Submission for Wednesday 16th March 2022 – 17 minutes. Max Marks: 5

Instructions: *Open notes and textbook; consultation and use of calculators, computers and internet not allowed. You may use any* ***known*** *result. This includes all propositions and observations in the lecture slides, and results from tutorials. If you use any other result from any other source, including the textbook, you have to give a full proof of that result.*

Let V = $\mathbb{R}^3$, let S = {$\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, $\mathbf{v}_5$} as below, and let W = Span S.

a. Find a subset B of S such that B is a basis for W. All your steps must be clearly shown and briefly explained. *(3 marks)*

b. Is W = V (YES/NO) ? Justify your answer briefly. (2 *mark*)

$\mathbf{v}_1 = (2, 10, 4)$; $\mathbf{v}_2 = (3, 15, 6)$; $\mathbf{v}_3 = (1, 6, 3)$; $\mathbf{v}_4 = (2, 11, 6)$; $\mathbf{v}_5 = (8, 42, 19)$

Answer

The quickest way to answer this question is to determine the RREF matrix of the matrix $A = [\bar{v}_1\ \bar{v}_2\ \bar{v}_3\ \bar{v}_4\ \bar{v}_5]$

The RREF matrix is

$$R = \begin{bmatrix} 1 & 3/2 & 0 & 0 & 5/2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

a) W = Col A, so a basis for W consists of the pivot columns of R, namely: $\bar{v}_1, \bar{v}_3, \bar{v}_4$

Rubric: 1.5 marks for Ans, 1.5 marks for calculation of R.

b) YES. Since dim W = 3 = dim V, W = V by Prop. 19.

Rubric: 1 mark for YES, 1 mark for Justify.

$$A = \begin{bmatrix} 2 & 3 & 1 & 2 & 8 \\ 10 & 15 & 6 & 11 & 42 \\ 4 & 6 & 3 & 6 & 19 \end{bmatrix}$$

$$\xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 5R_1} \begin{bmatrix} 2 & 3 & 1 & 2 & 8 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 2 & 3 & 1 & 2 & 8 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - 2R_3}$$

$$\begin{bmatrix} 2 & 3 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_2} \begin{bmatrix} 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 \to \frac{1}{2} R_1} \begin{bmatrix} 1 & \frac{3}{2} & 0 & 0 & \frac{5}{2} \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

free variable

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