

Quiz 2

Time: 10:30AM - 11:50AM

Instruction:

- Uploading of your answer sheet needs to be completed on or before 12:00PM in your google classroom section page. Any upload after 12:00PM will result in severe penalty including the possibility of assignment of 0 as your Quiz 2 score.
- You may refer to the text book or class-notes. If you need to write verbatim from the book or class notes, you need to provide clear citation. You are not allowed to discuss with your peers or anyone else. Plagiarism policies are strictly enforced.
- Your answer sheet needs to contain your name, roll number and tutorial section in the header.
- Your answers needs to be correct and properly explained in order to obtain full score.
- You can score a maximum of 10 points.

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1. (a) Change the following integral to an equivalent polar integral: [1 point]

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) \, dx dy$$

- (b) Change the following integral to an equivalent Cartesian integral: [1 point]

$$\int_0^{\frac{\pi}{2}} \int_0^1 r^3 \sin \theta \cos \theta \, dr d\theta$$

2. Set up a triple integral (no need to evaluate) to calculate the volume of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 2)$. [2 points]
3. What domain D in space maximizes the value of the integral [3 points]

$$\iiint_D (1 - x^2 - y^2 - z^2) \, dV$$

Explain your answer. You may use the following fact about triple integrals: For two non overlapping regions D_1 and D_2 ,

$$\iiint_{D_1} f(x, y, z) \, dV + \iiint_{D_2} f(x, y, z) \, dV = \iiint_{D_1 \cup D_2} f(x, y, z) \, dV$$

4. What relationship must hold between the constants a, b and c to make [3 points]

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(ax^2+2bxy+cy^2)} \, dx dy = 1$$

(Hint: Let $s = \alpha x + \beta y$ and $t = \gamma x + \delta y$, where $(\alpha\delta - \beta\gamma)^2 = ac - b^2$. Then $ax^2 + 2bxy + cy^2 = s^2 + t^2$.)