

Discrete Mathematics CSE 121 : Homework 3

In every proof/derivation clearly state your assumptions and give details of each step.

1. Are the statements $P \rightarrow (Q \vee R)$ and $(P \rightarrow Q) \vee (P \rightarrow R)$ logically equivalent?
2. Simplify the statements below to the point that negation symbols occur only directly next to predicates.
 - (a) $\neg \forall x \forall y (x < y \vee y < x)$.
 - (b) $\neg (\exists x P(x) \rightarrow \forall y P(y))$.
3. Prove or disprove that for all integers n , n is divisible by 3 if and only if n^2 is divisible by 3.
4. Prove that $\sqrt{3}$ is irrational.
5. Express the statement “There is exactly one student in this class who has taken exactly one mathematics class at this school” using the uniqueness quantifier. Then express this statement using quantifiers, without using the uniqueness quantifier.
6. Prove or disprove that for every x there is a unique y such that $(x + 1)^3 - x^3 = 3y + 1$.
7. Show that the two statements $\neg \exists x \forall y P(x, y)$ and $\forall x \exists y \neg P(x, y)$, where both quantifiers over the first variable in $P(x, y)$ have the same domain, and both quantifiers over the second variable in $P(x, y)$ have the same domain, are logically equivalent.