ECE 351 DSP: Practice Problems 5

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1) Consider a Type I linear phase system with group delay 3. Let $0.5e^{j\frac{\pi}{4}}$ and $-\frac{1}{3}$ be two of its zeros. Furthermore, the system satisfies $H(1) = \frac{5}{3}(5 - 2\sqrt{2})$. Show the cascaded form realization of this system.

2) Consider the IIR system $H(z) = \frac{\frac{7}{4} - \frac{1}{4}z^{-1} + \frac{5}{16}z^{-2}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-2})}$. How will you realise this system in the parallel form?

3) Let $X(k)$ be the N-point DFT of some sequence $x[n], 0 \leq n \leq N-1$. Define the $2N$-point sequence $y[n]$ as $y[n] = x[\frac{n}{2}]$ if n is even and $y[n] = 0$ otherwise. What is the relation ship between the DFT of $y[n]$ and $X(k)$?

4) Suppose you are given chips that performs 8-point DFTs and 3-point DFTs. Draw a block diagram of how you will use these chips to perform a 24-point DFT.

5) Consider the problem of designing an IIR filter with pass bands $[0, \frac{\pi}{24}]$ and $[\frac{15\pi}{24}, \frac{17\pi}{24}]$ rad/sample, and stop bands $[\frac{\pi}{12}, \frac{7\pi}{12}]$ and $[\frac{9\pi}{12}, \pi]$ rad/sample. Let the pass-band and stop-band ripples be δ_1 and δ_2 respectively. Describe the design procedure, including what analog low-pass filter you will use,[1] whether you will require frequency transformations, bilinear transforms, and anything else you think is necessary.

6) Consider an 3-bit uniform quantizer with quantization levels $\pm\frac{(2i+1)\Delta}{2}, 0 \leq i \leq 3$, and the quantization intervals being $I_i = [i\Delta, (i+1)\Delta], -4 \leq i \leq 3$. Consider an incoming signal $x[n]$ whose magnitude is distributed as follows. The probability that $x[n]$ lies in any of the quantization intervals is uniformly distributed. Next, conditioned on the fact that $x[n]$ lies in $[i\Delta, (i+1)\Delta]$, the distribution of $x[n]$ follows the pdf

$$f(x|x \in I_i) = \begin{cases} \frac{2}{\Delta}(2i + 1 - \frac{2}{\Delta}x), & \text{if } x \in [i\Delta, \frac{(2i+1)\Delta}{2}] \\ \frac{2}{\Delta}(\frac{2}{\Delta}x - 2i - 1), & \text{if } x \in [\frac{(2i+1)\Delta}{2}, (i+1)\Delta]. \end{cases}$$

Calculate the SQNR of the quantizer for this signal $x[n]$.

[1]For example, if you are using the Chebyshev I filter, give an expression for its Ω_p, N, ϵ in terms of δ_1 and δ_2.

ANSWERS

1) See figure below.

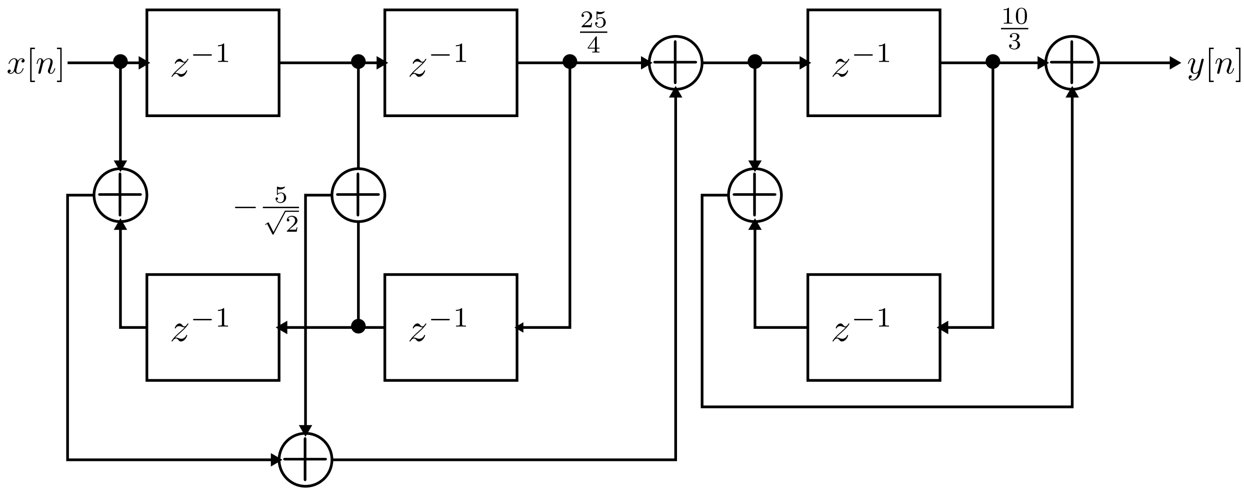

Fig. 1: Solution to question 1.

2) See figure below.

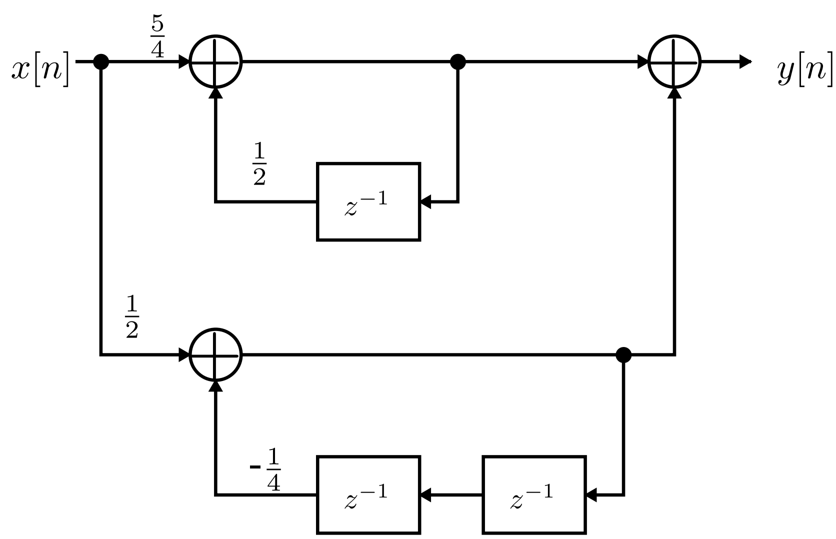

Fig. 2: Solution to question 2.

3)

$$Y(k) = \begin{cases} X(k), & 0 \leq k \leq N-1 \\ X(k-N), & N \leq k \leq 2N-1. \end{cases}$$

4) See figure below.

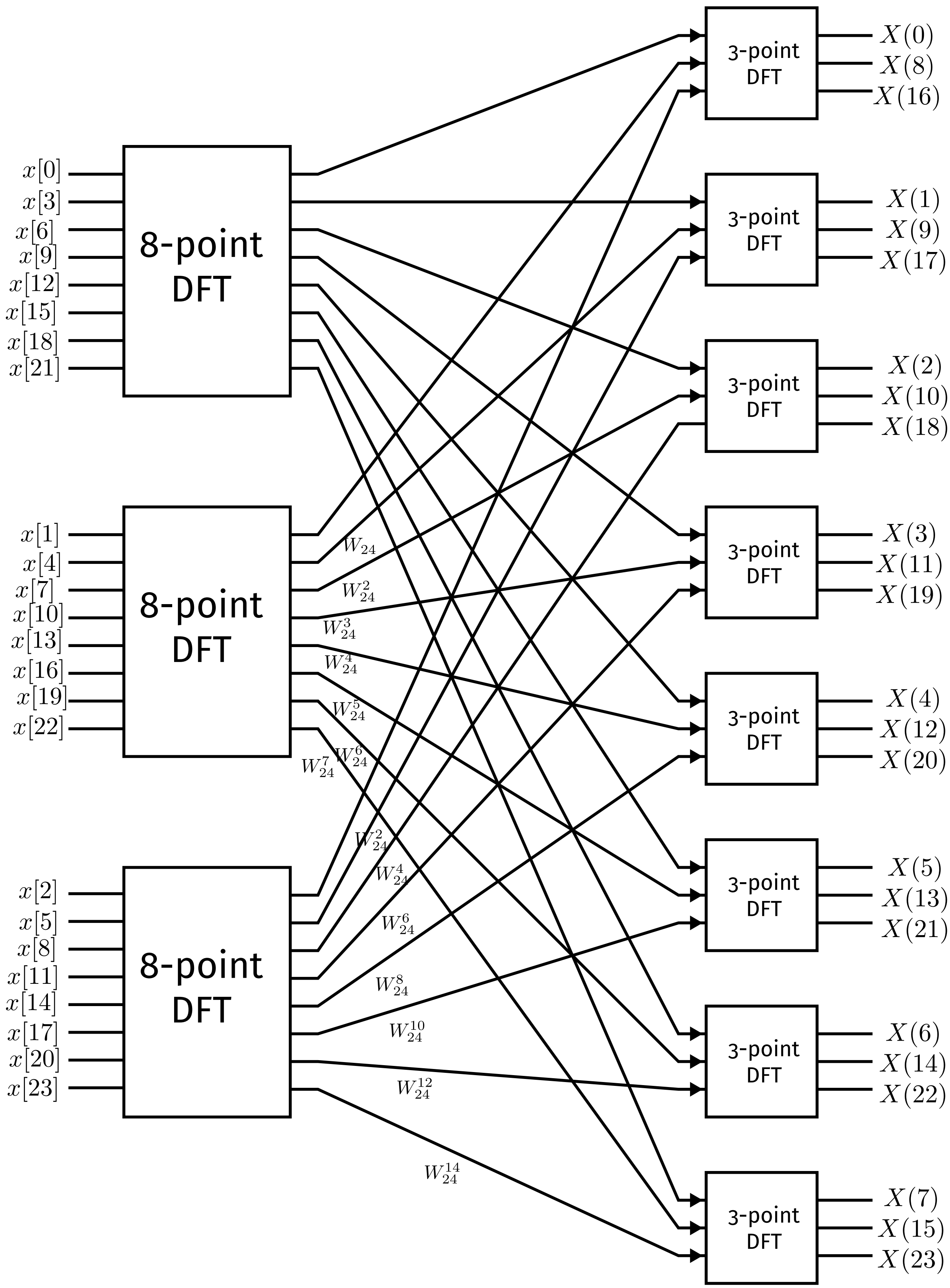

Fig. 3: Solution to question 4.

5) First, design Butterworth filter with $\Omega_p = 2\tan\frac{\pi}{16}$, $\epsilon = \sqrt{\frac{1}{(1-\delta_1)^2} - 1}$, and $N = \frac{\log\left(\frac{\sqrt{1-\delta_2^2}}{\delta_2 \epsilon}\right)}{\log\left(\frac{\tan\frac{\pi}{8}}{\tan\frac{\pi}{16}}\right)}$. Then apply bilinear transform with $T = 1$. Finally, make a comb filter out of the resulting filter with $L = 3$.

6) 43.