Quiz 1
DM, Monsoon 2021

Duration : 60 mins
Max marks : 10

1. (2 marks) Use the Well Ordering Principle to prove that any integer greater than or equal to 18 can be represented as the sum of nonnegative integer multiples of 5, 8 and 11.

2. (2 marks) How many integers in the set $\{1, 2, \ldots, 212\}$ are not divisible by any of the primes $2, 3, 5, 7$?

3. (2 marks) Use the Well Ordering Principle to prove that $\sqrt{10}$ is irrational.

4. (2 marks) Give a combinatorial proof that

$$\sum_{k=0}^{m} \binom{m}{k}\binom{n}{n-k}k = m\binom{m+n-1}{n-1}$$

for positive integers m, n.

5. (2 marks) Let

$$s_n = \sum_{\substack{A\subseteq[n]\\A\neq\emptyset}} \frac{1}{p(A)},$$

where the sum is taken over all nonempty subsets of $[n] = \{1, \ldots, n\}$ and, for each subset A, $p(A)$ denotes the product of all elements of A. For instance, when $n = 3$, there are 7 nonempty subsets $\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}$ and therefore

$$s_3 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{1\cdot 3} + \frac{1}{1\cdot 2\cdot 3} = 3.$$

Guess s_n as a function of n. Using induction, or otherwise, prove your answer.