

# Disc. Maths - T2

November 5, 2019

1. Rewrite each statement below in “if – then” form.

- (a) I am on time for lecture if I catch the 7 am bus.
  - i. If I catch the 7am bus, then I am on time for lecture.
- (b) David studies hard or he fails the examination.
  - i. If David studies hard, then he passes the examination.
- (c) The program is readable only if it is well structured.
  - i. If the program is well structured, then it is readable.
- (d) This door will not open unless a security code is entered.
  - i. If a security code is entered, then the door will open.
- (e)  $2x - 5 = 11$  implies  $x = 8$ .
  - i. If  $x = 8$ , then  $2x - 5 = 11$
- (f) Having two  $45^\circ$  angles is a sufficient condition for this triangle to be a right triangle.
  - i. If the triangle is a right triangle, then it has two  $45^\circ$  angles.
- (g) Solving all tutorial's questions is a necessary condition for Alan to pass this subject.
  - i. If Alan solve all tutorial questions, then Alan passes the subject.
- (h) To be a citizen in this country, it is sufficient that you were born in this country.
  - i. If you were born in this country, then you are a citizen in this country.
- (i) It is necessary to have a valid password to log on to the server.
  - i. If you have a valid server, then you can log on to the server.

2. Using truth tables, verify the following,

(a)  $(p \wedge q) \rightarrow r = (p \rightarrow r) \wedge (q \rightarrow r)$

$p$	$q$	$r$	$(p \vee q)$	$(p \vee q) \rightarrow r$	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

(b)  $(p \wedge (p \rightarrow q)) \rightarrow q = t$

$p$	$q$	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$	$t$
0	0	1	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	1	1	1	1	1

(c)  $(p \rightarrow q) \wedge (\sim p \rightarrow q) \wedge \sim q = c$

$p$	$q$	$p \rightarrow q$	$\sim p$	$(\sim p \rightarrow q)$	$(p \rightarrow q) \wedge (\sim p \rightarrow q)$	$\sim q$	$(p \rightarrow q) \wedge (\sim p \rightarrow q) \wedge \sim q$	$c$
0	0	1	1	0	0	1	0	0
0	1	1	1	1	1	0	0	0
1	0	0	0	1	0	1	0	0
1	1	1	0	1	1	0	0	0

3. Write negations for each of the following statements.

- (a) If P is a square, then it is a rectangle.
  - i. P is a square and is not a rectangle.
- (b) If the sun is shining, then I shall play tennis or swimming this afternoon.
  - i. Even if the sun is shining, I shall not play tennis and not swim this afternoon.
- (c) If I am free and I am not tired, then I will go to the supermarket.
  - i. Even if I am free and I am not tired, I will not go to the supermarket.
- (d) If  $x = 17$  or  $x = 8$ , then  $x$  is prime.
  - i. There are cases where  $x = 17$  or  $x = 8$ , and  $x$  is not a prime.

4. State the converses, inverses and contrapositives for each of the following implications.

Converse:  $q \rightarrow p$

Inverse:  $\sim p \rightarrow \sim q$

Contrapositive:  $\sim q \rightarrow \sim p$

- (a) If I am late, then I will not take the train to work.  $p \rightarrow q$ 
  - i. Converse: If I will not take the train to work, then I am late.
  - ii. Inverse: If I am not late, then I will take the train to work.
  - iii. Contrapositive: If I will take the train to work, then I am not late.
- (b) If I have enough money, then I will buy a car and I will buy a house.  $p \rightarrow (q \wedge r)$ 
  - i. Converse: If I will buy a car and I will buy a house, then I have enough money.
  - ii. Inverse: If I do not have enough money, then I will not buy a car or I will not buy a house.
  - iii. Contrapositive: If I will not buy a car or I will not buy a house, then I do not have enough money.
- (c) A positive integer is a prime only if it has no divisors other than 1 and itself.
  - i. Converse: If a positive integer has no divisors other than 1 and itself, then it is a prime number.
  - ii. Inverse: A positive integer is not a prime only if it has divisors other than 1 and itself.
  - iii. Contrapositive: If a positive integer has divisors other than 1 and itself, then it is not a prime number.
- (d) If  $x$  is non-negative, then  $x$  is positive or  $x$  is 0.
  - i. Converse: If  $x$  is positive or  $x$  is 0, then  $x$  is non-negative number.
  - ii. Inverse: If  $x$  is negative, then  $x$  is not positive and  $x$  is not 0.
  - iii. Contrapositive: If  $x$  is not positive and  $x$  is not 0, then  $x$  is not negative.

5. "If Jim studies hard, then he will pass his final examination." Assuming that this statement is true, which of the following must also be true?

- (a) Jim passed his final examination implies he studies hard.  $q \rightarrow p$ 
  - i. False
- (b) Jim studied hard or he failed his final examination.  $p \vee q$ 
  - i. False
- (c) Jim will fail his final examination only if he does not study hard.  $\sim q \rightarrow \sim p$ 
  - i. True
- (d) Jim will fail his final examination unless he studied hard.  $q \wedge p$ 
  - i. False
- (e) A necessary condition for Jim to pass his final examination is that he studied hard.  $p \rightarrow q$ 
  - i. False
- (f) Studying hard is sufficient for Jim to pass his final examination.  $p \rightarrow q$ 
  - i. True

6. Given  $p \rightarrow q \equiv \sim p \vee q$  and  $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ .

Thus, rewrite the following statement form without using  $\rightarrow$  or  $\leftrightarrow$ .

- (a)  $p \wedge \sim q \rightarrow r$

$$\begin{aligned}(p \wedge \sim q) \rightarrow r &= \sim (p \wedge \sim q) \vee r \\ &= \sim p \vee q \vee r\end{aligned}$$

$$(b) (p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$$

$$\begin{aligned} &\equiv (\sim p \vee (q \rightarrow r)) \leftrightarrow (\sim (p \wedge q) \vee r) \\ &\equiv (\sim p \vee \sim q \vee r) \leftrightarrow (\sim (p \wedge q) \vee r) \\ &\equiv (\sim p \vee \sim q \vee r) \leftrightarrow (\sim p \vee \sim q \vee r) \\ &\equiv \sim (\sim p \vee \sim q \vee r) \vee (\sim (p \wedge q) \vee r) \wedge \sim (\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r) \\ &\equiv \sim (\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r) \wedge \sim (\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r) \\ &\equiv \sim (\sim p \vee \sim q \vee r) \vee c \vee (\sim p \vee \sim q \vee r) \\ &\equiv \sim (\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r) \vee c \\ &\equiv t \vee c \\ &\equiv t \end{aligned}$$

7. Obtain the PDNF and PCNF of each of the following expressions:

$$(a) \sim (p \vee q)$$

i. PDNF

$$\begin{aligned} \sim (p \vee q) &= (\sim p \wedge \sim q) \\ &= (\bar{p} \wedge \bar{q}) \end{aligned}$$

ii. PCNF (Note, PDNF is all consist of 1 in truth table, PCNF is all consists of 0 is truth table, but with their parameters inverted.)

$$\sim (p \vee q) = (\bar{p} + \bar{q}) (p + \bar{q}) (\bar{p} + q)$$

$$(b) \sim (p \wedge q)$$

i. NOTE: For this question, by using DeMorgan's law, you will find PCNF first (remember, sum-of-products, conjunctive = products)

ii. PCNF

$$\begin{aligned} \sim (p \wedge q) &= \sim p \vee \sim q \\ &= (\bar{p} + \bar{q}) \end{aligned}$$

iii. PDNF (Same as part A, in truth table, PCNF is all 0, PDNF is all 1. Take the rest of the terms and invert them)

$$\sim (p \wedge q) = (\bar{p}\bar{q}) (p\bar{q}) (\bar{p}q)$$

$$(c) \sim (p \rightarrow q)$$

i. PDNF

$$\begin{aligned} \sim (p \rightarrow q) &= \sim (\sim p \vee q) \\ &= p \wedge \sim q \\ &= p\bar{q} \end{aligned}$$

ii. PCNF

$$\sim (p \rightarrow q) = (p + q) (p + \bar{q}) (\bar{p} + \bar{q})$$

$$(d) \sim (p \leftrightarrow q)$$

i. PDNF

$$\begin{aligned} \sim ((\sim p \vee q) \wedge (\sim q \vee p)) &= \sim (\sim p \vee q) \vee \sim (\sim q \vee p) \\ &= (p \wedge \sim q) \vee (q \wedge \sim p) \\ &= p\bar{q} + \bar{p}q \end{aligned}$$

ii. PCNF

A. DeMorgan-the-DeMorgan way (lol)

$$\begin{aligned} p\bar{q} + \bar{p}q &= \overline{\overline{p\bar{q} + \bar{p}q}} \\ &= \overline{\overline{p\bar{q}} \cdot \overline{\bar{p}q}} \\ &= \overline{\overline{p\bar{q}} \cdot \overline{\bar{p}q}} \\ &= \overline{(\bar{p} + q) \cdot (p + \bar{q})} \\ &= \overline{\bar{p}p + \bar{p}\bar{q} + pq + q\bar{q}} \\ &= (p + \bar{p}) (p + q) (\bar{p} + \bar{q}) (\bar{q} + q) \\ &= (p + q) (\bar{p} + \bar{q}) \end{aligned}$$

B. The find-the-terms-not-found-in-PDNF-then-invert-the-parameters way

$$\sim ((\sim p \vee q) \wedge (\sim q \vee p)) = (p + q) (\bar{p} + \bar{q})$$

8. Construct a truth table for the expression  $A \equiv (p \rightarrow q) \wedge (\sim p \vee r)$ . Based on the truth table, write the PDNF of A, the PDNF of  $\sim A$ , the PCNF of A, and the PCNF of  $\sim A$ .

$p$	$q$	$r$	$(p \rightarrow q)$	$\sim p$	$(\sim p \vee r)$	$(p \rightarrow q) \wedge (\sim p \vee r)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	0	0	1	0
1	1	0	1	0	0	0
1	1	1	1	0	1	1

(a) PDNF of A:

i.  $\bar{p}\bar{q}\bar{r} + \bar{p}\bar{q}r + \bar{p}q\bar{r} + \bar{p}qr + pqr$

ii. PCNF of A:

A.  $(\bar{p} + q + r)(\bar{p} + q + \bar{r})(\bar{p} + \bar{q} + r)$

iii. PDNF of  $\sim A$ :

A.  $p\bar{q}\bar{r} + p\bar{q}r + pq\bar{r}$

iv. PCNF of  $\sim A$ :

A.  $(p + q + r)(p + q + \bar{r})(p + \bar{q} + r)(p + \bar{q} + \bar{r})(\bar{p} + \bar{q} + \bar{r})$

9. Without constructing truth tables, obtain the PDNF of A, the PDNF of  $\sim A$ , the PCNF of A, and the PCNF of  $\sim A$ , (in any order), if the normal forms exist.

(a)  $A \equiv q \wedge (p \vee \sim q)$

i. PDNF of A:

$$A \equiv q(p + \bar{q})$$

$$\equiv pq + q\bar{q}$$

$$\equiv pq + c$$

$$A \equiv pq$$

ii. PDNF of  $\sim A$ :

$$A \equiv p\bar{q} + \bar{p}q + \bar{p}\bar{q}$$

iii. PCNF of A:

$$A \equiv (p + q)(\bar{p} + q)(p + \bar{q})$$

iv. PCNF of  $\sim A$ :

$$A = (\bar{p} + \bar{q})$$

(b)  $A \equiv (\sim p \vee \sim q) \rightarrow (p \leftrightarrow \sim q)$

i. PDNF of A:

$$A \equiv \sim(\sim p \vee \sim q) \vee (p \leftrightarrow \sim q)$$

$$\equiv (p \wedge q) \vee [(\sim p \vee \sim q) \wedge (q \vee p)]$$

$$\equiv pq + [(\bar{p} + \bar{q})(p + q)]$$

$$\equiv pq + p\bar{p} + \bar{p}q + p\bar{q} + \bar{q}q$$

$$\equiv pq + c + \bar{p}q + p\bar{q} + c$$

$$\equiv pq + \bar{p}q + p\bar{q}$$

ii. PDNF of  $\sim A$ :

$$\sim A \equiv \bar{p}\bar{q}$$

iii. PCNF of A:

$$A \equiv pq$$

iv. PCNF of  $\sim A$ :

$$\sim A \equiv \bar{p}q + p\bar{q} + \bar{p}\bar{q}$$

(c)  $A \equiv p \rightarrow [p \wedge (q \rightarrow p)]$

i. PDNF of  $A$ :

$$\begin{aligned} A &\equiv p \rightarrow [p \wedge (q \rightarrow p)] \\ &\equiv \sim p \vee (p \wedge (q \rightarrow p)) \\ &\equiv \sim p \vee (p \wedge (\sim q \vee p)) \\ &\equiv \bar{p} + p(\bar{q} + p) \\ &\equiv \bar{p} + p\bar{q} + pp \\ &\equiv \bar{p} + p\bar{q} + p \\ &\equiv \bar{p}(q + \bar{q}) + p\bar{q} + p(q + \bar{q}) \\ &\equiv \bar{p}q + \bar{p}\bar{q} + p\bar{q} + pq + p\bar{q} \\ &\equiv pq + p\bar{q} + \bar{p}q + \bar{p}\bar{q} \end{aligned}$$

ii. PDNF of  $\sim A$ :

$$\sim A \equiv c$$

A. Note: No possible minterms

iii. PCNF of  $A$ :

$$A \equiv t$$

A. Note: No possible maxterms

iv. PCNF of  $\sim A$ :

$$A \equiv (p + q)(p + \bar{q})(\bar{p} + q)(\bar{p} + \bar{q})$$

(d)  $A \equiv (q \rightarrow p) \wedge (\sim p \wedge q)$

i. PDNF of  $A$ :

$$\begin{aligned} A &\equiv (\sim q \vee p) \wedge (\sim p \wedge q) \\ &\equiv (\bar{q} + p)(\bar{p}q) \\ &\equiv \bar{p}\bar{q}q + p\bar{p}q \\ &\equiv c \text{ (No possible minterms)} \end{aligned}$$

ii. PDNF of  $\sim A$ :

$$A \equiv pq + \bar{p}q + p\bar{q} + \bar{p}\bar{q}$$

iii. PCNF of  $A$ :

$$A \equiv (p + q)(\bar{p} + q)(p + \bar{q})(\bar{p} + \bar{q})$$

iv. PCNF of  $\sim A$ :

$$A \equiv t \text{ (No possible maxterms)}$$