Ans 1. Mixed extension of the game:

$(N, \{\Delta A_i\}_{i \in N}, \{U_i\}_{i \in N})$

where U_i is the expected utility ~~fn~~ of player i.

$N = \{1, 2\}$; $A_i = \{A, B\}$ $\forall i \in N$.

We solve for MSNE using indifference principle.

Let q be player 1's belief about ~~player 2~~ the prob. of choosing A.

$U_1(A, (q, (1-q))) = 1q + 3(1-q)$

$U_1(B, (q, (1-q))) = 1 \cdot q + 4(1-q)$

Player 1 will mix his actions when

$U_1(A, (q, (1-q))) = U_1(B, (q, (1-q)))$

$\Rightarrow q + 3(1-q) = q + 4(1-q)$

$\Rightarrow q = 1, \quad (1-q) = 0.$

Similarly ~~for~~ p find player 2's belief p about player 1's choice prob. of A.

MSNE $= (\sigma_1, \sigma_2)$

where $\sigma_1 = (1, 0)$, $\sigma_2 = (1, 0)$

this is equivalent to PSNE (A, A).

The game has another PSNE: (B, B).

$\therefore$ set of all MSNE $= \{((1,0), (1,0)), ((0,1), (0,1))\}$.

Ans. 2. a) False. Suppose player 1 believes that player 2 chooses 'C' with prob q and 'DC' (don't confess) with prob $(1-q)$.

[Refer to matrix in slides — write it the matrix as part of the answer]

$\therefore \; U_1(C, (q, (1-q))) = -3q + (-1)(1-q)$

$U_1(DC, (q, (1-q))) = -10q + (-2)(1-q)$

Using indifference principle, player 1 will mix only when

$U_1(C, (q, (1-q))) = U_1(DC, (q, (1-q)))$

$\Rightarrow -3q - 1 + q = -10q - 2 + 2q$

$\Rightarrow 8q - 2q = -1$

$\Rightarrow q < 0$ which is impossible.

$\therefore$ player 1 does not mix.

Since payoffs are symmetric, player 2 will also not mix, ~~i.e. assign positive probability to~~

Since don't confess is strictly dominated and players will not mix, don't confess will not be assigned positive probability.

The PSNE (C, C) can be written as MSNE $((1,0), (1,0))$.

b) Let $a_i \in A_i$ be a weakly dominated action for a player $i \in N$ in game ~~$\{A_i\}$~~ $(N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N})$.

$\therefore \exists\, a_i' \in A_i$ s.t

(i) $u_i(a_i', a_{-i}) \geq u_i(a_i, a_{-i}) \;\forall\, a_{-i}$

(ii) $u_i(a_i', a_{-i}') > u_i(a_i, a_{-i}')$

for some~~for~~ $a_{-i}' \in A_{-i}$.

Consider $a_i^* \in A_i$ such that

$$u_i(a_i^*, a_{-i}) > u_i(a_i', a_{-i})$$

$$\forall \; a_{-i} \in A_{-i}.$$

Then, $a_i^* \in BR_i(a_{-i}) \;\; \forall \;\; a_{-i} \in A_i$

a_i and a_i' are never-best responses.

Ans. 3. Set of strategies for player 1:

$$S_1 = \{LD, LU, RD, RU\}$$

Set of strategies for player 2:

$$S_2 = \{ll, lr, rl, rr\}$$

Note that if player 2 could not observe player 1's actions, his set of actions would be $\{l, r\}$ since he cannot distinguish between the 2 decision nodes.

Game in strategic (matrix) form:

Player 2

Player 1	l	r
LD	5, 3	3, 2
LU	3, 2	5, 3
RD	4, 0	4, 0
RU	4, 0	4, 0

Subgame perfect equilibria if player 2 observes player 1's actions:

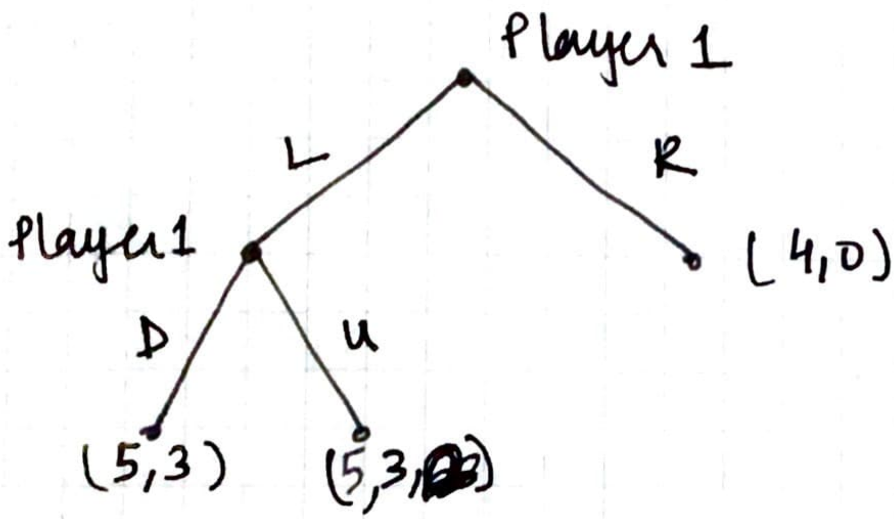
~~Let play~~ At the first decision node, player 2 will choose l:

$$BR_2(s_1 = LD) = l$$

At the 2nd decision node, player 2 will choose r:

$BR_2(s_1 = LU) = r$

using backward induction, we ~~know~~ reduce the game tree to :



At the 2nd decision node, ~~player 1 will choose D,~~ and ~~again by backward induction~~, ~~we thus~~ at the
player 1 can choose either D or U and generate the same payoff

SPNE = $\{(LD, lr), (LU, lr)\}$.

From the game matrix, we get the following Nash equilibria: ~~$\{(LD, l), (LD, r)\}$~~.

$\{(LD, l), (LU, r)\}$.

The difference is due to the information set of the 2nd player.

Ans. 4. strategic form game:

$N = \{1, 2, \ldots, n\}$ set of bidders

$A_i = \mathbb{R}_+$ bids (actions) for each $i \in N$

$u_i : \times_{i \in N} A_i \to \mathbb{R}$ is as follows:

$$u_i(b_i) = \begin{cases} v_i - b_i, & \text{if } b_i \text{ is the highest bid} \\ 0, & \text{otherwise.} \end{cases}$$

$\therefore \; (N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N})$ is a strategic game.

Bidding v_i for any $i \in N$ is <u>not</u> a strictly or weakly dominant strategy.

In fact, v_i is weakly dominated by any bid $b_i < v_i$ for all $i \in N$.

<u>Proof</u>:
$$u_i(b_i = v_i) = \begin{cases} 0 & \text{if } b_i = \max_{j \in N} b_j \\ 0 & \text{if } b_i < \max_{j \in N} b_j \end{cases}$$

Consider $b_i' < v_i$

$$u_i(b_i') = \begin{cases} (v_i - b_i') > 0 & \text{if } b_i' = \max_{j \in N} b_j \\ 0, & \text{if } b_i' \neq \max_{j \in N} b_j \text{ or } (<) \end{cases}$$

$\therefore$ b_i' generates the same payoff as $b_i = v_i$ when $b_i' \neq \max_{j \in N} b_j$ and a strictly higher payoff when $b_i' = \max_{j \in N} b_j$.

Therefore $b_i' < v_i$ weakly dominates $b_i = v_i$.

Let index i denote the ordering of bidders according to their valuation v_i i.e.

$$v_1 > v_2 > v_3 > \dots > v_n.$$

Consider the following action profile:

$$(b_1 = v_2,\ b_2 = v_2 - \varepsilon,\ b_3 = v_3 - \varepsilon,\ \dots,\ b_n = v_n - \varepsilon)$$

for some $\varepsilon > 0$.

Note that player 1 gets $u_1 = \cancel{v_1 - b_2}\ v_1 - b_1$
$= v_1 - v_2$.

& All other players get 0.
If player 2 raises his bid to $b_2 = v_2$, he still gets 0. $\therefore$ this is a Nash Equilibrium

• Note that while $(b_1 = v_2, b_2 = v_2 - \varepsilon, b_3 = v_3 - \varepsilon, \ldots, b_n = v_n - \varepsilon)$ is an equilibrium v_2 is not known to player 1.

Other equilibrium strategies are:

$$b_i = v_i/2 \quad \forall\ i \in N$$

$$b_i = \frac{(n-1)}{n} v_i \quad \forall\ i \in N.$$

[you can mention any of these, but must show that no player can unilaterally deviate and increase their payoff.]