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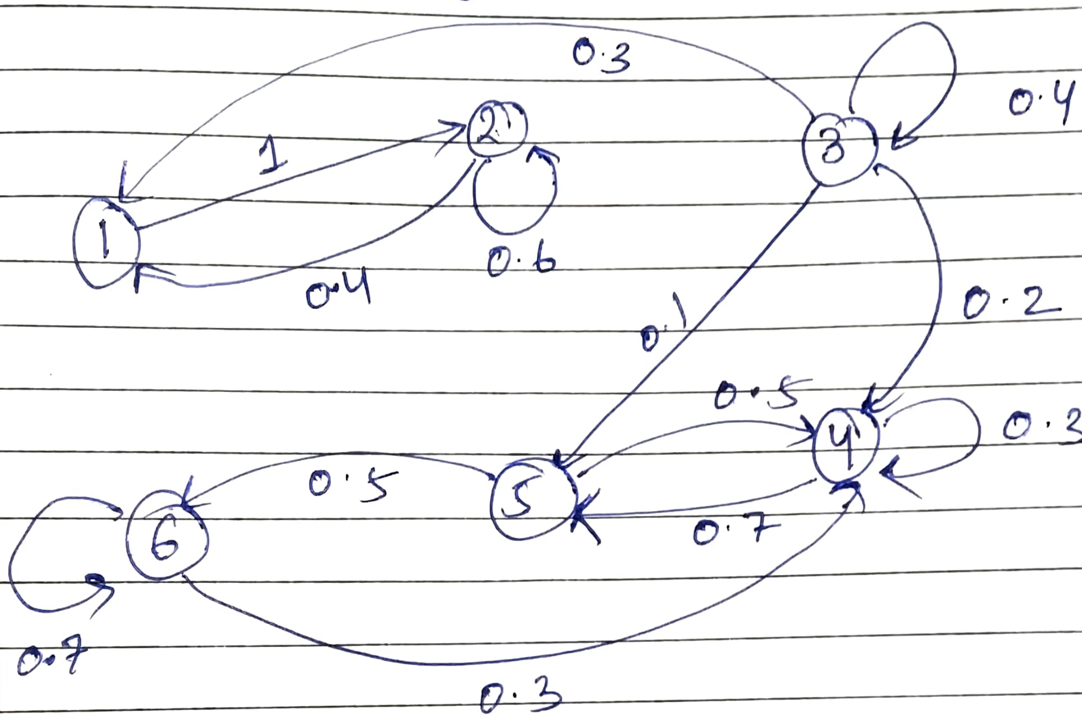
Sl (a) given
$S = \{1, 2, 3, 4, 5, 6\}$

transition prob matrix of a Markov process begining at $X_0 = 1$



(f) $P(X_0 = 1, X_1 = 2, X_2 = 3) = ?$

$= P(X_2 = 3 \mid X_1 = 2, x_0 = 1)\, P(x_1 = 2 \mid x_0 = 1) \times P(X_0 = 1)$

$= P(X_2 = 3 \mid x_1 = 2)\, P(x_1 = 2 \mid x_0 = 1)\quad P(X_0 = 1)$

process begins at $x_0 = 1$

according to marton property.

$= 0 \times 1 \times 1 = 0$

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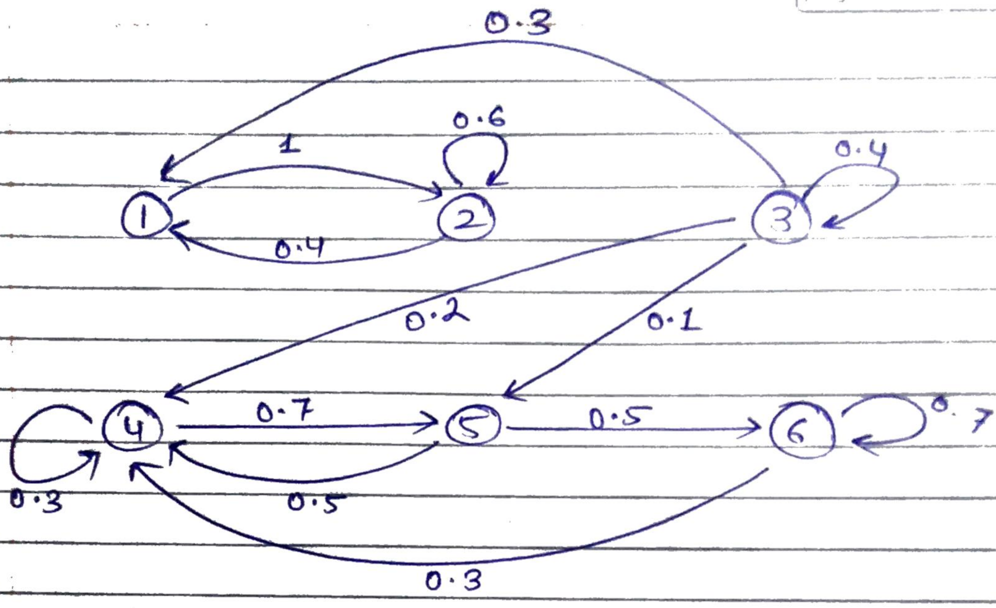
Sol 1



(b) Classes:

$C_1 : \{1, 2\}$

we know,

$1 \rightarrow 2$ and $2 \rightarrow 1$

therefore, $1 \leftrightarrow 2$

$C_2 : \{3\}$

No directed towards 3, thus no state can reach 3.

$C_3 : \{4, 5, 6\}$

~~As~~, $4 \rightarrow 5$ and $5 \rightarrow 4$

therefore $4 \leftrightarrow 5$

Also, $4 \rightarrow 6$ (through 5) and $6 \rightarrow 4$

therefore $5 \leftrightarrow 6$

therefore we have 3 classes

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$C_1 = \{1, 2\}$
$C_2 = \{3\}$
$C_3 = \{4, 5, 6\}$

And $C_1 \cup C_2 \cup C_3 = S$

(c)

(1) 1: recurrent
as once it reaches 1, it will keep looking & won't come out of loop

2: recurrent
similar to 1

3: transient
As it is possible to leave 3 to go to other states.

4: Recurrent
once MC reaches state 4, it will not come out of loop

5: Recurrent
similar to 4

6: Recurrent
similar to 4.

Q1.

(d) Period of State 1, $d(1) = \gcd\{n : P_{11}^{n} > 0\}$

$= \gcd\{2,3,4,5,6,7,8\}$

$= \cancel{2}\,1$

Similarly,

$d(2) = \gcd\{n : P_{22}^{n} > 0\}$

$= \gcd\{1,2,3,4 \dots\}$

$= 1$

$d(3) = \gcd\{n : P_{33}^{n} > 0\}$

$= \gcd\{1,2,3,4 \dots\}$

$= 1$

$d(4) = 1$

$d(5) = \gcd\{2, 3, 4, 6, 8, 9, 10, 12, \dots\}$

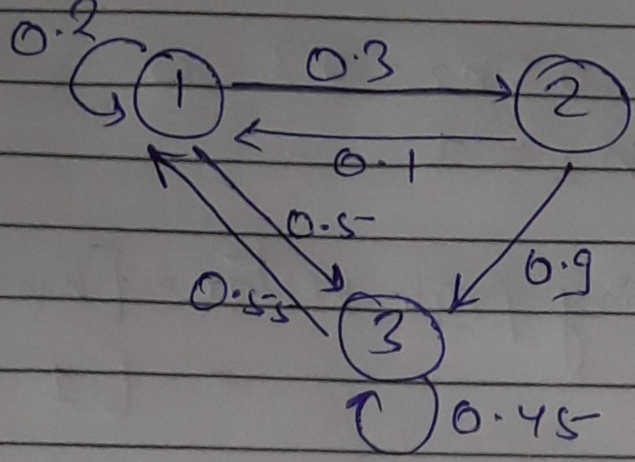
$= 1$

$d(6) = 1$

(e) The markov chain has finite space and it is not irreducible ($\because$ there are three communicating classes), though the MC is aperiodic.

So, the limiting distribution will not exist.

Ans 2



$A = \{2\}$

$$h_A = \begin{bmatrix} h_{1A} \\ h_{2A} \\ h_{3A} \end{bmatrix}$$

$$h_{1A} = \sum_{i=1}^{3} P_{1i} h_{iA}$$

$$= P_{11} h_{1A} + P_{12} h_{2A} + P_{13} h_{3A}$$

$$h_{1A} = 0.2 h_{1A} + 0.3 \times h_{2A} + 0.5 h_{3A}$$

now as we would be starting from state 2 hence

$$h_{2A} \Rightarrow 1$$

$\therefore$

$$h_1 A = 0.2 h_1 A + 0.3 + 0.5 h_3 A$$

$$0.8 h_1 = 0.5 h_3 + 0.3 \quad — (1)$$

Illy

$$h_3 A = \sum_{i=1}^{3} P_{3i} h_{iA}$$

$$h_{3A} = 0.55 h_{1A} + \cancel{0.h_{2A}} + 0.45 h_{3A}$$

$$h_{3A} = 0.55 h_{1A} + 0.45 h_{3A}$$

$$h_1 = h_3 \quad - (2)$$

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putting (2) in (1) we get

$0.8h_3 - 0.5h_3 - 0.3 = 0$

$h_3 = 1$

hence hitting prob. vector = $[h_{1A}\ h_{2A}\ h_{3A}]$

$= [1\ 1\ 1]$

(6) Given

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0 & 0.9 \\ 0.55 & 0 & 0.45 \end{bmatrix}$$

$\pi^0 = [0.2\ \ 0.3\ \ 0.5]$

$\pi^1 = \pi^0 P^1$

$$\therefore \pi^1 = [0.2\ 0.3\ 0.5] \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0 & 0.9 \\ 0.55 & 0 & 0.45 \end{bmatrix}$$

Solving we get

$\pi^1 = [0.345\ \ 0.06\ \ 0.595]$

Ques-3. State Space $= j = \{0, 1, 2, 3, \ldots\}$

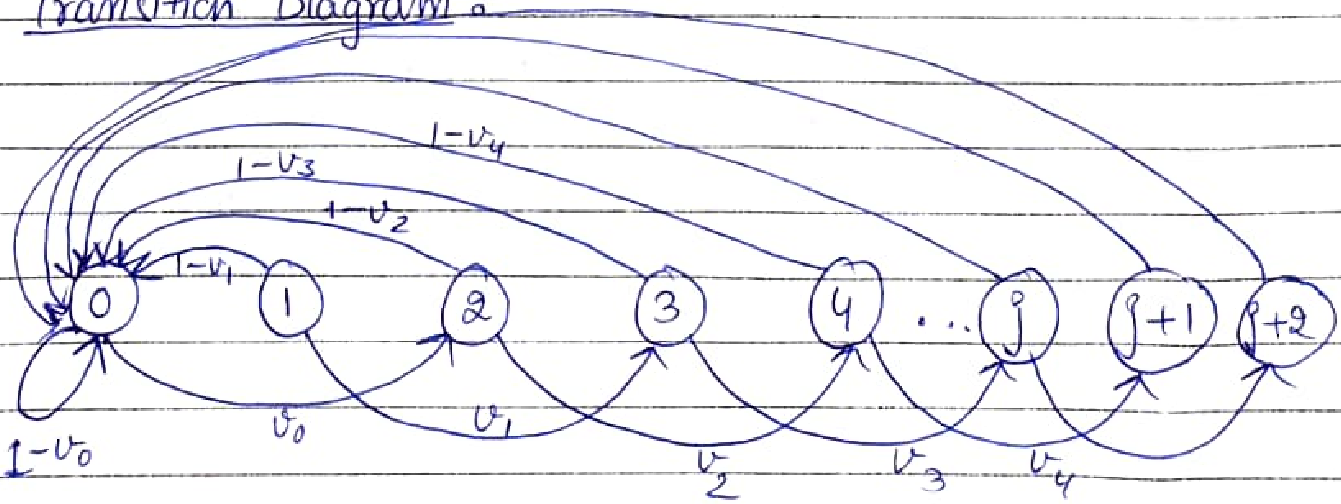
Given: $P_{j(j+2)} = v_j \Rightarrow$ Going from j to $j+2$ in one step with probability v_j.

$P_{j0} = 1 - v_j \Rightarrow$ Going from j to 0 in one step with probability $(1 - v_j)$.

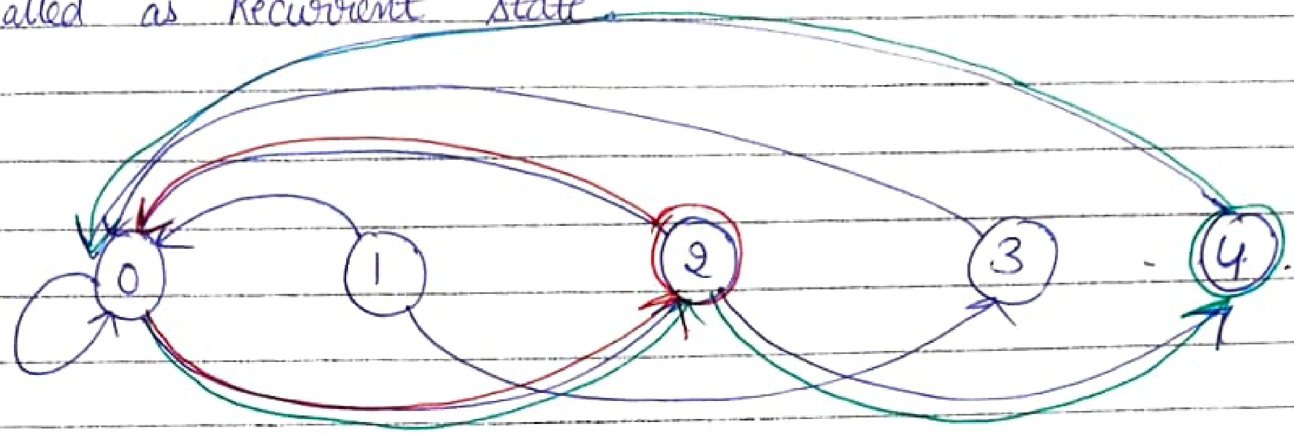
Transition Probability Matrix:

$$
P = \begin{array}{c|cccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \cdots & j+2 & \cdots \\
\hline
0 & 1-v_0 & 0 & v_0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\
1 & 1-v_1 & 0 & 0 & v_1 & 0 & 0 & 0 & \cdots & 0 & \cdots \\
2 & 1-v_2 & 0 & 0 & 0 & v_2 & 0 & 0 & \cdots & 0 & \cdots \\
3 & 1-v_3 & 0 & 0 & 0 & 0 & v_3 & 0 & \cdots & 0 & \cdots \\
4 & 1-v_4 & 0 & 0 & 0 & 0 & 0 & v_4 & \cdots & 0 & \cdots \\
5 & 1-v_5 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\
6 & 1-v_6 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \\
j & 1-v_j & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & v_j & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \cdots
\end{array}
$$

Transition Diagram:



Recurrent States ⇒ If MC enters a state then it will not come out of that loop (class) then that state is called as Recurrent State.



If we observe even states, then we can see that once we reach even state we can form a loop from that state to 0 state to some intermediate states to that state.

ex- State = 0 : $0 \leftrightarrow 0$

$0 \leftrightarrow 2$,

$0 \leftrightarrow 4$

⋮

State = 2 : $2 \rightarrow 0$; $0 \rightarrow 2$ $(0 \leftrightarrow 2)$

State = 4 : $4 \rightarrow 0$; $0 \rightarrow 2$; $2 \rightarrow 4$

⋮

$\rightarrow \{0, 2, 4, \ldots\}$

∴, all even states are Recurrent States

Transition States ⇒ At some point the state will leave their class and will go to some other state.

from the transition diagram, we can observe that once we reach at any odd state then we can go to state 0 and from there we can never come back to that odd state.

ex - State = 1 : $1 \rightarrow 0$ (or) $1 \rightarrow 3$

$0 \not\rightarrow 1$ $\quad$ $3 \rightarrow 0$

$0 \not\rightarrow 1$

$\therefore$, all odd states are Transient states.

$\hookrightarrow \{1, 3, 5, \ldots\}$