MTH 204 Quiz 1

Maximum Points: 20 (Maximum Time: 20 mins)

February 26, 2021

Question 1.

(2 points) Mention the correct option in the answer sheet (Do not show your work).

Let $u(x, y)$ be a harmonic function, i.e.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Which of the following is an exact differential?

1. $\frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy = 0$
2. $\frac{\partial u}{\partial x} dx - \frac{\partial u}{\partial y} dy = 0$
3. $\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy = 0$
4. $\frac{\partial u}{\partial y} dx - \frac{\partial u}{\partial x} dy = 0$

Question 2.

(2 points) Mention the correct option in the answer sheet (Do not show your work).

Consider the ODE

$$x^2 \frac{d^2 y}{dx^2} - 6y = 0.$$

Which of the following form a basis of solutions for this ODE?

1. $\{x^2, x^3\}$
2. $\{x^2, x^{-3}\}$
3. $\{x^{-2}, x^3\}$
4. $\{x^{-2}, x^{-3}\}$

Question 3.

(2 points) Fill in the blanks to make the following sentence correct (Just write your answer, do not show work).

The ODE

$$(6x^5 - xy)dx + (-x^2 + xy^2)dy = 0$$

can be converted into an exact ODE by multiplying it with ______.

Question 4.

(2 points) Fill in the blanks to make the following sentence correct (Just write your answer, do not show work).

If $y(x) = e^{-x^2}$ is a solution of the ODE

$$x\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + \beta x^3 y = 0$$

for some $\alpha, \beta \in \mathbb{R}$, then the value of $\alpha\beta$ is ______.

Question 5.

(2 points) Mention whether the following statement is TRUE (Do not show work).

One particular solution of ODE

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = -e^x$$

is xe^x.

Question 6.

(2 points) Mention whether the following statement is TRUE or FALSE (Do not show work).

Consider the following ODE

$$\frac{dy}{dt} = y\left(1 - \frac{y}{10}\right)$$

For the initial condition $y(0) = 20$, if the solution is $y(t)$ then

$$\lim_{t\to\infty} y(t) = 20.$$

Question 7.

(4 points) Show your full work for this problem.

Consider the ODE

$$\frac{dy}{dt} + 5y = 10 + 29\cos 2t.$$

If $y(0) = 0$ then find $y(\pi)$.

Question 8.

(4 points) Show your full work for this problem.

Find the general solution of the ODE

$$\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = 10\cos x + 5\sin x.$$