Worksheet #5
Date : 14/02/2024 **Name:** ____________________
MTH204: ODEs/PDEs
Semester: Winter 2024 **Section:** ____________________

Problem 1. If μ and λ are the distinct roots of the characteristic polynomial of the operator $L = D^2 + aD + bI$, show that a particular solution is

$$y = \frac{e^{\mu x} - e^{\lambda x}}{\mu - \lambda}$$

Problem 2.

(a) Find the ODE whose basis of solutions are the functions x^2 and $x^2 \ln(x)$.

(b) Show the linear independence of these two functions using wronskian for $x \in (0, \infty)$.

(c) Solve the initial value problem that satisfies $y(1) = 4$ and $y'(1) = 6$.

Problem 3. Solve

$$(D^2 + 6D + 9I)y = 16\frac{e^{-3x}}{x^2 + 1}$$

by variation of parameters.

Problem 4. Show that the functions 1, $e^{-x}\cos 2x$, $e^{-x}\sin 2x$ are solutions of

$$y''' + 2y'' + 5y' = 0$$

and form a basis on any interval.

Problem 5. Solve

$$(D^3 - 9D^2 + 27D - 27I)y = 27\sin 3x$$

Problem 6. Solve

$$x^3 y''' + xy' - y = x^2,$$

given that $y(1) = 1$, $y'(1) = 3$, $y''(1) = 14$.