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| **Quiz #4** | **Name:** _____ |
| **Date : 24/04/2024** | **Section:** _____ |
| **MTH204: ODEs/PDEs** | **Maximum Time:** 15 Minutes |
| **Semester: Winter 2024** | **Maximum Marks:** 10 |

**DO NOT SHOW ANY WORK HERE. JUST WRITE WHAT IS BEING ASKED. THERE IS NO STEP MARKING.**

**Problem 1.** [1] What is the Laplace Transform of $u(t-\pi)$, where $u$ denotes the unit step function?

$$\frac{e^{-\pi s}}{s} \quad +1$$

**Problem 2.** [2] What is the inverse Laplace Transform of $e^{-\pi s}/s^4$? (Hint. Laplace transform of $t^n$ is $n!/s^{n+1}$.)

$$\frac{1}{6}(t-\pi)^3 u(t-\pi)$$

$+1$ $+\frac{1}{2}$ $+\frac{1}{2}$

**Problem 3.** [2] What is the values of the integral

$$\int_0^\infty \sin(\pi - t)\delta(t + \pi/2)\, dt$$

where $\delta$ is the Dirac's delta function?

$$0 \quad +2$$

**Problem 4.** [2+2] Consider the equation

$$y - y * \sin(t) = t$$

where $*$ is the convolution of two functions.

(a) Take the Laplace Transform of the above equation and find an expression for Laplace transform of $y$. (Hint. Laplace transform of $\sin(t)$ is $1/(s^2+1)$.)

$$\mathcal{L}(y) = \frac{s^2+1}{s^4} = \frac{1}{s^2} + \frac{1}{s^4}$$

+1 +1

(b) Take inverse Laplace transform of expression obtained in (a) to find $y(t)$. (Hint. Laplace transform of $t^n$ is $n!/s^{n+1}$.)

$$y(t) = t + \frac{t^3}{6}$$

+1 +1

**Problem 5.** [1] For the $2\pi$-periodic function

$$f(x) = \begin{cases} x^2, & -\pi \le x < 1 \\ \frac{x}{2}, & 1 \le x < \pi \end{cases}$$

where will its Fourier Series converge at the point $x = 1$?

$$\frac{1}{2}\left(f(1^-) + f(1^+)\right) = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{3}{4}$$ +1