c) Let carrot and stick strategy be as follows:

$$s_{1t}^* = \begin{cases} A & \text{if } s_{2(t-1)} = a \\ B \text{ (or } C) & \text{if } s_{2(t-1)} = b \text{ (or } c) \end{cases}$$

Similarly define s_{2t}^*. Let $s^* = (s_{1t}^*, s_{2t}^*)$

Suppose 1 deviates from A in $t = 1$ [one-shot deviation]

In t_1, $s_{11} = B$, $s_{21} = a$.

$$U_{1t}(s_{1t}', s_{2t}^*) = 5 + 0.(-1)\delta + 4\delta^2 + 4\delta^3 + \ldots$$

$$U_{1t}(s_{1t}^*, s_{2t}^*) = 4 + 4\delta + 4\delta^2 + \ldots$$

For s^* to be an equilibria strategy,

$$U_{1t}(s_{1t}^*, s_{2t}^*) \geq U_{1t}(s_{1t}', s_{2t}^*)$$

$\Rightarrow$

In $t=1$, $s_{11} = B$ [one-shot deviation]

following s^*_{2t}, $s^*_{21} = a$

In $t=2$, $s^*_{12} = A$ [carrot]

$s^*_{22} = b$ [stick]

In $t=3$, $s^*_{13} = B$ [stick]

$s^*_{23} = a$ [carrot]

In $t=4$, $s^*_{14} = A$ [carrot]

$s^*_{24} = b$ [stick]

and so on...

$$U_1(s'_{1t}, s^*_{2t}) = 5 - 1\delta + 5\delta^2 - 1\delta^3 + \dots\dots$$

$$= 5 + \frac{5\delta^2}{1-\delta^2} - \frac{\delta}{1-\delta^2}$$

$$U_1(s^*_{1t}, s^*_{2t}) = 4 + 4\delta + 4\delta^2 + \dots\dots$$

$$= \frac{4}{1-\delta}$$

OSD optimality: $U_1(s^*_{1t}, s^*_{2t}) \geq U_1(s'_{1t}, s^*_{2t})$ for player 1

Find δ.

Is punishment (B, b) optimal?

Consider player 2's payoff if punishment is (C, C):

In $t=1$, $s_{11} = B$ [one-shot deviation]

$s_{21} = a$

Let s_{it}'' denote strategy for i with punishment (C, C). [with one-shot deviation at $t=1$]

In $t=2$, $s_{12}^* = A$ [stick]
$s_{22}^* = C$ [stick]

In $t=3$, $s_{13}^* = C$ [carrot]
$s_{23}^* = a$ [carrot]

In $t=4$, $s_{14}^* = A$ [stick]
$s_{24}^* = C$ [stick]

and so on...

$U_2(s_{1t}'', s_{2t}^*) = -1 + 2\delta + 2\delta^2 + 2\delta^3 + \dots$

(punishment (B,B)) $U_2(s_{1t}', s_{2t}'') = -1 + 5\delta - \delta^2 + 5\delta^3 - \delta^4 + \dots$

check one-shot deviation for player 2

If $U_2(s_{1t}'', s_{2t}^*) > U_2(s_{1t}', s_{2t}^*)$ for ~~every~~ t

then (C, C) as punishment dominates (B, b).

Find relevant values of δ.