Quiz 6 (Solutions).

Sol1: →

(a) $x[n] = \sin\left[\frac{\pi(n-1)}{4}\right]$

To find the period, $x[n] = x[n+N]$

$\sin\left[\frac{\pi(n-1)}{4}\right] = \sin\left[\frac{\pi(n+N-1)}{4}\right] = \sin\left[\frac{\pi(n-1)}{4} + \frac{\pi N}{4}\right]$

So, $\frac{\pi N}{4} = 2\pi i$, when i is integer.

1 mark

Thus, $\boxed{N = 8}$

(b)

Now, $x[n] = \frac{1}{2j} e^{j[\pi(n-1)/4]} - \frac{1}{2j} e^{-j[\pi(n-1)/4]}$

$= \frac{1}{2j} e^{-j(\pi/4)} e^{j(\pi n/4)} - \frac{1}{2j} e^{j\pi/4} e^{-j(\pi n/4)}$

2 mark

$\therefore \quad a_1 = \frac{e^{-j(\pi/4)}}{2j}, \quad a_7 = -\frac{e^{j(\pi/4)}}{2j}$

2 mark

All other coefficients a_k are zero, in the range $0 \leq k \leq 7$.

1 mark.

Sol 2.

(ii) $x(n) \xleftrightarrow{FS} a_k$.

$x[n] - x[n-1] \longleftrightarrow a_k\left(1 - e^{-j(2\pi k/N)}\right]$. } 2 marks

(b) $x^*[-n]$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x^*[-n]e^{-jk(2\pi/N)n}.$$

$$a_k^* = \frac{1}{N}\sum_{n=0}^{N-1} x[-n]\,e^{jk(2\pi/N)n}.$$

2 marks

$$= \frac{1}{N}\sum_{n=0}^{-N+1} x[n]e^{-jk(2\pi/N)n} = a_k.$$

$\therefore \boxed{a_k = a_k^*}$

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