Calculus L2 - LIMITS AND CONTINUITY

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1 Example 2.1

1.

(a) $\lim_{x\to 2} 5 = 5$

(b) $\lim_{x\to 2} (4x-5)$

$$\begin{aligned}\lim_{x\to 2}(4x-5) &= 4(2)-5\\ &= 3\end{aligned}$$

(c) $\lim_{x\to 0}\left(2x^2+3x-5\right)$

$$\lim_{x\to 0}\left(2x^2+3x-5\right) = -5$$

(d) $\lim_{x\to 0}(x+1)(x-1)$

$$\lim_{x\to 0}(x+1)(x-1) = -1$$

(e) $\lim_{x\to 2}\left(\frac{x-1}{x^2-1}\right)$

$$\boldsymbol{\lim_{x\to 2}\left(\frac{x-1}{x^2-1}\right) = \frac{1}{3}}$$

(f) $\lim_{x\to 1}\left(\frac{x-1}{x^2-1}\right)$

$$\begin{aligned}\lim_{x\to 2}\left(\frac{x-1}{x^2-1}\right) &= \lim_{x\to 1}\left(\frac{x-1}{x^2-1}\right)\\ &= \frac{0}{0}\,(indeterminant)\end{aligned}$$

$$\begin{aligned}\lim_{x\to 2}\left(\frac{x-1}{x^2-1}\right) &= \lim_{x\to 1}\left(\frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}\right)\\ &= \lim_{x\to 1}\left(\frac{1}{(x+1)}\right)\end{aligned}$$

$$\boldsymbol{\lim_{x\to 2}\left(\frac{x-1}{x^2-1}\right) = \frac{1}{2}}$$

(g) $\lim_{x\to 3}\left(\frac{x^2-x-6`}{x-3}\right)$

$$\lim_{x\to 3}\left(\frac{x^2-x-6`}{x-3}\right)=\frac{0}{0}\,(indeterminant)$$

$$\begin{aligned}\lim_{x\to 3}\left(\frac{x^2-x-6`}{x-3}\right)&=\lim_{x\to 3}\left(\frac{\cancel{(x-3)}\,(x+2)}{\cancel{x-3}}\right)\\&=\lim_{x\to 3}(x+2)\\&=5\end{aligned}$$

(h) $\lim_{x\to -1}\left(\frac{x^2+4x+3}{x+1}\right)$

$$\lim_{x\to -1}\left(\frac{x^2+4x+3}{x+1}\right)=\frac{0}{0}\,(indeterminate)$$

$$\begin{aligned}\lim_{x\to -1}\left(\frac{x^2+4x+3}{x+1}\right)&=\lim_{x\to -1}\left(\frac{(x+3)\,\cancel{(x+1)}}{\cancel{x+1}}\right)\\&=-1+3\\&=2\end{aligned}$$

(i) $\lim_{x\to 8}\left(\sqrt{x+1}\right)$

$$\begin{aligned}\lim_{x\to 8}\left(\sqrt{x+1}\right)&=\sqrt{8+1}\\&=\sqrt{9}\\&=3\end{aligned}$$

(j) $\lim_{x\to 1}\left(\frac{7+6x}{3x-2}\right)^4$

$$\begin{aligned}\lim_{x\to 1}\left(\frac{7+6x}{3x-2}\right)^4&=\left(\frac{7+6\,(1)}{3\,(1)-2}\right)^4\\&=\left(\frac{13}{1}\right)^4\\&=28561\end{aligned}$$

(k) $\lim_{x\to 0}\left(\frac{x}{1-\sqrt{1+x}}\right)$

$$\begin{aligned}
\lim_{x\to 0}\left(\frac{x}{1-\sqrt{1+x}}\right) &= \lim_{x\to 0}\left(\frac{x}{1-\sqrt{1+x}}\right)\cdot\left(\frac{1+\sqrt{1+x}}{1+\sqrt{1+x}}\right)\\
&= \lim_{x\to 0}\left(\frac{x\left(1+\sqrt{1+x}\right)}{1^2-(1+x)}\right)\\
&= \lim_{x\to 0}\left(\frac{\cancel{x}\left(1+\sqrt{1+x}\right)}{-\cancel{x}}\right)\\
&= \lim_{x\to 0}\left(-\left(1+\sqrt{1+x}\right)\right)\\
&= -2
\end{aligned}$$

(l) $\lim_{x\to 0}\left(\frac{\sqrt{1+x^2}-1}{x^2}\right)$

$$\begin{aligned}
\lim_{x\to 0}\left(\frac{\sqrt{1+x^2}-1}{x^2}\right) &= \lim_{x\to 0}\left(\frac{\sqrt{1+x^2}-1}{x^2}\right)*\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1}\right)\\
&= \lim_{x\to 0}\left(\frac{1+x^2-1^2}{x^2\left(\sqrt{1+x^2}+1\right)}\right)\\
&= \lim_{x\to 0}\left(\frac{\cancel{x^2}}{\cancel{x^2}\left(\sqrt{1+x^2}+1\right)}\right)\\
&= \lim_{x\to 0}\left(\frac{1}{\sqrt{1+x^2}+1}\right)\\
&= \frac{1}{\sqrt{1+0^2}+1}\\
&= \frac{1}{2}
\end{aligned}$$

2 Example 2.2

Since $\lim_{x\to 0}(u(x))$is between the functions