

Game Theory Mid-Sem 2023

Indicative Solutions

Ans. 1. a) $i=1, 2, \dots, n$; $j=1, 2, \dots, n$
 When $n=1$ we have the following game:

		Thief	
		Inside	Trunk
Police	Inside	1, -1	-1, 1
	Trunk	-1, 1	1, -1

There are no PSNE.

$$MSNE = \left\{ \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right\} \text{ [Show calculation]}$$

b) $n=100$

$T_i = \{b_1, b_2, \dots, b_{100}\}$ types of i

$T_j = \{d_1, d_2, \dots, d_{100}\}$ types of j

prob. of i being matched with j is $\frac{1}{100}$

" " a match $\frac{1}{100} \times \frac{1}{100}$

$S_i : T_i \rightarrow \{\text{Inside, Trunk}\}$ for each i

$S_j : T_j \rightarrow \{\text{Inside, Trunk}\}$ for each j

		Thief	
		Inside	Trunk
Police	Inside	$1, -1$	$-1, 1 + b_i$
	Trunk	$-1 + d_j, 1$	$1 + d_j, -1 + b_i$

Bayesian game: $(N, \{s_i, s_j\}, \{T_i, T_j\}, \{u_i, u_j\})$
 $i, j \in \{1, 2, \dots, 100\}$

where $i, j \in \{1, 2, \dots, 100\}$.

Best response:

$BR_j(s_i = \text{Inside} \mid d_j > 0) = \text{Trunk if } d_j > 2$
 Inside otherwise

$BR_j(s_i = \text{Trunk} \mid d_j > 0) = \text{Trunk}$

$BR_j(s_i = \text{Inside} \mid d_j < 0) = \text{Inside}$

$BR_j(s_i = \text{Trunk} \mid d_j < 0) = \text{Trunk if } d_j > 2$
 Inside, otherwise

Similarly for write BR_i .

Pure strategy Bayes' Nash equilibrium:

(i) If $b_i < -2$ then $s_i = \text{Inside}$

If $b_i \in (-2, 2)$ then $s_i = \text{Inside or Trunk with equal prob.}$

If $b_i > 2$, then $s_i = \text{Trunk}$.

(ii) If $d_j < -2$, then $s_j = \text{Inside}$
If $d_j \in [-2, 2)$ then $s_j = \text{Inside or Trunk}$
If $d_j \geq 2$ then $s_j = \text{Trunk}$.

Ans 2. a) False.

In prisoner's dilemma, Nash equilibrium payoffs are lower than in ~~(Don't confess, Don't confess)~~ [show payoffs].

OR In bilateral trading, efficiency loss occurs due to imperfect information / incomplete info.
[see lecture slide 103-104]

b) True. Continuity of $\succeq \Rightarrow$ Continuous U_i exists.
Weierstrass theorem \Rightarrow optimal solution exists
 ~~$B(\sigma)$ U_i is~~

By Kakutani's FPT, $B(\sigma)$ has a fixed point.

ΔA_i is the space of mixed actions.
~~the set of mixed strategies~~

The fixed point $f \in \Delta A_i$.

Mixed strategies permit all convex combinations
~~to be~~ as feasible actions / strategies.

3. [Hint: Similar to Median voter theorem]

$$N=2 \text{ or } N=\{i,j\}$$

$$A_i = [0,1]$$

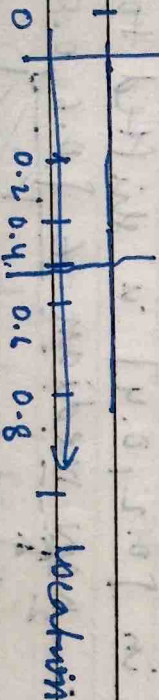
$$U_i(a_i, a_j) =$$

$$\begin{cases} \frac{(a_i + a_j)}{2} \times 1 & \text{if } a_i < a_j \\ \frac{1 - (a_i + a_j)}{2} \times 1 & \text{if } a_i > a_j \\ 0.5 & \text{if } a_i = a_j \end{cases}$$

$(N, \{A_i\}_{i \in N}, \{U_i\}_{i \in N})$ is the strategic form game

Since consumers are distributed uniformly over $[0,1]$, at each interval $x \in [0,1]$, the max is uniform. Median = 0.5

max of consumer



Median = 0.5

For any i , if $a_i < \text{median} < a_j$ then it can increase its sale by choosing $a'_i = a_j - \delta$ for some $\delta > 0$. $U_i(a'_i, a_j) > U_i(a_i, a_j)$

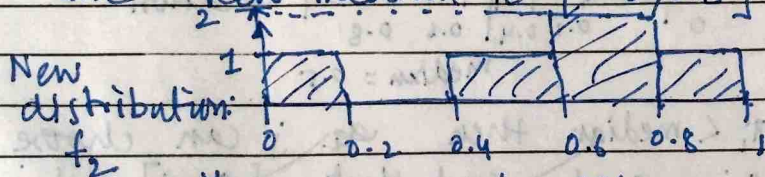
If $a_i < a_j < \text{median}$, then $a'_i = a_j + \delta$ generates higher sales for some $\delta > 0$. $U_i(a'_i, a_j) > U_i(a_i, a_j)$

If $\text{median} < a_i < a_j$ then $a'_j = a_i - \delta$ generates higher payoff for j .

If $\text{median} < a_j < a_i$ then $a'_i = a_j - \delta$ [similarly].

If $a_i = a_j = \text{median}$, then no dev. profitable deviation exists.

If max in $[0.2, 0.4]$ is shifted to $[0.6, 0.8]$ then new median $\in [0.6, 0.8]$



Yes, answer changes
New equilibrium is median of distribution f_2 .

Ans. 4. First price auction

$$N = \{1, 2, \dots, n\}$$

$A_i = b_i \in \mathbb{R}$ for each $i \in N$

$$u_i(b_i, b_{-i}) = \begin{cases} v_i - b_i, & \text{if } b_i > b_{-i} \\ 0, & \text{otherwise} \end{cases}$$

$(N, \{b_i\}_{i \in N}, \{v_i\}_{i \in N}, \{u_i\}_{i \in N})$ is a strategic form game. $\{v_i\}_{i \in N}$ is not necessary to mention.

Nash equilibrium:

$$b_1 \in [v_2, v_1]$$

$$b_j \leq b_1 \quad \forall j \neq 1$$

$$b_j = b_1 \quad \text{for some } j \neq 1$$

Solⁿ: Suppose $v_i > v_j \quad \forall j \neq i, \therefore i = 1$

Player 1 will bid $b_1 \in [v_2, v_1]$

$$[b_1 < v_2 \Rightarrow 1 \text{ may lose}; b_1 > v_1 \Rightarrow \text{lose}]$$

$$\text{Since } u_i(b_i, b_{-i}) = v_i - b_i,$$

Player 1 will bid not bid v_1 to win

$$\text{If } b_j > b_1$$

$$\text{for some } j \neq 1, \text{ then } u_i(b_i, b_{-i}) < 0.$$

$$\therefore b_j \leq b_1 \quad \forall j \neq 1.$$

If $b_j < b_1 \quad \forall j \neq 1$ then Player 1 can increase $u_1(b_1, b_{-1})$ by choosing $b_1 - \epsilon \geq b_j$.

$\therefore b_1 = b_j$ for some $j \neq 1$.