Game Theory Mid-Sem 2023

Indicative Solutions

Ans. 1. a) $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, n$

When $n = 1$ we have the following game:

		Thief: Inside	Thief: Trunk
Police	Inside	1, −1	−1, 1
	Trunk	−1, 1	1, −1

There are no PSNE.

MSNE = $\{(1/2, 1/2), (1/2, 1/2)\}$ [Show calculation].

b) $n = 100$

$T_i = \{b_1, b_2, \ldots, b_{100}\}$ types of i

$T_j = \{d_1, d_2, \ldots, d_{100}\}$ types of j.

prob. of i being matched with j is $\frac{1}{100}$.

" " a match (i,j) is $\frac{1}{100} \cdot \frac{1}{100}$

$S_i : T_i \rightarrow$ {Inside, ~~Out~~ Trunk} for each i

$S_j : T_j \rightarrow$ {Inside, Trunk} for each j

		Thief	
		Inside	Trunk
Police	Inside	$1, -1$	$-1, 1+b_i$
	Trunk	$-1+d_j, 1$	$1+d_j, -1+b_i$

Bayesian game : $(N, \{S_i, S_j\}_{i,j \in \{1,}, \{T_i, T_j\}, \{u_i, u_j\})$

where $i, j \in \{1, 2, \ldots, 100\}$.

Best response:

$BR_j(S_i = \text{Inside} \mid d_j > 0) =$ Trunk if $d_j > 2$
Inside otherwise

$BR_j(S_i = \text{Trunk} \mid d_j > 0) =$ Trunk.

$BR_j(S_i = \text{Inside} \mid d_j < 0) =$ Inside

$BR_j(S_i = \text{Trunk} \mid d_j < 0) =$ Trunk if $d_j > 2$
Inside, otherwise

Similarly ~~for~~ write BR_i.

Pure strategy Bayes' Nash equilibrium:

(i) If $b_i < -2$ then $S_i =$ Inside

If $b_i \in (-2, 2)$ then $S_i =$ Inside or Trunk with equal prob.

If $b_i \geq 2$, then $S_i =$ Trunk.

(ii) If $d_j \leq -2$, then s_j = Inside

If $d_j \in (-2, 2)$ then s_j = Inside or Trunk with equal probabilit

If $d_j \geq 2$ then s_j = ~~outs~~ Trunk.

Ans 2. a) False.

In prisoner's dilemma, Nash equilibrium payoffs are lower than in ~~(Don't confess~~ (Don't Confess, Don't Confess) [show payoffs].

OR In bilateral trading, efficiency loss occurs due to imperfect information/incomplete info.

[see lecture slide 103-104].

b) True. Continuity of $\succsim$ $\Rightarrow$ Continuous u_i exists.

Weierstrass theorem $\Rightarrow$ optimal solution exists

~~$B(\sigma)$ u_i is~~

By Kakutani's FPT, $B(\sigma)$ has a fixed point.

ΔA_i is the space of mixed actions.

~~the m a mixed strategy~~

The fixed point $f \in \Delta A_i$.

Mixed strategies permit all convex combinations ~~to~~ be as feasible actions/strategies.

3. [Hint : Similar to Median voter theorem]

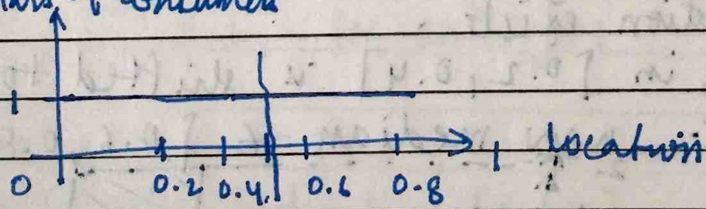
$N = 2$ or $N = \{i, j\}$

$A_i = [0,1]$

$$U_i(a_i, a_j) = \begin{cases} \left(\frac{a_i + a_j}{2}\right) \times 1 & \text{if } a_i < a_j \\ \left\{1 - \left(\frac{a_i + a_j}{2}\right)\right\} \times 1 & \text{if } a_i > a_j \\ 0.5 & \text{if } a_i = a_j \end{cases}$$

$(N, \{A_i\}_{i \in N}, \{U_i\}_{i \in N})$ is the strategic form game.

Since consumers are distributed uniformly over $[0,1]$, ~~at~~ in each interval $x \in [0,1]$, the mass is uniform. Median $= 0.5$



median = 0.5

For any i, if $a_i <$ median $< a_j$ then it can increase its sale by choosing $a_i' = a_j - \delta$ for some $\delta > 0$. $u_i(a_i', a_j) > u_i(a_i, a_j)$

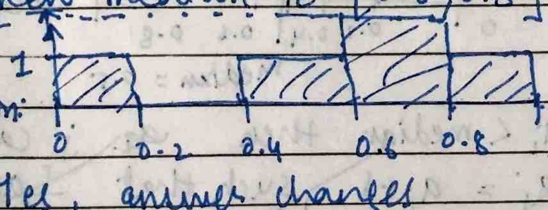
If $a_i < a_j <$ median, then $a_i' = a_j + \delta$ generates higher sales for some $\delta > 0$. $u_i(a_i', a_j) > u_i(a_i, a_j)$

If median $< a_i < a_j$ then $a_j' = a_i - \delta$ generates higher payoff for j.
If median $< a_j < a_i$ then $a_i' = a_j - \delta$ ...
[similarly].

If $a_i = a_j =$ median, then no ~~dev~~ profitable deviation exists.

If mass in [0.2, 0.4] is shifted to [0.6, 0.8] ~~one~~ new median $\in$ [0.6, 0.8]

New distribution: f_2



- Yes, answer changes
New equilibrium is median of distribution f_2.

Ans.4. First price auction

$N = \{1, 2, \dots, n\}$

$A_i = b_i \in \mathbb{R}$ for each $i \in N$

$$u_i(b_i, b_{-i}) = \begin{cases} v_i - b_i, & \text{if } b_i > b_j, i > j \\ 0, & \text{otherwise.} \end{cases}$$

$(N, \{b_i\}_{i\in N}, \{v_i\}_{i\in N}, \{u_i\}_{i\in N})$ is a strategic form game. $\{v_i\}_{i\in N}$ is ~~not~~ necessary to mention.

Nash equilibria:

$b_1 \in [v_2, v_1]$

$b_j \leq b_1 \quad \forall \ j \neq 1$

$b_j = b_1$ for some $j \neq 1$

Sol^n: Suppose $v_i > v_j \ \forall \ j \neq i$, $\therefore$ $i = 1$

bidder 1 will bid $b_1 \in [v_2, v_1]$

[$b_1 < v_2 \Rightarrow$ 1 may lose; $b_1 > v_1 \Rightarrow$ lose

Since $u_i(b_1, b_{-i}) = v_1 - b_1$,

Player 1 ~~will bid~~ need not bid v_1 to win]

If $b_j > b_1$ for some $j \neq 1$, then $\cancel{u_j(b_j, b_{-j})}$

$U_j(b_j, b_{-j}) < 0$.

$\therefore b_j \leq b_1 \ \forall \ j \neq 1$.

If $b_j < b_1$ $\forall$ $j \neq 1$ then Player 1 can increase $u_1(b_1, b_{-1})$ by choosing $b_1 - \varepsilon \geq b_j$.

$\therefore$ $b_1 = b_j$ for some $j \neq 1$.