Eco 311/511: Game Theory
Assignment (Take Home/open book)

November 9, 2023

Instructions: Answer all questions. Copied answers will result in a score of 0 in the assignment.

1. Consider the following definition of a "rationalizable" action:

   Definition 1 *An action $a_i \in A_i$ is rationalizable in the strategic game $(N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N})$ if for each $j \in N$ there is a set $Z_j \subseteq A_j$ such that*

   - $a_i \in Z_i$
   - *every action $a_j \in Z_j$ is a best response to a belief μ_j of player j that assigns positive probabilities to actions in some subset of Z_{-j}.*

   (a) In the following two-player simultaneous move game, find the set of rationalizable actions (pure actions, not probabilistic) for each player. (2+2)

   Player 2

Player 1	a	b	c	d
A	$(7,0)$	$(2,5)$	$(0,7)$	$(0,1)$
B	$(5,2)$	$(3,3)$	$(5,2)$	$(0,1)$
C	$(0,7)$	$(2,5)$	$(7,0)$	$(0,1)$
D	$(0,0)$	$(0,-2)$	$(0,0)$	$(9,-1)$

   (b) Do you think a weakly dominated action can be rationalizable? Why/Why not? What about strictly dominated actions? (3)

   (c) Every action profile (a_i, a_{-i}) where $a_i \in Z_i$, $a_{-i} \in Z_{-i}$ is a Nash equilibrium. True or False? Why? (3)

2. Consider a sequential game with $N = \{1, 2, \ldots, \infty\}$ players. Suppose each player $i \in N$ moves only in time period i: he chooses an action $a_i \in \{Yes, No\}$. For example, player $i = 1$ chooses $a_1 \in \{Yes, No\}$ in time period 1. In time period 2, player 2 chooses $a_2 \in \{Yes, No\}$, and so on for each $i \in N$. Each player can observe the actions of all the preceding players in the game. In every time period i, if the new player i chooses Yes, 1 dollar is deposited in the account of each player $j \leq i$. If i chooses No, then 2 dollars is deposited in i's account and the accounts of all $j < i$ players is reduced to 0.

(a) How will you find equilibrium in this game? Briefly explain the key technique used and find the equilibria. (3+2)

(b) Suppose each player $i > 1$ forms a belief about the "types" of players who will join the game after him by observing the actions of the previous players. In this setting, can the action profile $(Yes, Yes, \ldots)$ be sustained in equilibrium? Explain your answer. (5)

Ans. 1 a) First eliminate d for Player 2. Next eliminate D for player 1. Or, show that d is dominated by a mixed strategy consisting of a and c, and then eliminate D.
Rationalizable actions: Player 1: {A,B,C}, Player 2: {a,b,c}.

b) Weakly dominated actions can be best response. Therefore, are rationalizable. Strictly dominated actions are never rationalizable.
c) Nash equilibria actions are a subset of rationalizable actions.

Ans 2. a) Technique: OSD optimality. Players compute utility from the entire game, not only from the ith round. There are multiple equilibria:
1. s_i=Yes if s_j=Yes for all j<i; No, if s_j=No for any j<i. Show that this punishment strategy enforces outcome {Yes, Yes,….} (is OSD optimal). Note that at outcome {Yes, Yes, Yes,…} the payoff is infinite for each player as each of them know that an infinite stream of players are there after them. You may use a discount factor delta and show the values for which different strategies are sustained in equilibria (if it is close to 0 then this strategy will not work since players wont care about payoffs received by future players choosing Yes).
2. s_i=No for all i. Show that outcome {No, No, ….} is OSD optimal with/without discounted payoffs.

b) Let type of each player be "yes" or "no" (revealed by their actions, but unknown ex-ante). For a generic player i, define belief b_i (prob [type of j=yes]) for each j>i. Belief b_i can be updated with each new player i,i+1,… The update process can be Bayesian/Markovian or any other algorithm. Compute expected payoff from {Yes, Yes, …} and range of b_i such that outcome {Yes, Yes, …} occurs in equilibrium. You may use a discount factor delta while computing payoffs.
Alternative approaches are possible and well reasoned answers have been positively rewarded.

Ans. First eliminate d for Player 2, then D for player 1. Rationalizable actions: Player 1: {A,B,C}, Player 2: {a,b,c}