# MTH 377/577 CONVEX OPTIMIZATION
## Winter Semester 2022
## Indraprastha Institute of Information Technology Delhi
### Problem Set 3: Convex Optimization

Q1. (Exercise 5.5 in [BV]). Find the dual function of the linear optimization problem. Also write the dual problem, making any implicit constraints explicit.

$$\begin{aligned} \min \quad & c^T x \\ & Gx \leq h \\ & Ax = b \end{aligned}$$

Q2. For any set $C \subseteq \mathbb{R}^n$, define what is a supporting hyperplane to $C$. What does the supporting hyperplane theorem for convex sets assert ? What does the supporting hyperplane theorem for convex functions assert ? For a differentiable convex function $f$, derive the hyperplane that supports $\mathbf{epi} f$ at the point $(\mathbf{x}, f(\mathbf{x}))$.

Q3. Suppose we have two sets in $\mathbb{R}^2$ given by

$$\begin{aligned} C &= \{(x, y) \in \mathbb{R}^2_+ : xy \geq 1\} \\ D &= \{(x, y) \in \mathbb{R}^2 : y = 0\} \end{aligned}$$

Can $C$ and $D$ be weakly separated ? If yes, what is the separating hyperplane ? If not, argue why not. Can they be strictly separated ? If yes, what is the separating hyperplane ? If not, argue why not

Q4. Explain, via a diagram, what is the separating hyperplane view of convex optimization. In particular, identify the two convex sets and the separating hyperplane.

Q5. The Nash bargaining problem for two players with 'disagreement point' $(1, 1)$ is a set consisting of pairs $(x, y)$ of real numbers satisfying $x \geq 1$ and $y \geq 1$. This set is interpreted as feasible utilities/payoffs of these players that they can achieve in negotiations. Suppose we capture the bargaining power of the players by a pair of weights $(\alpha, 1-\alpha)$ where

$\alpha \in (0,1)$ is interpreted as the bargaining weight of player 1 in negotiations, and $1-\alpha$ as the bargaining weight of player 2 in negotiations. For given bargaining weighs $(\alpha, 1-\alpha)$, the Nash bargaining solution is defined as the solution of the following optimization problem in which the bargaining problem is specified by the constraints.

$$\begin{aligned} \max_{x,y} \quad & x^{\alpha} y^{1-\alpha} \\ \text{subject to } & x + 3y \leq 12 \\ & 3x + y \leq 12 \\ & x \geq 1, y \geq 1 \end{aligned}$$

Write down a detailed solution to compute the Nash bargaining solution as a function of bargaining weights of players.