MTH 371: Quiz II
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Instructions

- Show all your work to score full marks. Detailed answers are a must.
- You can use a calculator. No phones or other electronic devices may be used.

Questions

1. The transition probability matrix of a Markov process beginning at $X_0 = 1$ with $S = \{1, 2, 3, 4, 5, 6\}$ is given by

	1	2	3	4	5	6
1	0	1	0	0	0	0
2	0.4	0.6	0	0	0	0
3	0.3	0	0.4	0.2	0.1	0
4	0	0	0	0.3	0.7	0
5	0	0	0	0.5	0	0.5
6	0	0	0	0.3	0	0.7

   (a) (1 point) Draw the corresponding state transition diagram.

   (b) (2 points) Find the classes.

   (c) (2 points) Identify the recurrent and transient states.

   (d) (2 points) Write the period of all the states.

   (e) (2 points) Will the limiting distribution exist, explain.

   (f) (2 points) Find $P(X_0 = 1, X_1 = 2, X_3 = 3)$.

2. The transition probability matrix of a Markov process with $S = \{1, 2, 3\}$ is given by

	1	2	3
1	0.2	0.3	0.5
2	0.1	0	0.9
3	0.55	0	0.45

   Answer the following questions

   (a) (2 points) Consider $\{A\} = \{2\}$, find the hitting probability vector.

   (b) (2 points) If initial distribution is [0.2 0.3 0.5] then find $\pi^{(1)}$.

3. (2 points) For $j = 0, 1, \ldots,$ let $P_{jj+2} = v_j$ and $P_{j0} = 1 - v_j$, construct the transition probability matrix of the corresponding Markov Chain. Identify the recurrent and transient states in the Markov Chain.