

# Eco 311/511: Game Theory

## Problem set 2

September 21, 2023

1. Suppose there is a Firm and a Worker. Worker can be of High ability (H), in which case he would like to Work when he is hired, or of Low ability (L), in which case he would rather Shirk. Firm would want to Hire the worker that will work but not the worker that will shirk. Worker knows his ability level. Firm does not know whether the worker is H or L. Firm believes that the worker is H with probability  $p$  and L with probability  $1 - p$ . The firm knows that the worker knows his own ability level. Denote the ability/type of the worker by  $t_w$ . The payoff matrix is as follows: (Firm is the row player, Worker is the Column player)

$t_w = \text{high}$	<i>work</i>	<i>shirk</i>	$t_w = \text{low}$	<i>work</i>	<i>shirk</i>
<i>hire</i>	1,2	0,1	<i>hire</i>	1,1	-1,2
<i>dont</i>	0,0	0,0	<i>dont</i>	0,0	0,0

Suppose  $p = \frac{3}{4}$ . Formulate the above situation as a Bayesian game and find a Bayesian Nash equilibrium in this game (there may be multiple equilibria, you are required to find any one that is compatible with given  $p$ ).

2. Why is Quasi-concavity of preferences necessary for the existence of Nash equilibrium?
3. **Information may hurt the players:** There are two states and two corresponding payoff matrices, given as follows:

	<i>L</i>	<i>M</i>	<i>R</i>
<i>T</i>	1, 2	1, 0	1, 3
<i>B</i>	4, 4	0, 0	0, 5

$\omega_1$ , prob. 1/2

	<i>L</i>	<i>M</i>	<i>R</i>
<i>T</i>	1, 2	1, 3	1, 0
<i>B</i>	4, 4	0, 5	0, 0

$\omega_2$ , prob. 1/2

$\omega_1, \omega_2$  are the probabilities of each possible state of nature being realized. Player 2 is the Column player, Player 1 is the Row player, What strategy will each player choose in equilibria? Now, suppose that Player 2 is informed about which state will be realised. Show that this leads to both players being worse off in the new equilibrium.

4. Represent the famous game Rock-Paper-Scissors as a strategic form game. Does the game have any PSNE? Show that the game has a unique mixed strategy Nash equilibrium.