Worksheet 4

October 20, 2021

1. Let f(x) $\in$ **Z**[x], let n $\geq$ 1, and let p be a prime. For all x, t $\in$ **Z**.

$$f(x + p^n t) = f(x) + f'(x)p^n t \,(mod\, p^{n+1}).$$

Hints:

1. Step 1: Write the Taylor expansion of f.
2. Step 2: Show $p^{n+1} \mid \frac{f^{(j)}(x)}{j!} p^{jn} t^j \qquad for\ j \geq 2.$

Substep 1: Show $\frac{f^{(j)}(x)}{j!}$ is an integer. In order to show this prove that the product of k consecutive integers is divisible by k! (k $\geq$ 1)

2. Solve the congruence $x^2 + 5x + 18 \equiv 0 (mod\ 49)$.