Worksheet-6
Course Name: Math-III (Section-A)
Total marks = 20
Date: 02/11/2022

1. Sketch the region of integration and reverse the order of integration and then evaluate the following integral: $\int_0^1 \int_x^1 e^{y^2} dydx$ (2+3+3)
2. Calculate the volume beneath the surface $z = 3 + x^2 - 2y$ over the region D defined by $0 \leq x \leq 1$ and $-x \leq y \leq x$ (5)
3. Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$ (5)
4. Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$. (2)

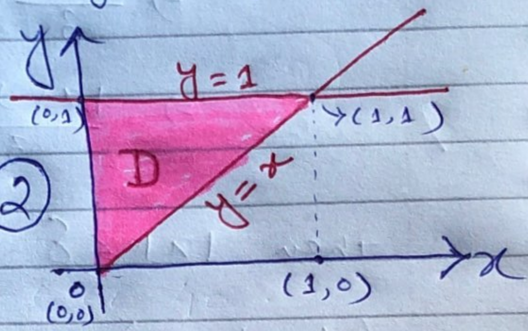
Rubric & Solution of Work Sheet - 6.

Q.1. Given integral is :

$$\int_0^1 \int_x^1 e^{y^2} \, dy \, dx.$$

So, according to the limits of integration of the given integral, the region of integration is :

$$D : \begin{cases} 0 \le x \le 1 \\ x \le y \le 1 \end{cases}$$



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Now reversing the order of integration will give us :

$$0 \le y \le 1$$
$$0 \le x \le y.$$

3

And the integral with the order changed is

$$\int_{y=0}^{y=1} \int_{x=0}^{x=y} e^{y^2} \, dx \, dy.$$

$$= \int_0^1 \left[x e^{y^2} \right]_{x=0}^{x=y} dy.$$

$$= \int_0^1 y e^{y^2} \, dy.$$

3

$$= \left[\frac{1}{2} e^{y^2} \right]_0^1$$

$$= \frac{1}{2} (e - 1).$$

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Q.2. Here the Surface is $z = 3 + x^2 - 2y$ and region is $D : 0 \le x \le 1$ & $-x \le y \le x$. (1)

So, the volume V is the double integral of $3 + x^2 - 2y$ over D. which is — (1)

$$V = \iint_D (3 + x^2 - 2y)\, dx\, dy$$

$$= \int_{x=0}^{x=1} \left(\int_{y=-x}^{y=x} (3 + x^2 - 2y)\, dy \right) dx$$

$$= \int_0^1 \left[3y + x^2 y - y^2 \right]_{-x}^{x} dx$$

$$= \int_0^1 (6x + 2x^3)\, dx$$

$$= \left[3x^2 + \frac{x^4}{2} \right]_0^1$$

$$= 3 + \frac{1}{2}$$

$$= \frac{7}{2}$$ (3)

Q.3. Given curves are : $y^2 = 4ax$ and $x^2 = 4ay$.

The point of intersection of these two curves are :

$$(y^2)^2 = 16a^2 \cdot 4ay.$$

$$\Rightarrow y\left(y^3 - (4a)^3\right) = 0.$$

2

$$\therefore \; y = 0 \text{ or } y = 4a.$$

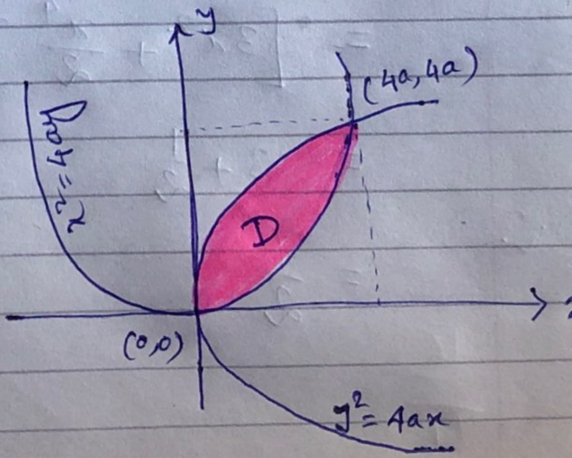
Hence, $x = \dfrac{y^2}{4a} \Rightarrow x = 0$ or $4a$.

$\therefore$ The point of intersections are $(0,0)$ & $(4a,4a)$

$\therefore$ The area between the two curves

= Area of the shaded region.

$$= \iint_D dx\,dy. \quad 1$$



$$= \int_{x=0}^{x=4a} \int_{y=\frac{x^2}{4a}}^{y=\sqrt{4ax}} dy\,dx.$$

$$2 \quad = \int_{x=0}^{x=4a} \left(\sqrt{4ax} - \frac{x^2}{4a}\right) dx = \left[(4a)^{\frac{1}{2}} \frac{2}{3} x^{3/2} - \frac{1}{4a}\frac{1}{3}x^3\right]_0^{4a}$$

$$= \frac{32a^2}{3} - \frac{16a^2}{3} = \frac{16a^2}{3} \quad \text{(Proved)}$$

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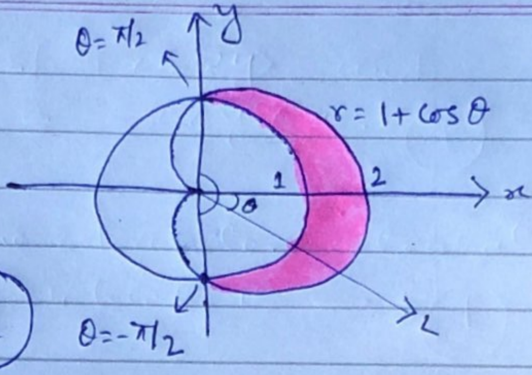
Q.4. Area of the region that lies inside the Cardioid $r = 1+\cos\theta$ & outside the circle $r = 1$



$$= \int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=1}^{r=1+\cos\theta} r\,dr\,d\theta \qquad 1$$

$$= \int_{-\pi/2}^{\pi/2} \left[\frac{r^2}{2}\right]_1^{1+\cos\theta} d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \left[\frac{(1+\cos\theta)^2}{2} - \frac{1}{2}\right] d\theta$$

$$= \frac{1}{2}\left[\cancel{\theta} + 2\sin\theta + \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) - \cancel{\theta}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}\left[2 + \frac{1}{2}\left(\frac{\pi}{2}\right) + 2 + \frac{1}{2}\frac{\pi}{2}\right]$$

$$= 2 + \frac{\pi}{4}$$

$$= \frac{8+\pi}{4} \qquad 1$$

—— x ——