MULTIVARIATE CALCULUS
QUIZ 2

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Abstract. This is the second of the two quizzes that you have to take for the successful completion of this course at IIIT-Delhi.

This is a closed book test and you are supposed to be done with it in thirty minutes at most.

1. Problem

Evaluate

$$\iint_S x^2y^2\,dA,$$

where S is the region in the first quadrant bounded by $xy = 1$, $xy = 2$, $x = y$ and $x = y/4$.

[*10 points*]

2. Problem

Let

$$F(x, y) = \frac{x}{x^2 + y^2}\vec{j} - \frac{y}{x^2 + y^2}\vec{i},$$

defined on $\mathbb{R}^2 \setminus \{(0, 0)\}$.

(a) Compute

$$\frac{\partial F_1}{\partial y} \text{ and } \frac{\partial F_2}{\partial x}.$$

What do you conclude from this?

(b) Compute

$$\int_C F \cdot dr$$

where C is the curve from $(a, 0)$ to $(-a, 0)$ following the circle $x^2 + y^2 = a^2$ in

(i) counterclockwise sense;

(ii) clockwise sense.

(c) Why is this *seemingly* incorrect?

[*20 points*]

3. Problem

Consider F as defined in Problem 3, restricted to the set $\{(x, y) \in \mathbb{R}^2 : x > 0\}$. Can you assert that F is conservative? If yes, find a potential function for it. If no, why?

[*10 points*]

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4. Problem

Find the area bounded by $r(t) = (\sin 2t, 2\sin t)$, $t \in [0, \pi]$ in $\mathbb{R}^2$ using Green's theorem. [*10 points*]

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