MTH-204: Worksheet 3

8 February, 2023

1. **Exactness.** Is the following ODE exact? If yes, solve the ODE. $\qquad$ (1)

$$(2x + 3y)\mathrm{d}x + (3x + 4y)\mathrm{d}y = 0$$

2. **Epidemics.** A model for the spread of contagious diseases is obtained by assuming that the rate of spread is proportional to the number of contacts between infected and non-infected persons, who are assumed to move freely among each other. Set up the model. Find the equilibrium solutions and indicate their stability or instability. Solve the ODE. Find the limit of the proportion of infected persons as t $\to \infty$ and explain what it means. (2)

3. **Harvesting renewable resources. Fishing.** Suppose that the population $y(t)$ of a certain kind of fish is given by the logistic equation (2)

$$y' = Ay - By^2$$

and fish are caught at a rate Hy proportional to y. Solve this so-called *Schaefer model*. Find the equilibrium solutions y_1 and $y_2(> 0)$ when $H < A$. The expression $Y = Hy_2$ is called the **equilibrium harvest** or **sustainable yield** corresponding to H. Why?

4. **Harvesting.** In problem 3, find the solution and draw an approximate plot to get an idea of the solution satisfying $y(0) = 1$ when (for simplicity) $A = 2, B = 1$ and $H = 0.5$. What is the limit? What does it mean? What if there were no fishing? (1)

5. **Intermittent harvesting.** In problem 3, assume that you fish for 2 years, then fishing is banned for the next 2 years. Thereafter you start again. And so on. This is called *intermittent harvesting*. Describe qualitatively how the population will develop if intermitting is continued periodically. Find the solution for the first 6 years, assuming that $A = 2, B = 1, H = 0.5$ and $y(0) = 1$. Draw an approximate plot to get an idea of the solution. (2)

6. **Drug injection.** Find and solve the model for drug injection into the bloodstream if, beginning at $t = 0$, a constant amount $A\ g/min$ is injected and the drug is simultaneously removed at a rate proportional to the amount of the drug present at time t. (2)