

MTH 377/577 Convex Optimization

Quiz 1

Feb 14, 2024

Answer all questions. Max. Marks = 20. Time = 60 minutes

1. Show that all affine sets are convex. Are all convex sets affine? Provide a formal argument in support of your answer. (3)
2. Find the conic hull of the cone generated by the vectors $(1, 2), (3, 1)$. Is this cone a Polyhedral? Write down the dual for this cone. (3)
3. Show that for any two convex sets $A, B \in R^n \times R^m$, their partial sum is also a convex set. (3)
4. Let $f : [a, b] \rightarrow R$ be a convex function. Is the following statement true or false?

$$\max\{f(x) : x \in [a, b]\} = \max\{f(a), f(b)\}$$

Provide a formal argument. (3)

5. If there exists a supporting hyperplane at every point on the boundary of a closed, non-empty set, then the set is weakly convex. True/False? Show formally. (3)
6. Check whether the following functions are convex or not: (2)
 - (a) $f(x) = -\ln(x)$ where $x \in R_+$
 - (b) $f(x) = |x|$, where $x \in R$
7. Use Farkas' lemma to decide if the following system has a non-negative solution. Show your calculations: (3)

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$