MTH100 **Monsoon Semester – 2021-22** **Section B**

Submission for Tuesday 22nd March 2022 – 17 minutes. **Max Marks: 5**

Instructions: *Open notes and textbook; consultation and use of calculators, computers and internet not allowed. You may use any* **known** *result. This includes all propositions and observations in the lecture slides, and results from tutorials. If you use any other result from any other source, including the textbook, you have to give a full proof of that result.*

a) For the matrix A = [$\mathbf{v}_1$ $\mathbf{v}_2$ $\mathbf{v}_3$ $\mathbf{v}_4$ $\mathbf{v}_5$] given below, find a basis for Col A which consists of columns of A, ***but not including*** $\mathbf{v}_1$. You must briefly describe your method, and clearly show all your calculations. *(3 marks)*

b) For a general m×n matrix B = [$\mathbf{u}_1$ $\mathbf{u}_2$ …… $\mathbf{u}_n$] with non-zero columns, which are **not linearly independent**, explain with reference to any known results, why it is always possible to have a basis of Col B consisting of columns of B, ***not including*** $\mathbf{u}_1$. *(2 marks)*

$$A = \begin{bmatrix} 1 & 2 & 2 & 1 & 3 \\ 2 & 6 & 9 & 1 & 4 \\ 3 & 7 & 17/2 & 2 & 5 \end{bmatrix}$$

→ b) was an error – it was meant to be PROVE/DISPROVE

SOLUTION & RUBRIC

a) There are different possible answers and methods. So:

Any Correct Answer → 2.5 marks

Method → 2.5

(steps and brief explanation to be shown. No marks for method if answer is wrong.)

One such method is presented; it keeps the calculations to the minimum.

~~and follows the idea given for the~~

a) - continued

We permute the columns of A, moving $\bar{v}_1$ to the last place, giving the matrix $A' = [\bar{v}_4\ \bar{v}_2\ \bar{v}_3\ \bar{v}_5\ \bar{v}_1]$

Clearly Col A = Col A'

(I also shifted $\bar{v}_4$ to 1st column; this is only to simplify the calculations, and is not necessary.) - We now-reduce A'

From the calculations (see page ~~2~~ 3) it follows that the 1st, 2nd, 3rd columns of A' form a basis for Col A' = Col A. So:-

Answer: $\{\bar{v}_4, \bar{v}_2, \bar{v}_3\}$ is one possible basis for Col A, which does not include $\bar{v}_1$.

Rubric is in the box on page 1.
There are other possible selections of 3 lin. indep. columns, not including $\bar{v}_1$.

Remark: There was an error in part b) → it was meant to be PROVE/DISPROVE - So all the marks are for a).

Row-reducing $A' =$

$$\begin{bmatrix} 1 & 2 & 2 & 3 & 1 \\ 1 & 6 & 9 & 4 & 2 \\ 2 & 7 & 17/2 & 5 & 3 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - R_1}$$

$$\begin{bmatrix} 1 & 2 & 2 & 3 & 1 \\ 0 & 4 & 7 & 1 & 1 \\ 0 & 3 & 9/2 & -1 & +1 \end{bmatrix} \xrightarrow{R_2 \to R_2 - R_3}$$

$$\begin{bmatrix} 1 & 2 & 2 & 3 & 1 \\ 0 & 1 & 5/2 & 2 & 0 \\ 0 & 3 & 9/2 & -1 & -1 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_2}$$

$$\begin{bmatrix} 1 & 2 & 2 & 3 & 1 \\ 0 & 1 & 5/2 & 2 & 0 \\ 0 & 0 & -3 & -7 & -1 \end{bmatrix}$$

Without obtaining an RREF matrix, it is clear from the echelon form above, that columns 1, 2, 3 of A' are pivot columns, hence indicate a basis of Col A' = Col A.