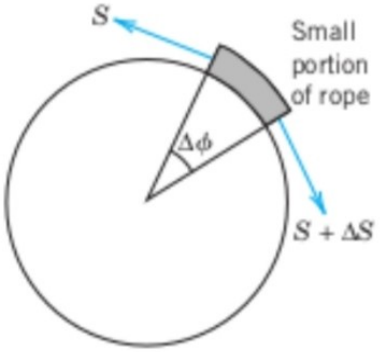
MTH204: Worksheet 2

February 1, 2023

1. **Solution curves of $\mathbf{y' = g(y/x)}$.** Show that any (nonvertical) straight line through the origin of the xy-plane intersects all these curves of a given ODE at the same angle. **(2)**

2. **Rope.** To tie a boat in a harbor, how many times must a rope be wound around a bollard (a vertical rough cylindrical post fixed on the ground) so that a man holding one end of the rope can resist a force exerted by the boat 100 times greater than the man can exert? First guess. Experiments show that the change ΔS of the force S in a small portion of the rope is proportional to S and to the small angle $\Delta\phi$ in figure given below. Take the proportionality constant 0.11. The result should surprise you! **(2)**



3. **Family of Curves.** A family of curves can often be characterized as the general solution of $y' = f(x, y)$. **(3)**
   (a) Show that for the circles with center at $(0, 1)$ we get $y' = -\frac{x}{y-1}$.
   (b) Graph some of the parabolas $x = (y - c)^2$. Find an ODE for them.
   (c) Find an ODE for the straight lines through the point $(0, 1)$.
   (d) You will see that the product of the right sides of the ODEs in (a) and (c) equals -1. Do you recognize this as the condition for two families to be orthogonal (i.e., to intersect at right angles)? Do your graphs confirm this?
   (e) Sketch families of curves of your own choice and find their ODEs. Can every family of curves be given by an ODE?

4. Solve the IVP:

$$2xyy' - y^2 + x^2 = 0,\ y(1) = 1.$$

(3)