MTH 201 Probability and Statistics: End Semester Exam

It is given that $\Phi(.5) = .6915$, $\Phi(1) = .8413$, $\Phi(2.06) = .98$, $\Phi(2) = .9772$, $\Phi(1.96) = .975$ and $\Phi(2.6) = .995$

Total: 100 points

1) (a) A student takes a multiple-choice test consisting of two problems, The first one has three possible answers and the secone one has five. The student chooses, at random, one answer as the right one from each of the two problems. Find the expected number, $E[X]$, of the right answers X of the student. Find $Var[X]$.

(b) A player has 15 dollars. If head appears on the first toss, he receives 1 dollar and withdraws from the game. If tail appears, he loses 1 dollar and he bets 2 dollars. If he wins the second toss, he withdraws . Otherwise, he continues playing on a bet of 4 dollars. If he loses, he bets the remaining amount of 8 dollars. What is the expected gain of the player ?

[20 points]

2) Professor HIJBIJBIJ is interested in measuring in light years, the distance from his observatory to a distant star. He knows that because of changing atmospheric conditions, each time a measurement is made, it will not yield the exact distance but merely an estimate. As a result Professor HIJBIJBIJ plans to make a series of measurements and then use the average value as his estimated value of the actual distance. If Professor HIJBIJBIJ believes that the values of the measurements are independent and identicall distributed random variables having a common mean d (actual distance) and a common variance $4(lightyear)^2$ and normal approximation is a good approximation, how many meaurements are needed to be 96 percent certain that the estimated value is accurate to within 0.5 light years ?

[15 points]

3) Let X_1, X_2,... denote an iid sequence of random variables, each with expected value 75 and standard deviation 15.

(a) How many samples n do we need to guarantee that the sample mean $M_n(X)$ is between 74 and 76 with probability 0.99?

(b) If each X_i has a Gaussian distribution, how many samples would we need to guarantee $M_n(X)$ is between 74 and 76 with probability 0.99?

[15 points]

4) A clerk leaves at A and works at C and he starts work at 9:00 AM. The clerk always takes the train from A to B which is supposed to reach B at 8:40 AM. Buses from B leaves for C every 15 minutes and the bus which leaves at 8:45 AM. is supposed to arrive at 8:56 AM.

~~The tram~~ on average experiences delays of 2 minutes and has a standard deviation of 4 minutes. The bus always leaves on time but arrives on avrage after a 2 minutes delay and a standard deviation of 3 minutes. What is the probability that the clerk arrives late?

The clerk's employer drives to his office (independently of his clerk) and he leaves at 8:45 AM. The driving time to the office has a mean value of 12 minutes and and a standard deviation of 2 minutes.

Find the probability that (a) both, clerk and employer, arrive late.

(b) the employer arrives earlier than the clerk.

(Assume that the distributions involved are normal and give all your answes in terms of standard normal CDF)

[15 points]

5) Random variables N and K have joint PMF

$$P_{N,K}(n,k) = \begin{cases} \frac{100^n e^{-100}}{(n+1)!} & \text{if } k = 0, 1, ..., n \\ & n = 0, 1, ..., \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal PMF $P_N(n)$, the conditional PMF $P_{K|N}(k|n)$ and the conditional expected value $E[K|N = n]$ Express the random variable $E[K|N]$ as a function of N and use the iterated expectation to find E[K].

[15 points]

6) Using Chebyshev inequality show that if a fair die is thrown 3600 times, then the probability that the number of threes lies between 550 and 650 is atleast $\frac{4}{5}$.

[10 points]

7) Professor HIJBIJBIJ has written 18 books for a publisher and every year one edition of each book is published. At the end of the year, each edition of a book is equally likely to bring a profit , a loss or a no profit no loss situation.The publisher decides to give annual royalty to Professor HIJBIJBIJ on the basis of a point system. Professor gets 3 points for each profit making edition of a book, 0 points for each loss making edition and 1 point for a no profit no loss situation. The publisher assumes that the sale of each book is independent of any other book. Let B_i be the number of points Professor HIJBIJBIG earned in the year 2023 for the i-th book and let T be the total number of points earned in 2023.

Find the moment generating function of B_i and that of T.

From the above result, find E[T] and Var[T].

[10 points]

→ Tree
→ BST
→ BSF
→ DSF
→ AVL
→ Graph

linked list
Stack
Queue