**Instructions:**
- Do not plagiarize. Do not assist your classmates in plagiarism.
- Show your full solution for the questions to get full credit.
- Attempt all questions that you can.
- In the unlikely case a question is not clear, discuss it with an invigilating TA. Please ensure that you clearly include any assumptions you make, even after clarification from the invigilator.

1. (1 point) A chessboard is an $8 \times 8$ grid. A *rook* can move on a chessboard any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces. Is the *Manhattan distance* an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves?{*Hint*: Manhattan distance is the sum of the number of vertical squares and the number of horizontal squares between any two squares A and B.}

   **Solution**:
   **False**. A rook can move across the board in move one, although the Manhattan distance from start to finish is 7.

2. (2 points) Prove each of the following statements by specifying the evaluation function $f(n)$, or give a counterexample.
   (a) (0.5 point) Breadth-first search is a special case of A* search.
   (b) ($3 \times 0.5 = 1.5$ points) Breadth-first search, Depth-first search and Uniform-cost search are all special cases of Best-first search.

   **Solution**:

   (a) When all step costs are equal, $g(n) \propto depth(n)$, and $h(n) = 0$, A* search reproduces breadth-first search.
   (b) BFS: $f(n) = depth(n)$; DFS: $f(n) = -depth(n)$; UCS:$f(n) = g(n)$.

   **Rubric**: 50% points for a correct evaluation function example, and 50% for the explanation / proof.

3. (3 points) A uniform distribution is one where every value that a random variable can possibly take is equally likely, i.e., has equal probability.
   (a) ($2 \times 0.5 = 1$ points) Give an example each of a discrete random variable and a continuous random variable that is uniformly distributed.
   (b) ($2 \times 0.5 = 1$ points) Draw the CDF for both, the discrete and continuous cases. {*Note*: No partial credit if the axes are not labeled correctly with `min` and `max` values.}
   (c) ($2 \times 0.5 = 1$ points) Draw the PMF/PDF for discrete and continuous cases.
   {*Note*: No partial credit if the axes are not labeled correctly with `min` and `max` values.}

   **Solution**:

   (a) A fair die is an example of a discrete, uniform random variable, with the Probability Mass Function (PMF) as

$$U_D(d) = \begin{cases} \frac{1}{6}, & d = \{1, 2, 3, 4, 5, 6\} \\ 0, & \text{otherwise} \end{cases}$$

   The temperature in Delhi for the month of October is uniformly distributed over the interval 25-35 degrees Celcius, i.e., the Probability Density Function (PDF) is given as

$$U_T(t) = \begin{cases} \frac{1}{35-25}, & t \in [25, 35] \\ 0, & \text{otherwise} \end{cases}$$

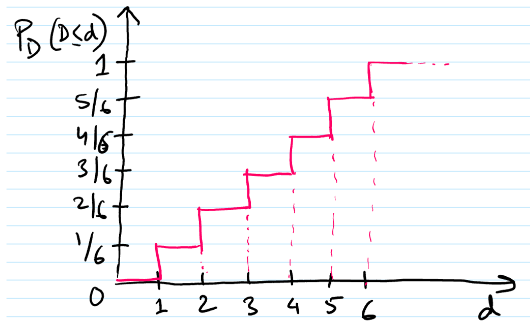


Figure 1: Discrete Uniform CDF

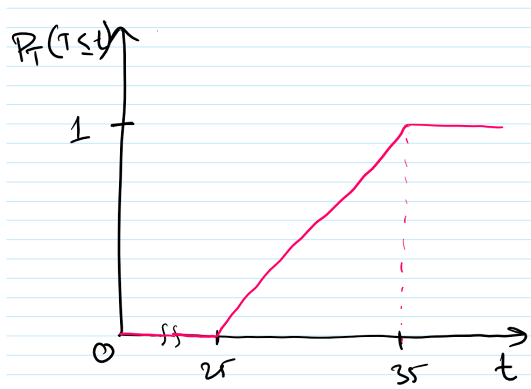


Figure 2: Continuous Uniform CDF

(b) Discrete Uniform CDF (Fig.1) and Continuous Uniform CDF (Fig. 2).

(c) Discrete Uniform PMF (Fig.3) and Continuous Uniform PDF (Fig. 4).

4. (2 points) For each of the following statements, either prove it is true or give a counterexample.

   (a) (1 point) If $P(a|b,c) = P(b|a,c)$, then $P(a|c) = P(b|c)$

   (b) (1 point) If $P(a|b,c) = P(a)$, then $P(b|c) = P(b)$

**Solution**:

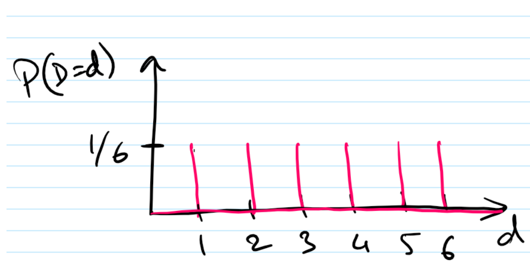


Figure 3: Discrete Uniform PMF

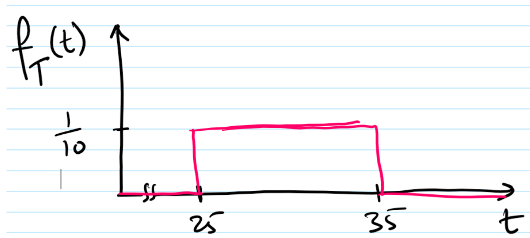


Figure 4: Continuous Uniform PDF

(a) **True**. By the product rule we know $P(a,b,c) = P(b,c)P(a|b,c) = P(a,c)P(b|a,c)$, which by assumption reduces to $P(b,c) = P(a,c)$. Dividing through by $P(c)$ gives the result, i.e., $P(b,c)/P(c) = P(b|c) = P(a,c)/P(c) = P(a|c)$.

(b) **False**. While the statement $P(a|b) = P(a)$ implies that $a$ is independent of $b$, it does not imply that $a$ is conditionally independent of $b$ given $c$. A counter-example: $a$ and $b$ record the results of two independent coin flips, and $c$ equals the *xor* (or sum if coin flips lead to $\{0,1\}$) of $a$ and $b$.