Worksheet 12 Solution

Problem 1: Since the PDE is only depending on y, we can treat x as if it were a parameter, then we can solve the PDE as if it were an ODE on y

$$u_y = -y^2 u$$

$$\frac{du}{u} = -y^2$$

$$\log|u| = \frac{-y^3}{3} + C(x)$$

$$\boxed{u = C(x)\, e^{\frac{-y^3}{3}}}$$

Problem 2:

Laplace equation is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Given: $u(x,y) = a\ln(x^2+y^2) + b$

$$\frac{\partial u}{\partial x} = \frac{2ax}{x^2+y^2} \quad ; \quad \frac{\partial u}{\partial y} = \frac{2ay}{x^2+y^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2a(x^2-y^2)}{(x^2+y^2)^2} \quad ; \quad \frac{\partial^2 u}{\partial y^2} = \frac{2a(y^2-x^2)}{(x^2+y^2)^2}$$

Adding these together,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2a(x^2-y^2)}{(x^2+y^2)^2} + \frac{2a(y^2-x^2)}{(x^2+y^2)^2} = 0$$

So, $u(x,y)$ satisfies laplace's equation.

- for the circle $x^2+y^2=1$, $u=110$

  $a\ln(1)+b = 100 \implies \boxed{b=100}$

- for the circle $x^2+y^2=100$, $u=0$

  $a\ln(100)+b=0$

  $\implies 2a\ln(10)+b=0$

  $\implies 2a(2\cdot 3026) = -110$

$$\boxed{a = -23.865}$$

Problem 3 :

$$u = x^2 + t^2$$

$$u_t = 2t$$

$$u_{tt} = 2 \quad ; \quad u_{xx} = 2$$

The wave equation states $2 = (c^2)(2)$

which is true for $c = 1$