Q.1) $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

here σ is known

$H_0: \mu = \mu_0$ v/s $H_1: \mu \neq \mu_0$

We usually find Pivot quantity using sufficient statistic.

Consider $\bar{X}$

→ we know that $\bar{x}$ is sufficient statistic

and some f^n of $\bar{X}$ is probably a pivot.

Now, let us consider

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

[Z distribution because we know σ]

here, Z is a function of x (Since, $\bar{x} = \frac{1}{n}\sum x_i$)

& θ (here unknown parameter is μ)

$$Z = Q(x, \theta) \quad -(1)$$

But, we know $Z \sim N(0,1)$

Thus, distribution of Z doesn't depend on the parameter θ (which is actually μ). $-(2)$

Using ① & ②

$\Rightarrow Z$ is our pivot.

Now,

$$P(a \leq Q(x, \theta) \leq b) = 1-\alpha$$

$$\Rightarrow P(a \leq Z \leq b) = 1-\alpha$$

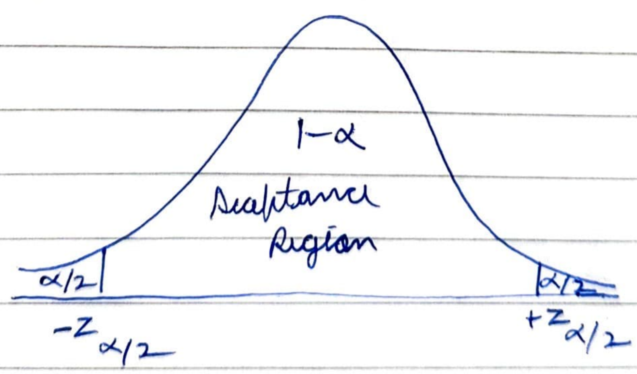
$$\Rightarrow P\left(a \leq \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq b\right) = 1-\alpha$$

$$\Rightarrow P\left(\frac{\sigma a}{\sqrt{n}} \leq \bar{X}-\mu \leq \frac{\sigma b}{\sqrt{n}}\right) = 1-\alpha$$

$$\Rightarrow P\left(\frac{\sigma a}{\sqrt{n}} - \bar{X} \leq -\mu \leq \frac{\sigma b}{\sqrt{n}} - \bar{X}\right) = 1-\alpha$$

$$\Rightarrow P\left(\bar{X} - \frac{\sigma a}{\sqrt{n}} \leq \mu \leq \bar{X} - \frac{\sigma b}{\sqrt{n}}\right) = 1-\alpha$$

Now, we try and find values of a & b
And, we observe



Thus $b = Z_{\alpha/2}$

& $a = -Z_{\alpha/2}$

Putting these values, we get

$$P\left(\bar{X} - \frac{\sigma z_{\alpha/2}}{\sqrt{n}} \le \mu \le \bar{X} + \frac{\sigma z_{\alpha/2}}{\sqrt{n}}\right) = 1-\alpha$$

Thus our required confidence interval is :-

$$\mu \in \left[\bar{X} - \frac{\sigma z_{\alpha/2}}{\sqrt{n}} , \bar{X} + \frac{\sigma z_{\alpha/2}}{\sqrt{n}}\right]$$

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Q2.

ans) Given $X_i \overset{iid}{\sim}$ Binomial(n_i, p) , $i = 1, \dots, K$

We know m.g.f of Binomial distrn

$$M_X(t) = (1-p+pe^t)^n$$

Now, $U = \sum_{i=1}^{K} X_i$

So,

$$M_U(t) = M_{\sum_{i=1}^{K} X_i}(t)$$

$$= E\left[e^{t(X_1 + \dots + X_K)}\right]$$

$$= \prod_{i=1}^{K} E\left[e^{tX_i}\right] \qquad \left[X \text{ is independent } \cancel{\text{iid}}\right]$$

$$= \prod_{i=1}^{K} (1-p+pe^t)^{n_i}$$

$$= (1-p+pe^t)^{\sum_{i=1}^{K} n_i}$$

So, for U,

$$n' = \sum_{i=1}^{K} n_i \quad , \quad p' = p$$

$\therefore$, $U \sim$ Binomial$\left(\sum_{i=1}^{K} n_i, p\right)$