Quiz #2	**Name:** _____
Date : 21/02/2024	**Section:** _____
MTH204: ODEs/PDEs	**Maximum Time:** 20 Minutes
Semester: Winter 2024	**Maximum Marks:** 15

DO NOT SHOW ANY WORK HERE. JUST WRITE WHAT IS BEING ASKED. THERE IS NO STEP MARKING.

Problem 1. [2] For the eigenvalue 2 of the differential operator $D^2 - 4D + 6$, write a basis for the eigenspace.

$\{e^{2t}, te^{2t}\}$

Problem 2. [1] The motion of an undamped mass-spring system is described by the ODE

$$2y'' + 36y = \sin(\omega t), \quad \omega > 0.$$

For what value(s) of ω will the system exhibit resonance?

$\sqrt{18} = 3\sqrt{2}$

Problem 3. [1+3] Consider the following differential equation which models a damped mass-spring system

$$y'' + 14y' + 58y = 0.$$

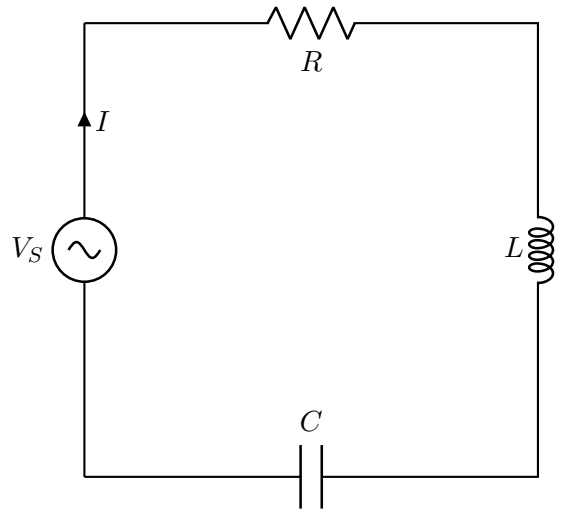
(a) Is the system overdamped, underdamped, or critically-damped?

Underdamped

(b) Suppose an external force $F(t) = e^{-3t}$ is applied to the system. Write the solution to the resulting non-homogeneous ODE that satisfies $y(0) = 0$ and $y'(0) = 0$.

$e^{-7t}\left(-\frac{1}{25}\cos(3t) - \frac{4}{75}\sin(3t)\right) + \frac{1}{25}e^{-3t}$

Problem 4. [1+4] Consider the RLC circuit shown below with $R = 0$, $L = 2$, $C = \frac{1}{18}$, and external voltage, $V_S = 6\tan(3t)$, all in SI units. Let $I(t)$ be the current at any time t after switching on the circuit at $t = 0$.



(a) Write a second order ODE for the current, $I(t)$, in circuit.

$$2I'' + 18I = 6\tan(3t)$$

(b) Find a particular solution for $I(t)$ from the ODE obtained in (a).

$$-\frac{\cos(3t)}{3}\ln\left(\sec(3t) + \tan(3t)\right)$$

Problem 5. [3] Write a particular solution of the ODE

$$y''' - 12y'' + 48y' - 64y = 12 - 32e^{-8t} + 2e^{4t}.$$

$$-\frac{3}{16} + \frac{1}{54}e^{-8t} + \frac{1}{3}t^3e^{4t}$$