Worksheet 8

November 30, 2021

1. Prove that for $1 \leq k \leq n$,

   (i) $<a_0, a_1, \dots, a_n> = <a_0, a_1, \dots, a_{k-1}, <a_k, a_{k+1}, \dots, a_n>>$,

   Hint : Use Induction on the number of terms in the innermost continued fraction on the right hand side $<a_k, a_{k+1}, \dots, a_n>$.

   (ii) Use part(i) to prove that:

   $$<a_0, a_1, \dots, a_n> = a_0 + \frac{1}{<a_1, \dots, a_n>}$$

2. Convert each of the following into finite simple continued fractions

   (i) 0.23

   (ii) $\frac{233}{177}$

3. Find $\frac{p}{q}$ if $\frac{p}{q} = [3, 7, 15, 1]$. Convert $\frac{p}{q}$ to a decimal and compare with the value of π.

4. (i) Writing the simple continued fraction of proper fractions. Ex. $\frac{29}{67}$.

1 Ex. $\frac{29}{67}$;

$$\frac{29}{67} = [0, 2, 3, 4, 2]$$
$$= [a_1, a_2, a_3, a_4, a_5]$$

$$0 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{2}}}}$$

$67 \overline{)\,29\,}(\,0 = a_1$
$\quad 0$
$29 \overline{)\,67\,}(\,2 = a_2$
$\quad 58$
$9 \overline{)\,29\,}(\,3 = a_3$
$\quad 27$
$2 \overline{)\,9\,}(\,4 = a_4$
$\quad 8$
$1 \overline{)\,2\,}(\,2 = a_5$
$\quad 2$
$\quad 0$

(ii) Write the continued fraction expansion for $\frac{67}{29}$.

(iii) Compare (i) & (ii) above to conclude a general result for the relation between the continued fraction of

$\frac{p}{q}$ & $\frac{q}{p}$, where p>q.