Discrete Mathematics CSE 121 : Homework 6

In every proof/derivation clearly state your assumptions and give details of each step.

1. Prove the correctness of Prim's algorithm.

2. A spanning forest of a graph G is a forest that contains every vertex of G such that two vertices are in the same tree of the forest when there is a path in G between these two vertices. Devise an algorithm for constructing the spanning forest of a graph based on depth-first searching.

3. The complete m-partite graph $K_{n_1,n_2,...,n_m}$ has vertices partitioned into m subsets of $n_1, n_2, ..., n_m$ elements each, and vertices are adjacent if and only if they are in different subsets in the partition. Draw $K_{2,2,2}$. How many vertices and how many edges does the complete m-partite graph $K_{n_1,n_2,...,n_m}$ have?

4. Show that if G is a graph with n vertices, then no more than $n/2$ edges can be colored the same in an edge coloring of G.

5. A connected graph G is called chromatically k-critical if the chromatic number of G is k, but for every edge of G, the chromatic number of the graph obtained by deleting this edge from G is $k-1$. Show that W_n, the wheel graph on $n$vertices, is chromatically 4-critical whenever n is an odd integer, $n \geq 3$.