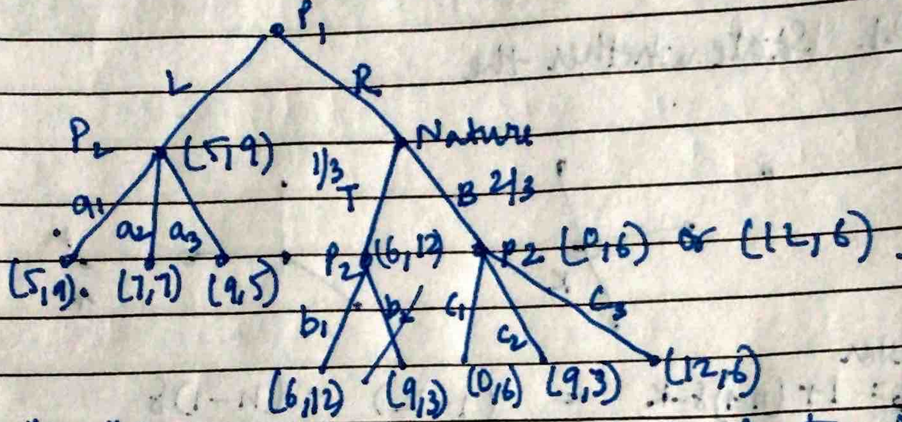
Ans. 1. False.

		Player 2	
		a	b
Player 1	A	(3, 4)	(2, 0)
	B	(0, 0)	(2, 2)

Action B for player 1 is weakly dominated. But (B, b) is a Nash equilibrium.

Ans. 2



Let (s_1^*, s_2^*) be a subgame perfect strategy profi

$$u_1(R, s_2^*) = \frac{1}{3}\times 6 + \frac{2}{3}\times\{(0) \text{ or } 12\}$$

$$= 2 + \left(\frac{2}{3}\times 0 \text{ or } \frac{2}{\cancel{3}}\times \cancel{12}^{4}\right)$$

$$= 2 + [0 \text{ or } 8]$$

$$= 2 + \frac{2}{3}\{p\cdot 0 + (1-p)\,12\}$$

$$= 2 + \frac{2(1-p)}{3}\times 12$$

$$= 2 + \frac{2(1-p)\times 4}{\cancel{3}}$$

$$= 2 + 8(1-p).$$

$$u_1(L, s_2^*) = 5$$

$$2 + 8(1-p) \geq 5$$

$$\Rightarrow \quad 8 - 8p \geq 3$$

$$\Rightarrow \quad 8p \leq 5$$

$$\Rightarrow \quad p \leq 5/8$$

~~s_1^*~~

~~Let p be the prob. that player 1~~

Suppose player 1 believes that player 2 will choose c_1 with prob. p, and c_3 ~~prob. wit~~ with prob $(1-p)$.

$$s_1^* = \begin{cases} R & \text{if} \quad p \leq 5/8 \\ L & \text{otherwise} \end{cases}$$

$$s_2^* = \begin{cases} a_1 & \text{if} \quad s_1 = L \\ b_1 & \text{if} \quad T \\ c_1 \text{ with prob. } p,\ c_3 \text{ with prob } (1-p) & \text{if } R. \end{cases}$$

Ans. 3. Yes. $\frac{1}{2}$ (Monopoly profits) is greater than min max value. Show steps as demonstrated in class for prisoner's dilemma. Payoffs ~~are~~ for 1 period are:

~~cournot~~

Cournot model:

$Q = a - P$

$P = a - Q$ [Inverse demand]

$P = a - (q_1 + q_2)$ [2 firms' output].

For firm 1:

Revenue $= Pq_1 = aq_1 - (q_1 + q_2)q_1$

Profit for firm 1, $\pi_1 = aq_1 - (q_1 + q_2)q_1 - cq_1$

$$\frac{d\pi_1}{dq_1} = a - 2q_1 - q_2 - c$$

π_1 is max at $d\pi_1/dq_1 = 0$ [first order condition]

$\therefore\ a - 2q_1 - q_2 - c = 0$ at $q_1 = q_1^*$

$\Rightarrow\ 2q_1 = a - q_2 - c$

$\Rightarrow\ q_1 = \frac{a - q_2 - c}{2}$ at $q_1 = q_1^*$

$$R_1(q_2) = \frac{a - q_2 - c}{2}$$

Similarly $R_2(q_1) = \frac{a - q_1 - c}{2}$

Solving for q_1^*, q_2^*, we get $q_i^* = \frac{a-c}{3}$

$$P = a - 2\frac{(a-c)}{3} = \frac{a+2c}{3}$$

Profit for firm i, $i \in \{1,2\}$ is

$$\pi_i = p(q_i - c) = \left(\frac{a+2c}{3}\right)\left(\frac{a-c}{3} - c\right).$$

$$= \left(\frac{a+2c}{3}\right) \cdot \left(\frac{a-4c}{3}\right).$$

For Monopoly, $Q = a - P$

Inverse demand : $P = a - Q$

Profit $\pi_m = aQ - Q^2 - cQ$

$$\frac{d\pi_m}{dQ} = a - 2Q - c$$

by First order Condition $Q = Q^*$ at $\frac{d\pi_m}{dQ} = 0$

$$\therefore a - 2Q^* - c = 0 \Rightarrow Q^* = \frac{a-c}{2}$$

$$\therefore P = a - \frac{(a-c)}{2} = \frac{a+c}{2}$$

$$\pi_m = P(Q-c) = \left(\frac{a+c}{2}\right)\left(\frac{a-c}{2} - c\right)$$

$$= \left(\frac{a+c}{2}\right)\left(\frac{a-3c}{2}\right).$$

Half of monopoly profit is

$$\frac{1}{2}\left(\frac{a+c}{2}\right)\left(\frac{a-3c}{2}\right) = \frac{1}{8}(a+c)(a-3c).$$

Suppose $\frac{1}{8}(a+c)(a-3c) > [\pi_i \text{ under cournot}]$

Note that $\pi_m > \frac{1}{2}\pi_m > \pi_i$ under cournot

$\therefore$ at appropriate $\delta > 0$, $\frac{1}{2}\pi_m$ can be sustained in repeated game.

[Refer Folk theorem and find δ]

[show all steps]

Ans. 4. See lecture slides 171 - 175

[2 player signaling game].