Worksheet 9

December 2, 2022

1. Prove that for $1 \leq k \leq n$,

   (i) $<a_0, a_1, \ldots, a_n> = <a_0, a_1, \ldots, a_{k-1}, <a_k, a_{k+1}, \ldots, a_n>>$,

   Hint : Use Induction on the number of terms in the innermost continued fraction on the right hand side $<a_k, a_{k+1}, \ldots, a_n>$.

   (ii) Use part(i) to prove that:

   $$<a_0, a_1, \ldots, a_n> = a_0 + \frac{1}{<a_1, \ldots, a_n>}$$

2. Convert each of the following into finite simple continued fractions

   (i) 0.23

   (ii) $\frac{233}{177}$

3. Find $\frac{p}{q}$ if $\frac{p}{q} = [3, 7, 15, 1]$. Convert $\frac{p}{q}$ to a decimal and compare with the value of π.

4. (i) Writing the simple continued fraction of proper fractions. Ex. $\frac{29}{67}$.

1 Ex. $\frac{29}{67}$;

$$\frac{29}{67} = [0, 2, 3, 4, 2]$$
$$= [a_1, a_2, a_3, a_4, a_5]$$

$$0 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{2}}}}$$

$67\,)\,29\,(\,0 = a_1$
0
$29\,)\,67\,(\,2 = a_2$
58
$9\,)\,29\,(\,3 = a_3$
27
$2\,)\,9\,(\,4 = a_4$
8
$1\,)\,2\,(\,2 = a_5$
2
0

(ii) Write the continued fraction expansion for $\frac{67}{29}$.

(iii) Compare (i) & (ii) above to conclude a general result for the relation between the continued fraction of

$\frac{p}{q}$ & $\frac{q}{p}$, where p>q.