Quiz 1

1. Use the Euclidean Algorithm to obtain integers x and y satisfying the following equation

$$\gcd(56, 72) = 56x + 72y.$$

Sol^n:- By Euclidean Algorithm -

$72 = 1 \cdot 56 + 16$ —(i)

$56 = 3 \cdot 16 + 8$ —(2)

$16 = 2 \cdot 8 + 0$ —(3)

$\Rightarrow \gcd(56, 72) = 8$

from -(i) $16 = 72 - 56$ —(4)

from (2) - $8 = 56 - 3 \cdot 16$

$8 = 56 - 3(72 - 56)$ (from 4)

$8 = 4 \cdot 56 + (-3) \cdot 72$

Thus - $x = 4, \quad y = -3$

General sol^n- $x = 4 + 9n, \quad y = -3 - 7n$

2. If $a \equiv b \mod n_1$ and $a \equiv c \mod n_2$, prove that $b \equiv c \mod n$, where $n = \gcd(n_1, n_2)$.

Sol^n-

$$a \equiv b \mod n_1$$

$$\Rightarrow \quad n_1 \mid a-b \quad -(i)$$

$$a \equiv c \mod n_2$$

$$\Rightarrow \quad n_2 \mid (a-c) \quad -(2)$$

$$n = \gcd(n_1, n_2)$$

$$\Rightarrow \quad n \mid n_1 \quad , \quad n \mid n_2 \quad -(3)$$

from (i), (2), and (3)

$$n \mid a-b \quad , \quad n \mid a-c$$

$$\Rightarrow \quad n \mid a-b-(a-c)$$

$$\Rightarrow \quad n \mid -(b-c)$$

$$\Rightarrow \quad n \mid b-c$$

$$\Rightarrow \quad b \equiv c \pmod n. \quad \square$$