Mid-Sem

October 15, 2022

Max marks: 120

1. Compute the Legendre symbol $\left(\frac{46}{83}\right)$.

   Be sure to justify all steps in your calculation.

2. Find integers x and y such that
$$182x + 1155y = \gcd(182, 1155).$$

3. Prove that if $m, n \in \mathbb{N}$ and $\gcd(m, n) = 1$, then
$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \mod (mn).$$

4. Show that for any integer n,
$$n^{13} \equiv n \mod (2730).$$

5. Solve the simultaneous congruences
$$x \equiv 4 \mod (91),$$
$$x \equiv 5 \mod (31).$$

6. Let $f(x) = x^3 + x^2 - 5$. Show that for $j = 1, 2, 3, \cdots$ there is a unique $x_j \mod 7^j$ such that
$$f(x_j) \equiv 0 \mod 7^j.$$

7. Find a complete set of quadratic residues r modulo 13 with $1 \le r \le 12$.

8. Show that for any integers c & k and prime p,
$$\left(\frac{c}{p}\right) = \left(\frac{c + kp}{p}\right).$$

9. Determine whether $x^2 \equiv 150 \mod (1009)$ is solvable.

10. T/F. Justify.

    (a) The numbers $3, 3^2, 3^3, 3^4, 3^5, 3^6$ form a reduced residue system modulo 7.

    (b) If n is an odd integer, then $\phi(2n) = \phi(n)$ and if n is an even integer, then $\phi(2n) = 2\phi(n)$.

    (c) The numbers -13, -9, -1, 9, 18 and 21 form a complete residue system modulo 7.

11. State one named Theorem from the course so far.