MTH210 – SUBMISSION_20220922

If P and Q are two partitions of a non-empty set X, we say that $\mathbb{P}$ ***is finer than*** $\mathbb{Q}$ if every subset in $\mathbb{P}$ is contained in some subset in $\mathbb{Q}$, i.e. for any $A \in \mathbb{P}$, there is a $B \in \mathbb{Q}$ such that $A \subseteq B$. Let $\leq$ denote the ***is finer than*** relation and let Σ denote the family (set) of all partitions of X, i.e. $\Sigma = \{\mathbb{P} : \mathbb{P}$ is a partition of X$\}$.

a) Show that $\leq$ is a partition ordering on Σ. (*3 marks*)
b) Draw the Hasse diagram of $<\Sigma, \leq >$ if $X = \{1, 2, 3\}$. (*2 marks*)

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List of Common Errors and Marks Deductions:

1. Using an undefined symbol.
2. Writing an equation in which the LHS and RHS are nt comparable. For example, the LHS is a set, and the RHS is an integer.
3. Writing a meaningless or completely illogical statement.

Deduct 0.5 marks for each occurrence of an error of the above type. **However, since this is only the second submission, a maximum of 1.5 marks to be deducted for the above, and the total marks for the submission should remain non-negative.**

a) For proving that ≤ as above is a partial ordering:

1. Reflexive property $\rightarrow$ 1 mark

2. Anti-symmetric property $\rightarrow$ 1 mark

3. Transitive property $\rightarrow$ 1 mark

4. General Instruction: The proof for each property should be in technical language using the definitions, as in the given solution; however, it can be more concise (the given solutions typically are more detailed than strictly necessary).

5. Partial Credit: A "***wordy***" answer for this problem, which does ***not*** use technical terms and notation, is not very satisfactory, since the elements involved are themselves sets. For example, a wordy answer for the reflexive property would be something like: "Since every subset in a partition is a subset of itself, a partition is finer than itself". However, some credit may be given as follows:

- If an answer has three correct wordy proofs $\rightarrow$ 1 mark
- If an answer has one or two correct wordy proofs $\rightarrow$ 0.5 marks
- If an answer has mixed technical and wordy proofs, award marks for the technical proofs only, as explained above.

b) For the Hasse diagram:

- Correct diagram $\rightarrow$ 2 marks
- **But** if the diagram has the correct shape, but the subsets making up each partition have not been clearly identified $\rightarrow$ 0.5 marks.