Quiz-2

Date : / /

(1) Let x is number of sloths that Sasha can have Then-

$$x \equiv 4 \pmod{5}$$
$$x \equiv 6 \pmod{8}$$
$$x \equiv 8 \pmod{9}$$

$$N = 5 \cdot 8 \cdot 9 = 360 \qquad n_1 = 5, \quad n_2 = 8, \ n_3 = 9$$

$$m_1 = \frac{N}{n_1} = 72$$

$$m_2 = \frac{N}{n_2} = 45$$

$$m_3 = \frac{N}{n_3} = 40$$

By CRT-

$$x = \sum_{i=1}^{3} b_i a_i m_i$$

$$= b_1 \times 4 \times 72 + b_2 \cdot 6 \times 45 + b_3 \cdot 8 \cdot 40$$

where-

$$72 b_1 \equiv 1 \pmod{5}$$
$$45 b_2 \equiv 1 \pmod{8}$$
$$40 b_3 \equiv 1 \pmod{9}$$

or-

$$2 b_1 \equiv 1 \pmod{5} \Rightarrow b_1 = 3$$
$$5 b_2 \equiv 1 \pmod{8} \Rightarrow b_2 = 5$$
$$4 b_3 \equiv 1 \pmod{9} \qquad b_3 = 7$$

$$x = 3 \times 4 \times 72 + 5 \times 6 \times 45 + 7 \times 8 \times 40$$
$$x = 864 + 1350 + 2440$$
$$x = 4454$$
$$x \equiv 134 \pmod{360}$$

$\Rightarrow$ Smallest numbers of sloths $= 134$

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(2) $\quad 113$ is a prime.

and $(47, 113) = 1$

By Farmat's theorem.

$$(47)^{112} \equiv 1 \pmod{113}$$

$$(47)^{2\cdot 112} \equiv 1 \pmod{113}$$

$$47^{224} = 47^{222}\cdot 47^2 = 2209 \cdot 47^{222}$$

$$\Rightarrow (47)^{224} \equiv 62\cdot(47)^{222} \pmod{113}$$

$$\Rightarrow (47)^{222} \equiv (62)^{-1} \pmod{113}$$

$$\Rightarrow (47)^{222} \equiv 31 \pmod{113}$$

We can write

$113 = 62\cdot(1) + 51 \quad \Rightarrow 51 = 113 - 62\cdot 1$

$62 = 51\cdot(1) + 11 \quad \Rightarrow 11 = 62 - 51\cdot 1$

$51 = 11\cdot(4) + 7 \quad \Rightarrow 7 = 51 - 4\cdot 11$

$11 = 7(1) + 4 \quad \Rightarrow 4 = 11 - 7\cdot 1$

$7 = 4\cdot(1) + 3 \quad \Rightarrow 3 = 7 - 4\cdot 1$

$4 = 3\cdot(1) + 1 \quad \Rightarrow 1 = 4 - 3\cdot 1$

or- $\quad 4 + 3(-1) = 1$

$7 - 4\cdot 1 = 3$

$\Rightarrow 4 - (7-4) = 1$

$\Rightarrow 1 = 2\cdot 4 - 7$

$1 = 2\cdot(11 - 7) - 7$

$1 = 2\cdot 11 - 3\cdot 7$

$1 = 2\cdot 11 - 3(51 - 4\cdot 11)$

$1 = 14\cdot 11 - 3\cdot 51$

$1 = 14\cdot(62 - 51\cdot 1) - 3\cdot 51$

$1 = 14\cdot 62 - 17\cdot 51$

$1 = 14\cdot 62 - 17(113 - 62)$

$1 = 31\cdot 62 - 17\cdot 113$

$$\Rightarrow (62)^{-1} \equiv 31 \pmod{113}$$