(1) Find all incongruent solutions.

a) $x^2 \equiv 23 \mod 77$

b) $x^2 \equiv 11 \mod 39$

(2) T/F The linear congruence $124x \equiv 10 \pmod{1040}$ has a solution.

(3) Determine if the system has a solution.

$$x \equiv 1 \pmod 5$$
$$x \equiv 3 \pmod 6$$
$$4x \equiv 2 \pmod{14}$$

(4) Find $\gcd(587, 345)$ using Euclidean algorithm.

(5) T/F If $a \equiv b \pmod m$ & $c \neq 0$, then $ac \equiv bc \pmod{mc}$

(6) Prove if p is prime & $p \mid a^3$, then $p \mid a$.

(7) If $a \equiv b \pmod m$ & $d = \gcd(a,b)$, then (Prove)

$$\frac{a}{d} \equiv \frac{b}{d} \mod m$$

(8) T/F If $d = \gcd(x,y)$, then $d = \gcd(ax + by, y)$ for all integers $a, b > 0$.

(9) T/F The linear congruence

$$124x \equiv 10 \pmod{1040}$$ has a solⁿ.

(10) T/F If p_1, p_2, p_3 are consecutive prime nos. Then $2p_1p_2p_3 + 1$ is a prime no.

(11) Let p be an odd prime. Find a formula for the Legendre symbol $\left(\frac{-2}{p}\right)$

(12) Find the least non-negative residue of

3^{2011} modulo 22.

(13) Let $n \geq 1$. Find $\gcd(n, 2n^2+1)$

(14) Let $p \equiv 1 \mod 4$ be a prime & a is a quadratic residue mod p. Decide with justification if then automatically $p - a$ is a quadratic residue mod p.

(15) Calculate $\phi(7!)$

(16) Let $f(x) = 2x^4 + 3x^2 + x$ and $g(x) = 5x^2 + x + 3 \in \mathbb{Z}_{11}[x]$. $= \{ k(x)$ with coefficients $\in$ mod 11$\}$

Determine polynomials $q(x), r(x) \in \mathbb{Z}_{11}[x]$ with $\deg r < \deg g$ such that $f = gq + r$.

Write the coefficient of q, r in reduced form modulo 11.

(17) Determine all solutions of the congruence

$$f(x) = x^2 + 3 \equiv 0 \pmod{7^e} \text{ for } e = 1, 2, 3.$$

(18) Determine all solutions of the congruence

$$x^3 + 2x + 1 \equiv 0 \pmod{5^4}.$$

(19) Determine all solutions of the congruence

$$2x^2 + 3x + 2 \equiv 0 \pmod{7^2}.$$

(20) Determine an integer x with $0 \leq x < 88$ s.t

$$x \equiv 9^{1283} \pmod{88}$$

(21) Determine all solutions of

$$x^2 - 4x + 9 \equiv 0 \pmod{25}.$$

(22) Compute the Legendre symbols

$$\left(\frac{21}{53}\right) \quad \left(\frac{143}{409}\right). \quad \left(\frac{46}{83}\right) \quad \left(\frac{2}{127}\right) \left(\frac{11}{127}\right)$$

(23) State the law of quadratic reciprocity.

(24) " Euler's thm.

(25) " Fermat's thm.

(26) Prove $n^{13} \equiv n \mod 2730$ for any integer n.

(27) Prove that if m is an odd positive integer, then the sum of any complete set of residues modulo m is $0 \pmod{m}$.

(28) If m is any integer & $m > 2$, then prove the analogous result

as (27) for any reduced system of residues modulo m.

(29) Solve $11x \equiv 21 \pmod{105}$

(30) Solve the simultaneous system

$$x \equiv 3 \pmod 6$$
$$x \equiv 5 \pmod{35}$$
$$x \equiv 7 \pmod{143}$$
$$x \equiv 11 \pmod{323}$$

(31) Show that $61! + 1 \equiv 63! + 1 \equiv 0 \pmod{71}$.

(32) Let p be an odd prime no. The numbers
$1, 2^2, 3^2, \ldots, \left(\frac{p-1}{2}\right)^2$ are distinct modulo p.
Prove this.

(33) The above set of nos. in (32) are distinct modulo p & give a complete set of (non zero) quadratic residues mod p.
Use this to find a complete set of quadratic residues of modulo 19.

(34) Prove that the no. of solutions of the congruence
$x^2 \equiv c \pmod p$ is
$1 + \left(\frac{c}{p}\right)$.

(35) Find all primes p such that $x^2 \equiv 13 \pmod p$ has a solⁿ.