Time: 15 minutes. Max marks: 10

Name: **Roll No.:**

Instructions:
- Do not plagiarize. Do not assist your classmates in plagiarism.
- Show your full solution for the questions to get full credit.
- Attempt all questions that you can.
- A statement is true if it is *always* true. MCQs may have multiple correct answers.
- In the unlikely case a question is not clear, discuss it with an invigilator. Please ensure that you clearly include any assumptions you make, even after clarification from the invigilator.

1. ($5 \times 1 = 5$ points) Answer True or False and provide the justification.

   (a) If $\mathbf{T}$ and $\mathbf{R}$ are the 3D translation and rotation transformations (i.e., the 4×4 matrix) for the world-to-camera transformation (extrinsic parameters), we can obtain the extrinsic matrix $\mathbf{T}_{ext} = \mathbf{R} \cdot \mathbf{T} = \mathbf{T} \cdot \mathbf{R}$ by multiplying $\mathbf{R}$ and $\mathbf{T}$ in any order.
   Solution: False. Matrix multiplication is in general not commutative, and the translation & rotation transformations do not form a special case, unlike say scaling and rotation, or two consecutive rotations.

   (b) Parallel lines intersect at *points at infinity*, which have finite values in homogeneous coordinates.
   Solution: True. $[x, y, 0]$ is the finite valued homogeneous coordinates of points at infinity.

   (c) The homogeneous coordinates represent points in a projective plane as an equivalence class (i.e., a set whose elements are mapped to the same entity) of vectors. If true, justify by writing the set of equivalent vectors, else justify why these are not a set of vectors.
   Solution: True. Each point is a scaled version of the canonical homogeneous coordinates, which in-turn is obtained by appending a '1' to the corresponding cartesian coordinates of the points.

   (d) Lines are represented in homogeneous coordinates using the equation of an *arbitrary* plane passing through the said line.
   Solution: False. The plane passing through the line should also be passing through the origin.

   (e) If θ is the angle of rotation in a 3D rotation, the trace of the 3D rotation matrix would be $1+2\cos\theta$.

   Solution: True. Using the Rodriguez's formula, if you add the traces (trace is a linear operator) of the constituent matrices, i.e., $\mathbf{I}$, $\mathbf{N}$, and $\mathbf{N}^2$, where $\mathbf{I}$ is the identity matrix and $\mathbf{N}$ is the skew-symmetric matrix for the axis vector $\widehat{\mathbf{n}}$. Rodriguez's formula is:

$$\mathbf{R} = \mathbf{I} + \sin\theta\mathbf{N} + (1-\cos\theta)\mathbf{N}^2$$

$$\begin{aligned}
\mathbf{N} &= \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix} \\
\mathbf{N}^2 &= \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix} = \begin{bmatrix} -n_3^2 - n_2^2 & & \\ & -n_3^2 - n_1^2 & \\ & & -n_1^2 - n_2^2 \end{bmatrix} \\
Tr(\mathbf{I}) &= 3 \\
Tr(\mathbf{N}) &= 0 \\
Tr(\mathbf{N}^2) &= -n_3^2 - n_2^2 - n_3^2 - n_1^2 - n_1^2 - n_2^2 \\
Tr(\mathbf{R}) &= Tr(\mathbf{I}) + Tr(\mathbf{N})\sin\theta + Tr(\mathbf{N}^2)(1-\cos\theta) \\
&= 3 + 0 + (1-\cos\theta)(n_3^2 + n_2^2 + n_1^2)(-2) \\
&= 3 - 2 + 2\cos\theta \qquad = 1 + 2\cos\theta
\end{aligned}$$

since $||\widehat{\mathbf{n}}|| = (n_1^2 + n_2^2 + n_3^2) = 1$.

Partial credit for a reasonable justification instead of a rigorous proof as to why $(1 + 2\cos\theta)$ would be the trace of any rotation matrix.

2. ($5 \times 1 = 5$ points) Check ***all*** the correct answers. {*Rubric*: If any incorrect answer is checked, no credit is awarded}.

   (a) How many degrees of freedom (dof) does the extrinsic parameter matrix in the image formation pipeline have?
   (A) 1 (B) 3 (C) 4 (D) 6 (E) None of the above
   Solution: (D) The extrinsic matrix has 6 dof - 3 for 3D rotation and 3 for 3D translation.

   (b) If the camera and the world coordinate systems are identical, what would be the dof of the extrinsic parameter matrix?
   (A) 1 (B) 3 (C) 4 (D) 6 (E) None of the above
   Solution: (E) It will be zero as the world-to-camera transformation will be identity when the world and camera coordinate systems are identical.

   (c) If the camera sensor has pixels as perfect squares, what would be the dof of the intrinsic camera matrix?
   (A) 1 (B) 3 (C) 4 (D) 6 (E) None of the above
   Solution: (B) - one for focal length and two for principal point; aspect ratio and the skew parameter are fixed to 1 and 0 respectively.

   (d) If the axis of rotation is known, how many dof does the corresponding 3D rotation representation have?
   (A) 1 (B) 3 (C) 4 (D) 6 (E) None of the above
   Solution: (A) - 3D rotation has 3 dof, of which two are due to the unknown axis of rotation. Once the axis is known, there is only 1 unknown, i.e., the angle of rotation.

   (e) Let θ be the angle of rotation corresponding to the rotation matrix $\mathbf{R}$. Which of the following is an eigenvalue of $\mathbf{R}$? (*Note*: $\det \mathbf{R}$ is the determinant of $\mathbf{R}$)
   (A) 1 (B) 3 (C) $\cos\theta$ (D) $\sin\theta$ (E) $\det \mathbf{R}$
   Solution: (A,E) - Rotation matrices always have an eigenvalue equal to 1. Also for rotation matrices, $\det \mathbf{R} = 1$ which also turns out to be equal to the eigenvalue. {*Rubric*: Credit to be given if only (A) or both (A,E) are marked. No credit if only (E) is marked. No credit if (C,D) are marked.}