Solution & Rubric

Worksheet-1

Date - 7/9/22

Q.1. (a) Domain : $x^2+y^2 < 16$ for all (x,y) (0.5)

Range : suppose $z = \frac{1}{\sqrt{16-x^2-y^2}}$

$\left[\because \text{At } (0,0),\ z = \frac{1}{4} \right.$

$\left. \& \text{ for } (x,y) > (0,0)\ \ z > \frac{1}{4} \right].$

$\therefore$ Range of $f \Rightarrow z \geqslant \frac{1}{4}$ (0.5)

so, Range of $f = [\frac{1}{4}, \infty)$

(b) Level curves :

$x^2+y^2 < 4^2$

$\therefore$ circles centered at origin $(0,0)$ with radius < 4. (1)

(c) Boundary is the circle

$x^2+y^2 = 16$. (1)

(d) $\because x^2+y^2 < 16$ (1)

Hence the domain is open and bounded.

Q.2. Let, $z = 16-(x^2+y^2)$.

At $(2\sqrt{2}, \sqrt{2})$: $z = 16-((2\sqrt{2})^2+(\sqrt{2})^2) = 6$ $\rightarrow$ (2)

Hence the equation for the level

Curve is : $f(x,y) = 6$

$\Rightarrow 16-(x^2+y^2) = 6.$

$\therefore \boxed{x^2+y^2 = 10}$ 2

Q.3. Here $f(x,y,z) = \ln(x^2+y^2+z^2)$

Let, $w = \ln(x^2+y^2+z^2)$

So, at $(-1,2,1) \Rightarrow w = \ln((-1)^2+(2)^2+(1)^2)$

$\therefore w = \ln 6.$ → 2

Hence the level surface is :

$f(x,y,z) = \ln 6$

$\Rightarrow \ln(x^2+y^2+z^2) = \ln 6.$

$\therefore \boxed{x^2+y^2+z^2 = 6}$ → 2

Q.4. a open set in $\mathbb{R}^2$

$\{(x,y) \in \mathbb{R}^2 : (x-1)^2+(y-2)^2 < 5\}$ 1

* For 4a, 4b, 4c other possible (correct) answers can be considered

b ~~{(x,y)}~~ closed set in $\mathbb{R}^2$

$\{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 4\}$. → 1

c Neither open nor closed :

2 (i) set of all rational numbers in $\mathbb{R}$

(ii) set of all irrational numbers in $\mathbb{R}$

Q.5. (a) $w = \sqrt{x^2 - y}$

Domain $= \{ (x,y) \in \mathbb{R}^2 : x^2 \geq y \}$ 0.5

Range $= \{ w \in \mathbb{R} : w \geq 0 \}$ or $w \in \underbrace{[0,\infty)}_{\text{Set of all non-negative real nos.}}$. 0.5

Q.5. (b) $w = \dfrac{1}{\sqrt{xy} - 1}$

Domain $= \{ (x,y) \in \mathbb{R}^2 : xy \geq 0 \text{ and } \sqrt{xy} \neq 1 \}$. 0.5

Range $= \{ w \in \mathbb{R} : w \leq -1 \text{ or } w > 0 \}$. 0.5

Hint: $w = \dfrac{1}{\sqrt{xy}-1} \Rightarrow \sqrt{xy} = \dfrac{1}{w} + 1$

Now $\sqrt{xy} \neq 1$ and $xy \geq 0$

$\Rightarrow w \leq -1$ and $w > 0$

Q.5. (c) Domain $= \{ (x,y,z) \in \mathbb{R}^3 : (x-1)^2 + (y-1)^2 + (z-1)^2 \neq 0 \}$

OR

$\{ (x,y,z) \in \mathbb{R}^3 : (x,y,z) \neq (1,1,1) \}$. 0.5

Range $= \{ w \in \mathbb{R} : w > 0 \}$ OR $w \in \underbrace{(0,\infty)}_{\text{(all +ve real nos.)}}$. 0.5

Q.5. (d) Domain $= \{ (x,y) \in \mathbb{R}^2 : x \neq 0 \}$ 0.5

Range $= \{ w \in \mathbb{R} : -\pi/2 < w < \pi/2 \}$. 0.5

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