ECE 351 DSP: Practice Problems 3

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1) Let $x[n]$ be an N-point sequence where N is divisible by 3. You need to design a one-stage radix-3 decimation in time algorithm to calculate the N-point DFT $X(k)$ by using 3 different $\frac{N}{3}$-point DFTs $F_1(k), F_2(k), F_3(k)$ of signals $f_1[n], f_2[n], f_3[n]$.
   a) Express $f_1[n], f_2[n], f_3[n]$ in terms of $x[n]$.
   b) Express $X(k)$ in terms of $F_1(k)$, $F_2(k)$, and $F_3(k)$.

2) Let $x[n]$ be any N-point sequence and let $y[n]$ be an $3N$-point sequence defined as $y[n] = x[\frac{n}{3}]$ whenever n is a multiple of 3, and $y[n] = 0$ otherwise. Express the $3N$-point DFT $Y(k)$ of $y[n]$ in terms of the N-point DFT $X(0), X(1), \ldots, X(N-1)$ of $x[n]$.

3) Consider the difference equation $y[n] = \frac{1}{2}y[n-1] - y[n-2] + 2y[n-3] + x[n] - \frac{1}{3}x[n-1] + 2x[n-2]$.
   a) Draw the lattice-ladder representation of this system.
   b) Is this system stable?

4) Consider the system given in Figure 1. Suppose you wish to compute the output of this system given an input $x[n] = [1, -1, 2, 4]$ using DFT, multiplication, and IDFT. What is the minimum number of points (i.e., N) for the DFT needed to successfully do this?

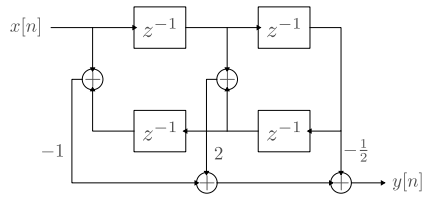

Fig. 1: Figure for Q.4

5) Figure 2 shows the transpose form representation of a system. Find its impulse response.

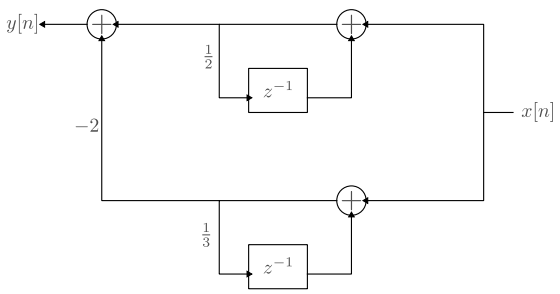

Fig. 2: Figure for Q.5

I. Answers

1) a)

$$f_1[n] = x[3n]$$
$$f_2[n] = x[3n+1]$$
$$f_3[n] = x[3n+2],$$

where $0 \leq n \leq \frac{N}{3} - 1$.

b)

$$X(k) = F_1(k) + F_2(k)W_N^k + F_3(k)W_N^{2k}$$
$$X(k + \frac{N}{3}) = F_1(k) + e^{-j\frac{2\pi}{3}}F_2(k)W_N^k + e^{-j\frac{4\pi}{3}}F_3(k)W_N^{2k}$$
$$X(k + \frac{2N}{3}) = F_1(k) + e^{-j\frac{4\pi}{3}}F_2(k)W_N^k + e^{-j\frac{8\pi}{3}}F_3(k)W_N^{2k},$$

where $W_N \triangleq e^{-j\frac{2\pi}{N}}$ and $0 \leq k \leq \frac{N}{3} - 1$.

2)

$$Y(k) = \begin{cases} X(k), & 0 \leq k \leq N-1 \\ X(k-N), & N \leq k \leq 2N-1 \\ X(k-2N), & 2N \leq k \leq 3N-1. \end{cases}$$

3) a) See Figure 3.

b) No.

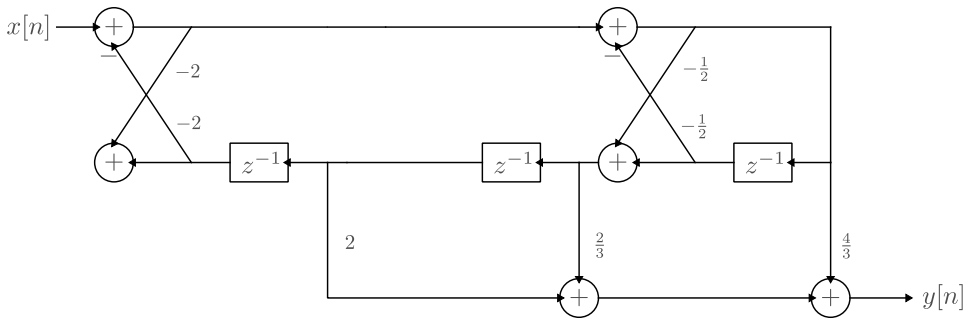

Fig. 3: Answer for Q3.a)

4) 8.

5) $h[n] = (\frac{1}{2})^n u[n] - 2(\frac{1}{3})^n u[n]$.