# MTH-204: Worksheet 5

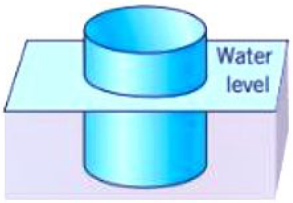
22 February, 2023

1. **Abel's formula.** Prove Abel's formula (2)

$$W(y_1(x), y_2(x)) = c\, e^{-\int_{x_0}^{x} p(t)\mathrm{d}t}$$

where $c = W(y_1(x_0), y_2(x_0))$. Apply it for $y_1(x) = e^{-x}cos\,\omega x$, $y_2(x) = e^{-x}sin\,\omega x$ and $x_0 = 0$.
*Hint:* Write $y'' + py' + qy = 0$ for $y_1$ and for $y_2$. Eliminate $q$ algebraically from these two ODEs, obtaining a first-order linear ODE. Solve it.

2. **Archimedian principle.** This principle states that the buoyancy force equals the weight of the water displaced by the body (partly or totally submerged). The cylindrical buoy of diameter 30 cm in the figure below is floating in water with its axis vertical. When depressed downward in the water and released, it vibrates with period 3 sec. What is its weight? (2)




3. (a) **Coefficient formulas.** Show how $a$ and $b$ in (1)

$$y'' + ay' + by = 0 \qquad \text{(A)}$$

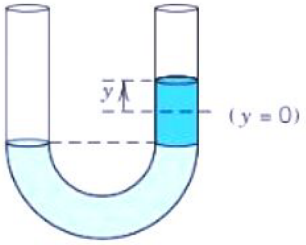
can be expressed in terms of $\lambda_1$ and $\lambda_2$. Explain how these formulas can be used in constructing equations for given bases.

(b) **Finding ODE.** Find an ODE of the form (A) for the given bases using above formula. (1)

1. $e^{2.6x},\ e^{-4.3x}$
2. $e^{-3.1x}cos\,2.1x,\ e^{-3.1x}sin\,2.1x$

4. **Shock absorber.** What is the smallest value of the damping constant of a shock absorber in the suspension of a wheel of a car (consisting of a spring and an absorber) that will provide (theoretically) an oscillation-free ride if the mass of the car is 2500 $kg$ and the spring constant equals 2500 $kg/sec^2$? (1)

5. **Vibration of water in a tube.** If 3 litre of water (about 3.17 US quart) is vibrating up and down under the influence of gravitation in a U-shaped tube of diameter 4 cm, what is the frequency? Neglect friction. First guess. (2)




6. **Double root.** If $D^2+aD+bI$ has distinct roots $\mu$ and $\lambda$, show that a particular solution is $y = (e^{\mu x} - e^{\lambda x})/(\mu - \lambda)$. Obtain from this a solution $xe^{\lambda x}$ by letting $\mu \to \lambda$ and applying $l'Hôpital's\ rule$. (1)