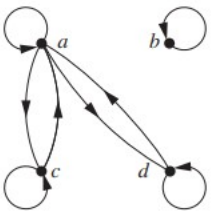
Discrete Mathematics CSE 121 : Homework 2

In every proof/derivation clearly state your assumptions and give details of each step.

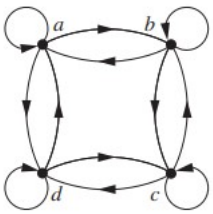
1. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

2. Let R be the relation on the set of all URLs (or Web addresses) such that xRy if and only if the Web page at x is the same as the Web page at y. Prove or disprove that R is an equivalence relation.

3. For the following relations, represented by either a digraph or a matrix, determine if it is (a) partial order, (b) linear order, (c) equivalence relation.



(a)



(b)

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d)

4. Is the assertion "This statement is false" a proposition?

5. Express these system specifications using the propositions p "The user enters a valid password," q "Access is granted," and r "The user has paid the subscription fee" and logical connectives (including negations).

   (a) "The user has paid the subscription fee, but does not enter a valid password."

   (b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."

   (c) "Access is denied if the user has not paid the subscription fee."

   (d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

6. A collection of logical operators is called functionally complete if every compound proposition is logically equivalent to a compound proposition involving only these logical operators. Show that $\neg$ and $\wedge$ form a functionally complete collection of logical operators. [Hint: First use a De Morgan's law to show that $p \vee q$ is logically equivalent to $\neg(\neg p \wedge q)$.]