MTH100 Monsoon Semester – 2021-22 Section B

Submission for Tuesday 25th January 2022 – 15 minutes. Max Marks: 5

Instructions: Open notes and textbook; consultation and use of calculators, computers and internet not allowed.

a) Find the RREF matrix of the matrix A below. **YOUR STEPS MUST BE SHOWN.** Otherwise, you will not get credit. *(3 mark*

b) Suppose that A is actually the augmented matrix [B:**c**] for a nonhomogeneous system B**x** = **c**. Either find the general solution of the system in vector form OR state that the system is inconsistent and explain why. *(2 mark*

$$A = \begin{bmatrix} 2 & 3 & 1 & 4 & 0 \\ 3 & 1 & 2 & 1 & -1 \\ 5 & 4 & 3 & 6 & -1 \end{bmatrix}$$

SOLUTION-CUM-RUBRIC

a) The RREF matrix of A is R =

$$\begin{bmatrix} 1 & 0 & 5/7 & 0 & -3/7 \\ 0 & 1 & -1/7 & 0 & 2/7 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

RUBRIC:– R correct $\longrightarrow$ 1 mark

Steps shown correctly $\longrightarrow$ 2 marks

NB: If R is incorrect, no marks for steps. In this case, 0 marks awarded.

See next page for steps; this is not the only possible correct sequence of steps.

(b) The system is consistent. Hence, the general solution in vector form is:–

$$\bar{x} = \begin{bmatrix} -3/7 \\ 2/7 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -5/7 \\ 1/7 \\ 1 \\ 0 \end{bmatrix} = \bar{u} + x_3 \bar{v},$$

See next page for steps.

where $\bar{u}$ is a particular solution and $\bar{v}$ is a solution of the associated homogeneous system $B\bar{x} = \bar{0}$.

Rubric $\rightarrow$ $\bar{u}$ correct 1 mark; $\bar{v}$ correct 1 mark

a) $A = \begin{bmatrix} 2 & 3 & 1 & 4 & 0 \\ 3 & 1 & 2 & 1 & -1 \\ 5 & 4 & 3 & 6 & -1 \end{bmatrix}$

$\xrightarrow[R_2 - R_1]{R_2 \to} \begin{bmatrix} 2 & 3 & 1 & 4 & 0 \\ 1 & -2 & 1 & -3 & -1 \\ 5 & 4 & 3 & 6 & -1 \end{bmatrix}$

$\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -2 & 1 & -3 & -1 \\ 2 & 3 & 1 & 4 & 0 \\ 5 & 4 & 3 & 6 & -1 \end{bmatrix}$

$\xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & -2 & 1 & -3 & -1 \\ 0 & 7 & -1 & 10 & 2 \\ 0 & 14 & -2 & 21 & 4 \end{bmatrix}$

$\xrightarrow[R_3 - 2R_2]{R_3 \to} \begin{bmatrix} 1 & -2 & 1 & -3 & -1 \\ 0 & 7 & -1 & 10 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$\xrightarrow[R_2 \to R_2 - 10R_3]{R_1 \to R_1 + 3R_3} \begin{bmatrix} 1 & -2 & 1 & 0 & -1 \\ 0 & 7 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$\xrightarrow{R_2 \to \frac{1}{7}R_2} \begin{bmatrix} 1 & -2 & 1 & 0 & -1 \\ 0 & 1 & -\frac{1}{7} & 0 & \frac{2}{7} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$\xrightarrow[2R_2]{R_1 \to R_1 +} \begin{bmatrix} 1 & 0 & \frac{5}{7} & 0 & -\frac{3}{7} \\ 0 & 1 & -1/7 & 0 & 2/7 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$= R$

b) The RREF matrix of the augmented matrix $[B : \bar{c}]$ is R above. Converting it to a linear system with only basic variables on LHS, and inserting a dummy equation, we get:

$$x_1 = -\frac{3}{7} - \frac{5}{7}x_3$$

$$x_2 = \frac{2}{7} + \frac{1}{7}x_3$$

$$x_3 = 0 + x_3$$

$$x_4 = 0 + 0x_3$$

OR

$$\bar{x} = \begin{bmatrix} -\frac{3}{7} \\ \frac{2}{7} \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -5/7 \\ 1/7 \\ 1 \\ 0 \end{bmatrix}$$

$$= \bar{u} + x_3 \bar{v}$$

Check:

$B\bar{u} = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 3 & 1 & 2 & 1 \\ 5 & 4 & 3 & 6 \end{bmatrix} \begin{bmatrix} -3/7 \\ 2/7 \\ 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = \bar{c}$, as required.

$B\bar{v} = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 3 & 1 & 2 & 1 \\ 5 & 4 & 3 & 6 \end{bmatrix} \begin{bmatrix} -\frac{5}{7} \\ \frac{1}{7} \\ 1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \bar{0}$, as required.