Ans. 1 a) Since $p_A < 400$, Hire is a strictly dominant action
1+2 for player A.
Since $p_B < 400$, Hire is a " " "
for player B. $\therefore$ (Hire, Hire) is Nash equilibrium.

b) Suppose both players use the following strategy:
In $t = 1$, play NH
For $t \geq 1$: play NH if (NH, NH) was played in every period
[redacted]
s, $s = 1, 2, \ldots, t-1$
play H otherwise.
[Trigger strategy: play NH if ~~the other player has p~~ NH has been played in all previous periods. Otherwise play H.] We check with B because deviation is more profitable for him.

By following this strategy,
payoff for B = $U_B((NH, NH), (NH, NH), \ldots)$

$$= 1000 + 1000\delta + 1000\delta^2 + \ldots = \frac{1000}{(1-\delta)}$$

[redacted]

If B deviates at $t = 1$:

$$U_B(\underbrace{(NH, H)}_{\text{dev } t=1}, (H, H), (H, H), \ldots) = 1700 + 900\delta + 900\delta^2 + \ldots$$

$$= 1700 + \frac{900\delta}{1-\delta}$$

The proposed strategy profile will be N.E iff

$$1700 + \frac{900\delta}{1-\delta} \leq \frac{1000}{1-\delta} \Rightarrow \delta \geq 7/8.$$

[redacted]

1. b) contd.

Alternatively, single deviation:
consider that B deviates in $t=1$ and then sticks to NH in all subsequent periods.

strategy: play NH every period. If other player plays H, play H forever.

$$U_B\left((NH, H), (H, NH), (H, NH), \ldots\right)$$

$$= 1700 + 800\delta + 800\delta^2 + \ldots$$

$$= 1700 + \frac{800\delta}{1-\delta}$$

To rule out the above deviation, we must have

$$1700 + \frac{800\delta}{1-\delta} \leq \frac{1000}{1-\delta} \Rightarrow \delta \geq 7/9 .$$

Ans. 2.

$Q = 12 - p$ ✓

$Q = q_1 + q_2$ $\quad p = 12 - q_1 - q_2$

Firm 2's profit, $\pi_2 = (12 - q_1 - q_2)\, q_2 - 0$

maximising w.r.t q_2 and using F.O.C

$\partial \pi_2 / \partial q_2 = 0$ at $q_2 = q_2^*$,

[redacted]

$12 - q_1 - 2q_2 = 0 \Longrightarrow$

$\Rightarrow q_2^*(q_1) = 6 - q_1/2$

reaction function | best response function.

Firm 1's profit max. problem: $\quad \pi_1 = (12 - q_1 - q_2(q_1))\, q_1$

$\max\limits_{q_1} \; (12 - q_1 - q_2(q_1))\, q_1$

[redacted]

using ~~first order condition~~ expression for $q_2(q_1)$ in the above:

$\left(12 - q_1 - \left(6 - \frac{q_1}{2}\right)\right) - \left(1 - \frac{1}{2}\right) q_1$,

$= 6 - q_1$

using the expression for $q_2(q_1)$:

$\left(12 - q_1 - (6 - q_1/2)\right) q_1$

$\frac{d\pi_1}{dq_1} = 0$ By first order condition: $12 - 2q_1 - 6 + q_1 = 0$

$\Rightarrow 6 - q_1 = 0 \Rightarrow q_1^* = 6$.

1. b) contd.

$$\therefore q_2^* = 6 - \frac{6}{2} = 3$$

1 $\therefore$ the quantities chosen in equilibrium are $(6, 3)$.

$\therefore q_2^* = 6 - \frac{6}{2} = 3$

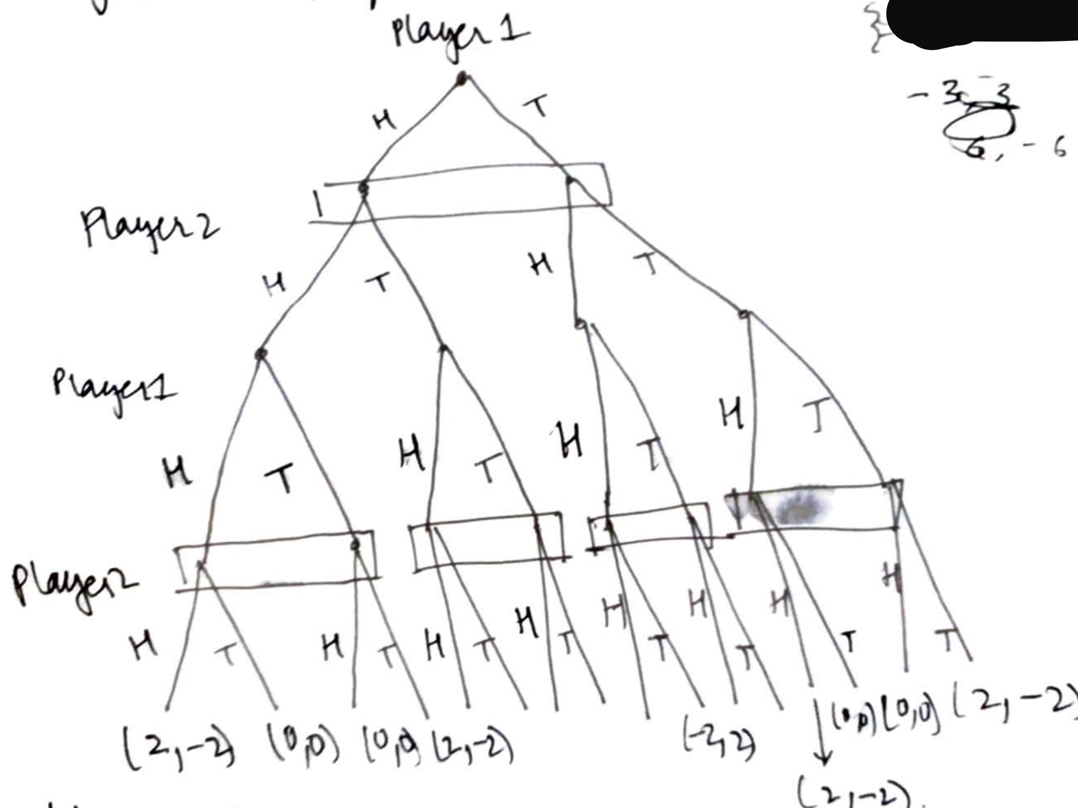
① $\therefore$ the quantities chosen in equilibrium are (6, 3).

Ans. 3. Matching pennies game: [redacted]

Player 2

Player 1	H	T
H	(1, -1)	(-1, 1)
T	(-1, 1)	(1, -1)

Matching pennies played twice in extensive form [redacted]



④ Indicative payoffs shown above:

~~when~~ ~~(HH, HH), (H, H)~~

(HH, HH), ~~(HH~~, (HT, HT), (TT, TT), (TH, TH)

then payoff is (2, -2)

(HT, TH), (HH, TT), (TT, HH), (TH, HT)

then (-2, 2).

(HH, HT), (HH, TH), (TT, HT), (TT, TH),

(HT, TT), (TH, TT), (HT, HH), (TH, HH)

then (0, 0).

1/2 The game will have ~~a~~ ~~no~~ 5 PNE in mixed strategies.

[redacted]

Ans. 3

Explanation: ① Every finite game has a mixed strategy Nash equilibrium. In 1 period Matching pennies, we know that $(1/2, 1/2)$ is MSNE. For stage games, the equilibrium in the 1-stage game repeated is equilibrium of the stage game also. $\therefore$ $\{(1/2, 1/2), (1/2, 1/2)\}$ will be a ~~PSNE~~ SPNE : [redacted]

Ans. 4. $I = [0, 1]$

For $n = 2$

$$u_i(a_i, a_{-i}) = |a_i - a_{-i}|$$

$$BR_i(a_{-i}) = \begin{cases} 0 & \text{if } a_{-i} \geq 0.5 \\ 1 & \text{if } a_{-i} \leq 0.5 \end{cases}$$

For $n = 3$

Consider $a = (a_1, a_2, a_3) = (0, 1/2, 1)$

$$u_1(a) = |a_1 - a_2| + |a_1 - a_3|$$
$$= \frac{1}{2} + 1 = \frac{3}{2}$$

If 1 moves to any $x > 0$, ~~etc~~ then its distance from both 2 and 3 decrease. $\therefore$ no incentive to deviate.

$$u_2(a) = |a_2 - a_1| + |a_2 - a_3|$$
$$= 1/2 + 1/2 = 1$$

For a unit increase in distance from a_1, distance from a_3 will fall by the same amount, and vice-versa $\therefore$ no incentive to deviate.

Argument for 3 is similar to that for 1.

$\therefore$ $a = (0, 1/2, 1)$ is a Nash equilibrium in pure strategies.