Submission for Wednesday 2nd February 2022 – 15 minutes. Max Marks: 5

Instructions: Open notes and textbook; consultation and use of calculators, computers and internet not allowed.

IMPORTANT: You may use any **known** result. This includes all propositions and observations in the lecture slides, and results from tutorials. If you use any other result from any other source, including the textbook, you have to give a full proof of that result. Note that we have not covered *determinants* as of now. So, use of any result related to determinants is forbidden.

a) Use row reduction (Gauss-Jordan Elimination, Corollary 1.1) to determine the inverse of the given 4×4 matrix A below. **Do not use any other method. <u>YOUR STEPS MUST BE SHOWN.</u>** otherwise, you will not get credit.   (*4 marks*)

b) Use your answer above to exhibit the inverse of a general n×n matrix A with the same structure (n ≥ 2). You **<u>do not need</u>** to show your steps.   (*1 mark*)

$$A = \begin{bmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & a_2 & 0 \\ 0 & a_3 & 0 & 0 \\ a_4 & 0 & 0 & 0 \end{bmatrix} \quad a_i \neq 0, \ i = 1,2,3,4$$

SOLUTION - CUM - RUBRIC

a) We apply row-reduction to the enlarged matrix $[A : I_4] =$

$$\begin{bmatrix} 0 & 0 & 0 & a_1 & \vdots & 1 & 0 & 0 & 0 \\ 0 & 0 & a_2 & 0 & \vdots & 0 & 1 & 0 & 0 \\ 0 & a_3 & 0 & 0 & \vdots & 0 & 0 & 1 & 0 \\ a_4 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_4$
$$\begin{bmatrix} a_4 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 1 \\ 0 & 0 & a_2 & 0 & \vdots & 0 & 1 & 0 & 0 \\ 0 & a_3 & 0 & 0 & \vdots & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a_1 & \vdots & 1 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$
$$\begin{bmatrix} a_4 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 1 \\ 0 & a_3 & 0 & 0 & \vdots & 0 & 0 & 1 & 0 \\ 0 & 0 & a_2 & 0 & \vdots & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & a_1 & \vdots & 1 & 0 & 0 & 0 \end{bmatrix}$$

(PTO)

(a) — cont'd :-

Now, normalize each row, since $a_i \neq 0$
to give:

$$\left[\begin{array}{cccc:cccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1/a_4 \\
0 & 1 & 0 & 0 & 0 & 0 & 1/a_3 & 0 \\
0 & 0 & 1 & 0 & 0 & 1/a_2 & 0 & 0 \\
0 & 0 & 0 & 1 & 1/a_1 & 0 & 0 & 0
\end{array}\right]$$

$$\underbrace{\hspace{3cm}}_{I} \qquad \underbrace{\hspace{3cm}}_{A^{-1}}$$

(b) A^{-1} looks like:

$$\left[\begin{array}{ccccc}
0 & 0 & \cdots & 0 & a_n^{-1} \\
0 & 0 & \cdots & a_{n-1}^{-1} & 0 \\
\vdots & & & & \\
a_1^{-1} & 0 & \cdots & \cdots & 0
\end{array}\right]_{n \times n}$$

1 mark