MTH-204: Worksheet 6

22 March, 2023

1. To get a feel for higher order ODEs, show that the given functions are solutions and form a basis on any interval. Use Wronskians.
   (a) $e^{3x},\ sin\,x,\ cos\,x, \quad y''' - 3y'' + y' - 3y = 0$ (2½)
   (b) $1, x^2, x^5, \quad x^3y''' - 4x^2y'' + 4xy' = 0$ (2½)

2. (a) Extend the method to a variable-coefficient ODE (3)

$$y''' + p_2(x)y'' + p_1(x)y' + p_0(x)y = 0.$$

   Assuming a solution y_1 to be known, show that another solution is $y_2(x) = u(x)\ y_1(x)$ with $u(x) = \int z(x)\mathrm{d}x$ and z obtained by solving

$$y_1z'' + (3y_1' + p_2y_1)z' + (3y_1'' + 2p_2y_1' + p_1y_1)z = 0.$$

   (b) Reduce (2)

$$(2-x)y''' + (2x-3)y'' - xy' + y = 0,$$

   using $y_1 = x$ (perhaps obtainable by inspection).