Eco 311/511: Game Theory
Problem set 1

September 8, 2023

1. Which of the following binary relations are "preferences"? Why/Why not?

   (a) "is greater than ($>$)" defined on $\mathcal{R}$, the set of real numbers.

   (b) "is a neighbour of" defined on the set $N = \{1, 2, 3, 4\}$. Each of the four individuals lives in a separate house on one side of a street as depicted below:

   $$- - - - - - - - - - - - - - - street - - - - - - - - - - - -$$
   $$[House1] - - - - [House2] - - - - [House3] - - - - [House4]$$

2. Find the values of $x, y \in R$ such that the following 2 player simultaneous move game is a Prisoner's Dilemma game, and find the Nash equilibria:

   Player 1

Player 2		A	B
	A	(x,3)	(6,2)
	B	(2,6)	(4,y)

   (a) Find values of x, y such that the game has a weakly dominant strategy for each player but does not have a strictly dominant strategy. What will be the Nash equilibria in this case?

   (b) Find an example of payoffs that you can assign to (A, A) and (B, B) for the two players so that there is no strategy that strictly dominates the other strategy for any player and (B, B) is a Nash equilibrium ?

(c) Consider a strategic game $< N, \{A_i\}_{i \in N}, \{U_i\}_{i \in N} >$ with $N = 2$. Show that for any player $i \in N$, if a pure strategy a_i is assigned non-zero probability in an MSNE, it must be a best response to some strategy a_j of player $j \neq i$, $j \in N$.

(d) Consider a market for homogeneous products that has 2 firms: firm 1, firm 2. The firms compete by choosing their respective prices p_1, p_2. The demand function for firm $i \in \{1, 2\}$ is as follows:

$$q_i(p_i, p_{-i}) = \begin{cases} 1, & \text{if } p_i < p_{-i} \\ \frac{1}{2}, & \text{if } p_i = p_{-i} \\ 0, & \text{if } p_i > p_{-i} \end{cases}$$

Suppose the cost of production is 0 for both the firms. Find the prices that the firms will charge in equilibria and the quantities that they will sell.