

Worksheet 1

September 29, 2021

1. Use the Euclidean algorithm to compute $\gcd(1160718174, 316258250)$.
2. Let $b = r_0, r_1, r_2, \dots$ be the successive remainders in the Euclidean algorithm applied to a and b . Show that after every 2 steps, the remainder is reduced by at least one half. In other words, verify that

$$r_{i+2} < \frac{1}{2}r_i \quad \forall i = 0, 1, 2, \dots$$

3. It is believed that there are infinitely many primes of the form $N^2 + 1$, but no one knows for sure.
 - a) Do you think there are infinitely many primes of the form $N^2 - 1$?
 - b) Do you think there are infinitely many primes of the form $N^2 - 2$?
 - c) How about $N^2 - 3$? $N^2 - 4$?
 - d) Which values of a do you think give infinitely many primes of the form $N^2 - a$?