ECE250: Signals and Systems
Assignment 4

Issued on: November 3, 2023 | **Max-Marks : 70** | Due by: November 14 ,2023 (1:30 pm)

Guidelines for submission

Theory Problems:

- Submit a hard copy of your solutions in the wooden box kept on the 3rd Floor of Old Academic Block (right side of the lift).
- Write your Name, Roll No. on the hard copy of your solutions.
- Do all questions in sequence. Use A4 sheets (Plain). Staple your sheets properly
- Submission Policy: Expect no extensions. Late submissions will not be evaluated and hence will be awarded zero marks strictly.
- **Clarifications:**
  - Symbols have their usual meaning. Assume the missing information & mention it in the report. Use Google Classroom for any queries. In order to keep it fair for all, no email queries will be entertained.
  - There could be multiple ways to approach a question. Please justify your answers. Questions without justification will get zero marks.

Programming Problems:

- Use Matlab or python to solve the programming problems.
- For your solutions, you need to submit a zipped file on Google classroom with the following:
  - program files (.m) or (.ipynb) with all dependencies.
  - a report (.pdf) with your coding outputs and generated plots. The report should be self-complete with all your assumptions and inferences clearly specified.
- Before submission, please name your zipped file as: "A4_RollNo_Name.zip".
- Codes/reports submitted without a zipped file or without following the naming convention will NOT be checked.
- **Important Note**: Do not use inbuilt functions in MATLAB or PYTHON. Use mathematical equations/derivations to solve the required.

Institute Plagiarism Policy: This will be subjected to a strict plagiarism check.

Theory Problems (60 points)

[CO4] **Q1.** A signal x(t) is represented by, $x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin\left(\frac{k\pi}{4}\right)}{\frac{k\pi}{4}} \delta(t - \frac{k\pi}{4})$. Determine the following

(a) If $x(t) = \frac{\sin(t)}{\pi t} g(t)$, then determine g(t). **[4 points]**

(b) Prove $X(j\omega)$ is periodic signal. Specify $X(j\omega)$ over one time period. **[3 Points]**.

[CO4] **Q2.** Determine which, if any, of the real signals given in the following figure-1 have Fourier Transforms that satisfy each of the following conditions: **[6x3 Points]**.

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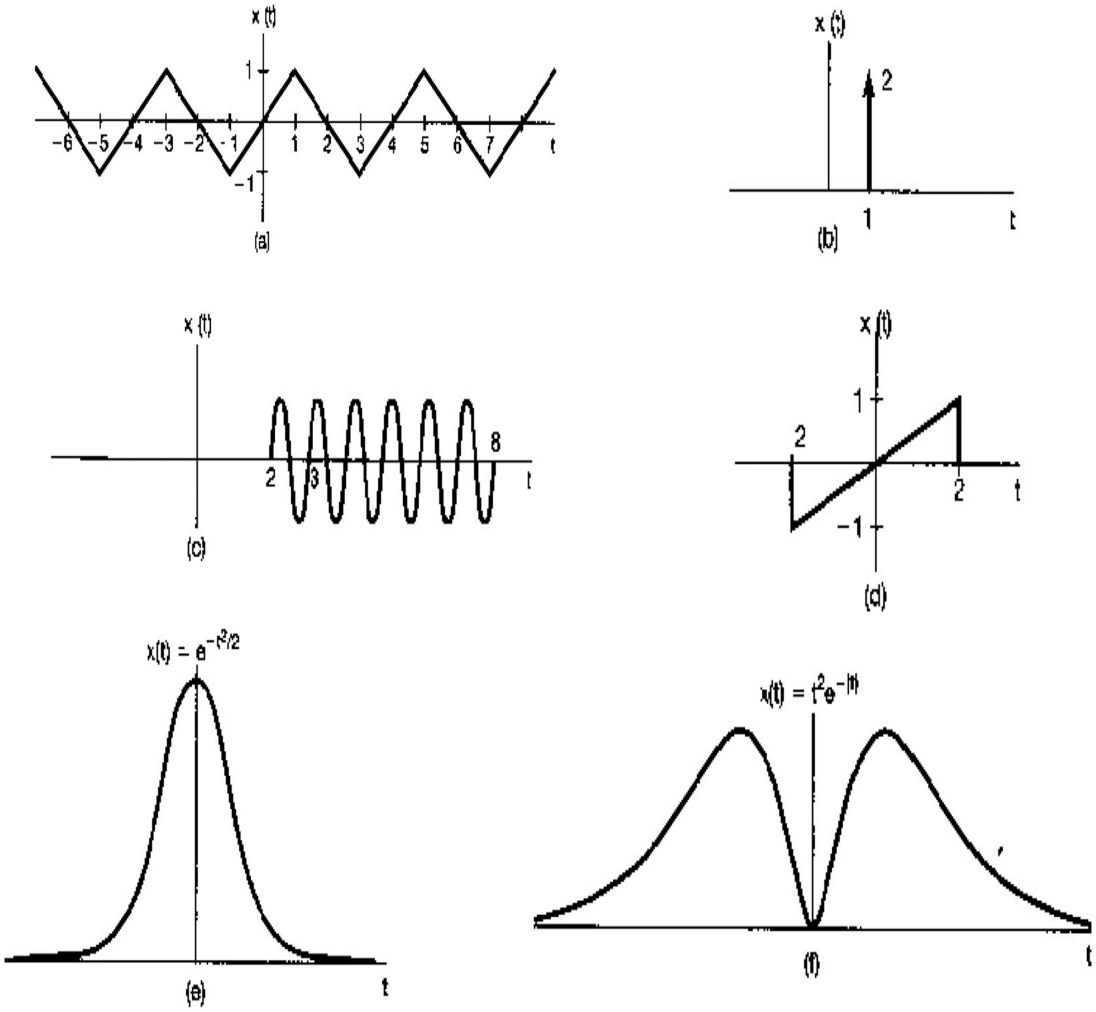

Figure 1: Figure-Q1

(a) $Re\{X(j\omega)\} = 0$
(b) $Img\{X(j\omega)\} = 0$
(c) $\exp(j\alpha\omega)X(j\omega)$ is real (for real value of α)
(d) $\int_{-\infty}^{\infty} X(j\omega)d\omega = 0$
(e) $\int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0$
(f) $X(j\omega)$ is periodic

[CO4] **Q3.** Consider the LTI system "S" with impulse response $h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}$. Determine the output of "S" for each of the following inputs: **[4x4 Points]**

(a) $x(t) = \cos(6t + \pi/2)$
(b) $x(t) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k . \sin(3kt)$
(c) $x(t) = \frac{\sin(4(t-1))}{\pi(t-1)}$
(d) $x(t) = \left(\frac{\sin(2t)}{\pi t}\right)^2$

[CO4] **Q4.** The input and output of a causal LTI system are related by the differential equation-

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t) \tag{1}$$

(a) Find the impulse response of the system. **[2 Points]**
(b) What is the response of the system if $x(t) = t.e^{(-2t)}u(t)$ **[4 Points]**
(c) Repeat part (a) for the causal LTI system described by the following equation **[4 Points]**

$$\frac{d^2y(t)}{dt^2} + \sqrt{2}\frac{dy(t)}{dt} + y(t) = 2\frac{d^2x(t)}{dt^2} - 2x(t) \tag{2}$$

[CO4] **Q5.** Let **[5 Points]**

$$x[n] = \left(\frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n}\right)^2 * \frac{\sin(\omega_c n)}{\pi n} \tag{3}$$

where "*" denotes convolution and $|\omega_c| \leq \pi$. Determine the stricter constraint on "ω_c", which ensure that-

$$x[n] = \left(\frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n}\right)^2 \tag{4}$$

[CO4] **Q6.** An input $x[n]$ with length "3" is applied to an LTI system having an impulse response $h[n]$ of length "5". The output is $y[n]$. **[4 points]**

$$y[n] \leftrightarrow Y(e^{j\omega}) \tag{5}$$
$$|h[n]| \leq L, \qquad |x[n]| \leq B \tag{6}$$

Find the maximum value of $Y(e^{j0})$.

Programming Problems (10 points)

[CO4] **Q1.** Let $X(j\omega)$ be the Fourier transform of the signal $x(t)$ given in Fig.2.

(a) Determine and Plot the frequency domain signal $X(j\omega)$. **[1 Point]**

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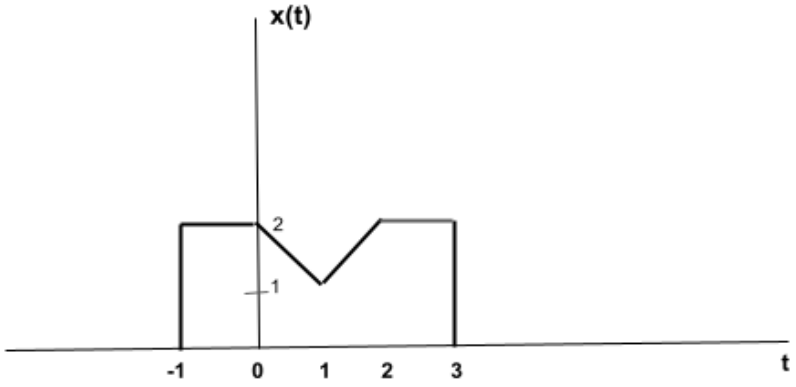

Figure 2: Q1

(b) Plot the magnitude spectrum of the frequency domain signal $X(j\omega)$. **[1 Point]**

(c) Plot the phase spectrum of the frequency domain signal $X(j\omega)$. **[1 Point]**

(d) Plot the inverse Fourier transform of real part of $\{X(j\omega)\}$. **[1 Point]**

[CO4] **Q2.** Let $x[n]$ be a discrete-time signal with Fourier Transform $X(e^{j\omega})$, which is the given Fig. 3. Plot the frequency response, magnitude spectrum and phase spectrum of $w[n] = x[n]p[n]$, for these $p[n]$

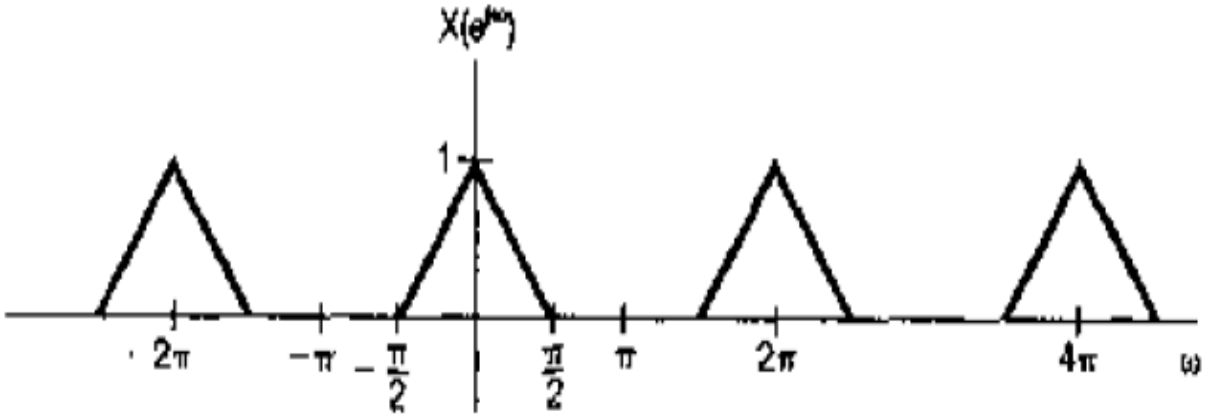

Figure 3: Q2

(a) $p[n] = \cos(\pi n)$ **[2 Points]**

(b) $p[n] = \sin(\pi n/2)$ **[2 Points]**

(c) $p[n] = \sum_{k=-\infty}^{\infty} \delta(n - 2k)$ **[2 Points]**