Algorithms Under Uncertainty : Quiz 1
Full Marks : 25

23/8/2023

Write solutions in the space provided. NO extra pages will be provided. Write brief and precise solutions. Meaningless rambles fetch negative credits.

Problem 1. (10 points) Recall that the deterministic MARKING algorithm for paging is k-competitive and this result is tight for any deterministic algorithm. This result is a bit pessimistic primarily since we are comparing the online algorithm against an offline optimal which has full knowledge of the input. In order to balance this out, people have considered the so-called 'resource augmentation' analysis model. Consider the same MARKING algorithm. But, assume that the cache size provided to the offline optimal solution is k while the size of the cache provided to the MARKING algorithm is $2k$. Prove that in this model, MARKING is 2-competitive.

Solution: Just like the proof done in lecture, consider the sequence σ to be decomposed in phases where each phase i now has $2k$ many distinct requests. (Since MARKING has a cache of size $= 2k$).

Now, number of misses for MARKING in any phase is ~~exactly~~ at most $2k$.

On the other hand, in a phase, ~~there is~~ offline opt must make at least k misses since number of requests is $2k$ while cache size is k.

Hence, competitive ratio is at most $2k/k = 2$.

Problem 2. (5 points) Show that for any given number of machine $m \geq 2$ (you cannot assume any specific value for m), the greedy algorithm is at least $2 - 1/m$-competitive for online load balancing with *identical machines* (that is any job j has the same load p_j on all machines).

Sol. Let us consider a sequence of ~~unit~~ $m(m-1)$ unit sized jobs followed by a single job of $p_j = m$.

Greedy first fills up all machines with load $(m-1)$ on each. Hence, final load on one m/c will be exactly $2m-1$.

Opt will distribute all unit jobs on $(m-1)$ machines and put large job on an empty m/c.

Hence, ratio is $2m-1/m = (2-1/m)$. ▨

Problem 3. (10 points) Let us consider the scheduling with restricted assignments problem, but a very special case. Assume that any job j can be scheduled on at least $m/4$ machines and $p_j = 1$ for all those machines. Prove that under this assumption, the greedy algorithm done in class is 5-competitive.

Sol. Let, if possible, the load on some machine i^* exceed $5\lambda^*$ where, $\lambda^* =$ Opt load.

Now, ~~the~~ consider the last job being assigned to i^*. Just before this, load on i^* is $\geq 5\lambda^*$ (since final load $> 5\lambda^*$)

But then, due to greedy property, every ~~other~~ machine to which i^* can be assigned has load $\geq 5\lambda^*$.

Finally, acc. to the problem, number of such m/c's is $\geq m/4$.

Hence, total load over all is $\geq 5\lambda^* \times m/4$
$> m\lambda^*$.

This is clearly a contradiction sine opt is λ^* ▨