ECE 351 DSP: Final Exam

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Total: 20 points

1) Is the system shown in Figure 1 stable?

[2 points]

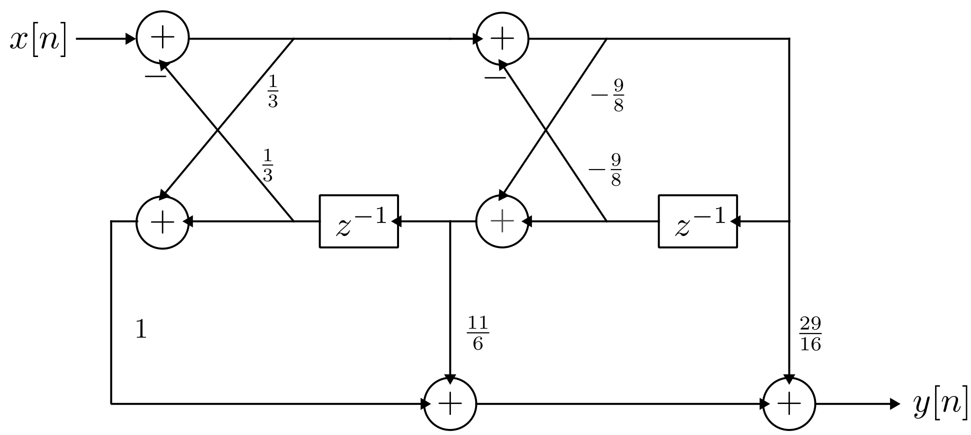

Fig. 1: Figure for Q.1

Solution: Since $| -\frac{9}{8} |> 1$, the system is unstable. ■

2) Is the system shown in Figure 2 stable?

[2 points]

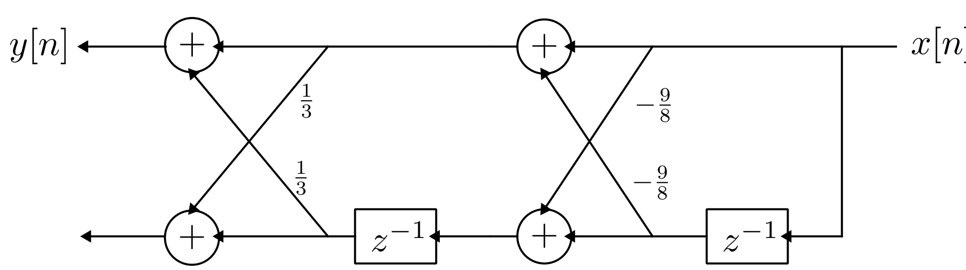

Fig. 2: Figure for Q.2

Solution: This is an FIR system, and hence it is stable. ■

3) Can the system shown in Figure 3 represent a high pass filter?

[2 points]

Solution: This is a cascaded form with 3 delays, and hence $M = 4$. Since M is even, this cannot represent a high pass filter. ■

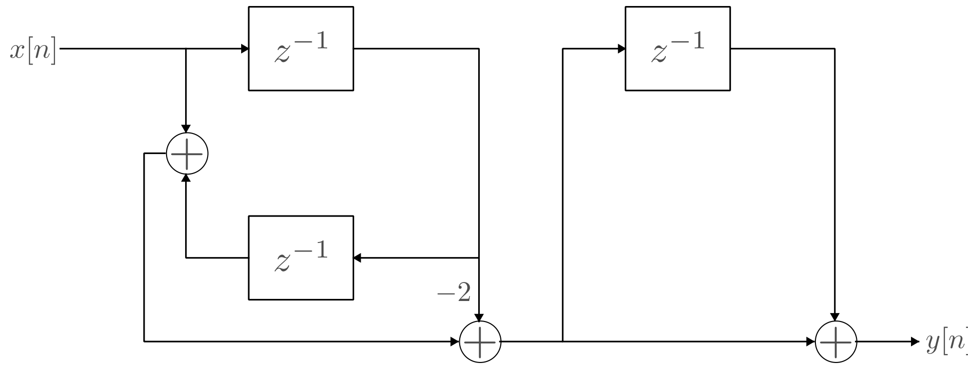

Fig. 3: Figure for Q.3

4) Consider the system shown in Figure 4.

   a) Show that the system is mixed phase.

   b) Decompose the system into a min-phase system and an all pass filter.

[3+3=6 points]

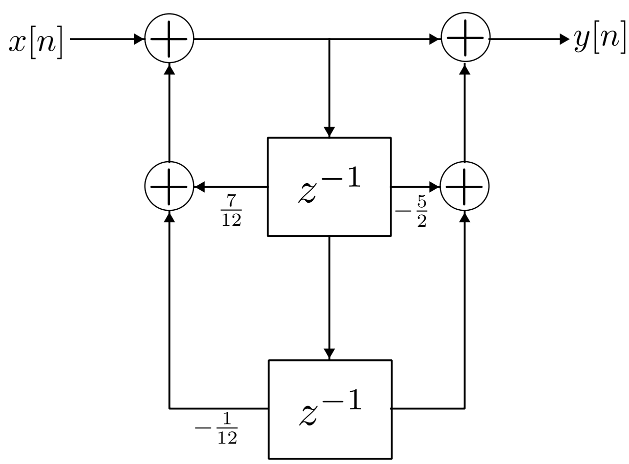

Fig. 4: Figure for Q.4

Solution: a) The above is a direct form IIR system. Hence, we have

$$H(z) = \frac{1 - \frac{5}{2}z^{-1} + z^{-2}}{1 - \frac{7}{12}z^{-1} + \frac{1}{12}z^{-2}} = \frac{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})}.$$

So the poles of the system are $\frac{1}{3}$ and $\frac{1}{4}$ and the zeros are at $\frac{1}{2}$ (inside the unit circle) and 2 (outside the unit circle). Hence, the system is mixed-phase.

b)

$$\begin{aligned} H(z) &= \frac{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})} \\ &= \frac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})} \frac{(1-2z^{-1})}{(1-\frac{1}{2}z^{-1})} \\ &= -2\frac{(1-\frac{1}{2}z^{-1})^2}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})} \frac{(-\frac{1}{2}+z^{-1})}{(1-\frac{1}{2}z^{-1})}. \end{aligned}$$

So the min phase system has transfer function $-2\frac{(1-\frac{1}{2}z^{-1})^2}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})}$, and the all pass filter has transfer function $\frac{-\frac{1}{2}+z^{-1}}{1-\frac{1}{2}z^{-1}}$. ■

5) Consider the sampling rate conversion system shown in Figure 5.

a) On feeding the upsampler with $[1, -2, 4]$, it returns $[1, 0, 0, 0, -2, 0, 0, 0, 4]$. Find I.

b) On feeding the downsampler with $[1, -2, -1, 4, 8, 9, -3, -6, 0, -2, 3, 6, 7]$ it returns $[1, 9, 3]$. Find D.

c) What are the sampling frequencies of $x[n]$ and $y[n]$?

[1+1+2=4 points]

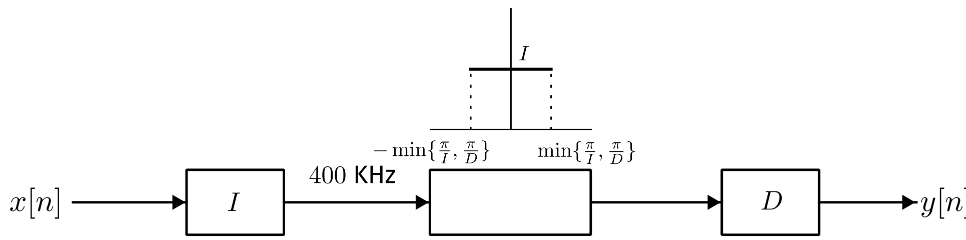

Fig. 5: Figure for Q.5

Solution: a) Since 3 zeros are added between each sample, we have $I = 4$.

b) Since every 5th sample is being retained, we have $D = 5$.

c) Notice that $IF_x = 400$ and $I = 4$. Hence, the sampling rate of $x[n]$ is $F_x = 100KHz$. Now, $F_y = \frac{I}{D}F_x$. Hence, $F_y = 80KHz$. ■

6) Consider an analog low pass filter $H_{\text{LP}}(s)$ with pass band edge at 1 rad/s and stop band edge at $\sqrt{3}$ rad/s.

a) Consider applying the frequency transformation $H(s) = H_{\text{LP}}(\frac{1}{s})$. What is the resulting filter and where is its stop band edge(s)?

b) Now, suppose a bilinear transform is applied on this filter with $T = 2$. What is the stop band edge of the resulting digital filter?

c) Next, assume the digital filter is combed with $L = 3$. Where are its stop band edges in $[0, \pi]$?

[1+1+2=4 points]

Solution: a) This is a low-pass to high-pass frequency transformation. Note that the general form of this transformation is given by $s \to \frac{\Omega_p \Omega_{p'}}{s}$, where Ω_p and $\Omega_{p'}$ are respectively the passband edges of the initial low-pass and the final high-pass filter. Thus, in this case, $\Omega_{p'} = 1$ and $\Omega_p = 1$. Now, let Ω_s and $\Omega_{s'}$ be the stop band edges of the original and transformed filter respectively. Then, we have $\Omega_{s'} = \frac{\Omega_p \Omega_{p'}}{\Omega_s} = \frac{1}{\sqrt{3}}$.

b) We have that bilinear transformation with $T = 2$ gives $\omega = 2\tan^{-1}\Omega$. So the stop band edge is at $2\tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{3}$ rad/sample.

c) Note that combing with $L = 3$ results in the stop-band edge appearing at $\frac{\pi}{3L} + \frac{2\pi l}{L}$ for every $l \in \mathbb{Z}$. Furthermore, the stop-band edge at $-\frac{\pi}{3}$ also reappears at $-\frac{\pi}{3L} + \frac{2\pi l}{L}$ for every $l \in \mathbb{Z}$. Of these both sets of stop band edges, only $L = 3$ will appear in the range $[0, \pi]$. These are $\frac{\pi}{9}$, $-\frac{\pi}{9} + \frac{2\pi}{3} = \frac{5\pi}{9}$, and $\frac{\pi}{9} + \frac{2\pi}{3} = \frac{7\pi}{9}$. ■