Worksheet-2 Solution + Rubric (I)

Q.1

(a) $\lim_{(x,y)\to(\pi/2,0)} \frac{\cos y+1}{y-\sin x} = \frac{2}{0-1} = -2$ (2)

(b) $\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x} = \lim_{(x,y)\to(0,0)} e^y \frac{\sin x}{x} = e^0 \cdot \lim_{x\to 0} \frac{\sin x}{x}$

$= e^0 \cdot 1 = 1\cdot 1 = 1$ (2)

(c) $\lim_{(x,y,z)\to(1,-1,-1)} \frac{2xy+yz}{x^2+z^2} = \frac{2(1)(-1)+(-1)(-1)}{1^2+(-1)^2} = \frac{-2+1}{2} = -\frac{1}{2}$ (2)

(d) $\lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1} = \lim_{(x,y)\to(4,3)} \frac{\cancel{(x-y-1)}}{\cancel{(x-y-1)}(\sqrt{x}+\sqrt{y+1})}$ $(x \neq y+1)$

$= \lim_{(x,y)\to(4,3)} \frac{1}{\sqrt{x}+\sqrt{y+1}}$

$= \frac{1}{\sqrt{4}+\sqrt{3+1}} = \frac{1}{4}$ (2)

Q.2 Given $f(x_0,y_0) = 3$.

- If f is continuous at (x_0,y_0): $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$

  Hence, $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = 3$. (2)

- If f is not continuous at (x_0,y_0):

  Then, Either, limit of the function f at (x_0,y_0) does not exist.

  OR, if the limit of the function f at (x_0,y_0) exists, then it is not equal to 3. (2)

Q.3. Given, $f(x,y) = \dfrac{x^2+y^2}{x^2-3x+2}$ (II)

Now if, $x^2-3x+2 = 0$

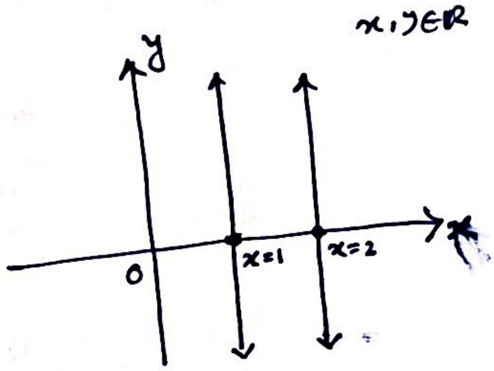
$\Rightarrow (x-2)(x-1) = 0$ (2)

$\therefore x = 2$ or 1.

Hence, f is Continuous at $\mathbb{R}^2 - \{(1,y)\in\mathbb{R}^2 : y\in\mathbb{R}\}$
$\cup \{(2,y)\in\mathbb{R}^2 : y\in\mathbb{R}\}$

Alternative answer

$x,y\in\mathbb{R}$



f is not Continuous at all points on the vertical line $x=1$ and $x=2$ on $\mathbb{R}^2$.

So, f is Continuous at every point in $\mathbb{R}^2$ which is not on these two line. (2)

Q.4. III

Given, $f(x,y) = \dfrac{xy}{|xy|}$

Now, along the line $y = mx$ $(m \neq 0)$, 2

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{mx^2}{x^2|m|} = \frac{m}{|m|}$$

$$= \begin{cases} 1, & \text{if } m > 0 \\ -1, & \text{if } m < 0 \end{cases}$$

~~Hence, the limit for other function~~

Hence as $(x,y)\to(0,0)$, $f(x,y)$ has no limit.

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