MTH 201: QUIZ 3

It is given that $\Phi(.5) = .6915$, $\Phi(1.04) = .85$, $\Phi(1) = .8413$, $\Phi(2) = .9772$, $\Phi(.67) = .7486$ and $\Phi(3) = .9987$

Total: 100 points

1) PROFESSOR HIJBIJBIJ has N Golden Retreivers and they all look almost the same. To avoid confusion, Professor HIJBIJBIJ decides to put collars (with name of the dog) on their neck. Professor HIJBIJBIJ bought N collars, inscribed each dog's name , mixed the collars throughly and put them in a big bucket. He then asked his dogs to go to the bucket one by one, pick one collarr at random (by their mouth) and come to him. He then put the collar on the dog's neck with out looking at the name on the collar.

(a) Find the expected number of dogs that got their own collar.

(b) Find the variance of the number of dogs that got their own collar.

(Hint: Think in terms of sum of random variables)

[25 points]

2) (a) The number of mistakes made by a typesetter on any given page is a random variable having the Poisson distribution with parameter λ, independent for each page. A three-page article is prepared by one of four typesetters, for whom the values of λ are 1, 2, 3, and 4, respctively. The typesetter is selected randomly with uniform probability. Find the expected number of misprints throughout the article.

(b) Let X and Y are independent continuous random variables with CDFs $F_X(x)$ and $F_Y(y)$. Let $U = min(X, Y)$ and $V = max(X, Y)$. Find the joint CDF and joint PDF of U and V.

[40 points]

3) Using the Central Limit Theorem, estimate the probability that the number of heads in 10.000 independent tosses of a fair coin differs by less than 1 percent from 5000.

What is the probability that the number of heads will be greater than 5100.

[20 points]

4) Random variable Y has the moment generating function $\phi_Y(s) = \frac{1}{(1-s)}$. Random variable V has the moment generating function $\phi_V(s) = \frac{1}{(1-s)^4}$. Y and V are independent. Let $W = Y + V$.

(a) What are $E[Y]$, $E[Y^2]$, and $E[Y^3]$?

(b) What is $E[W^2]$?

$Y+V$

[15 points]