Rubric for Quiz 1

(1)

(1)

Let us denote the following events by appropriate symbols

S : The patient is serious

M : The patient is moderate.

LN : The doctor is Laser Nephrologist

CN : The doctor is Conservative Nephrologist

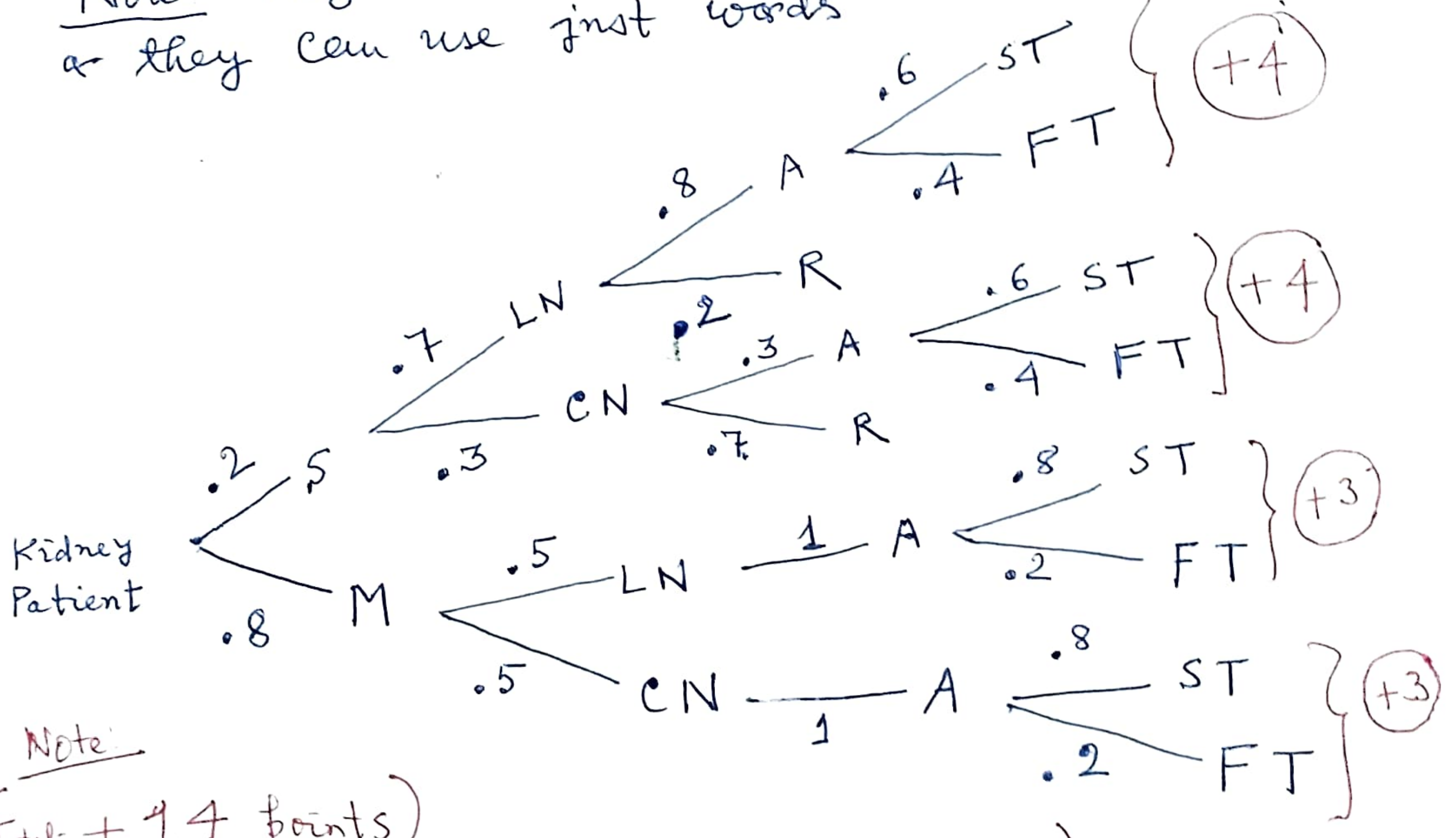
A : The patient has been accepted by the doctor

R : The patient has not been accepted by the doctor

ST : The treatment is successful

FT : The treatment has failed.

Note: They can use their own symbol or they can use just words




Note:

Total = + 14 points

Give proportional points for each branch

$$P[LN \cap A \mid S \cap ST] = \frac{P[LN \cap A \cap S \cap ST]}{P[S \cap ST]}$$

$$= \frac{(.2)(.7)(.8)(.6)}{(.2)(.7)(.8)(.6) + (.2)(.3)(.3)(.6)}$$

$$\underbrace{= \frac{.0672}{.078} = \frac{672}{780}}_{+3} = \underbrace{\boxed{\frac{56}{65}}}_{+2}$$

+7

Note: They can directly write the third step from the diagram and that will be O.K.

Total: +12 points

$$P[ST \mid CN \cap A] = \frac{P[ST \cap CN \cap A]}{P[CN \cap A]}$$

$$= \frac{(.2)(.3)(.3)(.6) + (.8)(.5)(1)(.8)}{(.2)(.3)(.3)(.6+.4) + (.8)(.5)1(.8+.2)}$$

$$\underbrace{= \frac{.0108 + .320}{.018 + .4} = \frac{.3308}{.4180}}_{+3} = \underbrace{\boxed{\frac{827}{1045}}}_{+2}$$

+7

Note: They can directly write the third step from the diagram and that will be O.K.

Total: +12 points

$$P[ST \mid LN \cap A] = \frac{P[ST \cap LN \cap A]}{P[LN \cap A]}$$

$$= \frac{(.2)(.7)(.8)(.6) + (.8)(.5)(1)(.8)}{(.2)(.7)(.8)(.6+.4) + (.8)(.5)(1)(.8+.2)}$$

$$\underbrace{= \frac{.0672 + .32}{.112 + .4} = \frac{.3872}{.5120}}_{+3} = \underbrace{\boxed{\frac{121}{160}}}_{+2}$$

+7

Total: +12 points

(Note: They can directly write the third step from the diagram and that will be O.K.)

2. Let $n$ be the number of coins required in 50 games so that the net gain or loss is less than \$ 1.

Then
$$-1 < n - \frac{(50-n)}{2} < 1$$
$$\Rightarrow -2 < 2n-(50-n) < 2$$
$$\Rightarrow -2 < 3n-50 < 2$$
$$\Rightarrow 48 < 3n < 52$$
$$\Rightarrow 16 < n < 17.3$$
$$\Rightarrow n = 17$$

+6

Now,
$$P[\text{Win in 1 game}] = P[HH] = \frac{1}{4}$$

So, P[Net gain or loss is less than \$ 1]
$$= \binom{50}{17}\left(\frac{1}{4}\right)^{17}\left(\frac{3}{4}\right)^{33}$$

+3

Hence P[Net gain or loss is atleast \$ 1]
$$= \boxed{1 - \binom{50}{17}\left(\frac{1}{4}\right)^{17}\left(\frac{3}{4}\right)^{33}}$$

+3

Similarly if $n$ is the number of coins required in 50 games so that net gain or loss is less than \$ 5, then

$$-5 < n - \frac{(50-n)}{2} < 5$$

$$\Rightarrow \quad -5 < \frac{2n-(50-n)}{2} < 5$$

$$\Rightarrow \quad -5 < \frac{3n-50}{2} < 5$$

$$\Rightarrow \quad 40 < 3n < 60$$

$$\Rightarrow \quad 13.4 < n < 20$$

+6

So P[Net gain or loss is less than \$ 5]

$$= \sum_{n=14}^{19} \binom{50}{n} \left(\frac{1}{4}\right)^{n} \left(\frac{3}{4}\right)^{50-n}$$

+4

Hence P[Net gain or loss is atleast \$ 5]

$$= \boxed{1 - \sum_{n=14}^{19} \binom{50}{n} \left(\frac{1}{4}\right)^{n} \left(\frac{3}{4}\right)^{50-n}}$$

+3

(Total = 25 points)

Note: Some students may write the answer as

$$\sum_{n=0}^{13} \binom{50}{n} \left(\frac{1}{4}\right)^{n} \left(\frac{3}{4}\right)^{50-n} + \sum_{n=20}^{50} \binom{50}{n} \left(\frac{1}{4}\right)^{n} \left(\frac{3}{4}\right)^{50-n}$$

or

$$\sum_{\substack{n=0 \\ n \notin \{14,15,\ldots,19\}}}^{50} \binom{50}{n} \left(\frac{1}{4}\right)^{n} \left(\frac{3}{4}\right)^{50-n}$$

(all are correct)

3

A will win the series if A wins $m$ games before B wins $n$ games. So, by $(m+n-1)$th game there must be a winner.

Let $X_K$ be the event,

$X_K = \{$A wins $m$ games in exactly $m+k$ games$\}$

$K = 0, 1, 2, \dots, n-1$

Then $X_K$ s are mutually exclusive events

and $\{A \text{ wins}\} = X_0 \cup X_1 \cup \dots \cup X_{n-1}$

+13

So, $P[A \text{ wins}] = P[X_0 \cup X_1 \cup \dots \cup X_{n-1}]$

+2

$$= \sum_{k=0}^{n-1} P[X_k]$$

For A to win $m$ games in exactly $m+k$ games, A must win the last game and $(m-1)$ games in any order among the first $(m+k-1)$ games

$$= \sum_{k=0}^{n-1} \binom{m+k-1}{m-1} p^{m-1} q^k p$$

+8

Note: Some students can reach this step without writing all the earlier steps. However they should give proper logical explanation to reach ~~....~~ here ~~........~~

$$= \sum_{k=0}^{n-1} \binom{m+k-1}{m-1} p^m q^k$$

+2

$$= \boxed{p^m \sum_{k=0}^{n-1} \binom{m+k-1}{m-1} q^k}$$

(They can write the answer in either form)

(In place of $q$, they can write $1-p$)

Total = 25 points

Note:

They can also write in the form:

$$p^m \sum_{k=0}^{n-1} \frac{(m+k-1)!}{(m-1)!\;k!} q^k$$

or, $p^m \left(1 + \frac{m}{1} q + \frac{m(m+1)}{1.2} q^2 + \cdots + \frac{m(m+1)\cdots(m+n-2)}{1.2\cdots(n-1)} \times q^{n-1}\right)$

All these answers are acceptable.