Quiz 8 (Solutions)

a) $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n} \quad ②$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} e^{-j\omega(n+1)} = e^{-j\omega} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} e^{-j\omega n}$$

$$= e^{-j\omega}\left(\frac{1}{1-\left(\frac{1}{2}\right)e^{-j\omega}}\right) \quad - ③$$

$$\frac{1}{2e^{-j\omega}-1} \qquad \frac{1}{2e^{j\omega}-1}$$

b) $x[n] = \left(\frac{1}{2}\right)^{|n-1|}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{(1-n)} e^{-j\omega n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n}$$

↳ part (a)

$$\sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{(1-n)} e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} e^{j\omega n} = \left(\frac{1}{2}\right)\left(\frac{1}{1-\frac{1}{2}e^{j\omega}}\right) \quad - ②$$

$$\therefore X(e^{j\omega}) = e^{-j\omega}\left(\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{1-\frac{1}{2}e^{j\omega}}\right) \quad - ③$$