Discrete Mathematics CSE 121 : Homework 1

In every proof/derivation clearly state your assumptions and give details of each step.

1. The defining property of an ordered pair is that two ordered pairs are equal if and only if their first elements are equal and their second elements are equal. Surprisingly, instead of taking the ordered pair as a primitive concept, we can construct ordered pairs using basic notions from set theory. Show that if we define the ordered pair (a, b) to be $\{\{a\}, \{a, b\}\}$, then $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. [Hint: First show that $\{\{a\}, \{a, b\}\} = \{\{c\}, \{c, d\}\}$ if and only if $a = c$ and $b = d$.]

2. Describe a procedure for listing all the subsets of a finite set.

3. Prove Russell's paradox. More specifically, let S be the set that contains a set X if the set X does not belong to itself, so that $S = \{X | X \notin X\}$.

   (a) Show the assumption that S is a member of S leads to a contradiction.

   (b) Show the assumption that S is not a member of S leads to a contradiction.

   By parts (a) and (b) it follows that the set S cannot be defined as it was. This paradox can be avoided by restricting the types of elements that sets can have.

4. For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

   (a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

   (b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

   (c) $\{(2, 4), (4, 2)\}$

(d) $\{(1,2),(2,3),(3,4)\}$

(e) $\{(1,1),(2,2),(3,3),(4,4)\}$

(f) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

5. A relation R on the set A is irreflexive if for every $a \in A, (a,a) \notin R$. That is, R is irreflexive if no element in A is related to itself. For the relations in Question 4, determine which are irreflexive.

6. A relation R is called asymmetric if $(a,b) \in R$ implies that $(b,a) \notin R$. For the relations in Question 4, determine which are asymmetric.