

MTH 204 Quiz 1

(Time : 15 mins, Maximum Marks : 10)

February 1, 2023

Question 1.

[5 points] Let $F(t)$ be the number of Facebook friends you have at time t . Assume that $F(t)$ satisfies the logistic growth equation:

$$\frac{dF}{dt} = 0.1F \left(1 - \frac{F}{100} \right).$$

Find $F(t)$ with an arbitrary constant C in it.

Solⁿ:

$$\frac{dF}{dt} = 0.1F \left(1 - \frac{F}{100} \right)$$

$$\Rightarrow \frac{dF}{0.1F \left(1 - \frac{F}{100} \right)} = dt$$

$$\Rightarrow \frac{1000 dF}{F(100-F)} = dt$$

$$\Rightarrow \left(\frac{10}{F} + \frac{10}{100-F} \right) dF = dt$$

$$\Rightarrow \left(\frac{1}{F} + \frac{1}{100-F} \right) dF = \frac{dt}{10}$$

$$\Rightarrow \ln F - \ln(100-F) = \frac{t}{10} + \ln C$$

$$\Rightarrow \frac{F}{100-F} = Ce^{t/10}$$

$$\Rightarrow F = (100-F)Ce^{t/10}$$

$$\Rightarrow (1 + Ce^{t/10}) F(t) = 100Ce^{t/10}$$

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Question 2.

[5 points] A population of insects in a region will grow at a rate that is proportional to their current population. In the absence of any outside factors the population will triple in two weeks time. On any given day there is a net migration into the area of 15 insects and 16 are eaten by the local bird population and 7 die of natural causes. Let $P(t)$ be the population of insects after t days. Set up a first order differential equation satisfied by $P(t)$ (just set up, you don't have to solve).

Solⁿ: $\left\{ \begin{array}{l} P(t) \text{ is the population of insects after } t \text{ days, so} \\ \textcircled{2} \quad \frac{dP}{dt} = \text{Birth rate} - \text{Death rate} + \text{Migration in} - \text{Migration out} \end{array} \right.$ $\xrightarrow{\textcircled{1}}$

$\textcircled{1} \left\{ \begin{array}{l} \text{Now, Birth rate} \propto P(t) \\ \Rightarrow \text{Birth rate} = kP(t) \text{ for proportionality constant } k. \end{array} \right.$

$\textcircled{2} \left\{ \begin{array}{l} \text{So, } \textcircled{1} \text{ becomes} \\ \frac{dP}{dt} = kP(t) - (16+7) + 15 = kP(t) - 8 \\ \therefore \text{ODE is} \\ \boxed{\frac{dP}{dt} = kP(t) - 8} \end{array} \right.$