Quiz-10 (Solutions)

Sol.

(a) $x(t)$ is real $\rightarrow$ Complex poles always come with conjugate.

$\rightarrow$ $x(t)$ is even

$$x(t) = x(-t)$$

$$X(s) = X(-s).$$

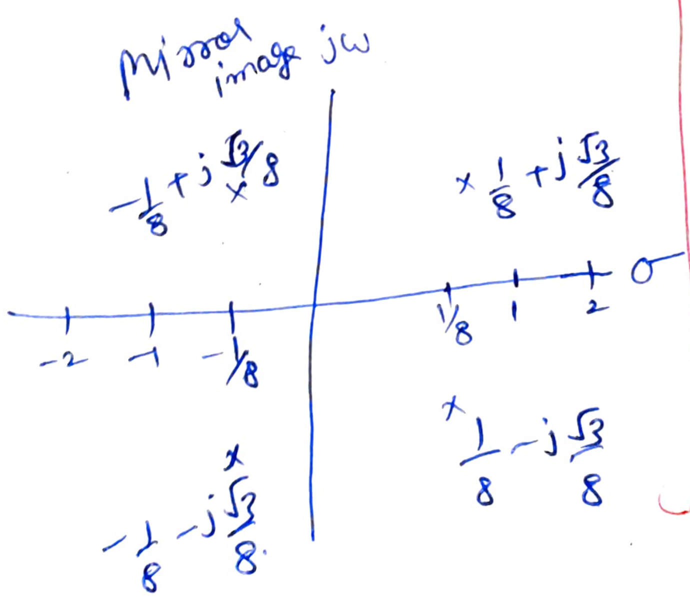
$\rightarrow$ One pole at $s = \frac{1}{4} e^{j\pi/3}$

$$= \frac{1}{4} \cos\left(\frac{\pi}{3} + j \sin\frac{\pi}{3}\right)$$

$$= \frac{1}{4}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{8} + j\frac{\sqrt{3}}{8}$$

So, another pole at $s = \frac{1}{8} - j\frac{\sqrt{3}}{8}$



Sol)

$$X(s) = \frac{A}{\left(s+\frac{1}{8}-j\frac{\sqrt{3}}{8}\right)\left(s+\frac{1}{8}+j\frac{\sqrt{3}}{8}\right)\left(s-\frac{1}{8}-j\frac{\sqrt{3}}{8}\right)\left(s-\frac{1}{8}+j\frac{\sqrt{3}}{8}\right)}$$

2 marks.

$$X(s) = \frac{A}{\left[\left(s+\frac{1}{8}\right)^2 - j^2\left(\frac{\sqrt{3}}{8}\right)^2\right]\left[\left(s-\frac{1}{8}\right)^2 - j^2\left(\frac{\sqrt{3}}{8}\right)^2\right]}.$$

$$= \frac{A}{\left(s^2+\frac{s}{4}+\frac{1}{64}+\frac{3}{64}\right)\left(s^2-\frac{s}{4}+\frac{1}{64}+\frac{3}{64}\right)}.$$

$$= \frac{A}{\left[\left(s^2+\frac{1}{16}\right)+\frac{s}{4}\right]\left[\left(s^2+\frac{1}{16}\right)-\left(\frac{s}{4}\right)\right]}.$$

$$= \frac{A}{\left(s^2+\frac{1}{16}\right)^2-\left(\frac{s}{4}\right)^2}$$

$$= \frac{A}{s^4+\frac{s^2}{8}+\frac{1}{256}-\frac{s^2}{16}}$$

$$X(s) = \frac{A}{s^4 + \frac{s^2}{16} + \frac{1}{256}}$$

$$X(s) = \frac{A}{\frac{256s^4 + 16s^2 + 1}{256}}$$

1 mark

$$X(s) = \frac{A(256)}{256s^4 + 16s^2 + 1}.$$

Another Condition

$$\int_{-\infty}^{\infty} x(t)\,dt = 8$$ → Absolutely integrable.

So. $X(0) = 8$.

$$X(0) = \frac{A(256)}{0 + 0 + 1}.$$

2 Marks

$$\frac{8}{256} = A.$$

$$\boxed{A = 1/32}$$

$$\boxed{\text{So, } X(s) = \frac{256 \times \frac{1}{32}}{256s^4 + 16s^2 + 1} = \frac{8}{256s^4 + 16s^2 + 1}.}$$

→ ROC contain $j\omega$ axis.

$$\boxed{\text{So, } -\frac{1}{8} < Re\{s\} < \frac{1}{8}}$$ 1 mark.

(b) $X(s)$ can be written as.

$$X(s) = \frac{A}{(s+a)(s-a)(s+a^*)(s-a^*)}.$$

where

$$A = \frac{1}{32}, \quad a = \frac{1}{8} + j\frac{\sqrt{3}}{8}$$

$$\frac{A}{(s+a)(s-a)(s+a^*)(s-a^*)} = \frac{C_1}{s+a} + \frac{C_2}{s-a} + \frac{C_3}{s+a^*} + \frac{C_4}{s-a^*}$$

$$C_1 = \frac{A}{(-2a)(-a+a^*)(-a-a^*)}, \quad C_2 = \frac{A}{(2a)(a+a^*)(a-a^*)}$$

$$C_3 = \frac{A}{(a-a^*)(-a^*-a)(-2a^*)}, \quad C_4 = \frac{A}{(a+a^*)(a^*-a)(2a^*)}.$$

2 marks

Put the values of A and a to compute C_1, C_2, C_3, C_4.

$$x(t) = C_1 e^{-at} u(t) + C_3 e^{-a^* t} u(t)$$
$$- C_2 e^{at} u(-t) - C_4 e^{a^* t} u(-t)$$

2 marks