Tutorial 3 - Continuous Probability Distribution

August 27, 2019

1. Since the thickness is uniformly distributed, let X be the thickness of photoresist where $X \sim U\left(0.2050, 0.2150\right)$

   (a) $P\left(X > 0.2125\right)$

$$f\left(x\right) = \begin{cases} \frac{1}{0.01} & 0.2050 \leq x \leq 0.2150 \\ 0 & otherwise \end{cases}$$
$$c = 100$$

$$\begin{aligned} P\left(X > 0.2125\right) &= 1 - P\left(X < 0.2125\right) \\ &= 1 - \int_{0.2050}^{0.2125} 100\,dx \\ &= 1 - 100\left(0.2125 - 0.2050\right) \\ &= 0.25 \end{aligned}$$

   (b) $P\left(X > a\right) = 0.1$

$$\begin{aligned} \int_{a}^{0.2150} 100\,dx &= 0.1 \\ 100\left(0.2150 - a\right) &= 0.1 \\ 21.5 - 100a &= 0.1 \\ 100a &= 21.4 \\ a &= 0.2140\mu m \end{aligned}$$

   (c)

$$\begin{aligned} \mu &= \frac{0.2050 + 0.2150}{2} \\ &= 0.21\mu m \end{aligned}$$

$$
\begin{aligned}
\sigma &= \sqrt{\frac{(b-a)^2}{12}} \\
&= \sqrt{\frac{(0.2150-0.2050)^2}{12}} \\
&= 0.002886\mu m
\end{aligned}
$$

2. Let X be the lifetime of a mechanical assembly in a vibration test with $X \sim E\,(400)$

   (a)

$$
\begin{aligned}
P\,(X < 100) &= \int_0^{100} \frac{1}{400} e^{-\frac{x}{400}}\,dx \\
&= 0.2212
\end{aligned}
$$

3. (Important notes: if don't know what to write the formulas, can just write the number for Z scores)

   (a) Important note 2 (lol): Always ignore past 2 decimal places in tables, no need to do interpolation, too small to matter

   (b) P(Z < -0.6)

$$
X \sim N\,(1,0)
$$

$$
P\,(Z > 0.6) = 0.27425
$$

   (c) $P\,(Z > -1.28)$

$$
\begin{aligned}
1 - P\,(Z > 1.28) &= 1 - 0.1003 \\
&= 0.8997
\end{aligned}
$$

   (d) $P(0.81 < Z < 1.94)$

$$
P\,(Z < 1.94) - P\,(Z < 0.81) = 0.18278
$$

   (e)

$$
\begin{aligned}
P(-0.68 < Z < 0) &= 0.5 - P\,(Z > 0.68) \\
&= 0.5 - 0.2483 \\
&= 0.2517
\end{aligned}
$$

   (f) $P(-0.46 < Z < 2.21)$

$$
1 - P\,(Z > 0.46) - P\,(Z > 2.21) = 0.66365
$$

   (g) $P\,(0.81 < Z < 1.94)$

$$
\begin{aligned}
P\,(Z > 0.81) - P\,(Z > 1.94) &= 0.209 - 0.0262 \\
&= 0.1828
\end{aligned}
$$

4.

(a)

$$
\begin{aligned}
P(-1.2 \le Z \le K) &= 0.523 \\
P(Z \le -1.2) + P(Z \ge K) &= 1 - 0.523 \\
P(Z \le -1.2) + P(Z \ge K) &= 0.477 \\
P(Z \ge K) &= 0.477 - 0.11507 \\
&= 0.36193 \\
K &= 0.35
\end{aligned}
$$

(b)

$$
\begin{aligned}
P(K \le Z \le 1.8) &= 0.355 \\
P(Z > 1.8) + P(Z < K) &= 1 - 0.355 \\
P(Z > 1.8) + P(Z < K) &= 0.645 \\
P(Z < K) &= 0.645 - 0.03593 \\
&= 0.60907 \\
P(Z > K) &= 0.39093 \\
K &= 0.27
\end{aligned}
$$

(c)

i. Calculuation

$$
\begin{aligned}
P(Z \le -0.8) + P(Z > K) &= 0.616 \\
P(Z > K) &= 0.616 - 0.21186 \\
&= 0.40414 \\
K &= 0.24
\end{aligned}
$$

(d)

$$
\begin{aligned}
P(Z \le K) &= 4 \cdot P(Z \le K) \\
5 \cdot P(Z \le K) &= 1 \\
P(Z \le K) &= \frac{1}{5} \\
\frac{1}{5} &= P(Z \ge -K) \\
-K &= 0.8 \\
K &= -0.84
\end{aligned}
$$

5. $\mu = 20.02, \sigma = 0.05$

(a)

$$\begin{aligned}
P(X < 19.9) &= P\left(Z < \frac{19.9 - 20.02}{0.05}\right) \\
&= P(Z < -2.4) \\
&= 8.1974 * 10^{-3} \\
Percentage &= 8.1974 * 10^{-3} * 100\% \\
&= 0.82\% \, or \, 0.0082
\end{aligned}$$

(b)

$$\begin{aligned}
P(X > 20.1) &= P\left(Z > \frac{20.1 - 20.02}{0.05}\right) \\
&= P(Z > 1.6) \\
&= 0.054799 \\
Percentage &= 5.4799\% \, or \, 0.0054779
\end{aligned}$$

6. $\mu_a = 1000, \sigma_a = 100,\ \mu_b = 900, \sigma_b = 50$

   (a) Let X be the breaking strength of the rope with $X \sim N(1000, 10000)$

$$\begin{aligned}
P(X < 750) &= P\left(Z < \frac{750 - 1000}{100}\right) \\
&= P(Z < -2.5)
\end{aligned}$$

   (b) Let X_2 be the breaking strength of th rope with $X_2 \sim N(900, 2500)$

$$\begin{aligned}
P(X_2 < 750) &= P\left(Z < \frac{750 - 900}{50}\right) \\
&= P(Z < -3)
\end{aligned}$$

   (c) Conclusion. All things considered, the company should pick supplier B, because the ropes that are lower than 750kg of breaking strength are less

7. $\mu = 5000, \sigma = 1000$

   (a)

      i.

$$\begin{aligned}
P(5500 \leq X \leq 6500) &= P\left(\frac{5500 - 5000}{1000} \leq Z \leq \frac{6500 - 5000}{1000}\right) \\
&= P\left(\frac{1}{2} \leq Z \leq \frac{3}{2}\right) \\
&= 0.24173
\end{aligned}$$

ii.

$$P(X < 5000) = P(Z < 0)$$
$$= 0.5$$

(b)

$$P(X < 7500) = P\left(Z < \frac{7500 - 5000}{1000}\right)$$
$$= P\left(Z < \frac{2500}{1000}\right)$$
$$= P\left(Z < \frac{5}{2}\right)$$
$$= 0.99379$$

8. $\mu = 500, \sigma = 20$

(a) $n = 2000$, let X be the packets weight with $X \sim N(500, 400)$

i.

$$P(X > 520) = P\left(Z > \frac{520 - 500}{20}\right)$$
$$= P(Z > 1)$$
$$= 0.15866$$

A. $2000 * 0.15866 \approx 317\, packets$

ii.

$$P(X < 470) = P\left(Z < \frac{470 - 500}{20}\right)$$
$$= P\left(Z < -\frac{3}{2}\right)$$
$$= 0.066807$$

A. $2000 * 0.066807 = 133.614 \approx 134 packets$

iii.

$$P(520 \leq X \leq 530) = P\left(\frac{520 - 500}{20} \leq Z \leq \frac{530 - 500}{20}\right)$$
$$= P(1 \leq Z \leq 1.5)$$
$$= 0.09185$$

A. $2000 * 0.09185 = 183.7 \approx 184 packets$

9. $\mu = 45 minutes, \sigma = 8 minutes$

(a)

$$\begin{aligned} P(X > 50) &= P\left(Z > \frac{50-45}{8}\right) \\ &= P\left(Z > \frac{5}{8}\right) \\ &= 0.26599 \end{aligned}$$

(b)

$$\begin{aligned} P(40 \leq X \leq 52) &= P\left(\frac{40-45}{8} \leq X \leq \frac{52-45}{8}\right) \\ &= P\left(-\frac{5}{8} \leq Z \leq \frac{7}{8}\right) \\ &= 0.54322 \end{aligned}$$

(c)

$$\begin{aligned} P(X \leq a) &= 0.9 \\ P\left(Z \leq \frac{a-45}{8}\right) &= 0.9 \\ P\left(Z > \frac{a-45}{8}\right) &= 0.1 \\ \frac{a-45}{8} &= 1.2815 \\ a &= 55.252 minutes \end{aligned}$$

(d)

$$\begin{aligned} P(X \leq a) &= 0.3 \\ P\left(Z \leq \frac{a-45}{8}\right) &= 0.3 \\ -P\left(Z \geq \frac{a-45}{8}\right) &= 0.3 \\ \frac{a-45}{8} &= -0.52466 \\ a &= 40.8027 minutes \end{aligned}$$

10. $p = 0.1, n = 500$

    (a) X are the males out of 500 males who suffer from a certain disease with $X \sim B(500, 0.1)$

        i. Cannot use Poisson because $np < 5$

ii. To approximate with normal, $np > 5$, $nq > 5$. The rule is plus or minus to make it slightly bigger, dont need to remember details.

$$\begin{aligned} P(X > 60) &= P(X > 60.5) \\ &= P\left(Z > \frac{60.5 - 50}{\sqrt{45}}\right) \\ &= P\left(Z > \frac{10.5}{\sqrt{45}}\right) \\ &= P(Z > 1.57) \\ P(Z > 1.57) &= \mathbf{0.058763} \end{aligned}$$

(b) QUESTION: HOW TO KNOW IF WE SHOULD TAKE SAMPLE DISTRIBUTION?

11. $p = 0.1$, $n = 1000, np = 100, \mu = 100, \sigma^2 = npq = 1000 * 0.1 * 0.9 = 90$

(a) Let X be the number of chocolates produced with mis-shapes such that $X \sim N(100, 100)$

$$\begin{aligned} P(X < 80) &= P(X < 79.5) \, \text{make it slightly bigger} \\ & P\left(Z < \frac{79.5 - 100}{\sqrt{90}}\right) \\ &= P(Z < -2.16) \\ &= P(Z > 2.16) \\ &= \mathbf{0.01539} \end{aligned}$$

(b) $P(90 < X < 115)$

$$\begin{aligned} P(90 < X < 115) &= P(89.5 < X < 115.5) \\ &= P\left(\frac{89.5 - 100}{\sqrt{90}} < X < \frac{115.5 - 100}{\sqrt{90}}\right) \\ &= P(-1.11 < Z < 1.63) \\ & 1 - 0.1335 - 0.0516 \\ &= \mathbf{0.8149} \end{aligned}$$

(c) P($X \geq 120$)

$$\begin{aligned} P(X \geq 120) &= P(X > 119.5) \\ &= P\left(Z > \frac{119.5 - 100}{\sqrt{90}}\right) \\ &= P(Z > 2.06) \\ &= \mathbf{0.0197} \end{aligned}$$

12. X is the number of bacteria on a plate viewed under a microscope where $X \sim P_o(\lambda = 60)$

    (a) Since $\lambda > 20$, and n is unobtainable using $X \sim N(60, 60)$ as approximation

        i. $P(55 \leq X \leq 75)$

$$\begin{aligned} P(55 < X < 75) &= P(55.5 < X < 74.5) \\ &= P(-0.5809 < Z < 1.8719) \\ &= 0.68874 \end{aligned}$$

    (b) $P(X < 38)$

$$\begin{aligned} P(X < 38) &= P(X < 37.5) \\ &= P\left(Z < \frac{37.5 - 60}{\sqrt{60}}\right) \\ &= P(Z < -2.905) \\ &= P(Z > 2.905) \\ &= 0.00185 \end{aligned}$$

        i. Plates $= 0.00185 * 2000 \approx 4$

13. Since a minute is an interval, and only the average rate is given. Let X be the customers arrivig at a department store in any particular minute, where $X \sim P_o(18.6)$. Since $\lambda$is big (> 10), and n cannot be easily obtained, we can approximate it with $X \sim N\left(18.6, 4.3128^2\right)$

    (a)

$$\begin{aligned} P(X \leq 25) &= P(X < 25.5) \\ &= P\left(Z < \frac{25.5 - 18.6}{4.3128}\right) \\ &= P(Z < 1.5998) \\ P(X \leq 25) &= \mathbf{0.94518} \end{aligned}$$