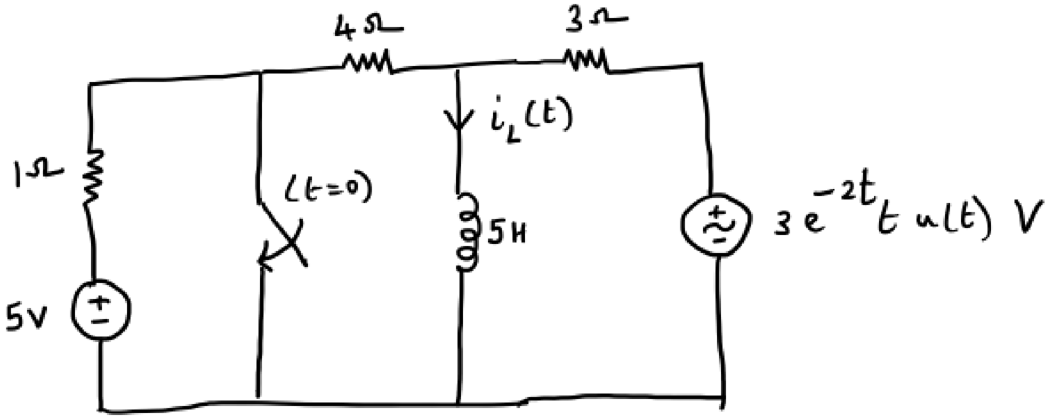
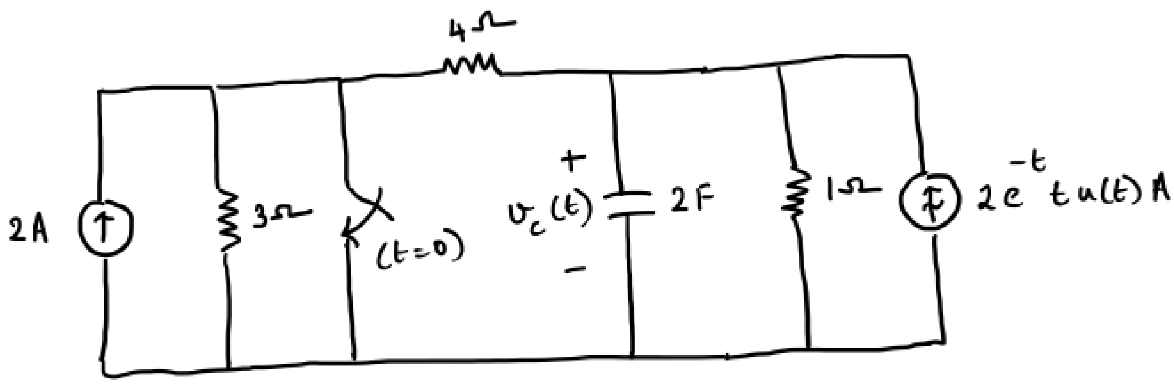
Set-1

(**10 marks**) Consider the circuit shown below with voltage source excitation $v_s(t) = 3e^{-2t}tu(t)V$. What is the current through the inductor, $i_L(t \geq 0)$?

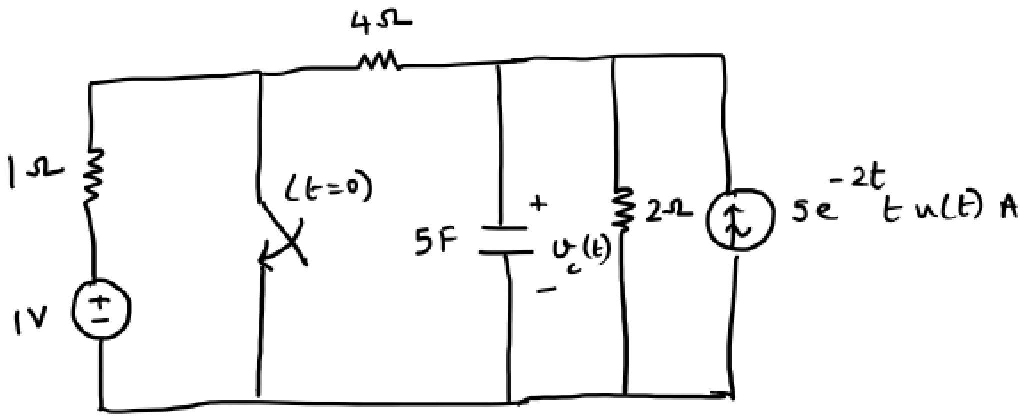


Set -2

(**10 marks**) Consider the circuit shown below with current source excitation $i_s(t) = 2e^{-t}tu(t)A$. What is the voltage across the capacitor, $v_C(t \geq 0)$?

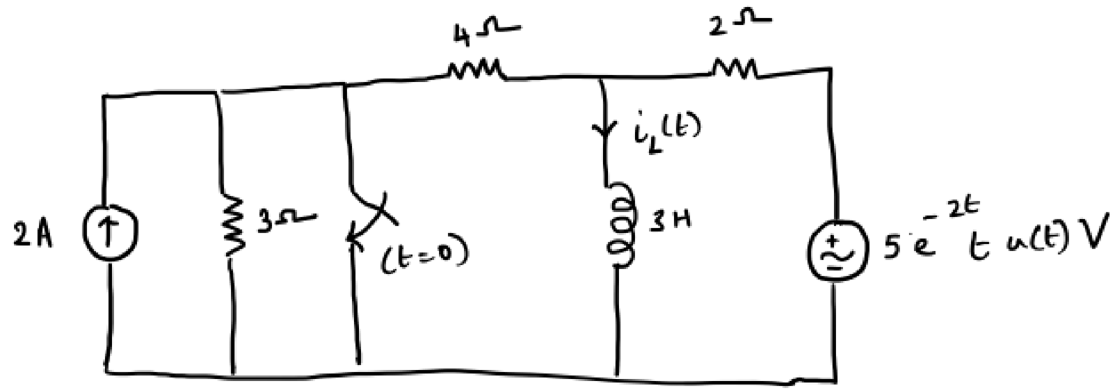


(**10 marks**) Consider the circuit shown below with current source excitation $i_s(t) = 5e^{-2t}tu(t)A$. What is the voltage across the capacitor, $v_C(t \geq 0)$?



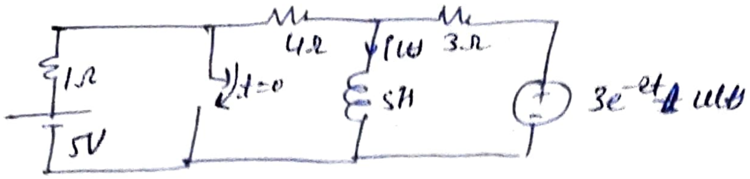
Set -4

(**10 marks**) Consider the circuit shown below with voltage excitation $v_s(t) = 5^{-2t}tu(t)V$. What is the current through the inductor, $i_L(t \geq 0)$?



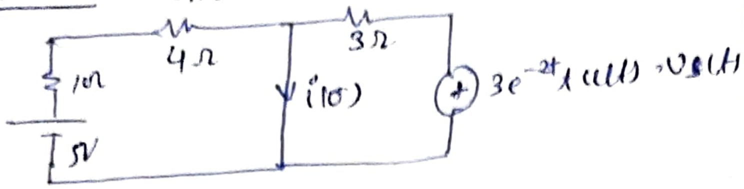
Q1 given : $v_s(t) = 3e^{-2t}\,t\,u(t)$

Set-1



Sol: finding $i(0^-)$ in the inductor

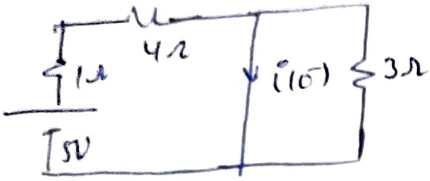
at $t = 0^-$ (Inductor will be acting like short ckt as it attained steady state)



Since at $t<0$

$\therefore V_s(t=0^-) = 0V$ as $u(t) = 0$ at $t = 0^-$

⇓

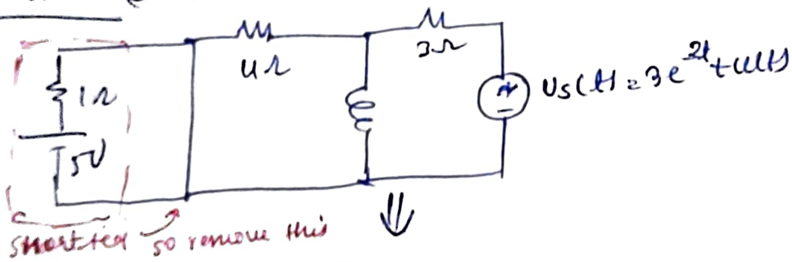


$\Rightarrow i(0^-) = \frac{5V}{(1+4)\Omega} = \frac{5}{5} = 1A$

$\therefore \boxed{i(0^-) = 1A}$

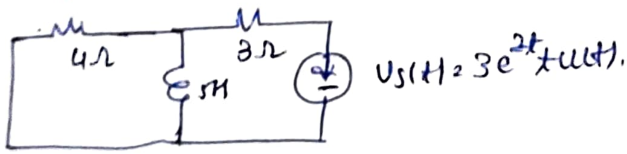
2 marks

Now at $t > 0$ (case)

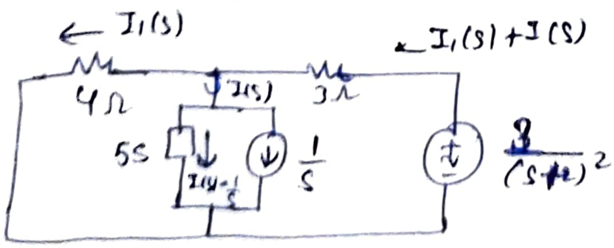


⇓

2 marks



↓ use Laplace Transform



(Replacing Inductor by its Laplace domain equivalent.)

2 marks

use KVL :

$$-\frac{3}{(s+2)^2} + 3(I_1(s) + I(s)) + 4I_1(s) = 0$$

→ 1 mark

$$\frac{9}{(s+2)^2} = 3I_1(s) + 3I(s) + 4I_1(s) = 7I_1(s) + 3I(s)$$

$$\frac{3}{(s+2)^2} = 3I(s) + 7I_1(s) \quad (1)$$

A10 $\frac{-3}{(S+2)^2} + 3(I_1(s)+I(s)) + 5s\left[I(s) - \frac{1}{s}\right] = 0$

$= \frac{3}{(S+2)^2} + 3I(S) + 3I_1(S) + 5SI(S) - 5 = 0$

$5 + \frac{3}{(S+2)^2} = (3+5S)I(S) + 3I_1(S)$ — ②

$$\begin{bmatrix} 3 & 7 \\ 3+5S & 3 \end{bmatrix} \begin{bmatrix} I(S) \\ I_1(S) \end{bmatrix} = \begin{bmatrix} 3/(S+2)^2 \\ 5 + 3/(S+2)^2 \end{bmatrix}$$

$$\begin{pmatrix} I(S) \\ I_1(S) \end{pmatrix} = \frac{1}{-(12+35S)} \begin{bmatrix} 3 & -7 \\ -(3+5S) & 3 \end{bmatrix} \begin{bmatrix} 3/(S+2)^2 \\ 5+3/(S+2)^2 \end{bmatrix}$$ — 1 mark

$I(S) = \frac{1}{-(12+35S)}\left(\frac{9}{(S+2)^2} - 35 - \frac{21}{(S+2)^2}\right) = \frac{1}{(12+35S)}\left[35 + \frac{12}{(S+2)^2}\right]$

$I(S) = \frac{35}{35S+12} + \frac{12}{(12+35S)((S+2)^2}$

$= \frac{1}{S+12/35} + \frac{12/35}{(S+12/35)(S+2)^2} = \frac{1}{S+\frac{12}{35}} + \frac{0.125}{S+12/35} + \frac{0.126}{S+2} + \frac{-0.207}{(S+2)^2}$

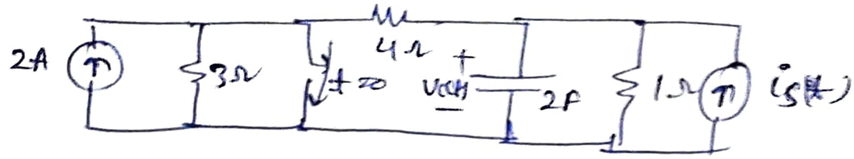
↓ take inverse Laplace we get

$i(t) = e^{-12/35t}u(t) + 0.125e^{-12/35t}u(t) + 0.126e^{-2t}u(t) - 0.207te^{-2t}u(t)$

— 2 marks

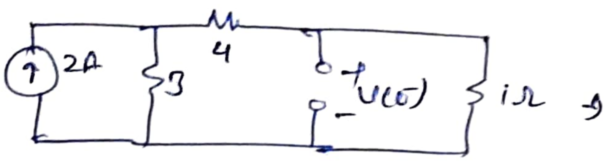
Q2 given: $i_s(t) = 2e^{-t}u(t)$

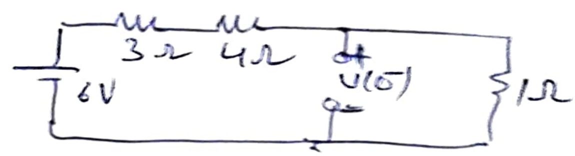
Set - 2

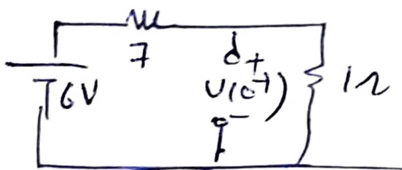


For $V(0^-)$ @

: at $t = 0^-$ (Capacitor is at steady state. and $i_s = 0A$ as $t = 0^-$. $u(0^-) = 0$



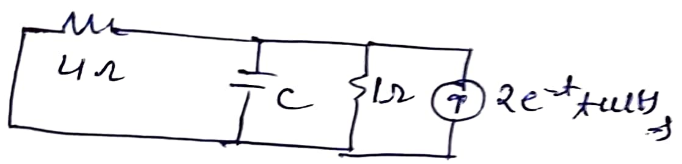


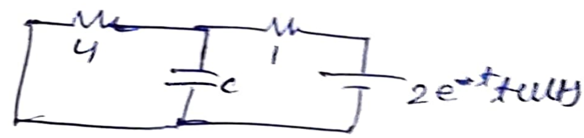


$V(0^-) = \cancel{6}^{3}\left[\frac{1}{\cancel{8}4}\right] = \frac{3}{4} = 0.75\,V.$

$\boxed{V(0^-) = V(0^+) = 0.75V}$ → 2 marks

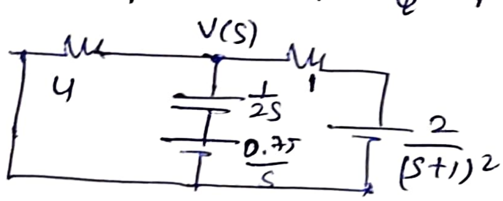
@ $t > 0$





use Laplace transform for capacitor and Volt source

2marks



(I need to find V(S) only as it's volt on capacitor)

use Nodal. 2marks

$$\frac{V(S)}{4} + \frac{V(S) - 0.75/S}{1/2S} + V(S) - \frac{2}{(S+1)^2} = 0 \quad \rightarrow 1 \text{ mark}$$

$$\frac{V(S)}{4} + 2SV(S) - 1.5 + V(S) - \frac{2}{(S+1)^2} = 0$$

$$V(S)\left[1 + \frac{1}{4} + 2S\right] = 1.5 + \frac{2}{(S+1)^2}$$

$$V(S)\left[\frac{5}{4} + 2S\right] = 1.5 + \frac{2}{(S+1)^2} \Rightarrow V(S) = \frac{1.5}{2S + 5/4} + \frac{2}{(S+1)^2(2S + 5/4)}$$

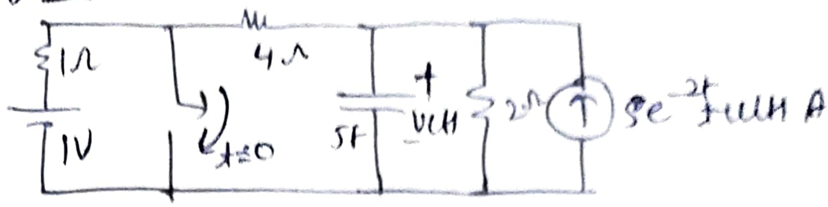
$$V(S) = \frac{0.75}{S + 5/8} + \frac{1}{(S+1)^2(S+5/8)} = \frac{0.75}{S+5/8} + \frac{-7.107}{S+1} + \frac{-2.67}{(S+1)^2} + \frac{7.11}{(S+5/8)} \quad 1 \text{ mark}$$

Taking inverse Laplace transform:

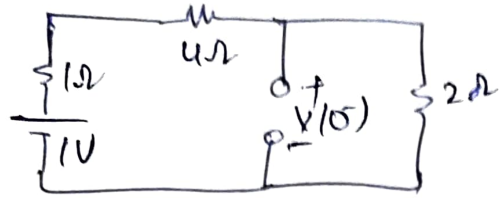
$$u(t) = 0.75e^{-518t}u(t) + 7.11e^{-518t}u(t) - 7.104e^{-t}u(t) - 2.67te^{-t}u(t).$$

— 2 marks

Ans 3 ~~at t>0~~ given: $I_s(t) = 5e^{-2t}u(t)$ **Set - 3**



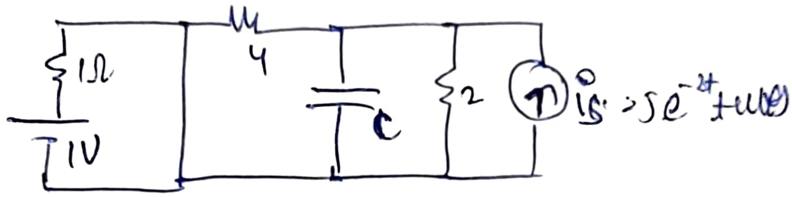
@ $t = 0^-$ [capacitor comes at steady state]

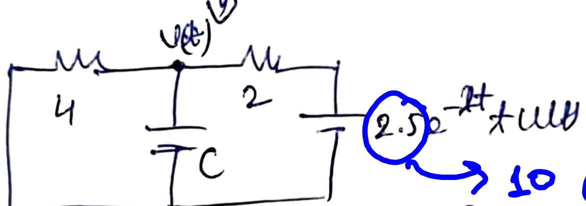


$\Rightarrow V(0^-) = 1\left(\frac{2}{2+5}\right) = \frac{2}{7} = 0.28\,V$

$\boxed{v(0^-) = v(0^+) = 0.28\,V}$ – 2 marks

for $t>0$

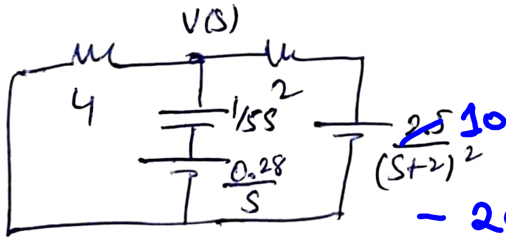




→ 2 marks

→ 10 (2 × 5 = 10)

Take Laplace of circuit



– 2 marks

use Nodal

$\frac{V(S)}{4} + \frac{V(S) - 0.28/S}{1/5S} + \frac{V(S) - \frac{10}{(S+2)^2}}{2} = 0$ – 1 mark

$\frac{V(S)}{4} + 5SV(S) - 1.4 + \frac{V(S)}{2} - \frac{\cancel{12.5}\ 5}{(S+2)^2} = 0$

$V(S)\left[\frac{1}{4} + \frac{1}{2} + 5S\right] = 1.4 + \frac{\cancel{12.5}\ 5}{(S+2)^2} \Rightarrow V(S)\left[0.75 + 5S\right] = 1.4 + \frac{\cancel{12.5}\ 5}{(S+2)^2}$

$V(S) = \frac{1.4}{5S+0.75} + \frac{\cancel{12.5}\ 5}{(5S+0.75)(S+2)^2}$

will change accordingly

$= \frac{0.28}{S+0.15} + \frac{0.25}{(S+2)^2(S+0.15)} = \frac{0.28}{S+0.15} + \frac{-0.073}{S+2} + \frac{-0.135}{(S+2)^2} + \frac{0.073}{S+0.15}$ → 1 mark

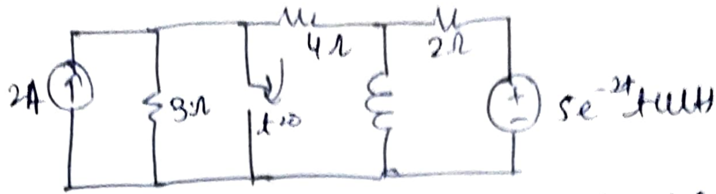
2 marks take Inv Laplace we get

$v(t) = 0.28e^{-0.15t}u(t) + -0.073e^{-2t}u(t) - 0.135te^{-2t}u(t) + 0.073e^{-0.15t}u(t)$. Ans

Ans 4

given: $v_s(t) = 5e^{-2t}u(t)\,V$

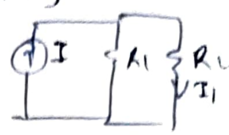
Set-4



(a) For $i(0^-)$ [@ $t = 0^-$] ~~capacitor~~ Inductor is at steady state

$i(0^-) = 2\left[\frac{3}{3+4}\right] = \frac{6}{7} = 0.85A$

(using current division rule

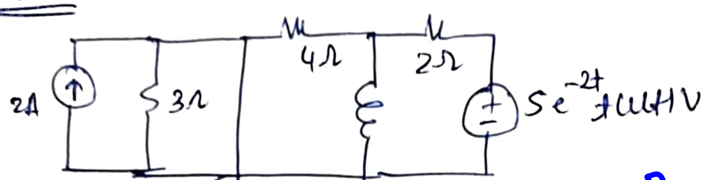


$I_1 = I\left[\frac{R_1}{R_1+R_2}\right]$

$$\boxed{i(0^-) = i(0^+) = 0.85A}$$

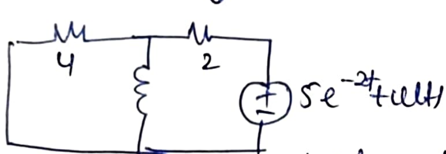
→ 2 marks

at $t > 0$

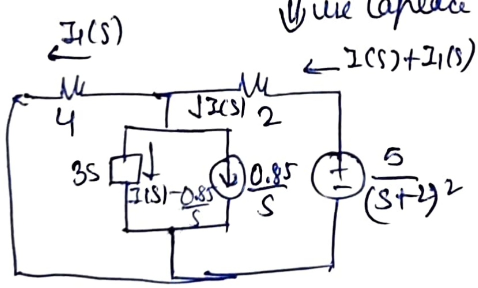


2 marks

⇓



⇓ use Laplace of the circuit



→ 2 marks

Use KVL

$$\frac{-5}{(S+2)^2} + 2\left(I(S)+I_1(S)\right) + 4I_1(S) = 0$$

$$\frac{5}{(S+2)^2} = 2I(S) + 2I_1(S) + 4I_1(S)$$

$$\frac{5}{(S+2)^2} = 2I(S) + 6I_1(S) \quad (1)$$

→ 1 mark.

Also: $\frac{-5}{(S+2)^2} + 2[I(S)+I_1(S)] + 3S\left[I(S) - \frac{0.85}{S}\right] = 0$

$$\frac{5}{(S+2)^2} = 2I(S) + 2I_1(S) + 3SI(S) - 2.55$$

$$2.55 + \frac{5}{(S+2)^2} = (2+3S)I(S) + 2I_1(S) = (2)$$

$$\begin{bmatrix} 2 & 6 \\ 2+3S & 2 \end{bmatrix}\begin{bmatrix} I(S) \\ I_1(S) \end{bmatrix} = \begin{bmatrix} 5/(S+2)^2 \\ 2.55+5/(S+2)^2 \end{bmatrix}$$

$$\begin{bmatrix} I(s) \\ I_1(s) \end{bmatrix} = \frac{1}{-(8+18s)} \begin{bmatrix} 2 & -6 \\ -(2+3s) & 2 \end{bmatrix} \begin{bmatrix} 5/(s+2)^2 \\ 5/(s+2)^2 + 0.255 \end{bmatrix}$$ → 1 mark

$$I(s) = \frac{1}{-(8+18s)} \left[\frac{10}{(s+2)^2} - \frac{30}{(s+2)^2} - 1.53 \right] = \frac{20}{(s+2)^2(8+18s)} + \frac{1.53}{8+18s}$$

$$I(s) = \cancel{1.53} \frac{0.085}{s+8/18} + \frac{1.11}{(s+8/18)(s+2)^2}$$

$$= \frac{0.085}{s+8/18} + \frac{-0.46}{s+2} \frac{-0.71}{(s+2)^2} + \frac{0.46}{s+8/18}$$

↓ Take Inverse Laplace

$$i(t) = 0.085e^{-8/18t}u(t) - 0.46e^{-2t}u(t) - 0.71te^{-2t}u(t) + 0.46e^{-8/18t}u(t)$$

→ 2 marks