MTH 372: Quiz I

Instructor: Monika Arora

Instructions

- Show all your work to score full marks.
- You can use the following information
    1. The pdf of X from Gamma (α, β) is given by

$$f(X) = \frac{\beta^\alpha}{\Gamma\alpha}\, x^{\alpha-1}\, e^{-\beta x} \quad 0 < x < \infty, \;\; \alpha > 0, \;\; \beta > 0.$$

1. (5 points) Let X_1, X_2 be iid from Poisson $(\lambda), \lambda > 0$. Consider a statistic $T = X_1 + 2X_2$. Using the formal definition of sufficiency, verify if the statistic $T = X_1 + 2X_2$ is sufficient for λ.

2. (10 points) Let $X_1, \ldots, X_n$ be iid from Gamma $(\alpha, \beta), \alpha > 0, \beta > 0$. Answer the following
    (a) Does it belong to the exponential family of distributions.
    (b) Find sufficient statistic(s) for $\boldsymbol{\theta} = (\alpha, \beta)$.

3. (5 points) Suppose $X_1, \ldots, X_n$ be iid from Uniform $(\theta - \frac{1}{2}, \theta + \frac{1}{2})$ distribution where $\theta > 0$ is unknown. Find sufficient statistic(s) for θ.