

1. (3 points) (a) Describe the notion of PEAS to specify the task environment. (b) Briefly explain what is a model-based agent?

Sol:

PEAS stands for Performance Measure, Environment, Actuators, and Sensors, and it is a framework used to specify the task environment for an intelligent agent. Each component of PEAS helps define the characteristics and requirements of the agent's interaction with its environment:

- **Performance measure:** This component specifies how the agent's performance will be evaluated or measured. It defines the criteria for success or failure.
- **Environment:** The environment represents the external context in which the agent operates. It includes everything that the agent interacts with or receives inputs from. The environment can be physical or virtual.
- **Actuators:** Actuators are the mechanisms or tools through which the agent affects the environment. They allow the agent to perform actions or manipulate the environment to pursue its goals. For example, actuators for a robot might include motors for movement or an arm for picking up objects.
- **Sensors:** Sensors are the means by which the agent perceives the state of the environment. They provide input to the agent by capturing data or information from the surroundings. Sensors can include cameras, microphones, temperature sensors, or any other sensory devices relevant to the agent's task.

A model-based agent is an intelligent agent that uses an internal model or representation of its environment to make decisions and plan actions. Updating the internal state information as time passes requires two kinds of knowledge to be encoded in the agent program in some form. First, we need some information about how the world changes. Second, we need some information about how the state of the world is reflected in the agent's percepts. This internal model allows the agent to predict the consequences of its actions before actually executing them. In other words, a model-based agent maintains a mental or computational representation of the world it interacts with.

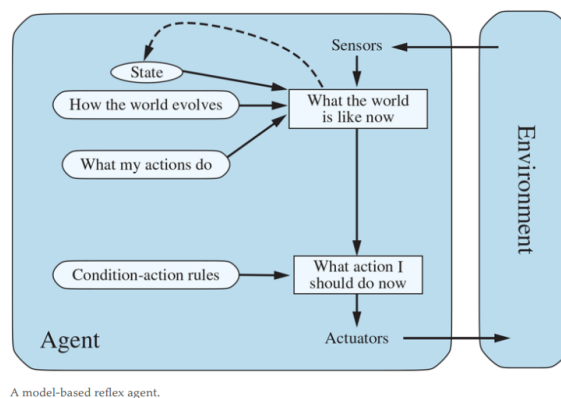


Figure 1: Model Based Agent

2. (1 point) Let α and β be two sentences, and $M(\alpha)$ and $M(\beta)$ as the set of models that satisfy α and β respectively. Recall that a model is a “possible world”, i.e., a set of values that each variable in the two sentences would take. Argue that $\alpha \models \beta$ if and only if $M(\alpha) \subseteq M(\beta)$.

Solution:

In order to prove a bi-implication (if and only if OR iff) statement, we need to prove both the forward and the reverse implications.

1. If $\alpha \models \beta$, then $M(\alpha) \subseteq M(\beta)$:

if α logically entails β , then the set of models that satisfy α ($M(\alpha)$) is a subset of the set of models that satisfy β ($M(\beta)$).

- α logically entails β , which means that in every possible world (model) where α is true, β must also be true. - Now, consider any model m that satisfies α , i.e., α is True, then ($m \in M(\alpha)$). Since α logically entails β , this model m must also satisfy β (i.e., $m \in M(\beta)$). - This means that for any model m that satisfies α , it must also satisfy β . In other words, every $m \in M(\alpha)$ is also in $M(\beta)$. This proves that $M(\alpha) \subseteq M(\beta)$.

2. If $M(\alpha) \subseteq M(\beta)$, then $\alpha \models \beta$:

if the set of models that satisfy α is a subset of the set of models that satisfy β , then α logically entails β .

- We start with the premise that $M(\alpha) \subseteq M(\beta)$, which means that every model that satisfies α also satisfies β . - Now, let $m \in M(\alpha)$ be any model that satisfies α . Because $M(\alpha) \subseteq M(\beta)$, it follows that m also satisfies β . This demonstrates that whenever α is true, β is also true. In other words, α logically entails β .

Thus, we proved both forward and backward implications and thus the bi-implication.

3. (2 points) Write the following sentence using propositional logic (i.e., use propositional symbols and connective): *If 7 is even then Rahul Gandhi is the CM of Delhi.* Is the above statement true in propositional logic? Re-write the propositional logic statement using only disjunctions and negations. Show the equivalence between the original sentence and the re-written sentence.

Solution:

Breaking down the given sentence into propositional logic as follows:

P : 7 is even

Q : Rahul Gandhi is the CM of Delhi

We can write the given sentence as : If P then Q

In propositional logic, we can write as $P \rightarrow Q$

Yes, the given statement is true in propositional logic.

The given sentences can be rewritten using disjunction and negation as follows : $\neg P \vee Q$

The truth table for both formats of the propositional logic is as follows :

| P | Q | $P \Rightarrow Q$ | $\neg P$ | $\neg Q$ | $\neg P \vee Q$ |
|-----|-----|-------------------|----------|----------|-----------------|
| T | T | T | F | F | T |
| T | F | F | F | T | F |
| F | T | T | T | F | T |
| F | F | T | T | T | T |

From the above truth table we can see $P \rightarrow Q \equiv \neg P \vee Q$

4. (3 points) Q4: Which of the following are correct?

- (a) $(A \Leftrightarrow B) \models (\neg A \vee B)$.
- (b) $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.
- (c) $False \models True$

Solution:

(a) Entailment: $KB \models Q$

We have seen that Q is entailed by KB (a set of premises or assumptions) if and only if there is no logically possible world in which Q is false while all the premises in KB are true. To determine if $(A \Leftrightarrow B) \models (\neg A \vee B)$ is correct, we need to check if the logical implication holds, which means that whenever $(A \Leftrightarrow B)$ is true, $(\neg A \vee B)$ must also be true. So, from above table we observe that this is **Correct**.

(b) Since, there is atleast one entry in truth table of $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is true, the compound preposition is satisfiable, Hence from above table, we observe that there are two values which are 1, so

| A | B | $A \Rightarrow B$ | $B \Rightarrow A$ | $\neg A$ | $A \Leftrightarrow B$ | $\neg A \vee B$ | $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ |
|-----|-----|-------------------|-------------------|----------|-----------------------|-----------------|--|
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |

this is correct.

(c) Recall that entailment is validated by a logical implication, i.e., if the *premise* is True, then the *conclusion* is True. However, if the premise is False, the statement, i.e., the logical implication, is still True, regardless of the truth of the conclusion. Therefore, the statement above is logically correct.

The following example (taken and modified from a Quora post) makes this point quite well.

If you look at the line that says F T T (where the column headings are A , C , and $A \Rightarrow C$), that is not actually saying that a false premise leads to a true conclusion. It is merely saying that the combination of a false premise and a true conclusion does not invalidate the argument that the premise implies the conclusion. The only way that particular argument is invalidated ($A \Rightarrow C$ is False) is if the premise is True, but the conclusion is False.

For example, consider the following implication: “This animal is a tiger (A) implies that this animal is a carnivore (C)”.

Now if we find a snake, which is a carnivore (False premise, True conclusion - F T T in the truth table), and try to claim that we have proven the above implication argument, we have not quite proven OR disproven anything. Similarly, if we find a cow (False premise, False conclusion - F F T in the truth table) and claim it disproves the implication argument because a cow is not a carnivore, again, we have not quite proven OR disproven anything.

In formal logic, unless the implication argument is proven to be False, it is held to be True (as in fact it is, i.e., tigers are indeed carnivores). The only way to prove that the implication argument is False is to find a tiger that is not a carnivore.

Therefore, when the premise is False, we have no way to disprove the implication argument, and hence it is taken to be true. Therefore the implication $False \Rightarrow True$ is indeed True, and so is the entailment: $False \models True$.

On the other hand, $True \Rightarrow False$ is a False or incorrect implication argument. Why?

5. (1 point) Show that all Horn clauses are implications.

Solution:

Horn clauses are defined as clauses (disjunction of literals) with at most one positive literal. For example, $(\neg A \vee \neg B \vee \neg C \vee D)$ or $(\neg A \vee \neg B \vee \neg C \vee \neg D)$ are Horn clauses. The former can also be written as $(\neg(A \wedge B \wedge C) \rightarrow D)$ and the latter can be written as $(\neg(A \wedge B \wedge C) \rightarrow \neg D)$ by recursively applying De Morgan’s law to the first three literals. Using the equivalence between $p \rightarrow q$ and $\neg p \vee q$, the former sentence becomes equivalent to the implication $((A \wedge B \wedge C) \rightarrow D)$ and the latter becomes equivalent to $((A \wedge B \wedge C) \rightarrow \neg D)$. We can have an arbitrary long Horn clause by replacing the literals A, B, C by A_1, A_2, A_3, \dots , and the expression will still simplify to an implication so long as there is at most one positive literal (D). Hence, all the Horn clauses are implications.