MTH210 – SUBMISSION_20220929

For $n \in \mathbb{Z}^+$, put $X_n = \{1,2,\ldots,n\}$. Put $V = \{0,1\}$, and finally, put V_n = Cartesian product of n copies of V, i.e. V_n is the set of all ordered n-tuples with 0-1 entries.

Construct a partial order R on V_n so that $<V_n , R >$ is isomorphic to $<\mathbb{P}(X_n), \subseteq >$.

Note: *You must firstly define the relation R, secondly show that it is a partial order, and thirdly prove the isomorphism.*

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List of Common Errors and Marks Deductions:

1. Using an undefined symbol.
2. Writing an equation in which the LHS and RHS are nt comparable. For example, the LHS is a set, and the RHS is an integer.
3. Writing a meaningless or completely illogical statement.

Deduct 0.5 marks for each occurrence of an error of the above type. **However, the total marks for the submission should remain non-negative.**

Marks to be awarded as follows:

Firstly, for the correct and explicit definition of the relation R $\rightarrow$ 1 mark.

Secondly, for showing that R is a partial ordering on V_n :

1. Reflexive property $\rightarrow$ 0.5 marks

2. Anti-symmetric property $\rightarrow$ 0.5 marks

3. Transitive property $\rightarrow$ 0.5 marks.

Thirdly, for showing the isomorphism:

- For correctly and explicitly defining the mapping $\psi \rightarrow$ 1 mark.
- For showing that the mapping ψ is a bijection $\rightarrow$ 1 mark

  (**NB**: This step has 3 components: showing that the two sets involved $\mathbb{P}(X_n)$ and V_n have the same cardinality, showing that the mapping is injective, and showing that it is bijective; deduct 0.5 marks for each component not explicitly stated, with the proviso that the marks for this step should be non-negative.)
- For showing that the mapping ψ is order-preserving $\rightarrow$ 0.5 marks.

Partial Credit: The answers are short and to the point, so no scope for partial credit. While proving isomorphism, the technically correct way to proceed is to define a mapping and then show that it fulfils the requirements for an isomorphism. However, this time, it would be acceptable if the student's answer describes how to obtain the n-tuple corresponding to a subset of X_n, even if the terms mapping or function are not used.