1.

(a) 1 - the rest $= 0.31$

(b)

i. $P(X = 1) = \mathbf{0.13}$

ii. $P(X \leq 1)$

$$\begin{aligned} P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &= 0.03 + 0.13 \\ \boldsymbol{P(X \leq 1)} &\boldsymbol{= 0.16} \end{aligned}$$

iii. $P(X \geq 3)$

$$\begin{aligned} P(X \geq 3) &= 0.31 + 0.19 + 0.12 \\ \boldsymbol{P(X \geq 3)} &\boldsymbol{= 0.62} \end{aligned}$$

iv. $P(X < 3)$

$$\begin{aligned} P(X < 3) &= 0.03 + 0.13 + 0.22 \\ \boldsymbol{P(X < 3)} &\boldsymbol{= 0.38} \end{aligned}$$

v. $P(X > 3)$

$$\begin{aligned} P(X > 3) &= 0.19 + 0.12 \\ \boldsymbol{P(X > 3)} &\boldsymbol{= 0.31} \end{aligned}$$

vi. $P(2 \leq X \leq 4)$

$$\begin{aligned} P(2 \leq X \leq 4) &= 0.22 + 0.31 + 0.19 \\ &= 0.72 \end{aligned}$$

(c)

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.03 & 0 \leq x < 1 \\ 0.16 & 1 \leq x < 2 \\ 0.38 & 2 \leq x < 3 \\ 0.69 & 3 \leq x < 4 \\ 0.88 & 4 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$$

2.

3.

(a)

i.

Number of order received per day	2	3	4	5	6
Number of days	12	21	34	19	14

(b)

i. $P(X=3)=0.21$
ii. $P(X\geq 3)=1-0.12=0.88$
iii. $P(2\leq X\leq 4)=0.12+0.21+0.34=0.67$
iv. $P(X<4)=0.33$

4.

(a)

$$\begin{aligned}
1 &= \int_2^5 c\,(1+x)\,dx\\
1 &= c\int_2^5 (1+x)\,dx\\
1 &= c\left[x+\frac{x^2}{2}\right]_2^5\\
1 &= c\left(5+\frac{5^2}{2}-2-\frac{2^2}{2}\right)\\
1 &= c\left(5+\frac{25}{2}-2-2\right)\\
1 &= c\left(\frac{27}{2}\right)\\
c &= \frac{2}{27}
\end{aligned}$$

(b)

i.

$$\begin{aligned}
P(X<4) &= \frac{2}{27}\int_2^4 (1+x)\,dx\\
&= \frac{2}{27}\left[x+\frac{x^2}{2}\right]_2^4\\
&= \frac{2}{27}\left[4+\frac{16}{2}-2-2\right]\\
&= \frac{2}{27}\,[8]\\
&= \frac{16}{27}
\end{aligned}$$

ii.

$$
\begin{aligned}
P\left(3 \leq X < 4\right) &= \frac{2}{27}\int_3^4 \left(1+x\right) dx \\
&= \frac{2}{27}\left[x+\frac{x^2}{2}\right]_3^4 dx \\
&= \frac{2}{27}\left[4+\frac{4^2}{2}-3-\frac{3^2}{2}\right] \\
&= \frac{2}{27}\left[4+\frac{16}{2}-3-\frac{9}{2}\right] \\
&= \frac{2}{27}\left[4+8-3-\frac{9}{2}\right] \\
&= \frac{1}{3}
\end{aligned}
$$

iii.

$$
F\left(x\right) = \begin{cases} 0 & 2 < x \\ & 2 \leq x \leq 4 \\ 1 & x > 4 \end{cases}
$$

$$
\begin{aligned}
F\left(x\right) &= \frac{2}{27}\int_2^x \left(1+x\right) dx \\
&= \frac{2}{27}\left[x+\frac{x^2}{2}\right]_2^x \\
&= \frac{2}{27}\left(x+\frac{x^2}{2}-2+2\right) \\
&= \frac{2}{27}x+\frac{x^2}{27}
\end{aligned}
$$

5.

(a)

$$
\begin{aligned}
\int_0^2 f\left(x\right) dx &= \int_0^1 x\, dx + \int_1^2 2-x\, dx \\
&= \left[\frac{x^2}{2}\right]_0^1 + \left[2x-\frac{x^2}{2}\right]_1^2 \\
&= \frac{1}{2}+\left(2\left(2\right)-\frac{2^2}{2}-2\left(1\right)+\frac{1}{2}\right) \\
&= \frac{1}{2}+\left(4-2-2+\frac{1}{2}\right) \\
&= 1
\end{aligned}
$$

(b)

$$
\begin{aligned}
P(X < 1.2) &= \int_0^{1.2} f(x)\,dx \\
&= \int_0^1 x\,dx + \int_1^{1.2} (2-x)\,dx \\
&= \frac{1}{2} + \left[2x - \frac{x^2}{2}\right]_1^{1.2} \\
&= \frac{1}{2} + \left[2(1.2) - \frac{(1.2)^2}{2} - \left(2(1) - \frac{1}{2}\right)\right] \\
&= \frac{1}{2} + [0.18] \\
&= 0.68
\end{aligned}
$$

(c)

$$
F(x) = \begin{cases} 0 & x \le 0 \\ \frac{x^2}{2} & 0 < x < 1 \\ 2x - \frac{x^2}{2} - 1 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}
$$

i. Between $0 < x \le 2$:

$$
\begin{aligned}
F(x) &= \int_0^2 f(x)\,dx \\
&= \int_0^1 x\,dx + \int_1^x 2 - x\,dx \\
&= \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^x \\
&= \frac{1}{2} + \left(2x - \frac{x^2}{2}\right) - \left(2 - \frac{1}{2}\right) \\
&= \frac{1}{2} + \left(2x - \frac{x^2}{2}\right) - \frac{3}{2} \\
&= \left(2x - \frac{x^2}{2}\right) - 1 \\
&= 2x - \frac{x^2}{2} - 1
\end{aligned}
$$

6.

7.

8. By investing in a particular stock, a person can make a profit in 1 year of RM4,000 with probability of 0.3 or take a loss of RM1,000 with probability 0.7. What is this person's expected gain?

(a) Calculation

$$\begin{aligned}\mu &= 0.3 * 4000 + 0.7 * -1000 \\ &= RM500\end{aligned}$$

(b) Conclusion

i. $\therefore$The expected gain is RM500.

9.

(a)

$$\begin{aligned}\frac{8}{52}\left(3-e\right)+\frac{8}{52}\left(5-e\right)+\frac{36}{52}(-e) &= 0 \\ \frac{24}{52}-\frac{8}{52}e+\frac{40}{52}-\frac{8}{52}e-\frac{36}{52}e &= 0 \\ \frac{64}{52}-\frac{52}{52}e &= 0 \\ \frac{64}{52}-e &= 0 \\ \frac{64}{52} &= e \\ \frac{64}{52} &= e \\ e &= \frac{16}{13}\end{aligned}$$

Therefore, she should pay $\$\frac{16}{5}$per game assuming that the game is fair.

10.

(a) Mean

$$\begin{aligned}\mu = E\left[X=x\right] &= \int_1^2 2x\left(x-1\right)dx \\ &= \int_1^2 2x\left(x-1\right)dx \\ &= \left[\frac{2x^3}{3}-x^2\right]_1^2 \\ &= \left[\frac{16}{3}-4-\left(\frac{2}{3}-1\right)\right] \\ &= \frac{5}{3} litres\end{aligned}$$

(b) Variance

$$\begin{aligned}
\sigma^2 &= E\left(X^2\right) - \mu^2 \\
&= \int x^2 f(x)\, dx - \mu^2 \\
&= \int_1^2 2x^2 (x-1)\, dx - \left(\frac{5}{3}\right)^2 \\
&= \int_1^2 2x^3 - 2x^2 dx - \left(\frac{5}{3}\right)^2 \\
&= \left[\frac{1}{2}x^4 - \frac{2}{3}x^3\right]_1^2 - \frac{25}{9} \\
&= \frac{1}{2}(2)^4 - \frac{2}{3}(2)^3 - \left(\frac{1}{2}(1)^4 - \frac{2}{3}(1)^3\right) - \frac{25}{9} \\
&= \frac{1}{18} litres^2
\end{aligned}$$

11.

(a) Calculation

$$\begin{aligned}
\mu &= \frac{2}{5}\int_0^1 x(x+2)\, dx \\
&= \frac{2}{5}\int_0^1 x^2 + 2x dx \\
&= \frac{2}{5}\left[\frac{x^3}{3} + \frac{2x^2}{2}\right]_0^1 \\
&= \frac{2}{5}\left[\frac{1}{3} + 1\right] \\
&= \frac{2}{5}\left(\frac{4}{3}\right) \\
&= \frac{8}{15} individuals
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= \frac{2}{5}\int_0^1 x^3 + 2x^2 dx - \mu^2 \\
&= \frac{2}{5}\left[\frac{x^4}{4} + \frac{2x^3}{3}\right]_0^1 - \left(\frac{8}{15}\right)^2 \\
&= \frac{2}{5}\left(\frac{1}{4} + \frac{2}{3}\right) - \left(\frac{8}{15}\right)^2 \\
&= \frac{37}{450} indivuduals^2
\end{aligned}$$

12.

(a) Mean

$$\begin{aligned}\mu &= \int_0^1 x\left(x\right)dx + \int_1^2 x\left(2-x\right)dx \\ &= 1 \\ &= 100 hours\end{aligned}$$

(b) Variance

$$\begin{aligned}\sigma^2 &= \int_0^1 x^2\left(x\right)dx + \int_1^2 x^2\left(2-x\right)dx - \left(1\right)^2 \\ &= \frac{1}{6}\left(variance\,of\,X\right)\end{aligned}$$