MTH 377/577 Convex Optimization
Mid-sem

March 3, 2024

Answer all questions. Time = 60 minutes. All students must attempt Q1,2,3,4,5. Q6 is for MTH377. Q7 is for MTH577.

1. State whether the following are true/false. Provide a reason for your answer: (2X2)

   (a) A polyhedron is always convex. −True

   (b) An intersection of finite number of halfspaces is always conxex. ↳ True.

2. Let $A = \{(x_1, x_2) : x_1 \geq 0, x_1^2 + x_2^2 \leq 4\}$ and $B = \{(x_1, x_2) : x_1, x_2 \geq 0; x_2 \geq \frac{2}{x_1}\}$. Does a separating hyperplane for A and B exist? (3)

3. Consider the following optimization problem:

$$max \ 3x_1^2 + 2x_2^2$$

such that

$$x_1 + x_2 - 3 \geq 0$$

Using what you have learnt about multipliers $\lambda_0, \lambda_1, \ldots$ check whether a local maxima exists. Show all the steps. (3)

4. Write down the dual for the following optimization problem: (3)

$$min \ - x_1 + 2x_2 + 8x_3 + 2x_4$$

such that