(10) false

<u>Take</u> - $p_1 = 5$, $p_2 = 7$, $p_3 = 11$

then- $2p_1 \cdot p_2 \cdot p_3 + 1 = 771$

$= 3 \times 257$

(Not prime).

(32) If.

$a^2 \equiv b^2 \pmod p$

$1 \leq a, b \leq \frac{p-1}{2}$, $a \neq b$

$\Rightarrow \quad a^2 - b^2 \equiv 0 \pmod p$

$\Leftrightarrow \quad (a-b)(a+b) \equiv 0 \pmod p$

But $\quad a+b < \frac{p-1}{2} + \frac{p-1}{2} = p-1$

$\Rightarrow \quad \gcd(a+b, p) = 1$

$\Rightarrow \quad p \nmid a+b$

$\Rightarrow \quad a-b \equiv 0 \pmod p$

$\Rightarrow \quad a \equiv b \pmod p$.

$\Rightarrow \quad a = b$

$\Rightarrow$ which is a contradiction.

33) for -

$$a = 1^2, 2^2, \ldots \left(\frac{p-1}{2}\right)^2$$

$$a^{(p-1)/2} = 1^{p-1}, 2^{p-1}, \ldots, \left(\frac{p-1}{2}\right)^{p-1}$$

but for

$$b = 1, 2, \ldots, \frac{p-1}{2}, \quad \gcd(b, p) = 1$$

as $1 \leq b \leq p-1$ & $p$ is a prime.

By Fermat's thm

$$b^{p-1} \equiv 1 \pmod{p}$$

$$\therefore \quad a^{(p-1)/2} \equiv 1 \pmod{p}$$

$\Rightarrow$ By Euler's criterion

$1^2, 2^2, \ldots, \left(\frac{p-1}{2}\right)^2$ are quadratic residue of $p$.

(34) <u>Case I:-</u> If $c$ is quadratic residue of $p$ then

$x^2 \equiv c \pmod{p}$ has

a solution. If $b$ is a solution. then

$$(p-b)^2 = p^2+b^2-2pb$$
$$\equiv b^2 \equiv c \pmod{p}$$

is also a solution.

$\Rightarrow x^2 \equiv c \pmod{p}$ has exactly 2 solutions.

and $1+\left(\frac{c}{p}\right) = 2$

So, in this case it is true.

<u>Case-2:-</u> If $c$ is nonresidue of $p$, then

$x^2 \equiv c \pmod{p}$ has $0$ $sol^n$.

Also, $1+\left(\frac{c}{p}\right) = 1-1 = 0$ $\square$.