Quiz I

Indicative Solutions

Ans. 1. No, continuity of preferences is not necessary for the existence

1 of Nash equilibria. Continuity of preferences implies that a payoff | utility function that is continuous exists.

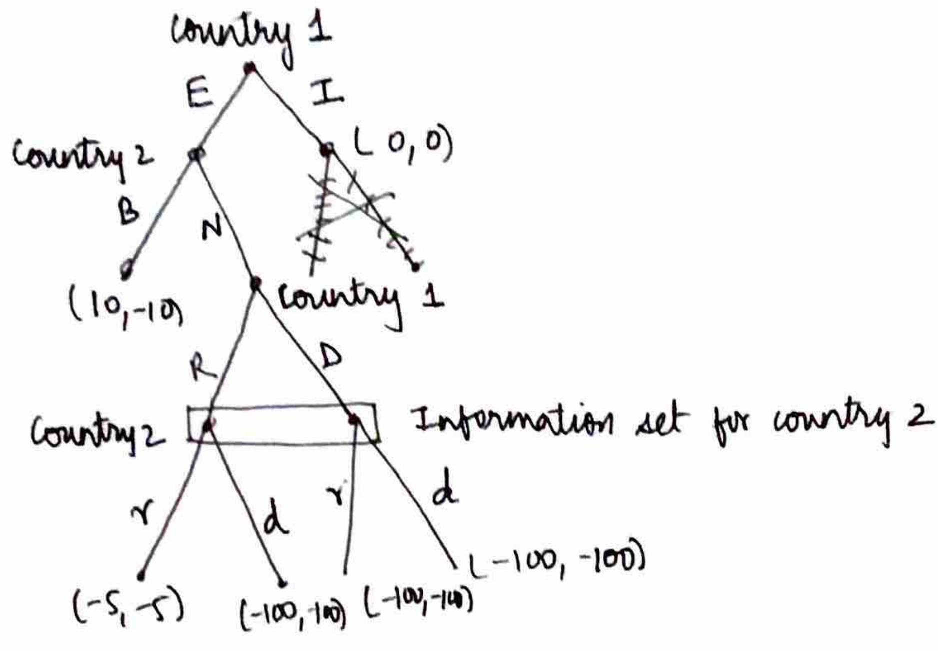
This implies that we can find an optimal solution using Weierstrass' Theorem : $\max_x f(x)$ or $\min_x f(x)$

3

Counter-example: In prisoner's dilemma game, payoffs are not continuous ~~and we do not know if underlying preferences~~ but a Nash equilibrium exists. We do not know — underlying preferences here may not be continuous.

Ans. 2. Extensive form game :

2



2 SPNE = $\{(IR, Nr)\}$ using backward induction

2 PSNE = $\{(ED, Br), (ED, Bd), (ER, Nr), (IR, Nr), (IR, Nd), (ID, Nr), (ID, Nd)\}$

[show matrix | payoffs $A_1 = \{ER, ED, IR, ID\}$

$A_2 = \{Br, Bd, Nr, Nd\}$].

Ans.3. Let $N = \{1,2\}$ be the set of players.

$A_i = \{t \mid t = 0,1,2,\ldots,\bar{t}\}$ $\forall i \in N$ be the set of actions denoting the time period in which player i concedes.

$$u_i(a_i, a_j) = \begin{cases} v_i - a_i & \text{if } a_i > a_j \\ -a_i & \text{if } a_i \leq a_j \end{cases}$$

strategic game $= (N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N})$

Best responce :

$$BR_i(a_j) = \begin{cases} a_j + 1 & \text{if } v_i > a_j + 1 \\ 0 & \text{otherwise} \end{cases}$$

Similarly $BR_j(a_i)$.

Yes, $a_i > v_i$ is a dominated strategy.

Nash equilibria: if $v_i > v_j$: $(v_i, 0)$

$v_j > v_i$: $(0, v_j)$.

Reasoning: note that for $(v_i, 0)$, $v_i > v_j$ the game ends ~~on~~ in $t = 0$ as player 2 (j) concede. Player i gets $u_i = v_i$ and has no incentive to deviate.

Since $v_i > v_j$, player j will bear a loss if he wants to ~~dev~~ concede at a time where he can win i.e. at $t > v_j$. $\therefore$ He cannot gain by deviating from $t = 0$.

Similar argument for $(0, v_j)$ when $v_i < v_j$.

Ans.4 (Indicative).

Player 2

	R	P	S
R	(0,0)	(-1,1)	(1,-1)
P	(1,-1)	(0,0)	(-1,1)
S	(-1,1)	(1,-1)	(0,0)

- The game does not have a pure strategy Nash equilibrium.
- MSNE $= (1/3, 1/3, 1/3)$
- Reasoning : [this is indicati
  (i) Using existence theorem -it is a finite game.
  (ii) Using indifference principle.