MULTIVARIATE CALCULUS
SOLUTION TO QUIZ 1

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Abstract. This is the solution to the first of the two quizzes that you have to take for the successful completion of this course at IIIT-Delhi.

1. Problem

Identify each of the following subsets of the two-dimensional Euclidean space $\mathbb{R}^2$ as an open set or a closed set or neither.

(a) $\{(x, y) : |x| + |y| < 1\}$;
(b) $\{(x, y) : x^2 + y^2 \leq 1, x \geq 0 \text{ and } y \geq 0\}$;
(c) $\{(x, 0) : 0 < x < 1\}$;
(d) $\{(x, y) : x^2 + y^2 < 1\} \cup \{(1, 0)\}$;
(e) $\{(0, y) : y \in \mathbb{Q}\}$.

[*10 points*]

Solution. All of these are subsets of $\mathbb{R}^2$, so when we say that a particular item in the above question is *an open (respectively a closed)set*, we mean to say that it is *open (respectively closed) in* $\mathbb{R}^2$.

(a) Open subset of $\mathbb{R}^2$.
(b) Closed subset of $\mathbb{R}^2$.
(c) Neither open nor closed in $\mathbb{R}^2$.
(d) Neither open nor closed in $\mathbb{R}^2$.
(e) Neither open nor closed in $\mathbb{R}^2$.

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2. Problem

Suppose that $g : \mathbb{R} \to \mathbb{R}$ is a continuous function. Find the partial derivatives of the function f in two variables given by

$$f(x, y) = \int_a^{x+y} g(t)dt,$$

where $a \in \mathbb{R}$. [*05 points*]

Solution.

$$\frac{\partial f}{\partial x} = g(x + y) \text{ and } \frac{\partial f}{\partial y} = g(x + y).$$

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3. Problem

State whether the following statement is true or false. The gradient of the function

$$f(x,y) = x^2 + y^2$$

at the point $(1,1)$ is

$$2\vec{i} + 2\vec{j} + 3\vec{k}.$$

[*05 points*]

Solution. False. ∎

4. Problem

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function in two variables defined via

$$f(x,y) = \begin{cases} 1 & \text{if } xy = 0 \\ 0 & \text{if } xy \neq 0 \end{cases}.$$

True or False: The partial derivative of the function f with respect to y at the point $(2,0)$ exists. [*05 points*]

Solution. False. ∎

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