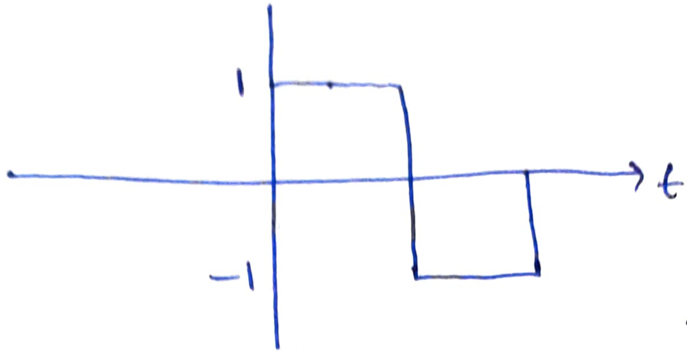
Sol:→ $\Rightarrow x(t) = u(t) - 2u(t-2) + u(t-5)$



— 1

t is replaced with τ.

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

$\Rightarrow h(t) = e^{2t} u(1-t)$

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

$h(t) = e^{2t} u(1-t)$

$\Downarrow$

$$u(1-t) = \begin{cases} 1, & 1-t \geq 0 \\ & 1 \geq t \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} e^{2t} & 1 > t \\ 0 & \text{otherwise.} \end{cases}$$ — 1

$$h(t-\tau) = \begin{cases} e^{2(t-\tau)}, & 1 \geq t-\tau \\ 0, & \text{otherwise} \end{cases}$$

$$h(t-\tau) = \begin{cases} e^{2(t-\tau)}, & \tau \geq t-1 \\ 0, & \text{otherwise.} \end{cases}$$

Case I :→ $t-1 \leq 0$ or $t \leq 1$

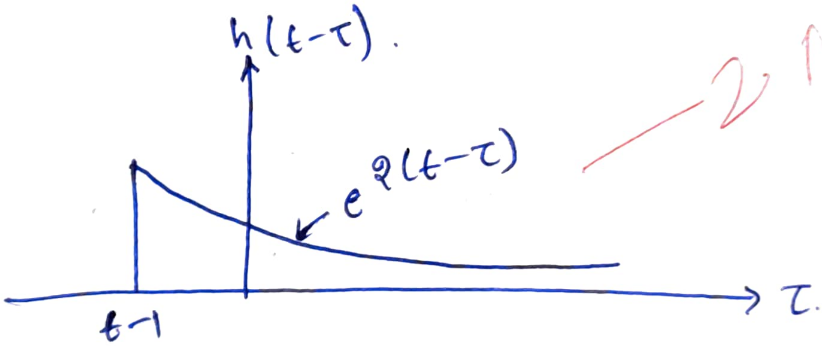
$$h(t-\tau) = \begin{cases} e^{2(t-\tau)}, & \tau \geq t-1 \\ 0, & \text{otherwise.} \end{cases}$$

Put $\tau = t-1$

$$h(t-\tau) = \begin{cases} e^{2(t-(t-1))}, & \tau \geq t-1 \\ 0, & \text{otherwise} \end{cases}$$

At $\tau = t-1$, $h(t) = e^{2}$

$\tau = \infty$, $h(t) = e^{-\infty} = 0$



2 Marks

$$x(t) \circledast h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

$$= \int_{-\infty}^{t-1} 0 \times 0\, d\tau + \int_{t-1}^{0} 0 \times e^{2(t-\tau)} d\tau + \int_{0}^{2} 1 \times e^{2(t-\tau)} d\tau + \int_{2}^{5} (-1)\, e^{2(t-\tau)} d\tau$$

$$+ \int_{5}^{\infty} 0 \times e^{2(t-\tau)} d\tau$$

$$= 0 + 0 + \int_{0}^{2} e^{2(t-\tau)} d\tau + \int_{2}^{5} (-1)\, e^{2(t-\tau)} d\tau + 0$$

③

$$= \left[\frac{e^{2(t-\tau)}}{-2}\right]_0^2 - \left[\frac{e^{2(t-\tau)}}{-2}\right]_2^5$$

$$= -\frac{1}{2}\left(e^{2(t-2)} - e^{2t}\right) + \frac{1}{2}\left[e^{2(t-5)} - e^{2(t-2)}\right]$$

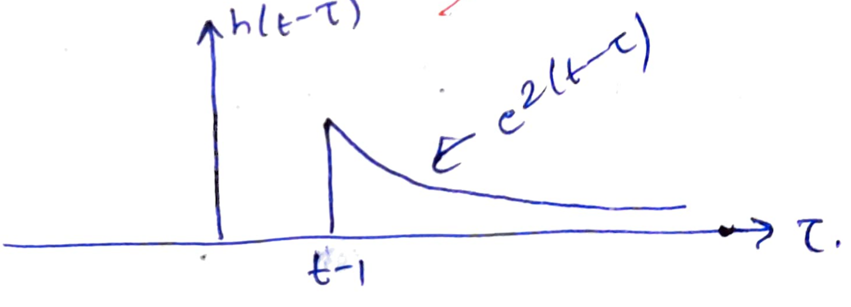
$$= -\frac{1}{2}e^{2(t-2)} + \frac{1}{2}e^{2t} + \frac{1}{2}e^{2(t-5)} - \frac{1}{2}e^{2(t-2)}$$

$$= e^{2t}\left(-e^{-4} + \frac{1}{2} + \frac{1}{2}e^{-10}\right).$$

<u>Case II</u> :→ $0 < t-1 \leq 2 \Rightarrow 1 < t \leq 3$

2 Marks

$h(t-\tau)$

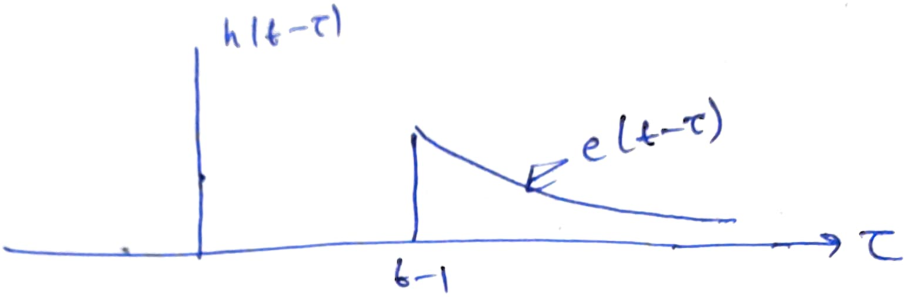


$$= \int_{t-1}^{2} 1 \times e^{2(t-\tau)}\,d\tau + \int_{2}^{5} (-1)\, e^{2(t-\tau)}\,d\tau$$

$$= \left[\frac{e^{2(t-\tau)}}{-2}\right]_{t-1}^{2} - \left[\frac{e^{2(t-\tau)}}{-2}\right]_2^5$$

$$= -e^{2(t-2)} - e^{2} + \frac{1}{2}e^{2(t-5)}$$

Case III :→ $2 < t-1 \leq 5 \Rightarrow 2 < t-1 \leq 6.$



$$= \int_{t-1}^{5} (1)\, e^{2(t-\tau)}\, d\tau$$

2 Marks

$$= 0 - \left[\frac{e^{2(t-\tau)}}{-2}\right]_{t-1}^{5}$$

$$= \frac{1}{2}\left[e^{2(t-5)} - e^{2(t-(t-1))}\right]$$

$$= \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right].$$

Case IV :→ $0, \ t > 6.$

2 Marks.

$$y(t) = \begin{cases} e^{2t}(-e^{-4} + \frac{1}{2} + \frac{1}{2}e^{-10} ; & t \leq 1 \\ \frac{1}{2}e^{2(t-5)} - e^{2} - e^{2(t-1)}, & 1 < t \leq 3 \\ \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right] ; & 3 < t \leq 6 \\ 0 ; & t > 6. \end{cases}$$