MTH100 **Monsoon Semester – 2021-22** **Section B**

Submission for Saturday 29th January 2022 – 15 minutes. Max Marks: 5

Instructions: Open notes and textbook; consultation and use of calculators, computers and internet not allowed.

a) Show that the matrix A below satisfies the equation $A^3 - 3A^2 - 7A + 18I_3 = \mathbf{0}$. Here **0** indicates the 3×3 matrix with all zero entries. *(3 marks)*

b) Hence, find A^{-1}. **Do not use any other method.** *(2 marks)*

YOUR STEPS FOR BOTH PARTS MUST BE SHOWN. otherwise, you will not get credit.

$$A = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix}$$

SOLUTION-CUM-RUBRIC BELOW

a) We get $A^2 = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix}\begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 13 & -9 & 3 \\ 1 & 3 & 0 \\ 1 & -1 & 7 \end{bmatrix}$ (1)

$A^3 = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix}\begin{bmatrix} 13 & -9 & 3 \\ 1 & 3 & 0 \\ 1 & -1 & 7 \end{bmatrix}$

$= \begin{bmatrix} 42 & -34 & 37 \\ 3 & 5 & 7 \\ 10 & -10 & -11 \end{bmatrix}$ (2)

PT

(a) Cont'd :

Let us now check the equation:

$$A^3 - 3A^2 - 7A + 18I_3$$

$$= \begin{bmatrix} 42 & -34 & 37 \\ 3 & 5 & 7 \\ 10 & -10 & -11 \end{bmatrix} -$$

$$3\begin{bmatrix} 13 & -9 & 3 \\ 1 & 3 & 0 \\ 1 & -1 & 7 \end{bmatrix} -$$

$$7\begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix} + 18\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= [0]$ ✓ (3)

$-10 + 3 + = 0$

RUBRIC: Calculating (1) Correctly → 1 mark ✓

Calculating (2) correctly → 1 mark ✓

Doing Calculation (3) correctly → 1 mark

(b) From the given equation:-

$A^3 - 3A^2 - 7A + 18I_3 = [0]$,

we get:

$$A(A^2 - 3A - 7I_3) = -18I_3$$

$$\Rightarrow A\left[\frac{-1}{18}(A^2 - 3A - 7I_3)\right] = I_3$$

(2)

$$\Rightarrow A^{-1} = -\frac{1}{18}\left[A^2 - 3A - 7I_3\right] \quad (4)$$

$$= (-1/18)\left\{\begin{bmatrix} 13 & -9 & 3 \\ 1 & 3 & 0 \\ 1 & -1 & 7 \end{bmatrix} - 3\begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix} - 7I_3\right\}$$

$$= \left(\frac{-1}{18}\right)\begin{bmatrix} -3 & -6 & -9 \\ 1 & -10 & -3 \\ -2 & +2 & 6 \end{bmatrix} \quad (5)$$

Rubric for (b)

Getting Expression (4) → 1.5 marks

Getting Final answer (5) → 0.5 marks

Check:-

$$\begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix}\begin{bmatrix} -3 & -6 & -9 \\ 1 & -10 & -3 \\ -2 & +2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -18 & 0 & 0 \\ 0 & -18 & 0 \\ 0 & 0 & -18 \end{bmatrix}$$ ✓