

MTH210 – END-SEM EXAMINATION – 20221213

TIME: 2 HOURS

MAXIMUM MARKS: 80

NB: You may use any known result (i.e. theorems, propositions and lemmas, and tutorial problems) without proof; however, it should be identified clearly. This does not apply if you have been asked to prove a known result. Marks will depend on the correctness and completeness of your proofs. All questions have equal marks.

RUBRIC

List of Common Errors and Marks Deductions:

1. Using an undefined symbol.
2. Writing an equation in which the LHS and RHS are not comparable. For example, the LHS is a set, and the RHS is an integer.
3. Writing a meaningless or completely illogical statement.

Deduct 1 mark for each occurrence of an error of the above type. **However, the total marks for any question should remain non-negative.**

REMARK: *For some of the problems or parts, there are correct alternative methods other than the given solution. Answers which use any of these alternative methods will be awarded marks on merit. Details are not covered in the rubric.*

Q1. a) YES \rightarrow 1 mark. Justify \rightarrow 2 marks; 1 mark (0.5 + 0.5) for defining the join and meet correctly, and 1 mark (0.5 + 0.5) for noting that they are finite sets and hence in $F(N)$.

b) Least (minimum) element YES \rightarrow 1 mark + 1 mark for stating that \emptyset is the least element. Largest (maximum) element NO \rightarrow 1 mark + 2 marks for the proof (**no partial credit** for a wrong proof – it is very easy).

c) not defined (or not existent) \rightarrow 1 mark. Reason \rightarrow 1 mark.

Q2. Two inclusions have to be proved. Correct proof per inclusion \rightarrow 4 marks. Additional \rightarrow 2 marks for stating that equality follows from the two proved inclusions. There should not be any scope for partial credit in this question. Marks should be deducted for common errors (listed above), since this is a technical proof.

Q3. Correct proof → 10 marks. It is a straightforward application of PHP. The main idea is to calculate the number of 5-element subsets, and to calculate the range of possible values of the sums of the set elements. If the main idea is not captured, **then no partial credit**. If the basic idea is captured, but one or both of the calculations is done incorrectly, deduct 3 marks **for each such wrong calculation**. If in doubt, consult the instructor.

Q4. a) Correct answer is 9 → 1 mark; justify → 2 marks. No scope for partial credit.

b) Basic idea of the justification (backward movement) → 3 marks. Stepwise explanation → 4 marks. If the initial step correctly starting with B(10,2) is explained, but the rest of the idea or step-wise explanation is not given, not more than 2 marks may be given for b).

Q5. Correct final answer → 2 marks; defining the four atomic propositions correctly → 1 mark (0.25 for each); defining the two compound propositions correctly → 1 mark (0.5 for each); construction of the main truth table correctly → 2; correct logic to obtain the final answer → 4 marks. **Note:** Partial credit as detailed above would apply in the case of an incomplete or wrong answer only. **Remark:** The given method using a truth-table is recommended. However, it is possible to give a correct answer without using a truth-table. Such answers to be given marks on merit. However, additional assumptions cannot be introduced. For example, it cannot be assumed that if there is no lion in cage A, there must be a lion in cage B. Consult instructor if in doubt.

Q6. a) NO → 2 marks. The justification consists of 4 cases, each case → 1 mark: the cases are (i) all odd values, should be done together; (ii) $x = 0$; (iii) $x = 2$; (iv) $x = 4$. If odd cases are done individually, 1 mark to be given if all of them done correctly, else 0 mark.

a) **Alternative solution:** No → 2 marks. Precise explanation → 4 marks. The proof is very short, so no scope for partial credit.

b) The proof consists of two statements: Writing statement (1) correctly → 2 marks; writing statement (2) correctly → 2 marks. **No partial credit other than this.**

Q7. Correct proof → 10 marks. It is a straightforward graph-theoretic application of PHP. The main idea is to observe that any vertex of odd degree must appear(at least once) as an end-vertex of one of the paths in the

decomposition. If the main idea is not captured, **then no partial credit**. If the basic idea is captured, but not expressed clearly, then ≤ 2 **grace marks** may be awarded. It is a very easy question, so marking **must not be lenient**. If in doubt, consult the instructor.

Q8. Correct answer is: 2 components \rightarrow 2 marks. Observation that any path in the graph must consist of vertices of equal parity \rightarrow 2 marks. Correct inductive proof \rightarrow 6 marks. For a deficient proof $\rightarrow \leq 2$ **grace marks** may be awarded.