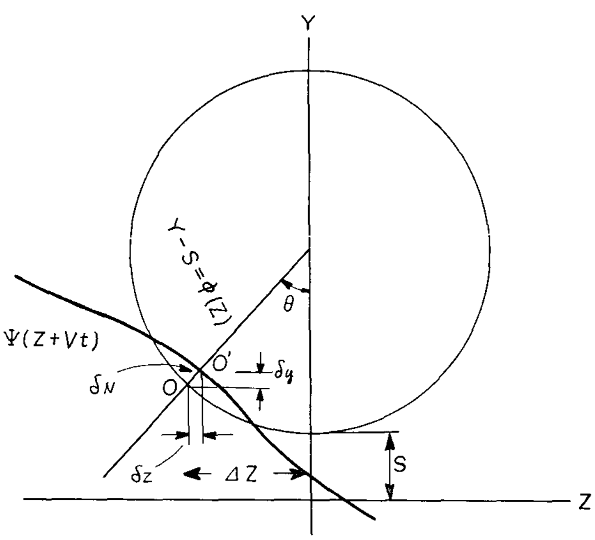

FIG. 5. Displacement of the spherical center resulting from elastic deformation of the groove wall.

There, the modulation of the groove wall is expressed by the waveform $a\psi(Z+Vt)$ which is moving toward $-Z$ at a speed V, and the shape of the profile of the sphere is expressed as the function $\phi(Z)$, which is swinging in the Y direction in response to the modulation. Then, with a contact force F_N being applied, the sphere penetrates along the local wall normal to a depth δ_N.

When the sphere is displaced a distance S from the origin of Y, the vertical position of a point 0 at a position ΔZ from the center is just $\phi(\Delta Z)+S$ so that the equation of the circular profile is

$$Y-S = \phi(\Delta Z), \tag{6}$$

and, for the sphere, one also has

$$\begin{aligned}\phi(0) &= \phi'(0) = \phi'''(0) = 0,\\ \phi''(0) &= 1/r,\ \phi^{IV}(0) = 3/r^3,\end{aligned} \tag{7}$$

etc. The Y coordinate of the point 0′ is

$$a\psi(\Delta Z-\delta_z+Vt),$$

so that for 0 one has

$$Y = S+\phi(\Delta Z) = a\psi(\Delta Z-\delta_z+Vt)-\delta_y. \tag{8}$$

Since the tangents to the curves $\phi(Z)$ and $a\psi(Z+Vt)$ at 0 and 0′ are parallel, one also has the relation

$$\phi'(\Delta Z) = a\psi'(\Delta Z-\delta_z+Vt). \tag{9}$$

The force F_N is

$$F_N = F/\cos\theta = F[1+(\phi'(\Delta Z))^2]^{1/2},$$

so that from Eq. (5) one obtains

$$\delta_N = \delta[1+(\phi'(\Delta Z))^2]^{1/3}, \tag{10}$$

in which δ is now given by

$$\delta = [(1+r/2\rho)F^2/(rH^2)]^{1/3}, \tag{11}$$

the same form as (5), but with F replacing F_N. The expressions for δ_z and δ_y then are

$$\begin{aligned}\delta_z &= \phi'(\Delta Z)[1+(\phi'(\Delta Z))^2]^{-1/2}\delta_N\\ &= \delta[\phi'(\Delta Z)-(1/6)(\phi'(\Delta Z))^3+\ldots],\end{aligned} \tag{12}$$

$$\begin{aligned}\delta_y &= [1+(\phi'(\Delta Z))^2]^{-1/2}\delta_N\\ &= \delta[1-(1/6)(\phi'(\Delta Z))^2+\ldots].\end{aligned} \tag{13}$$

These are to be substituted into Eqs. (8) and (9) to obtain the pair of equations

$$\begin{aligned}&S+\phi(\Delta Z) =\\ &\quad a\psi(\Delta Z-\delta\{\phi'(\Delta Z)-(1/6)(\phi'(\Delta Z))^3+\ldots\}+Vt)\\ &\quad -\delta[1-(1/6)(\phi'(\Delta Z))^2+\ldots],\end{aligned} \tag{14}$$

and

$$\begin{aligned}&\phi'(\Delta Z) =\\ &\quad a\psi'(\Delta Z-\delta\{\phi'(\Delta Z)-(1/6)(\phi'(\Delta Z))^3+\ldots\}+Vt).\end{aligned} \tag{15}$$

With the help of Eq. (7) an expansion around $Z=0$ is to be made of Eq. (15) to obtain

$$\begin{aligned}&\Delta Z\phi''(0)+(1/6)(\Delta Z)^3\phi^{IV}(0)+\ldots\\ &= a\psi'(Vt)+[\Delta Z-\delta\{\Delta Z\phi''(0)+(1/6)(\Delta Z)^3\phi^{VI}(0)+\ldots\\ &\quad -(1/6)(\Delta Z\phi''(0)+(1/6)(\Delta Z)^3\phi^{IV}(0)+\ldots)^3\}]\\ &\quad \times a\psi''(Vt)+(1/2)[\Delta Z-\delta\{\Delta Z\phi''(0)\\ &\quad +(1/6)(\Delta Z)^3\phi^{IV}(0)+\ldots-(1/6)(\Delta Z\phi''(0)\\ &\quad +(1/6)(\Delta Z)^3\phi^{IV}(0)+\ldots)^3\}]^2a\psi'''(Vt)+\ldots.\end{aligned} \tag{16}$$

The assumption that $\Delta Z = 0$, and $\delta = \delta_o$, for $a = 0$, and that ΔZ is near zero, with δ near δ_o, for small a, is made to allowing the writing of

$$\Delta Z = a\Delta_1 + a^2\Delta_2 + a^3\Delta_3 + \ldots \qquad (17)$$
$$\delta = \delta_o + a\delta_1 + a^2\delta_2 + \ldots .$$

Then, (16) and (17) may be combined to write

$$\begin{aligned} 0 = {} & a[\Delta_1\phi''(0) - \psi'(Vt)] \\ & + a^2[\Delta_2\phi''(0) - \Delta_1\psi''(Vt) + \delta_o\Delta_1\phi''(0)\psi''(Vt)] \\ & + a^3[\Delta_3\phi''(0) + \ldots] + \ldots . \end{aligned} \qquad (18)$$

In this equation, each coefficient of a given power of a should vanish separately. This observation results in

$$\begin{aligned} \Delta_1 &= \psi'(Vt)/\phi''(0) = r\psi' \\ \Delta_2 &= \psi'(Vt)\psi''(Vt)[1 - \delta_o\phi''(0)]/[\phi''(0)]^2 \\ &= r^2\psi'\psi''[1 - \delta_o/r. \end{aligned} \qquad (19)$$

The locus S of the center of the sphere is similarly obtained by expanding (14) in a power series around $Z = 0$. The expansion for ΔZ is then inserted, using (19). Rewriting $a\psi(Vt)$ as $q(Vt)$, one then obtains

$$\begin{aligned} S(t) = {} & -\delta + q(Vt) + [(r/2) - (5/6)\delta](q')^2 \\ & + [(r^2/2) - (5r/3)\delta + (1/2)\delta^2 + (2/3)\delta_o\delta] \\ & \times (q')^2 q'' + \ldots , \end{aligned} \qquad (20)$$

which becomes, for $\delta = \delta_o = 0$, the rigid-body result

$$S(t) = q(Vt) + (r/2)(q')^2 + (r^2/2)(q'^2)q'' + \ldots , \qquad (20')$$

as obtained by Lewis and Hunt.

EFFECT OF LOCAL CURVATURE IN THE GROOVE WALL

In Eq. (11) it is seen that the penetration δ depends upon the radius of local curvature in the groove wall. The positive sign for ρ is taken when the wall is convex (center of curvature within the material), and negative when concave, as required in the Hertzian formula. This is

$$\rho = -[1 + (dY/dZ)^2]^{3/2}/d^2Y/dZ^2. \qquad (21)$$

In this, on puts $Z = Vt$ and $a\psi(Vt) = a\psi(Z)$, so that, regarding $a^2\psi'^2(Z)$ to be small, one writes

$$\begin{aligned} (1/\rho) &= -a\psi''(1 + a^2\psi'^2)^{-3/2} \\ &= -a\psi'' + (3/2)a^3\psi'^2\psi'' - \ldots . \end{aligned} \qquad (22)$$

Combining this with (11), one has

$$\begin{aligned} \delta = {} & [F^2/rH^2]^{1/3}[1 - (r/6)a\psi'' - (r/36)a\psi''^2 \\ & + (r/4)a^3\psi'^2\psi'' - (5r^3/648)a^2\psi''^3 + \ldots]. \end{aligned} \qquad (23)$$

In this, $a\psi''$ should be understood to be $a\psi''(\Delta Z - \delta_z + Vt)$. Thus, in (23) one should write

$$\begin{aligned} a\psi''(\Delta Z - \delta_z + Vt) = {} & \\ & a\psi'' + a^2(r - \delta)\psi'\psi'' \\ & + a^3[\{r^2 - r(\delta_o - \delta) + \delta_o\delta\}\psi'\psi''\psi''' \\ & + (1/2)(r - \delta)^2\psi'^2\psi^{IV}] + \ldots . \end{aligned} \qquad (24)$$

The effect upon $S(t)$, however, in using (24) instead of simply $a\psi''$ will appear only in the a^4 and higher power terms of the expansion of (8) into a power series. This is because (24) already contains δ multiplied by a^2 and higher powers, and the principal appearance of δ in the expansion will be in a^2 and higher power terms.

EFFECTS OF ELASTICITY AND INERTIA

Consider a model consisting of a sphere having mass M constrained to move along only the Y axis while the wall moves to the left, as in Fig. 6, at speed V. It is assumed

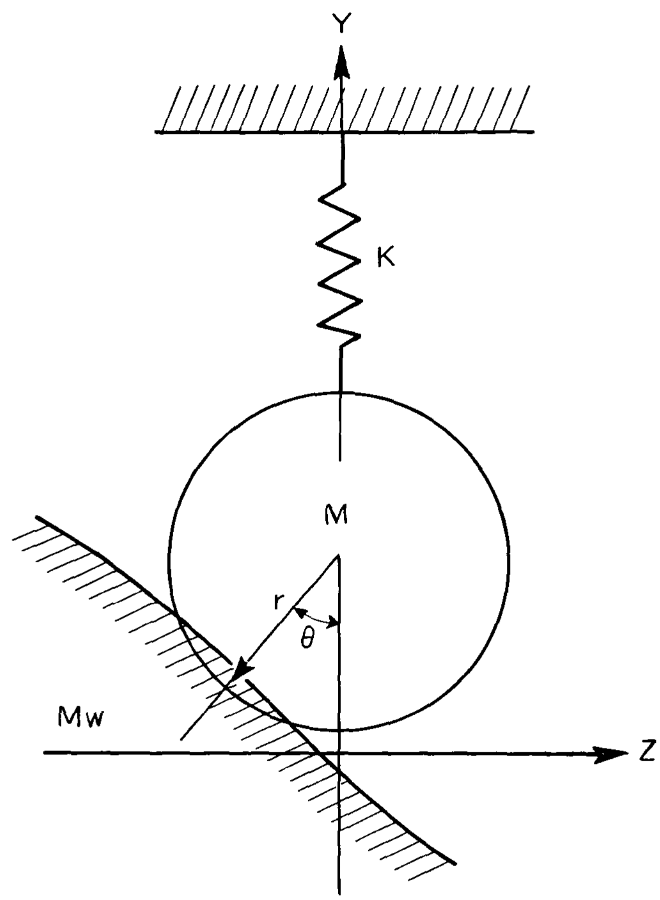

FIG. 6. Model of a spherical stylus and the groove wall as constituting an oscillatory system.

that the wall is elastic, but that its surface is so smooth that the friction opposing the movement is negligible, that a deformation of the type shown in Fig. 7 need not be

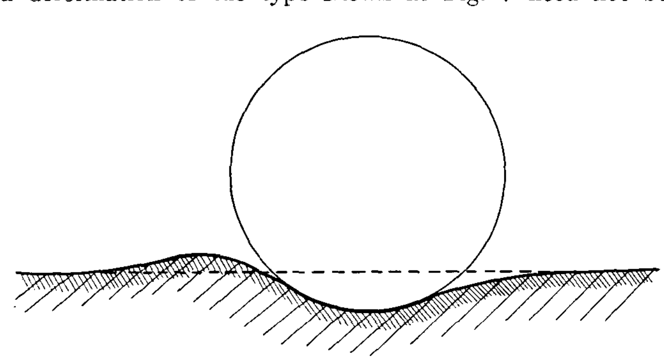

FIG. 7. A possible type of elastic deformation.

considered, and that there is no energy loss within the wall.

Assigning a mass M_w to the wall and writing $S_f(t)$ for an oscillatory term in $S(t)$, one finds the kinetic energy to be

$$T=(1/2)MS_f'^2(t)+(1/2)M_wV^2, \tag{25}$$

and the potential energy to be

$$U=(F_G/\sqrt{2})S(t)+(1/2)KS_f^2(t) \tag{26}$$
$$+\frac{2}{5}\left(\frac{r}{C}\right)^{1/2}\left(\frac{\delta_y^{5/2}}{(\cos\theta)^{1/2}}+\frac{\delta_z^{5/2}}{(\sin\theta)^{1/2}}\right).$$

In (26) the elastic constant of the spring suspending the sphere is K, the tracking force is F_G, and the combination $(1-r/2\rho)/H^2$ is represented by C. Letting Q denote a force in the Z direction applied to the wall, the set of differential equations below

$$\frac{d}{dt}\frac{\partial T}{\partial S'(t)}+\frac{\partial U}{\partial S(t)}=0, \tag{27}$$
$$\frac{d}{dt}\frac{\partial T}{\partial V}+\frac{\partial U}{\partial Z}=Q,$$

is obtained.

Upon insertion of (25) and (26) into (27),

$$\frac{F_G}{\sqrt{2}}+MS_f''(t)+KS_f(t)+\frac{(r/C)^{1/2}\delta_y^{3/2}}{(\cos\theta)^{1/2}}\frac{\partial\delta_y}{\partial S(t)}=0, \tag{28}$$
$$M_w\frac{dV}{dt}+\frac{(r/C)^{1/2}\delta_z^{3/2}}{(\sin\theta)^{1/2}}\frac{\partial\delta_z}{\partial Z}=Q.$$

In (28), $\partial\delta_y/\partial S(t)=-1$, and $\partial\delta_z/\partial Z=1$, and $dV/dt=0$, because the velocity of the wall is constant. With the help of these observations, there is obtained

$$\delta_y=(C/r)^{1/3}(\cos\theta)^{1/3}\times[MS_f''(t)+KS_f(t)+F_G/\sqrt{2}]^{2/3}, \tag{29}$$
$$\delta_z=(C/r)^{1/3}(\sin\theta)^{1/3}Q,$$

or putting $\delta_z/\delta_y=\tan\theta$, just

$$Q=(MS_f''(t)+KS_f(t)+F_G/\sqrt{2})^{2/3}(\tan\theta)^{2/3}, \tag{30}$$
$$\delta_z=(C/r)^{1/3}\sin\theta(\cos\theta)^{-2/3}\times(MS_f''(t)+KS_f(t)+F_G/\sqrt{2})^{2/3}.$$

As given by Eqs. (29) and (30), δ_y and δ_z should be equal to those same components given by Eqs. (12) and (13), in which $F=MS_f''(t)+KS_f(t)+F_G/\sqrt{2}$. For this reason, it must be understood that the relation derived in Section 3 is valid only when deformations of the kind shown in Fig. 7 need not be considered, and the static storage of energy in the wall may be carried over without change to the case of the moving wall. In Eq. (30), Q relates to a torque to be supplied by the turntable motor in rotating the disc recording.

In reproduction, the spherical stylus should always make contact with the wall, requiring $F>0$. This requires

$$|KS_f(t)+MS_f''(t)|<F_G/\sqrt{2}. \tag{31}$$

Using this relation, we obtain

$$F^{2/3}=(F_G/\sqrt{2})^{2/3}\{1+(2/3)[KS_f(t)+MS_f''(t)] \div(F_G/\sqrt{2})-(1/9)[KS_f(t)+MS_f''(t)^2 \div(F_G^2/\sqrt{2})+\ldots\}. \tag{32}$$

Rewriting $F_G/\sqrt{2}$ as f, and using the relation

$$\delta_o=[f^2/rH^2]^{1/3},$$

then, with the help of Eqs. (17), (23), (24), and (25), one expands (20) into a series as follows: One denotes

$$S(t)=-\delta_o+S_f(t),$$

and the series for $S_f(t)$ is

$$\begin{aligned}
S_f(t)= & \\
& -\delta_o\{(2/3)[KS_f(t)+MS_f''(t)]/f-(1/9)\\
&\quad\times[KS_f(t)+MS_f''(t)]^2/f^2+\ldots\}\\
& +a[\psi(Vt)+(r\delta_o/6)\{1+(2/3)[KS_f(t)+MS_f''(t)]/f\\
&\quad-(1/9)[KS_f(t)+MS_f''(t)^2/f^2+\ldots\}\psi'']\\
& +a^2[\{(r/2)-(5\delta_o/6)(1+(2/3)[KS_f(t)\\
&\quad+MS_f''(t)]/f+\ldots)\}\psi'^2+r(\delta_o/6)(1+(2/3)\\
&\quad\times[KS_f(t)+MS_f''(t)]/f-\ldots)\\
&\quad\times\{r-\delta_o(1+(2/3)[KS_f(t)+MS_f''(t)]/f-\ldots)\}\\
&\quad\times\psi'\psi'''+r^2(\delta_o/36)(1+(2/3)[KS_f(t)\\
&\quad+MS_f''(t)]/f-\ldots)\psi''^2]\\
& +a^3[\{(r^2/2)-(5r\delta_o/3)(1+\ldots)+(\delta_o^2/2)(1+\ldots)^2\\
&\quad+(2\delta_o^2/3)(1+\ldots)\}\psi'^2\psi''+(r\delta_o/6)(1+\ldots)^2\\
&\quad\times\{(r-\delta_o)(r-\delta_o(1+\ldots))\}\psi'\psi''\psi'''\\
&\quad+(r\delta_o/12)(1+\ldots)\{r-\delta_o(1+\ldots)\}^2\psi'^2\psi^{IV}\\
&\quad+(r^2\delta_o/18)(1+\ldots)\{r-(\delta_o/2)(1+\ldots)\}\\
&\quad\times\psi'\psi''\psi'''-(r\delta_o/9)(1+\ldots)\psi'^2\psi''\\
&\quad+(5r^3\delta_o/648)(1+\ldots)\psi''^3]\\
& +a^4[\ldots]+\ldots.
\end{aligned} \tag{33}$$

As a power series in a, the solution $S_f(t)$ is as shown in Eq. (33). Denoting terms of first, second, etc., degrees as $S_{1f}(t)$, $S_{2f}(t)$, . . . , the solution may be written

$$S_f(t)=S_{1f}(t)+S_{2f}(t)+\ldots. \tag{34}$$

If we first extract the $S_{1f}(t)$ term, we obtain

$$S_{1f}(t)=-(2\delta_o/3f)[KS_{1f}(t)+MS_{1f}''(t)]+a[\psi(Vt)+(r/6)\delta_o\psi''(Vt)], \tag{35}$$

a differential equation for $S_{1f}(t)$. Denoting $2\delta_o/3f$ as $1/k$, we can write it as

$$MS_{1f}''(t)+(K+k)S_{1f}(t)=ka[\psi+(r/6)\delta_o\psi'']. \tag{36}$$

This equation describes the situation, shown in Fig. 8, for which the wall B is in oscillation, with a possibility for resonance, as given by the term $a[\psi+(r/6)\delta_o\psi'']$.

The solution of the differential equation (36) is

$$S_{1f}(t)=A\cos nt+B\sin nt,$$

in which

$$A = -(a/n)\int[\psi+(r/6)\delta_o\psi'']\sin nt\,dt+C_1, \tag{37}$$
$$B = (a/n)\int[\psi+(r/6)\delta_o\psi'']\cos nt\,dt+C_2,$$
$$n = \sqrt{[(k+K)/M]}.$$

Assuming $k >> K$, and $k\psi >> M\psi''$, and performing the integrals for A and B, we obtain

$$S_{1f}(t) = a[\psi+(r/6)\delta_o\psi''- (1/k)(K\psi+MV^2\psi'') -(1/k^2)(K\psi''+MV^2\psi^{IV})+\ldots]. \tag{38}$$

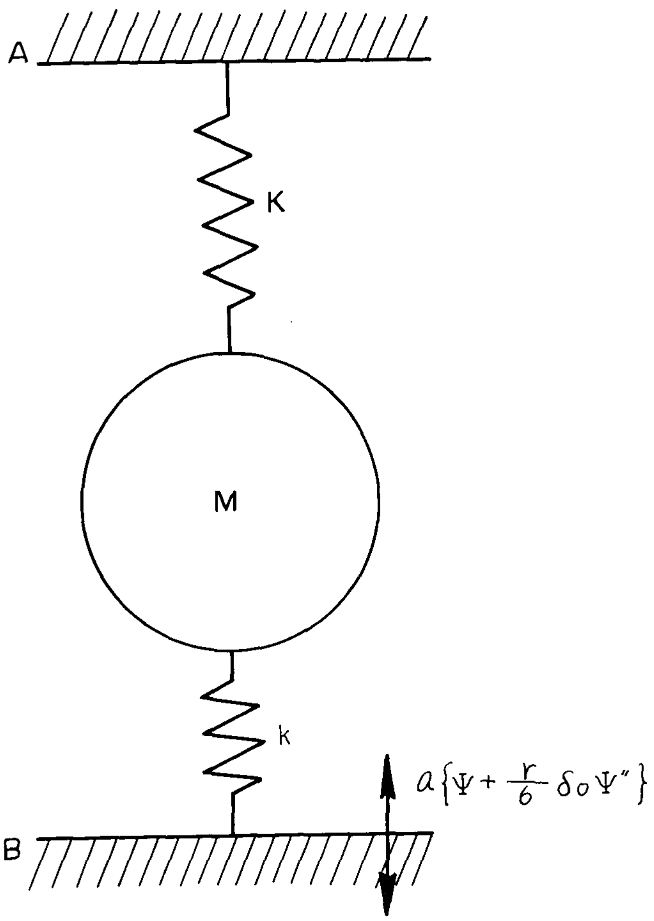

FIG. 8. An equivalent oscillatory system for the linear term in the equation of stylus motion.

In this equation, if all but the initial three terms be disregarded, the result is seen to be the same as that obtained by Kornei.[2] Thus, it is understood that Kornei's result is an approximation of (37), and, as explained in a later section, the properties of the solution obtaining for $k\psi \approx M\psi''$, i.e., in the vicinity of the resonance between k and M, had been overlooked.

Secondly, if the term $S_{2f}(t)$ be extracted, there is obtained

$$S_{2f}(t) = -\delta_o\{(2/3f)[KS_{2f}(t)+MS_{2f}''(t)] -(1/9f^2)[KS_{1f}(t)+MS_{1f}''(t)]^2\} +a\{(r\delta_o/9f)[KS_{1f}(t)+MS_{1f}''(t)]\}\psi'' +a^2\{[(r/2)-(5/6)\delta_o]\psi'^2+(r^2/6)\delta_o\psi'\psi''' +(r^2/36)\delta_o\psi''^2\}. \tag{39}$$

By substituting Eq. (38) in the above, and eliminating terms of order δ_o^2, for the conditions $k >> K$ and $k\psi >> M\psi''$, one obtains

$$S_{2f}(t) = a^2\{(r/2)\psi'^2-\delta_o[(5/6)\psi'^2-(r^2/6)\psi'\psi'' -(r_2/36)\psi''^2+(r/3f)(K\psi'^2+MV^2 \times(2\psi''^2+2\psi'\psi''))-(1/9f^2) \times(K\psi+MV^2\psi'')^2-(r/9f) \times(K\psi+MV^2\psi'')\psi'']\}. \tag{40}$$

In a similar manner, an equation for $S_{3f}(t)$, involving a^3, can be obtained.

The solution (37) may be used to represent general kinds of wave motion, but we now seek the solution for ψ being a sine wave. For this it will be better practice to express the solution as a Fourier series, i.e., in terms of the magnitudes of the fundamental and harmonic component waves, rather than leaving it as a series of ascending powers of a. For this let there be adopted the notation

$$\psi = \cos\omega t \tag{41}$$
$$S_f(t) = S_1\cos\omega t + S_2\cos 2\omega t + S_3\cos 3\omega t + \ldots.$$

Then, S_1 includes the terms of order a, a^3, a^5, . . . ,
S_2 includes the terms of order a^2, a^4, a^6, . . . ,
S_3 includes the terms of order a^3, a^5, a^7, . . . ,

etc. However, for $a << 1$, then, in S_1, the terms of order a^3 and higher will be negligible compared to that of order a, and, in S_2, the terms of order a^4 and higher will be negligible, etc. As actual modulation amplitudes in disc records, such terms should be very small.

Inserting Eq. (41) into (33), the reproduced fundamental component is found to be

$$S_1 = a[1-(r/6)\delta_o\omega^2/V^2][1-j\omega Z_{m1}/k], \tag{42}$$

in which

$$j\omega Z_{m1} = k[M\omega^2-K]/[M\omega^2-(k+K)], \tag{43}$$

and Z_{m1} is the mechanical impedance presented to the wall in the model of Fig. 8, and is equivalent to the impedance presented by the network shown in Fig. 9. Figure 10 shows the frequency plot of this impedance for $K << k$. It is,

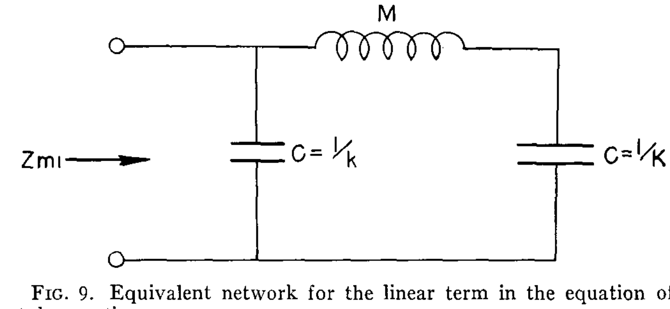

FIG. 9. Equivalent network for the linear term in the equation of stylus motion.

of course, essential in actual playback that the wall stiffness greatly exceed the stylus suspension stiffness, placing the value of ω_c, shown in Fig. 10, in the range between 10 and 20 kHz.

Condition (31) may be expressed, if harmonic components are disregarded, as

$$f > |a(1-r\delta_o\omega^2/6V^2)j\omega Z_{m1}|,$$

which implies

$$|j\omega Z_{m1}| < f/A, \text{ for } A = a[1-r\delta_o\omega^2/6V^2], \quad (44)$$

a condition for the avoidance of intermittent tracing, so there will always be contact with the groove wall.

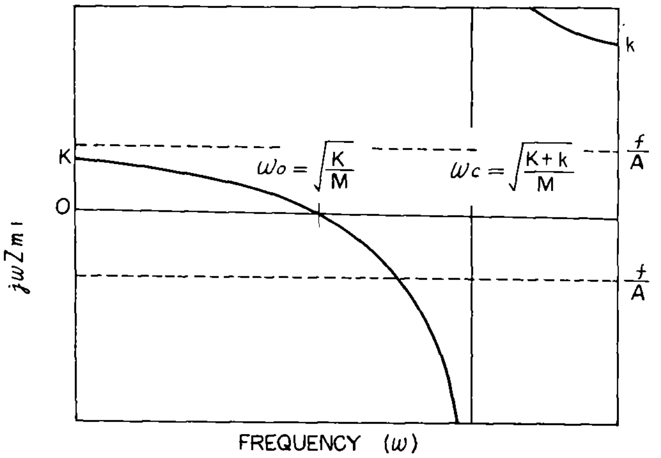

FIG. 10. Frequency plot of the impedance $j\omega Z_{m1}$.

Figure 11 exhibits a frequency plot of $|1-j\omega Z_{m1}/k|$. The response at the resonant frequency ω_c, in actual playback, is always finite, as shown, because of energy dissipation or losses in the resonant system. If the loss term is proportional to velocity, the equation

$$F = f + K_1S_f(t) + R_mS_f'(t) + MS_f''(t)$$

obtains, in which R_m is the resistive coefficient. As shown in Fig. 11, the resistive coefficient controls the blunting of the resonance peak at ω_c, the more so as $\epsilon = R_m/M\omega_c$ is being increased.

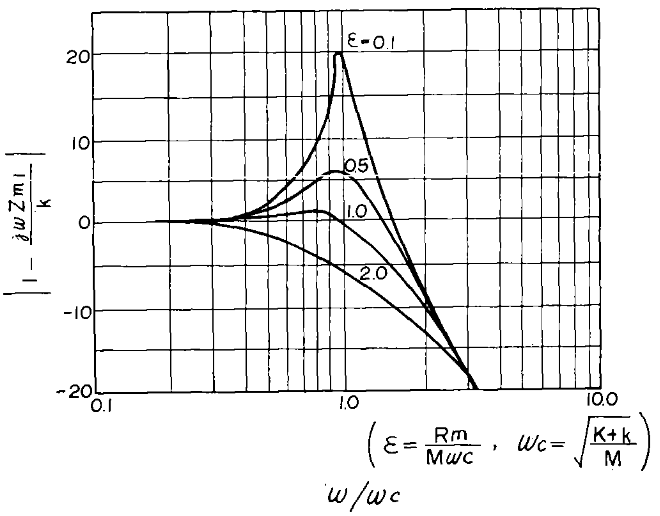

FIG. 11. Frequency of plot of $|1-j\omega Z_{m1}/k|$.

Actual systems are more complicated, involving a cartridge and arm, as well. One should also combine the impedances of these elements with the combined groove-wall-stylus impedance to obtain a more realistic response. In the course of actual measurements, one often finds several other resonance peaks, identifiable as an arm resonance, usually at 100 Hz or lower, together with resonances within the cartridge, depending upon the actual style of construction.

The second-harmonic component S_2 can be obtained in a similar manner from Eqs. (33) and (41). It is

$$\begin{aligned} S_2 = -(a^2\omega^2/2V^2)\{(r/2)-\delta_o[(5/6)+(7r^2/36) \\ \times(\omega/V)^2+(1/9f^2)(V/\omega)^2(1-r\delta_o\omega^2/6V^2)^2 \\ \times(j\omega Z_{m1})^2-(r/9f)(1-r\delta_o\omega^2/6V^2)j\omega Z_{m1}]\} \\ \times(1-j\omega Z_{m2}/k), \end{aligned} \quad (45)$$

in which

$$j\omega Z_{m2} = k[4M\omega^2-K]/[4M\omega^2-(k+K)]$$

is the mechanical impedance for the second harmonic, with a resonance at $\omega^2 = (k+K)/4M$, i.e., at $\omega = \omega_c/2$.

The various terms in Eq. (45) may be qualitatively described, and estimates obtained (quoted in parentheses) for the relative magnitudes of the contributions from each:

1) $-(a^2r/4)(\omega/V)^2$, a term representing a geometrical distortion present even for a perfectly rigid wall. (6%)

2) $(5a^2/12)\delta_o\omega^2/V^2$, a term representing a reduction in the geometrical distortion because of stylus force. Figure 12

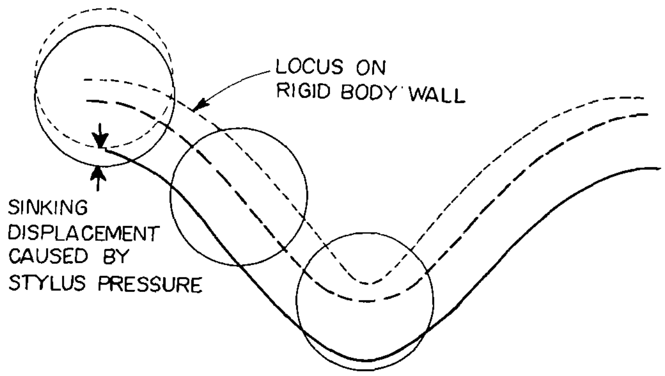

FIG. 12. Reduction in distortion caused by the static stylus force.

illustrates this action providing for a more faithful reproduction in the elastic-wall case. (1.3%)

3) $(7a^2r^2/72)\delta_o\omega^4/V^4$, a term representing a distortion generated by stylus pressure, also acting to reduce the geometrical distortion, because of a greater deformation when the wall curvature is the less. (0.06%)

4) $(a^2/18f^2)(1-r\delta_o\omega^2/6V^2)^2(j\omega Z_{m1})^2\delta_o$, a term representing a distortion generated by the nonlinearity in the wall stiffness, as shown in Fig. 13, also acting to reduce the geometrical distortion. Upon the application of an elastic force, the drop at A would be less and the rise at B would be greater. Upon the application of an inertial force the rise at A would be greater and the drop at B would be less. Thus, in any case, the sphere traces the locus shown dashed. (0.13%)

5) $-(a^2r/18f)(\omega^2/V^2)(1-r\delta_o\omega^2/6V^2)j\omega Z_{m1}\delta_o$, a term representing a distortion generated by elastic and/or inertial forces acting on the wall, and resulting from a deeper deformation where the wall curvature is the less. It is negative for elastic forces and positive for inertial. (0.006%)

The percentages indicated above are derived from the following set of parameter values:

$a = 2 \times 10^{-3}$ cm	$M = 5 \times 10^{-3}$ gm
$K = 1 \times 10^{6}$ dyn/cm	$\delta_o = 2.1 \times 10^{-4}$ cm
$f = 5 \times 10^{3}$ dyn	$r = 1.7 \times 10^{-3}$ cm
$k = 4 \times 10^{7}$ dyn/cm	$\omega = 4800/s$

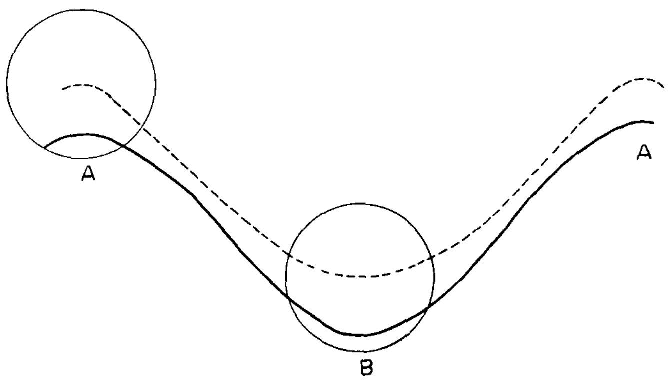

FIG. 13. Distortion caused by the nonlinearity of the wall stiffness.

The ω value corresponds to 800 Hz. The groove-speed value used was $V = 18$ cm/s. These values approximate those for a typical magnetic pickup. The figures quoted in the parentheses are, of course, percentages of the "a" value. These same parameter values have been used to make a plot, shown as Fig. 14, of second-harmonic content *versus* groove speed.

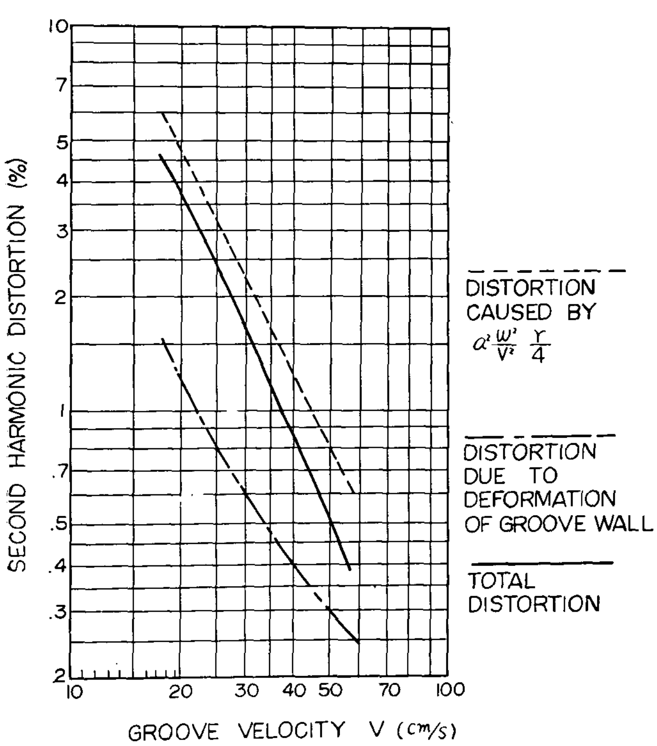

FIG. 14. Second-harmonic content *versus* groove speed.

The third-harmonic component may also be obtained, proceeding in the same manner as before. It is

$$\begin{aligned} S_3 = {} & (a^3/4)(\omega/V)^4\{(r^2/2) - \delta_o[(16r/9) \\ & + (203r^3/648)(\omega/V)^2 - (2/9f^2)(V/\omega)^4 \\ & \times (1 - r\delta_o\omega^2/6V^2)S_2'(j\omega Z_{m1})(j\omega Z_{m2}) \\ & + (4/81f^3)(V/\omega)^4(1 - r\delta_o\omega^2/6V^2)^3(j\omega Z_{m1})^3 \\ & - (5/9f)(V/\omega)^2(1 - r\delta_o^2/6V^2)j\omega Z_{m1} \\ & + (r/9f)(V/\omega)^2 S_2' j\omega Z_{m2} - (7r^2/54f) \\ & \times (1 - r\delta_o\omega^2/6V^2)j\omega Z_{m1}]\}(1 - j\omega Z_{m3}/k), \end{aligned} \tag{46}$$

in which

$$S_2' = (S_2/a^2)/(1 - j\omega Z_{m2}/k),$$
$$j\omega Z_{m3} = k[9M\omega^2 - K]/[9M\omega^2 - (k+K)],$$

the latter being the impedance for the third harmonic, with a resonance at $\omega^2 = (k+K)/9M$, i.e., at $\omega = \omega_c/3$. In Eq. (46) the first term represents the geometrical distortion, and the others represent deformation distortions. A plot of third harmonic content *versus* groove speed is shown in Fig. 15 for the same parameter values cited above.

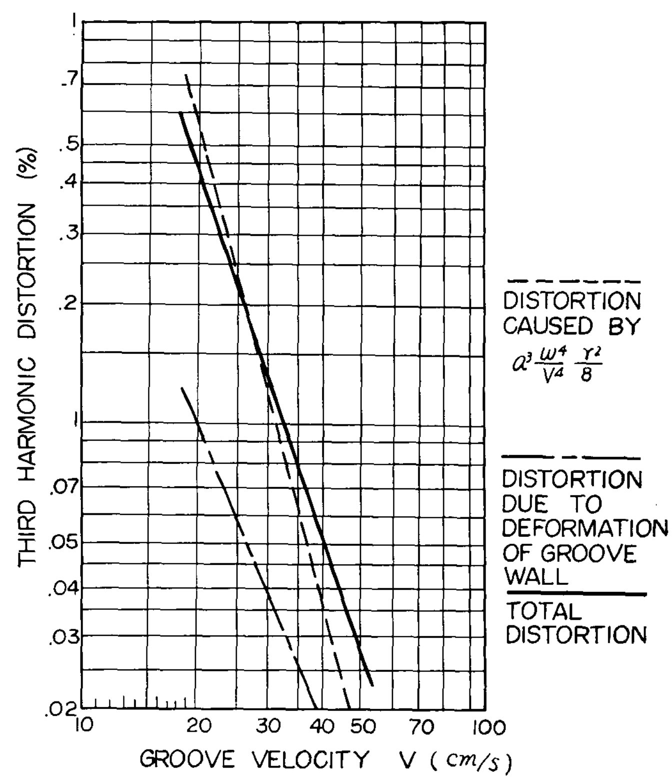

FIG. 15. Third-harmonic content *versus* groove speed.

In addition to showing plots of harmonic content *versus* groove speed, as in Figs. 14 and 15, for a fixed frequency, 800 Hz, it is also possible to show the dependence of harmonic content upon frequency at a fixed groove speed. For this, $V = 50$ cm/s was chosen, together with the fixed amplitude value, $a = 0.4 \times 10^{-3}$, for all fundamental frequencies from 400 Hz to 20 kHz, to prepare the plot shown as Fig. 16.

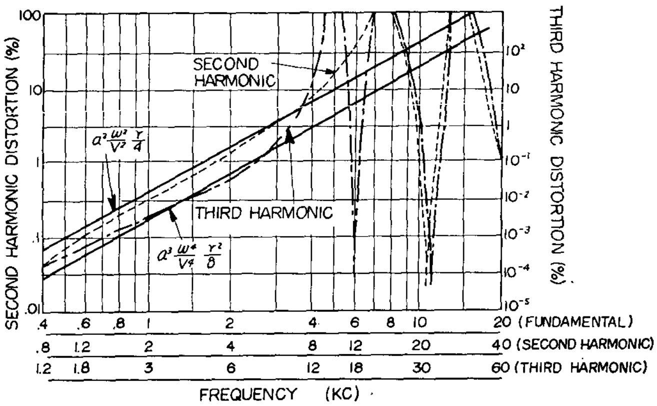

FIG. 16. Variation of harmonic content with frequency for $a = 0.4 \times 10^{-3}$ cm and $V = 50$ cm/sec.

FRICTION BETWEEN A SPHERICAL STYLUS AND THE GROOVE WALL

In considering the friction between a spherical stylus and a groove wall, the forces are assumed to act at equilibrium as shown in Fig. 17. Taking the frictional force to

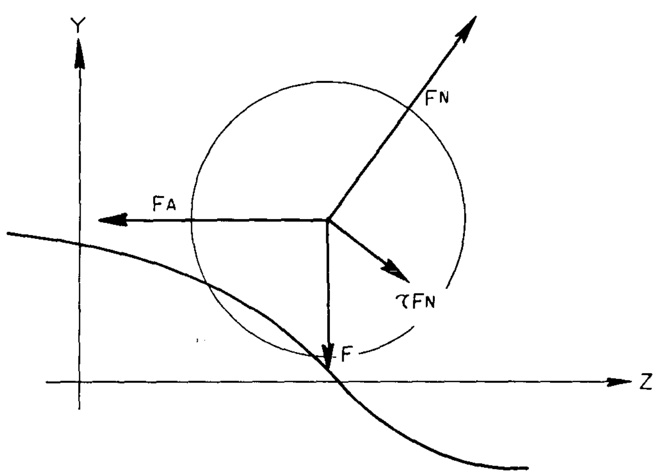

FIG. 17. Resolution of forces including the frictional force.

act in a direction parallel to the tangent to the wall, and denoting the frictional coefficient by τ, one writes the equilibrium condition as

$$\frac{F_N}{\sqrt{[1+(\phi'(\Delta Z))^2]}}[1+\tau\phi'(\Delta Z)] = F, \tag{47}$$

$$\frac{F_N}{\sqrt{[1+(\phi'(\Delta Z))^2]}}[\tau-\phi'(\Delta Z)] = F_A.$$

The deformation δ_N, and its components δ_y and δ_z, produced by the force F_N are then given by

$$\begin{aligned}\delta_N &= \delta[1+(\phi'(\Delta Z))^2]^{1/3}[1+\tau\phi'(\Delta Z)]^{-2/3}, \\ \delta_z &= \delta[\phi'(\Delta Z)-(2/3)\tau(\phi'(\Delta Z))^2+\ldots], \\ \delta_y &= \delta[1-(2/3)\tau\phi'(\Delta Z)+(5/9)\tau^2(\phi'(\Delta Z))^2 \\ &\quad -(1/6)(\phi'(\Delta Z))^2+\ldots].\end{aligned} \tag{48}$$

From Eqs. (8) and (9), one obtains

$$S(t) = a\psi\{\Delta Z-\delta[\phi'(\Delta Z)-(2/3)\tau(\phi'(\Delta Z))^2 + \ldots]+Vt\}-\phi(\Delta Z)-\delta_y. \tag{49}$$

Then, from Eqs. (9) and (48) one obtains

$$\phi'(\Delta Z) = a\psi'\{\Delta Z-\delta[\phi'\Delta Z) -(2/3)\tau(\phi'(\Delta Z))^2+\ldots]+Vt\}. \tag{50}$$

In a manner similar to that by which (14) and (15) were expanded into the series (20), equations (49) and (50) may also be expanded into the series

$$S(t) = -\delta+q(Vt)+\{(r/2)-\delta[(5/6)+(5/9)\tau^2]\}q'^2 + (2\tau/3)\delta q'+(2\tau/3)\delta(r-\delta_0)q'q''+\ldots. \tag{51}$$

This series appears to be identical with (20), except that it has the additional terms involving τ with q', $q'q''$, $q'q''^2$, q'^3, etc., arising because of the friction. Letting $q = a\cos kVt$, the derivatives of q give rise to $a^2\sin 2kVt$, $a^3 \sin 3kVt$, etc., all showing a phase shift of $\pi/2$.

The extension of these considerations would demand the oscillator model of Fig. 8 to contain a frictional resistance. Because the friction would also vary in accordance with the modulation of the other groove wall, an additional distortion could be incurred, which would be significant for frequencies near ω_c, where the elasticity of the groove wall plays a dominant role. In this way, a mechanism, due to friction, may be found by which the motions caused by the two groove walls may be shown to be strongly correlated at resonance.

REMARKS

The above represents a part of a paper originally published in Japanese in the *Journal of the Acoustical Society of Japan* in January, 1962. The English-language manuscript was prepared by Professor Duane H. Cooper, at the University of Illinois, from a translation furnished by the author. For this help, the author wishes to express his thanks. Since its original publication, some of the material has fallen out of date, so that it was felt that republishing certain sections of the original article would not be worthwhile at this time. The various sections omitted were concerned with:

1. An analysis of the two-dimensional stylus motions under the influence of tracing error, using a complex-variable approach to obtain a compact representation of the two degrees of freedom. The results obtained were similar to those recently published by Cooper,[8] but were somewhat more extensive.

2. An analysis of intermodulation distortion for tracing error in the absence of deformation. The results agree with those published by Roys[9] and Cooper.[10]

3. An analysis of the effects of the overall curvature of the groove track as it spirals around the center hole of the disc. In effect, each of the two groove walls has a different groove speed. The consequence was a source of distortion that would be the more serious at low frequencies and at inner grooves, and one which conceivably could be enhanced by arm resonance. Present-day arm resonances

in the better systems are below the audio band, however, so that exploration of this question was thought to be somewhat less timely than the deformation problem.

4. A summary for which these remarks will equally well serve was the final deletion.

Apart from these deletions, no attempt has been made to bring this material up to date, or to correlate it with more recent references. The reader may wish to consult the 1963 paper by Kantrowitz.[11]

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THE AUTHOR

Takeo Shiga was born in Tokyo, Japan, in 1924. He graduated from Tokyo University receiving a degree in engineering physics in 1946, and from the same university, a doctorate in applied physics in 1962.

In 1946 he joined Nippon Columbia, Ltd., a company which is affiliated with Columbia Records of CBS, and which is a manufacturer of phonograph records, stereo phonographs, and television receivers, where he has been responsible for the development of loudspeakers, phonograph pickups, and stereo phonographs. In recent years his studies have been of stereophonic sound, the analysis of musical and vocal sound, and the development of acoustical transducers at the Research Laboratories of Nippon Columbia, where he is presently Director of Acoustical Research Development.

Dr. Shiga is a member of the Acoustical Society of Japan and of the Audio Engineering Society.

COMMENTS ON DEFORMATION DISTORTION IN DISC RECORDS

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THE paper "Deformation Distortion in Disc Records" by Takeo Shiga which appeared in the July 1966 issue of the *Journal* (**14**, 208) still appears to be the most nearly definitive work on the subject to date. Although it is more than two years since publication, it would seem that the corrections as given below would help in maintaining the paper's usefulness to interested readers.

Considering the complexity of the original article and the circumstances of its original publication it is a tribute to the staff of The Journal of the Audio Engineering Society that so few errors were found.

CORRECTIONS

Eq. (16), second line: for superscript VI read IV.

Eq. (19): the last line should read $r^2\psi'\psi''[1-\delta_0/r]$.

Eq. (23): for $(r/36)a\psi''^2$ read $(r^2/36)a^2\psi''^2$, and for $(5r^3/648)a^2\psi''^3$ read $(5r^3/648)a^3\psi''^3$.

Eq. (24): for $a^2(r-\delta)\psi'\psi''$ read $a^2(r-\delta)\psi'\psi'''$.

Eq. (26), third line following: for $(1-r/2\rho)/H^2$ read $(1+r/2\rho)/H^2$.

Eq. (33), fourth line on the right-hand side: for $MS_f''(t)^2/f^2$ read $MS_f''(t)]^2/f^2$ (also *second line of 32*); *twelfth line on the right-hand side:* for $(r\delta_0/6)(1+\ldots)^2$ read $(r\delta_0/6)(1+\ldots)$.

Eq. (40), first line: for $(r^2/6)\psi'\psi''$ read $(r^2/6)\psi'\psi'''$; *second line:* for $(r_2/36)\psi''^2$ read $(r^2/36)\psi''^2$; *third line:* for $2\psi'\psi''$ read $2\psi'\psi'''$.

Eq. (44), displayed equation immediately following: for K_1 read K.

Eq. (46), fifth line: for $(1-r\delta_0{}^2/6V^2)$ read $(1-r\delta_0\omega^2/6V^2)$.