Worksheet-11
Course Name: Math-III (Section-A)
Total marks = 20
Date: 7/12/2022

1. Use Green's theorem to find the counterclockwise circulation and outward flux for the field $\mathbf{F} = (tan^{-1}\frac{y}{x})\hat{i} + \ln(x^2 + y^2)\hat{j}$ and the curve C: The boundary of the region defined by the polar coordinate inequalities $1 \leq r \leq 2, 0 \leq \theta \leq \pi$. (5+5=10 marks)

2. First parameterized the cylindrical surface $y^2 + z^2 = 4, z \geq 0, 1 \leq x \leq 4$ and then evaluate the integral of the function $\mathbf{F}(x, y, z) = z$ over the parameterized cylindrical surface. (5+5=10 marks)

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Rubric + solution for Worksheet - 11.

Q.1. Given $F = \left(\tan^{-1}\frac{y}{x}\right)\hat{i} + \ln(x^2+y^2)\,\hat{j}$

So, $M = \tan^{-1}\frac{y}{x}$, $N = \ln(x^2+y^2)$

2 $\quad \frac{\partial M}{\partial x} = \frac{-y}{x^2+y^2}$, $\frac{\partial M}{\partial y} = \frac{x}{x^2+y^2}$, $\frac{\partial N}{\partial x} = \frac{2x}{x^2+y^2}$, $\frac{\partial N}{\partial y} = \frac{2y}{x^2+y^2}$

$\therefore$ Using Green's theorem, we have,

$$\text{Flux} = \iint_R \left(\frac{-y}{x^2+y^2} + \frac{2y}{x^2+y^2}\right) dx\,dy.$$

$$= \iint_R \frac{y}{x^2+y^2}\, dx\,dy.$$

$$= \int_0^{\pi}\int_1^2 \frac{r\sin\theta}{r^2}\cdot r\,dr\,d\theta.$$

4

$$= \int_0^{\pi}\int_1^2 \sin\theta\, dr\,d\theta.$$

$$= \left[-\cos\theta\right]_0^{\pi} = 2.$$

$$\text{circulation} = \iint_R \left(\frac{2x}{x^2+y^2} - \frac{x}{x^2+y^2}\right) dx\,dy.$$

$$= \iint_R \frac{x}{x^2+y^2}\, dx\,dy.$$

4

$$= \int_0^{\pi}\int_1^2 \frac{r\cos\theta}{r^2}\, r\,dr\,d\theta.$$

$$= \left[\sin\theta\right]_0^{\pi} = 0.$$

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Q.2. Let, the parametrization be

$$\vec{r}(x,y) = x\hat{i} + y\hat{j} + z\hat{k}$$

(2) But, $y^2 + z^2 = 4 \Rightarrow z = \sqrt{4-y^2}$

$$\therefore \vec{r}(x,y) = x\hat{i} + y\hat{j} + \sqrt{4-y^2}\,\hat{k}$$

(2) $\therefore \vec{r}_x = \hat{i}\ , \quad \vec{r}_y = \hat{j} - \dfrac{y}{\sqrt{4-y^2}}\hat{k}$.

$$\therefore \vec{r}_x \times \vec{r}_y = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & -\frac{y}{\sqrt{4-y^2}} \end{vmatrix}$$

(2)

$$= \frac{y}{\sqrt{4-y^2}}\hat{j} + \hat{k}$$

(2) $\therefore |\vec{r}_x \times \vec{r}_y| = \sqrt{\dfrac{y^2}{4-y^2} + 1} = \dfrac{2}{\sqrt{4-y^2}}$

$$\therefore \iint_S f(x,y,z)\,dS = \int_1^4 \int_{-2}^{2} z \cdot \frac{2}{\sqrt{4-y^2}}\,dx\,dy.$$

(2)

$$= \int_1^4 \int_{-2}^{2} \sqrt{4-y^2}\,\frac{2}{\sqrt{4-y^2}}\,dx\,dy.$$

$$= 2 \times (4-1)(2+2) = 12 \times 2$$

$$= 24.$$

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