ECE 351 DSP: Assignment 1

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Total: 30 points

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A word on the notation: I shall represent finite duration causal signals as arrays. For example, $x[n] = [1, 2, 3]$ means $x[0] = 1$, $x[1] = 2$, and $x[2] = 3$, and $x[n] = 0$ for all other n.

1) Can the region marked in red in Figure 1 be the ROC of some signal's z-transform?

[2 points]

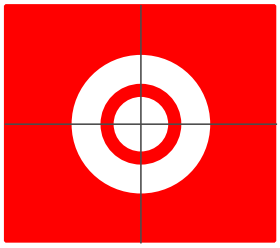

Fig. 1: Figure for Prob 1

2) Consider the system with unit sample response $h[n] = (\frac{3n+1}{n+2})^{2n} u[n]$.

   a) Prove that the system is unstable.

   b) Find a bounded input signal which will result in an unbounded output from this system.

[3+3=6 points]

3) Consider the recursive system given by $y[n] = \frac{1}{12} y[n-1] + \frac{1}{12} y[n-2] + x[n] - \frac{1}{2} x[n-1]$.

   a) Find $H(z)$ for the system.

   b) Find $h[n]$ for the system.

[2+5=7 points]

4) The signal $x[n] = [1, 0, 1]$ is passed through an LTI system with unit sample response $h[n] = [2, -1]$.

   a) Find $y[n]$.

b) Let $Y(\frac{2\pi}{3}k)$ be the DTFT of $y[n]$ evaluated at $\omega = \frac{2\pi}{3}k, k = 0, 1, 2$. Then find the vector z where

$$z = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}(1-\sqrt{3}j) & -\frac{1}{2}(1+\sqrt{3}j) \\ 1 & -\frac{1}{2}(1+\sqrt{3}j) & -\frac{1}{2}(1-\sqrt{3}j) \end{pmatrix} Y,$$

where $Y = [Y(0), Y(\frac{2\pi}{3}), Y(\frac{4\pi}{3})]^{\mathrm{T}}$.

[**Hint**: Remember that this is not a linear algebra course. So this question is not looking at your matrix multiplication skills. Carefully study the matrix multiplication and you will be able to see what is going on.]

[2+3=5 points]

5) Consider the two sequences $x[n] = \binom{3n}{n}(\frac{1}{9})^n$,[1] $0 \leq n \leq 44$, and $h[n] = [-1, 2, 1, -2]$. Write a python code to compute $x[n] * h[n]$ by the overlap-and-save method using 16-point DFTs. Plot the output sequence.

Keep the following points in mind:

- To implement the internal 'small' convolutions using the 16-point DFTs, you can use built-in functions, such as fft, ifft, and fftconvolve from scipy.fft and scipy.signal. For example, let x and y be 2 size 16 arrays you are looking to convolve using size 16 DFTs as internal computation for the overlap-and-save method. Then take their 16-point DFTs using X=scipy.fft.fft(x) and Y=scipy.fft.fft(y). Then, multiply X and Y arrays term-wise to give a new array Z. The result of the convolution is then obtained by taking the 16-point IDFT of Z, which can be obtained through scipy.fft.ifft(Z).
- Recall that you are plotting a discrete time sequence. So use matplotlib.pyplot.stem instead of matplotlib.pyplot.plot.
- You do not need to show on-paper calculations for this question.

[10 points]

[1] Take $\binom{0}{0} = 1$.