ECE 351 DSP: Practice Problems 5

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1) Write a code to design a Kaiser window filter with the following specifications. Pass bands [0,$\frac{\pi}{4}$] rad/sample and [$\frac{2\pi}{3}$, $\frac{3\pi}{4}$] rad/sample, and stop bands [$\frac{\pi}{3}$, $\frac{\pi}{2}$] and [$\frac{5\pi}{6}, \pi$] rad/sample, and both ripples being at -20 dB. Now, suppose you build a comb filter from this filter with $L = 3$. Identify its pass bands and stop bands in $\omega \in [0, \pi]$. Verify your answer by plotting the comb filter's frequency response (magnitude only).

   [**Hint**: Note that combing 'squeezes' the frequency response from $-\pi$ to π by a factor of L and then repeats it periodically. So, please take into account that the pass bands and stop bands given in the question are from 0 to π, and also that if $[\omega_1, \omega_2]$ is a pass or stop band, then so is $[-\omega_2, -\omega_1]$.]

2) Write a python code to design an equiripple filter with the following specs: Pass bands 50-100 Hz, Stop bands 0-30 Hz and 120-150 Hz, with the sampling frequency being 300 Hz. The ripple requirements are -30 dB in the 0-30Hz band, -20 dB in the 50-100 Hz band, and -25 dB in the 120-150Hz band. The code needs to plot the magnitude response of the filter. Next, let $x[n] = 2\cos(\frac{\pi}{4}n + \frac{\pi}{2})(u[n] - u[n-10])$ be an input to this filter. The code needs to compute and plot the output $y[n]$ of this filter.

3) Write a code to design an IIR filter with the following specifications: Pass bands [0,$\frac{1}{6}$] sample^{-1}, [$\frac{5}{12}$, $\frac{1}{2}$] sample^{-1}, and stop band [$\frac{1}{4}$, $\frac{1}{3}$] with pass band ripple -30 dB and stop band ripple -50 dB. Plots its magnitude response in dB. Also, the code should output the difference equation of the IIR filter.

4) Show that the transformation for the low-pass to band-pass conversion is an increasing function in the range $\Omega \geq \Omega_0$ and is a decreasing function in the range $\Omega \leq \Omega_0$, where Ω_0 is the geometric mean of the upper and lower pass band edges.

I. Answers

1) Pass bands: $[0, \frac{\pi}{12}], [\frac{2\pi}{9}, \frac{\pi}{4}], [\frac{5\pi}{12}, \frac{4\pi}{9}], [\frac{7\pi}{12}, \frac{3\pi}{4}], [\frac{8\pi}{9}, \frac{11\pi}{12}]$.

   Stop bands: $[\frac{\pi}{9}, \frac{\pi}{6}], [\frac{5\pi}{18}, \frac{7\pi}{18}], [\frac{\pi}{2}, \frac{5\pi}{9}], [\frac{7\pi}{9}, \frac{5\pi}{6}], [\frac{17\pi}{18}, \pi]$.