MTH204: Worksheet 4

February 15, 2023

1. **Cauchy-Riemann equations.** Show that for a family $u(x, y) = c$ = constant the orthogonal trajectories $v(x, y) = c^*$ = constant can be obtained from the following $Cauchy - Riemann\ equations$ (which are basic in complex analysis) and use them to find the orthogonal trajectories of $e^x \sin y$ = constant. (Here, subscripts denote partial derivatives.)

$$u_x = v_y,\ u_y = -v_x.$$

(2)

2. **Congruent OTs.** If $y' = f(x)$ with f independent of y, show that the curves of the corresponding family are congruent, and so are their OTs. **(1)**

3. **Temperature field.** Let the isotherms (curves of constant temperature) in a body in the upper half-plane $y > 0$ be given by $25x^2 + 36y^2 = c$. Find the orthogonal trajectories (the curves along which heat will flow in regions filled with heat-conducting material and free of heat sources or heat sinks). **(1)**

4. **Motion.** In a straight-line motion, let the velocity be the reciprocal of the acceleration. Find the distance $y(t)$ for arbitrary initial position and velocity. **(1)**

5. **Existence?** Does the initial value problem

$$(x - 5)y' = y,\ y(5) = a$$

have a solution? Does your result contradict our present theorems? **(2)**

6. **Length of x-interval.** In most cases the solution of an initial value problem $y' = f(x, y),\ y(x_0) = y_0$ exists in an x-interval larger than that guaranteed by the present theorems. Show this fact for $y' = 3y^3,\ y(1) = 1$ by finding the best possible a (choosing b optimally) and comparing the result with the actual solution. **(2)**

7. **Hanging cable.** It can be shown that the curve $y(x)$ of an inextensible flexible homogeneous cable hanging between two fixed points is obtained by solving

$$y'' = k\sqrt{1 + (y')^2},$$

where the constant k depends on the weight. This curve is called catenary (from Latin catena = the chain). Find and graph $y(x)$, assuming that $k = 1$ and those fixed points are $(-1, 0)$ and $(1, 0)$ in a vertical xy-plane. **(1)**