# MTH 204 Quiz 3

(Time : 15 mins, Maximum Marks : 10)

March 22, 2023

**Question 1.**

[10 points] Solve the following system of ODEs:

$$\frac{dx}{dt} = 3x - 2y,$$
$$\frac{dy}{dt} = 2x - y.$$

Classify the critical point $(0,0)$ and draw trajectories in the phase plane around $(0,0)$ to support your classification.

Sol$^n$:

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = Ay$$

$$\det(A-\lambda I) = \begin{vmatrix} 3-\lambda & -2 \\ 2 & -1-\lambda \end{vmatrix} = (3-\lambda)(-1-\lambda)+4$$

$$= -3+\lambda-3\lambda+\lambda^2+4$$

$$= \lambda^2-2\lambda+1 = (\lambda-1)^2$$

2 pts.

$$|A-\lambda I| = 0 \Rightarrow \lambda = 1, 1$$

3 pts.

Eigenvectors: $3x-2y = \lambda x$

$\lambda=1$: $3x-2y = x \Rightarrow 2x-2y=0 \Rightarrow x=y \longrightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} = u_1$

For generalized one, $[A-\lambda I]u_2 = \lambda u_1$

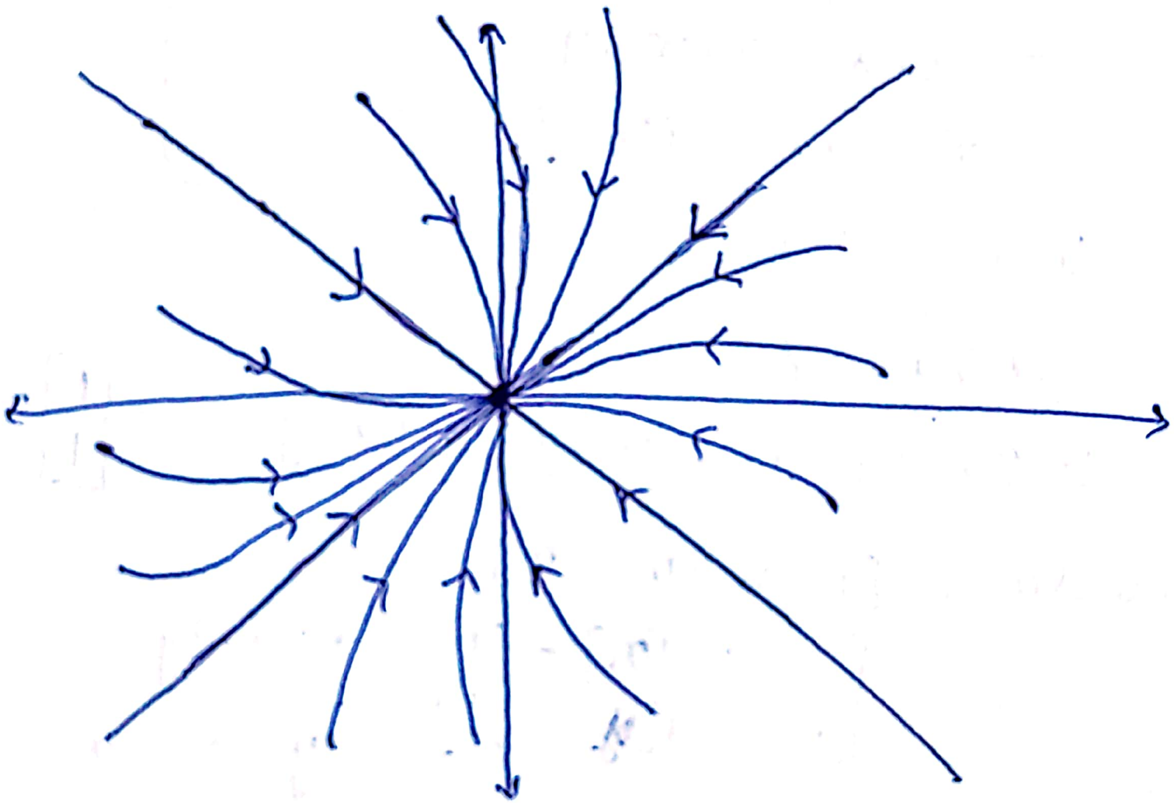
$$\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow 2x-2y=1$$

$y=0 \Rightarrow x = \frac{1}{2} \longrightarrow \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} = u_2$

$$\therefore \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^t + c_2\left(t\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^t + \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}e^t\right)$$

1.5 pts.

Since eigendimension is 1, so it is an improper node. } 1.5 pts.



2 pts.