# Discrete Mathematics CSE 121 : Homework 4

In every proof/derivation clearly state your assumptions and give details of each step.

1. Let $P(n)$ be the statement that a postage of $n$ cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 18$.
   (a) Show statements $P(18)$, $P(19)$, $P(20)$, and $P(21)$ are true, completing the basis step of the proof.
   (b) What is the inductive hypothesis of the proof?
   (c) What do you need to prove in the inductive step?
   (d) Complete the inductive step for $k \geq 21$.
   (e) Explain why these steps show that this statement is true whenever $n \geq 18$.
2. Show that $n$ lines separate the plane into $(n^2 + n + 2)/2$ regions if no two of these lines are parallel and no three pass through a common point.
3. Show that an $n \times n$ checkerboard with one square removed can be completely covered using right triominoes if $n > 5$, $n$ is odd, and $3 \nmid n$.
4. Prove that if $A_1$, $A_2$,...,$A_n$ and $B$ are sets, then $(A_1 - B) \cup (A_2 - B) \cup \ldots \cup (A_n - B) = (A_1 \cup A_2 \cup \ldots \cup A_n) - B$.
5. What is wrong with this "proof"?

   **Theorem.** For every positive integer $n$, if $x$ and $y$ are positive integers with $max(x, y) = n$, then $x = y$.

   **Proof.**
   Basis Step: Suppose that $n = 1$. If $max(x, y) = 1$ and $x$ and $y$ are positive integers, we have $x = 1$ and $y = 1$.

Inductive Step: Let $k$ be a positive integer. Assume that whenever $max(x, y) = k$ and $x$ and $y$ are positive integers, then $x = y$. Now let $max(x, y) = k + 1$, where $x$ and $y$ are positive integers. Then $max(x - 1, y - 1) = k$, so by the inductive hypothesis, $x - 1 = y - 1$. It follows that $x = y$, completing the inductive step.

6. Give a recursive definition of each of these sets of ordered pairs of positive integers. [Hint: Plot the points in the set in the plane and look for lines containing points in the set.]

   (a) $S = (a, b)|a \in Z^+, b \in Z^+,\ and\ a + b\ is\ odd.$

   (b) $S = (a, b)|a \in Z^+, b \in Z+,\ and\ a|b.$

   (c) $S = (a, b)|a \in Z^+, b \in Z^+,\ and\ 3|(a + b).$

7. Recursively define the set of bit strings that have more zeros than ones.

8. Give a recursive algorithm for finding a mode of a list of integers. (A mode is an element in the list that occurs at least as often as every other element.)

9. Give a recursive algorithm for finding the reversal of a bit string.

10. Use strong induction to show that when a simple polygon $P$ with consecutive vertices $v_1, v_2, \ldots, v_n$ is triangulated into $n - 2$ triangles, the $n - 2$ triangles can be numbered $1, 2, \ldots, n - 2$ so that $v_i$ is a vertex of triangle $i$ for $i = 1, 2, \ldots, n - 2$.