

Midterm Examination

Date: 03.11.2021

Time: 10:00AM - 12:00PM

Instruction:

- *Uploading of your answer sheet needs to be completed on or before 12:00PM in your google classroom section page. Any upload after 12:00PM will result in severe penalty including the possibility of assignment of 0 as your midterm score.*
 - *You may refer to the text book or class-notes. If you need to write verbatim from the book or class notes, you need to provide clear citation. You are not allowed to discuss with your peers or anyone else. Plagiarism policies are strictly enforced.*
 - *Your answer sheet needs to contain your name, roll number and tutorial section in the header.*
 - *Your answers needs to be correct and properly explained in order to obtain full score.*
 - *This question paper has **2 pages**, containing **10 questions**. Every question is worth 3 points. You can score a maximum of 30 points.*
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1. Assume $f(a, b) = 0$. What can you say about

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y)$$

if f is continuous at (a, b) ? What if you assume that f is not continuous at (a, b) ?

2. Assume that $w = f(s^3 + t^2)$ and $f'(x) = e^x$. Find $\partial w / \partial t$ and $\partial w / \partial s$.
3. Find the parametric equation for the line tangent to the curve of intersection of the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the point $(6, 4, 3)$.
4. In what direction is the derivative of $f(x, y) = xy + y^2$ at $(3, 2)$ equal to zero.
5. Define the notion of linearization for real valued functions of three variables. Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = xz - 3yz + 2$ at $(1, 1, 2)$.
6. Find all the local maxima, and local minima, and saddle points of the function $f(x, y) = 4xy - x^4 - y^4$.
7. Find the absolute maxima and absolute minima of $f(x, y) = 4x - 8xy + 2y + 1$ on the triangular plate bounded by the lines $x = 0, y = 0, x + y = 1$ in the first quadrant.
8. Find the minimum distance from the surface $x^2 - 2y^2 - 2z^2 = 1$ to the origin.

9. Show that the maximum value of $a^2b^2c^2$ on a sphere of radius r centered at the origin of a Cartesian abc -coordinate system is $(r^2/3)^3$.

10. Suppose that f and g are functions of x and y such that

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x} \quad \text{and} \quad \frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$$

and suppose that

$$\frac{\partial f}{\partial x} = 0, \quad f(1, 2) = g(1, 2) = 5, \quad \text{and} \quad f(0, 0) = 4.$$

Find $f(x, y)$ and $g(x, y)$.
