MTH100 Monsoon Semester – 2021-22 **Section B**

Submission for Wednesday 9th February 2022 – 15 minutes. Max Marks: 5

Instructions: Open notes and textbook; consultation and use of calculators, computers and internet not allowed.

IMPORTANT: You may use any **known** result. This includes all propositions and observations in the lecture slides, and results from tutorials. If you use any other result from any other source, including the textbook, you have to give a full proof of that result.

a) Construct an LU Factorization of the matrix A given below. (*2 marks*)

b) Then, use the LU Factorization to solve the nonhomogeneous system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (1, a, a^2)$, $a \neq 0$. **(Do not use any other method to solve the equation.)** (*3 marks*)

(Remark: Show all your steps clearly, with brief explanations. Else, you will not be given credit.)

$$A = \begin{bmatrix} 2 & -3 & -1 \\ 6 & 0 & 6 \\ -4 & 6 & 5 \end{bmatrix}$$

SOLUTION - CUM - RUBRIC [Calculations on page 2]

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & -1 \\ 0 & 9 & 9 \\ 0 & 0 & 3 \end{bmatrix} = LU$$

1 mark for each one correct.
Cut 0.5 marks for each if steps not shown.

Solution is $\bar{x} =$ → 1 mark

$$\left(\frac{-2a^2+a-4}{6}, \frac{-3a^2+a-9}{9}, \frac{a^2+2}{3} \right)$$

x_1 x_2 x_3

{ 1 mark for steps for solving $L\bar{y} = \bar{b}$
{ 1 mark for steps for solving $U\bar{x} = \bar{y}$

Step marks to be given only if answer to that stage is correct (may be left up to step shown with ✓ mark].

Calculations

$$A = \begin{bmatrix} 2 & -3 & -1 \\ 6 & 0 & 6 \\ -4 & 6 & 5 \end{bmatrix} \xrightarrow[R_3 \to R_3 + 2R_1]{R_2 \to R_3 - 3R_1} \begin{bmatrix} 2 & -3 & -1 \\ 0 & 9 & 9 \\ 0 & 0 & 3 \end{bmatrix}$$

$\uparrow U$

No further steps reqd.

Inserting $(-1) \times$ the factor for i-j-th position in I_3, we get $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

Check: $LU = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & -1 \\ 0 & 9 & 9 \\ 0 & 0 & 3 \end{bmatrix} =$

$$\begin{bmatrix} 2 & -3 & -1 \\ 6 & 0 & 6 \\ -4 & 6 & 5 \end{bmatrix} = A \checkmark$$

Solving $L\bar{y} = \bar{b}$, $\left.\begin{aligned} y_1 &= 1 \\ 3y_1 + y_2 &= a \\ -2y_1 + y_3 &= a^2 \end{aligned}\right\}$ we get:

$\bar{y} = \begin{bmatrix} 1 \\ a-3 \\ a^2+2 \end{bmatrix}$. Solving $U\bar{x} = \bar{y}$, we get

$$2x_1 - 3x_2 - x_3 = 1$$
$$9x_2 + 9x_3 = a - 3$$
$$3x_3 = a^2 + 2,$$

to give: $x_3 = \frac{1}{3}(a^2+2)$, $x_2 = \frac{1}{9}\left[(a-3) - 3(a^2+2)\right]$
$= \frac{1}{9}\left[-3a^2 + a - 9\right]$, $x_1 = \frac{1}{2}\left[1 + 3x_2 + x_3\right] \checkmark$
$= \frac{1}{6}\left[-2a^2 + a - 4\right]$