Monsoon 2023 - CSE 643 Artificial Intelligence
Mid-Sem Solutions - Sep. 25, 2023

Maximum score: 20 **Time: 60 minutes**

1. (5 points) **Intelligent Agent**. What is a goal-based agent? What is a utility-based agent? How are they different? Give an example of each: a goal-based agent and a utility-based agent.

Solution:

Goal-based agent: A goal-based agent is one that has some information of one or more *goal* states of the environment. This goal state may depict a desirable condition of the environment. For example, the goal for a robotic arm may be to sort products in a basket by size.

Utility-based agent: A utility-based agent is provided with a utility function (a single function or a mixture of functions) that serve as a performance measure in its environment. For example, a taxi driver who has to reach the destination in the shortest time, or with the smoothest ride, or the shortest time with a sufficiently smooth ride. Another agent could be a student who has to secure the highest CGPA without plagiarizing and with a high diversity of courses.

Rubric: 1 point for each sub-question with 0.5 points for each example.

2. (3+3=6 points) **Propositional Logic and Inference**.

According to some astrologers, a person who is a Virgo (V) is enlightened (E) if he/she is creative (C), but otherwise is not enlightened.

(a) (3 points) Which of the following are correct representations of this assertion?

(i) $(V \wedge E) \iff C$

(ii) $V \Rightarrow (E \iff C)$

(iii) $V \Rightarrow ((C \Rightarrow E) \vee \neg E)$

(b) (3 points) Which of the sentences in (a) can be expressed in Horn form (as a Horn clause)? If they can be, write them in Horn form, else justify why they cannot be written in Horn form.

Solution:

(a) Correct representations of "a person who is a Virgo is enlightened if he/she is creative, but otherwise is not englightened":

(i) $(V \wedge E) \iff C$
No. this sentence asserts that all creative people are Virgo, and that all creative people are enlightened, which is not what was stated.

(ii) $V \Rightarrow (E \iff C)$
Yes. This sentence says that if a person is a Virgo then they are enlightened if and only if they are creative (which is the same as a Virgo is enlightened if he/she is creative, otherwise not enlightened).

(iii) $V \Rightarrow ((C \Rightarrow E) \vee \neg E)$
No, this is equivalent to $\neg V \vee \neg C \vee E \vee \neg E$ which is a tautology, true under any assignment.

(b) Horn form.

(i) The sentence cannot be reduced to a single Horn clause (disjunction of literals with at most one positive literal). Note that it can be written as a conjunction of Horn clauses.

$$
\begin{aligned}
(V \land E) \iff C &\equiv ((V \land E) \Rightarrow C) \land (C \Rightarrow (V \land E)) \\
&\equiv ((V \land E) \Rightarrow C) \land (C \Rightarrow V) \land (C \Rightarrow E) \\
&\equiv (\neg(V \land E) \lor C) \land (\neg C \lor V) \land (\neg C \lor E) \\
&\equiv (\neg V \lor \neg E \lor C) \land (\neg C \lor V) \land (\neg C \lor E)
\end{aligned}
$$

(ii) The sentence cannot be reduced to a single Horn clause, however, again, it can be written as a conjunction of Horn clauses.

$$
\begin{aligned}
V \Rightarrow (E \iff C) &\equiv V \Rightarrow ((E \Rightarrow C) \land (C \Rightarrow E)) \\
&\equiv \neg V \lor ((\neg E \lor C) \land (\neg C \lor E)) \\
&\equiv (\neg V \lor \neg E \lor C) \land (\neg V \lor \neg C \lor E)
\end{aligned}
$$

(iii) Since this statement is a tautology, it trivially becomes an implication $True \Rightarrow True$ and therefore becomes a Horn clause.

3. (9 points) **First Order Logic**.
Let's assume the following is known. (1) All 4-credit courses have a written final exam. (2) Not all 2-credit courses have written final exams. (3) If a course does not have a written final exam, it is not a 4-credit course.

(a) (3 points) Write statements (1)-(3) above using quantifiers.

(b) (3 point) Write the De Morgan's law for conjunctions and disjunctions. Justify why can it be extended to quantifiers.

(c) (3 points) Can it be shown that sentence (1) entails (3) by applying De Morgan's law to quantifiers? If yes, prove it, else justify your answer.

Solution:
(a) The statements can be written using quantifiers using the following predicates:
$4CreditCourse(C)$ - The course C is a four credit course.
$2CreditCourse(C)$ - The course C is a two credit course.
$WrittenFinal(C)$ - The course C has a written final exam.
We now write the statements in first order logic using quantifiers
(1) $\forall\ x\ \ 4CreditCourse(x) \implies WrittenFinal(x)$. [1 mark]
(2) $\exists\ x\ \ 2CreditCourse(x) \land \neg WrittenFinal(x)$. [1 mark]
(3) $\forall\ x\ \ \neg WrittenFinal(x) \implies \neg 4CreditCourse(x)$. [1 mark]

Remark: One may be tempted to use an implication in statement (2) or conjunctions in statements (1) and (3), but that wouldn't be correct. Why? (See Sec. [RN-Ch-8.2.6] "Quantifiers" in the textbook for an explanation.
(b) For P and Q as symbols, the DeMorgan's law is given by:
$\neg(P \land Q) \equiv \neg P \lor \neg Q$ [0.5 mark]
$\neg(P \lor Q) \equiv \neg P \land \neg Q$ [0.5 mark]
If the domain $\mathcal{D}$ is defined as $\mathcal{D} = \{P_1, P_2, \ldots\}$, then a universal quantification for membership of $\mathcal{D}$ can be written as a series of conjunctions using the predicate $in\mathcal{D}(P_i)$:
$\forall\ x\ \ in\mathcal{D}(x) \equiv in\mathcal{D}(P_1) \land in\mathcal{D}(P_2) \land \ldots$

Similarly, the existential quantifier can be written as a series of disjunctions:
$\exists\ x\ \ in\mathcal{D}(x) \equiv in\mathcal{D}(P_1) \vee in\mathcal{D}(P_2) \vee \ldots$ [1 mark]

Since the quantifiers can be written as a series of conjunctions or disjunctions, De Morgan's law can directly be applied to them to obtain the following equivalences for the predicate Q:
$\neg\exists\ x\ \ Q \equiv \forall\ x\ \ \neg Q$ [0.25 mark]
$\neg\forall\ x\ \ Q \equiv \exists\ x\ \ \neg Q$ [0.25 mark]
$\exists\ x\ \ Q \equiv \neg\forall\ x\ \ \neg Q$ [0.25 mark]
$\forall\ x\ \ Q \equiv \neg\exists\ x\ \ \neg Q$ [0.25 mark]

(c) Statement (1) is $\forall\ x\ \ 4CreditCourse(x) \implies WrittenFinal(x)$. We wish to show that statement (3) $\forall\ x\ \ \neg WrittenFinal(x) \implies \neg 4CreditCourse(x)$ is entailed by statement (1).
While there are multiple ways to show the entailment, by applying De Morgan's law to the quantifiers, we can show an equivalence (a stronger condition than entailment).
Applying a double negation to (1), we get [0.25 x 12 mark]

$\neg\neg\forall\ x\ \ 4CreditCourse(x) \implies WrittenFinal(x)$
$\neg\,(\neg\forall\ x\ \ 4CreditCourse(x) \implies WrittenFinal(x))$
Applying De Morgan's to the universal quantifier inside the parentheses
$\neg\,(\exists\ x\ \ \neg\,(4CreditCourse(x) \implies WrittenFinal(x)))$
Since $P \implies Q \equiv \neg P \vee Q$
$\neg\,(\exists\ x\ \ \neg\,(\neg 4CreditCourse(x) \vee WrittenFinal(x)))$
$\neg\,(\exists\ x\ \ (\neg\neg 4CreditCourse(x) \wedge \neg WrittenFinal(x)))$
$\neg\,(\exists\ x\ \ (4CreditCourse(x) \wedge \neg WrittenFinal(x)))$
Applying De Morgan's to the existential quantifier
$\forall\ x\ \ \neg\,(4CreditCourse(x) \wedge \neg WrittenFinal(x))$
$\forall\ x\ \ (\neg 4CreditCourse(x) \vee \neg\neg WrittenFinal(x))$
$\forall\ x\ \ (\neg 4CreditCourse(x) \vee WrittenFinal(x))$
Swapping the disjuncts
$\forall\ x\ \ (\neg\neg WrittenFinal(x) \vee \neg 4CreditCourse(x))$
$\forall\ x\ \ (\neg WrittenFinal(x) \implies \neg 4CreditCourse(x))$

The last sentence is equivalent to statement (3).
4. (*Extra credit*: 5 points) Following are two sentences in First Order Logic: (1) $\forall x\ \exists y(x \geq y)$ (2) $\exists y\ \forall x(x \geq y)$.

(a) Assume that the variables range over all the natural numbers $0, 1, 2, \ldots, \infty$ and that the "$\geq$" predicate means "is greater than or equal to". Under this interpretation, translate (1) and (2) into English.

(b) Is (1) true under this interpretation?

(c) Is (2) true under this interpretation?

(d) Does (1) logically entail (2)?

(e) Does (2) logically entail (1)?

Solution:
(a) Sentence (1) is: "For every natural number there is some other natural number that is smaller than or equal to it."

Sentence (2) is: "There is a particular natural number that is smaller than or equal to any natural number."

(b) (1) is true under this interpretation. Since the predicate is "$\geq$", i.e., greater than or equal to, the existential quantifier can always take the value of the universal variable itself, thus making (1) true for every value of x.

(c) (2) is also true under this interpretation. The *particular* natural number could be chosen to be 0. (d) No, (1) does not logically entail (2).

To prove this entailment, we set our knowledge base to consist of (1) and the negation of (2), which we will call ($\neg$2), and try to derive a contradiction. First we have to convert (1) and ($\neg$2) to canonical form (eliminating quantifiers, etc.). For ($\neg$2), this involves moving the $\neg$ in past the two quantifiers. This involves introducing Skolem functions in both sentences. After Skolemization:

$(1) x \geq F_1(x)$

$(\neg 2) \neg F_2(y) \geq y$

Now we can try to resolve these two together, but the occurs check rules out the unification. It looks like the substitution should be $\{x/F_2(y), y/F_1(x)\}$, but that is equivalent to $\{x/F_2(y), y/F_1(F_2(y))\}$, which fails because y is bound to an expression containing y. So the resolution fails, there are no other resolution steps to try, and therefore (2) does not follow from (1).

(e) Yes, (2) logically entails (1).

To prove that (2) entails (1), we start with a knowledge base containing (2) and the negation of (1), which we will call ($\neg$1):

$(\neg 1) \neg F_1 \geq y$

$(2) x \geq F_2$ This time the resolution goes through, with the substitution $\{x = F1; y = F2\}$, thereby yielding False, and proving that (2) entails (1).

Rubric: (a) has two parts of 0.25 points each. (b) and (c) are 0.25 points each and (d) and (e) are 1.5 points each.