ECE 351 DSP: Assignment 3

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Total: 30 points

Submission deadline: During class on 20.11.2024

A word on the notation: I shall represent finite duration causal signals as arrays. For example, $x[n] = [1, 2, 3]$ means $x[0] = 1$, $x[1] = 2$, and $x[2] = 3$, and $x[n] = 0$ for all other n.

Coding has to be done in Python. MATLAB codes will be marked zero.

For every filter design question, please label the axes correctly (either mention the units or use the proper symbol among ω, f, F and Ω). Also, ensure to draw horizontal lines corresponding to pass band and stop band ripples and vertical lines corresponding to pass band and stop band edges. Incorrect labelling or missing lines will lead to deduction of points, and specifically for equiripple filters, missing lines will lead to 0 points.

1) Consider the lattice representation of an FIR system with $K_1 = -1, K_2 = 2, K_3 = 0.5$. Find the corresponding $h[n]$.

[10 points]

Solution:

Since we have 3 lattice coefficients, $h[n]$ is of length 4, and hence $A_3(z) = H(z)$. Now, $A_0(z) = B_0(z) = 1$. By the lattice recursion equations, $A_1(z) = z^{-1}K_1B_0(z) + A_0(z) = 1 - z^{-1}$. Hence, $B_1(z) = -1 + z^{-1}$. Thus,

$$\begin{aligned} A_2(z) &= z^{-1}K_2B_1(z) + A_1(z) \\ &= 2z^{-1}(-1 + z^{-1}) + 1 - z^{-1} \\ &= 1 - 3z^{-1} + 2z^{-2}. \end{aligned}$$

Therefore, $B_2(z) = 2 - 3z^{-1} + z^{-2}$. Thus,

$$\begin{aligned} A_3(z) &= z^{-1}K_3B_2(z) + A_2(z) \\ &= 0.5z^{-1}(2 - 3z^{-1} + z^{-2}) + 1 - 3z^{-1} + 2z^{-2} \\ &= 1 - 2z^{-1} + 0.5z^{-2} + 0.5z^{-3}. \end{aligned}$$

Hence, $H(z) = 1 - 2z^{-1} + 0.5z^{-2} + 0.5z^{-3}$, and so $h[n] = [1, -2, 0.5, 0.5]$.

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2) Write a python code to design a Kaiser window low-pass filter with pass band edge at $\frac{\pi}{8}$ rad/sample, stop band edge at $\frac{\pi}{4}$ rad/sample, pass band ripple of 40dB, and stop band ripple of 50dB.

   a) Plot the magnitude response of the filter in dB.

   b) Now consider the input

$$x[n] = \begin{cases} 2\cos\frac{\pi}{16}n + 3\cos\frac{\pi}{2}n, & 0 \leq n \leq 15 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

   to the filter designed in part a). Compute using any python function of your choice the output of this signal, and plot the input and the output signals side by side.

[6+4 = 10 points]

3) Write a python code to design an equiripple filter with pass band edges at 40 Hz, 60 Hz, 100 Hz, and stop band edges at 30 Hz, 80 Hz, 90 Hz, pass band ripples at 20dB, stop band ripples at 25 dB, and sampling frequency of 240 Hz. Plot the magnitude response of the filter in dB.

[10 points]