

## Tutorial 3 - Continuous Probability Distribution

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1. Since the thickness is uniformly distributed, let  $X$  be the thickness of photoresist where  $X \sim U(0.2050, 0.2150)$

(a)  $P(X > 0.2125)$

$$f(x) = \begin{cases} \frac{1}{0.01} & 0.2050 \leq x \leq 0.2150 \\ 0 & \text{otherwise} \end{cases}$$
$$c = 100$$

$$\begin{aligned} P(X > 0.2125) &= 1 - P(X < 0.2125) \\ &= 1 - \int_{0.2050}^{0.2125} 100 \, dx \\ &= 1 - 100(0.2125 - 0.2050) \\ &= 0.25 \end{aligned}$$

(b)  $P(X > a) = 0.1$

$$\begin{aligned} \int_a^{0.2150} 100 \, dx &= 0.1 \\ 100(0.2150 - a) &= 0.1 \\ 21.5 - 100a &= 0.1 \\ 100a &= 21.4 \\ a &= 0.2140 \mu m \end{aligned}$$

(c)

$$\begin{aligned} \mu &= \frac{0.2050 + 0.2150}{2} \\ &= 0.21 \mu m \end{aligned}$$

$$\begin{aligned}
\sigma &= \sqrt{\frac{(b-a)^2}{12}} \\
&= \sqrt{\frac{(0.2150 - 0.2050)^2}{12}} \\
&= 0.002886 \mu m
\end{aligned}$$

2. Let  $X$  be the lifetime of a mechanical assembly in a vibration test with  $X \sim E(400)$

(a)

$$\begin{aligned}
P(X < 100) &= \int_0^{100} \frac{1}{400} e^{-\frac{x}{400}} dx \\
&= 0.2212
\end{aligned}$$

3. (Important notes: if don't know what to write the formulas, can just write the number for Z scores)

(a) Important note 2 (lol): Always ignore past 2 decimal places in tables, no need to do interpolation, too small to matter

(b)  $P(Z < -0.6)$

$$X \sim N(1, 0)$$

$$P(Z > 0.6) = 0.27425$$

(c)  $P(Z > -1.28)$

$$\begin{aligned}
1 - P(Z > 1.28) &= 1 - 0.1003 \\
&= 0.8997
\end{aligned}$$

(d)  $P(0.81 < Z < 1.94)$

$$P(Z < 1.94) - P(Z < 0.81) = 0.18278$$

(e)

$$\begin{aligned}
P(-0.68 < Z < 0) &= 0.5 - P(Z > 0.68) \\
&= 0.5 - 0.2483 \\
&= 0.2517
\end{aligned}$$

(f)  $P(-0.46 < Z < 2.21)$

$$1 - P(Z > 0.46) - P(Z > 2.21) = 0.66365$$

(g)  $P(0.81 < Z < 1.94)$

$$\begin{aligned}
P(Z > 0.81) - P(Z > 1.94) &= 0.209 - 0.0262 \\
&= 0.1828
\end{aligned}$$

4.

(a)

$$\begin{aligned}
 P(-1.2 \leq Z \leq K) &= 0.523 \\
 P(Z \leq -1.2) + P(Z \geq K) &= 1 - 0.523 \\
 P(Z \leq -1.2) + P(Z \geq K) &= 0.477 \\
 P(Z \geq K) &= 0.477 - 0.11507 \\
 &= 0.36193 \\
 K &= 0.35
 \end{aligned}$$

(b)

$$\begin{aligned}
 P(K \leq Z \leq 1.8) &= 0.355 \\
 P(Z > 1.8) + P(Z < K) &= 1 - 0.355 \\
 P(Z > 1.8) + P(Z < K) &= 0.645 \\
 P(Z < K) &= 0.645 - 0.03593 \\
 &= 0.60907 \\
 P(Z > K) &= 0.39093 \\
 K &= 0.27
 \end{aligned}$$

(c)

i. Calculation

$$\begin{aligned}
 P(Z \leq -0.8) + P(Z > K) &= 0.616 \\
 P(Z > K) &= 0.616 - 0.21186 \\
 &= 0.40414 \\
 K &= 0.24
 \end{aligned}$$

(d)

$$\begin{aligned}
 P(Z \leq K) &= 4 \cdot P(Z \leq K) \\
 5 \cdot P(Z \leq K) &= 1 \\
 P(Z \leq K) &= \frac{1}{5} \\
 \frac{1}{5} &= P(Z \geq -K) \\
 -K &= 0.8 \\
 K &= -0.84
 \end{aligned}$$

5.  $\mu = 20.02, \sigma = 0.05$

(a)

$$\begin{aligned}P(X < 19.9) &= P\left(Z < \frac{19.9 - 20.02}{0.05}\right) \\&= P(Z < -2.4) \\&= 8.1974 * 10^{-3} \\Percentage &= 8.1974 * 10^{-3} * 100\% \\&= 0.82\% \text{ or } 0.0082\end{aligned}$$

(b)

$$\begin{aligned}P(X > 20.1) &= P\left(Z > \frac{20.1 - 20.02}{0.05}\right) \\&= P(Z > 1.6) \\&= 0.054799 \\Percentage &= 5.4799\% \text{ or } 0.054799\end{aligned}$$

6.  $\mu_a = 1000, \sigma_a = 100, \mu_b = 900, \sigma_b = 50$

(a) Let  $X$  be the breaking strength of the rope with  $X \sim N(1000, 10000)$

$$\begin{aligned}P(X < 750) &= P\left(Z < \frac{750 - 1000}{100}\right) \\&= P(Z < -2.5)\end{aligned}$$

(b) Let  $X_2$  be the breaking strength of the rope with  $X_2 \sim N(900, 2500)$

$$\begin{aligned}P(X_2 < 750) &= P\left(Z < \frac{750 - 900}{50}\right) \\&= P(Z < -3)\end{aligned}$$

(c) Conclusion. All things considered, the company should pick supplier B, because the ropes that are lower than 750kg of breaking strength are less

7.  $\mu = 5000, \sigma = 1000$

(a)

i.

$$\begin{aligned}P(5500 \leq X \leq 6500) &= P\left(\frac{5500 - 5000}{1000} \leq Z \leq \frac{6500 - 5000}{1000}\right) \\&= P\left(\frac{1}{2} \leq Z \leq \frac{3}{2}\right) \\&= 0.24173\end{aligned}$$

ii.

$$\begin{aligned}P(X < 5000) &= P(Z < 0) \\&= 0.5\end{aligned}$$

(b)

$$\begin{aligned}P(X < 7500) &= P\left(Z < \frac{7500 - 5000}{1000}\right) \\&= P\left(Z < \frac{2500}{1000}\right) \\&= P\left(Z < \frac{5}{2}\right) \\&= 0.99379\end{aligned}$$

8.  $\mu = 500, \sigma = 20$

(a)  $n = 2000$ , let  $X$  be the packets weight with  $X \sim N(500, 400)$

i.

$$\begin{aligned}P(X > 520) &= P\left(Z > \frac{520 - 500}{20}\right) \\&= P(Z > 1) \\&= 0.15866\end{aligned}$$

A.  $2000 * 0.15866 \approx 317 \text{ packets}$

ii.

$$\begin{aligned}P(X < 470) &= P\left(Z < \frac{470 - 500}{20}\right) \\&= P\left(Z < -\frac{3}{2}\right) \\&= 0.066807\end{aligned}$$

A.  $2000 * 0.066807 = 133.614 \approx 134 \text{ packets}$

iii.

$$\begin{aligned}P(520 \leq X \leq 530) &= P\left(\frac{520 - 500}{20} \leq Z \leq \frac{530 - 500}{20}\right) \\&= P(1 \leq Z \leq 1.5) \\&= 0.09185\end{aligned}$$

A.  $2000 * 0.09185 = 183.7 \approx 184 \text{ packets}$

9.  $\mu = 45 \text{ minutes}, \sigma = 8 \text{ minutes}$

(a)

$$\begin{aligned}P(X > 50) &= P\left(Z > \frac{50 - 45}{8}\right) \\&= P\left(Z > \frac{5}{8}\right) \\&= 0.26599\end{aligned}$$

(b)

$$\begin{aligned}P(40 \leq X \leq 52) &= P\left(\frac{40 - 45}{8} \leq Z \leq \frac{52 - 45}{8}\right) \\&= P\left(-\frac{5}{8} \leq Z \leq \frac{7}{8}\right) \\&= 0.54322\end{aligned}$$

(c)

$$\begin{aligned}P(X \leq a) &= 0.9 \\P\left(Z \leq \frac{a - 45}{8}\right) &= 0.9 \\P\left(Z > \frac{a - 45}{8}\right) &= 0.1 \\ \frac{a - 45}{8} &= 1.2815 \\a &= 55.252 \text{ minutes}\end{aligned}$$

(d)

$$\begin{aligned}P(X \leq a) &= 0.3 \\P\left(Z \leq \frac{a - 45}{8}\right) &= 0.3 \\-P\left(Z \geq \frac{a - 45}{8}\right) &= 0.3 \\ \frac{a - 45}{8} &= -0.52466 \\a &= 40.8027 \text{ minutes}\end{aligned}$$

10.  $p = 0.1, n = 500$

(a)  $X$  are the males out of 500 males who suffer from a certain disease with  $X \sim B(500, 0.1)$

i. Cannot use Poisson because  $np < 5$

- ii. To approximate with normal,  $np > 5$ ,  $nq > 5$ . The rule is plus or minus to make it slightly bigger, don't need to remember details.

$$\begin{aligned}
 P(X > 60) &= P(X > 60.5) \\
 &= P\left(Z > \frac{60.5 - 50}{\sqrt{45}}\right) \\
 &= P\left(Z > \frac{10.5}{\sqrt{45}}\right) \\
 &= P(Z > 1.57) \\
 P(Z > 1.57) &= \mathbf{0.058763}
 \end{aligned}$$

(b) QUESTION: HOW TO KNOW IF WE SHOULD TAKE SAMPLE DISTRIBUTION?

11.  $p = 0.1$ ,  $n = 1000$ ,  $np = 100$ ,  $\mu = 100$ ,  $\sigma^2 = npq = 1000 * 0.1 * 0.9 = 90$

- (a) Let  $X$  be the number of chocolates produced with mis-shapes such that  $X \sim N(100, 100)$

$$\begin{aligned}
 P(X < 80) &= P(X < 79.5) \text{ make it slightly bigger} \\
 &= P\left(Z < \frac{79.5 - 100}{\sqrt{90}}\right) \\
 &= P(Z < -2.16) \\
 &= P(Z > 2.16) \\
 &= \mathbf{0.01539}
 \end{aligned}$$

(b)  $P(90 < X < 115)$

$$\begin{aligned}
 P(90 < X < 115) &= P(89.5 < X < 115.5) \\
 &= P\left(\frac{89.5 - 100}{\sqrt{90}} < Z < \frac{115.5 - 100}{\sqrt{90}}\right) \\
 &= P(-1.11 < Z < 1.63) \\
 &= 1 - 0.1335 - 0.0516 \\
 &= \mathbf{0.8149}
 \end{aligned}$$

(c)  $P(X \geq 120)$

$$\begin{aligned}
 P(X \geq 120) &= P(X > 119.5) \\
 &= P\left(Z > \frac{119.5 - 100}{\sqrt{90}}\right) \\
 &= P(Z > 2.06) \\
 &= \mathbf{0.0197}
 \end{aligned}$$

12.  $X$  is the number of bacteria on a plate viewed under a microscope where  $X \sim P_o(\lambda = 60)$

(a) Since  $\lambda > 20$ , and  $n$  is unobtainable using  $X \sim N(60, 60)$  as approximation

i.  $P(55 \leq X \leq 75)$

$$\begin{aligned} P(55 < X < 75) &= P(55.5 < X < 74.5) \\ &= P(-0.5809 < Z < 1.8719) \\ &= 0.68874 \end{aligned}$$

(b)  $P(X < 38)$

$$\begin{aligned} P(X < 38) &= P(X < 37.5) \\ &= P\left(Z < \frac{37.5 - 60}{\sqrt{60}}\right) \\ &= P(Z < -2.905) \\ &= P(Z > 2.905) \\ &= 0.00185 \end{aligned}$$

i. Plates =  $0.00185 * 2000 \approx 4$

13. Since a minute is an interval, and only the average rate is given. Let  $X$  be the customers arrivig at a department store in any particular minute, where  $X \sim P_o(18.6)$ . Since  $\lambda$  is big ( $> 10$ ), and  $n$  cannot be easily obtained, we can approximate it with  $X \sim N(18.6, 4.3128^2)$

(a)

$$\begin{aligned} P(X \leq 25) &= P(X < 25.5) \\ &= P\left(Z < \frac{25.5 - 18.6}{4.3128}\right) \\ &= P(Z < 1.5998) \\ P(X \leq 25) &= \mathbf{0.94518} \end{aligned}$$