MTH210 – SUBMISSION_20221201

TIME: 15 minutes MARKS: 5

No consultation – open notes – books and internet not allowed.

The k-cube (or hypercube) is the graph Q_k, whose vertices are ordered k-tuples (for k ≥ 1) with entries from {0,1}, in which two vertices are adjacent if and only if they differ in exactly one position.

a. Find the number of vertices in the k-cube. (0.5 marks)

b. Find the degrees of the vertices of the k-cube. (1 mark)

c. Find the number of edges in the k-cube. (1 mark)

d. Is the k-cube bipartite (YES/NO) ? (2.5 marks)

For b., c., and d., justify your answer briefly.

RUBRIC: Award marks as follows:

- a. → 0.5 marks for the correct numerical answer, namely $\mathbf{2^k}$.
- b. → 0.5 marks for the correct numerical answer, namely **k**, and 0.5 marks for the reasoning.
- c. → 0.5 marks for the correct numerical answer, namely $\mathbf{k2^{k-1}}$, and 0.5 marks for the reasoning.
- d. → 0.5 marks for the correct answer YES, and 2 marks for the correct justification. A confused version, which however captures the idea correctly, may be given partial credit of 0.5 marks out of the 2 marks available. This is an easy answer, and by now students should be able to write clear answers using mathematical wording as far as possible.

MTH210 – SUBMISSION_EXTRA

Consider the statement A:
The equation $x^n + y^n = z^n$ has a solution in which x, y, z are positive integers, and n is a positive integer > 2.

a. ***Express A as a statement S in predicate logic, i.e. using a propositional function P with suitable variables and quantifiers. You must explicitly specify P and the variables required, along with the domain of discourse for each variable. (3 marks)***
b. ***Is S a proposition (YES/NO) ? Justify. (1 mark)***
c. ***If YES, is it TRUE or FALSE ? Justify in exactly one sentence. (1 mark)***

RUBRIC: Award marks as follows:

- a. $\rightarrow$ 0.5 marks for defining P correctly, $\rightarrow$ 0.5 marks for defining all four domains of discourse correctly, $\rightarrow$ 2 marks for the final statement S correctly.
- b. $\rightarrow$ 0.5 marks for YES, $\rightarrow$ 0.5 marks for the reasoning.
- c. $\rightarrow$ 0.5 marks for FALSE, $\rightarrow$ 0.5 marks for the reasoning.

Note: Binary marking for this submission – no partial credit.