MTH 204 MidSem Exam

Maximum Points: 30 (Maximum Time: 60 mins)

March 8, 2021

Question 1.

(2+2=4 points) Fill in the blanks (No need to show your work).

1. For the ODE
$$\left(\frac{d^6y}{dx^6}\right)^{\frac{1}{2}} = \sin\frac{d^3y}{dx^3}$$
the order is ______ and the degree is ______.

2. The ODE
$$(2y + x^2)\ dx + (ax + by)\ dy = 0$$
is exact for $a =$ ______ and $b =$ ______.

Question 2.

(1+1+1+1+1=5 points) Mention if the following statements are True or False (No need to show your work).

1. The equation
$$y'''' + 2y = x$$
is non-linear.

2. The equation
$$y' + 2xy = 0$$
is non-autonomous.

3. The equation
$$y'' + xy' + x^2(y + 1) = 0$$
is non-homogeneous.

4. The equation
$$x^2y'' + y = 0$$
is an Euler-Cauchy equation.

5. If we draw the direction field for the equation
$$y' = 2$$
then all vectors will point in the same direction.

Question 3.

(1+2+2+1=5 points) (Show your work for full points) Consider the ODE

$$2y\ dx + x\ dy = 0. \tag{1}$$

1. Show that it is not exact.
2. Find a function $F(x)$ such that multiplying $F(x)$ into (1) will change it to an exact ODE.
3. Find a function $u(x, y)$ such that the resulting exact ODE after multiplying $F(x)$ into (1) can be written in the form
$$du = 0.$$
4. Write the general solution of (1).

Question 4.

(2+2=4 points) (Show your work for full points) Consider the autonomous ODE

$$\frac{dy}{dx} = y^3 - 6y^2 - y + 30.$$

One of its equilibrium solution is $y = -2$.

1. Find the other two equilibrium solutions.
2. Classify all three equilibrium solutions as stable or unstable.

Question 5.

(1+1+1+1+1+1=6 points) (Show your work for full points) A spring with a mass of 10 kg has natural length 1 m. A force of 30 N is required to maintain it stretched to a length of 1.5 m. The spring is stretched to a length of 1.5 m and then released with initial velocity zero.

1. Find the spring constant k.
2. Find the natural frequency ω of the mass-spring constant.
3. Write the governing ODE for this mass-spring system.
4. Write the general solution of this ODE.
5. Write two initial conditions from the problem descriptions above for this ODE.
6. Find the two arbitrary constants in the general solutions using above initial conditions.

Question 6.

(1+3+3+1+3=11 points) (Show your work for full points) Consider the initial value problem (IVP)

$$y''' + 3y'' + 3y' + y = 6e^{-x}, \quad y(0) = -5, \quad y'(0) = 1, \quad y''(0) = -12.$$

1. Write the corresponding homogeneous equation for the IVP.
2. Find the characteristic equation, solve it, and write the general solution of the homogeneous ODE.
3. Find a particular solution of non-homogeneous equation using method of undtermined coefficients.
4. Write the general solution of the non-homogeneous ODE with three arbitrary constants c_1, c_2, and c_3.
5. Find c_1, c_2, and c_3 using three initial conditions.