

# Discrete Mathematics CSE 121 : Homework 5

In every proof/derivation clearly state your assumptions and give details of each step.

1. Let  $f_1(x)$  and  $f_2(x)$  be functions from the set of real numbers to the set of positive real numbers. Show that if  $f_1(x)$  and  $f_2(x)$  are both  $\Theta(g(x))$ , where  $g(x)$  is a function from the set of real numbers to the set of positive real numbers, then  $f_1(x) + f_2(x)$  is  $\Theta(g(x))$ . Is this still true if  $f_1(x)$  and  $f_2(x)$  can take negative values?.
2. Show that  $n \log n$  is  $O(\log n!)$ .
3. The number of multiplications of entries used to multiply a  $p \times q$  matrix and a  $q \times r$  matrix is  $pqr$ . What is the best order to form the product  $ABCD$  if A, B, C, and D are matrices with dimensions  $30 \times 10$ ,  $10 \times 40$ ,  $40 \times 50$ , and  $50 \times 30$ , respectively?
4. Show that isomorphism of simple graphs is an equivalence relation.
5. The converse of a directed graph  $G = (V, E)$ , denoted by  $G^{conv}$ , is the directed graph  $(V, F)$ , where the set  $F$  of edges of  $G^{conv}$  is obtained by reversing the direction of each edge in  $E$ . Show that if  $G$  and  $H$  are isomorphic directed graphs, then the converses of  $G$  and  $H$  are also isomorphic.
6. Does the graph in Figure 1 have a Hamilton path? If so, find such a path. If it does not, give an argument to show why no such path exists.
7. Devise an algorithm for constructing Euler paths in directed graphs.
8. Extend Dijkstra's algorithm for finding the length of a shortest path between two vertices in a weighted simple connected graph so that the length of a shortest path between the vertex a and every other vertex of the graph is found.

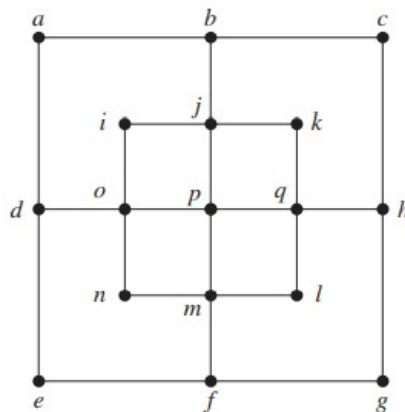


Figure 1: