MTH100 **Monsoon Semester – 2021-22** **Section B** 1

Submission for Tuesday 8th February 2022 – 15 minutes. Max Marks: 5

Instructions: Open notes and textbook; consultation and use of calculators, computers and internet not allowed.

IMPORTANT: You may use any **known** result. This includes all propositions and observations in the lecture slides, and results from tutorials. If you use any other result from any other source, including the textbook, you have to give a full proof of that result.

a) Construct an LU Factorization of the matrix A given below. (**Remark: Show all your steps clearly, with brief explanations. Else, you will not be given credit.**) *(5 marks)*

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} \quad a \neq 0,\ b \neq a,\ c \neq b,\ d \neq c.$$

SOLUTION-CUM-RUBRIC

$$A = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}}_{U}$$

Rubric → U → correct 1 mark; L correct ⇒ 1 mark

Marks for steps – ONLY TO BE GIVEN IF BOTH L and U ~~are~~ are correct.

Steps For U → 1.5 marks

Steps for L → 1.5 marks. Any method can be used for L, but must be stated.

CALCULATIONS

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} \xrightarrow[i=2,3,4]{R_i \to R_i - R_1} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix}$$

$$\xrightarrow[i=3,4]{R_i \to R_i - R_2} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{bmatrix} \xrightarrow[R_4 - R_3]{R_4 \to}$$

$$\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-d \\ 0 & 0 & 0 & d-c \end{bmatrix} = U$$

Inserting $(-1) \times$ the concerned factor in i-j-th position of I_4, we get $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Remark: Any method: either the theory method or short-cut, but must be <u>briefly explained</u>.

Check: $LU =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$$

$$= \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} \checkmark$$