Quiz 1 - Linear Algebra - CSE/ECE 344/544

Maximum score: 25 **Time: 30 mins**

Name: **Roll no:**

Instructions:

1. Attempt all True/False questions with justification. A statement is true if it is *always* true.
2. There will be partial grading for answers without justification. For incorrect answers, there will be negative marking (-1 point for each incorrectly answered T/F question).
3. Please do not copy. Institute's plagiarism policy is strictly enforced.

Questions:

1. (2 points each) Mark the following statements as True or False. Justify each answer.

   a. Any system of n linear equations in n variables has at most n solutions.

   b. If $\mathbf{A}$ is an m x n matrix and the equation $\mathbf{Ax}=\mathbf{b}$ is consistent for some $\mathbf{b}$, then the columns of $\mathbf{A}$ span $\mathbf{R}^m$.

   c. If none of the vectors in the set $\mathbf{S}=\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ in $\mathbf{R}^3$ is a multiple of one of the other vectors, then $\mathbf{S}$ is linearly independent.

   d. Left multiplying a matrix $\mathbf{B}$ by a diagonal matrix $\mathbf{A}$, with nonzero entries on the diagonal, scales the rows of $\mathbf{B}$.

   e. If $\mathbf{BC} = \mathbf{BD}$, then $\mathbf{C} = \mathbf{D}$.

   f. If $\mathbf{A}$ and $\mathbf{B}$ are n x n, then $(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$.

   g. If $\mathbf{AB} = \mathbf{BA}$ and if $\mathbf{A}$ is invertible, then $\mathbf{A}^{-1}\mathbf{B} = \mathbf{BA}^{-1}$.

   h. If $\mathbf{x}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$, then $\mathbf{x}$ must be orthogonal to $\mathbf{u}$-$\mathbf{v}$.

i. If a square matrix $\mathbf{U}$ has orthonormal columns, then it also has orthonormal rows. (Orthonormal vectors are unit vectors that are mutually orthogonal.)

j. If a matrix $\mathbf{U}$ has orthonormal columns, then $\mathbf{U}\mathbf{U}^{\mathbf{T}} = \mathbf{I}$.

2. (2 points) Prove that $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$.

3. (3 points) Given $\mathbf{u}$ in $\mathbf{R}n$ with $\mathbf{u}^{\mathrm{T}}\mathbf{u} = 1$, let $\mathbf{P} = \mathbf{u}\mathbf{u}^{\mathrm{T}}$ (outer product) and $\mathbf{Q} = \mathbf{I}$ - $2\mathbf{P}$. Justify the following statements:

a. $\mathbf{P}^2 = \mathbf{P}$ b.$\mathbf{P}^{\mathrm{T}} = \mathbf{P}$ c.$\mathbf{Q}^2 = \mathbf{I}$