A.1.(a) Consider $x, y \in R$ s.t $x = y$
then neither $x > y$ nor $y > x$
$\therefore$ $>$ is not complete
$\therefore$ $>$ is not a preference relation
[A preference relation is a complete & transitive binary relation]

(b) Consider House 1, House 3.
(i) neither House 1 is a neighbour of House 3
nor House 3 " " " " House 1
$\therefore$ the binary relation is not complete.

(ii) consider House 1, House 2, House 3
House 1 is a neighbour of House 2
House 2 " " " " 3
but not House 1 is a neighbour of House 3.
$\therefore$ the binary relation is not transitive.

A2. Set $x = 3$, $y = 4$. The game is Prisoner's Dilemma for these payoffs.
Nash Equilibrium $= \{(A, A)\}$.

a) The game has a weakly dominant action for each player when

$$x = 2, \ y = 6.$$

In this case Nash equilibrium is the same, i.e., (A, A).

b) The following payoffs result in a game that has no strictly dominated actions and (B, B) is a Nash equilibrium:

$u_1(A, A) = 3$, $\quad u_2(A, A) = 2$

$u_1(B, B) = 6$, $\quad u_2(B, B) = 6$

The game has the following representation:

		Player 1	
		A	B
Player 2	A	(2, 3)	(6, 2)
	B	(2, 6)	(6, 6)

A3. See Lemma 3.2 in Osborne & Rubinstein for detailed explanation.

[Indicative steps:] Consider a strategic form game $\langle N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$.

Let $\sigma = (\sigma_i, \sigma_j)$ be an MSNE.

Suppose player i assigns probability ~~p ∈ [0,1]~~ $p > 0$ to action $a_i \in A_i$ in σ_i, but

$\nexists$ $a_j \in A_j$ such that $BR_i(a_j) = a_i$.

Consider $a_j \in A_j$ such that player j assigns prob. $q > 0$ to a_j in σ_j.

Let $BR_i(a_j) = a_i^*$; $a_i^* \in A_i$.

By assumption we know that $a_i^* \neq a_i$.

Player i's utility from (a_i^*, a_j) & (a_i, a_j) are as follows:

$$u_i(a_i^*, a_j) > u_i(a_i, a_j)$$

since $a_i^* = BR_i(a_j)$.

This implies

$$p \cdot q \cdot u_i(a_i^*, a_j) > p \cdot q \, u_i(a_i, a_j).$$

since $p, q > 0$.

Player i can get higher utility by assigning prob p to a_i^* instead of a_i.

A4. (Indicative)

$N = \{1, 2\}$, For each $i \in N$, $A_i = \{p_i \in [0, \infty)\}$
and $c_i = 0$.

The utility function for each $i \in N$ is :

$$u_i(p_i, p_{-i}) = \begin{cases} p_i & \text{if } p_i < p_{-i} \\ \frac{1}{2} p_i & \text{if } p_i = p_{-i} \\ 0 & \text{if } p_i > p_{-i} \end{cases}$$

The Best response for each $i \in N$ is :

$$BR_i(p_{-i}) = \begin{cases} p_{-i} - \delta, \text{ where } \delta \in [0, p_{-i}] & \text{if } p_{-i} > 0 \\ 0 & \text{if } p_{-i} \leq 0 \end{cases}$$

Player i will change a lower price than p_{-i} if $p_{-i} > 0$.

If $p_{-i} = 0$ then Player i also changes $p_i = 0$.

$\therefore$ the set of Nash equilibria is

$$\{(p_i, p_{-i}) \mid p_i = p_{-i} = 0\},$$

[If $p_{-i} < 0$ then also the best response of i is to change $p_i = 0$].