Worksheet-2

① To show - $7 \mid 5^{2n} + 3\cdot 2^{5n-2}$

We prove by induction.

for $n=1$

$$5^{2\cdot 1} + 3\cdot 2^{3} = 49$$

$$7 \mid 49$$

Let for $n = k > 1$

$$7 \mid 5^{2k} + 3\cdot 2^{5k-2}$$

or $5^{2k} + 3\cdot 2^{2k-2} \equiv 0 \pmod 7$ - (i)

For $n = k+1$

$$5^{2k+2} + 3\cdot 2^{5k+3}$$

$$= 25\cdot 5^{2k} + 32\cdot 3\cdot 2^{5k-2}$$

$$= 25\left(5^{2k} + 3\cdot 2^{5k-2}\right) + 7\cdot 3\cdot 2^{5k-2}$$

$$\equiv 0 \pmod 7 \quad (\text{from (i)})$$

$$\Rightarrow \quad 7 \mid 5^{2k+2} + 3 \cdot 2^{5k+3} \quad \square.$$

(2) $$2^3 = 8 \equiv 1 \pmod 7$$

$$2^{50} = 4 \cdot (2^3)^{16} \equiv 4 \pmod 7$$

$$41 \equiv -1 \pmod 7$$

$$(41)^{65} \equiv (-1)^{65} \equiv -1 \equiv 6 \pmod 7$$

(3) Since a is not divisible by 2 & 3. So for some $k \in \mathbb{Z}$

$$a = 6k \pm 1$$

$$a^2 = 36k^2 + 1 \pm 12k$$

<u>Case-I:-</u> $k = 2m$ (even)

then-

$$a^2 = 24 \cdot 6m^2 + 1 \pm 24m \equiv 1 \pmod{24}$$

Case - 2 :- $K = 2m+1$ (odd)

$a^2 = 36(4m^2+4m+1)+1 \pm 12(2m+1)$

$\equiv 36+1 \pm 12 \equiv 1 \pmod{24}$ $\square$.

(4)

$ab \equiv cd \pmod n$

$b \equiv d \pmod n$

$\Rightarrow ab \equiv cb \pmod n$

$n \mid b(a-c)$

Since $\gcd(b,n) = 1$

$\Rightarrow n \mid a-c$

$\Rightarrow a \equiv c \pmod n$. $\square$