MTH 377/577 Convex Optimization
Problem Set 1

January 30, 2024

1. Consider the following definition:

   Definition 1 *A set $A \subset R^n$ is said to be **closed** if it contains all its limit points. That is, if $\{x^k\}_{k \geq 1}$ is any convergent sequence of points in S, then $lim_{k \to \infty} x^k$ is in S as well.*

   Using the above definition, prove that the set $\{x \in R : 0 \leq x \leq 1\} = [0, 1]$ is closed.

2. Prove or disprove: the union of 2 or more convex sets is convex.

3. Sketch the cone generated by the columns of the following matrix:

$$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

4. Use Farkas Lemma to decide if the following system has a non-negative solution:

$$\begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

5. Consider the set $A = \{(x_1, x_2) \in R^2 | x_1 + x_2 \leq 4\}$. Is this set a Polyhedron? Find a set S such that A is the convex hull of S.

6. Let A be a $m \times n$ matrix $K = \{y : y = Ax, ||x|| \leq 1\}$. Show that K is convex.

7. Let A be a circle with radius 2 and center $(0, 0)$. Let $B = \{(x_1, x_2) \in R^2 | 2 \leq x_1 + x_2 \leq 4\}$. Can you find a separating hyperplane between A and B? Provide an argument in support of your answer.