

MTH 377/577 Convex Optimization

Make-up Exam

May 3, 2024

Answer all questions. Time = 45 minutes.

1. Consider the following minimization problem: $\min x_1$ s.t. $(x_1 - 1)^2 + (x_2 - 1)^2 \leq 1$, $(x_1 - 1)^2 + (x_2 + 1)^2 \leq 1$, $x_1, x_2 \geq 0$. Find the optimal solution. Write down the Lagrangian function for the problem and comment whether constraint qualification conditions are satisfied. (4)
2. Let $f, g : I \rightarrow \mathbb{R}$ where I is an interval. Assume that f and $f + g$ both are convex. Does this imply that g is convex? Or concave? What if $f + g$ is convex and f is concave? (4)
3. Write down the dual for the following: Minimize $f = -x - 3y$ subject to $x + y = 6$, $-x + y \leq 4$ (4)
4. Suppose that a set $A \subseteq \mathbb{R}^3$ is not convex. Another set $B \subseteq \mathbb{R}^3$ is convex and $A \cap B = \emptyset$. Can you construct a strict separating hyperplane between A and B ? Why/Why not? Support your answer with the help of an example. (4)
5. What do you understand by a fixed point? Provide a formal example of an application of any fixed point theorem in the setting of an optimization problem. (4)