

MTH 377/577 CONVEX OPTIMIZATION
Winter Semester 2022
Indraprastha Institute of Information Technology Delhi
Problem Set 1: Convex Sets

Q1. (Exercise 2.1 in [BV]). Let $C \subset \mathbb{R}^n$ be a convex set with $x_1, \dots, x_k \in C$. and let $\theta_1, \dots, \theta_k \in \mathbb{R}$ satisfy $\theta_i \geq 0$ and $\theta_1 + \dots + \theta_k = 1$. Show $\theta_1 x_1 + \dots + \theta_k x_k \in C$.

Q2. (Exercise 2.4 in [BV]). Show that the convex hull of a set S is the intersection of all convex sets that contain S .

Q3. (Exercise 2.12 in [BV]). Which of the following sets are convex ?

- (a) A slab i.e. a set of the form $\{x \in \mathbb{R}^n : \alpha \leq \langle a, x \rangle \leq \beta\}$.
- (b) A rectangle i.e. a set of the form $\{x \in \mathbb{R}^n : \alpha_i \leq x_i \leq \beta_i, i = 1, \dots, n\}$.
- (c) An open ball centered at x_0 i.e. a set of the form $\{x \in \mathbb{R}^n : \|x - x_0\| < r\}$.

Q4. Sketch the convex hull of the following set $\{(x, y) : y = x^2, x \in [0, 1]\}$.

Q5. A consumer with income $m > 0$ likes to consume certain quantities of n commodities labeled $1, \dots, n$. Let $x_i \in \mathbb{R}_+$ denote the quantity she may choose to consume of commodity i . When the market prices of commodities are p_1, \dots, p_n , what is the affordable set or the budget set of the consumer ? Is it convex ? Why or why not ? (The affordable set is the consumption vector she can afford at her income.)

Q6. Show that the set $\{(x, y) \in \mathbb{R}_+^2 : y \leq \sqrt{1 - x^2}\}$ is a convex set. (Remarks: A diagram is not enough.)

Q7. On an exact graph paper, draw the line $2x + y = 4$. Is this line is a hyperplane in \mathbb{R}^2 ? If yes, write it as $H(a, b)$ specifying the normal vector a and the scalar b . On the same graph, draw the normal vector a . How would you verify that the vector a is indeed normal/orthogonal to the line-hyperplane $H(a, b)$.

Q8. On an exact graph paper, draw the hyperplane that goes through the point $(3, 0)$ and has the normal vector $(1, 1)$. Indicate the positive and the negative halfspaces associated with it. What is the subspace associated with this hyperplane ?