ECE 634/CSE 646 InT: Practice Problems 2

Instructor: Manuj Mukherjee

1) Let $X = (X_1, X_2, \ldots, X_n)$ be n independent and uniform bits.

  a) Show that $H(X) = n$.

  b) Let A be an $n \times n$ binary matrix. Let $Y = AX$, where the matrix multipliation is in modulo 2. Show that $H(Y) = \text{rank}(A)$.

2) Suppose a person has access to two coins with respective probability of heads being p and q. The person chooses one of these coins uniformly at random, and begins tossing them. The person records the number of the first toss that yielded heads and passes this number to you. Suppose you use the optimal hypothesis test to declare which coin the person chose based on the number supplied to you. Calculate an upper bound to the probability of error of your hypothesis test.

3) Let $X_i, i \geq 1$ constitute a time-homogenous[1] discrete time Markov chain, where X_is take values in $\{0, 1\}$, and $\mathrm{P}(X_2 = l | X_1 = l) = \frac{2}{3}, l = 0, 1$. Given that X_1 is a uniformly random bit, find $H(X_n | X_{n-1})$.

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[1]Time-homogenous means that that the distribution $\mathrm{P}_{X_n | X_{n-1}}$ does not depend on n