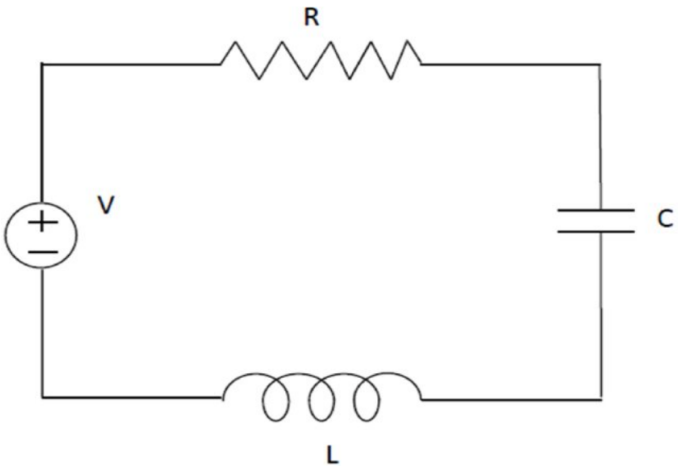
MTH204: Worksheet 6

March 15, 2023

Consider a RLC-circuit



$R \rightarrow$ Resistance (ohm, Ω) : Voltage drop=RI, $L \rightarrow$ Inductance (Henry, H) : Voltage drop=$L\frac{dI}{dt}$,
$C \rightarrow$ Capacitance (Farad, F) : Voltage drop =$\frac{Q}{C}$,
where $I(t)$ is the current in the circuit at any time t, $Q(t)$ is the amount of charge in capacitor at any time t and $V(t)$ is the external voltage provided to circuit. Here, $V(t) = V_0 \sin(\omega t)$.
Relationship in the capacitor between charge and current is $I = \frac{dQ}{dt}$.

1. Write a second order ODE model for the current $I(t)$ in this RLC circuit. **(2)**

2. Solve the characteristic equation of homogeneous part (find roots in terms of R, L and C). **(2)**

3. Start with a particular solution
$$I_p = a\cos(\omega t) + b\sin(\omega t),$$
and find a and b. **(2)**

4. Write your particular solution found in the previous question as
$$I_p = C\sin(\omega t - \delta).$$
Find C and δ explicitly in terms of V_0, R, L, C and ω. **(2)**

5. Find the current in an RLC circuit with $R = 13\Omega$, $L = 0.3H$, $C = 10^{-2}F$ which is connected to a source of EMF, $V(t) = 110\sin(100\pi t)$. Assume that current and capacitor charge are 0 at $t = 0$. **(2)**