

## Tutorial 2 - Discrete Probability Distribution

June 28, 2019

1.

(a)

$$P(x; 25) = \frac{1}{25}$$

(b)

$$\begin{aligned}\mu &= \frac{1}{25} * (1 + 2 + 3 \dots + 25) \\ &= \frac{1}{25} * \frac{25 * (25 + 1)}{2} \\ &= \frac{325}{25} \\ &= 13\end{aligned}$$

$$\sigma^2 = 52$$

(c)

$$\begin{aligned}Prime &= \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\ &= \frac{9}{25}\end{aligned}$$

2. (Geometric) Let  $X$  be the number of applicants interviewed until the first applicant with advanced ytsininh in vompuyrtp tohtsmminh.  $X \sim \text{Geometric}(p = 0.2)$

(a)

i.  $P(X = 5) = g(5; 0.2)$

$$\begin{aligned}g(5; 0.2) &= (0.8)^4 (0.2) \\ &= 0.08192\end{aligned}$$

ii.  $P(X < 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$$\begin{aligned}P(X = 5) &= 0.2 + 0.8 * 0.2 + 0.8^2 * 0.2 + 0.8^3 * 0.2 \\ &= 0.5904\end{aligned}$$

(b)

i. Mean

$$\begin{aligned}\mu &= \frac{1}{0.2} \\ &= 5tries\end{aligned}$$

ii. Median

$$\begin{aligned}\sigma^2 &= \frac{1-p}{p^2} \\ &= \frac{1-0.2}{0.2^2} \\ &= 20tries^2\end{aligned}$$

3.

(a)

$$\begin{aligned}P(X \geq 3) &= P(X = 3) + P(X = 4) \\ &= {}^4C_3 * 0.4^3 * 0.6 + 0.4^4 \\ &= 0.1792\end{aligned}$$

(b)

$$\begin{aligned}P(X = 0) &= {}^4C_0 * 0.6^4 \\ &= 0.1296\end{aligned}$$

(c)

$$\begin{aligned}P(X = 1, 2) &= P(X = 1) + P(X = 2) \\ &= {}^4C_1 * 0.4^1 * 0.6^3 + {}^4C_2 * 0.4^2 * 0.6^2 \\ &= 0.6912\end{aligned}$$

4.  $p = 0.5$

(a)

$$\begin{aligned}P(X \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= \binom{10}{0} 0.5^{10} + \binom{10}{1} 0.5^9 * 0.5 + \binom{10}{2} 0.5^8 * 0.5^2 \\ &= 0.0546875\end{aligned}$$

(b)

$$\begin{aligned}P(X > 3) &= 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) \\ &= 1 - 0.0546875 - \binom{10}{3} 0.5^{10} \\ &= 0.9453125 - 0.1171875 \\ &= 0.828125\end{aligned}$$

(c)

$$\begin{aligned}P(4 < x < 8) &= P(X = 5) + P(X = 6) + P(X = 7) \\&= \binom{10}{5} 0.5^{10} + \binom{10}{6} 0.5^{10} + \binom{10}{7} 0.5^{10} \\&= 252 * 0.5^{10} + 210 * 0.5^{10} + 120 * 0.5^{10} \\&= 0.5684\end{aligned}$$

5.

(a)

$$p = 0.3, q = 0.7, n = 20$$

$$\begin{aligned}P(3 \leq X \leq 6) &= P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) \\&= \binom{20}{3} \cdot 0.3^3 \cdot 0.7^{17} + \binom{20}{4} \cdot 0.3^4 \cdot 0.7^{16} + \binom{20}{5} \cdot 0.3^5 \cdot 0.7^{15} + \binom{20}{6} \cdot 0.3^6 \cdot 0.7^{14} \\&= 0.57253\end{aligned}$$

(b)

$$\begin{aligned}\mu &= np \\&= 20 * 0.3 \\&= 6customers\end{aligned}$$

$$\begin{aligned}\sigma^2 &= np(1 - p) \\&= 6 * q \\&= 4.2customers^2 \\ \sigma &= 2.0494customers\end{aligned}$$

(c)

$$\begin{aligned}P(X = 6) &= \binom{20}{6} * 0.3^6 * 0.7^{14} \\&= 0.1916\end{aligned}$$

6.  $\mu = 2, \sigma^2 = 1.6$

(a)

$$\begin{aligned}2 &= np \\1.6 &= np(1 - p) \\&= 2(1 - p) \\0.8 &= 1 - p \\-p &= -0.2 \\p &= 0.2\end{aligned}$$

(b)

$$\begin{aligned}2 &= np \\2 &= 0.2n \\n &= 10\end{aligned}$$

$$\begin{aligned}P(X=6) &= \binom{10}{6} * 0.2^6 * 0.8^4 \\&= 210 * 0.2^6 * 0.8^4 \\&= 0.005505\end{aligned}$$

7.

(a) Let X be the number of marriages that occur in June  $X \sim P_o(5)$ ,  $x = 0, 1, 2, 3..$

$$\begin{aligned}P(X < 3) &= P_o(X=0) + P_o(X=1) + \dots + P_o(X=2) \\&= e^{-5} \left( \frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} \right), \text{ where } \lambda = 5 \\&= 0.1246\end{aligned}$$

(b) Let X be the number of marriages that occur in July and August  $X \sim P_o(10)$ ,  $x = 0, 1, 2..$

$$\begin{aligned}P(X=10) &= e^{-10} \frac{\lambda^{10}}{10!} \\&= 0.1251\end{aligned}$$

(c) Let X be the number of marriages that occur between the 3 months  $X \sim P_o(15)$ ,  $x = 0, 1, 2..$

$$\begin{aligned}P(14 \leq X \leq 18) &= P_o(X=14) + P_o(X=15) + \dots + P_o(X=18) \\&= e^{-15} \left( \frac{15^{14}}{14!} + \frac{15^{15}}{15!} + \frac{15^{16}}{16!} + \frac{15^{17}}{17!} + \frac{15^{18}}{18!} \right) \\&= 0.45625 \text{ **Double check : true answer = 0.4561**}\end{aligned}$$

8. (in test, won't tell you Poisson, but will tell you week/duration)

$$\lambda = 2$$

(a) Let X be the number of claims received per week such that  $X \sim P_o(2)$ ,  $x = 0, 1, 2, 3..$

$$\begin{aligned}P(X > 3) &= 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) \\&= 1 - e^{-2} \left( 1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} \right) \\&= \mathbf{0.1429}\end{aligned}$$

- (b) Let  $X$  be the number of claims received per fortnight such that  $X \sim P_o(4), x = 0, 1, 2, 3, \dots$

$$\begin{aligned} P(X \geq 2) &= 1 - P(0) - P(1) \\ &= 1 - e^{-4}(1 + 4) \\ &= \mathbf{0.9084} \end{aligned}$$

- (c) Let  $X$  be the number of claims received per 5 days such that  $X \sim P_o\left(\frac{2}{5}\right), x = 0, 1, 2, 3, \dots$

$$\begin{aligned} P(X = 0) &= e^{-\frac{2}{5}}(1) \\ &= \mathbf{0.6703} \end{aligned}$$

9.

- (a) Let  $X_1$  be the number of cars arriving in a particular 5 minutes interval  $X_1 \sim P_o(2.5), x = 1, 2, 3, \dots$

i.

$$\begin{aligned} P(0) &= P_o(X = 0) \\ &= e^{-2.5} \left( \frac{2.5^0}{0!} \right) \\ &= 0.08208 \end{aligned}$$

- ii. Let  $X_2$  be the number of cars arriving in a particular 10 minutes interval  $X_2 \sim P_o(5), x = 1, 2, 3, \dots$

$$\begin{aligned} P(X < 3) &= P_o(X = 0) + P(X = 1) + P(X = 2) \\ &= 0.1247 \end{aligned}$$

- (b) Let  $X_3$  be the number of cars arriving in a particular 15 minutes interval  $X_3 \sim P_o(7.5), x = 1, 2, 3, \dots$

$$\begin{aligned} \mu &= \lambda \\ &= 7.5 \end{aligned}$$

$$\begin{aligned} \sigma^2 &= \lambda \\ &= 7.5 \\ \sigma &= 2.7386 \end{aligned}$$

10.  $\lambda = 5$

- (a) Let  $X$  be the number of vegetables a salad contains in a tossed salad.  $X \sim P_o(5), x = 0, 1, 2, 3, \dots$  **Note: Salad can contain no vegetables (Darn it)**

$$\begin{aligned} P(X > 5) &= 1 - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4) - P(X = 5) \\ &= 1 - e^{-5} \left( \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \frac{\lambda^5}{5!} \right) \\ &= 1 - e^{-5} \left( 1 + 5 + \frac{5^2}{2} + \frac{5^3}{6} + \frac{5^4}{24} + \frac{5^5}{120} \right) \\ &= 0.3840 \end{aligned}$$

- (b) Binomial (depends on Poisson)  $X_1$  is the number of days out of 4 days which the number of vegetables contained in a tossed salad is 3.  $X \sim B(4, 0.3840)$

$$\begin{aligned} P(X_1 = 3) &= {}^4C_3 (0.3840)^3 (1 - 0.3840)^1 \\ &= 0.1395 \end{aligned}$$

- (c) Geometric (depends on Poisson).  $X_2$  is the numbers of days until the first day with a tossed salad containing more than 5 vegetables

$$\begin{aligned} P(X_2 = 5) &= (1 - 0.3840)^4 (0.3840) \\ &= 0.0553 \end{aligned}$$

11.  $p = 0.01, n = 100$ . Since  $n > 50$  and  $np = 1 < 5$ , Let  $X$  be the number of faulty light bulbs in a box  $X \sim P_o(1), x = 1, 2, 3, \dots$

- (a)

$$\begin{aligned} P(X = 0) &= \frac{e^{-1} \lambda^0}{0!} \\ &= 0.3679 \end{aligned}$$

- (b)

$$\begin{aligned} P(X = 2) &= \frac{e^{-1} \lambda^2}{2!} \\ &= 0.1839 \end{aligned}$$

- (c)

$$\begin{aligned} P(X > 3) &= 1 - P(X \leq 3) \\ &= 1 - P(0) - P(1) - P(2) - P(3) \\ &= 1 - 0.3679 - \frac{e^{-1}}{1!} - 0.1839 - \frac{e^{-1}}{3!} \\ &= 0.019007 \end{aligned}$$

12.

$$p = 0.005 \text{ (Defective)}$$

$$n = 200$$

$$np = 1 < 5$$

$$n > 50$$

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= e^{-1} \left( 1 + 1 + \frac{1}{2} \right)$$

$$= 0.9197$$