Quiz4 (ADA-2023)
Answer keys

Q1. Given a graph G = (V, E) with positive edge weights, the Bellman-Ford algorithm and Dijkstra's algorithm can produce different shortest-path trees despite always producing the same shortest-path weights. True or False ?

Solution: True.

Q2. Dijkstra's Algorithm has lesser time complexity compared to Bellman Ford's algorithm, not considering the space complexity.

a) True
b) False

Q3. Given G= (N, M) where N is the number of vertices and M is the number of edges, it is known that G is a complete undirected graph. What is the total number of **spanning trees** for G ?

a. N * 2
b. N ^ 2
c. N ^ (N - 2)
d. N ^ (M - 1)

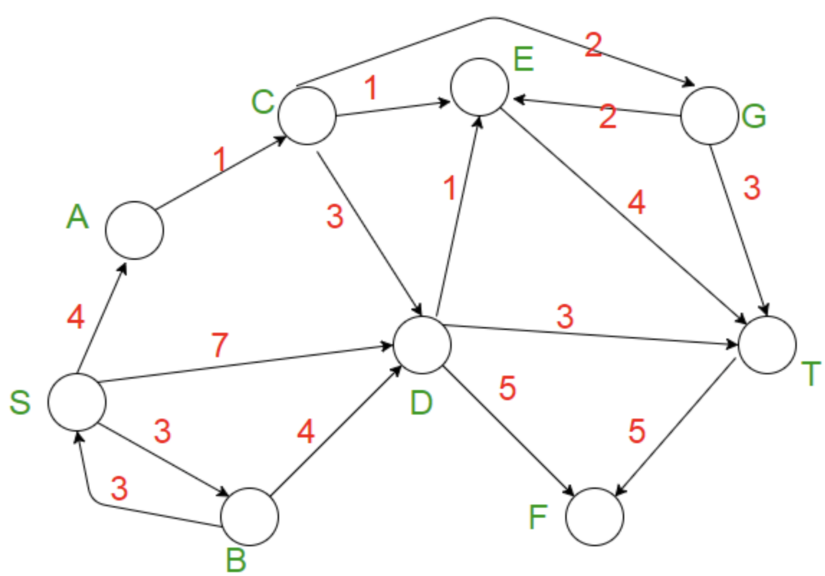
Q4. Read the following statements -
I) Let G = (V,E) be a flow network, with a source s, a sink t, and a positive integer capacity c on every edge e. If f is a maximum (s, t)-flow in G, then for all e ∈ E that goes out of s, f(e) = c.
II) Let G = (V,E) be a flow network with source s, sink t and all the edges of G have odd capacities. Then for any maximum (s, t)-flow f and for any edge e ∈ E, f(e) is either zero or odd.
Which of the following statements is/are **incorrect**?

A. I only
B. II only
C. Both I and II
D. Neither I or II

Q5. Read the following statements -
I) Let G be a directed graph with positive edge weights $w : E(G) \rightarrow R$.
Suppose that we modify the graph G into G′ as follows. For every edge $e \in E(G)$, we set $w'(e) = w(e)/2$ to be the modified weights in G′. Then, every shortest path from s to t in G is a shortest path from s to t in G′.
II) Let G = (V,E) be a flow network, with a source s, a sink t, and a positive capacity c on every edge e. Suppose (A, B) is the unique s-t minimum cut in G w.r.t these capacities. Consider a modified graph G′ where the capacity of every edge is increased by 1. Then (A, B) is still a minimum (s, t)-cut for G′.
Which of the following statements is/are correct?

A. I only
B. II only
C. Both I and II
D. Neither I or II

Q6. Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.



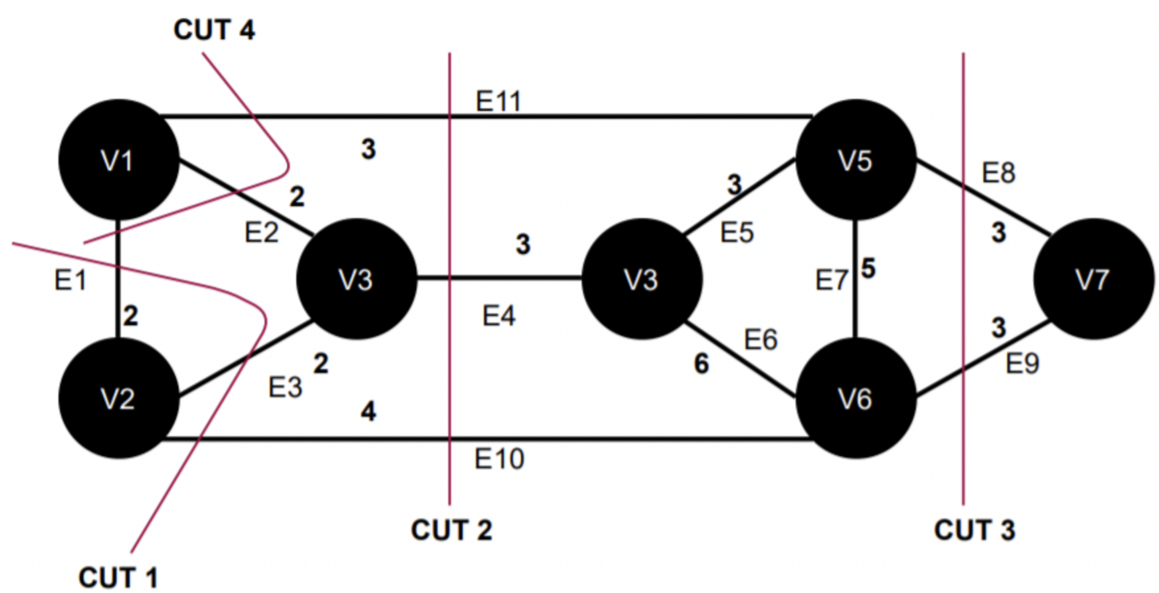
A. SDT
B. SBDT

C. SACDT
D. SACET

Q7. Consider the following statements given below:
S1 : If a graph contains a negative weight cycle then Dijkstra's algorithm may or may not terminate.
S2 : The Bellman-Ford algorithm guarantees that it will always produce a shortest path between two given vertices u and v in any weighted graph.
Which of the above statements are **incorrect**?

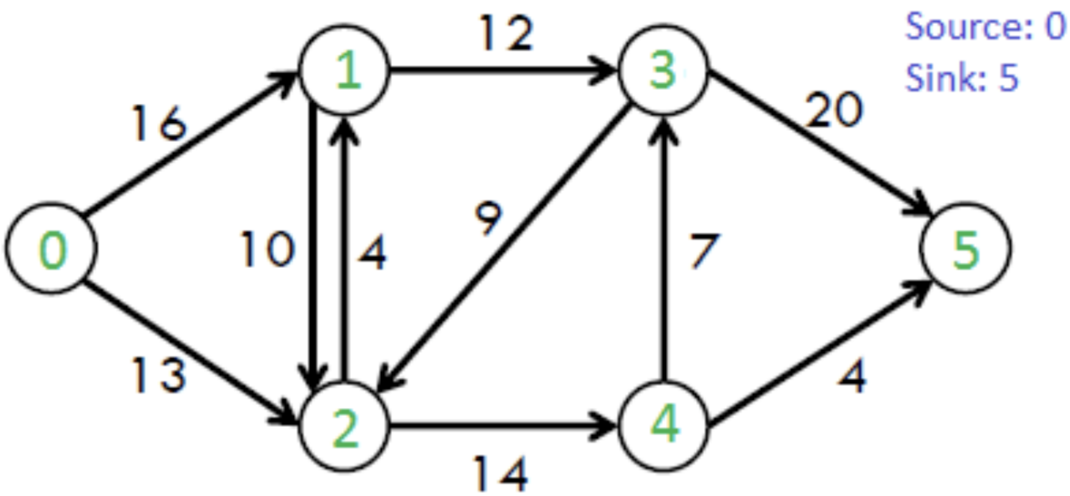
1. Only S1
2. Only S2
3. Both S1 and S2
4. None of these

Q8. Which cut is the minimum cut of the following network?



A. CUT 3
B. CUT 1
C. CUT 2
D. CUT 4
E. None of the above

Q9. The value of max-flow in the following graph is:



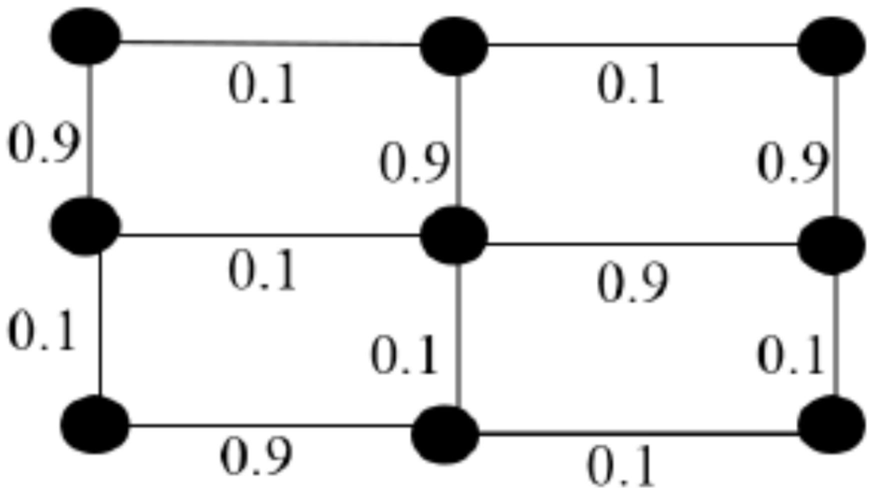
A. 22
B. 23
C. 24
D. 25

Q10. Suppose that f is a feasible but not maximum s, t-flow in a network with nonnegative capacity. Read the statements below and choose the correct statement.

S1. Residual graph Gf will never have a path from s to t that was not a path from s to t in the original graph G.
S2. The residual graph Gf can have a path P∗ from s to t with min{capacity(e) : e ∈P∗} > 0 that is not there at all in the original graph.
S3. The residual graph Gf will always have a path P^* from s to t with min{capacity(e) :e∈P^*} that is not there at all in the original graph.
S4. None of the above

A. S1
B. S2
C. S3
D. S4

Q11. Consider the following undirected graph with edge weights as shown:



The total number of possible minimum spanning trees are-

A. 2
B. 3
C. 4
D. 6

Q12. Read the following statements:

I) In a flow network, the maximum amount of flow passing from the source to the sink is equal to the total weight of the edges in a minimum cut.

II) Let f be a flow on a network (G,c) with net flow v and let C be a vertex cut (S, T) with capacity k. Then v > k.

Which of the above statements is/are correct:

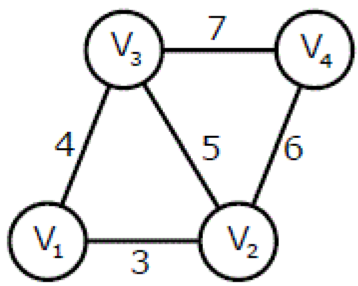
A. Only I
B. Only II
C. Both I and II
D. Neither I nor II

Q13. Each maximum flow defines a minimum capacity cut, using the method discussed in class. Claim: This minimum capacity cut is unique, i.e., for a given maximum flow, there could not be more than one minimum capacity cut for this flow.

A. True

B. False

Q14. An undirected graph G(V, E) contains n (n > 2) nodes named v1 , v2 ,....vn. Two nodes vi , vj are connected if and only if 0 < |i – j| <= 2. Each edge (vi, vj) is assigned a weight i + j. A sample graph with n = 4 is shown below. What will be the cost of the minimum spanning tree (MST) of such a graph with n nodes?



a. (11n^2 – 5n) / 12
b. n^2 – n + 1
c. 6n – 11
d. 2n + 1

Q15. Let G be an undirected connected graph with distinct edge weight. Let e_{max} be the edge with maximum weight and e_{min} the edge with minimum weight. Which of the following statements is false?

A. Every minimum spanning tree of G must contain e_{min}
B. If e_{max} is in a minimum spanning tree, then its removal must disconnect G
C. No minimum spanning tree contains e_{max}
D. G has a unique minimum spanning tree