MTH 204 Quiz 2

Maximum Points: 20 (Maximum Time: 35 mins)

April 16, 2021

Question 1.

(2 points) Mention the correct option in the answer sheet (Do not show your work).

The power series solution, near $x = 0$, of the following Legendre Equation

$$(1 - x^2)y'' - 2xy' + 30y = 0, \quad y(0) = 0 \quad and \quad y'(0) = 1$$

is ?

1. $y = x - \frac{14}{3}x^3 + \frac{21}{5}x^5 + \dots$
2. $y = 1 - 15x^2 + 20x^4 + \dots$
3. $y = 1 + x - 15x^2 - 7x^3 + \dots$
4. $y = x - \frac{28}{3}x^3 + \frac{56}{5}x^5 + \dots$

Question 2.

(2 points) Mention the correct option in the answer sheet (Do not show your work).

We have

$$(2n + 1)P_n = \frac{d}{dx}(P_{n+1}) + \frac{d}{dx}(P_{n-1})$$

where P_n is the Legendre polynomial of degree n, then what is the value of

$$\int_0^1 P_n(x)dx$$

when n is even?

1. 0
2. 1
3. $\frac{n}{2!}$
4. $NOTA$

Question 3.

(2 points) Fill in the blanks to make the following sentence correct (Just write your answer, do not show work).

Cosider the following higher order ODE,

$$4y'' - 15y' - 4y = 0$$

Convert it into a system of ODEs _____.

Question 4.

(2 points) Fill in the blanks to make the following sentence correct (Just write your answer, do not show work).

The roots of the indicial equation of the ODE

$$9x(1-x)y'' - 12y' + 4y = 0$$

are _____ and _____.

Question 5.

(2 points) Mention whether the following statement is TRUE (Do not show work).

The critical point of the following system

$$y_1' = 5y_1$$
$$y_2' = 6y_2$$

is stable node.

Question 6.

(2 points) Mention whether the following statement is TRUE or FALSE (Do not show work).

The $y(t) = \begin{bmatrix} -2 \\ 2 \end{bmatrix} e^{-2t}$ is a particular solution to the following ODE

$$\mathbf{y} = \mathbf{A}\ \mathbf{y} + \mathbf{g} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \mathbf{y} + \begin{bmatrix} -6 \\ 2 \end{bmatrix} e^{-2t}$$

Question 7.

(4 points) Show your full work for this problem.

Find the general solution to the following systems

$$y_1' = 6y_1 + 9y_2$$
$$y_2' = y_1 + 6y_2$$

Question 8.

(4 points) Show your full work for this problem.

Find the type of all critical points by linearization

$$y_1' = 4y_1 - y_1^2$$
$$y_2' = y_2$$