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Indicative Answer Key : Assignment

Ans 1 a) set of agents : $N = \{1,2,3\}$

Let A_i denote the set of actions for each $i \in N$.

$A_1 = \{A, B, C\}$. Suppose agent 1 chooses $a_1 \in A_1$.

$A_2 = A_1 \setminus \{a_1\}$, $a_1 \in A_1$

$A_3 = A_2 \setminus \{a_2\}$, $a_2 \in A_3$

or $A_2 \subset A_1$, $A_3 \subset A_2$.

$$u_i(a_i, a_{-i}) = \begin{cases} 2 & \text{if } a_i \succ_i a_j \ \forall j \neq i \\ 1 & \text{if } a_j \succ_i a_i \text{ for } \cancel{\text{some}} \text{ one } j \neq i \\ 0 & \text{if } a_j \succ_i a_i \ \forall j \neq i \end{cases}$$

In the above payoff function, a_{-i} is a generic action for all players other than i. An alternate formulation is

$$u_i(a_i, a_j, a_k) = \begin{cases} 2 & \text{if } a_i \succ_i a_l \ \forall l \in \{j, k\} \\ \text{— and so on —} \end{cases}$$

$\succ_i$ ~~is~~ denotes the preferences of agent i. The utility values 2, 1, 0 can vary as long as the ~~preferences~~ ~~are resp~~ order is respected.

The strategic game is $(N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N})$.

b) Yes, the game has Pure Strategy Nash Equilibria.

Consider an allocation (a_1, a_2, a_3) such that

$$a_1 = \arg\max_{a_i \in A_1} \cancel{u_1(a_1, a_{-1})}. \ U_1(a_i, a_2, a_3)$$

$$a_2 = \arg\max_{a_i \in A_2} (a_1, a_i, a_3)$$

$$a_3 = \arg\max_{a_i \in A_3} (a_1, a_2, a_i).$$

Teacher's Signature

Given the timing/sequence of the moves, (a_1, a_2, a_3) is PSNE

Note that the above formulation is general and ~~independent~~ ~~of the~~ valid for all preferences.

c) Case I: when each agent's most preferred object is different:

1	2	3
A	B	C
B	C	A
C	A	B

In this case, the allocation (A, B, C) is PSNE under the new assumption

Case II: when 2 or more agents have the same most preferred object:

1	2	3
A	A	C
B	C	B
C	B	A

In cases such as the above, agents will ~~choose to~~ prefer to exchange if they do not know or care about dissolution. In this case ~~there is~~ none of the allocations (a_1, a_2, a_3) will survive.

If agents care about dissolution, i.e., are rational about stating their choice to exchange, Then the same set of PSNE as in (b) survives:

~~B~~ since $u_i(a_1, a_2, a_3) > 0$ for any $(a_1, a_2, a_3) \in A_1 \times A_2 \times A_3$

Ans. 2. Set of players : $N = \{b, s\}$

Types : $\{H, L\}$

Seller's actions ~~$A_s = \{$~~ ~~S_s~~ $A_s = \{$sell, not sell$\}$

(s) Seller's strategy : $s_s : \{H, L\} \longrightarrow \{$sell, not sell$\}$.

(b) buyer's strategy : $s_b \in \{$buy, not buy$\}$.

Let p be the price at which the car is sold.

Payoffs when type = H

		Seller: Sell	Seller: Not sell
Buyer	Buy	$b_H - p, p - S_H$	$0, S_H$
	Not buy	$0, S_H$	$0, S_H$

Payoffs when type = L

		seller: sell	seller: not sell
Buyer	buy	$b_L - p, p - S_L$	$0, S_L$
	not buy	$0, S_L$	$0, S_L$

Prior belief that type is H is 0.4. Compute expected payoff from all pure strategy profiles. Let s_b = buy

(i) $s_s(H)$ = not sell , $s_s(L)$ = sell, ~~s_b = buy~~.

Expected payoff for seller : $0.4\,S_H + 0.6p - 0.6\,S_L$
$[0.4\,S_H + 0.6(p - S_L)]$.

(ii) $s_s(H) = s_s(L)$ = sell

Expected payoff for seller: $0.4(p - S_H) + 0.6(p - S_L)$
$= p - 0.4\,S_H - 0.6\,S_L$

(iii) $s_s(H)$ = sell , $s_s(L)$ = not sell

expected payoff for seller : $0.4(p - S_H) + 0.6\,S_L$
$= 0.4p - 0.4\,S_H + 0.6\,S_L$

(iv) $s_s(H)$ = not sell, $s_s(L)$ = not sell

expected payoff for seller $= 0.4\,S_H + 0.6\,S_L$

Expected payoff for buyer from $s_b = $ buy

$$0.4(b_H - p) + 0.6(b_L - p)$$
$$= 0.4 b_H + 0.6 b_L - p$$

buyer will choose $s_b = $ buy when $0.4 b_H + 0.6 b_L - p > 0$

$$\Rightarrow 0.4 b_H + \cancel{0.6 b_L}\ 0.6 b_L > p.$$

If type $= L$, seller will sell when $p \geq s_L$

If type $= H$, " " " " $p \geq s_H$.

~~Interim payoff for seller is p from selling, s_H or s_L from not selling~~

$$s_l^* = \begin{cases} \text{sell} & \text{if } L \\ \text{not sell} & \text{if } H \end{cases}, \quad s_b^* = \text{buy}$$

$s^* = (s_l^*, s_b^*)$ is ~~Nash~~ Bayes Nash equilibrium when

$$s_L \leq p \leq s_H$$
$$20 s_L \leq p \leq 2 s_H$$
$$p \leq 0.4 b_H + 0.6 b_L$$

[the conditions are derived by comparing (i) with (ii), (iii), (iv) — show the computations yourself].

Similarly find the conditions ~~in all cases~~ for all strategy profiles such that they are BNE.

Ans. 3. ~~[Indicative proof] Suppose for some $a_k \in A_1$, $\sigma_1(a_k) > 0$~~ and $\sigma_1'(a_k) = 0$

~~$\Rightarrow$~~

i.e., σ_1

Ans-3 [Indicative short proof]

Suppose ~~(σ_1', σ_2') is not MSNE~~. (σ_1, σ_2') is not MSNE

∴ $u_1(\sigma_1', \sigma_2') > u_1(\sigma_1, \sigma_2')$.

Since it is a zero sum game

$$u_2(a_k, b_l) = -u_1(a_k, b_l) \quad \forall (a_k, b_l) \in A_1 \times A_2.$$

∴ $u_1(\sigma_1', \sigma_2') > u_1(\sigma_1, \sigma_2')$

⇒ $u_2(\sigma_1, \sigma_2') > u_2(\sigma_1', \sigma_2') = u_2(\sigma_1, \sigma_2)$

∴ Player 2 will deviate to σ_2' from (σ_1, σ_2)

[since (σ_1, σ_2) and (σ_1', σ_2') both earn ~~minmax~~ same values for players]

∴ (σ_1, σ_2) is not MSNE.

This is a contradiction.

∴ (σ_1, σ_2') must be MSNE.

Arguments using minmax value are also correct — in any MSNE, the players earn the minmax/maxmin value.

Suppose $u_2(\sigma_1', \sigma_2') > u_2(\sigma_1, \sigma_2)$

$u_1(\sigma_1, \sigma_2) > u_1(\sigma_1', \sigma_2') \geq u_1(\sigma_1, \sigma_2')$

1 can increase payoff by mixing in favor of σ_1 when at σ_1'

Suppose (σ_1, σ_2), (σ_1', σ_2') are 2 MSNE of a zero sum game

$$u_2(\sigma_1, \sigma_2) \geq u_2(\sigma_1, \sigma_2') > u_2(\sigma_1', \sigma_2')$$

$$u_1(\sigma_1', \sigma_2') \geq u_1(\sigma_1, \sigma_2') \geq u_1(\sigma_1, \sigma_2)$$

σ_1 is not BR to σ_2'