QUIZ-9 (Solutions)

Sol.

(a) Since the LTI system is causal and stable, a single input-output pair is sufficient to determine the frequency response of the system. In this case

Input is $\left(\frac{4}{5}\right)^n u[n]$ & output is $n\left(\frac{4}{5}\right)^n u[n]$

The frequency response is given by

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

1 Mark.

where $X(e^{j\omega})$ & $Y(e^{j\omega})$ are the fourier transforms of $x[n]$ & $y[n]$ respectively

$$x[n] = \left(\frac{4}{5}\right)^n u[n] \xrightarrow{FT} X(e^{j\omega}) = \frac{1}{1-\frac{4}{5}e^{-j\omega}}$$

2 marks

using the differentiation in frequency property, we have

$$y[n] = n\left(\frac{4}{5}\right)^n u[n] \xrightarrow{FT} Y(e^{j\omega}) = j\,\frac{d\,X(e^{j\omega})}{d\omega} = \frac{\frac{4}{5}e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2}$$

2 marks

$$\therefore H(e^{j\omega}) = \frac{\left(\frac{4}{5}\right)e^{-j\omega}}{1-\frac{4}{5}e^{-j\omega}}$$

1 mark.

(b) $\because$ $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})}$

we can write

$$Y[e^{j\omega}]\left(1-\frac{4}{5}e^{-j\omega}\right) = \frac{4}{5}e^{-j\omega}X(e^{j\omega})$$

2 marks

Taking IFT on both sides

$$y[n] - \frac{4}{5}y[n-1] = \frac{4}{5}x[n-1]$$

2 marks.