Quiz 7 (Solutions).

Sol:→ Given,

$$IFT\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t).$$

Taking the fourier transformation of both sides we obtain

$$(1+j\omega)X(j\omega) = \frac{A}{(2+j\omega)}$$

$$X(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)}.$$

1 mark

$$X(j\omega) = A\left\{\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right\}.$$

2 marks.

Taking the inverse fourier transform of above equation.

$$x(t) = Ae^{-t}u(t) - Ae^{-2t}u(t).$$

1 mark.

Using Parseval's relation, we have.

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt.$$

Using the fact given $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$ we have.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = 1$$

2 marks

Substituting the previously obtained expression for $x(t)$ in the above equation, we have

$$\int_{-\infty}^{\infty} |A^2 e^{-2t} + A^2 e^{-4t} - 2A^2 e^{-3t}| \, u(t)\, dt = 1$$

1 mark

$$\int_{0}^{\infty} (A^2 e^{-2t} + A^2 e^{-4t} - 2A^2 e^{-3t}) \, dt = 1$$

So,

$$A^2/12 = 1$$

2 marks.

and $A = \sqrt{12} = 2\sqrt{3}$.

We choose A to be $2\sqrt{3}$ or $\sqrt{12}$ instead of $-2\sqrt{3}$ or $-\sqrt{12}$ because we know that $x(t)$ is non negative.

1 mark.

So.

$$\boxed{x(t) = 2\sqrt{3}\, e^{-t} u(t) - 2\sqrt{3}\, e^{-2t} u(t).}$$