MTH 204 Quiz 2

(Time : 15 mins, Maximum Marks : 10)

February 15, 2023

Question 1.

[6 points] Consider the initial value problem

$$\frac{dy}{dx} = y^{1/3}, \quad y(0) = 0.$$

Find at least three distinct solutions of the above IVP. Describe what hypothesis of the uniqueness theorem this IVP doesn't satisfy.

Sol^n

(4.5 points 1.5 for each sol^n)

$$\frac{dy}{dx} = y^{1/3}, \quad y(0) = 0$$

Clearly, $y=0$ is a solution of the IVP

If $y \neq 0$, then by separable variable

$$\frac{dy}{y^{1/3}} = dx \Rightarrow \frac{3}{2} y^{2/3} = x + c$$

$$\Rightarrow \quad y(x) = \sqrt{\left(\frac{2}{3}(x+c)\right)^3}$$

The set of all solutions

$$y(x) = \begin{cases} 0 & , x < -c \\ \left(\frac{2}{3}(x+c)\right)^{3/2} & , x \geq -c \end{cases}$$

(1.5 points)

Now, $\frac{dy}{dx} = f(x,y) \Rightarrow f(x,y) = y^{1/3}$

$f(x,y)$ is continuous in a rectangle centered at $(0,0)$

$\frac{\partial f}{\partial y} = \frac{1}{3} y^{-2/3}$ is not continuous and bounded in a rectangle centered at $(0,0)$.

Hence, IVP does not satisfy the condition of continuity & boundedness of $\frac{\partial f}{\partial y}$ in a rectangle centered at $(0,0)$.

Question 2.

[4 points] Find and classify the stability behavior of equilibrium points of

$$\frac{dy}{dt} = -(y-10)^2(y-4).$$

Sol^n

If $\frac{dy}{dt} = f(y) \Rightarrow f(y) = -(y-10)^2(y-4)$

For equilibrium points : $f(y) = 0$

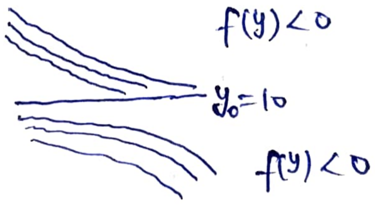
$\Rightarrow y = 4$ & 10

So, 4 and 10 are equilibrium points. (1 point)

Classification → $y_0 = 10$

$f(y) < 0$ below $y_0 = 10$

& $f(y) < 0$ above $y_0 = 10$



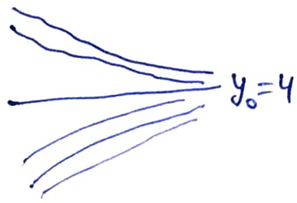
(1 point)

So, $y_0 = 10$ is semi-stable ($f(y)$ does not change sign along $y_0 = 10$) (0.5 point)

$y_0 = 4$

(1 point) ~~f(y)~~ $f(y) > 0$ below $y_0 = 4$

& $f(y) < 0$ above $y_0 = 4$



(0.5 point) So, $y_0 = 4$ is stable.