MTH 377/577 Convex Optimization Problem Set 2

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- 1. Let $A \in \mathbb{R}^{m \times n}$ and $C = \{x \in \mathbb{R}^n : Ax \leq 0\}$. Is C a convex cone? Show why/why not.
- 2. Is the intersection of affine sets (i) affine (ii) convex? Provide a formal argument.
- 3. A function $f: \mathbb{R}^n \to \mathbb{R}$ is convex if and only if its epigraph is a convex set. True/False? Provide a formal argument/proof.
- 4. Let $P = \{(x_1, x_2) : x_1 + x_2 \leq 4\}$ and $Q = \{(2, 2)\}$. Let C be the finite cone generated by vectors (-1, 1), (1, -1), (-1, -1). Show that if $x \in Q + C$, then $x \in P$.
- 5. Suppose C is a convex set. Is the following function convex?

$$G(x) = \begin{cases} 0 \text{ if } x \in C \\ \infty, \text{ if } x \notin C \end{cases}$$