Quiz #1	**Name:** _____
Date : 31/01/2024	**Section:** _____
MTH204: ODEs/PDEs	**Maximum Time:** 20 Minutes
Semester: Winter 2024	**Maximum Marks:** 15

DO NOT SHOW ANY WORK HERE. JUST WRITE WHAT IS BEING ASKED. THERE IS NO STEP MARKING.

Problem 1. [1×5] Circle **T** or **F** depending on if the following statements are **T**rue or **F**alse, respectively.

(a) The order of the ODE

$$\frac{d^2}{dx^2}(y - \sin(y'')) = y^2 - x$$

is 2. **T** (**F**)

(b) The ODE $M(x, y)\, dx + N(x, y)\, dy = 0$ is exact if

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y} \quad \text{for all points } x, y. \qquad \textbf{T} \qquad (\textbf{F})$$

(c) The ODE

$$y' + \frac{1}{x}y - \sqrt{y} = 0$$

is a Bernoulli's equation. (**T**) **F**

(d) Given an IVP $y' = f(x, y)$, $y(x_0) = y_0$ if f and $\partial f/\partial x$ are continuous and bounded in a rectangle R centered at (x_0, y_0) then a unique solution is guaranteed in a sub-rectangle in R. **T** (**F**)

(e) The principle of superposition can be applied to the ODE $y'' + y' - 3y + 2 = 0$. **T** (**F**)

Problem 2. [1×4] Coider the IVP

$$y'' - 8y' + 17y = 0, \quad y(0) = -4,\ y'(0) = -1.$$

(a) What are the two roots of the characteristic polynomial?

$4+i,\ 4-i$

(b) What is the general solution of the ODE?

$c_1 e^{4x}\cos x + c_2 e^{4x}\sin x$

(c) What are the values of arbitrary constants?

$c_1 = -4, \quad c_2 = 15$

(d) What is the solution of the IVP?

$y(x) = e^{4x}(-4\cos x + 15\sin x)$

Problem 3. [1×5] After 10 minutes in Aditi's room, her tea has cooled to $40°$ Celsius from $100°$ Celsius. The room temperature is $25°$ Celsius.

(a) According to Newton's law of cooling, the rate of change of temperature is directly proportional to the temperature difference between room temperature and temperature of tea. Use it to write an ODE for temperature, $T(t)$, of tea at time t. This ODE will include a proportionality constant.

$$\frac{dT(t)}{dt} = k(T-25)$$

(b) What is the general solution of this ODE?

$$T(t) = ce^{kt} + 25$$

(c) Write two values of $T(t)$ at two different times which is known to you.

$$T(0) = 100, \quad T(10) = 40$$

(d) What are the values of proportionality constant and arbitrary constant in the general solution? (You can leave your answers in $\log$.)

$$c = 75, \quad k = \frac{1}{10}\log\left(\frac{1}{5}\right) = -\frac{1}{10}\log(5)$$

(e) How much longer will it take to cool to $35°$? (You can leave your answers in $\log$.)

$$t = -10\,\frac{\log\left(\frac{2}{15}\right)}{\log(5)} = -10\,\frac{\log(2)-\log(15)}{\log(5)}$$

Problem 4. [1] Consider the ODE $y' = x - y$. Draw just one direction field vector at the point $(2, 4)$ in plane.