MTH 377/577 Convex Optimization
Final Problem Set

April 28, 2024

1. Solve the following optimization problem. Show all the steps.

$$\text{minimize } x^2 + y^2 - 14x - 6y$$

subject to

$$x + y \leq 2$$
$$x + 2y \leq 3$$

2. Let $A \subset R^{m \times n}$ and $C = \{x \in R^n : Ax \leq /\}$. Prove that C is a convex cone.

3. Suppose that S consists of $(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1)$ and $(1,1,1)$. Show that $conv(S) = \{(x_1, x_2, x_3) \in R^3; 0 \leq x_i \leq 1 \text{ for } i = 1,2,3\}$

4. Consider the following maximization problem:

$$max_{x,y} V(x,y) = x^\alpha y^\beta$$

such that

$$ax + by = K$$

where $\alpha, \beta \in (0,1)$ and $K \in R_+$. Write down the Lagrangian function, first order conditions and solve for optimal values of x and y. Compute the change in x^* as (i) α varies, (ii) β varies, (iii) K varies. Use Envelope theorem to explore the change in $V(x,y)$ as α varies.

5. Consider the following 2 player zero sum game.

$$\begin{bmatrix} (4,-4) & (2,-2) \\ (1,-1) & (3,-3) \end{bmatrix}$$

Read any entry (a_i, a_j) in the matrix as: a_i is the payoff for the row player, and a_j is the payoff for the column player. Using pessimistic play, fimd the minmax payoffs for both the players.

6. Suppose $n = 2$. Is the cut-and-choose protocol envy free? Support your answer with a proof/formal argument.