Eco 311/511: Game Theory
Quiz 1

September 18, 2024

Instructions: Answer all questions. Copying/plagiarism will result in a score of 0. Max marks: 20. Time: 1 hr 15 minutes

1. Is continuity of preferences necessary for the existence of Nash equilibrium? Provide an argument in support of your answer. (4)

2. Consider two countries, 1 and 2 that are engaged in a military incident. The game proceeds as follows:

   - Country 1 chooses to either ignore the incident (I), resulting in maintenance of the status quo with payoffs $(0, 0)$, or to escalate the situation (E).
   - If country 1 escalates, country 2 can back down (B) resulting in payoffs of $(10, -10)$, or it can choose nuclear confrontation (N).
   - If country 2 chooses N, the players play a simultaneous move game in which they can either retreat (R for country 1, r for country 2) or choose Doomsday (D for country 1, d for country 2). If both retreat then they suffer a small loss due to the mobilization process and payoffs are (-5, -5), while if either party chooses Doomsday then the world destructs and payoffs are (-100, -100).

   Write down the extensive form representation of the above game and find the subgame perfect equilibria. Is the set of subgame perfect equilibria same as the set of pure strategy Nash equilibria? (2+2+2)

3. Two players (1 and 2) are involved in a dispute over an object. The value of the object to player $i \in \{1, 2\}$ is $v_i > 0$. Let $t = 0, 1, 2, 3, \ldots, \bar{t}$ denote the time periods, where $\bar{t} > max\{v_1, v_2\}$. Each player chooses when to concede the object to the other player; if the first player to

concede does so at time t_i, the other player obtains the object at that time. If both players concede in the same time period, no player gets the object. Time is valuable: until the first concession each player loses one unit of payoff per unit time onwards from $t = 1$. Formulate this situation as a strategic game. Write down the best response of the players. Does the game have any dominated strategies? Does the game have any pure strategy Nash equilibria? If yes, find the equilibria. If no, provide an explanation. (1+2+1+2)

4. Formulate the famous game *Rock Paper Scissors* as a two-player simultaneous move game in strategic (matrix) form. Does the game have a pure strategy Nash equilibrium? Show that the game has a mixed strategy Nash equilibrium. (4)