~~$\{x_1, x_2, \ldots, x_n\}$~~

Ans. 1. Strategic game

Players : $N = \{1, 2\}$ or $\{$sender, receiver$\}$

$A_1 = \{L, R\}$

type of player 1, $\theta_1 \in \{t_1, t_2\}$

strategy of player 1 $s_1 : \{t_1, t_2\} \longrightarrow \{L, R\}$.

$A_2 = \{u, d\}$.

Payoffs: as given in diagram.

a) For separating equilibrium:

Let $s_1(t_1) = L$, $s_1(t_2) = R$

then $p = 1$, $q = 0$.

$a_2(L) = u$, $a_2(R) = u$

$u_1(L, u; t_1) = 3 > u_1(R, u; t_1) = 1$

$u_1(R, u; t_2) = 3 > u_1(L, u, t_2) = 2$

$\therefore$ 1 has no incentive to change his strategy.

$u_2(p=1, u) = 2$. belief is ~~confirmed is~~ consistent.

$u_2(q=0, u) = 3 > 0$ belief is consistent.

~~u is dominant action for player.~~
~~for both~~

$\therefore$ separating PBE.

$s_1 = \begin{cases} L \text{ if } \theta = t_1 \\ R \text{ if } \theta = t_2 \end{cases}$

$a_2 = u$ for both L & R.

$(\checkmark LR \checkmark, \checkmark u \checkmark u ; p=1, q=0)$ is the unique separating PBE.

consider ~~ano~~ another possibility:

S.t. $s_1 = \begin{cases} R & \text{if } t_1 \\ L & \text{if } t_2 \end{cases}$

then $p=0$, $q=1$.

$a_2(L) = d$, $a_2(R) = d$

0 Here $u_1(L, d; t_1) = 2 > u_1(R, d; t_1)$

$\therefore$ 1 has incentive to change her action.

$\therefore$ Not ~~PBE~~. Perfect Bayesian Equilibrium.

b) Pooling equilibrium:

Suppose $s_1 = \begin{cases} L & \text{if } t_1 \\ L & \text{if } t_2 \end{cases}$

or $s_1 = L \ \forall \ \theta \in \{t_1, t_2\}$.

$a_2(L) = u$ since if $p = 1/2$:

$\frac{1}{2} \times 2 + \frac{1}{2} \times 0 > \frac{1}{2} \times 0 + \frac{1}{2} \times 1$

$u_1(L, u; t_1) = 3 > u_1(R, u; t_1) = 1$

$u_1(L, u; t_2) = 2 < u_1(R, u; t_2) =$

This cannot be pooling equilibrium unless ~~a\in R~~ $a_2(R) = d$.

$p = 1, q = 2$

$\rightarrow q$ or $(1-q)$

Player 2 sees R and thinks he is at

$a_2(R) = d$ when

$q \times 1 + (1-q) \cdot 1 \geq q \cdot 0 + (1-q) \times 3$

$\Rightarrow 1 \geq 3 - 3q$

$\Rightarrow 3q \geq 2$

1 $\Rightarrow q \geq 2/3$

1.5 $\therefore$ $(LL, ud; p = \frac{1}{2}, q \geq 2/3)$

$\therefore$ is a pooling PBE.

$\Rightarrow$ ~~this~~ R is strictly dominated by L for t_1 but not for t_2.

$\therefore$ this equilibrium is not **sequentially rational**.

Ans. 2. Co-author model: True — 1

$i \in N$, $d_i(g)$: no. of ~~con~~ direct links for i.

g: network

$g = \{ ik, jk \in\}$ if they have

1 $$u_i(g) = \sum_{j : ij \in g} \left[\frac{1}{d_i(g)} + \frac{1}{d_j(g)} + \frac{1}{d_i(g) d_j(g)} \right]$$

for $d_i(g) > 0$, $u_i(g) = 1$ if $d_i(g) = 0$

For efficiency, we calculate the total utility for all the agents

$$\sum_{i \in N} u_i(g) = \sum_{i : d_i(g) > 0} \sum_{j : ij \in g} \left[\frac{1}{d_i(g)} + \frac{1}{d_j(g)} + \frac{1}{d_i(g) d_j(g)} \right]$$

1

$\frac{1}{2} \geq 1$

$\frac{1}{3} + \frac{1}{5} \leq 1$

n

(1) $\sum_{i\in N} u_i(g) \leq 2n + \sum_{i: d_i(g)>0} \sum_{j: ij\in g} \frac{1}{d_i(g)\, d_j(g)}$

equality holds only if $d_i(g) > 0$
$\forall\ i \in N$

(3) maximised when $\sum\sum \frac{1}{d_i(g) d_j(g)} \leq n$

with equality at $d_i(g) = d_j(g) =$

$\therefore$ Highest value of $\sum_{i\in N} u_i(g) = 3n$.

which occurs when $d_i(g) = 1$
$\forall\ i \in N$.

Ans. 3. $N = \{1, 2\}$ $\quad S_1 = S_2$

$S_i = [0, 1]$

when players are equally likely to get the good in case of a tie.

(3)

$$u_i(s_i, s_j) = \begin{cases} 1 - s_i & \text{if } s_i > s_j \\ \frac{1}{2} - s_i & \text{if } s_i = s_j \\ -s_i & \text{if } s_i < s_j \end{cases}$$

The game has no pure strategy Nash equilibrium.

If $s_i = s_j < 1$ then each player will raise their bid

If $s_i = s_j = 1$ then both prefer to bid 0 as $\frac{-1}{2} < 0$.

8 $1 - s_i$

$s_i = s_j$
$s_i > s_j$
$s_i < s_j$

If $s_i > s_j \geq 0$ then ~~j can~~ i ~~can~~ would want to lower his bid.

Assn 2: If $s_i = s_j$ results in no auction, then since its all pay, both get negative payoff, ~~or both can~~

(3) or if $s_i = s_j < 1$ then both will raise the ~~bid~~!

Assn 3: If $s_i = s_j$ ~~then~~ and payoff is 0

[s_i is not paid if $s_i = s_j$]

then If ~~$s_i = s_j = 1$~~ ~~then~~ equilibrium

If $s_i = s_j < 1$ then both raise their bid.

Ans. 4. $N = \{1, 2\}$ → 1

$A_i = \{MP, MPD\} \times \{H, T\} \times \{C, D\}$
$\forall i \in N$. 2

$t = 1$: Expected payoff from matching pennies at $(\sigma_1, \sigma_2) = ((1/2, 1/2), (1/2, 1/2))$ for each $i \in N$ is :

$$\frac{1}{2} \times \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{2} \times (-1) + \frac{1}{2} \times \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{2} (-1)$$
$$= 0.$$

In Prisoner's Dilemma, payoff of 1 is received from N.E.

3

$\therefore \{(MPD, D), (MPD, D)\}$ is a Nash Equilibrium.

then $s_i = s_j = 1$ is equilibrium.

(C, C) is sustained in infinite game when:

$$u_i(C, C) = 3 + 3\delta + 3\delta^2 + \dots$$

$$= \frac{3}{1-\delta}$$

$$u_i(\text{deviation } t, \text{on}) = 10 + 1\cdot\delta + 1\cdot\delta^2 + 1\cdot\delta^3 + \dots$$

$$= 10 + \frac{1}{1-\delta}$$

$$\frac{3}{1-\delta} > 10 + \frac{1}{1-\delta}$$

Date

If the game is played infinitely & $t=1$ onwards, we can get equilibrium

~~{(mPD, C), (mP~~

2 $\{(mPD, C, C, C, \dots), (mPD, C, C, C, \dots)\}$

For specific $\delta > 0$ using trigger strategy.

trigger strategy $\begin{cases} \text{choose C if other chooses C} \\ \text{if other chooses D in any foremost } t \text{ then always choose D.} \end{cases}$

Ans. 5. $x = (1/3, 1/3, 1/3)$ is equilibrium but it is not stable. 1

Projection matrix $\Phi = I - \frac{1}{n}\mathbf{1}\mathbf{1}'$, $\Phi \in R^{n \times n}$

In Rock-paper-scissors $n = 3$

$$\therefore \Phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1

$$= \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$F(x) = Ax = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

1

$x = (x_1, x_2, x_3)$

$$= \begin{bmatrix} x_3 - x_2 \\ x_1 - x_3 \\ x_2 - x_1 \end{bmatrix}$$

1

$$\phi F(x) = \begin{bmatrix} x_3 - x_2 \\ x_1 - x_3 \\ x_2 - x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

At $x = (1/3, 1/3, 1/3)$ there is no incentive to switch.

Consider $x' = (1/2, 1/4, 1/4)$:

1

$$\phi F(x) = \begin{bmatrix} 1/4 - 1/4 \\ 1/2 - 1/4 \\ 1/4 - 1/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/4 \\ -1/4 \end{bmatrix}$$

which implies x_3 will move to x_2.
we need x_1 to move to x_2 and x_3 $\therefore$ It is not converging to $x^* = (1/3, 1/3, 1/3)$.

$1/3, 1/3, 1/3$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

~~(0, 1, 0)~~.