ECE 351 DSP: Practice Problems 6

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1) Consider a signal between 100 Hz - 125 Hz. Suppose the sampling frequency can drift by $\pm$ 5 Hz.
   a) Find a good sampling frequency.
   b) Derive the interpolation to be used for the chosen frequency to convert $x[n]$ to $x(t)$.
2) Suppose you want to convert the sampling rate from 120 KHz to 80 KHz.
   a) Suppose the sequence $[1, -1, 0, 1, 5, 3, 2, -1, 6, 4]$ was fed to the upsampler. What will be its output?
   b) What is the sampling rate of the upsampler output?
   c) If the upsampler out in part (a) was directly fed to the downsampler, what will be the downsampler's output?
3) Let $x[n] = a^n u[n]$ is passed through a downsampler with $D = 2$. What is the DTFT of its output?
4) Suppose $x[n] = n2^n u[n]$ is passed through an upsampler with $I = 3$. Find the z-transform of its output.
5) Suppose you have a signal $x(t)$ which puts power in two frequency bands, 100-200 Hz, and 300-400 Hz.
   a) Is it possible to do integer band positioning type band pass sampling in this case, using a 200Hz sampling frequency.
   b) Suppose we use pass $x(t)$ parallely to two rectangular band pass filters, of pass bands respectively 100 Hz - 200 Hz, and 300-400 Hz. Next, we sample both streams at 200 Hz, to get $x_1[n]$ and $x_2[n]$. How can we reconstruct $x(t)$ from $x_1[n]$ and $x_2[n]$?

I. Answers

1) a) 89 Hz.

b) $x(t) = \frac{50}{89} \sum_{n=-\infty}^{\infty} x[n] \text{Sinc}(25\pi(t - \frac{n}{89})) \cos(225\pi(t - \frac{n}{89}))$.

2) a) $[1, 0, -1, 0, 0, 0, 1, 0, 5, 0, 3, 0, 2, 0, -1, 0, 6, 0, 4]$.

b) 240 KHz.

c) $[1, 0, 1, 0, 2, 0, 4]$.

3) $Y(\omega) = \frac{1}{1-a^2 e^{-j\omega}}$.

4) $\frac{2z^{-3}}{1-4z^{-3}+4z^{-6}}$.

5) a) No.

b) $x(t) = \sum_{n=-\infty}^{\infty} \Big(x_1[n] \cos(300\pi(t - 0.005n)) + x_2[n] \cos(700\pi(t - 0.005n)) \Big) \text{Sinc}(100\pi(t - 0.005n))$.