Ans. 2. Quasi-concave preferences imply that the upper contour set of any outcome is convex.

∴ We can construct a utility function such that ① linear combinations are permitted. ∴ We can find MSNE even if PSNE may not exist.

Ans. $P = 3/4$ $N = \{W, F\}$ $T_F = \{t_F\}$; $T_W = \{H, L\}$

Consider strategy profile $s^* = (s^*_F, s^*_W)$ where

$s^*_F(t_F) =$ hire

$s^*_W(H) =$ work

$s^*_W(L) =$ shirk.

The Firm has just 1 type : t_F. His belief about the type of the worker is:

$$p_F(\text{high} \mid t_F) = 3/4 \qquad p_F(\text{low} \mid t_F) = 1/4$$

Firm's expected utility from "hire" is:

$$U_F((\text{hire}, s^*_W) \mid t_F) = 1 \times \frac{3}{4} + (-1) \times \frac{1}{4} = \frac{1}{2}$$

Firm's expected utility from "don't" is:

$$U_F((\text{don't}, s^*_W) \mid t_F) = 0 \times \frac{3}{4} + 0 \times \frac{1}{4} = 0$$

Worker's type (t_W) is high (H) or low (L)

Consider t_W = high. $p_W(t_F \mid \text{high}) = 1$

Worker's expected payoff from "work" is 2.

" " " "shirk" is 1.

Since 2 > 1, the worker will choose "work" when $t_W = H$.

For $t_W = L$,

expected payoff of worker from choosing "work" is 1, but payoff from "shirk" is 2.

$\therefore$ if $t_W = L$, the worker chooses "shirk".

~~$\therefore$ s^* is an equilibrium strategy.~~

$\therefore$ s^* is a Bayesian Nash Equilibrium.

[Note: s_F = not hire, ~~s_W = shirk~~

$s_W(H) = s_W(L)$ = Shirk

is also a Bayes Nash Equilibrium]

(Check yourself)

Ans. Rock-Paper-Scissors:

		Player 2		
		R	P	S
Player 1	R	0,0	-1,1	1,-1
	P	1,-1	0,0	-1,1
	S	-1,1	1,-1	0,0

NO PSNE.

MSNE exists since this is a finite game.
Suppose player 1 assigns probabilities to 2's actions as follows:

$$p(R) = 1/3$$
$$p(P) = 1/3$$
$$p(S) = 1/3$$

Suppose player 2 assigns probabilities

$$q(R) = q(P) = q(S) = 1/3 .$$

$\{(1/3, 1/3, 1/3), (1/3, 1/3, 1/3)\}$ is MSNE.
[Check this yourself by computing expected payoffs & using indifference principle].