

MTH 377/577 Convex Optimization

End Semester Exam

May 3, 2024

Answer all questions. Time = 90 minutes.

- ① Let $S = \{(x, y, z) : z \geq x^2 + y^2\} \subset \mathbb{R}^3$. Is S a convex set? Provide a short proof in support of your answer. (2)
- ✓ 2. Solve the following optimization problem and show all the steps:
minimize $x^2 - y$ such that $y - x \geq -2$, $y^2 \leq x$ and $y \geq 0$. (4)
- ✓ 3. Does a separating or supporting hyperplane exist for $a = (3, 1)$ and $K = \{(x, y) : x^2 + y^2 < 10\}$? If yes, find one such hyperplane. If no, show why it doesn't exist. (3)
- ✓ 4. Consider the following 2 player zero-sum game. Formulate the optimization problems for both the players and find an equilibrium randomized strategy for both the players. (3)

$$\begin{bmatrix} (3, -3) & (-1, 1) \\ (-2, 2) & (1, -1) \end{bmatrix}$$

5. Consider a set of N agents and M indivisible objects that are to be allocated to the agents. Let $v_{ij} \geq 0$ for all $j \in N$ and $i \in M$ denote the value assigned to $i \in M$ by agent $j \in N$. Suppose that $x_{ij} = 1$ denotes that object i is allocated to agent j ; and $x_{ij} = 0$ denotes that i is not allocated to agent j . Assume that no agent receives more than one object and no object is allocated to more than one agent. Formulate the above as an optimization problem. What general property do you think the constraint matrix will satisfy? (2+2)
6. Consider the maximization problem: $\max f(x, y; a, b) = ax^2 - x + by^2 - y$. Find the change in the optimised value of f as a varies. Under what conditions on a, b is f concave in x and y ? When is it convex? (2+2)