ECE 634/CSE 646 InT: Assignment 1

Instructor: Manuj Mukherjee

Total: 10 points

1) *Han's inequality:* Prove that for any collection of discrete random variables $X_1, X_2, \ldots, X_n$,

$$H(X^n) \leq \frac{1}{n-1} \sum_{i=1}^{n} H(X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n).$$

[**Hint:** First show that for any $1 \leq i \leq n$, $H(X^n) \leq H(X_1, X_2, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n) + H(X_i|X^{i-1})$.]

[5 points]

2) *Submodularity of entropy:* Let T be any finite set of indices and $(X_t : t \in T)$ be a collection of jointly distributed random variables. For any $S \subseteq T$, we define the notation $X_S \triangleq (X_s : s \in S)$. Then, for any $S_1, S_2 \subseteq T$, show that

$$H(X_{S_1}) + H(X_{S_2}) \geq H(X_{S_1 \cup S_2}) + H(X_{S_1 \cap S_2}).$$

[**Hint:** Note that $S_2 = (S_2 \setminus S_1) \cup (S_2 \cap S_1)$ and this is a union of two disjoint sets. Accordingly, $H(X_{S_2})$ can be expanded using the chain rule.]

[5 points]