MTH 204 : Worksheet 2 (Solutions)

Q-1

Take a straight line passing through the origin
$y = mx$

To Prove - The angle between the line $y=mx$ and all the curves of a given ODE $y' = g(y/x)$ remains same.

At the point (x, y), on the line $y = mx$,
slope of tangent of the curve $= y' = g(y/x) = g(m)$
So the angle θ between $y = mx$ and the tangent line to the curve is given by

$$\tan\theta = \frac{m - g(m)}{1 + m\,g(m)}$$

which is constant.

Q2.

$$dS = 0.11\, S\, d\phi$$

$$\Rightarrow \frac{dS}{S} = 0.11\, d\phi$$

$$\Rightarrow \ln S = 0.11\phi + C$$

$$\Rightarrow S = K e^{0.11\phi}$$

Putting $S = 100$, $e^{0.11\phi_1} = \frac{100}{K}$

$S = 1$, $e^{0.11\phi_2} = \frac{1}{K}$

$$\Rightarrow e^{0.11(\phi_1 - \phi_2)} = 100$$

$$\Rightarrow \phi_1 = 2\pi + \frac{\ln 100}{0.11} \simeq 7.66 \times 2\pi$$

So, the rope must be wound 8 times around a bollard.

Q3.

$$y' = f(x,y)$$

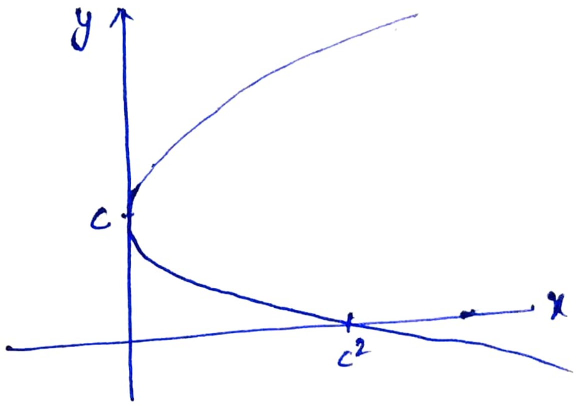
ⓐ Circles with center at $(0,1)$: $x^2 + (y-1)^2 = r^2$

$$\Rightarrow \quad 2x + 2(y-1)y' = 0$$

$$\Rightarrow \quad \boxed{y' = -\frac{x}{y-1}}$$

ⓑ $x = (y-c)^2$

$$\Rightarrow \quad 2(y-c)\, y' = 1$$



ⓒ Straight lines through the point $(0,1)$

$$y = mx + 1$$

$$y' = m$$

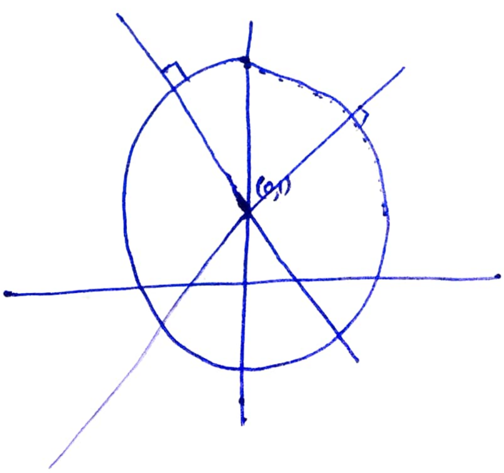
$$\Rightarrow \quad y = y'x + 1$$

$$\Rightarrow \quad y' = \frac{y-1}{x}$$

ⓓ ODE in ⓐ : $y' = \frac{-x}{y-1}$

" " ⓒ : $y' = \frac{y-1}{x}$

So, $\frac{-x}{y-1} \cdot \frac{y-1}{x} = -1$



Q4.

IVP : $2xyy' - y^2 + x^2 = 0$, $y(1) = 1$

$2xyy' - y^2 + x^2 = 0$

$\Rightarrow 2\frac{y}{x}y' - \frac{y^2}{x^2} + 1 = 0$

Take $\frac{y}{x} = u \Rightarrow y' = u + xu'$

$\Rightarrow 2u(u + xu') - u^2 + 1 = 0$

$\Rightarrow 2u^2 + 2xuu' - u^2 + 1 = 0$

$\Rightarrow 2xuu' = -(u^2+1)$

$\Rightarrow \frac{2u}{u^2+1}\,du = -\frac{1}{x}\,dx$

$\Rightarrow \ln(u^2+1) = -\ln x + \ln c$

$\Rightarrow u^2 + 1 = \frac{c}{x}$

$\Rightarrow \left(\frac{y}{x}\right)^2 + 1 = \frac{c}{x}$

$\Rightarrow y^2 + x^2 = cx$

$\Rightarrow y^2 + x^2 - cx = 0$

Since, $y(1) = 1$

$\Rightarrow 1 + 1 - c = 0$

$\Rightarrow \boxed{c = 2}$

Hence, $x^2 + y^2 - 2x = 0$