SOLUTION

MTH210 – SUBMISSION_20221013

TIME: 15 minutes MARKS: 5

No consultation – open notes – books and internet not allowed.

For $n \in \mathbb{Z}^+$, $n \geq 2$, put $X_n = \{1,2,\ldots,n\}$ and put $V = \{0,1\}$. How many functions are there from X_n to V, which :

a) Are injective ? (*1 mark*)

b) Assign 0 to both 1 and n ? (*2 marks*)

c) Assign 1 to exactly one of the positive integers < n ? (*2 marks*)

Note: *Show your calculations if any, and give a brief explanation of your approach (one or two sentences).*

ID: NAME: GROUP:

a) For $n=2$, answer $= 2$

For $n \geq 3$, answer $= 0$

Explain: If $n=2$, $|V| = |X_n|$, so the answer is same as no. of permutation, i.e. $2! = 2$

If $n \geq 3$, then cannot be any injective functions.

Alternate Explanation: Can be obtained from the formula in Proposition 15 with $m = 2$.

(PTO)

(b) Answer:- 2^{n-2}

~~An~~ Explanation: Any such function can be regarded as a function from the set $X' = \{2, \ldots, n-1\}$ to V, since the values at ~~x~~ 1 and n are fixed. Since $|X'| = n-2$, we get the answer by applying the formula of Proposition 14 with $|X| = n-2$ and $|Y| = 2$.

(c) Answer:- ~~∞~~ $2(n-1)$.

Explain: Any such function can be regarded as acting ~~as~~ like the characteristic function of a singleton subset of X_{n-1}, with the value at n having 2 possible choices, 0 or 1. Since there are $(n-1)$ distinct singleton subsets of X_{n-1}, we can choose the function in $(n-1) \times 2 = 2(n-1)$ ways.

Alternative: Apply the result of Q5/TUT05 - 20221006.