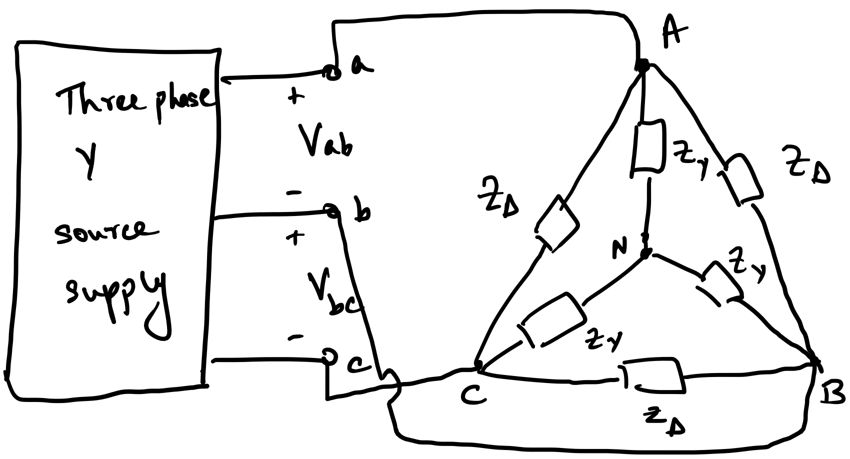
Set 1

Quiz 2 (12 marks)

The figure shows a balanced Y-connected load having $Z_Y = (5 + j5)\Omega$ in parallel with a Δ-connected load having $Z_\Delta = (15 - j9)\Omega$. The combined load is then connected to a 50Hz three-phase balanced Y- power supply having a line voltage $V_{ab} = 230\angle 15^\circ$ V. The wire impedance (of each wire) is 0Ω. Find the following –

(a) Phase voltages in time-domain
(b) Line currents in time-domain
(c) Complex power supplied by the source
(d) Power factor of the source
(e) Total time-averaged power consumed by the Y load
(f) Total time-averaged power consumed by the Δ load

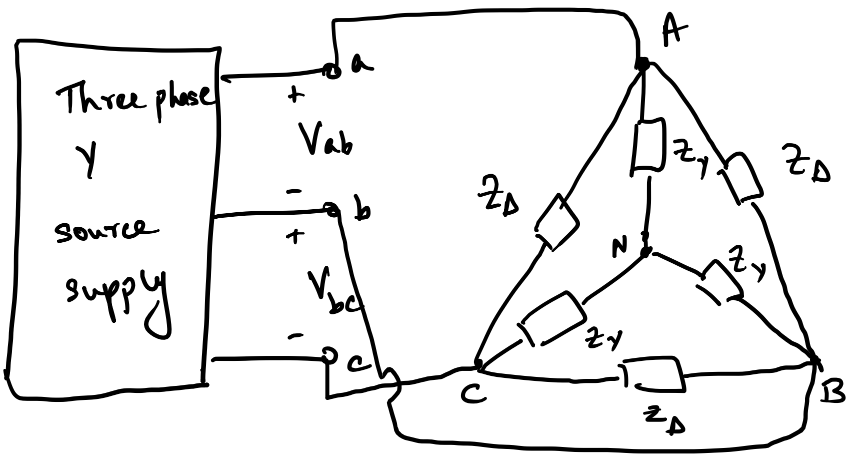


Set 2

Quiz 2 (12 marks)

The figure shows a 60Hz three-phase balanced Y- power supply having a phase voltage V_{an} = $110\angle -15°$ V connected to a balanced Y-connected load having $Z_Y = (15 - j9)\Omega$ in parallel with a Δ-connected load having $Z_\Delta = (5 + j5)\Omega$. The wire impedance (of each wire) to the combined load is 0Ω. Find the following –

(a) Line voltages in time-domain
(b) Line currents in time-domain
(c) Complex power supplied by the source
(d) Power factor of the source
(e) Total time-averaged power consumed by the Y load
(f) Total time-averaged power consumed by the Δ load

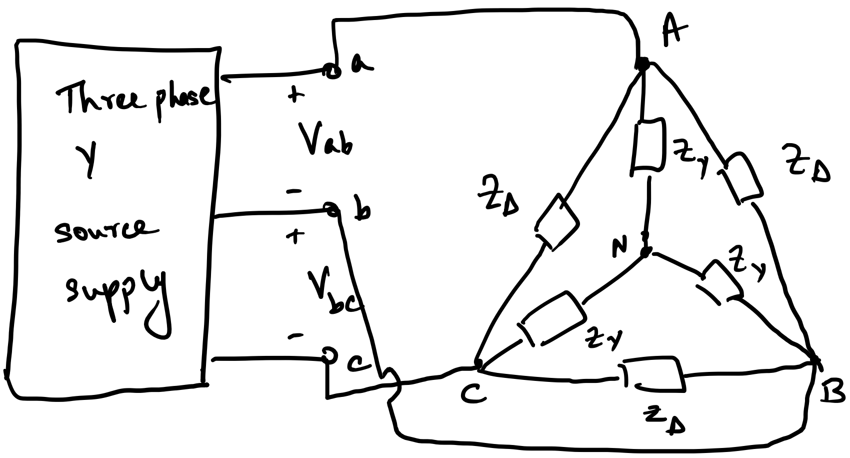


Set 3

Quiz 2 (12 marks)

The figure shows a 60Hz three-phase balanced Y- power supply that provides a time-averaged power 5KW that supplies a line current of $1\angle+30^\circ$ A connected to a balanced Y-connected load having $Z_Y = (5 + j5)\Omega$ in parallel with a Δ-connected load having $Z_\Delta = (5 + j5)\Omega$. The wire impedance (of each wire) to the combined load is 0Ω. Find the following –

(a) Phase voltages in time-domain
(b) Line voltages in time-domain
(c) Power factor of the source
(d) Reactive power supplied by the source
(e) Total time-averaged power consumed by the Y load
(f) Total time-averaged power consumed by the Δ load



Solutions of Set 1

Ques-

$Z_y = 5 + j5 \, \Omega$

$Z_\Delta = 15 - j9 \, \Omega$

$V_L = 230 \angle 15^\circ \, V$

$V_{AB} = V_L = 230 \angle 15^\circ \, V$

$V_{AB} = \sqrt{3} \, V_{an} \angle 30^\circ$ (the phase sequence) i.e (abc sequen

$230 \angle 15^\circ = \sqrt{3} \, V_{an} \angle 30^\circ$

$V_{an} = \frac{230}{\sqrt{3}} \angle (15^\circ - 30^\circ) = 132.79 \angle -15^\circ \, V$

$V_{bn} = \cancel{\frac{230}{\sqrt{3}}} \, V_{an} \angle -120^\circ = [132.79 \angle -15^\circ] \angle -120^\circ$

$= 132.79 \angle -135^\circ \, V$

$V_{cn} = V_{an} \angle -240^\circ = [132.79 \angle -15^\circ] \angle -240^\circ$

$= 132.79 \angle -255^\circ \, V$

(a) $f = 50 Hz$

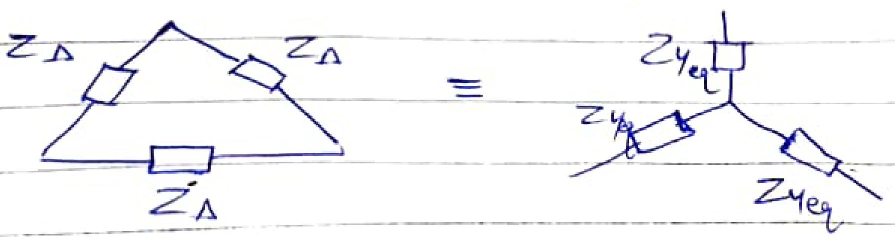
$2\pi f = 314$

Phase voltages in time domain,

$$\boxed{\begin{aligned} V_{an}(t) &= 132.79 \cdot \cos(314t - 15^\circ) \, V \\ V_{bn}(t) &= 132.79 \cdot \cos(314t - 135^\circ) \, V \\ V_{cn}(t) &= 132.79 \cdot \cos(314t - 255^\circ) \, V \end{aligned}}$$

(b) Line currents in time-domain

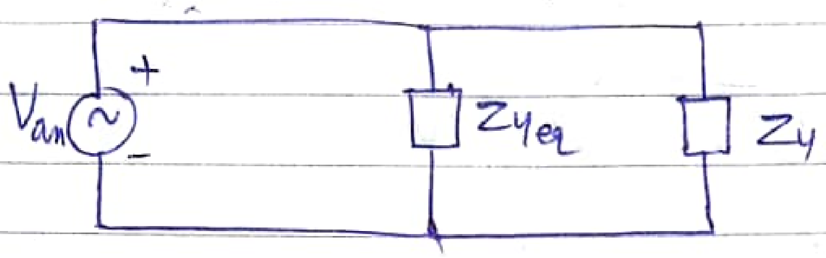
Convert 'Δ' load to 'Y' equivalent



$$\boxed{Z_{Yeq} = \frac{Z_\Delta}{3}}$$

$$Z_{Y_{eq}} = \frac{(15 - j9)}{3} = 5.83 \angle -30.96^\circ$$

using per phase equivalent circuit of an,



$$Z_{Y_{eq}} = 5.83 \angle -30.96^\circ$$

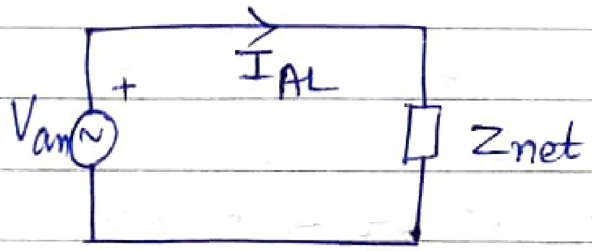
$$Z_Y = 5 + j5 = 7.07 \angle 45^\circ$$

from above ckt,

$$Z_{net} = \frac{(Z_{Y_{eq}})(Z_Y)}{Z_Y + Z_{Y_{eq}}}$$

$$= \frac{(7.07 \angle 45^\circ)(5.83 \angle -30.96^\circ)}{(7.07 \angle 45^\circ) + (5.83 \angle -30.96^\circ)}$$

$$\boxed{Z_{net} = 4.04 \angle 2.73^\circ}$$



$\Rightarrow$ $V_{an} = 132.79 \angle -15^\circ$

$Z_{net} = 4.04 \angle 2.73^\circ$

So,
$$I_{AL} = \frac{V_{an}}{Z_{net}} = \frac{132.79 \angle -15^\circ}{4.04 \angle 2.73^\circ}$$

$$\boxed{I_{AL} = 32.87 \angle -17.73^\circ \text{ A}}$$

Now,

$$I_{BL} = I_{AL} \angle -120^\circ = 32.87 \angle -137.73^\circ \text{ A}$$

$$I_{CL} = I_{AL} \angle -240^\circ = 32.87 \angle -257.73^\circ \text{ A}$$

Line currents in time-domain are,

$$\boxed{\begin{aligned} I_{AL} &= 32.87 \cdot \cos(314t - 17.73) \text{ A} \\ I_{BL} &= 32.87 \cdot \cos(314t - 137.73) \text{ A} \\ I_{CL} &= 32.87 \cdot \cos(314t - 257.73) \text{ A} \end{aligned}}$$

(c) $S_{3\phi} = \sqrt{3}\, V_L I_L^*$

$= \sqrt{3} \times (230 \angle 15^\circ) \times (32.87 \angle -17.73^\circ)^*$

$= \sqrt{3} \times (230 \angle 15^\circ) \times (32.87 \angle 17.73^\circ)$

$\boxed{S_{3\phi} = 13094.47 \angle 32.73^\circ}$

↳ complex power supplied by the source.

(d) Power factor of the source,

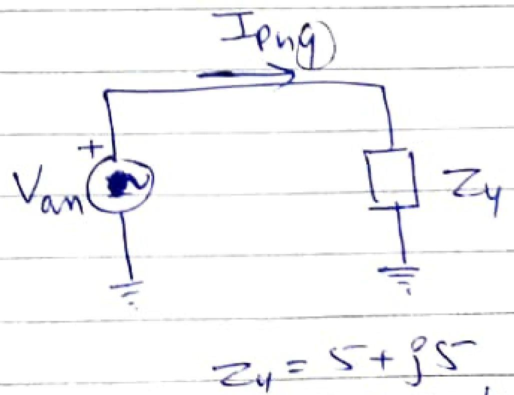
Power factor $= \cos(32.73)$ lag.

$\boxed{P.F = 0.841 \text{ lag}}$

(e) Total time-averaged power consumed by the Y-load,

for Y-load.

$V_{an} = 132.79 \angle -15^\circ$ V



$I_{ph(Y)} = \dfrac{V_{an}}{Z_Y}$

$= \dfrac{132.79 \angle -15^\circ}{7.07 \angle 45^\circ}$

$= 18.78 \angle -60^\circ$

$Z_Y = 5 + j5$

$= 7.07 \angle 45^\circ$

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$$S_{1\phi}(Y\ load) = \overline{V_{an}} \cdot \overline{I_{an}}^{*}$$
$$= (132.79\angle -15^\circ)(18.78\angle -60^\circ)$$

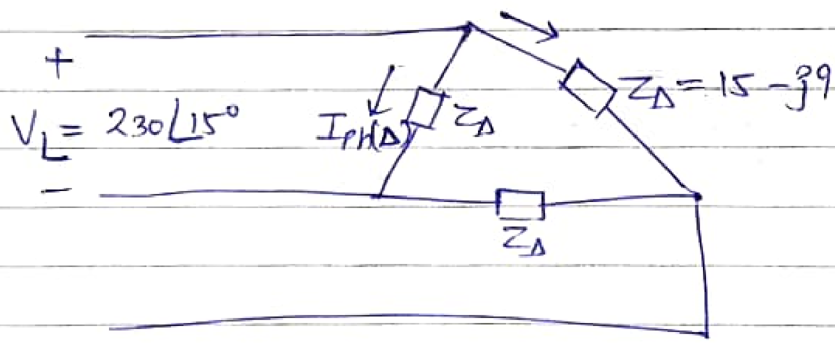
$$S_{1\phi} = 2493.79 \angle -75^\circ \text{ KVA}$$

$$P_{1\phi} = 2493.79 \cos(75^\circ) = 645.44 \text{ W}$$

$$P_{3\phi} = 3 \times P_{1\phi} = 3 \times 645.44 \text{ W}$$

$$\boxed{P_{3\phi} = 1936.32 \text{ W}}$$

(f) Total time-averaged power consumed by the Δ-load,



$$I_{ph(\Delta)} = \frac{230\angle 15^\circ}{Z_\Delta} = \frac{230\angle 15^\circ}{15 - j9} = 13.14\angle 45.96^\circ \text{ A}$$

$$S_{1\phi} = V_{ph} \cdot I_{ph}^{*} = (230\angle 15^\circ)(13.14\angle 45.96^\circ)^{*}$$
$$= (230\angle 15^\circ)(13.14\angle -45.96^\circ)$$
$$= 3022.2\angle -30.96^\circ$$

$$P_{1\phi} = 3022.2 \cdot \cos(30.96)$$
$$P_{3\phi} = 3 \times P_{1\phi} = 3 \times 3022.2 \cdot \cos(30.96)$$

$$\boxed{P_{3\phi} = 7774.85 \text{ W}}$$

SET 2 SOLUTIONS

Given,

Phase voltage $V_{an} = 110\angle -15° V$

$Z_Y = (15 - j9)\,\Omega$

$Z_\Delta = (5 + j5)\,\Omega.$

a) Line voltages in time domain:

we know,

$V_{ab} = \sqrt{3}\, V_{an} \angle 30°$ (the phase sequence → abc)

$\Rightarrow V_{ab} = \sqrt{3} \times (110\angle -15° + 30°)$ V

$= \boxed{190.52\angle -45° V}$

$V_{bc} = V_{ab}\angle -120°.$

$= \boxed{190.52\angle -165° V}$

$V_{ca} = V_{ab}\angle -240°$

$= \boxed{190.52\angle -285° V}$

$\because f = 50 Hz$

$\omega = 2\pi f = 314$ rad/s

$\therefore$ Line voltage in time domain are -

$$\boxed{\begin{aligned} V_{ab}(t) &= 190.52\cos(314t-45^\circ)\ V \\ V_{bc}(t) &= 190.52\cos(314t-165^\circ)\ V \\ V_{ca}(t) &= 190.52\cos(314t-285^\circ)\ V \end{aligned}}$$

b) Line currents in time domain:

$$Z_{Y eq} = \frac{Z_\Delta}{3}$$

$$= \frac{5+j5}{3} = 2.35\angle 45^\circ\ \Omega$$

$$Z_Y = 15 - j9 = 17.49\angle -30.96\ \Omega.$$

$$\therefore Z_{net} = (Z_{Y eq} \parallel Z_Y)$$

$$= \frac{Z_{Y eq}(Z_Y)}{(Z_Y)+(Z_{Y eq})}$$

$$= \frac{(2.35\angle 45^\circ)(17.49\angle -30.96)}{(2.35\angle 45^\circ)+(17.49\angle -30.96)}$$

$$\boxed{Z_{net} = 2.25\angle -9.72\ \Omega}$$

Now, $I_{aL} = \dfrac{V_{an}}{Z_{eq}}$

$$= \frac{160 \angle -15^\circ}{2.25 \angle -9.72^\circ}$$

$$= 48.8 \angle -5.28^\circ \text{ A.}$$

$$I_{bL} = I_{aL} \angle -120^\circ$$

$$= 48.8 \angle -5.28 - 120^\circ$$

$$= 48.8 \angle -125.28^\circ \text{ A.}$$

$$I_{cL} = I_{aL} \angle -240^\circ$$

$$= 48.8 \angle 245.28^\circ \text{ A}$$

In time domain,

$$\boxed{\begin{aligned} I_{aL} &= 48.8 \cos(314t - 5.28^\circ) \text{ A} \\ I_{bL} &= 48.8 \cos(314t - 125.28^\circ) \text{ A} \\ I_{cL} &= 48.8 \cos(314t - 245.28^\circ) \text{ A.} \end{aligned}}$$

c) Complex power supplied by source

$S_{3\phi} = \sqrt{3}\, V_L I_L^*$

$= \sqrt{3} \times (190.52 \angle 45^\circ) \times (48.8 \angle +5.28^\circ)$

$\boxed{S_{3\phi} = 16103.5 \angle -39.72^\circ \ VA.}$

d) Power factor of source -

$PF = \cos(-39.72^\circ)$

$= \boxed{0.769}$

e) Total time averaged power consumed by Y load

$V_{an} = 110 \angle -15^\circ V$

$I_{ph_Y} = \frac{V_{an}}{Z_Y} = \frac{110 \angle -15^\circ}{17.49 \angle -30.96^\circ} = 6.28 \angle 15.96^\circ$

Now, $S_{1\phi} = V_{an} \cdot I_{an}^*$

$= (110 \angle -15^\circ)(6.28 \angle 15.96^\circ)$

$= 690.8 \angle -30.96^\circ \ VA$

$\therefore P_{1\phi} = 690.8 \times \cos(30.96) = 592.3 W.$

$P_{3\phi} = 3 \times P_{1\phi} = 1776.9 W$

f) Power by Δ-load.

(Total time averaged power consumed by Δ)

$$I_{ph_\Delta} = \frac{190.52\angle -45^\circ}{5 + j5} = \frac{190.52\angle -45^\circ}{7.07\angle 45^\circ}$$

$$\therefore S_{1\phi} = (190.52\angle -45^\circ) \times (26.94\angle +90^\circ)$$

$$= 5132.6\angle 45^\circ \text{ VA}$$

$$\therefore P_{1\phi} = 5132.6\cos(45^\circ)$$

$$= 3628.8 \text{ W}$$

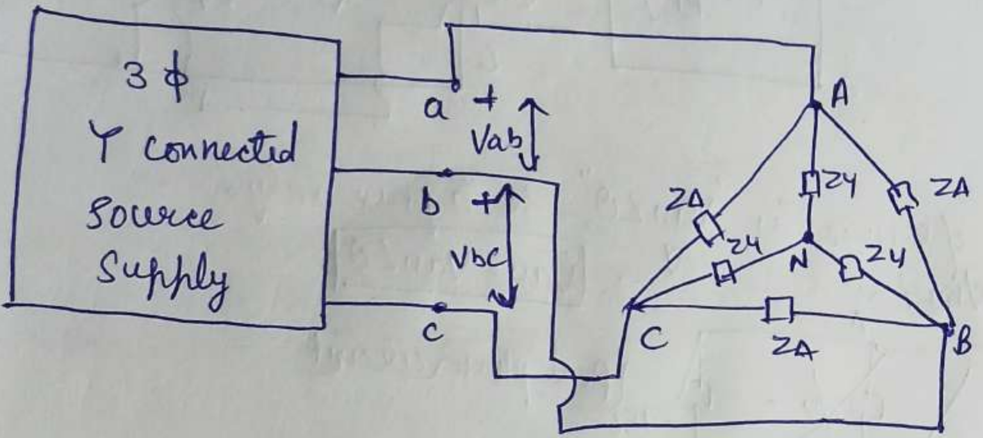
$$P_{3\phi} = 3 \times 3628.8 \text{ W}$$

$$= 10886.4 \text{ W}$$

SET-3 Solutions

Ans) **given:** $f = 60\ Hz$
$P = 5\ kW$
$I_L = 1\angle 30^\circ\ A$

$Z_Y = 5+j5\ \Omega = 7.07\angle 45^\circ\ \Omega$
$Z_\Delta = 5+j5\ \Omega = 7.07\angle 45^\circ\ \Omega$
$Z_{wire} = 0\ \Omega$



(a.) **For phase voltages** [We can find the phase Voltages via 2 ways]

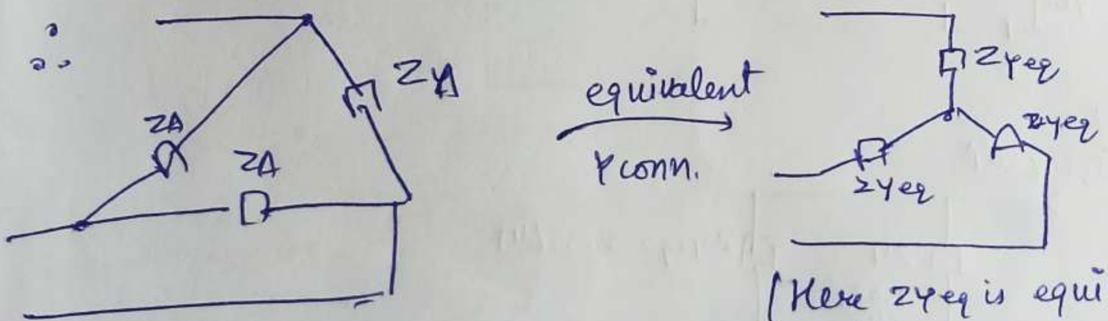
Method-(1)

Here since all Z_Y and Z_Δ are equal so they are in balanced state. Also source is also balanced. In this configuration we can perform per phase analysis to find phase voltages.

Problem is ~~only~~ this in doing per phase analysis is that it can be done when both loads are connected in Y configuration only. But in question one load is Δ and other is Y.

So convert "Δ" load to "Y" equivalent so that per phase analysis can be performed.

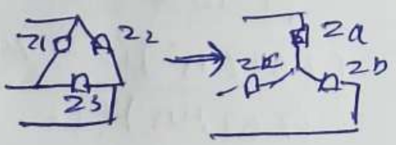
$\therefore$



[Here Z_{Yeq} is equivalent Y of "Δ" load]

$$\therefore Z_{Yeq} = \frac{Z_\Delta \times Z_\Delta}{Z_\Delta + Z_\Delta + Z_\Delta}$$

Using $\Delta \to Y$ conversion formula



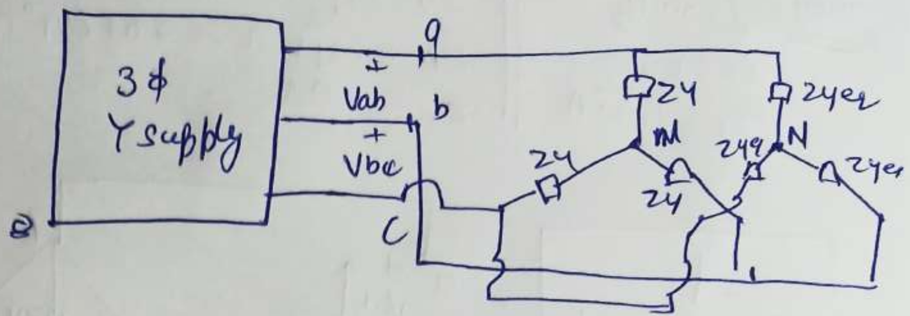
$$\boxed{Z_a = \frac{Z_1 Z_2}{Z_1+Z_2+Z_3}}$$

as $Z_1 = Z_2 = Z_3 = Z_\Delta$

so $\boxed{Z_a = Z_b = Z_c}$

$$Z_{Yeq} = \frac{Z_\Delta}{3} = \frac{5+j5}{3} = 2.35\angle 45^\circ\ \Omega$$

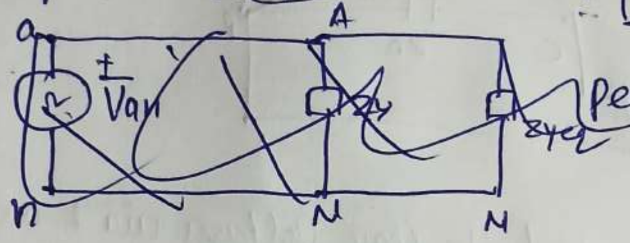
∴ ∴



Let phase supply of source is "$V_{an}\angle\theta$" for "a" phase only.

∴ ~~For b phase = $V_{bn}\angle$~~ 2θ ∴ $\boxed{\bar{V}_{an} = V_{an}\angle\theta}$

∴



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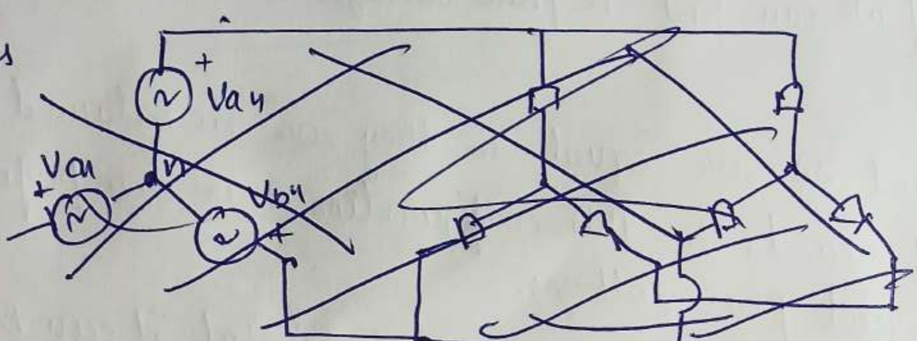
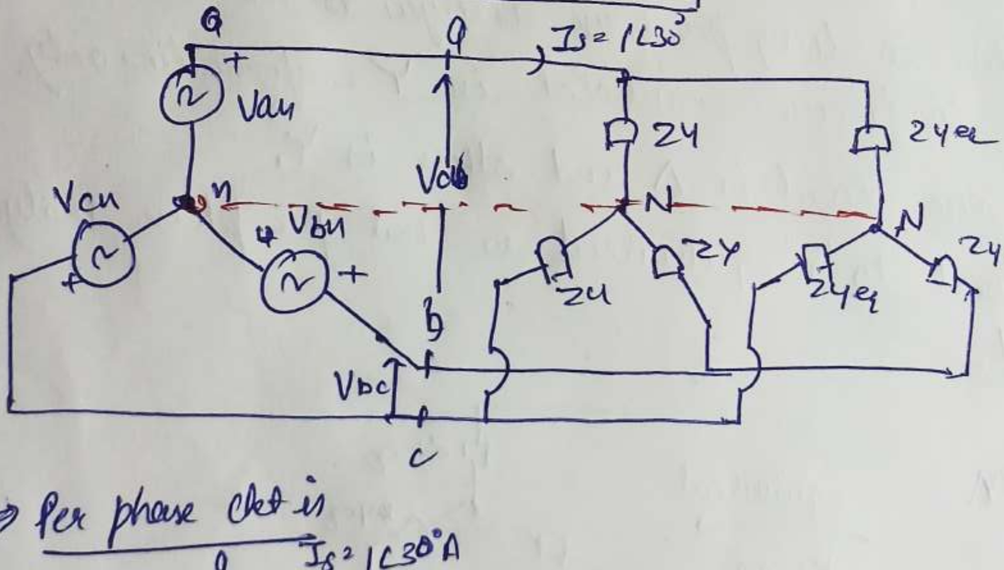
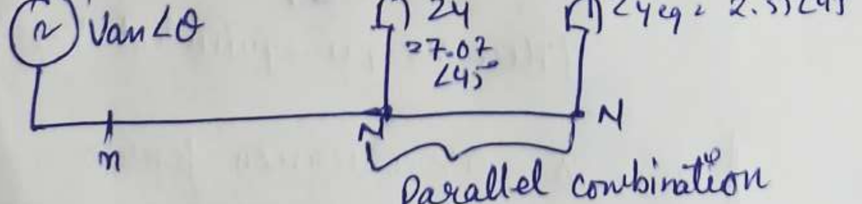




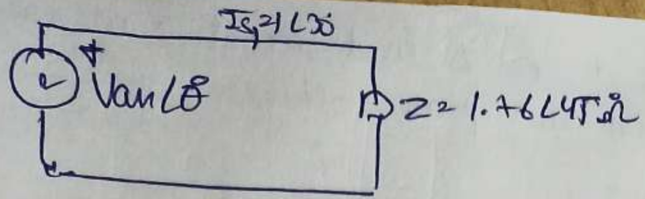
Now (n, N) can be connected for per phase analysis

⇒ Per phase ckt is



$$Z = \frac{Z_Y \cdot Z_{Yeq}}{Z_Y + Z_{Yeq}} = \frac{(7.07\angle 45^\circ)(2.35\angle 45^\circ)}{(7.07\angle 45) + (2.35\angle 45^\circ)} = 1.76\angle 45^\circ$$

$\rightarrow$



$\therefore\ V_{an}\angle\theta = \bar{I}_s Z = (1\angle 30°)(1.76\angle 45°) = 1.76\angle 75°\ V$

$\therefore\ V_{bn} = V_{an}\angle -120° = (\cancel{1.76\angle 75°}) \ 1.76\angle 75°-120 = 1.76\angle -45\ V$

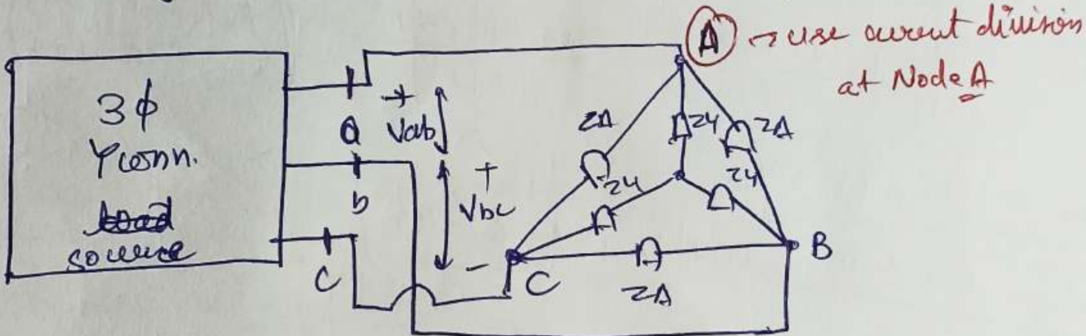
$V_{cn} = V_{an}\angle -240° = 1.76\angle 75-240° = 1.76\angle -165°\ V$

as $f = 60Hz \Rightarrow \omega = 2\pi f = 120\pi$ rad/sec

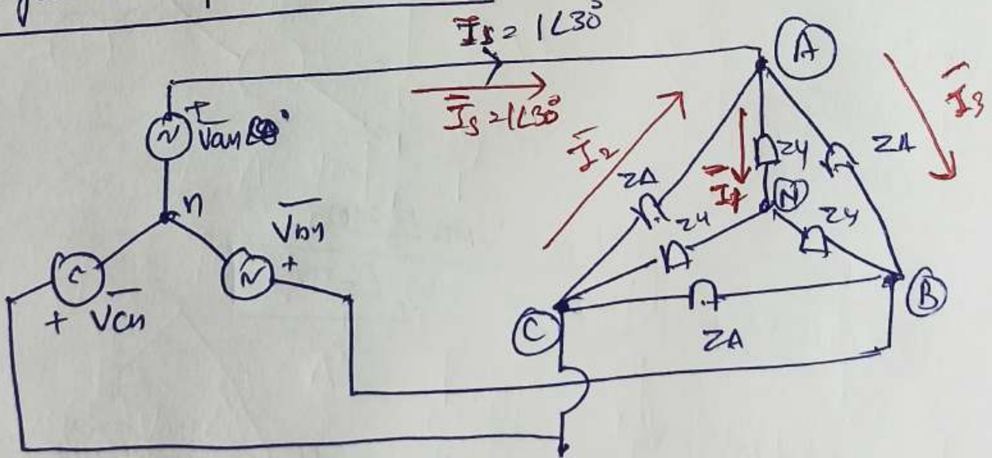
$\therefore\ V_{an}(t) = 1.76\cos(120\pi t + 75°)\ V$
$V_{bn}(t) = 1.76\cos(120\pi t - 45°)\ V$
$V_{cn}(t) = 1.76\cos(120\pi t - 165°)\ V$

} are Required phase voltage in Time Domain

Method (2) : Using current Division at Node A of the load.



Make full ckt of source + loads



Using KCL: $\bar{I}_s + \bar{I}_2 = \bar{I}_1 + \bar{I}_3$

$\bar{I}_s = \bar{I}_1 + \bar{I}_3 - \bar{I}_2$

Now for $\bar{I}_1 \Rightarrow \bar{I}_1 = \dfrac{\bar{V}_{an}}{ZY}$ (as $\bar{I}_1$ is phase current in (AN) : Use phase voltage only as its Y connection) for I_2 : $\bar{I}_2 = \dfrac{\bar{V}_{CA}}{Z_\Delta}$ || Here its "Δ" load current ∴ We use line voltage "VCA" not phase voltage.

Also we know : $\bar{V}_{AB} = \sqrt{3}\,\bar{V}_{an}\angle 30^\circ$

and $\bar{V}_{CA} = \bar{V}_{AB}\angle -240^\circ = \sqrt{3}\,\bar{V}_{an}\angle -240^\circ + 30^\circ$

$$\boxed{\bar{V}_{CA} = \sqrt{3}\,\bar{V}_{an}\angle -210^\circ\ V}$$

$$\therefore \bar{I}_2 = \frac{\sqrt{3}\,\bar{V}_{an}\angle -210^\circ}{Z_\Delta}$$

for "$\bar{I}_3$" : $\bar{I}_3 = \dfrac{\bar{V}_{AB}}{Z_\Delta} = \dfrac{\sqrt{3}\,V_{an}\angle 30^\circ}{Z_\Delta}$. [as $\bar{V}_{AB} = \sqrt{3}\,V_{an}\angle 30^\circ$

Now use I_1, I_2, I_3 values in KCL eqn:

$$\bar{I}_s = \bar{I}_1 + \bar{I}_3 - \bar{I}_2$$

$$1\angle 30^\circ = \frac{\bar{V}_{an}}{5+j5} + \frac{\sqrt{3}\,V_{an}\angle 30^\circ}{5+j5} - \frac{\sqrt{3}\,V_{an}\angle -210^\circ}{5+j5}$$ [Put $Z_Y = 5+j5$, $Z_\Delta = 5+j5$

$\boxed{\bar{V}_{an} = 1.76\angle 75^\circ\ V}$ as

$\therefore \boxed{V_{bn} = 1.76\angle -45^\circ}$ and $\boxed{V_{cn} = 1.76\angle -165^\circ}$

(b.) for line voltages

$$V_{AB} = \sqrt{3}\,V_{an}\angle 30^\circ$$
$$= \sqrt{3}\,[1.76\angle 75^\circ]\angle 30^\circ$$
$$\boxed{V_{AB} = 3.05\angle 105^\circ\ V}$$

and $V_{BC} = V_{AB}\angle -120^\circ$ || $V_{CA} = V_{AB}\angle -240^\circ$

$\boxed{V_{BC} = 3.05\angle -15^\circ\ V}$ || $\boxed{V_{CA} = 3.05\angle -135^\circ}$

$$\therefore V_{AB} = 3.05\cos(120\pi t + 105^\circ)\ V$$
$$V_{BC} = 3.05\cos(120\pi t - 15^\circ)\ V$$
$$V_{CA} = 3.05\cos(120\pi t - 135^\circ)\ V$$

} Line voltages in Time Domain,

(c.) For Power factor of source

Here $\theta = 75^\circ$ (angle of V_{an}), $\phi = 30^\circ$ (angle of I_s)

$$pf = \cos(\theta - \phi) = \cos(75 - 30^\circ)$$
$$= \cos 45 = \frac{1}{\sqrt{2}} = 0.707 \text{ lag.}$$

(d) for Reactive power

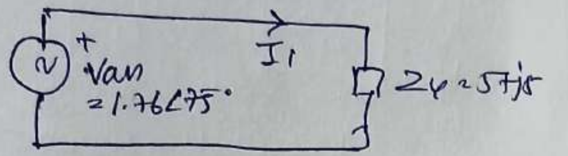
$Q = P \tan(\theta - \phi)$ [and $P = 5KW$ and $\theta - \phi = 75 - 30 = 45°$]

$= 5 \tan 45°$

$\boxed{Q = 5 KVARs}$

(e.) for average power consumed by Y load

using per phase ckt : for (Y) load only [not Δ load)



$I_1 = \frac{V_{an}}{Z_Y} = \frac{1.76\angle 75°}{5+j5} = 0.25\angle 30° A$

$\therefore P_{3\phi} = \frac{3}{2} V_m I_m \cos(\theta - \phi)$ || $I_m = I_1$, $V_m = V_{an}$ (and they are phase voltages and current)

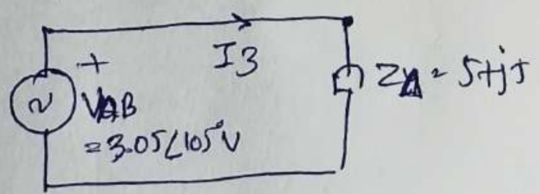
$= \frac{3}{2} \times 1.76 \times 0.25 \cos(75 - 30°) = 0.466 W$

$\boxed{P_{3\phi} = 0.466 W}$

(f.) for Average power consumed in Δ load

use per phase ckt for (Δ) load only

Here as its Δ load so we take line voltage of source as V_m as line and phase voltage of Δ connection are equal.



$\bar{I}_3 = \frac{\bar{V}_{AB}}{Z_\Delta} = \frac{3.05\angle 105°}{5+j5} = 0.43\angle 60°$

$\therefore P_{3\phi} = \frac{3}{2} V_m I_m \cos(\theta - \phi)$

|| Here $V_m = V_{AB} = 3.05 V$
$I_m = 0.43 A$
$\theta = 105°$
$\phi = 60°$

$= \frac{3}{2} \times 0.43 \times 3.05 \cos(105 - 60°)$

$\boxed{P_{3\phi} = 1.39 W}$ ac