MTH100 **Monsoon Semester – 2021-22** **Section B**

Submission for Tuesday 1st February 2022 – 15 minutes. **Max Marks: 5**

Instructions: Open notes and textbook; consultation and use of calculators, computers and internet not allowed.

IMPORTANT: You may use any **known** result. This includes all propositions and observations in the lecture slides, and results from tutorials. If you use any other result from any other source, including the textbook, you have to give a full proof of that result. Note that we have not covered *determinants* as of now. So, use of any result related to determinants is forbidden.

a) Suppose C is a 2×2 matrix such that C = AB where A is a 2×1 matrix and B is a 1×2 matrix. Can C be invertible (YES/NO) ? Justify your answer. (*2.5 marks*)

b) Suppose C is a 2×2 matrix such that C = AB where A is a 2×3 matrix and B is a 3×2 matrix. Can C be invertible (YES/NO) ? Justify your answer. (*2.5 marks*)

SOLUTION-CUM- RUBRIC

a) NO $\longrightarrow$ 0.5 MARKS

JUSTIFICATION: $\nearrow$ Another valid method on page ③

We use the following known result:-

A general 2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if and only if $ad - bc \neq 0$ (*)

[Tutorial 03_Week of Monday 20220124-Q5a)] $\longrightarrow$ 0.5 marks for citing this result.

So, now suppose C = AB, where

$A = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = [z \quad w]$.

a) - cont'd

$\therefore C = \begin{bmatrix} xz & xw \\ yz & yw \end{bmatrix}$ ①

Applying (*) to C, we get : $xzyw - xwyz = 0$. Hence, C is not invertible.

→ 1·5 marks for the argument.

Remarks: ① Any answer which uses determinants or the determinant formula without citing above result gets no marks (**) since use of determinants has been explicitly forbidden. (NB: the result in Tutorial Q3 does not use the word determinant.

② Any answer which uses Corollary 1·3 is wrong and gets 0 marks (**). Corollary 1·3 applies to factorization into square matrices only. It says : A square matrix $A = A_1 A_2 \cdots A_n$ is invertible if and only if each A_i is invertible (for square matrices only).

**: 0 marks for justification. 0.5 marks for NO can be given.

③ The justification can also be done by row-reduction (same as answer to Q5). This is lengthy.

b) YES

→ 1 mark

Justify : It is enough to give one correct example → 1·5 marks

There are many examples : One particularly simple one below :-

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$C = AB =$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$

I_2 - invertible !

a) Another method below:

Given $C = AB$ where A is a 2×1 matrix and B is a 1×2 matrix.

Consider the homogeneous system:

$$B\bar{x} = \bar{0} \qquad (1)$$

Since number of rows of B < number of columns of B, the system (1) has a non-trivial solution, say $\bar{v} \neq \bar{0}$.

This follows from:
Observation 3 on Homogeneous Systems
(LO5-2022011)

Since $B\bar{v} = \bar{0}$,

$$A(B\bar{v}) = A\bar{0} = \bar{0},$$

i.e. $C\bar{v} = \bar{0}$.

Since $\bar{v} \neq \bar{0}$, C is not invertible by VIT.

Rubric: 2 marks for this method
The results (underlined in red) should be cited.