MTH 204 MidSem Solution & Rubric

Q.1.

1. order $= 6$ (+1 if correct, 0 if not)
   degree $= 3$ or undefined (for both answers +1 otherwise 0)
2. $a = 2$ (+1 if correct, otherwise 0)
   $b =$ any value (+1 for anything unless they write "no value")

Q.2.

1. F
2. T
3. T
4. T
5. T

Q.3.

1. $M = 2y$, $N = x$

   $$\frac{\partial M}{\partial y} = 2, \quad \frac{\partial N}{\partial x} = 1$$

   Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ not exact

   (For any other proof of non-exactness, read if it makes sense. If makes sense +1, otherwise 0).

2. $$F(x) = e^{\int \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) dx} = e^{\int \frac{1}{x} \cdot 1\, dx} = e^{\log x} = x$$

   (For showing some work in finding $F(x)$ +1
   for getting correct $F(x)$ +1)

3. New eq. $2xy\, dx + x^2 dy = 0$

   Let $\frac{\partial u}{\partial x} = 2xy \quad \frac{\partial u}{\partial y} = x^2$

   $$u = x^2 y + g(y) \Rightarrow \frac{\partial u}{\partial y} = x^2 + g'(y) = x^2$$

   $$\Rightarrow g'(y) = 0 \Rightarrow g = \text{const.}$$

   $$\Rightarrow u(x,y) = x^2 y + C$$

   (for work in finding $u(x,y)$ +1
   for correct $u(x,y)$ +1)

4. General sol. of (1) is

$$u = C$$

$$\Rightarrow x^2 y = C \quad \text{where } C \text{ is an arbitrary cont.}$$

<u>Q.4.</u> 1. $y+2 \,\big)\, y^3 - 6y^2 - y + 30 \,\big(\, y^2 - 8y + 15$

$$\begin{array}{r} y^3 + 2y^2 \\ \hline -8y^2 - y \\ -8y^2 - 16y \\ \hline 15y + 30 \\ 15y + 30 \\ \hline 0 \\ \hline \end{array}$$

So, other two eq. sols. are sols. of

$$y^2 - 8y + 15 = 0 \quad \Big\} +1$$

$$\Rightarrow y^2 - 5y - 3y + 15 = 0$$

$$\Rightarrow y(y-5) - 3(y-5) = 0$$

$$\Rightarrow y = 3, 5 \quad \Big\} +1$$

2. Let $f(y) = y^3 - 6y^2 - y + 30$

$$f'(y) = 3y^2 - 12y - 1$$

$$+1 \left\{ \begin{array}{lll} f'(-2) = 12 + 24 - 1 = 45 > 0 & \text{unstable} \\ f'(3) = 27 - 36 - 1 = -10 < 0 & \text{stable} \\ f'(5) = 125 - 60 - 1 = 64 > 0 & \text{unstable} \end{array} \right. \Big\} +1$$

(If they argue using direction of vectors below and above these eq. sols. then +2 if the argument is completely correct otherwise +1)

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Q.5.

1. $30 = k(1.5 - 1)$ $\left(\begin{array}{l} +1 \text{ correct} \\ 0 \text{ otherwise} \end{array}\right)$

$\Rightarrow k = \dfrac{30}{0.5} = 60$

2. $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{60}{10}} = \sqrt{6}$ (,,)

3. $10\dfrac{d^2y}{dt^2} + 60y = 0$ or $\dfrac{d^2y}{dt^2} + 6y = 0$ (,,)

4. $y(t) = c_1\cos(\sqrt{6}\,t) + c_2\sin(\sqrt{6}\,t)$ $\left(\begin{array}{ll} +1 & \text{correct} \\ +0.5 & \text{partially correct} \\ 0 & \text{otherwise} \end{array}\right)$

5. $\underbrace{y(0) = 0.5}_{+0.5}$, $\underbrace{y'(0) = 0}_{+0.5}$

6. $\underbrace{c_1 = 0.5}_{+0.5}$ $\underbrace{c_2 = 0,}_{+0.5}$

Q.6.

1. $y''' + 3y'' + 3y' + y = 0$ $\}+1$

2. $\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0$ $\}+1$

$(\lambda+1)^3 = 0 \Rightarrow \lambda = -1$ $\}+1$

$y(t) = (c_1 + c_2 t + c_3 t^2)e^{-t}$ $\}+1$

3. Let $y_p(t) = At^3e^{-t}$ $\}+1$

+1
$$\left\{\begin{aligned} y_p'(t) &= A(3t^2e^{-t} + t^3 \cdot -e^{-t}) \\ &= A(3t^2 - t^3)e^{-t} \\ y_p''(t) &= A[(6t - 3t^2)e^{-t} - (3t^2 - t^3)e^{-t}] \\ &= A[t^3 - 6t^2 + 6t]e^{-t} \\ y_p'''(t) &= A[(3t^2 - 12t + 6)e^{-t} - (t^3 - 6t^2 + 6t)e^{-t}] \\ &= A(-t^3 + 9t^2 - 18t + 6)e^{-t} \end{aligned}\right.$$

Plugging in the eq. we get by eq. const. coeff.

$\underbrace{6A = 6 \Rightarrow A = 1}_{+1} \Rightarrow y_p = t^3e^{-t}$

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4. $y(t) = (c_1 + c_2 t + c_3 t^2) e^{-t} + t^3 e^{-t} \} +1$

5. $y(0) = c_1 = -5 \} +1$

$y'(t) = (c_2 + 2c_3 t) e^{-t} + (c_1 + c_2 t + c_3 t^2) . -e^{-t} + 3t^2 e^{-t} - t^3 e^{-t} = (-t^3 + (-c_3+3) t^2 + (2c_3 - c_2) t + (c_2 - c_1)) e^{-t}$

$y'(0) = c_2 - c_1 = 1$

$\Rightarrow c_2 = -4 \} +1$

$y''(t) = (-3t^2 + 2t(-c_3+3) + 2c_3 - c_2) e^{-t} - (-t^3 + (-c_3+3)t^2 + 2(c_3 - c_2)t + c_2 - c_1) e^{-t}$

$y''(0) = 2c_3 - c_2 - c_2 + c_1 = -12$

$2c_3 = -8 + 5 - 12 = -15$

$c_3 = -\frac{15}{2} \} +1$