MTH377/577 CONVEX OPTIMIZATION
Winter Semester 2022
Indraprastha Institute of Information Technology Delhi
MIDSEM EXAM (Time: 1 hour 15 minutes, Total Points: 30)

Please Note:

1. The stipulated time to workout the exam is 1 hour 15 minutes. This means you are supposed to stop writing at 4:15 pm. After that, scan and upload. Submission received after the deadline will incur an automatic penalty.

2. First attempt submitting via Google Classroom. If you encounter technical problems in submitting via Google Classroom, send it to shreyat@iiitd.ac.in directly via email. If you have sent it via email well within time, then there is no need to upload it on Google Classroom.

3. While you can consult all the material at hand, discussing the problems with any person is a violation of academic integrity.

Q1. (a) (2 points). Are the canonical basis vectors $e_1, \ldots, e_n$ in $\mathbb{R}^n$ affinely independent ? If yes, prove it. If not, argue why not.

(b) (2 point). Is the line passing through the point $(3/2, 1)$ and slope -2 a hyperplane ? If yes, identify this hyperplane with its (normal vector, scalar) and indicate the positive and negative halfspaces associated with it. If not, argue why not.

(c) (2 points). Consider the function $f : \mathbb{R}_{++} \mapsto \mathbb{R}$ defined by

$$f(x) = 2e^{(2x+5)\log(2x+5)} + \frac{5}{x}$$

If f convex or concave ? Why ?

(d) (2 points). Is the following optimization problem convex ? Argue why or why not.

$$\begin{aligned} \max_{x_1, x_2} \quad & 2\log(x_1 - 2) + 3\log(x_2 - 3) \\ \text{subject to} \quad & 2x_1 + 3x_2 \leq 25 \end{aligned}$$

(e) (2 points). Draw and precisely write as a set, the 1-sublevel set of e^x and the 0-superlevel set of $\log(x)$, where x is a real valued variable.

Q2. (5 points). Consider the function $f : \mathbb{R}^2 \mapsto \mathbb{R}$ defined by $f(x_1, x_2) = \log(e^{x_1} + e^{x_2})$. Is f convex or concave ? Either way, prove it.

Q3. (5 points). Let A be an $m \times n$ matrix. Is the set K defined below convex ? Why or why not ?

$$K = \{y \in \mathbb{R}^m : \exists x \in \mathbb{R}^n \text{ such that } ||x|| \leq 1 \text{ and } y = Ax\}$$

Q4. (5 points). Let f_1 and f_2 be concave functions from $\mathbb{R}^n$ to $\mathbb{R}$. Let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be defined as the pointwise minimum $f(x) = \min(f_1(x), f_2(x))$. Is f convex or concave ? Either way, prove it.

Q5. (5 points). Consider the function $f(x, y) = x^3 + y^2 - 4xy - 3x$. Find all the local minima, maxima or saddle points of f.