Quiz 1

October 6, 2021

1. Solve the equation $6x + 10y + 15z = 5$ for integer solutions.
2. Fix $m, n \in \mathbb{N}$ with $\gcd(m, n) = 1$. Suppose that $R = \{r_i : 1 \leq i \leq m\}$ is a complete residue system modulo m and that $S = \{s_i : 1 \leq j \leq n\}$ is a complete residue system modulo n . Prove that

$$T = nR + mS = \{nr_i + ms_j : 1 \leq i \leq m, 1 \leq j \leq n\}$$

is a complete residue system modulo mn.

Hint: Need to show that any two different element of T are incongruent, and we need to show that

$$|T| = mn.$$

1. Show if

$$nr_i + ms_j \equiv nr_i' + ms_j' \mod mn$$

then

$$nr_i + ms_j \equiv nr_i' + ms_j' \mod m,$$

and

$$nr_i + ms_j \equiv nr_i' + ms_j' \mod n.$$

2. Conclude form (1) that

$$r_i \equiv r_i' \mod m$$
$$s_j \equiv s_j' \mod n.$$

3. Complete the argument to prove the required statement.