DM: Tutorial 7

December 8, 2019

1. Let the universal set, $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and S, T be the subsets of U defined as $S = \{x | x \in U \text{ and } 3 \text{ divides } x\}$, $T = \{x | x \in U \text{and5divides} x\}$. List the elements in $S \times T$.

   (a) $S = \{0, 3, 6, 9\}$

   (b) $T = \{0, 5, 10\}$

   (c) $S \times T = \{(0, 0), (0, 5), (0, 10), (3, 0), (3, 5), (3, 10), (6, 0), (6, 5), (6, 10), (9, 0), (9, 5), (9, 10)\}$

2. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A_1 = \{1, 2, 3, 4\}$, $A_2 = \{5, 6, 7\}$, $A_3 = \{4, 5, 7, 9\}$, $A_4 = \{4, 8, 10\}$, $A_5 = \{8, 9, 10\}$, $A_6 = \{1, 2, 3, 6, 8, 10\}$. List the possible partitions of A.

   Partition of a set (Wikipedia): a partition of a set is a grouping of the set's elements into non-empty subsets, in such a way that every element is included in exactly one subset.
   $\{A_1, A_2, A_5\}, \{A_6, A_3\}$

3. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4\}$ and define a binary relation R from A to B as follows:
   For $(x, y) \in A \times B, (x, y) \in R \iff x \geq y$.
   Write R as a set of ordered pairs.

   $$R = \{(3, 3) . (4, 3), (4, 4), (5, 3), (5, 4)\}$$

4. For each of the following relation on N, list the ordered pairs that belong to the relation. **Note:** N refers to Natural numbers (AKA 1...∞)

   (a) $R = \{(x, y) : 2x + y = 9\}$

   $$\{(1, 7), (2, 5), (3, 3), (4, 1)\}$$

   (b) $S = \{(x, y) : x + y < 7\}$

   $$\{(1, 1) ... (6, 1)\}$$

   $$N = \{(3, 3), (4, 1)\}$$

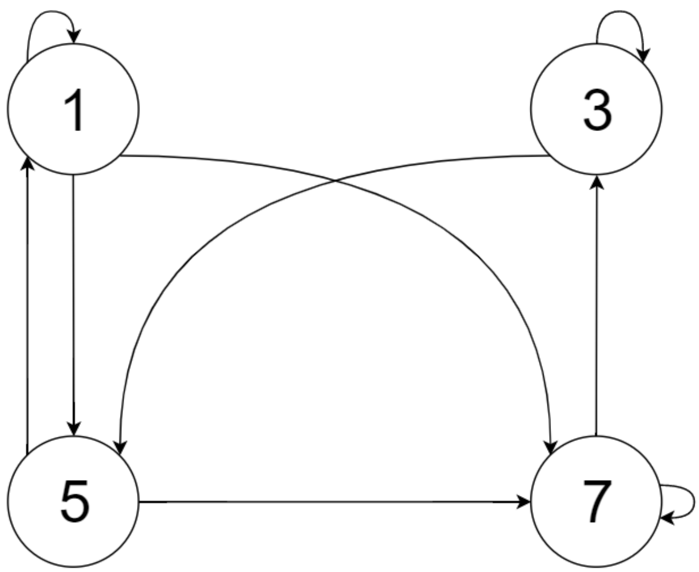
5. Let $A = \{1, 3, 5, 7\}$ and R be the relation on A whose matrix is given below.

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) Write R as a set of ordered pairs.

i. $R = \{(1,1), (1,5), (1,7), (3,3), (3,5), (5,1), (5,7), (7,3), (7,7)\}$

(b) Draw the digraph of R.

i. 

(c) Find the domain and range of R.

i. $Dom\,(R) = \{1, 3, 5, 7\}$

ii. $Ran\,(R) = \{1, 3, 5, 7\}$

(d) Give the in-degree and out degree of each vertex.

i.

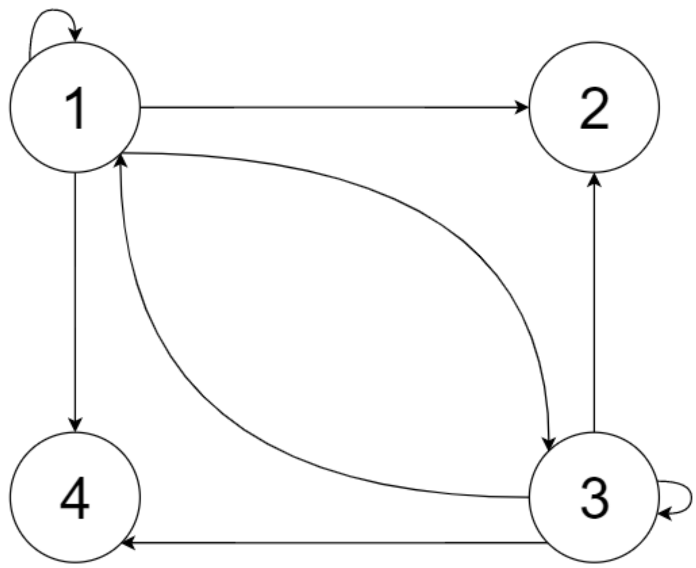
	1	3	5	7
In-degree	2	2	2	3
Out-degree	3	2	2	2

6. Let R be the relation on $\{1, 2, 3, 4\}$ given by u R v iff $u + 2v$ is odd. Represent R in each of the following ways:

(a) as a set of ordered pairs;

$$R = \{(1,1), (1,2), (1,3), (1,4), (3,1), (3,2), (3,3), (3,4)\}$$

(b) in graphical form;



i.

(c) in matrix form;

i. $\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(d) Give the in-degree and out-degree of each vertex.

i.

	1	2	3	4
In-degree	2	2	2	2
Out-degree	4	0	4	0

7. Find the domain, range, matrix, and, when $A = B$, the digraph of the relation R.

(a) $A = \{1, 2, 3, 4, 8\} = B$; a R b if and only if $a = b$.

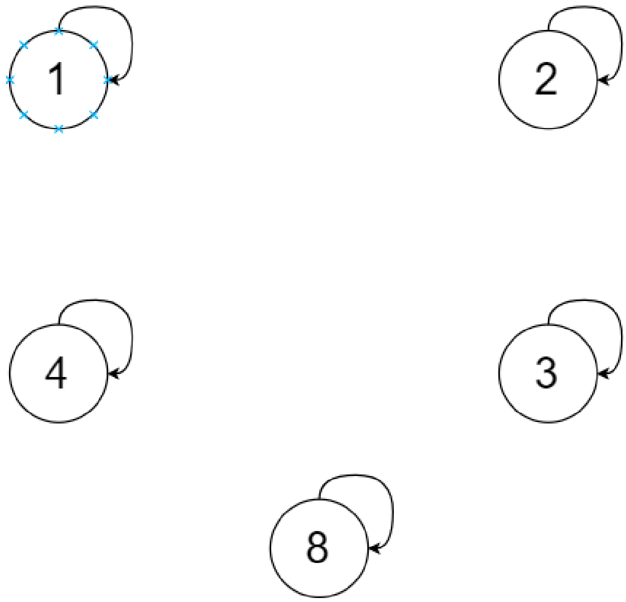
i. $Dom\,(R) : \{1, 2, 3, 4, 8\}$

ii. $Ran\,(R) : \{1, 2, 3, 4, 8\}$

iii. Matrix

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

iv. Digraph



A.

(b) $A = \{1, 2, 3, 4, 6\} = B$; a R b if and only if a is a multiple of b

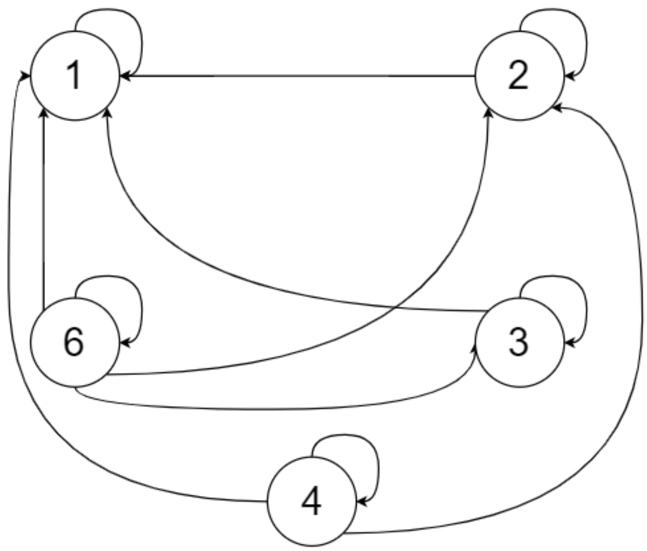
i. $Dom\,(R) : \{1, 2, 3, 4, 6\}$

ii. $Ran\,(R) : \{1, 2, 3, 4, 6\}$

iii. Matrix

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 6 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

iv. Digraph



A.

(c) $A = \{1,3,5,7,9\}$, $B = \{2,4,6,8\}$; a R b if and only if $b < a$.

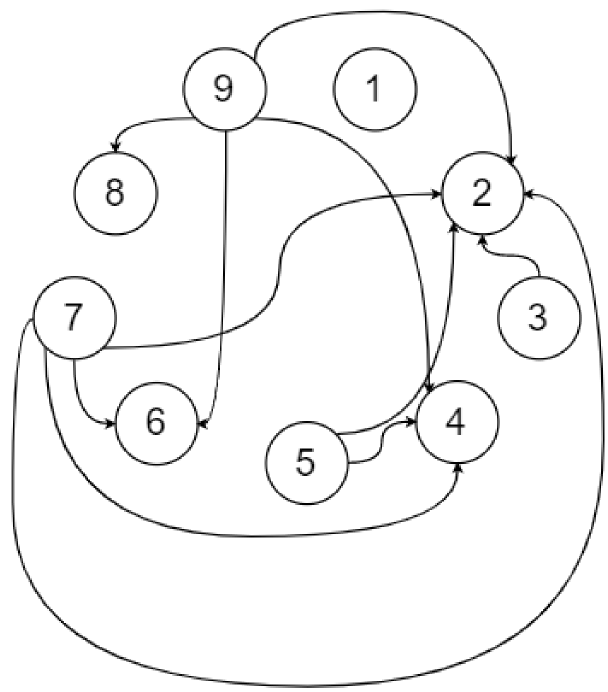
i. $Dom\,(R) : \{3,5,7,9\}$

ii. $Ran\,(R) : \{2,4,6,8\}$

iii. Matrix

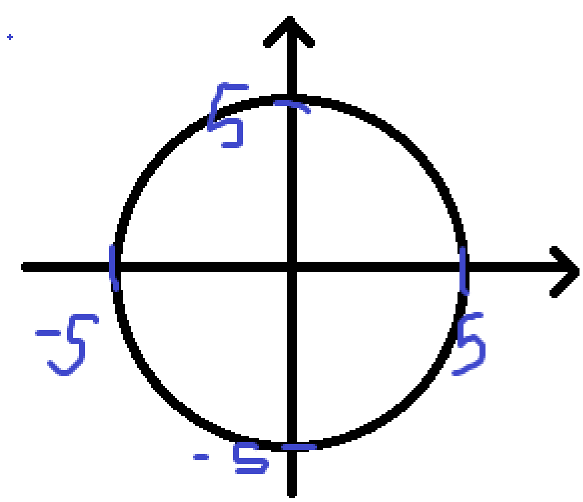
$$\begin{array}{c} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

iv. Digraph



A.

8. Let $A = R$, set of real numbers. Consider the following relation R on A: a R b if and only if $a^2 + b^2 = 25$. Find $Dom(R)$ and $Ran(R)$.



(a)

(b) Domain: $-5 \leq x \leq 5$

(c) Range: $-5 \leq y \leq 5$

9. Let $A = \{1, 2, 3, 4, 6\}$ and R be the relation defined as a R b if and only if a is a multiple of b. Find each of the following.

$$R = \{(1,1), (2,1), (3,1), (4,1), (6,1), (4,2), (6,2), (6,3)\}$$

(a) $R(3) = \{1, 3\}$

(b) $R(6) = \{1, 2, 3, 6\}$

(c) $R(\{2, 4, 6\}) = \{1, 2, 3, 4, 6\}$

10. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 3, 4, 6\}$, and $R = \{(1, 2), (1, 4), (2, 3), (2, 5), (3, 6), (4, 7)\}$. Compute the restriction of R to B.

$$R(B \times B) = \{(2,3), (3,6)\}$$

(a) Note: $(1, 2)$ and $(1, 4)$ originate from a single point in A. So, to be more specific it would be $(1, (2, 4))$, and hence excluded.