$$J = h[s(T)] + \int_0^T g[s(t), u(t)]dt = \frac{1}{T}\int_0^T j(t)^2 dt \qquad h[s(T)] = h[v(T), a(T)] = 0$$

$$\begin{cases} s = (p, v, a) \qquad \dot{s} = f(s,u) = (v, a, j) \qquad u = j \\ s(0) = (p_0, v_0, a_0) \\ s(T) = (p_f, ?, ?) \qquad s_1(T) = p_f \end{cases}$$

$$H(s, u, \lambda) = g(s,u) + \lambda^T f(s,u) = \frac{1}{T}j^2 + \lambda_1 v + \lambda_2 a + \lambda_3 j$$

$$\dot{\lambda} = -\nabla_s H(s^*, u^*, \lambda) = (0, -\lambda_1, -\lambda_2)$$

$$\lambda(t) = \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \lambda_3(t) \end{bmatrix} = \frac{1}{T}\begin{bmatrix} -2\alpha \\ 2\alpha t + 2\beta \\ -\alpha t^2 - 2\beta t - 2\gamma \end{bmatrix}$$

$$\frac{\partial H(s^*, j, \lambda)}{\partial j} = \frac{2}{T}j + \lambda_3 = 0$$

$$\Rightarrow j^* = -\frac{T}{2}\lambda_3 = \frac{1}{2}\alpha t^2 + \beta t + \gamma$$

$$s^*(t) = \begin{bmatrix} p^*(t) \\ v^*(t) \\ a^*(t) \end{bmatrix} = \begin{bmatrix} \frac{\alpha}{120}t^5 + \frac{\beta}{24}t^4 + \frac{\gamma}{6}t^3 + \frac{a_0}{2}t^2 + v_0 t + p_0 \\ \frac{\alpha}{24}t^4 + \frac{\beta}{6}t^3 + \frac{\gamma}{2}t^2 + a_0 t + v_0 \\ \frac{\alpha}{6}t^3 + \frac{\beta}{2}t^2 + \gamma t + a_0 \end{bmatrix}$$

$$h[s(T)] = 0$$

$$\lambda_2(T) = \frac{\partial h[s^*(T)]}{\partial v} = 0 \Rightarrow \beta = -\alpha T$$

$$\lambda_3(T) = \frac{\partial h[s^*(T)]}{\partial a} = 0 \Rightarrow \gamma = \frac{1}{2}\alpha T^2$$

$$p^*(T) = p_f$$

$$\Rightarrow \Delta p = p_f - p_0 - v_0 T - \frac{1}{2}a_0 T^2 = \frac{\alpha}{120}T^5 - \frac{\alpha}{24}T^5 + \frac{1}{12}\alpha T^5 = \frac{1}{20}\alpha T^5$$

$$\Rightarrow \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{1}{T^5}\begin{bmatrix} 20 \\ -20T \\ 10T^2 \end{bmatrix}\Delta p = \begin{matrix} \frac{20}{T^5}\Delta p \\ -\frac{20}{T^4}\Delta p \\ \frac{10}{T^3}\Delta p \end{matrix}$$

$$J = \frac{1}{T}\int_0^T j(t)^2 dt = \frac{1}{T}\int_0^T \frac{1}{4}\alpha^2 t^4 + \beta^2 t^2 + \gamma^2 + \alpha\beta t^3 + \alpha\gamma t^2 + 2\beta\gamma t \; dt$$

$$= \gamma^2 + \beta\gamma T + \frac{1}{3}\beta^2 T^2 + \frac{1}{3}\alpha\gamma T^2 + \frac{1}{4}\alpha\beta T^3 + \frac{1}{20}\alpha^2 T^4$$

$$= 100\cancel{\frac{\Delta p^2}{T^6}} - 200\cancel{\frac{\Delta p^2}{T^6}} + \cancel{\frac{400}{3}\frac{\Delta p^2}{T^6}} + \cancel{\frac{200}{3}\frac{\Delta p^2}{T^6}} - 100\cancel{\frac{\Delta p^2}{T^6}} + 20\frac{\Delta p^2}{T^6}$$

$$= \frac{20}{T^6} \delta^2 p = \frac{20}{T^6} \Big(p_f^2 + p_0^2 + v_0^2 T^2 + \frac{1}{4} a_0^2 T^4$$

$$- 2 p_f p_0 - 2 p_f v_0 T - p_f a_0 T^2 + 2 p_0 v_0 T + p_0 a_0 T^2 + v_0 a_0 T^3 \Big)$$

$$= \frac{20}{T^6} \left[\frac{1}{4} a_0^2 T^4 + v_0 a_0 T^3 + (v_0^2 - p_f a_0 + p_0 a_0) T^2 + 2 v_0 (p_0 - p_f) T + p_f^2 + p_0^2 - 2 p_f p_0 \right]$$

$$= 5 a_0^2 \frac{1}{T^2} + 20 v_0 a_0 \frac{1}{T^3} + 20 (v_0^2 - p_f a_0 + p_0 a_0) \frac{1}{T^4} + 40 v_0 (p_0 - p_f) \frac{1}{T^5} + 20 (p_f^2 + p_0^2 - 2 p_f p_0) \frac{1}{T^6}$$