MTH 371: Quiz I

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Instructions

- Show all your work to score full marks. Detailed answers are a must.
- You can use a calculator. No phones or other electronic devices may be used.
- This is a closed book exam.
- If needed, you can use the following information
  - The probability mass function of a random variable Y ~Binomial(n, p) is given by

    $$P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}, \;\; y = (1, 2, \ldots, n)$$

    with mean and variance $E(Y) = np$ and $V(Y) = np(1-p)$, respectively.
  - The probability mass function of a random variable Y ~Geometric(p) is given by

    $$P(Y = y) = (1-p)^{y-1} p, \;\; y = (1, 2, \ldots)$$

    with mean and variance $E(Y) = 1/p$ and $V(Y) = (1-p)/p^2$, respectively.
  - The probability mass function of a random variable Y ~Pascal(k, p) is given by

    $$P(Y = y) = \binom{y-1}{k-1} p^k (1-p)^{y-k}, \;\; y = (k, k+1, \ldots)$$

    with mean and variance $E(Y) = k/p$ and $V(Y) = k(1-p)/p^2$, respectively.
  - The probability mass function of a random variable Y ~Poisson(λ) is given by

    $$P(Y = y) = e^{-\lambda} \lambda^{y-1} / y!, \;\; y = (0, 1, 2, \ldots)$$

    with mean and variance $E(Y) = \lambda$ and $V(Y) = \lambda$, respectively.

I. Answer the following as True or False.

(a) (0.5 points) Let X_i be IID with moment generating function (m.g.f.), $M_{X_i}(s) = e^{s^2/2}$.The m.g.f. of $M_{X_1+X_2}(s) = e^{s^2}$.

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(b) (0.5 points) An Olympic pistol shooter hits the target with probability p. He is allowed n independent shots. The probability that he hits the target successfully in a shot remains same, i.e., p. We are interested in calculating the probability of number of hitting k successful targets out of n shots. The process can be modeled as discrete time and discrete state random process.

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(c) (0.5 points) Packet arrivals at a node in a communication network are modeled by a Bernoulli process with $p = 0.25$. The interarrival times, X_n in the process are IID Geometric. The sum of interarrival time is given by $S_n = X_1 + X_2 + \ldots X_n$. Though the X_i's are independent, the S_i's are not independent.

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(d) (0.5 points) Consider that cars arriving at a service center follows Poisson process. The number of arrivals, $\{N(t), t > 0\}$ follows a Poisson distribution with λt. The probability that there is exactly one arrival in $(0, 1]$ and $(1, 2]$ is $e^{-2\lambda}\lambda^2$.

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II. (2 points) In a Bernoulli process the interarrival times, $\{X_i\}$ are IID Geometric (p). Find

$$E\ (X_n - X_{n-1})^2$$

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III. You are visiting the rainforest, but unfortunately your insect repellent has run out. As a result, at each second, a mosquito lands on your neck with probability 0.5. If one lands, with probability 0.2 it bites you, and with probability 0.8 it never bothers you, independently of other mosquitoes. Assume that the time is considered discrete.

(a) (1 point) How will you model the process, explain.

(b) (1 point) Find the expected time between successive mosquito bites.

(c) (1 point) Find the probability of getting 2 bites in 5 seconds.

IV. Fred is giving out samples of canned dog food. He makes calls door to door, but he leaves a sample (one can) only on those calls for which the door is answered and a dog is in residence. On any call the probability of the door being answered is 3/4, and the probability that any household has a dog is 2/3. Assume that the events "Door answered" and "A dog lives here" are independent and also that the outcomes of all calls are independent. Answer the following questions

(a) (1.5 points) Find the probability that Fred gives away his first sample on his third call.

(b) (1.5 points) Find the probability that he gives away his second sample on his fifth call.