Worksheet #1
Date : 17/01/2024 **Name:** ______________________
MTH204: ODEs/PDEs
Semester: Winter 2024 **Section:** ______________________

Problem 1. Solve following ODEs using separation of variables.

(a)
$$y' + xe^{-x^2/2} = 0,$$

(b)
$$y' = 4e^{-x}\cos x.$$

Problem 2.

(a) Verify that $y^2 - 4x^2 = C$ is a solution of the ODE
$$yy' = 4x.$$

(b) Determine from y the particular solution of the ODE that satisfies the initial condition $y(1) = 4$.

(c) Graph the solution of the IVP.

Problem 3. An ODE may sometimes have an additional solution that cannot be obtained from the general solution and is then called a **singular solution**. The ODE
$$(y')^2 - xy' + y = 0$$
is of this kind. Show by differentiation and substitution that it has the general solution $y = cx - c^2$ and the singular solution $y = \frac{1}{4}x^2$.

Problem 4. Radium Ra^{228}_{88} has a half-life of about 3.6 days.

(a) Given 1 gram, how much will still be present after 1 day?

(b) After 1 year?

Problem 5. Graph a direction field by hand for the ODE
$$y' = 2y - y^2$$
in an integer grid centered at $(0, 0)$ of size 2.

Problem 6. Apply Euler's method to the ODE
$$y' = y$$
with $h = 0.1$ and $y(0) = 1$ and find three iterations.

Problem 7. Find the general solution of

$$y^3 y' + x^3 = 0.$$

Problem 8. The Gompertz model is $y' = -Ay\log(y)$ $(A > 0)$, where $y(t)$ is the mass of tumor cells at time t. The model agrees well with clinical observations. The declining growth rate with increasing $y > 1$ corresponds to the fact that cells in the interior of a tumor may die because of insufficient oxygen and nutrients. Solve the ODE.